

Job Competition, Hold-ups and Efficiency

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Abstract

We consider an auction-theoretic model of the labour market to study hold-up problems in markets with frictions. Firms have to invest in capital before they can advertise a job, while unemployed workers apply for jobs by bidding in auctions. This non-cooperative application procedure generates a distribution of applicants across jobs, and our aim is to investigate how the competition among applicants affects the investment decisions of firms.

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Our findings suggest that the economy always generates the right number of jobs. However, the type of job need not be efficient. In particular, if workers search at random, the expected wage costs of a firm increase with the investment level and firms under-invest in capital. In contrast, if workers can direct their search towards firms with different capital levels, the equilibrium is efficient. This result contrasts sharply with the predictions of models with ex post wage bargaining that never yield an efficient allocation. Moreover, our results extend the efficiency of auction mechanisms to an environment with non-contractible investments.

Keywords: hold-up, search, efficiency and auctions.

JEL Classification: C78, D44, D83.

1 Introduction

This article poses the question under which conditions the labour market generates an efficient distribution of jobs. The crucial feature of our economy is that firms need to invest in capital before they can enter the labour market to look for a worker. The investments in capital are therefore not contractible and surplus sharing may allow workers to appropriate a share of the returns. In contract theory this is known as a “hold-up problem”.¹

There exists an extensive literature that addresses this type of hold-up problem within the realm of a standard matching model, assuming pairwise meetings between firms and workers and bilateral wage negotiations. In that environment the distribution of jobs is never efficient (Davis (2001)). Either workers have some bargaining power and wages increase with capital, creating a hold-up problem, or all bargaining power is vested in firms, leading to very low wages and excessive entry of firms. Moreover, similar arguments have been advanced for investments in education (e.g. Laing, Palivos and Ping Wang (1995) and Moen (1998)), in general training (Acemoglu (1997)) and for complementary investments in education and capital (Acemoglu (1996) and Masters (1998)). In this study we contest these results. Our main objection against the use of ex post bargaining is that agents often receive several competing offers. The competition among potential trading partners may thus be an instrument to protect the returns on investments.

¹Williamson (1975) and Groult (1984) show that hold-up problems arise due to a lack of complete contingent contracts; with complete contracts all those who benefit from an investment could be forced to pay their share of the cost. The hold-up literature also proposed a number of solutions: parties can reallocate property rights (Williamson (1975), Hart and Moore (1990)), impose breach penalties (MacLeod and Malcomson (1993)), or enter into long-term relationships (Williamson (1975)). However, all these solutions require that agents can arrange their relationships before they make the investment. This is excluded by our assumption that investments are made before the firms meet workers.

It is not difficult to find examples. A familiar one is the academic job market. Most graduate students who enter this market prefer to apply at many universities in the hope to get competing offers. Similarly, the vacancy announcements of firms often attract several applicants. While formulating their wage demands, each of these applicants will take into account that the firm might hire a different candidate and this (implicit) threat tends to weaken their bargaining position *vis-à-vis* the firm.

In this essay we concentrate on the second example. In particular, our aim is to determine whether the competition among applicants can resolve the hold-up problem. For that purpose, we develop a model in which firms need to invest in capital before they can advertise a job. The matching process is based on the so-called “urn ball matching technology”. Underlying this approach is the assumption that workers cannot coordinate their application decisions. By chance, some vacancies therefore attract many applicants, while other identical ones attract none. Finally, to capture the competition among applicants, we assume that jobs are allocated via auctions. In accordance with the previous literature, wages are therefore determined after the investments are made, but workers may have to compete with other applicants to obtain the job they applied for.

The theoretical analysis builds on the recent literature on search and auctions (e.g. Kultti (1998), Shimer (1999) or Julien, Kennes and King (2000)). This literature has established that auction mechanisms can induce efficiency in the standard search environment. The characteristic of the efficient mechanisms is that sellers use a reservation bid at the level of their endogenous outside option value.

In order to focus on the investment decisions we initially restrict our attention to this class of efficient auction rules. The particular rule that we consider is a sealed second prize auction. In this case the wage negotiations have two possible outcomes: either the firm has two or more applicants

who receive their reservation wage irrespective whether they are hired or not, or the firm has a single applicant who appropriates the entire surplus of the match. It is shown that these payments correspond to the actual productivity of applicants. For a given investment level firms therefore have the right incentives to create jobs. However, the surplus of a job may accrue to the worker and this creates a scope for hold-ups.

1.1 Results

Our first result formalises the scope for hold-ups. It shows that the equilibrium with random applications is never efficient.

The problem with a random assignment of workers to jobs is that all firms face the same probability to attract a single applicant. Since the surplus of a job increases with the capital stock, this leads to a positive relation between the expected wage costs and the investment level of a firm. Firms will therefore not be able to appropriate the entire marginal returns on capital and this leads to under-investment.

We then proceed by showing that the hold-up problem is resolved when workers can direct their search towards different types of firms.

A sufficient condition for the equilibrium with directed search is that workers observe the capital stock of firms before they make their applications. This assumption creates a competitive environment in which each job opening needs to offer unemployed workers the same expected income. A firm that increases her investment level may therefore still need to pay a higher wage, but this is now exactly compensated by a larger number of expected applicants and hence a smaller probability that the firm will face a single applicant. From the viewpoint of an individual firm the expected wage costs are therefore independent of her investment level, resulting in efficient investments.

