ON THE DYNAMIC INEFFICIENCY OF GOVERNMENTS*

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Abstract

When the government must decide not only on broad public-policy programs but also on the provision of group-specific public goods, dynamic strategic inefficiencies arise. The struggle between opposing groups—that disagree on the composition of expenditures and compete for office—results in governments being endogenously short-sighted: a systematic under-investment in infrastructure and overspending on public goods arises as resources are more valuable when in power. I find that more ideologically homogeneous societies have higher capital accumulation and more efficient allocations. When there is an average advantage for one group over the other in the political dimension, the group that loses the elections more often tends to spend a higher share of output on public goods while investing even less than the other group. This creates economic cycles—that follow the political cycle—introducing fluctuations in real macroeconomic variables without assuming any exogenous productivity shocks.

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1 Introduction

Public policy and institutions are found to be relevant factors in explaining the large differences in income per-capita across countries.\textsuperscript{1} However, policies that enhance growth and promote development are not always chosen by the government (especially in under-developed economies). Why? A group of explanations attributes it to the instability that results from major political upheaval and coups d’etat.\textsuperscript{2} It turns out that even democratic countries, where changes in government follow a stable election process, exhibit large disparities in growth rates and implemented policies. Several authors suggest that this could be the result of failures or frictions in the decision-making process of the public sector.\textsuperscript{3} A basic point is that policymakers can engage in rent-seeking activities and may choose inefficient policies in order to increase their probability of re-election so as to get continued access to these office rents.\textsuperscript{4} However, one does not need to take such a cynical view of governments. Even governments that are not “intrinsically bad” can, due to the democratic process itself—where reelection is never certain—and the resulting natural lack of commitment, generate bad outcomes. This is the idea pursued in the present paper.

I present a theoretical model where the struggle between groups with different views that alternate in power results in governments being endogenously short-sighted—at least more so than the groups they represent. As a consequence, they tend to overspend and underinvest, causing growth to slow down. The party in power tries to tie the hands of its successor by strategically manipulating public investment. The political uncertainty, together with the assumption of a fundamental lack of commitment, create incentives to reduce the amount of public capital available for the next policymaker as a way to restrict its level of spending.\textsuperscript{5} In addition, asymmetries in the group’s relative political power can generate endogenous economic cycles where macroeconomic variables fluctuate in equilibrium, even in the absence of productivity shocks.

The analysis herein assumes that the role of the government is to provide public goods and invest in productive public capital.\textsuperscript{6} There are two groups in the economy that disagree over the composition of spending on public goods but not over its aggregate size or over the level of infrastructure investment. I characterize time-consistent outcomes as Markov-perfect equilibria.\textsuperscript{7} First, I present sufficient conditions under which the equilibrium is symmetric (across groups) and I solve for this equilibrium in closed form. Second, I consider the situation where the candidates of one group on average have an advantage over those from the other group, leading to an asymmetric equilibrium.

\textsuperscript{1}For example, Acemoglu, Johnson, Robinson (2001) study the relationship between property rights and different degrees of development.
\textsuperscript{2}Barro (1996) and Easterly and Rebelo (1993) provide evidence that coups reduce growth.
\textsuperscript{3}See Persson and Tabellini (1999) for an excellent review of the literature.
\textsuperscript{4}An analysis of the earlier models in this literature (e.g. Barro, Nordhaus, and Ferejohn) can be found in Drazen (2000). In terms of the more recent literature, see Rogoff (1990), and Martinez (2005).
\textsuperscript{5}This result is analogous to that in the models with debt, where the government chooses a level that is higher than optimal in order to constraint the opposition’s spending in case of losing elections; see, e.g., Persson and Svensson (1989).
\textsuperscript{6}There is empirical evidence that investments in infrastructure (public capital) increase the productivity of the private sector. Aschauer (1989) argues that a one-percent increase in public investment contributes to as much as 0.39 percent of GDP in the U.S. It has to be kept in mind that even though these estimates may well be overstating the positive effects on output, they are obtained for developed countries. The focus here is mainly on underdeveloped countries, where the benefits of basic infrastructure improvements arguably are far more important than are marginal additions to infrastructure in underdeveloped countries.
\textsuperscript{7}Thus, the equilibrium described herein is a “fundamental” equilibrium capturing the effects that are inherent in the dynamic game itself, whether of finite or infinite horizon. The equilibrium here is thus the limit of finite-horizon equilibria: its characteristics do not significantly depend on the time horizon, so long as the time horizon is long enough. See Dixit, Grossman, and Gul (1998) for efficient allocation rules that are not Markov in the political game.
I show that the dynamic inefficiency generated by the politician’s short-sightedness is mitigated by the degree of political stability. This result is consistent with the negative correlation between political instability and growth found in Alesina, Ozler, Roubini, and Swagel (1992). In the symmetric case, the stronger the advantage of the incumbent over the opposition, the higher the growth rate. This stability, however, comes at a cost: the persistence of one government leads to persistent underspending on public goods of the type preferred by the group out of power.

Even though there is no disagreement on the size of public spending and investment in public capital, in the asymmetric case there are incentives for the two groups to act differently when in power: no symmetric equilibrium can exist. The group that loses the elections more often tends to spend a higher share of output on public goods while investing less than its counterpart. The political uncertainty is then propagated into the economy, and endogenous economic cycles are generated. This decreases welfare not only because it reinforces dynamic inefficiency (investment is too low) but also because it introduces volatility in macroeconomic variables.

On a more methodological level, this paper provides an Euler equation faced by the government in power that we can use to analyze the trade-offs that arise in the presence of reelection uncertainty. It reveals that there is a wedge between the marginal cost and marginal benefit of investment when compared to the benevolent planner’s solution (so allocations are inefficient). This wedge arises from the existence of intertemporal strategic effects. On the one hand, the government wants to decrease the level of resources available to next period’s policymaker so as to restrict his level of spending. But this will cause a negative effect in the opposition’s investment level, which the incumbent may want to boost if it expects the opposition to invest too little. On the other hand, changes in policy may also modify reelection probabilities, which must be taken into account by forward-looking governments.

There are a number of papers emphasizing that parties may choose not to implement policies that increase welfare in the long run because their reelection is uncertain. In particular, the argument in this literature is that the government may be less inclined to improve the legal system, to overspend in public goods (which only benefit a specific group), to create an excessive level of debt or to under-invest in productive public capital (roads, harbors, schools, communication infrastructure etc.). The contribution of this paper lies in the analysis of a dynamic infinite-horizon political economy model, which is particularly important for assessing the long-run effects of current policy. A forward-looking government must take into account how future policymakers will react to current changes in investment and how this in turn will affect the availability of resources if power is regained. This dynamic strategic effect cannot be captured in two-period models.

This paper also contributes to a growing literature on political failures that result from a fundamental lack of commitment of the government. While existing models with repeated voting find strategic interactions, most of them have to rely on numerical methods to characterize the Markov-perfect equilibrium. Examples are Krusell, Quadrini, and Rios-Rull (1997), Krusell and Rios-Rull (1999), or Azzimonti, de Francisco, and Krusell (2003). Hassler, Mora, Storesletten, and Zilibotti (2004), on the other hand, find analytical solutions in an overlapping generations setup where policy is decided by majority voting, but assume away political uncertainty. Using a similar

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8Svensson (1993) analyzes the effects on the legal system. Persson and Svensson (1989), Alesina and Tabellini (1990), and Devereux and Wen (1998) study the interaction between changes in the identity of the policymaker and excessive debt creation. Finally, Persson and Tabellini (1999), and Besley and Coate (1998) present a model where the government under-invests.

9Asteriou, Economides, Philippopoulos, and Price (2000) analyze a dynamic model where under-investment and overspending arise but assuming that parties care about economic outcomes more when in power than when out of power.
environment, Hassler, Krusell, Storesletten, and Zilibotti (2005) explain the survival of the welfare state. As here, Hassler, Storesletten, and Zilibotti (2004) find that expenditures in a consumable public good can be inefficient, but in a model where two-period lived agents vote over redistributive policy. Unlike in their work, here part of the expenditures are devoted to productive investment, which allows us to analyze the effects of policy on economic growth. In a partial equilibrium model, Battaglini and Coate (2005) introduce productive public goods financed by the government. Instead of focusing on political parties, they assume that policy (i.e., spending on an unproductive public good, pork barrel expenditures, and taxation) is decided through legislative bargaining. In their work the probability of being able to choose expenditures is exogenous and only depends on the number of legislators, while it is endogenous here and depends on future expected policy. Another difference is that they restrict attention to symmetric environments, so the equilibrium policy is independent of the identity of the party in power.

The model that is most closely related this one, is that by Amador (2003). Amador analyzes an infinite-horizon economy where politicians also have a bias towards the present: they are too impatient. This results in inefficient overspending and undersaving. Since debt reduces this inefficiency (because governments can borrow when economic shocks are bad), there are sufficient incentives for the incumbent not to default (i.e., to repay previous debts). While some of the strategic effects of current savings on future savings are taken into account in Amador’s paper, the assumption that all parties are equally likely to win an election (whether they are in power or not) rules out the ‘incumbency advantage effect’ (to be described later). Moreover, since he only considers symmetric environments, endogenous policy cycles do not arise.

This paper also extends existing literature by endogenizing the probabilities of re-election in a dynamic setup. This makes possible an analysis of the channel from economic policy to political turnover, which is often ignored. Groups alternate in power based on a political institution where “ideology” or other non-economic issues play a role and where commitment is fundamentally lacking. In particular, I use a “probabilistic-voting” setup (see Lindbeck and Weibull, 1993) in order to provide micro-foundations for political turnover: the probability of being in power next period is endogenously determined via an electoral process.

Agents are assumed to be fully rational, and, unlike in much of the dynamic voting literature, voting is forward-looking as opposed to retrospective: politicians are chosen based on what voters expect them to do in the future, and voters do not collectively punish politicians for their past actions. By allowing agents to vote, the degree of political uncertainty is jointly determined with public policy, so that it depends on the primitives that shape it (i.e., on the intensity of ideology, on agent’s preferences, and on technology). A main prediction is that societies that are very heterogeneous in the non-economic dimension (high fractionalization) tend to grow at a lower rate. The model hence provides a rationality-based explanation of the empirical relationship found by Easterly and Levine (1997) between ethnic diversity and growth. In particular, they show that ethnic diversity increases the likelihood of adopting poor policies and underproviding growth-enhancing public goods (they find that ethnic diversity alone accounts for 28 percent of the growth differential between the countries of Africa and East Asia).

Finally, this paper is related to the literature on “political business cycles”. When incumbents only care about power, policy is chosen in order to attract as many votes as possible or to signal competence—if there is imperfect information—of the candidate, which results in sub-optimal decisions. The main prediction in that line of research is an electoral cycle in policies: the incumbent stimulates the economy before the elections to boost his chances of winning. In this paper, in-

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10 Rogoff (1990) assumes that voters are rational but backward-looking. See Drazen (2000) for a discussion on retrospective voting.

11 Martinez (2005) analyzes these political cycles in a theoretical model where the ability of the politician is learned
cumbents care about being in power but only as a means of implementing the policy desired by the
group they represent. In contrast to previous models (like Milesi-Ferreti and Spolaore, 1994), I
do not need to assume exogenous differences in preferences over the size of public expenditures in
order to generate cycles. In particular, both parties have the same utility over the size of spending
on public goods and on the level of investment. However, one party may spend more and invest
less just because it loses more often as a result of an ideological disadvantage.

The organization of the paper is as follows. The model is described the Section 2 and the
Markov-perfect equilibrium defined in Section 2.3, assuming exogenous (and symmetric) political
turnover. The asymmetric case is studied in Section 3. Analytical solutions for both, the symmetric
and asymmetric cases, are presented. Political turnover is endogenized in Section 4, and Section 5
concludes.

2 The basic model

2.1 Economic environment

Consider an infinite horizon economy populated by agents that work in the production sector for
a competitive salary and enjoy the consumption of private and public goods. While they have
identical income and identical preferences over private consumption, they differ in their preferences
over consumable goods provided by the government. In particular, assume that there are two types
of agents, A and B, of measure \( \mu^A \) and \( \mu^B \) (with the size of the population normalized to 1). The
lifetime utility of an individual of type \( J \) is:

\[
U^J = \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(g^J_t)],
\]

(1)

where \( c_t \) denotes the consumption of private goods and \( g^J_t \) the consumption of (public) good type
\( J \). Assume that preferences exhibit constant relative risk aversion, so

\[
u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}
\]

and

\[
v(g_t) = \frac{g_t^{1-\sigma} - 1}{1 - \sigma}.
\]

with \( \sigma \in (0, 1) \).

We can interpret \( g^J_t \) as the amount spent by the government in providing pure public goods
(like ‘clean air’ and ‘public television’), as expenditures in excludable private goods (maintenance
of beaches and parks) or as direct transfers to different regions within a country. Notice that an
agent in group \( B \) derives no utility from the provision of good \( A \) (and vice versa), so in principle
there will be disagreement in the population on the desired composition of public expenditures -but
not on its size, since both types have the same marginal rate of substitution between private and
public goods.

Private goods are produced by competitive firms that have access to a Cobb-Douglas technology:

\[
F(K_{gt}, L_t) = RK_t^\theta L_t^{1-\theta},
\]

where \( L_t \) is aggregate labor and \( K_{gt} \) is the stock of public capital (i.e. infrastructure, public health, and education, the knowledge produced by the public sector’s R&D and expenditures in national defense or law enforcement). Its level is determined by government
investments and acts as an externality in production. The idea behind this specification is that the

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12 In that sense, cycles are ‘partisan’.

13 We need to restrict the degree of risk aversion to this interval in order for the utility \( v(g) \) to be well defined when
\( g^J = 0 \).
better the infrastructure (roads, harbors, sewers, etc.), the more educated the population and the stronger the protection of property rights, the higher the productivity of the private sector.

