Paying for Observable Luck

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Abstract

In this paper I present a simple hidden action model in which the agent has explicit contractual incentives but also implicit incentives created by the possibility of bankruptcy. An observable exogenous shock affects the agent’s performance and determines the probability of liquidation. Furthermore, after signing the contract, but before choosing his action, the agent observes a private signal on the future shock. The observation of a bad signal strengthens the agent’s implicit incentive and reduces the conflict of interest with the principal. If the agent had no private information, the principal could completely filter out the observable luck. However, when the agent has private information, the contract optimally adjusts explicit to implicit incentives. As a result, observable luck is not completely removed from the agent compensation schedule. The model explains recent empirical evidence of asymmetric benchmarking in managerial compensation: managers appear to be insulated from bad luck but not from good luck. The result obtains in a model that shares most of the assumptions typically made in the empirical literature. In particular, asymmetric benchmarking arises even though the managerial productivity and the exogenous shock are independent.

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1 Introduction

The relationship between shareholders of a modern public corporation and their CEOs has long been studied within the framework of agency theory. This approach stresses the trade-off between insurance and incentive provision in the design of optimal contracts. Shareholders are typically well diversified and then better suited than their risk averse managers to bear the uncertainty affecting firm performance. As a consequence managerial exposure to risk finds its only rationale in the provision of incentives. Therefore, any noise that could be removed from CEOs’ compensation packages, without affecting underlying incentives, should indeed be filtered out. This intuitive idea is a direct consequence of the informativeness principle (Holmstrom, 1979) and it is usually believed to imply that managerial compensation should be insulated from events that are beyond their control such as, for example, macroeconomic fluctuations. To do this one should evaluate firm performance relative to some appropriate benchmark, that reflects stochastic elements which cannot be affected by managerial activity. For example, the widely recommended use of Relative Performance Evaluation (RPE) hinges on the idea that performances within a group of peers (e.g. CEOs within the same industry) are affected by common shocks. Hence, it is argued, compensation should be increasing in own performance but decreasing in others'. Other typical recommended benchmarks include stock price indexes, input and output prices.

In spite of its intuitive appeal, this apparently straightforward implication of agency theory has found very limited support in the data.\(^1\) In contrast, the recent works by Bannister and Newman (2003) and Garvey and Milbourn (2006) suggest the existence of a form of asymmetric benchmarking, which is also referred to as one sided RPE: managers appear to be insulated from bad luck but not from good luck. For example, in stock option plans the strike price typically coincides with the market price at the time of award and it is not linked to general stock price indexes. In this way managers can appropriate windfalls generated by a bull market, as it seems to have happened during the 1990s. However, when the stock price falls below the exercise price, it is often renegotiated down.\(^2\)

The typical principal-agent model that supports the idea that managerial compensation should be decreasing in an appropriate benchmark is indeed very simple. Usually, it

\(^1\)See for example the surveys by Rosen (1992), Murphy (1999) or Prendergast (1999).
\(^2\)A common justification for this conduct is that with a stock price decline the plan loses its motivational value (Murphy, 1999, and Garvey and Milbourn, 2006).
involves a standard hidden action model in which the firm performance is represented as the sum of three independent components: managerial productivity, an aggregate shock and a noise term. However, it is possible to modify this basic structure to produce contractual arrangements in which compensation is not decreasing in the benchmark. For example, in Himmelberg and Hubbard (2000), and Celentani and Loveira (2006), the managerial productivity is correlated with the aggregate state and, in this case, optimal benchmarking does not necessarily involve a smaller payment when the performance benchmark is high.

The lack of appropriate benchmarking in managerial compensation has however been considered by some authors as striking evidence of executive self dealing. Crystal (1991), Bertrand and Mullainathan (2001) and Bebchuk, Fried and Walker (2002) have pointed out that CEOs might have an important influence on the process leading to the adoption of their own compensation package. According to this view, CEOs would manipulate the members of the compensation committee in order to obtain the most favorable conditions, just trying to avoid the shareholders’ outrage. However, as Garvey and Milbourn (2006) have argued, it is not clear why the complete absence of any form of benchmarking should be desirable for CEOs. If compensation is linked to market movements, executives can only expect to receive the risk premium determined in the market and it is not clear whether it is high enough for the given managerial risk aversion. They also claim that asymmetric benchmarking is a more robust signal of self serving behavior since managers would prefer it to no benchmarking whatsoever.

This paper shows that asymmetric benchmarking can be a characteristic of optimal contracts adopted by completely independent and self-interested principals. The main contribution is to show that, contrary to the general presumption, asymmetric benchmarking can be obtained even when managerial productivity is independent of the aggregate shock. The model stresses that, besides the contractual (explicit) incentives, managers also respond to other sources of (indirect) incentives that shareholders should take into account. In particular, I consider the indirect discipline created by bankruptcy: when a firm performs poorly and gets liquidated its manager also suffers a cost. For example, it could take some time to find a new job and, having performed poorly in the previous one, new job conditions will presumably be less attractive. I also assume that, after signing the contract but before choosing his action, the agent

\[ \text{Schmidt (1997) offers the first formalization within a principal-agent model of the intuitive idea that the probability of bankruptcy reduces the conflict of interest between the firm's owners and the manager who is interested in keeping his job.} \]
can observe a private signal on the future state of the world (i.e. on the level of the benchmark). Bad luck, reflected in low levels of the benchmark, makes performance levels below the liquidation threshold more likely then sharpening indirect incentives. Since the agent observes the private signal before choosing an action, he can base his conduct on this information. As a consequence, the principal can provide the agent with different explicit incentives when different signals are observed. Therefore, the optimal contract should mitigate explicit incentives when the agent receives bad news because the relatively higher probability of being liquidated provides sharper indirect incentives. Because the signal observed by the manager is private, his compensation cannot be made contingent on it, but compensation depends on the firm’s performance and on the realized state that are both public. Furthermore, an ex-post low level of the benchmark makes it more likely that the signal received by the agent was a bad one. Hence, to provide the agent with weaker incentives when a bad signal is observed it is sufficient to make the compensation flatter when the benchmark is low. Asymmetric benchmarking then arises from this characteristic of the optimal contract: if the benchmark is high, compensation is more sensitive to the firm outcome so that a good performance leads to a high managerial payment, however, when the benchmark is low, the wage schedule is flatter so that a poor performance is not penalized that much.

In the literature there are several papers that have tackled the puzzling absence of RPE in managerial contracts. In a first strand of it, pioneered by Salas Fumás (1992) and then followed, among others, by Aggarwal and Samwick (1999) and Joh (1999), the main focus is on product market interactions. It is stressed that, especially in tough competitive environments, it could be in the interest of the firms’ owners to sign contracts that make managerial compensation increasing, instead of decreasing, in the market performance. In this way their commitment not to maximize profits is credible and can enforce some degree of collusion that, at the end, turns out to be better than straight competition. In a different vein, Oyer (2004) and Himmelberg and Hubbard (2000) stress the role of participation constraints. If the value of the managerial outside opportunity is positively correlated with wide industry movements, it could be necessary for the firms’ owners to pay more in case of good luck in order to keep the participation constrained satisfied. Himmelberg and Hubbard (2000) also stress the role of the managerial labor market. They notice that managerial talent is relatively more productive in good states of the world so that the demand for high level executives increases during a boom period. At the same time, the supply is relatively inelastic and, then, they predict a positive relation between managerial pay
and industry-wide performance, at least for the highest skilled managers.