Finally, in a last step we allow firms to choose and announce their reservation profit. In this environment application decisions respond both to capital and the reservation strategy of firms. As long as these variables are freely observable, we find that the equilibrium is efficient and that firms announce their true outside option value.

Our efficiency result contrasts sharply with the predictions of models with ex post bargaining. However, in order to attain efficiency, there needs to be a stronger competition for jobs with a larger capital stock. It is therefore crucial that unemployed workers observe all the payoff relevant information of jobs. Furthermore, we need to ensure that agents cannot renegotiate their agreement once the hiring procedure is concluded.

The insight that directed search may help to resolve hold-up problems is not entirely new. Acemoglu and Shimer (1999) develop a model of wage posting and ex ante investments in capital and they show that the equilibrium is efficient. However, their mechanism is entirely different. With wage posting firms choose the wage (or the sharing rule) at the same time as the investment. Hence, when a firm and a worker meet, there is no scope for wage negotiations and thus no scope for hold-ups. In contrast, in our model wages are determined after the investments are made, and the efficient allocation is entirely supported by the competition between rival applicants.

Finally, in order to avoid ex post renegotiation, we assume that agents can write perfectly enforceable contracts. This is a very strong assumption. However, in the last section we show that the same results obtain if renegotiation requires the consent of both parties.

The rest of the chapter is organised as follows. Section 2 introduces the environment and discusses the matching technology. Section 3 characterises the efficient resource allocation. This is followed in Section 4 by a discussion of the equilibrium with random search. The equilibrium with directed search is described in Section 5. This section also contains an analysis of

the mechanism design problem for firms that wish to protect their returns on non-contractible investments. Finally, Section 6 concludes the paper and discusses directions for future research.

2 The Environment

There is a continuum of risk-neutral workers with measure normalised to one and a larger continuum of risk-neutral firms. All agents live forever in discrete time and have a common discount factor $\beta \in (0, 1)$.

Workers are homogeneous and may be in one of two states, employed and producing or unemployed and searching. Firms, on the contrary, are inactive until they buy some capital $k > 0$ at constant marginal cost p , which allows them to attempt to hire a worker by posting a vacancy. If a firm employs a worker and k units of capital, it produces $f(k)$ units of output per period. The price of this good is normalised to one and f is assumed to be strictly increasing and concave and satisfies the usual Inada conditions with $f(0) = 0$.

Finally, job destruction is exogenous. At the end of each period the capital stock of a firm breaks down with probability s , leaving the worker unemployed and the firm inactive. Workers' income during unemployment is normalised to zero.

2.1 Job auctions

Each unemployment worker may apply for one job per period. An application strategy defines the probability that a worker applies for a job at each measurable set of firms. The chosen strategy will depend on the worker's information. However, to capture the notion of a large market economy, we assume that all workers adopt the same (mixed) strategy. Some vacancies therefore attract many applicants, while other identical vacancies get none.

This is our “coordination friction”.

During the interview stage firms solicit a wage bid from each applicant. The bid specifies the time path of wages at which the applicant is willing to work. Firms review these bids and decide whether to hire a worker, and if so, which one. The associated wage payments are determined by a pre-announced auction rule and we assume that contracts are enforceable. Finally, if a firm is indifferent between two or more bids, it selects one bid at random. The chosen worker begins her job at the start of the next period.

Further details of the hiring procedure are discussed in Section 4. In the remainder of this section we describe the outcome of the application process when workers cannot discriminate between jobs. From the viewpoint of applicants all jobs therefore look identical, and workers will apply for a job at a randomly selected firm. More precisely, suppose that in some labour market there are $q \in [0, \infty)$ unemployed workers per vacancy seeking employment. If workers cannot discriminate between jobs, q is also the (market) queue length or the expected number of applicants of each vacancy. The probability that a firm receives exactly n applications is then defined by:

$$\varphi(n, q) = \frac{q^n e^{-q}}{n!} \tag{1}$$

The above function is the Poisson distribution function with parameter q . Since $\varphi(0, q) = e^{-q}$, a vacant firm will thus attract at least one applicant with probability $\eta(q) = 1 - e^{-q}$. This expression is known as the “urn ball matching function”.² If firms randomize over applicants, a similar expression may be derived for workers. The probability that a worker is hired is then given by $\mu(q) = \eta(q)/q$. The matching probabilities with directed search are obtained by replacing q by the queue length associated with each particular

²The urn ball matching technology was first used by Butters (1977). For recent applications see Moen (1999) or Shimer (1999).

submarket.

3 The Efficient Allocation

We start the analysis with a derivation of the efficient resource allocation.³ This constrained efficient allocation is derived using an imaginary social planner who chooses the time path of the market queue length q , firms' capital investment k and the unemployment rate u to maximise the discounted value of output minus the costs of vacancy creation. Formally, the planner's problem can be expressed as:

$$\max_{\{q(t), k(t), u(t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(t)\mu(q(t))Y(k(t)) - \frac{u(t)}{q(t)} (1 - \beta(1 - s))pk(t) \right] \quad (2)$$

$$s.t. : u(t + 1) = u(t) + s(1 - u(t)) - (1 - s)\mu(q(t))u(t) \quad (3)$$

$$Y(k(t)) = \beta(1 - s) \left[\frac{f(k(t))}{1 - \beta(1 - s)} - pk(t) \right], \quad (4)$$

where $Y(k(t))$ denotes the value of a newly filled job with capital stock $k(t)$.