There are infinitely many competitive firms that produce a single consumption good and hire labor each period so as to maximize profits. In equilibrium, workers are paid their marginal product, \( w_t = F_2(K_{gt}, L_t) \) and firms distribute profits as dividends to the shareholders in the amount \( \pi_t = F(K_{gt}, L_t) - w_t L_t \).

The only tool available to raise revenues in this economy is a proportional income tax. Each period, the government sets a tax rate \( \tau_t \) and uses the proceeds from taxation to finance the provision of consumable public goods \( g_t^A \) and \( g_t^B \) and investments on productive public capital \( I_{K_{gt}} \). Since it is costly to observe whether an agent belongs to group \( A \) or \( B \), I will constrain \( \tau \) to be type-independent.

Given the tax rate, consumption is
\[
c_t = (1 - \tau_t)[w_t l_t + \pi_t],
\]
where \( l_t \) is the fraction of time devoted to work (the total time endowment is normalized to 1).

Assuming that there is no debt, the government must balance its budget every period, so its budget constraint reads as:
\[
g_t^A + g_t^B + I_{K_{gt}} = \tau_t \sum_j \mu^j[w_t l_t + \pi_t].
\]
The proceeds from taxation are displayed in the right hand side of the equation, while the left hand side contains the sum of expenditures in the provision public goods and productive capital.

Since leisure is not valued, the supply for labor is inelastic (so \( l_t = 1 \)). To simplify notation let \( F(K_g, 1) = f(K_g) \). Then, the government budget constraint can be written as:
\[
g_t^A + g_t^B + I_{K_{gt}} = \tau_t f(K_{gt}).
\]
Assuming a depreciation rate of \( \delta \), public capital evolves according to:
\[
K_{gt+1} = I_{K_{gt}} + (1 - \delta)K_{gt}.
\]
The current government inherits a certain level of infrastructure \( K_{gt} \) determined by investments undertaken by the previous government, and decides on the level of taxation, expenditures in each type of public good and investments in public capital.

### 2.2 Planning solutions

Before describing the outcome under political competition (where different parties alternate in power), it is useful to characterize the optimal allocation chosen by a benevolent social planner. The planner takes the initial level of infrastructure \( K_{g0} \) as given, and chooses sequences of \( \{c_t, K_{gt}, g_t^A, g_t^B\}_{t=0}^{\infty} \) that maximize the weighted sum of utilities, where the weight on type \( j \) agents is \( \lambda^j \) (with \( \lambda^A + \lambda^B = 1 \)). Its maximization problem follows.
\[
\max \sum_j \mu^j \lambda^j \sum_{t=0}^{\infty} \beta^t[u(c_t) + v(g_t^j)],
\]

\[\text{14}\]  The model can be easily extended to include an endogenous labor decision. Since this would involve more notation and no significant changes in the main results, I decided not to include the extension in this version of the paper.
subject to the resource constraint:
\[ g_t + c_t + K_{g_t+1} = f(K_{g_t}) + (1 - \delta)K_{g_t}, \]
where \( g_t = \sum_j g^j_t \) is the total amount spent in public goods \( A \) and \( B \).

As long as the planner gives a positive weight to each agent, the optimal allocation of public good \( j \) will be such that its marginal utility is proportional to the marginal utility of private consumption.\(^15\)

\[ u_c(c_t) = \zeta^j v_g(g^j_t), \quad \text{where} \quad \zeta^j = \frac{\mu^j\lambda^j}{\sum_j \mu^j\lambda^j} < 1 \]

By varying \( \lambda^j \) between 0 and 1 it is possible to trace the Pareto frontier that characterizes the optimal provision of public goods. Concavity of \( v \) implies that if type \( A \) agents have a higher weight in the social welfare function, more of their desired public good will be provided (at the expense of type \( B \) agents). As a benchmark I will consider utilitarian social welfare function that gives the same weight to each agent (that is \( \lambda^j = 1/2 \)).

The planner chooses the level of public capital that equates the marginal costs in terms of foregone consumption to the discounted marginal benefits of the investment.

\[ u_c(c_t) = \beta u_c(c_{t+1})(f(K_{g_{t+1}}) + 1 - \delta) \]

The planner’s Euler equation is completely independent of the choice of the social welfare function: changes in \( \lambda^j \) do not affect this margin. The results follows from assuming that both agents have the same trade-off between private and public consumption (i.e. \( u \) and \( v \) are equal for all agents).

### 2.3 Political equilibrium

"With the campaign over, Americans are expecting a bipartisan effort and results. I will reach out to everyone who shares our goals and I’m eager to start the work ahead”

G. W. Bush, 2004.\(^16\)

This quote illustrates that parties only care about the well-being of their consistency. It also suggests some lack of commitment of the policymaker (in the sense that promises made over the campaign are non-binding). These are two main features that will distinguish political parties from a benevolent social planner throughout the analysis that follows.

The role of the government in this economy is to provide public goods and productive public capital. Given the disagreement between groups over which public good should be provided, political parties will endogenously arise in a democratic environment. I analyze a stylized case where there are two parties, \( A \) and \( B \), representing each group in the population and competing for office every period. They alternate in power according to an exogenous and type-independent re-election probability, denoted by \( p \in [0, 1] \). The party in power at time \( t \) wins the election with probability \( p \) (remaining in power in period \( t + 1 \)), and loses it with probability \( 1 - p \). The elected party chooses the tax rate and the allocation of government resources between the different types of spending and investment so as to maximize the utility of its own type.\(^17\)

\[^{15}\text{If the planner only cares about the well-being of, say, agent } A, \text{ it will set } g^B_t = 0 \forall t \text{ and } g^A_t \text{ so as to equate the marginal rate of substitution between private and public goods to 1.}\]

\[^{16}\text{http:// CNN.com - Bush ‘Americans expect results’ - Nov 4, 2004}\]

\[^{17}\text{In that sense this is a partisan model. A politician from party } j \text{ is just like any other agent in that group, so he wants to maximize utility. In contrast, other models in the literature assume that politicians can extract rents from being in power, so their objective is to maximize the probability of winning the next election. See Drazen (2000) or Persson and Tabellini (2000) for a discussion on opportunistic models.}\]
There is no commitment technology, so promises made by any party before elections are not credible. The party in power plays a Nash game against the opposition taking their policy as given. Alternative realizations of history (defined by the sequence of policies up to time $t$) may result in different current policies. In principle, this dynamic game allows for multiple subgame-perfect equilibria, that can be constructed using reputation mechanisms. I will rule out such mechanisms and focus instead on Markov perfect equilibrium (MPE), defined as a set of strategies that depend only on the current—payoff relevant—state of the economy, $K_g$. Hence, a MPE is a set of strategies $\{h_j(K_g), g^A_j(K_g), g^B_j(K_g), \tau_j(K_g)\}$, where $h_j(K_g)$ determines tomorrow’s level of public capital chosen by party $j$ if in power.

Suppose that $j$ is the elected party. Given the stock of public capital $K_g$, its objective function today is:

$$V_j(K_g) = \max_{c, K'_g, g^A, g^B} \{u(c) + v(g^j) + \beta[pV_j(K'_g) + (1 - p)W_j(K_g)]\},$$

where $V_j$ denotes the utility of a type $j$ agent when its party is in power and $W_j$ his utility when out of power (to be described later). Consumption equals $c = f(K'_g) + (1 - \delta)K_g - g - K'_g$, total expenditures are $g = g^A + g^B$ and $K'_g$ is the level of tomorrow’s capital. Notice that primes denote next period variables.

As mentioned above, the only payoff relevant variable in this economy is the stock of public capital. Since $g^A_j$ and $g^B_j$ only affect today’s utility, tomorrow’s decisions are independent of the composition of expenditures. In that sense, the government faces a static decision problem: how to split total current expenditures into public goods $A$ and $B$. For any given level of $g$, if party $A$ is in power, it will choose to allocate it all in good $A$ (analogously with party $B$). This implies that $g^j_i = 0$, for $i \neq j$, which further simplifies the problem.

The symmetry in the Markov chain determining the alternation of government results in both parties facing exactly the same decision problem (when in power). It is natural then to look for symmetric Markov perfect equilibria, defined below.

**Definition:** A symmetric Markov perfect equilibrium is a set of policy functions $\{g(K_g), h(K_g)\}$ and value functions $\{V(K_g), W(K_g)\}$ that solves the incumbent’s maximization problem:

$$V(K_g) = \max_{g, K'_g} \{u(c) + v(g) + \beta[pV(K'_g) + (1 - p)W(K_g)]\} \quad (4)$$

$$W(K_g) = u(c(K_g)) + \beta[pW(h(K_g)) + (1 - p)V(h(K_g))], \quad (5)$$

where $c(K_g) = f(K_g) + (1 - \delta)K_g - g(K_g) - h(K_g)$.

Equation (4) represents the value function of the group currently in power while eq. (5) is the utility of that group when out of power, given the opposition’s policy decision.

In this equilibrium both parties invest following the same rule, so the evolution of public capital is completely deterministic: political uncertainty is not propagated into investment decisions made by the government. That is not the case for public goods: when party $A$ is in office, it only provides good $A$ (equivalently for party $B$). This introduces fluctuations in the composition of spending that mimic the political cycle. The concavity of the utility function implies that risk averse agents will suffer a welfare loss due to the volatility introduced by the electoral process.

Even though the composition of spending differs across incumbents’ types, the amount of resources spent is the same ($g^A_A = g^B_B = g$). This, together with the fact that investment is identical under either party, results in agents consuming the same level of privately produced goods and facing the same tax rate independently of the identity of the incumbent. In other words, there is no volatility in private consumption or macroeconomic variables.
The choice of expenditures is a static one, affecting only the *intra-temporal* margin. At the optimum, the government chooses \( g \) so that the marginal utilities of consumption of public and private goods (of its constituency) are the same:

\[
v_g(g) = u_c(c).
\]  
(6)

We can see that government spending in the MPE is sub-optimal from the standpoint of a utilitarian social planner by comparing eq. (2) to the equality above. This is the case for two reasons. First, the group out of power gets no provision of their preferred good. Second, there is overspending in the sense that the marginal rate of private consumption is too low when compared to that of the utilitarian optimum. Even the group in power would prefer a *lower* level of \( g \) if the difference was invested in productive capital instead.

The investment decision affects the *inter-temporal* margin; the costs of increasing public capital are paid today while the benefits are received in the future. The government chooses \( K_g' \) so that the marginal cost in terms of foregone consumption equals expected marginal benefits:

\[
u_c(c) = \beta \{ pV_1(K_g') + (1 - p)W_1(K_g') \}
\]  
(7)

As in the planner’s first-order condition, the cost of an extra unit of investment in public capital is given by a reduction in current utility via a decrease in consumption \(-u_c(c)\). The benefits, on the other hand, now depend on the identity of the party that wins the next election. When \( K_g' \) increases, expected future utility rises from the expansion of resources. Agents in a given group enjoy an increase of \( V_1(K_g') = \frac{\partial V(K_g)}{\partial K_g} \) utils if they win the next election (which occurs with probability \( p \)) and \( W_1(K_g') = \frac{\partial W(K_g)}{\partial K_g} \) otherwise (which occurs with probability \( 1 - p \)). Given that the identity of the decision-maker changes over time, the envelope theorem doesn’t hold in this environment, so the traditional Euler equation will not be satisfied.

### 2.4 Differentiable Markov Perfect Equilibrium (DMPE)

In order to further characterize the trade-offs faced by an incumbent when choosing investment, I will assume that policy functions are *differentiable*. Klein, Krusell, and Rios-Rull (2003) made this assumption (in a different context) arguing that there could be in principle an infinitely large number of Markov equilibria. By assuming differentiability, the problem delivers a solution that is the limit to the finite horizon problem. Moreover, it allows us to derive the government Euler equation (GEE) even if the envelope theorem does not hold: \(^{18}\)

\[
\begin{align}
&u_c(c(K_g)) - \beta u_c(c(K_g')) \left[ f_1(K_g') + (1 - \delta) \right] = \beta \{ -[1 - p] u_c(c(K_g')) g_k(K_g') + \right. \\
&\left. h_k(K_g')(2p - 1) [u_c(c(K_g')) - \beta u_c(c(K_g'')) \left[ f_k(K_g'') + (1 - \delta) \right] ] \}.
\end{align}
\]  
(8)

Disagreement

Incumbency Advantage

\( K_g' = h(K_g), \ K_g'' = h(h(K_g)), \) etc. \(^{19}\)

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\(^{18}\)See Appendix 6.1 for the derivation.

\(^{19}\)This is a functional equation that holds for any value of \( K_g \), as long as the functions are concave, satisfy general Inada conditions and the normalization \( v(0) = 0 \).
It is interesting to note that the specific functional form of the utility and production functions were not used to derive the GEE. Therefore, this equation describes the optimal behavior of an incumbent in political equilibrium with re-election uncertainty.

If \( p \) was equal to 1, the party currently in power would remain in office forever. In such case, the equation would reduce to

\[
uc(c(K_g)) = \beta uc(c(K'_g)) \left[ f_k(K'_g) + (1 - \delta) \right].
\]

This is exactly the same expression that was derived in Section 2.2. If the incumbent knew that he would never be out of power, he would invest as a social planner. This cannot be interpreted as implying that both equilibria are equivalent. The growth implications are the same (since efficiency is achieved), but at the expense of some proportion of the population enjoying lower utility as one type of public good is never provided.