None of the papers mentioned so far is able to explain the evidence of asymmetric benchmarking. In fact, their arguments always predict a positive relation between managerial pay and the level of the benchmark so that they can at most be useful to explain the observed mixed result on RPE. An exception is the recent work by Celentani and Loveira (2006). In a simple principal agent model they obtain that one sided RPE is optimal if the productivity of the managerial effort is sufficiently higher in the good state. Under this assumption, the observation of a good performance is more suggestive of managerial high effort in good times than in bad times and, similarly, poor performance is more suggestive of managerial effort in bad states than in good ones. The corresponding optimal contract then displays asymmetric benchmarking. In other words, the correlation between the aggregate state and managerial productivity can explain asymmetric benchmarking.4

In the model developed here, the relationship between firm performance, aggregate state and managerial activity is a simple linear equation of the kind commonly used in empirical studies and, even assuming independence between aggregate state and managerial productivity, asymmetric benchmarking emerges as an optimal contractual arrangement. The key element of the explanation is that for agents with a private information on the strength of their implicit incentives, benchmarking is not only used to filter out observable luck but also to adapt explicit incentives to the hidden information.

The paper is organized as follows. Section two introduces a standard principal agent model in which the agent performance is affected by an observable and uncontrollable shock (the benchmark). Furthermore, performances below a given threshold trigger the liquidation of the firm. In this framework the optimal benchmarking rule is obtained and discussed. In section three the possibility for the manager of observing a signal on the future level of the benchmark is introduced. I first study optimal contracts that induce the manager to exert unconditional effort, i.e., the level of effort he chooses is independent of his private information. I show that, in this case, the optimal benchmarking rule displays the kind of asymmetry observed in the data. In section four I characterize contracts that induce the agent to exert conditional effort, i.e., his effort choice depends on the realization of the private signal. Section 5 contains some numerical examples and, finally, section 6 concludes.

4They also show that the opposite assumption naturally leads to the opposite result: if the managerial effort is more productive in bad times then the managerial pay should appear to be insulated from good luck but not from bad luck.
2 The Baseline Analysis of an Uninformed Agent

A principal (she) has to hire a manager (he) to implement an investment project whose result $x$ depends on a managerial action $a \in \{0, 1\}$, on an aggregate state variable $y \in \{B, G\}$ and on a random term $\xi$ according to:

$$x = \gamma a + \delta I(y = G) + \xi. \quad (1)$$

Action $a$ can be interpreted as effort and can be high ($a = 1$) or low ($a = 0$). High effort has a utility cost of $c > 0$ for the manager while low effort involves no such cost. The term $\gamma > 0$ represents productivity of the managerial effort and is independent of the aggregate state. The noise term $\xi$ is normally distributed with zero mean and variance $\sigma^2$. Finally, the binary variable $y$ has a distribution $p(y)$ and it captures the stochastic elements affecting the project result that are observable and verifiable. Assuming that $\delta > 0$, $y = G$ can be interpreted as the favorable (Good) aggregate state while $y = B$ as the unfavorable (Bad) one, being $\delta$ the impact on the outcome of favorable aggregate conditions. The project outcome $x$ and the aggregate shock $y$ are both observable and verifiable and whenever $x$ is below a critical value $x \leq 0$, the principal goes bankrupt. In this case the agent incurs a liquidation cost $B > 0$ which is expressed in utility terms. The managerial action cannot be observed by the principal and the idiosyncratic noise term $\xi$ is not observable by anybody. All remaining parameters are commonly known.

In this framework a contract is a wage schedule of the form $w : \mathbb{R} \times \{B, G\} \rightarrow [w, \infty)$, notice in particular that the manager is wealth constrained so that feasible wage offers must be above the threshold $w$. The timing is as follows: the principal offers a contract to the agent that can either accept or reject it. If the contract is accepted, the manager has to decide whether to exert high or low effort. After such decision, which remains hidden to the principal, the aggregate state $y$ and the term $\xi$ are determined and then, the project outcome $x$ is realized according to (1). Both $x$ and $y$ are publicly observed and if the outcome is below the critical value, the principal goes bankrupt. Finally, the wage payment is carried out. The principal is risk neutral and maximizes total expected profits $E(x - w)$, while the agent is risk averse with a twice continuously differentiable Bernoulli utility $u : [w, \infty) \rightarrow \mathbb{R}$ strictly increasing and concave, and

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5 The notation $I(A)$ is used here to denote the indicator function of an event $A$, and it is equal to one if $A$ is true while it is zero otherwise.

6 Notice in particular that the agent receives the promised wage payment even in case of bankruptcy.
satisfying \( \lim_{w \to \infty} u'(w) = 0 \) and \( \lim_{w \to w^+} u'(w) = \infty \). The agent also has an outside opportunity which is worth \( \overline{U} \) to him. Notice that when the agent chooses action \( a \) in state \( y \), the outcome \( x \) is normally distributed with mean \( \gamma a + \delta I(y = G) \) and variance \( \sigma^2 \). Let \( F_y(\cdot, a) \) be the corresponding distribution function and \( f_y(\cdot, a) \) the associated density. It is then possible to define with \( b(a, y) \) the probability of bankruptcy in state \( y \) when the agent takes the action \( a \), that is \( b(a, y) = F_y(x, a) \). The total probability of bankruptcy when the agent takes action \( a \) is \( b_a = \sum_y p(y)b(a, y) \). It is immediate to check that \( b_0 > b_1 \), meaning that high effort reduces the probability of bankruptcy.

As in Schmidt (1997), the existence of some utility cost for the agent in case of bankruptcy, creates a natural incentive for the risk averse manager that mitigates the conflict of interest with the principal. Assuming that to induce the high level of effort is optimal, the contract to be offered in equilibrium solves:

\[
\min_{w(x,z) \geq w} \sum_y p(y) \int w(x,y)f_z(y,1)dx
\]  

subject to:

\[
\sum_y p(y) \int u[w(x,y)] f_y(x,1)dx \geq \overline{U} + c + b_1B \tag{3}
\]

\[
\sum_y p(y) \int u[w(x,y)] [f_y(x,1) - f_y(x,0)] dx \geq c - (b_0 - b_1)B. \tag{4}
\]

Notice in the incentive compatibility constraint (4) that if the right hand side is not strictly positive (i.e. \( B \geq \frac{c}{b_0 - b_1} \)) the agent would prefer to exert high effort even with a flat wage schedule. I will rule out this possibility assuming that \( B < \frac{c}{b_0 - b_1} \) so that the agent’s bankruptcy utility cost is not sufficient to perfectly align his interests with those of the principal. Let \( \lambda \) and \( \mu \) be multipliers for constraints (3) and (4) respectively and define with \( L_y(x) = \frac{f_y(x,1) - f_y(x,0)}{f_y(x,1)} \) the likelihood ratio corresponding to outcome \( x \) in state \( y \). Hence, the minimum cost contract \( w^* \) inducing the high level of effort satisfies the following condition:

\[
\frac{1}{u'[w^*(x,y)]} = \lambda + \mu L_y(x) \tag{5}
\]

for all pairs \((x, y)\) for which (5) has a solution \( w^*(x, y) \geq w \), otherwise \( w^*(x, y) = w \) whenever \( \lambda + \mu L_y(x) < 0 \). Notice that for a fixed \( a \) the density function of the outcome