To understand the objective function, it is convenient to imagine firms renting capital at a cost $(1 - \beta(1 - s))p$. The first term in parentheses then represents the payoff from newly created jobs. Each of these jobs produces $f(k(t))$ units of output per period. Moreover, production starts with one period delay and firms must continue to pay the rental cost of their equipment. The net expected present value of a filled job is therefore equal to $Y(k(t))$ as

³This section draws on Acemoglu and Shimer (1999). These authors consider the same problem in a continuous time matching model with pairwise encounters.

defined in (4). The second term in parentheses represents the costs of maintaining open vacancies, i.e. the rental cost of capital $(1 - \beta(1 - s))p$ times the number of vacancies $u(t)/q(t)$, times the capital used by each vacancy $k(t)$. The feature that it is optimal to have only one type of firm in a market follows from the concavity of $f(k)$.⁴ Finally, all payoffs are discounted to the initial period, while the evolution of the unemployment rate $u(t)$ satisfies eqn. (3).

In the following we restrict attention to stationary solutions. As shown in the Appendix, this is equivalent to the maximisation of the shadow value of unemployed workers in the above dynamic programme. This result is summarised in the following Proposition, which reformulates Proposition 1 in Acemoglu and Shimer (1999):

Proposition 1 *An efficient steady state solution exists. It is characterised by a pair $(q^S, k^S) \in (0, \infty)^2$ solving:*

$$\max_{\{k, q\}} \frac{\beta(1 - s)\eta(q) \left(\frac{f(k)}{1 - \beta(1 - s)} - pk \right) - (1 - \beta(1 - s))pk}{(1 - \beta(1 - s))q + \beta(1 - s)\eta(q)}. \quad (5)$$

Proof. See Appendix. ■

Maximisation problem (5) is not jointly concave in k and q , and so the first-order conditions are not sufficient for a maximisation. Nonetheless, because the efficient solution is an interior solution to the maximisation problem, the first-order conditions are necessary, and so they are useful in recognising inefficient allocations:

⁴Appendix A provides a formal proof of this assertion.

Corollary 2 *The efficient steady state allocation (q^S, k^S) satisfies:*

$$\frac{\beta(1-s)\eta(q^S)}{1-\beta(1-s)+\beta(1-s)\eta(q^S)} \left(\frac{f'(k^S)}{1-\beta(1-s)} \right) = p \quad (6)$$

$$\frac{\beta(1-s)(\eta(q^S) - q^S\eta'(q^S))}{1-\beta(1-s)+\beta(1-s)(\eta(q^S) + (1-q^S)\eta'(q^S))} \left(\frac{f(k^S)/k^S}{1-\beta(1-s)} \right) = p \quad (7)$$

The efficient allocation is illustrated in Figure 1. The graphs associated with eqns. (6) and (7) are denoted by $K^S(q)$ and $Q^S(k)$, respectively. Both graphs start in the origin and are strictly increasing in (q, k) space. This last feature is due to the fact that (5) is strictly concave in k and q separately. Moreover, for large enough values of q , $K^S(q)$ lies above $Q^S(k)$ and both curves intersect at least once on the interior.

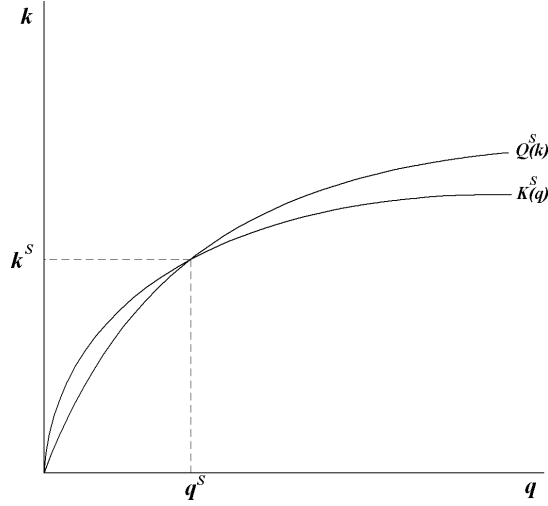


Figure 1: The constrained efficient allocation

For future reference we also provide the analytical interpretation of the efficiency conditions. First, consider equation (6). The first fraction on the left hand side is the expected current value of a unit of output when the vacancy is first filled. This value is discounted because of both impatience and the risk that the vacancy may break down before it is filled. The second fraction is the discounted marginal product of capital once the vacancy is filled. The left hand side of (6) therefore defines the marginal revenue of an additional unit of capital. In an efficient allocation this must be equal to the marginal cost of capital p . Similarly, equation (7) characterises the efficient market queue length. The planner chooses q to balance the expected social benefits of an additional vacancy with the costs of creating this vacancy. In doing so, the planner takes into account that an increase in the number of vacancies will reduce the rate at which outstanding vacancies are filled. The net-increase in the mass of new jobs due to the creation of an additional vacancy is therefore equal to $(\eta(q) - q\eta'(q)) = (1 - e^{-q} - qe^{-q})$.