When \( p < 1 \), there is a positive likelihood \((1-p)\) that the group in office loses power next period, which introduces a wedge in the first-order condition. This wedge is composed of two effects:

The first one is the disagreement effect (right hand side of eq. 8). When the incumbent is not re-elected, a marginal increase in public capital today changes the opposition’s spending in public goods tomorrow. This reports a cost in terms of foregone consumption next period with no utility benefit since the incumbent derives no utility from that public good. From today’s perspective it is optimal, then, to decrease investment with respect to the certainty case: the current incumbent wants to ‘tie the hands’ of its successor in order to restrict its spending. The disagreement over the composition of public goods together with the political uncertainty deter investment.\(^{20}\)

The second one is the incumbency advantage effect (second line of eq. 8). The party in power knows that not only spending will be altered when \( K'_g \) increases but also the level of investment carried over by future policymakers, captured by \( h_k(K'_g) \). This term was absent in the previous literature involving political instability, because most of the papers focused on two-period economies, so there were no incentives to invest in the last period, \( h(K'_g) = 0 \). Papers that did analyze infinite horizon economies assumed no persistence \((p = \frac{1}{2})\), which also causes the term to disappear.\(^{21}\)

The sign of the incumbency advantage term depends on current expectations about the behavior of future governments and on the degree of political instability. Assume that there is an ‘incumbency advantage’, which means that the party in power is more likely to win than the challenger \((p > 1/2)\). If future policymakers are expected to invest too little with respect to the planner’s solution, \( uc(c(K'_g)) < \beta uc(c(K''_g)) \left[ f_k(K''_g) + (1 - \delta) \right] \), the third term is negative so investment decreases. As long as there is low turnover \((p \text{ is relatively high})\), the disagreement effect is reinforced. To see this notice that eq.(8) holds for every period, so we can substitute its RHS (updated one period) into the incumbency advantage term. The result equals the sum of two disagreement effects and tomorrow’s incumbency advantage effect. Repeating this procedure, we can see that the gap in investment is just the sum of future disagreement effects from now on, weighted by the respective changes in investment and the probabilities of re-election.

The government’s Euler equation (eq. 8) depends on the derivative of an unknown equilibrium function: \( h_k(K'_g) \). In such an environment, the traditional methods to prove existence and uniqueness cannot be used. Most studies have to rely on numerical methods to characterize equilibrium functions. Even calculating the steady state level of investment in public capital is non trivial.\(^{22}\)

\(^{20}\)This effect is similar to that observed in Persson and Svensson (1989). Besley and Coate (1998) find that disagreements over redistribution policies can result in inefficient levels of investment. Milesi-Ferreti and Spolaore (1994) also obtain strategic manipulation, but for an alternative environment.


\(^{22}\)Krusell, and Rios Rull, (1999) and Azzimonti Renzo, Sarte, and Soares, (2003) use linear quadratic approxima-
In order to shed some more light on the characterization of Markov perfect equilibria under political uncertainty, it is useful to analyze our example economy.

### 2.5 The example economy

Under more specific assumptions over the production technology and the utility function it is possible to find an analytical solution. In this section, I characterize it and derive qualitative implications from the theory. Next, I discuss their validity by looking at empirical evidence.

Assume that the production function is linear, \( \theta = 1 \), with full depreciation, \( \delta = 1 \).

It is instructive to analyze the Pareto optimal allocations first (obtained by solving the planner’s problem). Under the assumptions above the economy collapses to an AK-model, so the standard results in the growth literature apply. There exists a balanced growth path in which output, consumption, investment, and unproductive expenditures grow at constant rate, denoted by \( \gamma^* \).

In order for the problem to be well defined (bounded utility) we need to assume that \( \frac{\beta R}{1 - \sigma} < 1 \). Replacing the functional forms into equation (3) and guessing that it is optimal to invest a constant proportion of output every period (so \( K' = s^* R K \)), we obtain an expression for the marginal propensity to invest \( s^* \):

\[
s^* = \frac{\beta}{1 - \sigma} R^{\frac{1 - \sigma}{\sigma}}.
\]

The optimal growth rate is then \( \gamma^* = s^* R \), which is greater than one as long as \( \beta R > 1 \). I will restrict the parameters so that this condition is satisfied.

The political equilibrium is characterized in the following proposition.

**Proposition 1** Under the assumptions of CRRA utility and linear production function with full depreciation there exists a balanced growth path in which output, consumption, investment and unproductive expenditures grow at rate \( \gamma = s R \) where \( s \) satisfies:

\[
s = \frac{\beta^{1/\sigma} R^{1-\sigma}}{\sigma} \left\{ \frac{1}{2} \left[ 1 + p + s (3p - 1) \right] - \beta \left[ 2p - 1 \right] (R s)^{1 - \sigma} \right\}^{1/\sigma}.
\]

The MPE is symmetric with

\[
\tau = \frac{1 + s}{2}, \quad g = \frac{1}{2} (1 - s) R K_g \quad \text{and} \quad h(K_g) = s R K_g,
\]

**Proof** See Appendix 6.2.

In equilibrium, the government taxes income at a time-invariant rate. A constant proportion \( s \) of tax revenues is invested in public capital and the remaining part is spent on the provision of public goods. Given that preferences over private and public goods have the same form, it is optimal to choose taxes and public spending so that consumption of public and private goods is the same:

\[
g = c = \frac{1}{2} (1 - s) R K_g.
\]

The marginal propensity to invest is implicitly defined by Equation (2.5) as a function of the probability of re-election \( p \), the degree of risk aversion \( \sigma \) and the productivity level \( R \).

**Corollary 1:** The Markov-perfect equilibrium is dynamically inefficient. That is, \( s < s^* \).

**Proof** The proof proceeds by contradiction. Assuming that \( s \geq s^* \) and replacing this into eq. we obtain:

\[
1 \geq \frac{1}{2} \left[ 1 + p + \beta^{1/\sigma} R^{1-\sigma} (1 - p) \right] < 1,
\]

In order to find steady states. 11
since \( s^* = \beta^{1/\sigma} R_{\frac{1}{\sigma}} < 1 \), which is a contradiction.

The intuition behind this result can be understood by looking at the trade-offs faced by the group in power. An incumbent who believes that he will be replaced with high probability does not have strong incentives to abstain from consumption today in order to invest in public capital. Knowing that it is very likely that tomorrow’s policymaker would prefer a different composition of spending, the incumbent tries to manipulate next period’s policy through the choice of the state variable. He can ‘tie the hands’ of his successor by decreasing the amount of available resources (i.e. investing a small amount today), which shrinks the tax base in the future. It is then reasonable to expect the propensity to invest under political uncertainty to be lower than that chosen by a planner.

There is yet another source of inefficiency in the political equilibrium. The uncertainty over the identity of tomorrow’s policymaker introduces volatility in the consumption of the public good that was absent in the planner’s solution. Welfare along the equilibrium’s balanced growth path is lower not only because the amount of resources is smaller, but also because individuals suffer from artificial fluctuations in the consumption of public goods (keep in mind that there are no productivity shocks in this economy).

Figure 1 depicts \( s \) as a function of the probability of re-election for different risk-aversion parameters.

The first observation that can be inferred from the graph is that, given \( \sigma \), the marginal propensity to invest decreases with the degree of political instability. This implies that the growth rate, \( \gamma = s(p) R \), is an increasing function of \( p \) (the efficient level of investment is attained when \( p = 1 \)). A qualitative prediction of the model is that politically unstable countries should exhibit lower growth-rates than more stable ones.

Figure 1: Investment as a function of \( p \)

The second observation that can be inferred from the graph is that the degree at which investment is affected by the probability of re-election depends non trivially on the value of \( \sigma \). When \( p < 1 \), the incumbent would like to smooth consumption over time and across states, since the identity of next period’s policymaker is unknown at that point. We can see that investment decreases with \( \sigma \) for every value of \( p \) when the economy is growing (as the ‘smoothing consumption’ effect dominates). Moreover, the higher the IES, the stronger the responsiveness of the growth rate
to increases in the probability of re-election (measured by the slope of $s(p)$). This feature of the model is much more difficult to test empirically, but it is qualitatively consistent with most growth models once we fix the probability of re-election.

**Corollary 2:** Stable economies exhibit higher investment as a fraction of GDP, and lower expenditures in unproductive public goods and transfers. As a result, they grow faster.

The current policymaker foresees that if he loses the next election, the opposition will spend part of the resources on a public good that reports no utility gains for his constituency. Hence, the benefits from an extra unit of investment are not fully internalized. This causes the incumbent to behave *myopically* and over-spend today on unproductive public goods (and under-invest in public capital). The effect is stronger the lower the probability of remaining in power.

According to the model, then, we should observe that economies with high political turnover (frequent changes of power) present a bias towards spending and relatively low levels of investment in infrastructure, education, public health or other productive activities.

A recent study by the Congressional Budget Office (1997) claims that a stable government is a necessary condition for economic growth. It reports, for example, that ‘Tunisia is one of the more stable and developed countries of North Africa and the Middle East. Governed by the Destourian Socialist Party since independence in 1957, Tunisia’s administration has been flexible and pragmatic in implementing development policy’. Asteriou, Economides, Philippopoulus, and Price (2000) find that increases in the probability of re-election (measured by the popularity of the government) increase growth in the UK, which is consistent with the main prediction of this model. Alesina, Ozler, Roubini, and Swagel (1992) study a sample of 113 countries for the period 1950-1982 and find that increases in political instability significantly reduce growth. Roubini and Sachs (1989) find that government spending (as a share of GDP) is positively related to a political instability index (measured by the degree of political power dispersion within the ruling group). Devereux and Wen (1998) also provide empirical support for the bias towards spending. They show that sociopolitical instability (using the Barro and Lee index) has a positive effect on the ratio of government spending to GDP.

**A note on Dictatorships**

The analysis suggests that the higher the probability of re-election, the larger the growth rate. In the limit, production efficiency is attained under a ‘dictator’, where $p = 1$. However, the resulting allocations do not coincide with those under an utilitarian social planner. This is because the public good that the dictator’s group doesn’t value is never provided when $p = 1$. And this hurts part of the population. Hence, dictatorships can be welfare reducing if the measure of people sharing the dictator’s preferences is too small.

The discussion above assumed that the dictator can stay in power forever. However, even dictators must have some support in sections of the population to remain in power. We can then re-interpret the model by considering democracy to be a continuous variable, where the degree of democracy is captured by $p$. The closer to 1, the less democratic the country is. The model would then predict that stable dictatorships grow faster than unstable ones. This is widely supported by the Latin American experience, with Pinochet in Chile, Somoza in Nicaragua and Porfirio Diaz in Mexico.

But as we mentioned before, efficiency does not imply welfare maximization. There is a trade-off between equity and efficiency in the model. While efficiency increases with the probability of re-election, equity decreases with it. The more democratic the country is, the less efficient but the more equal it is.
3 Asymmetric political power

Alesina and Roubini (1997) computed an average growth rate of output of 4.24% under a Democratic government and of 2.41% under a Republican one (for the sample 1949-1994). In a standard regression, they found that a change of regime to a Republican (Democratic) administration, leads to a fall (increase) in output growth (even after controlling for differences in the exchange rate system, shocks from the rest of the world, etc.). The effects of a change in regime also hold for a sample of industrial (and bipartisan) countries. In terms of fiscal policy, Persson and Tabellini (1999) report that tax rates fluctuate considerably over time (and across countries). These stylized facts suggest that the policy implemented and the resulting growth rate are not time-invariant. Moreover, they seem to depend on the identity of the party in power. Both observations are inconsistent with what the benchmark model presented before would predict. In this section, I will address that issue by relaxing the symmetry assumption.

Most of the political economy literature justified asymmetries in policy (and economic outcomes) with asymmetries in preferences, by assuming that parties disagree on the ‘size of government’. For example, Persson and Tabellini (2000), Persson and Svensson (1989), and Milesi-Ferretti and Soto (1994) consider models where left-wing parties enjoy a higher utility from public expenditures than the opposing right-wing. This modelling strategy makes empirical tests difficult, since the differences in policy are derived from differences in unobservable preferences.

I will take a different approach, where asymmetric policy results from differences in observable characteristics and arises as an equilibrium outcome. As before, preferences over the size of expenditures are identical for both groups (although they still disagree on its composition). However, one of the groups has greater political power than the other. In particular, I assume that type-B incumbents are more likely to be re-elected ($p_B > p_A$).

3.1 The Markov-perfect equilibrium

Due to the fact that different groups may choose different policies, allocations are—in principle—no longer symmetric. Therefore, we need to index policy rules by the incumbent’s affiliation. In that spirit, $h_B(K_g)$ and $g_B(K_g)$ denote optimal investment and spending rules—as functions of the state—made by a type B government. Since value functions are also type-dependent we can denote them by $V_B$ and $W_B$.\(^{23}\)

The optimization problem faced by incumbent $B$ follows.

$$V_B(K_g) = \max_{g, K_g' \geq 0} u(f(K_g) + (1 - \delta)K_g - g - K_g') + v(g) + \beta\{p_BV_B(K_g') + (1 - p_B)W_B(K_g')\}$$

$$W_B(K_g) = u(c_A(K_g)) + \beta\{p_AW_B(h_A(K_g)) + (1 - p_A)V_B(h_A(K_g))\} \tag{10}$$

Recall that $W_B$ denotes the utility of B-agents when the opposition is in power, so policy is given. In such case, consumption is given by $c_A(K_g) = f(K_g) + (1 - \delta)K_g - g_A(K_g) - h_A(K_g)$.

When in power, party $B$ chooses policy to maximize the right hand side of equation (10). The first-order condition with respect to public goods is analogous to the one derived under symmetric political power. The first-order condition with respect to productive capital is:

$$u_c(c_B(K_g)) = \beta\{p_BV_B(K_g') + (1 - p_B)W_B(K_g')\}.$$

\(^{23}\)The asymmetry in political power does not alter the result that candidates from party $j$ only spend on good $j$, so as in the previous section we do not need to keep track of the type of good but just of the total amount spent.
**Definition:** A Markov perfect equilibrium is a set of policy functions \( \{g_j(K_g), h_j(K_g)\} \) and value functions \( \{V_j(K_g), W_j(K_g)\} \) with \( j \in \{A, B\} \) that solves the incumbent’s maximization problem given by eq. (10) for \( j = B \).