\footnote{Hence, a manager receiving a wage payment equal to \( w \) obtains an ex-post utility equal to \( u(w) - cI(a = 1) - BI(x \leq x) \).}
in the favorable state is obtained shifting to the right the corresponding density in the
unfavorable state by the amount $\delta$, that is: $f_G(x, a) = f_B(x - \delta, a)$. This also implies
that $F_G(x, a) = F_B(x - \delta, a)$ and $L_G(x) = L_B(x - \delta)$, and then also $\lambda + \mu L_G(x) = \lambda + \mu L_B(x - \delta)$, which in particular implies the following condition:

$$w^*(x, G) = w^*(x - \delta, B). \quad (6)$$

Hence, if we denote with $EW_y^*$ the equilibrium expected wage in state $y$, an immediate
consequence of (6) is that:

$$EW_G^* = EW_B^* \quad (7)$$

To verify this last condition just notice that:

$$EW_G^* = \int w^*(x, G) f_G(x, 1) dx = \int w^*(x - \delta, B) f_B(x - \delta, 1) dx = \int w^*(\tilde{x}, B) f_B(\tilde{x}, 1) d\tilde{x} = EW_B^*$$

where the last equality in the first line follows from the change of variable $\tilde{x} = x - \delta$.

The expected wage payment is the same in both aggregate states so that, in expected
terms, there is no reward associated to observable luck. Figure 1 gives an illustration
of the optimal contract in this case.

This characteristics of the optimal wage schedule should not come as a surprise: the
variable $y$ represents in fact a benchmark against which managerial performance can
be evaluated. Put it another way, the aggregate term $y$ is beyond managerial control
but, nevertheless, it affects his performance. It is then optimal to filter it out by
benchmarking the managerial compensation according to the rule described in (6) that
simply states that in good times a given outcome $x$ induces the same wage payment that
would have been induced in the unfavorable state by the smaller outcome $x - \delta$. In this
way the risk induced by the uncertain aggregate state is completely removed so that the
risk averse agent can be hired and properly motivated at a lower cost. As commonly
obtained in this class of models the resulting wage schedule is then decreasing in the
benchmark.\footnote{Remember that the contract design problem has a statistical interpretation: the outcome $x$ is
used as a signal about the managerial action and wage payments increases with the likelihood of the
manager having exerted high effort. Given the simple (linear) structure in equation 1, an outcome $x$ in
favorable conditions has the same informational content about the managerial action as the outcome
$x - \delta$ in bad times.}
The benchmarking condition (6) has been obtained in a very simple model. In more sophisticated environments it isn’t necessarily so. For example, in Celentani and Loveira (2006) the productivity of the managerial effort can vary across aggregate states and in this case the simple benchmarking condition (6) no longer hold. In particular, they show that, if effort productivity is sufficiently higher in good times than in bad times, the optimal contract displays one sided RPE.\footnote{In terms of the present model, if managerial productivity in state \( y \) is denoted with \( \gamma_y \) and the variability of the noise term \( \xi \) in state \( y \) is denoted with \( \sigma^2_y \), it could be shown that if \( \frac{\gamma_y}{\sigma_y^2} > \frac{\gamma_B}{\sigma_B^2} \), the optimal contract displays one sided benchmarking.} In the next section I will stay with the hypothesis that the effort productivity is independent of the aggregate state and nevertheless asymmetric benchmarking arises whenever the agent can observe a private signal on the future state of the world.

3 Informed Agent Exerting Unconditional Effort

In the situation described in the previous section the probability of bankruptcy doesn’t play any role in shaping the benchmarking rule adopted in the optimal contract. However, the utility cost that the manager suffers in case of bankruptcy mitigates the conflict of interest with the principal. Notice also that, everything else being constant, the probability of bankruptcy is higher in bad times than in good times so that, if the agent were able to forecast the future aggregate state, the agency problem would be less severe in case bad times were anticipated to come. This idea suggests that if the agent is able to observe some signal about the future level of \( y \), the principal could find it optimal to take advantage of the disciplining effect of bankruptcy, then, in particular, reducing the managerial exposure to risk after the observation of bad news, and increasing it after good news. As it will be shown in this section, the optimal contractual arrangement resulting in this case exhibits asymmetric benchmarking.

Consider a private signal \( z \) that is received by the agent after signing the contract, but before choosing the level of effort. The signal can either be good, \( z = G \) or bad, \( z = B \), and its conditional probability, given the future state of nature is given by:

\[
\rho(z \mid y) = \begin{cases} 
1 - \varepsilon & \text{if } z = y \\
\varepsilon & \text{if } z \neq y 
\end{cases}
\]

where \( \varepsilon \in [0, \frac{1}{2}] \) measures how noisy the signal is. If \( \varepsilon = 0 \), the signal is perfectly
informative, while $\varepsilon = \frac{1}{2}$ corresponds to a completely noisy signal.\(^{10}\) Define with $\rho(z)$ the total probability of observing signal $z$, that is $\rho(z) = (1 - \varepsilon)p(y = z) + \varepsilon p(y \neq z)$. The posterior probability of the future aggregate state held by the manager after receiving signal $z$ is:

$$p(y \mid z) = \frac{\rho(z \mid y)p(y)}{\rho(z)}.$$  

(8)

It is a matter of simple computations to check that for any $\varepsilon \in (0, \frac{1}{2})$

$$p(y \mid z = y) > p(y) > p(y \mid z \neq y),$$

meaning that the observation of signal $z = y$ raises the conditional probability of observing state $y$ in the future while the observation of signal $z \neq y$ decreases it. Define the conditional probability of bankruptcy given signal $z$ and action $a$ as $b_a(z) = \sum_y p(y \mid z)b(a, y)$. Notice that for both $z = B, G$, $b_0(z) > b_1(z)$, i.e., whatever the observed signal, the high level of effort reduces the conditional probability of bankruptcy. However, the following lemma shows that the extent of such reduction is larger if bad times are anticipated to come.

**Lemma 1** The reduction in the conditional probability of bankruptcy induced by the high level of effort increases after $z = B$ and decreases after $z = G$:

$$b_0(B) - b_1(B) > b_0 - b_1 > b_0(G) - b_1(G)$$

**Proof** Computing explicitly probabilities $b_0$ and $b_a(z)$ and rearranging terms yield:

$$b_0(z) - b_1(z) = \int_{-\infty}^{\infty} \{p(G \mid z) [f_G(x, 0) - f_G(x, 1)] + p(B \mid z) [f_B(x, 0) - f_B(x, 1)]\} \, dx,$$

$$b_0 - b_1 = \int_{-\infty}^{\infty} \{p(G) [f_G(x, 0) - f_G(x, 1)] + p(B) [f_B(x, 0) - f_B(x, 1)]\} \, dx,$$

Notice that for $x \leq x \leq 0$ it results that $f_B(x, 0) - f_B(x, 1) > f_G(x, 0) - f_G(x, 1) > 0$ and then the lemma immediately follows from the fact that $p(G \mid G) > p(G) > p(G \mid B)$ and $p(B \mid B) > p(B) > p(B \mid G)$. \(\blacksquare\)

The availability of the private signal for the agent enlarges his action space. He can