In the next sections we compare the efficient allocation to the decentralised allocation. The investments in capital are now financed by firms and wages are determined by auctions. Initially, we impose two restrictions on the set of auction rules. First, we assume that the auction rule must be fair. In equilibrium jobs are therefore awarded to the worker who submits the lowest bid. Second, we assume that firms cannot announce a reserve bid above the value of their endogenously determined outside option. The auction rules in this class are revenue equivalent⁵ and induce the constrained efficient

⁵Formally, the revenue equivalence theorem also requires that the common equilibrium wage bids of workers increase with a worker's evaluation of the job (*e.g.* Riley and Samuelson (1982)). This condition is satisfied for most common auction rules and is certainly satisfied for the family of sealed first and second prize auctions considered here. Finally, note also that I have implicitly ruled out the possibility that a firm is able to extract payments from applicants who are not hired.

allocation if the value of firms is exogenous (*e.g.* Shimer (1999)). We use this setup to study the case of random search, in which applicants do not observe k before they contact a firm, and directed search. In this last case applicants observe the entire distribution of investments across firms before they make their application decision. The case in which firms also choose and announce the trading mechanism is discussed in Section 6.

4 Random Search

This section characterises the equilibrium with random search. Without loss of generality we restrict attention to the example of a sealed second prize auction. Hence, when a firm has more than one applicant, the worker is paid according to the second lowest bid. Instead, when a firm has a single bid, the worker receives her outside option plus the entire surplus of the match. Finally, to ensure that the dominant wage bid is unique, we assume that workers initially do not know n .

The structure of the equilibrium is self-evident. The reason is that all firms attract the same expected number of applicants q . The conditional distribution of n is therefore independent of k . Formally, let $J^V(k)$ denote the expected present value of a vacant firm with capital-intensity k . An equilibrium must then satisfy five conditions: *(i)* Firms have consistent beliefs about q . *(ii)* The investment level of firms maximises expected profits $J^V(k) - pk$. *(iii)* New entrants make zero profits. *(iv)* Match formation is voluntary. *(v)* The hiring decision and the wage correspond to the equilibrium of a sealed second prize auction. The model is solved backwards starting with wages, and for the moment I assume that condition *(iv)* is satisfied.⁶

⁶When I characterise the investment decisions, I will show that this is indeed an equilibrium feature. In equilibrium it is never optimal for a firm to create a job that is subsequently refused by all workers.

4.1 Wages

Consider a vacant firm with k units of capital that attracts $n > 0$ applicants. In the following we denote the equilibrium payoff of the firm and the worker by $J^F(k, n)$ and $J^E(k, n)$, respectively. These payoffs are discounted to the start of the first period of production. Moreover, we denote the joint value of the match by $S(k)$, so that $S(k) = J^F(k, n) + J^E(k, n)$. Finally, J^U denotes the expected present value of an unemployed worker. The unique reservation value of workers is then given by βJ^U .

Since workers are identical, there are two possible outcomes. In the first case, the firm has a single applicant who makes a wage bid equal to βJ^U . The firm hires the applicant and agrees to a wage stream with value $J^E(k, 1) = S(k) - J^V(k)$ in the next period. In the second case, the firm has several job candidates. Each of these applicants submits a wage bid equal to βJ^U and the firm selects one candidate at random and offers this worker a contract with value $J^E(k, n) = J^U \forall n > 1$. Since $S(k) = J^F(k, n) + J^E(k, n)$, the outcome of the auction can thus be summarised as follows:

$$J^E(k, n) = \begin{cases} S(k) - J^V(k) & \text{if } n = 1 \\ J^U & \text{if } n > 1 \end{cases} \quad (8)$$

Similarly,

$$J^F(k, n) = \begin{cases} J^V(k) & \text{if } n = 1 \\ S(k) - J^U & \text{if } n > 1 \end{cases} \quad (9)$$

Hence, the firm receives the entire value of the surplus if it has at least two applicants. Otherwise the surplus accrues to the single applicant.⁷

⁷As explained before, standard matching models take a different approach. Wages in

4.2 Bellman equations

We are now in a position to derive the value functions for all unattached agents. These functions depend on the value of the future matches in which an agent will be involved, and on the relative probability that the surplus of a match accrues to the firm or the worker. The value of $S(k)$ is defined by the following standard Bellman equation:

$$S(k) = f(k) + \beta[(1 - s)S(k) + sJ^U] \quad (10)$$

According to (10) the joint value of a match is equal to the payoffs in the current period, $f(k)$, plus the expected discounted payoffs in the next period. The probability that a match survives is equal to $(1 - s)$, in which case both parties continue to obtain $S(k)$. With complementary probability s , however, the capital equipment is destroyed. The match will be dissolved and the firm becomes inactive with payoffs equal to zero, while the worker returns to the pool of unemployed with payoffs equal to J^U . Rearranging terms we obtain the following expression:

$$S(k) = \frac{f(k) + s\beta J^U}{1 - \beta(1 - s)} \quad (11)$$

Next, consider the expression for $J^V(k)$. With random search the number of applicants of each firm is distributed according to $\varphi(n, q)$. Moreover, the expected payoff corresponding to any number of applicants n is defined by $J^F(k, n)$. The asset value of a vacant firm can therefore be expressed as the weighted sum of the payoffs in three distinct events:

these models are determined through bilateral negotiations between the firm and a single applicant. The auction model presented here is formally equivalent to a model in which the entire bargaining power is endogeneously assigned to either the firm or the worker.

$$J^V(k) = \beta(1 - s) [(e^{-q} + qe^{-q})J^V(k) + (1 - e^{-q} - qe^{-q})(S(k) - J^U)] . \quad (12)$$

First of all, with probability e^{-q} the firm receives no applications. Second, with probability qe^{-q} the firm receives exactly one application and settles for a payoff equal to $J^F(k, 1) = J^V(k)$, leaving the surplus to the worker. Finally, with complementary probability $(1 - e^{-q} - qe^{-q})$ the firm has more than one applicant. In this case the firm retains the surplus and receives $J^F(k, n) = S(k) - J^U$. Finally, in all three events the payoffs need to be discounted to account for time preference and the risk of breakdown.