**The GEE under asymmetry**

As in the symmetric case, we would like to have an expression which is independent of (derivatives of) the value functions, so as to cut down the number of unknown functions. Since \( h_A(K_g) \) is not necessarily equivalent to \( h_B(K_g) \), the government Euler equation cannot be derived unless an extra technical assumption is made: saving rules \( h_B(K_g) \) and \( h_A(K_g) \) are not only differentiable but also invertible (see details in Appendix 6.3). After some algebra, the GEE can be summarized as follows:

\[
MC = \beta(D + EMB + SME). \tag{13}
\]

The left hand side captures the fact that investing in public capital today involves incurring a **marginal cost** (\( MC \)) in terms of current foregone consumption,

\[
MC = u_c(c_B(K_g)).
\]

The right hand side of equation (13) is the discounted sum of three effects of present actions into future outcomes. Utility is affected directly through the **disagreement effect** and the formation of **expected marginal benefits**; and indirectly through the **strategic manipulation effect**.

The disagreement effect (\( DE \)) is just a modified version of that described in section 2.4. If party \( B \) loses the election, a portion of the extra resources will be devoted to provide a public good that its constituency doesn’t value. This results in a utility loss,

\[
DE = -(1 - p_B)u_c(c_A(K_g''))g_A(K_g').
\]

The **expected marginal benefits** of having more public capital in the economy next period (keeping the level of investment chosen by next period’s incumbent constant) are included in the \( EMB \) term:

\[
EMB = \left[p_Bu_c(c_B(K_g')) + (1 - p_B)u_c(c_A(K_g'))\right][f_k(K_g') + (1 - \delta)].
\]

If there was no political turnover, the benefits of an extra unit of public capital would be given by \( \beta u_c(c')f_k(K_g') \), the utility-denominated marginal return of investment. The effect of increasing \( K_g' \) in future investment would only be of second order and, due to the envelope theorem, there would be no need to keep track of it. In the present case, it is uncertain who will take the decisions tomorrow, so the theorem as we know it no longer holds. As long as parties differ, the reaction of the opposition to a change in \( K_g' \) will be sub-optimal from the standpoint of party \( B \) (because they value the future differently). This leaves room for **strategic manipulation**: future policy can be influenced by the current incumbent through it’s choice of public capital. The \( SME \) term captures these incentives.

\[
SME = (1 - p_B)h_A(K_g')\{[-u_c(c_A(K_g')) + u_c(c_A'K_g')]|[-u_c(c_B(K_g')) + \beta u_c(c_B(K_g''))[f_k(K_g'') + (1 - \delta)]\},
\]

where \( K_g'' = h_A(K_g') \) and \( c_B = f(h_B^{-1}(K_g'')) - K_g'' - g_B(h_B^{-1}(K_g'')) \).

When parties were assumed to have the same political power, the composition of expenditures was the only source of disagreement. The center of the conflict was what to spend the budget on,
instead of how much to spend. As we relax the symmetry assumption, party B realizes that it is more likely that they retain power next period, so investing in public capital today becomes more attractive. Party A values the future differently, since the chances of collecting the returns from investment appear to be much lower. As a result, there is now disagreement on how much to invest. This source of disagreement is captured in the first term of SME.

Note that no party has an advantage while in power whenever \( p_B = 1 - p_A \). If that is the case, the second term of SME disappears. The incumbency advantage effect depicted above is just a modified version of that discussed in section 2.4. Note that what is relevant to party B is how the opposition’s investment reacts to increases in \( K'_g \).

An intuitive explanation of the SME is presented next.

**Understanding strategic manipulation**

If party B were in power tomorrow, indirect benefits and costs derived from the increase in investment tomorrow as a result of an increase in investment today would cancel out. With probability \( (1 - p_B) \), he would be out of power, in which case the opposition chooses \( K''_g \) so as to maximize their own objective according to the rule \( h_A(K'_g) \).

When \( K'_g \) rises, the marginal increase in investment tomorrow is given by \( h_A(K'_g) \). This reduces utility by \(-u_c(c_A(K'_g))\) utils since consumption tomorrow decreases. Now, how much would the costs be if party B was taking the decision and had enough resources to invest exactly \( K''_g \)? Given party B’s investment rule, the level of resources that would make him invest exactly \( K''_g \) are precisely \( h_B^{-1}(K''_g) \), since \( h_B(h_B^{-1}(K''_g)) = K''_g \). The associated level of consumption in such a case would be

\[
\tilde{c}_B = f(h_B^{-1}(K''_g)) - K''_g - g_B(h_B^{-1}(K''_g)).
\]

Assuming that party B did have this resources, the utility-denominated costs of increasing investment would be \(-u_c(\tilde{c}_B)\). Then, the difference between the costs actually faced by the incumbent and the costs that would face was it the one making the decisions is \(-u_c(c_A(K'_g)) + u_c(\tilde{c}_B)\). This term appears because current and future governments disagree on how investment decisions should be taken whenever the incumbent loses an election.

Will this difference encourage or discourage investment today? If the opposition has a lower propensity to invest, \( K''_g < h_B(K'_g) \). Now, \( h_B \) is an increasing function so \( h_B^{-1}(K''_g) < h_B^{-1}(h_B(K'_g)) = h_B(K'_g) \), so the incumbent would need a lower amount of resources to invest \( K''_g \), the level chosen by the opposition. Since marginal utility is decreasing, this means that \(-u_c(c_A(K'_g)) + u_c(\tilde{c}_B) > 0\) so it is more costly for the other party to invest in terms of utility than it would be for the incumbent. This creates incentives for party B to increase the level of investment today (\( K'_g \)) which makes investments in the future less costly for the other party. In other words, if the opposition always invests less than what the incumbent would like, then the best strategy is to increase the level of public capital today.

If party B is more likely to win when out of power, so \( 1 - p_A > p_B \) the second term is non-zero. Moreover, if the value of initial resources was greater than the cost incurred in terms of foregone consumption when the other party is in power, there are incentives to increase party A’s investment in public capital. That is, it is optimal to manipulate the investment of the other party when out of power by increasing the amount of public capital today.\(^{24}\)

To summarize the discussion, there is a force pulling up current investment: the desire to manipulate A’s choices of public capital. But since A would, as a result, increase the expenditures on their public good, there is another force pulling investment down. In general, it is not clear which effect dominates. Would the competition between parties foster growth or slow it down?

\(^{24}\)The value of initial resources is given by: \( V_k(K''_g) = \beta u_c(c_B(h_A(K'_g)))f_k(K''_g) \).

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In order to answer this question we can characterize the Markov equilibrium under asymmetric political power for the example economy.

3.2 Growth and cycles

Consider an economy like the one described in section 2.5. While preferences and technology take the same form, the probability of re-election is now indexed by the incumbent’s affiliation. Without loss of generality, let $p_B > p_A$.

**Proposition 2** In a Markov-perfect equilibrium with asymmetric political power, the incumbent $B$ chooses:

$$\tau = \frac{1 + s_B}{2}, \quad g_B = \frac{1}{2}(1 - s_B)RK_g \quad \text{and} \quad h_B(K_g) = s_BRK_g,$$

where $s$ satisfies:

$$s_B = \beta^{\frac{1}{\rho}} R^{-\frac{1}{\rho}} \left\{ p_B + (1 - p_B) \left( \frac{1 - s_B}{1 - s_A} \right)^{\sigma} (1 - s_A) \frac{1}{2} + p_A \frac{s_A^{1-\sigma}}{s_B^{1-\sigma}} - (p_B + p_A - 1)(Rs_A)^{1-\sigma} \right\}. \quad (14)$$

$A$’s rules analogously defined (just replace the sub indexes).

**Proof** See Appendix 6.4.

As in the symmetric case, each incumbent would like to invest a constant proportion of resources each period and a tax rate that is time-invariant. However, in this case the marginal propensity and the tax rate depend on the identity of the party in power.

In principle, the party with higher political power can have a higher or a lower propensity to invest than the opposition depending on the values of parameters. For a set of reasonable parameters is is possible to show numerically that party $B$ undertakes greater investment, $s_A < s_B < s^\ast$, where $s^\ast$ denotes the saving propensity under a benevolent planner.\(^{25}\)

Despite the fact that $B$’s investment decision is closer than $A$’s decision to the one that would be undertaken by a planner, $B$’s saving propensity is lower than the first best, $s^\ast$. This is a result of the short-sightedness created by the political uncertainty and the disagreement over the composition of expenditures.

The political advantage of party $B$ creates greater incentives to invest when in power. The opposite occurs with party $A$; given its low chances of being in office next period, it is inclined towards unproductive expenditures.

This result implies that some caution should be taken when inferring unobservable characteristics from observable actions in at least two respects. First, the result was obtained assuming a source of asymmetry that is completely unrelated to how ‘competent’ candidates from different parties are. Suppose that we observe an economy where the average level of output and investment on infrastructure is higher under one of the parties, while spending on unproductive activities is lower. It would be incorrect to infer that candidates belonging to such party are more ‘capable’ or ‘efficient’ in dealing with economic issues. This observed outcome could be the result of equilibrium actions consistent with our model, where even though parties have identical investment technologies, political considerations make them choose different strategies. Second, note that in our model

\(^{25}\)By reasonable we mean values that are consistent with productivity levels observed in real economies, and for which the saving propensities and the probabilities of re-election satisfy desirable properties (in particular belonging to the [0,1] interval).
preferences over productive and unproductive spending are completely symmetric. In the absence of political advantages (as in section 2.3) both parties would invest equal proportions of available resources while spending the same amount on public goods. The difference in policy choices in the asymmetric case is a result of strategic considerations and does not rely on the assumption of differences in preferences. The nature of the economic cycle is then intrinsically different from the one found in traditional partisan cycle models, where one of the parties is assumed to derive higher utility from public goods than the other. Hence, it is inaccurate to label parties as being ‘right’ or ‘left’ just because they are observed to choose opposite tax and spending decisions.

An interesting feature of this model is that it delivers endogenous cycles in economic variables that are generated as parties that behave differently in equilibrium alternate in power. Even though there are no exogenous productivity shocks, output, investment, consumption, and taxes fluctuate in the long run.

From the government’s maximization problem, the evolution of public capital is:

$$K_g' = s_i RK_g.$$  

where $$s_i \in \{s_A, s_B\}$$ depends on the identity of the party currently in power.

When parties alternate in power, investment in public capital fluctuates following the political cycle. Every change in power implies a change in policy, and hence in the path of capital. Since the latter affects the productivity of the private sector, other macroeconomic variables (such as output and consumption) also fluctuate. Given that the result of any election cannot be predicted with certainty, the evolution of macroeconomic variables has a stochastic component, so political shocks are propagated into the real economy.

Recall that the optimal growth rate for this economy is given by $$\gamma^* = s^* R$$, where

$$\gamma^* > \gamma_B > \gamma_A, \quad \text{with } \gamma_i = s_i R.$$  

If the government was always to follow B’s optimal investment rule, the economy would grow faster than under A’s alternative, but this is still far from being optimal.

One does not need to rely on irrationality on the part of agents, myopic behavior, or unexpected actions taken by the government, for fluctuations to arise (as in Alesina’s model where cycles are generated by ‘surprise’ inflation). If a benevolent planner were in power instead, non-economic considerations would have no effect on the evolution of the economy. When there are two conflicting parties fighting for power, one of them with an advantage in the political dimension, the evolution of the economy is dramatically changed. Volatility in macro and micro economic variables is introduced, and welfare is reduced as a result of agents’ risk aversion.

I simulated the economy under specific parameter values, which are the same as the ones used in the symmetric example (I fixed $$\sigma = 0.9$$ here), except that the probabilities of re-election are allowed to differ. I used $$p_A = 0.5$$ and $$p_B = 0.7$$, so party A has no incumbency advantage, while party B wins 70 % of the time if in office at the time of the election. Figure 2 depicts the evolution of expenditures and capital for a simulated economy (the solid line). We can see how policy fluctuations are propagated into the real economy.

In the example explored in section 2.5, both parties had a propensity to invest large enough to ensure growth as long as their probability of re-election was greater than 0.3. That is no longer the case in the asymmetric environment. Even though party A has a fair probability of winning the election, its marginal propensity is much lower than before. Moreover, whenever A is in office the economy shrinks, since $$\gamma_A < 1$$. The (endogenous) short-sightedness is so large that induces this party to spend so much in unproductive goods that there is not enough investment to replace depreciated capital. In the figure, $$I_A$$ represents the evolution of investment if A happened to win.
every election (a very unlikely event). It is possible to see how the erosion of capital stock results in even lower levels of investment. Expenditures in public goods end up decreasing in size, as the tax base is reduced with the decrease in production resulting from poorer infrastructure. However, the overall trend is positive, because party B is in power more often.

In order to better understand the cycles, I have de-trended the investment and expenditure series (using an HP-filter) and plotted deviations from the trend for some periods in figure 3. The vertical lines represent changes of power.

The economy experiences booms when B is in office and short periods of recession when A happens to win an election. For example, consider what happens after t=6, when group B takes
office. There is an immediate jump in investment and a contraction of expenditures. This results in larger levels of public capital, and hence more production. Government investment grows over time (periods 6 to 12) and, as public capital becomes larger, the amount provided of the public good also increases. Group A gets into power in period 13 and we can see that the expenditures in public goods has a boost accompanied by a contraction of investment.

An empirical implication from this analysis is that we should observe a jump in unproductive expenditures when a party that doesn’t win often takes power together with a sudden decrease in investment.

4 Endogenizing political turnover

The previous sections described how the incentives faced by opposing parties facing re-election uncertainty affected their policy choices. We found that endogenous short-sightedness arises, where parties over-spend in public goods and under-invest in productive capital in equilibrium. When the political power is asymmetric, political cycles propagate into the real economy generating economic cycles. While the channel from politics to economics is well understood from that analysis, the model is silent on the other direction of causality. Can economic factors affect political turnover? Moreover, wouldn’t rational politicians choose policy so as to tilt probabilities in their favor?