\(^{10}\) In the latter case the agent is completely uninformed and the baseline analysis in the first section applies.
now condition the level of effort on his private forecast of the future aggregate state. Let \((a_B, a_G)\) be one such possible action profile where \(a_z \in \{0, 1\}\) represents effort chosen after the observation of signal \(z\). Notice that, because the private signal is correlated with the aggregate state and the agent can condition his action on it, the managerial productivity, represented by the term \(\gamma a\) in equation (1), can be correlated with the aggregate state \(y\) even if productivity of the managerial high effort, represented by the term \(\gamma\), is independent of \(y\). In particular, \((a_B, a_G) = (1, 1), (0, 0)\) are unconditional effort profiles and if the agent adopts one of them, there still is independence between the aggregate state \(y\) and the managerial productivity \(\gamma a\). However, if the agent chooses a conditional effort profile, i.e. \((a_B, a_G) = (0, 1), (1, 0)\), there would be a correlation between aggregate state and managerial product, even if the outcome is determined according to the simple equation (1). In the rest of this section I will focus on the minimum cost contract implementing the unconditional effort profile \((a_B, a_G) = (1, 1)\). It is important to notice that, even if no correlation is produced by this contract, incentives constraints must prevent the agent from choosing conditional action profiles that would create such correlation. For this reason benchmarking is not only used to filter out observable luck as in (6), but also to avoid such deviations. The minimum cost contract implementing high effort after both signals solves:

\[
\min_{w(x,z) \geq w} \sum_y p(y) \int w(x,y) f_z(y,1) dx \tag{9}
\]

subject to:

\[
\sum_y p(y) \int u[w(x,y)] f_y(x,1) dx \geq U + c + b_1 B \tag{10}
\]

and for \(z \in \{B, G\}\)

\[
\sum_y p(y \mid z) \int u[w(x,y)] [f_y(x,1) - f_y(x,0)] dx \geq c - [b_0(z) - b_1(z)] B. \tag{11}
\]

The objective function in (9) and the (IR) constraint (10) are the same as in the previous section, but now there are two incentive compatibility constraints in (11), one for each possible realization of the private signal. Remember from lemma 1 that \(b_0(B) - b_1(B) > b_0(G) - b_1(G)\) so that the IC constraint corresponding to the observation of \(z = B\) is now less demanding, while after the observation of \(z = G\) sharper explicit incentives are needed to induce the high level of effort. Let \(\lambda\) be the multiplier for the
IR constraint and $\mu(z)\rho(z)$ the multiplier for constraint IC with signal $z$. The optimal wage schedule $w^\varepsilon$ satisfies now the following condition:

$$\frac{1}{w'[w^\varepsilon(x,y)]} = \lambda + [(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)] L_y(x) \quad (12)$$

for all pairs $(x, y)$ for which (12) has a solution $w^\varepsilon(x, y) \geq w$. otherwise $w^\varepsilon(x, y) = w$ whenever $\lambda + [(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)] L_y(x) < 0$. The next proposition shows how the availability of the private information modifies the simple benchmarking rule (6).

**Proposition 1** For each $\varepsilon \in [0, \frac{1}{2})$ the unique minimum cost contract $w^\varepsilon(x, y)$ implementing $(a_B, a_G) = (1, 1)$ is a continuous function of $x$, such that:

1. if $x > \delta + \frac{\gamma}{2}$, then $w^\varepsilon(x, G) > w^\varepsilon(x - \delta, B)$;
2. if $x < \delta + \frac{\gamma}{2}$, then $w^\varepsilon(x, G) \leq w^\varepsilon(x - \delta, B)$.

**Proof** Problem (9) - (11) defining the optimal wage schedule $w^\varepsilon$ is not a convex program. Following a common practice, it is better to define an equivalent problem stated in terms of the utility levels $u(w(x, y))$. Let $U$ be the range of the utility function $u$, and $h: U \to [w, \infty)$ be its inverse. Define $\underline{u} = u(w)$ and notice that $h$ is twice continuously differentiable, strictly increasing, strictly convex and such that $\lim_{u \to \underline{u}} h'(u) = 0$ and $\lim_{u \to \sup U} h'(u) = \infty$. With a slight abuse of notation let’s write $u(x, y)$ to denote $u(w(x, y))$ and notice that $w(x, y) = h(u(x, y))$. Consider now the following problem:

$$\min_{u(x,z) \geq \underline{u}} \sum_y p(y) \int h(u(x,y)) f_z(y,1) dx \quad (13)$$

subject to

$$\sum_y p(y) \int u(x,y) f_y(x,1) dx \geq U + c + b_1 B \quad (14)$$

and for $z \in \{B, G\}$

$$\sum_y p(y \mid z) \int u(x,y) [f_y(x,1) - f_y(x,0)] dx \geq c - [b_0(z) - b_1(z)] B. \quad (15)$$

This is now a convex program and it is equivalent to (9) - (11) in the sense that $u^\varepsilon(x, y)$ solves (13) - (15) if and only if $w^\varepsilon(x, y) = h[u^\varepsilon(x, y)]$ solves (9) - (11). Let $\lambda$ be the multiplier for the IR constraint and $\mu(z)\rho(z)$ the multiplier for constraint IC with signal
z. The optimal utility schedule $u^\varepsilon$ satisfies now the following condition:

$$h'[u(x, y)] = \lambda + [(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)] L_y(x) \tag{16}$$

for all pairs $(x, y)$ for which (16) has a solution $u(x, y) \geq u$, otherwise $u(x, y) = u$ whenever $\lambda + [(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)] L_y(x) < 0$. The advantage of this formulation is that being (13) - (15) a convex program its solution is unique and multipliers $\lambda, \mu(B)$ and $\mu(G)$ are non negative.

To facilitate the subsequent exposition, define $x_B = \frac{\gamma_1}{2}$, and $x_G = \frac{\gamma_2}{2} + \delta$ and notice that the quantities $f_y(x_y, 1) - f_y(x_y, 0)$ and $L_y(x)$ have the same sign as $x - x_y$. Furthermore, simple algebra shows that for any $v \geq 0$, the following holds:

$$f_y(x_y + v, 1) - f_y(x_y + v, 0) =$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} \left\{ \exp \left[ -\frac{1}{2\sigma^2} \left( v - \frac{\gamma}{2} \right)^2 \right] - \exp \left[ -\frac{1}{2\sigma^2} \left( v + \frac{\gamma}{2} \right)^2 \right] \right\} \equiv g(v)$$

which is independent of $y$. Similarly it can be noticed that for $v \geq 0$:

$$f_y(x_y - v, 1) - f_y(x_y - v, 0) = g(-v) = -g(v)$$

i.e., the function $g$ is odd. Using this new notation it is possible to write the IC constraint associated to signal $z$ in the following more convenient way:

$$\sum_y p(y \mid z) \int_0^\infty [u(x_y + v, y) - u(x_y - v, y)] g(v)dv \geq c - [b_0(z) - b_1(z)] B.$$

The rest of the proof is organized in two steps.