The value function of an unemployed worker is slightly more complicated, because it takes into account that vacant firms may have different capital stocks:

$$J^U = \beta \left[(1 - e^{-q})J^U + e^{-q} \int [(1 - s)(S(k) - J^V(k)) + sJ^U] dG(k) \right] \quad (13)$$

With random search the value of an unemployed worker therefore depends on the distribution of capital stocks across vacant firms, which I denote by $G(k)$, and on the number of applicants with whom the worker will compete. It is easy to show that a worker is the only applicant with probability e^{-q} . In this case the worker is hired and unless the capital equipment breaks down she receives a payoff equal to $J^E(k, 1) = S(k) - J^V(k)$.⁸ The probability that

⁸Recall that this interpretation is based on the *auxiliary* assumption that payment start at the beginning of production period. In reality, however, contracts are enforceable and all payments could in principle be made immediately after the contract is signed. In that case the worker would receive $\beta(1 - s)(S(k) - J^V(k))$ in the first period and J^U in the next period in case the match breaks down. The expected value of payments is the same under both interpretations.

the firm has at least one additional applicant is thus equal to $1 - e^{-q}$, and in this case the worker receives J^U regardless whether she is hired or not.

4.3 Investments and hold-up

In this section we characterise the profit-maximising investments. To solve the investment problem of firms we substitute (11) into the right-hand side of equation (12):

$$J^V(k) = \frac{\beta(1-s)(1 - e^{-q} - qe^{-q})}{1 - \beta(1-s) + \beta(1-s)(1 - e^{-q} - qe^{-q})} \left(\frac{f(k) - (1-\beta)J^U}{1 - \beta(1-s)} \right) \quad (14)$$

The above equation defines the value of a vacant firm as the expected return from hiring an applicant at her reservation wage $(1-\beta)J^U$. This wage depends on the entire distribution of capital across vacant firms and is therefore not affected by the investment decision of a single firm.

The derivation of the equilibrium investments is now straightforward. Firms maximise expected profits $J^V(k') - pk'$ taking q and the investment decisions of all other firms as given. This leads to the following first order condition:⁹

$$\frac{\beta(1-s)(1 - e^{-q} - qe^{-q})}{1 - \beta(1-s) + \beta(1-s)(1 - e^{-q} - qe^{-q})} \left(\frac{f'(k)}{1 - \beta(1-s)} \right) = p \quad (15)$$

Equation (15) is our first equilibrium condition. It defines the investment level of firms as a strictly increasing function of the likelihood that a firm

⁹Notice that $J^V(k)$ is strictly concave in k for any given value of q . The second-order conditions are therefore satisfied and (15) defines a unique profit-maximising investment level for each value of q .

attracts at least two applicants. In any equilibrium firms therefore make the same investment and $G(k)$ is degenerate. Furthermore, what is more important, firms never invest the efficient amount.

Lemma 3 *For any given value of q , firms underinvest in capital.*

Proof. The proof of this Lemma is straightforward. With identical agents on both sides of the market, all jobs that attract at least one applicant will be filled. The social marginal returns from an investment in capital are therefore proportional to $\eta(q) = 1 - e^{-q}$. However, firms only consider their private marginal returns which are proportional to $1 - e^{-q} - qe^{-q}$. Since $f(k)$ is strictly concave, firms' investments are therefore below the efficient amount defined in (6). ■

The underlying source of the underinvestment is a hold-up problem. Firms anticipate that a share of the returns on capital may accrue to a single applicant, and in response they reduce the investment level until the private marginal return equals p .

Furthermore, it is important to understand the relationship between the hold-up problem and random search. In the efficient allocation the investment level is positively related to q . A more capital-intensive job is therefore associated with a longer queue than a less capital-intensive job. In contrast, in the equilibrium with random search q is the same for all firms. The probability that a firm attracts one or more applicants is therefore unrelated to the investment level of the firm. However, the implicit cost of search in terms of foregone output increases with the investment level of the firm. As a result, both the surplus of a job and the income of a single applicant $S(k) - J^V(k)$ rise with k , and when a firm decides how much to invest it will anticipate that a share of the marginal returns of the investment may accrue to the worker.¹⁰

¹⁰This is similar to the hold-up problem in Davis (2001), Moen (1996) and Acemoglu

Finally, notice that the risk of hold-up is especially strong at low values of q . The explanation is again simple. When the expected number of applicants is small, firms face a high conditional probability of meeting a single applicant.¹¹ The private marginal returns from an investment are therefore low compared to the social marginal gains.