In this section, I propose an environment where this issue can be addressed. In particular, I endogenize the re-election probabilities by adding a voting stage into the model. The determination of such probabilities is non-trivial and its characterization depends on the assumptions made on preferences over the political and economic dimensions. In some cases, it is possible to show that \( p \) is constant over time and independent of the stock of public capital (as assumed in the previous sections). In other cases, it depends on the identity of the party in power. By looking at a specific economy, the feedback from economic policy choices to re-election probabilities can be well understood. This is done in section 4.2.2.

4.1 Ideology and voting

The groups will alternate in power based on a political institution where “ideology” or other non-economic issues play a role. In particular, I use a “probabilistic-voting” setup (see Lindbeck and Weibull (1993)) in order to provide micro-foundations for political turnover: the probability of being in power next period is going to be endogenously determined via the electoral process.

Agents are assumed to differ, not only in their preferences over the composition of expenditures, but also in another dimension that is completely unrelated to the economic policy (religious or ethnic views, charisma of the politician, etc.). Preferences over this political dimension imply derived preferences over the candidates. The instantaneous utility is assumed to be separable in the consumption of public and private goods, and the political shocks are assumed to be additive. For an agent in group \( j \) we have

\[
\underbrace{u(c) + v(g^j)}_{\text{economic}} + \underbrace{\xi_i}_{\text{ideological}},
\]

were \( \xi_i \) summarizes the utility derived from political factors when group \( i \) is in power.

We can divide each period \( t \) into two stages: the Taxation Stage and the Election Stage.

At the Taxation Stage, the incumbent chooses \( \tau, g^A, g^B \) and \( K'_j \) knowing the state of the economy \( (K_g) \) and the distribution of the political shocks but not their realized values. Hence, policy is chosen under uncertainty. The probability of re-election can be calculated by forecasting how agents would take their voting decisions for different realizations of the shock.
After production, consumption and investment take place, $\xi'$ is realized. At the Election Stage, agents vote for the party that gives them higher expected lifetime utility. They need to forecast how the winner of the election would choose policy. The assumptions of rationality and perfect foresight imply that their predictions are correct in equilibrium (i.e., they cannot be systematically fooled by the government).

This timeline is represented below.

Figure 4: Time Line

Given the sequence of events and the separability assumption between the economic and the political dimensions, the only payoff-relevant state variable is the stock of public capital. In a Markov-perfect equilibrium, policy rules and voting decisions are functions of this state. Since there is no commitment and no retrospective voting, the values of $g^A$ and $g^B$ chosen by the incumbent are irrelevant in the voting decision. So is the history of ideology shocks, since there is no persistence.

The equilibrium objects that we are interested in are the investment rule of incumbent $i$, $h_i(K_g)$; his expenditure rules, $g^A_i(K_g)$ and $g^B_i(K_g)$, and the probability of re-election $p_i(K_g)$.

Taxation Stage

At this stage, the incumbent must decide on the optimal policy knowing that it will be replaced by a different policymaker with some probability. The maximization problem looks exactly like the one presented in section 3.1, with the exception that probabilities now depend on the state variable. To fix ideas, consider the problem faced by an incumbent from group $B$:

$$V_B(K_g) = \max_{g, K_g' \geq 0} u(c) + v(g) + \beta \{ p_B(K_g') V_B(K_g') + [1 - p_B(K_g') ] W_B(K_g') \}$$

$$(16)$$

$$W_B(K_g) = u(c_A(K_g)) + \beta \{ p_A(h_A(K_g)) W_B(h_A(K_g)) + [1 - p_A(h_A(K_g)) ] V_B(h_A(K_g)) \},$$

$$(17)$$

The optimality condition for expenditures is analogous to that in eq. (6), where the incumbent equates the marginal utility of public and private consumption of his group.

Even though parties represent their constituencies and have no derived value of being in office, they will try to manipulate the probability of being re-elected (which allows them to implement the desired policy in the future). This can be seen in the first-order condition with respect to $K_g'$,

$$u_c(c_B) = \beta \{ p_B(K_g') V_B(K_g') + [1 - p_B(K_g') ] W_B(K_g') + p_B(K_g') [V_B(K_g') - W_B(K_g')] \},$$

where $p_{B1}(K_g') = \frac{\partial p_B(K_g')}{\partial K_g'}$.

The only difference between this equation and eq. (12) is the last term. A change in investment today modifies the problem faced by voters, which in turn affects the probability of re-election.

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26 In this formulation, I follow Persson and Tabellini (2000) and assume that parties maximize utility net of shocks. The qualitative nature of results does not change if shocks are included, but the notation becomes much more cumbersome.
A rational incumbent realizes this and thus takes into account the effect of expanding $K'_g$ on its likelihood of winning. It is reasonable to expect that $V_B(K'_g) > W_B(K'_g)$, a party is better off while in power. However, the sign of $p_{B1}(K'_g)$ is, in principle, ambiguous. On the one hand, we expect that the richer the economy, the lower the political turnover (i.e. the probability of an incumbent losing the election decreases, so that the same party tends to stay in power for longer periods of time). This would imply that by increasing the level of productive capital, the incumbent can tilt the election in its favor, which creates an extra indirect benefit of having more $K'_g$. On the other hand, if $p_{B1}(K'_g)$ was negative, the last term represents an indirect cost to investment inducing the incumbent to spend resources on non-productive activities instead. Spending on public capital would decrease not only current consumption but also future utility via the indirect effect of reducing the probability of re-election.

**Election Stage**

At this stage, agents must decide which party to vote for.

The utility derived from political factors, $\xi_i$, has three components: an individual ideology bias (denoted by $\varphi^{ij}$), an overall popularity bias ($\psi$) and an incumbency advantage term ($\alpha$).\(^{27}\)

In particular,

$$\xi_i = (\psi + \varphi^{ij}) I_i + \alpha \tilde{I}_{i,i^-},$$

where $I$ and $\tilde{I}$ are indicator functions such that $I_B = 1$ and $I_A = 0$, $I_{i,i} = 1$ and $I_{i,i^-} = 0$, if $i \neq i^-$. The subindex $i$ denotes the identity of the party in power, and $i^-$ represents last period’s value of $i$.

The individual specific parameter $\varphi^{ij}$ measures voter $j$’s ideological bias towards the candidate from party $B$. The distribution of $\varphi^{ij}$ is assumed uniform and group-specific, \(^{28}\)

$$\varphi^{ij} \sim \left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J} \right], \text{ with } J = A, B.$$ 

These shocks are iid over time, hence ‘candidate specific.’ Each period, a given party presents a candidate and voters form expectations about the candidate’s position on certain moral, ethnic or religious issues, orthogonal to the provision of public goods. Examples are attitudes towards crime (gun control or capital punishment), drugs (i.e. whether to legalize the use of marijuana), immigration policies, abortion, etc.

A value of zero indicates neutrality in terms of the ideological bias, so agents only care about the economic policy, while a positive value indicates that agent $j$ prefers party $B$ over $A$. Since $\varphi^{ij}$ can take positive or negative values, there are members in each group that are biased towards both candidates. Therefore, individuals belonging to the same group may vote differently.

The parameter $\psi$ represents a general bias towards party $B$ at each point in time. It measures the average relative popularity of candidates from that party relative to those from party $A$. While the realization of $\varphi^{ij}$ is individual-specific, the value of $\psi$ is the same for all agents. Political scientists refer to this parameter as *valence*, referring to “issues on which parties or leaders are differentiated not by what they advocate but by the degree to which they are linked in the public’s mind with conditions or goals or symbols of which almost everyone approves or disapproves” (Stokes, 1992). More specifically, valence captures candidates’ personal characteristics such as honesty, leadership, integrity, charisma, trustworthiness, etc. Candidates with higher values of $\psi$ are preferable.

---

\(^{27}\)I build on the specification presented in Persson and Tabellini [2000]. Note that I am abusing notation since $\xi$ does not only depend on $i$, but also on $i^-$, $j$ and $J$.

\(^{28}\)This is a usual assumption in the literature. See Persson and Tabellini (2000).
The popularity shock is iid over time and can also take positive or negative values. Is distributed according to:

$$\psi \sim \left[ \frac{1}{2\Psi} + \eta, \frac{1}{2\Psi} + \eta \right].$$

A positive value for $\eta$ (the expected value of $\psi$) implies that candidates from party $B$ have an average (valence) advantage over those from the opposition. On the other hand, $\eta = 0$ implies that parties are symmetric, in the sense that their candidates are expected to be equally popular or charismatic.

As noted in the empirical literature, the party in power is more likely to win an election. Among the reasons, it has been argued that the public is familiar with the incumbent. This creates an incumbency advantage, which in the model is captured by the parameter $\alpha > 0$. Everything else equal, voters prefer an incumbent over a challenger.

Finally, agents are assumed to have perfect information about the candidates, so there are no informational asymmetries in this model. At the election stage, voters compare their lifetime utility under the alternative parties. The maximization problem of voter $j$ in group $A$ is given by

$$\max \left\{ V_A(K'_g, i), W_A(K'_g) + \psi' + \varphi'^j_A \right\};$$

where $V_A(K'_g, i) = V_A(K'_g) + \alpha \tilde{I}_{A,i}$ and $W_A(K'_g, i) = W_A(K'_g) + \alpha \tilde{I}_{B,i}$. If the incumbent today belongs to the same party (so $I_{BB} = 1$ and $I_{BA} = 0$) then there is some extra utility associated with the incumbency advantage effect. The maximization problem of an agent in group $B$ is analogously defined.

**Determination of probabilities**

Let’s turn now to the intermediate stage between taxation and voting. The shocks have not yet been realized and we are trying to determine the probability of re-election that each party will face.

Individual $j \in A$ votes for $B$ whenever the shocks are such that

$$V_A(K'_g, i) < W_A(K'_g) + \psi' + \varphi'^j_A.$$

We can identify the swing voter in group $A$ as the voter whose value of $\varphi'^j_A$ makes him indifferent between the two parties

$$\varphi'^i_A(K'_g) = V_A(K'_g, i) - W_A(K'_g, i) - \psi'.$$

Figure 5 illustrates this point (assuming $\psi = 0$, $\tilde{I}_{A,i} = 0$ and $\tilde{I}_{B,i} = 0$ for simplicity). The swing voter is found where the two solid lines intersect. All voters in group $A$ with $\varphi'^{j_A} > \varphi'^{i_A}(K'_g)$ also prefer party $B$ as can be seen in the graph.

The same type of analysis can be performed for agents in group $B$, so there is a swing voter in each group. The value of $\varphi'^j_B$ depends on the difference in utilities of having group $A$ vs group $B$ being in office, on the realization of the popularity shock, and on the identity of the party in power at the time of elections.

Given the assumptions about the distributions of $\varphi'^A$ and $\varphi'^B$ the share of votes for party $B$ is:

$$\pi_{iB} = \sum_j \mu^j P(\varphi'^{j_B} > \varphi'^i_A(K'_g))$$

$$= \frac{1}{2} - \sum_j \mu^j \varphi'^j_A(K'_g).$$
Figure 5: Utility as a function of \( \phi^J \)

Under majority voting, party \( B \) wins if it can obtain more than half of the electorate; that is, if \( \pi_{iB} > \frac{1}{2} \). This occurs whenever its relative popularity is high enough. There exists a threshold for the \( \psi \), denoted by \( \psi^*_i(K_g') \) such that \( B \) wins for any realization \( \psi > \psi^*_i(K_g') \). After performing some algebra using the expression above, we find that

\[
\psi^*_i(K_g') = \frac{1}{\phi} \left( \mu^A \phi^A \left[ V_A(K_g', i) - W_A(K_g', i) \right] + \mu^B \phi^B \left[ W_B(K_g', i) - V_B(K_g', i) \right] \right) .
\]

(18)

So the threshold is given by a weighted sum of the differences in the utility of the swing voter under each party. The weights depend on the dispersion in the ideology shocks and on the amount of supporters that each party has. The higher the heterogeneity within a constituency (\( \phi^J \)), the bigger the effect these factors have on the election outcomes. Also, the greater the number of individuals belonging to type \( J \), the stronger the group in the determination of the probability of re-election. Finally, note that the threshold depends on the level of public capital, though it is not clear in which direction. In principle, this level could increase or decrease with \( K_g' \).

Since \( \psi^*_i(K_g') \) depends on the realized value of \( \psi \), ex-ante the share of votes for party \( B \) \( (\pi_{iB}) \) is a random variable.

If \( B \) was currently in power, its probability of re-election would be given by:

\[
p_B(K_g') = P \left( \pi_{BB} > \frac{1}{2} \right) = P(\psi' > \psi^*_B(K_g')) ,
\]

which is equivalent to:

\[
p_B(K_g') = \frac{1}{2} + \Psi \left[ \eta - \psi^*_B(K_g') \right] .
\]

(19)

\( A \)'s probability of re-election is

\[
p_A(K_g') = \frac{1}{2} + \Psi \left[ \psi^*_A(K_g') - \eta \right] .
\]

(20)

Recall that \( \eta \) represents the popularity advantage of candidates from party \( B \) over those from party \( A \). So in principle, \( A \)'s probability of re-election decreases with \( \eta \).

The current level of consumption in private and public capital does not affect the voting decision (i.e. no retrospective voting). Voters do not 'punish' politicians/parties for their past behavior but decide instead in terms of their future policy choices.
4.2 Politico-economic equilibrium

We can now define a political equilibrium that takes into account agent’s voting decisions.

**Definition** A Markov-perfect equilibrium with endogenous political turnover is a set of value and policy functions such that:

i. Given the re-election probabilities, the functions $h_i(K_g)$, $g_i^B(K_g)$, $g_i^A(K_g)$, $V_i(K_g)$, and $W_i(K_g)$ solve incumbent $i$’s maximization problem.

ii. Given the optimal rules of the government, $p_A(K_g)$ and $p_B(K_g)$ solve eq. (19) and eq. (20).

In general $p_i(K_g)$ is a non trivial function of the state variable and requires the use of numerical methods for its characterization. However, when the political shocks are symmetric it is possible to show that the probability of re-election takes a very simple form: it is a constant. Moreover, as shown in Proposition 3, the resulting Markov-perfect equilibrium is symmetric.