**Step 1** Let’s show here that the proposition follows from $\mu(G) > \mu(B) \geq 0$ that, in turn, will be established in step two. If $\varepsilon \in (0, \frac{1}{2})$ and $\mu(G) > \mu(B) \geq 0$ it is also true that $[(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)] > [(1 - \varepsilon)\mu(B) + \varepsilon\mu(G)]$, so that, using the first order condition (16), recalling that $L_y(x)$ is larger for $y = B$ and has the same sign of $(x - x_y)$, for $x > x_y$, it results that:

$$h'[u(x, G)] = \lambda + [(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)] L_G(x) > \lambda + [(1 - \varepsilon)\mu(B) + \varepsilon\mu(G)] L_B(x - \delta) = h' [u(x - \delta, B)].$$
Hence, being \( h \) an increasing and convex function, it results that \( u^e(x, G) > u^e(x - \delta, B) \) which is the same as \( w^e(x, G) > w^e(x - \delta, B) \). As for \( x < x_z \) let’s distinguish two cases. Consider first the case in which \( \lambda + [(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)]L_G(x) \geq 0 \). Condition (16) still defines the optimal level of utility to be assigned to the manager for both \( y = B, G \) so that, since \( L_G(x) < 0 \), we have now:

\[
h'[u(x, G)] = \lambda + [(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)]L_G(x) < \lambda + [(1 - \varepsilon)\mu(B) + \varepsilon\mu(G)]L_B(x - \delta) = h'[u(x - \delta, B)].
\]

It follows then that \( w^e(x, G) < w^e(x - \delta, B) \). In the second complementary case in which \( \lambda + [(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)]L_G(x) < 0 \), we have that the wage offer is \( w^e(x, G) = \underline{w} \) and then for sure not larger then \( w^e(x - \delta, B) \). Finally, to establish continuity of the schedule \( w^e(x, z) \) it is sufficient to check that at the point \( \omega_y \equiv L_y^{-1}(\frac{\lambda}{(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)}) \) it results that:

\[
\lim_{x \to \omega_y^+} u(x, y) = \lim_{x \to \omega_y^-} h^{-1}(\lambda + [(1 - \varepsilon)\mu(z = y) + \varepsilon\mu(z \neq y)]L_y(x)) = \\
\lim_{v \to 0^+} h^{-1}(v) = \underline{u}
\]

which is clearly true since, being \( h \) the inverse of \( u \), \( h' \) converges to 0 as its argument converges to \( \underline{u} \).

**Step 2** Let’s show that \( \mu(G) > \mu(B) \geq 0 \). Assume by contradiction that \( \mu(B) \geq \mu(G) \geq 0 \) and notice that the two IC multipliers cannot be both zero and then \( \mu(B) > 0 \) follows, i.e. the IC constraint associated to signal \( B \) is binding. Now, similarly to what have been found in step 1, the first order condition (16) implies that

\[
[u^e(x, G) - u^e(x - \delta, B)](x - x_G) \leq 0
\]

so that we can write:

\[
\sum_y p(y | G) \int_0^\infty [u(x_y + v, G) - u(x_y - v, G)] g(v) dv = \\
\int_0^\infty p(G | G) [u(x_G + v, G) - u(x_G - v, G)] + p(B | G) [u(x_B + v, G) - u(x_B - v, G)] g(v) dv \leq \\
\int_0^\infty p(G | B) [u(x_G + v, B) - u(x_G - v, B)] + p(B | B) [u(x_B + v, B) - u(x_B - v, B)] g(v) dv = \\
c - [b_0(B) - b_1(B)] B < c - [b_0(G) - b_1(G)] B
\]

which clearly violates the incentive compatibility constraint corresponding to the signal.
$z = G$. Notice that the first (weak) inequality follows from the fact that being $v \geq 0$, we also have $u(x_G + v, G) \leq u(x_B + v, B)$ and $u(x_G - v, G) \geq u(x_B - v, G)$ so that $u(x_G + v, G) - u(x_G - v, G) \leq u(x_B + v, B) - u(x_B - v, B)$ and furthermore $P(G | G) > P(G | B), P(B | G) < P(B | B)$. The second (strict) inequality is a consequence of lemma 1. ■

Notice that proposition 1 states that the simple benchmarking rule obtained in (6) does not hold in the present context. In particular if the performance is above the critical level $x_G = \delta + \frac{\gamma}{2}$, compensation in case of good luck appears to be excessive if it is evaluated according to the simple benchmarking rule (6) obtained in the previous section, while performances below the same threshold leads to a compensation level in bad times that is above what would be predicted by the same simple rule (6). Overall, according to proposition 1, if the combined effect of the probability of bankruptcy and the managerial private signal is overlooked, then it appears that good luck is not filtered out completely while, in case of bad luck, the manager is overcompensated. This is equivalent to saying that the wage schedule $w^\varepsilon$ displays asymmetric benchmarking.

The following result provides an additional characterization of the optimal contract.

**Proposition 2** For each $\varepsilon \in [0, \frac{1}{2})$ the unique minimum cost contract $w^\varepsilon(x, y)$ implementing $(a_B, a_G) = (1, 1)$ has a performance threshold $\hat{x}(\varepsilon) > \delta + \frac{\gamma}{2}$ such that:

1. if $x > \hat{x}(\varepsilon)$, then $w^\varepsilon(x, G) > w^\varepsilon(x, B)$;
2. if $x < \hat{x}(\varepsilon)$, then $w^\varepsilon(x, G) \leq w^\varepsilon(x, B)$.

**Proof** For a given $\varepsilon$, define $\hat{x}(\varepsilon)$ as the solution to:

$$[(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)] L_G(x) = [(1 - \varepsilon)\mu(B) + \varepsilon\mu(G)] L_B(x)$$

that, after rearranging terms, can be written as:

$$H(x, \varepsilon) \equiv \mu(B) [L_G(x) - L_B(x)] + [\mu(G) - \mu(B)] \{L_G(x) - \varepsilon [L_G(x) + L_B(x)]\} = 0.$$  

(17)

Notice that $H$ is a continuous and strictly increasing function of $x$ and furthermore $H(x_G, \varepsilon) < 0$ and $\lim_{x \to \infty} H(x, \varepsilon) = [\mu(G) - \mu(B)] (1 - 2\varepsilon) > 0$. This implies that $\hat{x}(\varepsilon)$ exists and it is unique and, furthermore, $\hat{x}(\varepsilon) > x_G = \delta + \frac{\gamma}{2}$. Let’s show now that such quantity has the properties claimed in the proposition. To this end notice that for $x > \hat{x}(\varepsilon), y \in \{B, G\}$ so that condition (16) determines the optimal contract $u(z, G, \gamma)$.

---

11To see this just check that for $x \to \infty$, both $L_y(x)$ converge to 1 and remember that $\varepsilon < \frac{1}{2}$.
wage $w^\varepsilon(x, y)$. Furthermore, for any such $x$ it results that:

$$[(1 - \varepsilon)\mu(G) + \varepsilon\mu(B)] L_G(x) > [(1 - \varepsilon)\mu(B) + \varepsilon\mu(G)] L_B(x)$$

immediately implying that $w^\varepsilon(x, G) < w^\varepsilon(x, B)$, that is point (1) in the proposition. As for point (2) just notice that it is an implication of proposition 1 for any $x < x_G$ while for $x \in (x_G, \hat{x}(\varepsilon))$ an argument similar to the one used to establish point (1) applies. \hfill \blacksquare

This second proposition reveals that optimal contracting does not simply lead to a failure to filter out aggregate risk. It also requires that above the performance threshold $\hat{x}(\varepsilon)$ compensation be increasing in the benchmark, i.e., for any given $x > \hat{x}(\varepsilon)$ the manager receives a higher compensation if favorable aggregate conditions are observed while, for $x < \hat{x}(\varepsilon)$ the compensation is higher in bad states. The proof of both results relies on the same intuitive idea that can be grasped referring to figure 2 which displays how the possibility of observing a signal on the future aggregate state distorts the wage schedule that would be optimal in the absence of the signal.