4.4 Equilibrium

Having determined the optimal investment level for each possible value of q , we can now invoke the free entry condition to derive the equilibrium. First of all, since the distribution of capital stocks is degenerate we can rewrite J^U as:

$$J^U = \frac{\beta(1-s)e^{-q}}{1 - \beta(1-s) + \beta(1-s)e^{-q}} \left(\frac{f(k) - (1 - \beta(1-s))J^V(k)}{1 - \beta} \right) \quad (16)$$

This expression is similar to (14), except that J^U is determined by the likelihood that an unemployed worker manages to extract the surplus in a future match.

Next, we substitute (16) into the right-hand side of equation (14). Solving this expression for $J^V(k)$ and equating this value to the cost of a vacant job, pk , yields our free entry condition:

(1996), except that these studies examine models with pair-wise matching and ex post wage bargaining.

¹¹The conditional probability that a firm attracts exactly 1 applicant is defined as $\Pr\{n = 1 \mid n > 0\} = qe^{-q}/(1 - e^{-q})$. After applying l'Hôpital's rule, it follows that the limit of this probability goes to 1 as q approaches zero. Similarly, when q goes to infinity, firms almost surely receive two or more applicants. Hence, in that case the private and social marginal returns coincide.

$$\frac{\beta(1-s)(1-e^{-q}-qe^{-q})}{1-\beta(1-s)+\beta(1-s)(1-qe^{-q})} \left(\frac{f(k)/k}{1-\beta(1-s)} \right) = p. \quad (17)$$

This condition coincides with equation (7). Hence, the efficient allocations and the decentralised equilibrium allocations are located on the same upward sloping locus in (q, k) space, and conditional on an efficient value for the capital stock k^S the labour market will create the efficient number of jobs. However, from the preceding discussion of investments we know that the reverse does not hold. Given an efficient value for the market queue length q^S , firms are not willing to adopt the efficient capital stock due to the risk of holdup. Hence, in equilibrium only one of the two margins is efficient. This is illustrated in Figure 2.

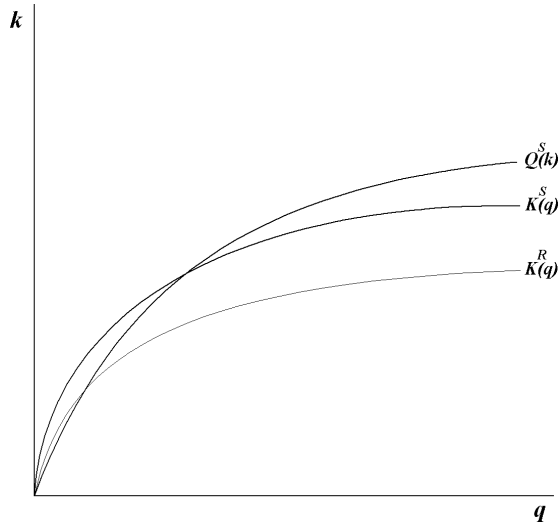


Figure 2: The equilibrium with random search

The investment locus associated with (15) is denoted by $K^R(q)$. Due to the hold-up problem this locus lies everywhere below $K^S(q)$. Hence, if the

efficient investment k^S is small, $K^R(q)$ may lie entirely below the free entry locus $Q^R(k)$. In this case there exists no (interior) equilibrium.

To rule out this last case, it suffices to impose a mild restriction on the concavity of $f(k)$. More precisely, let $\epsilon(k)$ denote the elasticity of $f(k)$ with respect to k , so that $\epsilon(k) = f'(k)k/f(k)$. A sufficient condition for the existence of an equilibrium is then given by:

Condition 1 $\lim_{k \rightarrow 0} \epsilon(k) > 1 - \beta(1 - s)$.

The properties of the decentralised equilibrium are summarised in the Proposition below:

Proposition 4 *(i) Given Condition 1, a steady state equilibrium with random search exists. (ii) The equilibrium can be summarised by a pair $(k^R, q^R) \in (0, \infty)^2$ that solves (15) and (17). (iii) The equilibrium never coincides with an efficient allocation. Either firms underinvest and $k^R < k^S$, or entry is too low and $q^R > q^S$. (iv) If $\epsilon(k)$ is non-increasing, the equilibrium is unique and firms underinvest in capital.*

Proof. See Appendix. ■

Proposition 4 extends results of Kultti (1998) and Shimer (1999). These articles showed that our class of auction rules induces an efficient allocation if the value of matches is exogenous. With ex ante investments the entry decisions are still optimal for a given value of the capital stock. However, ex ante firms anticipate that the wage costs increase with the investment level and due to this hold-up problem the equilibrium is not efficient.

The reason for the efficient entry margin is that applicants receive their actual marginal product. Consider the example of a single applicant. By contacting the firm, the applicant raises the joint wealth of both agents by $S(k) - J^V(k) - J^U$, and in order to hire this worker the firm would be

willing to pay an amount $S(k) - J^V(k)$. In contrast, if several workers apply at the same firm, only one new job is created. The marginal productivity of each individual worker is thus equal to zero, and the firm would not be willing to pay any of these workers more than her reservation value. Hence, firms' expected wage payments are equal to the shadow value of unemployed workers and this implies that Hosios' (1990) condition is satisfied.