4.2.1 Symmetric political shocks

Political shocks are symmetric under the following assumption.

**Assumption 1:** Both parties have identical political power $\mu^A\phi^A = \mu^B\phi^B \equiv \mu\phi$ and none exhibits a popularity advantage $\eta = 0$.

The term $\mu^J\phi^J$ affects the relative strength of one group over the other in the determination of re-election probabilities. If the supporters of party $A$ were more homogeneous than $B$’s supporters (i.e. $\phi^A > \phi^B$), then $A$’s chances of winning would be larger. Since $B$’s constituency would be less responsive to policy than to ideology it would have fewer swing voters and hence less political power. By assuming that $\mu^A\phi^A = \mu^B\phi^B$ we make sure that both groups are equally responsive to economic policy and have the same political power.

The distribution of the popularity bias towards group $B$ is symmetrically distributed around zero when $\eta = 0$. Therefore, no party has a popularity advantage over the other when this is assumed.

**Proposition 3:** If utility takes the form (15) and Assumption 1 holds, the Markov perfect equilibrium is symmetric:

i. $h_A(K_g) = h_B(K_g) \equiv h(K_g)$.

ii. $c(K_g) = f(K_g) + (1 - \delta)K_g - g(K_g) - h(K_g)$.

iii. $g_A^A(K_g) = g_B^B(K_g) \equiv g(K_g)$ and $g_A^B(K_g) = g_B^A(K_g) = 0$.

iv. $p_A(K_g) = p_B(K_g) = p \ \forall K_g$, with $p = \frac{1}{2} + \Psi\alpha$.

**Proof** See Appendix 6.5.

When the political shocks are symmetric both groups face the same probability of re-election in equilibrium ($p_A = p_B$). The inter-temporal maximization problem does not depend on the identity of the party in power, so both invest following the same rule. The amount of expenditures is also equivalent, resulting in a deterministic path of taxes and consumption.

Symmetric political shocks translate one to one into symmetric political power. The characterization of the Markov-perfect equilibrium is analogous to that presented in section 2.3. The qualitative effects are equivalent, the only difference now being that $p$ is determined in equilibrium.
It is worth noticing that while the probabilities are endogenously determined, they do not depend on the state variable \( (K_g) \), the rate of time preference or the technology parameter. According to this, as long as two economies are ideologically equivalent, power should alternate between the two groups with the same frequency. An immediate empirical prediction is that the degree of political turnover should not be affected by the economic conditions of a country. Alesina, Ozler, Roubini, and Swagel (1992) provide evidence consistent with this finding by reporting that changes in growth do not affect political instability.

This result should not be interpreted as implying that there is no ‘opportunistic behavior’. In equilibrium, it is optimal to set current policy so that the marginal effect of the last unit invested on the probability of re-election is actually zero. This equality does not hold off-equilibrium. There will be some strategic behavior as well. A party knows that when it is out of power, the opposition spends resources on a public good from which it derives no utility. Since part of this is financed through taxes, and taxes depend on the stock of public capital, there are incentives to invest so as to ‘tie the hands of successors.’ Therefore, the disagreement over the composition of public goods and the possibility of being replaced by a party with a different objective function can result in inefficient levels of investment.

In section 2.5, we found that the propensities to invest get closer to the planner’s optimal value when the incumbent is more likely to be re-elected. With the help of Proposition 3, we can now link increases in the probability to fundamentals of the economy: political turnover is going to be low whenever there is a strong incumbency advantage (\( \alpha \) is high) or a small degree of ‘political polarization’ (\( \Psi \) is low) in a country.

We can ask ourselves whether societies with extreme political views are expected to exhibit higher or lower growth than more homogeneous ones. The degree of heterogeneity is captured by the parameter \( \Psi \): the lower its value, the higher the variability in the political shock and hence the less likely it is for any incumbent to be re-elected. If candidates are very heterogeneous, alternation of power is more frequent, which makes policymakers more short-sighted and reduces their incentives to invest. Hence, the model predicts that the more heterogeneous the society in the non-economic dimension, the smaller the observed growth rate. In order to test this implication, we need an observable measure of ‘heterogeneity’. Easterly and Levine (1997) provide one, by constructing a variable that captures the degree of ethnic and cultural fractionalization in a country. Their main finding, a negative relationship between fractionalization and growth, is consistent with the prediction.

This model also provides a rationale for the positive correlation found between ethnically (or culturally) divided societies and large ratios of public expenditures to GDP. As an example, consider the African case, where the fraction of GDP devoted to public consumption is 0.16 (0.164 for Sub-Saharan Africa and 0.12 for North-Africa). In contrast, the corresponding number for OECD countries is 0.07 and 0.06 for East Asia. Artadi and Sala-i-Martin (2003) estimate that if Africa had had a level of public spending similar to that of the OECD over the last 40 years, its annual growth rate would have been 0.40 percentage points larger.

The deep political myopia generated by these ethnic divisions also causes incentives for policymakers to substitute away from productive investments and improvements in education or public health (affecting human capital) according to the model, all of which implies lower growth rates. Life expectancy in Africa in 1960 was just above 40 years whereas it was around 67 and 62 years for OECD and East Asia respectively. This numbers began to deteriorate in the late 1990s due to the adverse impact of AIDS and the prevalence of malaria. If Africa had no malaria over the last four decades, its annual growth rate would have been 1.25 percentage points larger according to Artadi and Sala-i-Martin’s estimations.
Botswana is among the few success stories in Africa, with a growth rate comparable to South Korea’s. Interestingly, it is one of the most (ethnically) homogeneous populations in Africa. Kenya and Ghana on the other hand, having experienced a rapid rotation of ethnic coalition governments during the 70s and 80s, chose a sequence of growth retarding policies instead.

4.2.2 Asymmetric political shocks

In this section, I relax the assumptions that gave rise to a symmetric equilibrium. Unfortunately, there is no equivalent to Proposition 3 under asymmetry. In general, the probability of re-election will depend on the state variable in the Markov-prefect equilibrium.

We can derive the GEE under asymmetric political shocks following the same steps presented in section 3.1. The difference being that, as $p$ is not necessarily constant, we have to keep track of all strategic effects that arise through manipulation of re-election probabilities.

$$MC = \beta\{DE + EMB + SME + \text{New MPE}\};$$ (21)

This GEE is equivalent to that derived in section 3.1 under exogenous probabilities (see eq. 13) with the exception that now $p$ is a function of public capital and that there is an extra term (which involves new strategic effects). The MPE reflects the opportunistic behavior of the party in power ($B$ for expositional ease):

$$MPE = p_B(K'_g)[V_B(K'_g) - W_B(K'_g) - \beta h_A(K'_g)[1 - p_A(K''_g) - p_B(K''_g)] \times$$
$$p_A(K''_g) \{V_B(K''_g) - W_B(K''_g)\}, \text{ with } K''_g = h_A(K'_g).$$

The first term incorporates marginal increases (or decreases) in the probability of re-election due to increases in initial resources.

The second term takes into account how changes in today’s public capital alter the probability to recovering power in the future, by affecting tomorrow’s investment decision by the opposition. That is, how the opposition would want to change their probability of re-election tomorrow. If party $B$ has the same probability to win an election whether in or out of power, $1 - p_A(h_A(K'_g)) = p_B(h_A(K'_g))$, and the term disappears.

The following two assumptions will allow us to characterize the equilibrium with endogenously determined re-election probabilities.

**Assumption 2:** Both parties have identical political power $\mu^A\phi^A = \mu^B\phi^B \equiv \mu$ but party $B$ has a popularity advantage $\eta > 0$.

Recall that $\psi$ represents the popularity of party $B$ relative to party $A$. If $\eta > 0$ party $B$ has an average popularity advantage over party $A$. This will in principle tilt $B$’s probability of winning an election in its favor.

**Assumption 3:** Let utility be logarithmic $\sigma = 1$, and normalize the utility function over public goods so that $v(g^J) = \max\{0, \log(g^J)\}$. Suppose that there is full depreciation, $\delta = 1$.

The utility function over public goods is zero for small values of $g$ and increasing and concave for $g \geq 1$. Although this assumption is necessary from a technical point of view, it captures the idea that agents don’t value expenditures that are too low. For example, there is a minimum size...
that a park should have in order to provide any consumption value.\(^{29}\) The kink in \(v\) will result in expenditures being discontinuous, as the following lemma shows.

**Lemma 1:** Let \(K_g^*\) be the value of capital for which the chosen level of unproductive expenditures is \(g(K_g^*) = 1\). Then,

\[
g(K_g) = \begin{cases} 
0 & \text{if } K_g \leq K_g^* \\
1 & \text{if } K_g > K_g^*
\end{cases}
\]

**Proof:** The first equality holds by definition. The second one, follows from the assumptions on utility (this will become clearer after proposition 4).

As a benchmark, let’s study first the efficient allocations chosen by a benevolent planner under Assumption 3.

**Lemma 2:** Under the welfare function described in section 2.2, the optimal investment rule is \(h^*(K_g) = \beta \theta RK_g^\theta\), consumption allocations are:

i. If \(K_g \leq K_g^*\)

\[g^*(K_g) = 0 \quad \text{and} \quad c^*(K_g) = (1 - \beta \theta) RK_g^\theta,\]

ii. If \(K_g > K_g^*\)

\[g^J*(K_g) = \frac{\zeta^J}{1 + \zeta^A + \zeta^B (1 - \beta \theta) RK_g^\theta}, \quad c^*(K_g) = \frac{1}{1 + \zeta^A + \zeta^B (1 - \beta \theta) RK_g^\theta},\]

and \(K_g^* = \left[\frac{(1 + \zeta^A + \zeta^B^3 (1 - \beta \theta)^3}{\zeta^A \zeta^B (1 - \beta \theta)}\right]^{1/\theta}\).

**Proof:** See Appendix 6.6.

Hence, when there is too little infrastructure in the economy (that is, \(K_g\) is very low), the planner will not have enough resources to provide the minimum level of unproductive expenditures that agents would value. In that case, it will rather set \(g^J = 0\) and use the extra resources to provide more consumption. Interestingly, savings are unaffected by the discontinuity: the savings function is continuous and differentiable for all values of \(K_g^*\).\(^{30}\)

We can now turn to the analysis of the political equilibrium under asymmetric shocks. It is useful to define \(K_g^{ss} = min\{K_g^{ssA}, K_g^{ssB}\}\), as the steady state value of public capital that would be attained if the party with the lower marginal propensity to invest happened to be in power long enough. Also, let \(K_g^*\) be the level of capital that makes any incumbent indifferent between providing the consumable public good \(g\) and not providing it. As long as the stock of capital is larger than this value (and smaller than \(K_g^{ss}\)), the solution to the government’s problem will be interior. The reason being that the party in power will choose \(K_g^J > K_g\), \(\forall K_g \in (K_g^*, K_g^{ss})\). Since the value function is increasing in capital, if it is worth providing the good today, it will be worth providing it tomorrow. In other words, once the economy becomes rich enough parties will choose to spend on at least one type of the consumable public good \(g_i\). Proposition 4 characterizes the political equilibrium in this case.

**Proposition 4:** Suppose that \(K_g \in (K_g^*, K_g^{ss})\). Under the assumptions 2 and 3, there exists an asymmetric differentiable Markov equilibrium where incumbent B chooses:

\(^{29}\)If we do not normalize \(v(0) = 0\), utility when out of power would be infinitely low making welfare comparisons impossible.

\(^{30}\)This is a result of the separability between the utility of consumption of private and public goods.
\[ g^B(K_g) = \frac{1}{2}(1-s_B)RK^\theta_g, \quad \tau_B = \frac{1+s_B}{2} \quad \text{and} \quad h_B(K_g) = s_BRK^\theta_g, \]

where the propensity \( s_B \) satisfies

\[ s_B = \theta \beta \left[ \frac{1+p_B + 2\theta \beta (1-p_B - p_A)}{2 - \theta \beta (2\bar{p}_A -(1-p_B))} \right], \tag{22} \]

and where \( \bar{p}_A \) and \( \bar{p}_B \) are constants such that the next two equations hold:

\[ \bar{p}_A = \frac{1}{2} + \Psi (\alpha - \eta) + \frac{3\Psi}{2(1 - 2\alpha \theta \Psi)} \left[ \ln \frac{(1-s_A)}{(1-s_B)} + \frac{\beta \theta}{1-\theta \beta} \ln \frac{s_A}{s_B} \right], \]

\[ \bar{p}_B = 1 - \bar{p}_A + 2\alpha \Psi. \]

\( A \)'s rules analogously defined (just replace the sub indexes).

**Proof** See Appendix 6.7.

When \( K_g > K^*_g \), both parties would like to devote resources to provide public goods. But since they disagree on the composition of expenditures, strategic manipulation of future policy through investment in public capital is now optimal. Their behavior is determined by equation (21).

Note that the re-election probabilities are jointly determined with the propensities to invest. We have a system of four non-linear equations in four unknowns \((\bar{p}_A, \bar{p}_B, s_A \text{ and } s_B)\). The feedback from policy decisions to political turnover is clearer than it was in the symmetric case (where \( \eta = 0 \)).

If \( s_A > s_B \), forward-looking voters realize that candidates from party \( A \) spend relatively more resources in productive activities than the opposition. This increases \( A \)'s chances of re-election. On the other hand, higher investment in public capital implies more taxes and lower consumption than under party \( B \). This force pushes down the likelihood of \( A \) being re-elected. Overall it is not clear whether \( \bar{p}_A \leq \bar{p}_B \) when \( s_A > s_B \). For a set of reasonable parameters, is is possible to show numerically that party \( B \) invests more and it is re-elected more often than the other party, \( s_A < s_B < s^* \) and \( \bar{p}_A < \frac{1}{2} < \bar{p}_B \).