If the agent receives some private information before choosing his action, the incentive he needs to choose high effort is sharper in case of good news because liquidation is perceived to be less likely and therefore is less effective as an incentive device. The way to provide the agent with sharper incentives after the observation of good news is to make the schedule $w(x, G)$ steeper, thus increasing his exposure to risk. The reason is that, after observing the signal $z = G$ it is relatively more likely that the future state will be $y = G$ and then the relevant wage schedule is likely to be $w(x, G)$. Similarly, the observation of $z = B$ weakens the incentive constraint that must be met to induce high effort, and lead to a flatter compensation schedule $w(x, B)$. It can be noticed in figure 2 that the overall result is then the form of asymmetric benchmarking described in proposition 1 and 2.

Let $EW^\varepsilon_y = \int w^\varepsilon(x, y)f_y(x, 1)dx$ be the expected wage payment in state $y$ when the available signal is affected by a noise term $\varepsilon$ and define the ex ante expected compensation cost as $EW^\varepsilon = \sum_y p(y)EW^\varepsilon_y$. The next result shows that the availability of a more precise private signal for the agent makes it more costly to induce unconditional high effort.

**Proposition 3** If for each $\varepsilon \in [0, \tfrac{1}{2})$ the principal offers in equilibrium the minimum cost contract implementing $(a_B, a_G) = (1, 1)$, then the agent expected wage $EW^\varepsilon$ is a decreasing function of $\varepsilon$. 
Proof The proof relies on the simple observation that the availability of a more informative signal shrinks the set of incentive compatible wage schedules. Define the following quantity:

$$\Delta_y = \int_0^\infty [u(x_y + v, y) - u(x_y - v, y)] g(v)dv.$$  

With this notation the IC constraint corresponding to signal $z$ can be written as follows:

$$p(G \mid z) \Delta_G + p(B \mid z) \Delta_B \geq c - [b_0(z) - b_1(z)] B.$$ 

The quantity $\Delta_y$ can be seen as an index measuring the pay performance sensitivity in state $y$. For example, if the contract specified a linear payment above the level $x_y$, the quantity $\Delta_y$ would be increasing in its slope. Using this notation it is possible to see that each IC constraint defines a hemiplane in the space $(\Delta_G, \Delta_B)$. Furthermore, straightforward calculations show how increasing the signal precision, i.e. decreasing $\varepsilon$, shrinks the intersection of the two hemiplanes defining the region corresponding to incentive compatible contracts. □

Endogenous independence between managerial productivity and the aggregate state also arises when the principal implements $(a_B, a_G) = (0, 0)$. In this case a constant wage equal to $\bar{U} + b_0 B$, i.e., high enough to meet the agent’s participation constraint, would be optimal.  

4 Informed Agent Exerting Conditional Effort

An endogenous correlation between managerial productivity and the aggregate state emerges when the principal implements either $(a_B, a_G) = (0, 1)$ or $(a_B, a_G) = (1, 0)$. The first conditional effort profile produces a positive correlation while the second a negative one. Notice that if the agent adopts the first profile his productivity in the good state is larger because he exerts high effort only after the observation $z = G$ which, for any $\varepsilon \in \left[0, \frac{1}{2}\right]$, is more likely when the future state of the world is $y = G$. Similarly,

\[12\] Notice however that if $B > \frac{c}{b_0(B) - b_1(B)}$, it is impossible to induce the agent to choose $a = 1$ if he observes the private signal $z = B$.

\[13\] Notice that for $\varepsilon = \frac{1}{2}$, even the adoption of a conditional action profiles would not induce any correlation between the aggregate state and the managerial product. In fact, with a completely noisy signal, both $(a_B, a_G) = (0, 1)$ and $(a_B, a_G) = (1, 0)$ are equivalent to the behavioral strategy of playing either level of effort with probability one half. The corresponding benchmarking rule would then be
if the agent adopts \((a_B, a_G) = (1,0)\) and the signal is not completely noisy, the agent will choose \(a = 1\) more often in state \(y = B\) than in state \(y = G\) then producing a negative correlation. The following proposition describes the characteristics of the optimal benchmarking rule corresponding to the minimum cost contract implementing \((a_B, a_G) = (0,1)\).

**Proposition 4** Given \(\varepsilon \in \left[0, \frac{1}{2}\right]\), if \(B < \frac{c}{b(B) - b_1(B)}\), the unique minimum cost contract \(w^{01}(x, y)\) implementing \((a_B, a_G) = (0,1)\) is a continuous function of \(x\). Furthermore, if the incentive compatibility constraint corresponding to signal \(z = B\) is slack, it results that:

1. if \(x > \delta + \frac{\gamma}{2}\), then \(w^{01}(x, G) > w^{01}(x - \delta, B)\);
2. if \(x < \delta + \frac{\gamma}{2}\), then \(w^{01}(x, G) \leq w^{01}(x - \delta, B)\).

**Proof** If the agent adopts the (conditional or unconditional) effort profile \((a_B, a_G)\), the outcome pdf in state \(y\) is:

\[
f_y[x, (a_B, a_G)] = [(1 - \varepsilon)f_y(x, a_{z=y}) + \varepsilon f_y(x, a_{z\neq y})].
\]

Define the corresponding likelihood ratio as \(L^{(a_B, a_G)}_y(x) = \frac{f_y(x, 1) - f_y(x, 0)}{f_y[x, (a_B, a_G)\]}\). The ex ante\(^{14}\) probability of bankruptcy is:

\[
b_{(a_B, a_G)} = \sum_y p(y) \left[ (1 - \varepsilon)b(y, a_{z=y}) + \varepsilon b(y, a_{z\neq y}) \right],
\]

and the ex ante probability of exerting high effort is:

\[
q(a_B, a_G) = \sum_y p(y) \left[ (1 - \varepsilon) I(a_{z=y} = 1) + \varepsilon I(a_{z\neq y} = 1) \right].
\]

It is now possible to state the problem characterizing the minimum cost contract implementing \((a_B, a_G) = (0,1)\):

\[
\min_{u(x,y) \geq y} \sum_y p(y) \int h[u(x, y)]f[x, (0,1)] dx
\]

subject to

\[
\sum_y p(y) \int u(x, y)f_y[x, (0,1)] dx \geq U + q(0,1)c + b_{(0,1)}B
\]

the same as without the private signal.

\(^{14}\)Ex ante here means before the observation of the private signal.
\[
\sum_y p(y | B) \int u(x, y) \left[ f_y(x, 1) - f_y(x, 0) \right] dx \leq c - [b_0(B) - b_1(B)] B \tag{20}
\]

\[
\sum_y p(y | G) \int u(x, y) \left[ f_y(x, 1) - f_y(x, 0) \right] dx \geq c - [b_0(G) - b_1(G)] B. \tag{21}
\]