It is straightforward to corroborate this last claim. Formally, Hosios' condition requires that the surplus share of workers is equal to the elasticity of the matching function with respect to u . With the urn ball matching technology the total mass of vacancies can be expressed as $M(v, u) = v\eta(q)$. The elasticity of the matching function with respect to u is thus given by $\frac{\eta'(q)q}{\eta(q)} = \frac{qe^{-q}}{1-e^{-q}}$. This expression corresponds to the probability that the surplus of a job accrues to the worker (see footnote 10).¹² In expected terms Hosios' condition is therefore satisfied. Nonetheless, with random search this surplus

¹²This is a generalisation of an important result in Mortensen (1982). For environments with a pairwise linear matching technology Mortensen showed that the efficient allocation can be decentralised if the property rights to a match are assigned to the agent who initiates the contact. Pissarides questions the usefulness of this result (*e.g.* Pissarides (2000)), because with pairwise matching it is in general not possible to determine who initiated the match. Moreover, in Mortensen's analysis search externalities are absent. The urn ball matching technology introduces both congestion externalities and a logical way to assign the property rights to a match. According to our results these rights should go to the agent on the short side, so that workers obtain the surplus if they are a single applicant, while the firm is "responsible" for the match creation if it faces more than one applicant. Now why is it that the urn ball matching technology generates efficiency with Mortensen's rule despite the presence of congestion externalities? The answer is simple. When creating a job firms disregard the returns that accrue to a single applicant. As a result, the private marginal returns are equal to $(1 - e^q - qe^{-q})$ times the surplus of the match. However, this is exactly the social marginal return from job creation since $(1 - e^q - qe^{-q}) = \eta(q) - q\eta'(q)$. In other words, the fact that firms disregard the returns of a single applicant compensates for the reduction in the matching rate of existing vacancies. The congestion externality is therefore perfectly internalized.

share creates a hold-up problem. The equilibrium allocation is therefore never efficient.

5 Directed search

This section characterises the equilibrium with directed search. The environment is the same as before. However, in contrast to Section 4, we now assume that workers have perfect information about the capital stock of firms. Unemployed workers can therefore direct their search to firms with different capital stocks.

The change to directed search has two implications. First, unemployed workers will choose the application strategy that maximises their *true* expected income. A firm that increases her investment in capital will therefore attract a longer queue of applicants who all wish to appropriate the higher surplus. Second, if workers decide to apply for different types of jobs, then the mixing probabilities are such that workers are indifferent between these jobs.

To accommodate these changes, we denote the queue length of a firm with k' units of capital by $q(k')$. Moreover, in comparison to the previous section, we impose two stronger conditions on the equilibrium allocation: (i) The application strategy of workers maximises the value of J^U given any distribution $G(k)$. (ii) Firms' beliefs about $q(k)$ are consistent with rational expectations beginning at any decision node. This last restriction is needed to rule out situations in which firms fail to deviate to a profitable investment level because they incorrectly conjecture too few workers would apply. The above conditions replace condition (i) in Section 4. The rest of the conditions is the same as before.

5.1 Equilibrium

We start the analysis by writing the Bellman equations. Once again we restrict attention to steady states. First, consider the value of an unemployed worker who applies for a job with capital-intensity k :

$$J^U(k) = \beta((1 - e^{-q(k)})J^U + e^{-q(k)}((1 - s)(S(k) - J^V(k)) + sJ^U)) \quad (18)$$

The derivation of (18) parallels equation (13), with the value of an unemployed worker now defined by the highest value she can obtain while unemployed. That is,

$$J^U = \sup_{k \in \mathcal{K}} J^U(k), \quad (19)$$

where \mathcal{K} denotes the set of equilibrium investment levels.

Similarly, the value of a vacant firm $J^V(k)$ is defined by eqn. (12) with the uniform queue length replaced by $q(k)$.

The distinctive feature of directed search is that all queue length adjusts so that workers are indifferent between jobs. Any submarket that attracts workers in equilibrium must therefore yield the same expected income J^U , and for each of these submarkets equations (12)-(18) can be solved in isolation. This yields

$$J^V(k) = \frac{\beta(1 - s)(1 - e^{-q(k)} - q(k)e^{-q(k)})}{1 - \beta(1 - s) + \beta(1 - s)(1 - q(k)e^{-q(k)})} \left(\frac{f(k)}{1 - \beta(1 - s)} \right) \quad (20)$$

for firms offering a job with $k \in \mathcal{K}$, and

$$J^U = \frac{\beta(1 - s)e^{-q(k)}}{1 - \beta(1 - s) + \beta(1 - s)(1 - q(k)e^{-q(k)})} \left(\frac{f(k)}{1 - \beta} \right) \quad (21)$$

for workers who apply for these jobs.

For a given value of J^U , equation (21) defines a unique relationship between the capital stock and the queue length in each submarket. Moreover, since the value of J^U is not affected by the decisions of a single firm, the same relationship also defines the queue length that results if some firm deviates by offering a job with $k' \notin \mathcal{K}$ units of capital. Denote this relationship by $q(k; J^U)$. From (21) it follows immediately that $q(k; J^U)$ is continuous, strictly increasing in q and strictly decreasing in J^U on $(\underline{k}, \infty) \times (0, \infty)$, where \underline{k} is defined by $J^U = \beta(1 - s)(f(\underline{k})/(1 - \beta))$. For capital levels below this value $J^V(k) < J^U$ for all $q \geq 0$. In equilibrium no worker will therefore apply for these jobs, and so $q(k) = 0 \forall k \leq \underline{k}$.