When \( \eta > 0 \), party \( B \) has an advantage over \( A \) because positive realizations of the popularity shock are more likely. This tilts the utility of all voters in \( B \)'s favor, which in turn increases its probability of re-election. From the GEE, this creates incentives to invest more. The opposite occurs with party \( A \): given it’s low chances of being in power next period, it is inclined towards unproductive expenditures. In this example, we see a *virtuous circle*: if individuals believe that one party has on average ‘better’ candidates (on aspects orthogonal to the management of economic policy), the strategic effects imply that they will indeed behave ‘better’ in choosing policy (spend less on unproductive activities).

When \( K_g \leq K^*_g \), there is so little infrastructure that it is not optimal for any party to spend on goods that are unproductive, so they set \( g^A = g^B = 0 \). The relevant question is what happens to investment in that case. The answer depends on how far from the threshold public capital is. Consider the period before reaching \( K^*_g \), that is \( K_g \leq K^*_g \) but \( K^*_g > K^*_g \). Then, the utility of an incumbent type \( B \) would be:

\[ \max \{ \ln(c) + \beta [p_B(K_g')V_B(K_g') + (1-p_B(K_g'))W_B(K_g')] \}, \]

\[ 31 \text{By reasonable we mean values that are consistent with productivity levels observed in real economies, and for which the saving propensities and the probabilities of re-election satisfy desirable properties (in particular belonging to the } [0,1] \text{ interval).} \]
From Proposition 4, we know how policy looks like from tomorrow on, and that one type of public good is provided, \( g^J > 0 \) for some \( J \). By taking first-order conditions, we find that the current policymaker would like to set \( g_A = g_B = 0 \) today, but \( K'_g = h_B(K_g) = s_B R K^\theta_g \) with \( s_B \) defined according to equation 22 (and \( s_A \) analogously defined). This implies that in the period before reaching \( K^*_g \) both parties invest as if the good was provided. The result is intuitive by looking at the GEE, where the disagreement about future expenditures and investment results in the rules described in Proposition 4.

If, on the other hand, investment is not large enough to reach the threshold in one period (but it can reach it in two periods): \( K'_g < K^*_g \) and \( K''_g > K^*_g \), utility would be instead:

\[
\max \{\ln(c) + \beta \left( \bar{p}_B(K'_g)[\ln c_B(K'_g) + \beta(p_B V_B(s_B R K'_g) + (1 - p_B) W_B(s_B R K'_g))]) + (1 - \bar{p}_B(K'_g))[\ln c_A(K'_g) + \beta((1 - p_A) V_B(s_A R K'_g) + p_A W_B(s_A R K'_g))]\}.
\]

Clearly, there are no expenditures on unproductive public goods in this case either. After performing some algebra, we find that the first-order condition with respect to \( K'_g \) is almost the same as eq. (21), the difference being that there is no ‘disagreement effect’: \( \text{DE}=0 \). Since the public good is not provided next period either, the incentives to manipulate the provision of the good disappear. However, investment is still inefficiently low (there is under investment). To understand why this happens, we need to look at the behavior of both parties in the future. Suppose that \( B \) (the party that wins more often) is currently in power. If they were to lose the next elections, party \( A \) would not waste resources on the provision of \( g^J \), but it would choose a level of investment that is too low from \( B \)’s standpoint. Foreseeing this, \( B \) would choose public capital today in order to manipulate \( A \)’s future investment to a level closer to \( B \)’s preferred level. It is the disagreement about tomorrow’s level of investment (which is driven by a disagreement on expenditures two periods from now) what causes current policy to differ with the identity of the policy maker. It is interesting to note though, that both parties will choose a level of \( K'_g \) closer to the planner’s choice. This is a result of the DE being zero in the incumbent’s Euler equation.

The following corollary summarizes the evolution of capital in the asymmetric environment.

**Corollary:** Consider an economy with \( K_{g0} < K^*_g \). Then capital increases at a fast rate but subject to short-run fluctuations (following the political cycle) until \( K^*_g \) is reached. At this point the average growth rate is reduced, but still positive, until it converges to an ergodic set. Once there, capital fluctuates around a constant mean. Moreover, the economy is dynamically inefficient.

Hence, we should observe rapid growth at early stages of development, and no unproductive expenditures (parks, regional transfers, public television, etc.). When the economy becomes rich enough, there are enough resources that these goods can be afforded. Unfortunately, opposing interests come into play which results in slower growth and greater volatility in policy. As in the example under exogenous—and asymmetric—political turnover (see section 3.2), we also find endogenous cycles in economic variables generated by the political cycle. The evolution of public capital when \( K_g > K^*_g \) follows:

\[
K'_g = s_i R K^\theta_g,
\]

where \( s_i \in \{s_A, s_B\} \) depends on the identity of the party currently in power. Assuming that \( \theta < 1 \), due to decreasing returns to public capital, this economy will not exhibit long run growth. If the government were always to follow \( B \)'s optimal investment rule, \( K_g \) would evolve according to the upper line in Panel A of Figure 6, converging eventually to \( K^*_{gB} \). If \( A \)'s rule was followed instead, not only would the steady state be lower \( (K^*_{gA} < K^*_{gB}) \) but convergence would take place at a slower pace. Under political uncertainty the evolution of capital is stochastic. A possible path, with a starting value of \( K_{g0} > K^*_g \), is represented by the arrows in Panel A of Figure 6.
Eventually, the economy reaches an ‘ergodic set’ in which public capital only takes values belonging to the interval $(K_{gA}^{ss}, K_{gA}^{ss})$. Once there, all macroeconomic variables evolve cyclically. This is illustrated in Panel B of Figure 6, which depicts a series of investment for a simulation of this economy.

It is interesting to analyze how the popularity bias affects the evolution of the economy. Consider an increase in the value of $\eta$. Everything else constant, the probability of re-election of party $B$ rises. Therefore, if the incumbent belongs to that group, it is more likely to be succeeded by a candidate of his own type and has incentives to invest more resources in productive activities. If $A$ was in power instead, a higher value of $\eta$ would decrease this party’s probability of re-election, so the short-sightedness would be strengthened, resulting in a propensity to invest even further away from first best. Figure (7) illustrates this observations numerically.

When $\eta = 0$, both parties are completely symmetric. The point at which the two curves intersect
represents the symmetric solution, similar to that analyzed in section 4.2.1 (with the difference that technology is Cobb-Douglas and $\sigma = 1$ in this case). As $\eta$ increases, the marginal propensity to invest of type $A$ falls below the symmetric level, while that of type $B$ lies above that value. Hence, the gap in the marginal propensities to invest is widened when $\eta$ increases. The model predicts that the greater the popularity bias towards one of the parties, the larger the volatility of macroeconomic variables when there is a change of government. On the positive side, the rise in the probability of re-election of $B$ causes public capital to grow faster on average and increases its mean in the long run.

5 Concluding Remarks

I present a model where disagreements about the composition of spending results in implementation of myopic policies by the government: investment in infrastructure is too low while spending on public goods is too high. Groups with conflicting interests try to gain power in order to implement their preferred fiscal plan. Since there is a chance of being replaced by the opposition, strategic manipulation of the level of investment is optimal. In particular, the incumbent invests so as to restrict spending in the future and to maximize his probability of reelection. In contrast to previous models, the degree of ‘impatience’ of the government is endogenous here and depends on preferences and technology.

The model provides a formal motivation for the empirical findings of Easterly and Levine (1997) within a dynamic framework with rational agents. I show how preferences over the political dimension (intensity of ideology), which is completely unrelated to economic issues, may affect the optimal level of spending and investment and hence macroeconomic variables. The higher the polarization of ethnic groups, for example, the lower the growth rate. Moreover, when one group has an advantage in the political dimension (i.e., its candidates are more “popular”) policy is not only inefficiently low but also fluctuates. The group that wins elections more often becomes less impatient and finds it optimal to choose a share of investment to GDP closer to the one resulting under a benevolent planner. Even though both groups have symmetric preferences over the size of spending and investment, in equilibrium the group with the disadvantage tends to spend more and invest less. The political cycle is propagated onto the real economy, so as parties alternate in power, different policies are implemented. In equilibrium, macroeconomic variables fluctuate even in the absence of economic shocks.

The forces that drive short-sightedness are disagreement of consecutive governments, political uncertainty, and the induced lack of commitment. Therefore, a way to improve the performance of democratic institutions would be to try to reduce the effect of either of these factors. Welfare analysis comparing different constitutional setups, e.g., legislating limits on how unevenly public goods can be spent, are interesting extensions.

One may conjecture from this analysis that political competition may have a negative impact on growth and welfare when checks and balances are absent. Establishing a democracy is just not enough to ensure growth. If there was an institution that allowed both parties to take part in the decision process, the inefficiency could be reduced or even eliminated. For example, consider an independent Congress where both groups had representation. Depending on each group’s bargaining power, positive amounts of both public goods could be provided every period, thus reducing the ‘disagreement effect’ less important. In order for the institution to have any impact, it needs to be de-facto independent from the government. In many under-developed countries, the Congress is directly controlled by the executive power, so checks and balances are very weak and the problem of low-growth remains. Analyzing this conjecture is something I want to do in future research.
An alternative is to allow for coalition governments, which I ruled out by assumption here. Everyone would be better off if the political parties could agree to follow the same investment policy, and thus eliminate the cycles. The economy would approach a steady state more quickly and with more infrastructure, thus allowing a higher level of consumption. Moreover, agents of both types could consume some of their preferred public good every period. Since utility is concave, smoothing would increase welfare. Notice that the resulting policy will not necessarily be the one that a benevolent planner would choose: its level would depend on the relative bargaining power of each party. Formally specifying and solving a dynamic bargaining game—subject to a Markov-perfect refinement—is, however, a nontrivial extension (though one I am also planning).

I argued that the higher the incumbency advantage, the weaker is the short-sightedness, leading to the higher levels of investment (and growth). This does not imply that increases in this parameter necessarily make agents better off, since the group that is out of power will not consume the public good—for which it derives utility—for longer periods of time. Therefore, it is not clear whether a technology for increasing the incumbency advantage (say, by allowing campaign contributions) would be welfare-improving.

Another possible extension would be to introduce a more realistic private sector (where agents can accumulate capital as well), so that the model becomes more reminiscent of developed economies, and quantify the degree of the dynamic inefficiency. Still another possibility is to assume that the disagreement revolves around the provision of regional public capital (so that only one of the groups benefits from the investment), which would generate cycles in the development of different areas within a country.
6 Appendix

6.1 Derivation of the GEE (Symmetric Case)

The FOC with respect to $K_g'$ is:

$$u_c(f(K_g)) + (1 - \delta)K_g - g(K_g) - K_g' = \beta \left\{ pV_K(K_g') + (1 - p)W_K(K_g') \right\}. \tag{23}$$

Denote the rule that solves this functional equation by $h(K_g)$.

We can obtain $V_K(K_g)$ by differentiating equation 4:

$$V_K(K_g) = u_c(c(K_g))[f_K(K_g) + (1 - \delta) - g_K(K_g) - h_K(K_g)] + \beta h_K(K_g)[pV_K(h(K_g)) + (1 - p)V_K(h(K_g))].$$

Using equations 6 and 23 the expression can be simplified to:

$$V_K(K_g) = u_c(c(K_g))[f_K(K_g) + (1 - \delta)]. \tag{24}$$

To find $W_K(K_g)$ differentiate equation 5:

$$W_K(K_g) = u_c(c(K_g))[f_K(K_g) + 1 - \delta - g_K(K_g) - h_K(K_g)] + \beta h_K(K_g)[pW_K(h(K_g)) + (1 - p)V_K(h(K_g))]. \tag{25}$$

We can use eq. (23) to solve for $W_K(h(K_g))$:

$$W_K(h(K_g)) = \frac{1}{1 - p} \left\{ \frac{u_c(c(K_g))}{\beta} - pV_K(h(K_g)) \right\}. \tag{26}$$

Replacing eq. (26) into eq. (25) and simplifying:

$$W_K(K_g) = u_c(c(K_g))[f_K(K_g) + 1 - \delta - g_K(K_g)] + \frac{1 - 2p}{1 - p} \beta h_K(K_g) \left[ -u_c(c(K_g)) + V_K(h(K_g)) \right].$$

Replacing eq. (24) in the expression above and updating one period we obtain:

$$W_K(K_g') = u_c(c(K_g'))[f_K(K_g') + 1 - \delta - g_K(K_g')] + \frac{1 - 2p}{1 - p} h_K(K_g') \left[ -u_c(c(K_g')) + \beta u_c(c(K_g''))[f_K(K_g'') + (1 - \delta)] \right], \tag{27}$$

where $K_g'' = h(h(K_g))$. Finally, we can update eq. (24) one period and replace it together with eq. (27) into eq. (23) to obtain the GEE:

$$u_c(c(K_g) - \beta u_c(c(K_g'))[f_K(K_g') + 1 - \delta] = \beta \left\{ -(1 - p)u_c(c(K_g'))g_K(K_g') + (1 - 2p)h_K(K_g') \left[ -u_c(c(K_g')) + \beta u_c(c(K_g''))[f_K(K_g'') + (1 - \delta)] \right] \right\}. \tag{28}$$

6.2 Proof of Proposition 1

Guess $h(K_g) = sRK_g$, so $c = (1 - s)RK_g - g$. The marginal utilities of consumption are $u_c(c) = c^{-\sigma}$ and $v_g(g) = g^{-\sigma}$. From eq. (6), $c = g = \frac{1}{2}(1 - s)RK_g$. Replacing this into eq. (8) and simplifying we obtain eq. (2.5). Since that expression is independent of the state variable, $K_g$, we verify that the propensity to invest, $s$, is indeed a constant. The tax level can be obtained from the government's resource constraint, $g + h(K_g) = \tau f(K_g)$. 34
6.3 Derivation of the GEE (Asymmetric Case)

The FOC with respect to $K'_g$ is:

$$u_c(c_B(K_g)) = \beta \{ p_B V_{BK}(K'_g) + (1 - p_B) W_{BK}(K'_g) \}$$

Denote the rule that solves this functional equation by $h_B(K_g)$.