The problem stated in terms of utility levels \(u(x, y)\) is a convex program which admits a unique solution \(u^{01}\). Let \(\lambda\) be the multiplier for the IR constraint (19) while \(\mu(B)\rho(B)\) and \(\mu(G)\rho(G)\) are multipliers for IC constraints (20) and, respectively, (21). The optimal utility schedule \(u^{01}\) satisfies now the following conditions:

\[
h' \left[ u^{01}(x, G) \right] = \lambda + [(1 - \varepsilon)\mu(G) - \varepsilon\mu(B)] L_G^{(0,1)}(x) \tag{22}
\]

\[
h' \left[ u^{01}(x, B) \right] = \lambda + [\varepsilon\mu(G) - (1 - \varepsilon)\mu(B)] L_B^{(0,1)}(x) \tag{23}
\]

for all pairs \((x, G)\) and \((x, B)\) for which (22) and, respectively, (23) have a solution above \(u\), otherwise \(u(x, y) = u\). Because \(L_{y}^{(0,1)}(x)\) is a continuous function, it is immediate to check that \(u^{01}\) is continuous too. Furthermore, if we assume that \(\mu(B) = 0\), it must be \(\mu(G) > 0\) and first order conditions (22), (23) can be rewritten as follows:

\[
h' \left[ u^{01}(x, G) \right] = \lambda + \mu(G) \left[ 1 - \frac{f_G(x, 0)}{f_G[x, (0, 1)]} \right] \tag{24}
\]

\[
h' \left[ u^{01}(x, B) \right] = \lambda + \mu(G) \left[ 1 - \frac{f_B(x, 0)}{f_B[x, (0, 1)]} \right]. \tag{25}
\]

Notice that \(f_G(x, 0) = f_B(x - \delta, 0)\) and for \(x > \delta + \frac{\gamma}{2}\) also \(f_G[x, (0, 1)] < f_B[x - \delta, (0, 1)]\). Furthermore, the solution is characterized by (24), (25) and it results:

\[
h' \left[ u^{01}(x, G) \right] = \lambda + \mu(G) \left[ 1 - \frac{f_G(x, 0)}{f_G[x, (0, 1)]} \right] >
\]

\[
\lambda + \mu(G) \left[ 1 - \frac{f_B(x - \delta, 0)}{f_B[x - \delta, (0, 1)]} \right] = h' \left[ u^{01}(x - \delta, B) \right]
\]

which is equivalent to \(w^{01}(x, G) > w^{01}(x - \delta, B)\) as claimed in point (1) of the proposition. Point (2) follows from a similar argument. \(\blacksquare\)

The intuition behind this first result is that if the agent’s private signal is not completely noisy, in order to induce high effort after the observation of \(z = G\) but not after \(z = B\), the wage schedule has to be steeper in state \(y = G\) than in state
The asymmetry in the benchmarking rule described in proposition 4 is then a consequence of this characteristic. Notice that if \( B \geq \frac{c}{b_0(B) - b_1(B)} \) it would not be possible to implement \((a_B, a_G) = (0, 1)\) because in this case the liquidation cost \( B \) is so large that the manager would choose high effort after \( z = B \) even if his wage were completely independent of the firm’s outcome. Notice that points (1) and (2) in the previous proposition have been shown under the hypothesis that the incentive constraint corresponding to signal \( z = B \) is not binding. This condition for example holds when the private signal is perfectly informative. In fact, if \( \varepsilon = 0 \) first order conditions (22), (23) implies that \( \mu(B) = 0 \),\(^{16}\) therefore leading to a constant wage in state \( B \).

The next result describes the characteristics of the optimal benchmarking rule corresponding to the minimum cost contract implementing \((a_B, a_G) = (1, 0)\).

**Proposition 5** Given \( \varepsilon \in \left[0, \frac{1}{2}\right] \), if \( B < \frac{c}{b_0(G) - b_1(G)} \), the unique minimum cost contract \( w^{10}(x, y) \) implementing \((a_B, a_G) = (1, 0)\) is a continuous function of \( x \). Furthermore, if the incentive compatibility constraint corresponding to signal \( z = G \) is slack, it results that:

1. if \( x > \delta + \frac{1}{2} \), then \( w^{10}(x, G) < w^{10}(x - \delta, B) \);
2. if \( x < \delta + \frac{1}{2} \), then \( w^{10}(x, G) \geq w^{10}(x - \delta, B) \).

The proof closely resembles the argument given for proposition 4 and is then omitted. The intuition here is similar to the previous one. If the principal wants to induce \((a_B, a_G) = (1, 0)\) and the agent’s signal brings some information on the future state, the wage schedule has to be steeper in state \( y = B \) than in state \( y = G \). Then, the kind of asymmetric benchmarking described in proposition 5 turns out to be optimal. Again, if \( B \geq \frac{c}{b_0(G) - b_1(G)} \), it would be impossible to prevent the agent from choosing \( a = 1 \) after the signal \( z = G \), and then also after \( z = B \). However, the assumption that \( B < \frac{c}{b_0(G) - b_1(G)} \) maintained throughout, implies that \( B < \frac{c}{b_0(G) - b_1(G)} \).

Proposition 4 and 5 resemble the main results in Celentani and Loveira (2006). They found that if the agent productivity is larger in the good state, in order to induce high effort, optimal payments must be increasing in the benchmark for large outcome realizations and decreasing in the benchmark for small outcome realizations. Proposition 4 contains a similar result for the optimal contract that induces the agent to adopt the conditional effort profile \((0, 1)\) that, in turns, produces a positive correlation between the aggregate shock and the managerial product. Notice, however, that in Celentani

\(^{15}\)If the signal is perfectly informative, the optimal wage schedule in state \( y = B \) is constant.

\(^{16}\)Otherwise the wage schedule \( w(x, B) \) would be decreasing in \( x \) and this cannot be optimal.
and Loveira (2006) the assumption over the managerial productivity leads to an optimal contract displaying asymmetric benchmarking, while in proposition 4 it is the contract that is designed to induce such positive correlation. In other words, in Celentani and Loveira (2006) the positive correlation creates asymmetric benchmarking while here the opposite happens: to induce positive correlation, i.e. the conditional effort profiles (0, 1), asymmetric benchmarking is needed. Notice also that in order to produce the desired correlation between managerial product and aggregate shock, the contract has to satisfy a larger number of incentive constraints (there are three possible deviations instead of one). Finally notice that at the same time that the contract is creating the desired positive correlation, it is also ruling out independence (i.e. the unconditional effort profiles) and the opposite negative correlation, i.e., the conditional effort profile (1, 0). Similar remarks apply to proposition 5. Celentani and Loveira (2006) show that if the agent productivity and the aggregate state are negatively correlated, optimal payments are increasing in the benchmark for small outcome realization and decreasing in the benchmark for large realizations. Proposition 5 shows that one needs a contract with similar characteristics to create this negative correlation and to rule out other possible statistical relationships between the managerial productivity and the aggregate state.

Results contained in proposition 1 and 2 goes one step further in this direction. They show that, even if the optimal contract is designed to produce independence between aggregate shock and agent’s productivity, wage payments can display asymmetric benchmarking. This happens because the optimal contract has to rule out possible correlations that could emerge when the agent observes a private signal on the strength of his indirect incentives.

5 Numerical Examples

This section contains numerical examples highlighting most of the findings of the paper. Consider an agent with a CRRA Bernoulli utility of the form \( u(w) = \frac{w^{1-r}}{1-r} \) and the following parameter values: \( r = 0.7, \ w = 0 \), reservation wage \( W = 1 \) (corresponding to a reservation utility \( U = 3.3 \)), \( p(G) = 0.6, \ \gamma = 100, \ \delta = 300, \ \sigma = 150, \ \varrho = 0, \ B = 2.5, \ c = 1.5 \). If the agent does not observe a private signal on the future state of the world, bankruptcy probabilities are \( b_0 = 0.21 \) and \( b_1 = 0.10 \). The minimum cost contract inducing high effort has an expected wage \( EW = 5.54 \) with a standard
deviation $\sigma_W = 3.06$. Inducing high effort, the principal obtains profits equal to 283.24, while inducing low effort obtains 176.13. Figure 3 shows the optimal wage schedules in both the good and bad state. Notice that, the horizontal difference between the two schedules is exactly 300.

Consider now the case in which the agent can observe a private signal on the future state of the world up to a noise term $\varepsilon = 0.2$. The conditional bankruptcy probabilities are then: $b_0(G) = 0.09$, $b_1(G) = 0.04$, $b_0(B) = 0.37$, $b_1(B) = 0.18$.

Table 1: Optimal Contract Inducing (1,1)

<table>
<thead>
<tr>
<th>Expected Wage</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EW$</td>
<td>$5.67$</td>
</tr>
<tr>
<td>$EW_G$</td>
<td>$6.12$</td>
</tr>
<tr>
<td>$EW_B$</td>
<td>$5.03$</td>
</tr>
</tbody>
</table>

Table 1 shows wage expectations and standard deviations in both the good and bad state of the minimum cost contract implementing $(a_B, a_G) = (1, 1)$. The principal obtains profits equal to 283.11. Notice in particular that the ex ante expected wage is larger with respect to the case in which the agent does not observe a private signal. Furthermore, when the agent has private information, his expected wage is larger in the good state than in the bad state. As in Bertrand and Mullainathan (2001), managerial pay is then positively affected by the observable shock. Notice also that the wage volatility is higher in the good state because the manager has to receive a sharper incentive in this case. Figure 4 shows the optimal wage schedule implementing high effort after both realizations of the private signal. Notice that above the threshold $\hat{x}(0.2) \approx 455$ the wage payment is larger in the good state than in the bad state. Notice also that outcomes above $\hat{x}(0.2)$ have a 40% of probability in the good state but only a 1% of probability in the bad state. Furthermore, for outcomes below the threshold $\hat{x}(0.2)$, the wage payment is larger in the bad state. Therefore, the figure describes a situation in which the optimal wage displays asymmetric benchmarking.

Table 2: Optimal Contract Inducing (0,1)

<table>
<thead>
<tr>
<th>Expected Wage</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EW$</td>
<td>$4.64$</td>
</tr>
<tr>
<td>$EW_G$</td>
<td>$5.26$</td>
</tr>
<tr>
<td>$EW_B$</td>
<td>$3.71$</td>
</tr>
</tbody>
</table>

Table 2 shows wage expectations and standard deviations corresponding to the
minimum cost contract implementing \((a_B, a_G) = (0, 1)\). With this contract profits are equal to 252.26. Notice that the ex ante expected wage is smaller with respect to previous cases because the agent is induced to provide effort only after the observation of signal \(z = G\). For the same reason the expected outcome is smaller too and, given other parameter values, profits decrease. Notice that the expected wage payment is both larger and more volatile in state \(y = G\). This is because the aggregate state \(G\) is relatively more likely after the observation of signal \(z = G\), which is the signal that triggers high effort. Figure 5 shows the optimal wage schedule implementing the conditional effort profile \((0, 1)\). In this example, the incentive constraint corresponding to the observation of signal \(z = B\) is not binding. Therefore, we observe the situation described in proposition 4. In particular, notice that for large outcome realizations (success), good luck is not completely removed, i.e. \(w(x, G) > w(x - \delta, B)\). In other examples it is possible to obtain a wage structure that induces larger payments in the good state for outcomes above a certain threshold.

Table 3: Optimal Contract Inducing \((1,0)\)

<table>
<thead>
<tr>
<th>Expected Wage</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(EW)</td>
<td>(3.96)</td>
</tr>
<tr>
<td>(EW_G)</td>
<td>(3.72)</td>
</tr>
<tr>
<td>(EW_B)</td>
<td>(4.32)</td>
</tr>
</tbody>
</table>

Table 3 shows wage expectations and standard deviations generated by the minimum cost contract implementing \((a_B, a_G) = (1, 0)\). Corresponding profits are equal to 232.61. The expected wage is even smaller than for the contract implementing \((0, 1)\). This is because high effort here is induced after the observation of signal \(z = B\) that sharpens indirect incentives. Notice that wage expectation and volatility are both larger in state \(B\). Figure 6 shows the optimal wage schedule implementing the conditional effort profile \((1, 0)\). The incentive constraint corresponding to signal \(G\) is not binding so that proposition 5 applies. Notice in particular that payments are increasing in the benchmark for small outcome realizations while they are decreasing for large outcome realizations.

Finally, notice that the contract inducing the unconditional effort profile \((1, 1)\) maximizes profits in this example and, therefore, it would be adopted by the principal.
6 Conclusion

The model presented in this paper describes a mechanism that explains how an optimal contractual arrangement between a principal and an agent could display asymmetric benchmarking even if managerial productivity and aggregate shocks are uncorrelated. There are two key elements behind this result. First, the manager has implicit incentives deriving from the possibility of bankruptcy and, second, after signing the contract but before choosing his action, he observes a private signal on the future state of the world. The signal affects managerial indirect incentives: the observation of bad news increases the conditional probability of bankruptcy in case of misbehavior and therefore reduces the conflict of interest with the principal. The availability of the private signal also allows the agent to adopt conditional or unconditional effort profiles. Therefore it introduces the possibility of observing some correlation between managerial product and aggregate state, even if the exogenous productivity of managerial high effort is constant across states. The optimal benchmarking rule is used not only to filter out the observable shock but also to adjust the contractual explicit incentives to the variable implicit incentives. As a result, even when the managerial productivity and the observable shocks are uncorrelated, the optimal contract can display the kind of asymmetric benchmarking observed in the data.

The focus of the paper is on managerial incentives but, because the model adopted is very simple, it could readily be applied in different contexts where RPE considerations are important. For example, the recognition that apparently suboptimal practices, like asymmetric benchmarking, can indeed correspond to the most desirable arrangement may be important for the analysis of yardstick competition (Shleifer 1985, Sobel 1999). Regulated firms can in fact be induced to efficiently reduce their costs by setting up incentive schemes that relies on relative performances. The findings in this paper are then of some interest to assess the most desirable form of such incentives structures.

In this model, optimal contracts are usually non linear in the outcome. An interesting possibility would be to restrict to contracts with a base salary and a call option on the firm’s stock. As in Aseff and Santos (2005) one could obtain the optimal contract within this class and evaluate how it performs relative to the optimal non linear contract. This modification of the model would also allow to study how the strike price of the optimal option plan is affected by the observable shock.

As a final remark notice that results in this paper rely on the interplay between explicit and implicit incentives. One could obtain similar results considering other
sources of indirect incentives as long as they are sharper in bad states of the world. For example, negative aggregate shocks reducing the value of the firm, could increase the probability of a takeover or could trigger a closer monitoring activity by large stakeholders. In both cases managers would have stronger implicit incentives in bad states and optimal contracts should not overlook their effects.

References


**Figure 1:** The reference benchmarking rule

- $w^*(x, B)$
- $w^*(x, G)$

**Figure 2:** Asymmetric benchmarking

- $w^*(x, B)$
- $w^*(x, G)$
- $w^f(x, B)$
- $w^f(x, G)$
- $x_B$ and $x_G$
Figure 3: Minimum cost contract implementing high effort with no signal
Figure 4: Minimum cost contract implementing (1,1)
Figure 5: Minimum cost contract implementing (0,1)
Figure 6: Minimum cost contract implementing (1,0)