When the probability of re-election is different for the two parties, the functional form of their value functions is in principle different ($V_A \neq V_B$ and $W_A \neq W_B$). For expositional purposes, let’s focus on the problem of party $B$ (I’ll abstract from the subindexes in its value function).

We can obtain $V_K(K_g)$ by differentiating equation 4 and simplifying:

$$V_K(K_g) = u_c(c_B(K_g))[f_K(K_g) + (1 - \delta)].$$

(29)

To find $W_K(K_g)$ differentiate equation (11):

$$W_K(K_g) = u_c(c_A(K_g))c_{AK}(K_g) + \beta h_{AK}(K_g) \{ p_A W_K(h_A(K_g)) + (1 - p_A) V_K(h_A(K_g)) \},$$

(30)

where $c_{AK}(K_g) = f_K(K'_g) + 1 - \delta - g_{AK}(K_g) - h_{AK}(K_g)$.

We can use eq. (6.3) to solve for $W_K(h_B(K_g))$:

$$W_K(h_B(K_g)) = \frac{1}{1 - p_B} \{ u_c(c_B(K_g))/\beta - p_B V_K(h_B(K_g)) \}.$$  \hspace{1cm} (31)

In order to replace the equation above in eq. (30) we need the value function to be evaluated in the investment choice of government $A$, $W_K(h_A(K_g))$. If the functions $h_i$ are invertible, we can achieve this by evaluating eq. (31) in $x = h_B^{-1}(h_A(K_g))$. Assuming that $h_A(K_g)$ and $h_B(K_g)$ are invertible we derive:

$$W_K(h_A(K_g)) = \frac{1}{1 - p_B(h_A(K_g))} \{ u_c(\tilde{c}_B(K_g))/\beta - p_B V_K(h_A(K_g)) \},$$

(32)

where $\tilde{c}_B(K_g) = f(h_B^{-1}(h_A(K_g))) + (1 - \delta) h_B^{-1}(h_A(K_g)) - g_B(h_B^{-1}(h_A(K_g)) - h_A(K_g)$.

Replacing eq. (32) into eq. (30) and simplifying:

$$W_K(K_g) = u_c(c_A(K_g))c_{AK}(K_g) + \frac{h_{AK}(K_g)}{1 - p_B} \{ p_A u_c(\tilde{c}_B(K_g)) + \beta (1 - p_A - p_B) V_K(h_A(K_g)) \}.$$  \hspace{1cm} (33)

Now, update eq.(33) by substituting $K_g$ with $K'_g = h_B(K_g)$ and replace the equation back into the first-order condition, eq.(6.3). After some algebra, and defining $K''_g = h_A(h_B(K_g))$, we find the final expression for the GEE when shocks are asymmetric:

$$u_c(c_B(K_g)) = \beta \{ p_B u_c(c_B(K'_g))[f_K(K'_g) + 1 - \delta] + [1 - p_B] u_c(c_A(K'_g))c_{AK}(K'_g)$$

$$+ h_{AK}(K'_g) [ p_A u_c(\tilde{c}_B(K''_g)) + \beta (1 - p_A - p_B) u_c(c_B(K''_g))[f_K(K''_g) + 1 - \delta] \}.$$  

(34)

6.4 Proof of Proposition 2

Guess $h_i(K_g) = s_i RK_g$. This implies that $c_i = (1 - s_i) RK_g - g_i$. The marginal utilities of consumption are $u_c(c) = e^{-c}$ and $v_g(g) = g^{-\sigma}$. From eq. (6), $c_i = g_i = \frac{1}{\tau}(1 - s_i) RK_g$. Replacing this into eq. (34) and simplifying we derive eq. (14). Since that expression is independent of the state variable, $K_g$, we verify that the propensity to invest, $s_i$, is indeed a constant. The tax level can be obtained from the government’s resource constraint, $g_i + h_i(K_g) = \tau_i f(K_g)$.  

35
6.5 Proof of Proposition 3

I will first show that when both parties face the same probability of re-election, they choose symmetric policy functions. Then I will show that whenever the policy functions are independent of the party in power (i.e., symmetric), voting decisions result in constant and symmetric probabilities of re-election.

Part 1

It is straightforward to show that if both parties face the same probability of re-election \((p)\) it is best for them to choose symmetric policy functions. Inspection of equations 16 and 17 reveals that they face exactly the same maximization problem when in power. The value functions when out of power are also identical. Therefore, they will choose the same investment, expenditure and taxation levels.

Part 2

Under assumption 1, the probabilities of re-election are:

\[
p_B(K'_{g}) = \frac{1}{2} - \Psi \psi^*_B(K'_{g}),
\]

\[
p_A(K'_{g}) = \frac{1}{2} + \Psi \psi^*_A(K'_{g}),
\]

where

\[
\psi^*_i(K'_{g}) = \frac{1}{2} \left( \left[ V_A(K'_{g}, i) - W_A(K'_{g}, i) \right] + \left[ W_B(K'_{g}, i) - V_B(K'_{g}, i) \right] \right).
\]

I will assume that both parties follow the same policy rules while in power and guess a constant probability of re-election:

\[
p_A(K_{g}) = p_B(K_{g}) \equiv p.
\]

Since policy rules are symmetric, \(c_A(K'_{g}) = c_B(K'_{g})\) and \(g_A(K'_{g}) = g_B(K'_{g})\). Therefore, \(V_A(K'_{g}, A) - V_B(K'_{g}, A) = \alpha\) and \(W_B(K'_{g}, A) - W_A(K'_{g}, A) = \alpha\). This implies that \(\psi^*_A(K'_{g}) = \alpha\). Using the same reasoning we can see that \(\psi^*_B(K'_{g}) = -\alpha\). Replacing these into eqs. 36 and 35, we verify that the probabilities of re-election are constant:

\[
p_B(K'_{g}) = p_A(K'_{g}) = \frac{1}{2} - \Psi \alpha.
\]

Therefore, when the government follows a symmetric rule, the probabilities of re-election are symmetric.

6.6 Proof of Lemma 2

The maximization problem is the same as in section 2.2, with the exception that the function \(v\) has a kink at \(g = 1\). The first thing to notice is that under this utility function the intertemporal decision is unaffected. Hence, \(h(K_{g})\) can be determined from the first-order condition in section 2.2. Since the value function is increasing in \(K_{g}\), once capital is bigger than the threshold that makes \(g^*(K_{g}) > 1\), a positive amount of the public good will always be provided. The optimal consumption and provision of \(g\) can be solved from eq.(2). If \(K_{g} \leq K^*_g\), then the planner chooses not to provide the good and consumption can be obtained from the budget constraint. The period utility with \(g = 0\) equals \(u_0 = \ln(1 - \beta \theta) RK^0_{g}\), while that with a positive provision is \(u_1 = \sum \ln \frac{1}{1 + \zeta_{i+1} + \zeta} (1 - \beta \theta) RK^0_{g} + \ln \frac{1}{1 + \zeta_{i+1} + \zeta} (1 - \beta \theta) RK^0_{g} \). The threshold is the value of \(K_{g}\) that makes

\[
u_0 = u_1.
\]
6.7 Proof of Proposition 4

We can prove the proposition by solving the problem backwards. It is useful to re-write the value function as follows:

\[
\hat{V}_i(K_g) = \begin{cases} 
\max\{\ln(c_i) + \beta [p_i(K_g^j)\hat{V}_i(K_g^j) + (1 - p_i(K_g^j))\hat{W}_i(K_g)]\} & \text{if } K_g \leq K_g^*, \\
\max\{\ln(c_i) + \ln(g_i) + \beta [p_i(K_g')\hat{V}_i(K_g') + (1 - p_i(K_g'))\hat{W}_i(K_g')]\} & \text{if } K_g > K_g^*,
\end{cases}
\]

(38)

The function \(\hat{W}_i(K_g)\) is exactly like \(W_i(K_g)\), but with \(V_i(K_g)\) replaced by \(\hat{V}_i(K_g)\).

Since \(K_g \in (K_g^*, K_g^{ss})\), the sequence of capital is increasing and the solution to the problem is interior.

Guess a constant probability of re-election, \(p_i(K_g) = \bar{p}_i\) and that each party chooses to invest a constant proportion of current resources, \(h_i(K_g) = s_i RK_g^\theta\). Under this guess, logarithmic utility, full depreciation and Cobb-Douglas production function the FOC with respect to publicly provided goods \((g)\) delivers:

\[g_B(K_g) = c_B(K_g) = \frac{1}{2}(1 - s_B)RK_g^\theta.\]

The investment rule that a given incumbent follows is no longer continuous or differentiable in its whole domain. So in principle, the GEE derived in section 4.2.2 does not hold for all \(K_g\). However, when capital is bigger than the threshold, it is always optimal to provide a positive amount of the public good. Under the guesses, investment and spending are increasing functions of current capital. Since \(K_g^* < K_g^{ss}\), then \(K_g' > K_g^*\) and tomorrow’s provision is positive as well, \(g(K_g') > 0\). This implies that the GEE holds when in power satisfies \(\hat{V}_i(K_g') = V_i(K_g')\), and \(\hat{W}_i(K_g') = W_i(K_g')\) as well. Hence, the GEE holds when \(K_g > K_g^*\).

Using equation (21) the GEE of government \(B\) reduces to:

\[
\frac{1}{c_B(K_g)} = \beta \left\{ \bar{p}_B \frac{f_K(K_g')}{c_B(K_g')} + (1 - \bar{p}_B) \frac{[f_K(K_g') - g_B(K_g')]}{c_A(K_g')} + (1 - \bar{p}_B)h_A(K_g') \times \right.
\]

\[
\left. \left[ - \frac{1}{c_A(K_g')} \frac{1}{1 - \bar{p}B} + \frac{\bar{p}_B}{1 - \bar{p}_B} \frac{1}{\bar{c}_B(K_g')} + \beta \frac{1 - \bar{p}_B - \bar{p}_A f_K(K_g')}{1 - \bar{p}_B} \right] \right\}.
\]

where \(K_g' = h_B(K_g) = s_B RK_g^\theta, K_g'' = h_A(h_B(K_g)) = s_ARK_g^\theta\) and \(\bar{c}_B(K_g') = \frac{1 - 2s_B}{s_B} K_g''.32\)

Replacing the guess into the equation above and simplifying, we obtain:

\[
s_B = \frac{\beta\theta[1 + \bar{p}_B + 2\beta\theta(1 - \bar{p}_A - \bar{p}_B)]}{2 - \beta\theta(2\bar{p}_A - (1 - \bar{p}_B))}.
\]

(39)

The propensity to invest of party \(A\) is analogously determined. This verifies that when the probabilities of re-election are indeed constant, so the incumbent chooses to invest a constant proportion of output.

Voters on the other hand take investment decisions as given and choose whom to vote for. This determines the probabilities of re-election in equilibrium. We need to guess on the functional form of their value functions, \(V_i\) and \(W_i\) as an intermediate step. Let them be equal to:

\[
V_j(K_g, t) = \nu_j^\prime + \nu_j \ln(K_g).
\]

(40)

\[
W_j(K_g, t) = \nu_j^\prime + \nu_j \ln(K_g).
\]

(41)

Recall that \(\bar{c}_B(K_g) = f(h_B^{-1}(h_A(K_g))) + (1 - \delta)h_B^{-1}(h_A(K_g)) - g_B(h_B^{-1}(h_A(K_g))) - h_A(K_g)).\)
The sub-index \( j \) denotes the group to which the agent belongs. The index \( \iota \) captures the fact that utility depends on the identity of the previous incumbent (through the incumbency advantage term). For example, \( V_A(K_g, B) \) is the utility of agent belonging to group \( A \) when his party is in power and \( B \) was in power last period; \( W_A(K_g, B) \) is his utility when \( A \)'s party was out of power last period and remains out of power this period as well. Note that \( \nu_j \) and \( \omega_j \) do not depend on \( \iota \) since \( \alpha \) only affects the level of utility but not its derivative.

Given the guesses of the value function we obtain

\[
\nu_j = \frac{\theta(2 - \theta\beta[2\overline{p}_\iota - (1 - \overline{p}_j)])}{1 - \theta\beta(\overline{p}_\iota + \overline{p}_j) - (\theta\beta)^2(1 - \overline{p}_\iota - \overline{p}_j)} \quad \iota \neq j. 
\]

\[
\omega_j = \frac{\theta(1 - \theta\beta[\overline{p}_j - 2(1 - \overline{p}_\iota)])}{1 - \theta\beta(\overline{p}_j + \overline{p}_\iota) - (\theta\beta)^2(1 - \overline{p}_j - \overline{p}_\iota)} \quad \iota \neq j.
\]

\[
\bar{\nu}_j = \frac{1}{(1 - \beta)[1 + \beta(1 - \overline{p}_\iota - \overline{p}_j)]} \left\{ \beta(1 - \overline{p}_j) \left[ \ln \left( \frac{1}{2}(1 - s_\iota)R \right) + \beta[\alpha(2\overline{p}_\iota - 1) + (1 - \overline{p}_\iota)\nu_j + (1 - \overline{p}_j)\nu_j \ln(s_\iota R) \right] \right\},
\]

\[
\bar{\omega}_j = \frac{1}{1 - \beta\overline{p}_\iota} \left\{ \ln \frac{1}{2}(1 - s_\iota)R + \beta \left[ \alpha(2p_\iota - 1) + (1 - p_\iota)\bar{\nu}_j + (p_\iota\omega_j + (1 - p_\iota)\nu_j) \ln(s_\iota R) \right] \right\}. 
\]

Finally, \( \bar{\nu}_j = \bar{\nu}_j - \alpha \) and \( \bar{\omega}_j = \bar{\omega}_j + \alpha \).

Replacing eq.(40) and eq.(41) into eq.(36) we obtain the expression that determines \( \overline{p}_\iota \).

Finally, we verify that probabilities are constant and that governments choose to invest a proportion of output. These rules are increasing in capital, differentiable and invertible as guessed.
References