We can now formalize the restriction on the out-of-equilibrium beliefs by the condition that firms' beliefs about $q(k)$ are consistent with $q(k; J^U)$ for all values of k , including the values of capital that are not actually observed in equilibrium.¹³ Individual firms take this relationship as given and choose k to maximise profits. Let $J^V(k, q(k; J^U))$ denote the asset value of a vacancy given the equilibrium relation $q(k; J^U)$. Profit maximisation and the assumption of free entry of firms then imply that in equilibrium the following condition must hold:

$$J^V(k, q(k; J^U)) - pk \leq 0 \forall k, \tag{22}$$

with equality if $k \in \mathcal{K}$.

¹³Since workers have perfect information, the queue length associated with each capital level $k' \notin \mathcal{K}$ becomes instantly observable as soon as a firm deviates to offer this capital stock. It seems reasonable therefore to assume that the out-of-equilibrium beliefs of firms are consistent with $q(k; J^U)$.

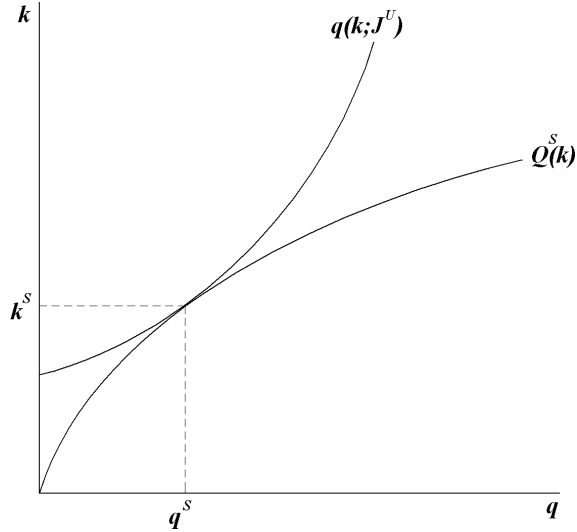


Figure 3: The equilibrium with directed search

The situation is depicted in Figure 3. Assume for a moment that a certain capital level k is offered in equilibrium. Free entry of firms then implies that $q(k)$ solves:

$$\frac{\beta(1-s)(1 - e^{-q(k)} - q(k)e^{-q(k)})}{1 - \beta(1-s) + \beta(1-s)(1 - q(k)e^{-q(k)})} \left(\frac{f(k)/k}{1 - \beta(1-s)} \right) = p \quad (23)$$

which corresponds to (5.10). The graph associated with this equation starts in the origin and is strictly increasing on the whole domain $(q, k) \in (0, \infty)^2$. Moreover, at (q, k) combinations below and to the right of this locus firms make strictly positive profits, while points above this curve correspond to losses. Workers, on the contrary, value high investment levels and short queues. Their indifference curves have a positive slope, and indifference curves that are located further away from the origin correspond to higher values of J^U . According to condition (22), equilibrium allocations therefore

correspond to a point of tangency between the zero profit locus of firms and the highest attainable indifference curve of workers.¹⁴ This feature of the model allows us to characterise equilibrium allocation as the outcome of the following constrained optimisation problem:

Lemma 5 $(q(k), \mathcal{K})$ with $k^D \in \mathcal{K}$ and $q^D = q(k^D)$ is a steady-state equilibrium with directed search iff (q^D, k^D) solves:

$$\max_{\{k, q\}} \frac{\beta(1-s)e^{-q}}{1 - \beta(1-s) + \beta(1-s)(1 - qe^{-q})} \left(\frac{f(k)}{1 - \beta} \right)$$

subject to:

(P1)

$$\frac{\beta(1-s)(1 - e^{-q} - qe^{-q})}{1 - \beta(1-s) + \beta(1-s)(1 - qe^{-q})} \left(\frac{f(k)/k}{1 - \beta(1-s)} \right) - p \geq 0$$

According to Lemma 5, any capital stock observed in equilibrium must maximise the expected income of a representative worker, subject to the condition that new entrants earn zero profits. Moreover, as shown in the previous section, along the free entry locus the expected income of unemployment workers is equal to the shadow value of workers. In an equilibrium with directed search firms therefore maximise the shadow value of unemployed workers. This optimisation problem is equivalent to the planner's problem, and the resulting allocations are therefore efficient.

¹⁴Our restrictions on the production technology are not sufficient to guarantee that the zero-profit curve is concave. As a result, there may be multiple equilibrium allocations. However, this is not important for the analysis since each of these allocations must correspond to a point of tangency between the free entry locus and the highest attainable indifference curve. The equilibrium welfare level of workers is thus uniquely determined.

Proposition 6 *If $(q(k), \mathcal{K})$ is an equilibrium with directed search with $k^D \in \mathcal{K}$ and $q^D = q(k^D)$, then (q^D, k^D) is an efficient allocation as defined in Proposition 1. Conversely, if (q^S, k^S) is an efficient allocation as characterised in Proposition 1, then there exists an equilibrium with directed search such that $k^S \in \mathcal{K}$ and $q^S = q(k^S)$.*

Proof. See Appendix. ■

The main difference with Proposition 4 is that firms make efficient investments. The ability of workers to direct their search to firms with different capital stocks therefore resolves the holdup problem. Notice that single applicants still earn a higher wage at more capital intensive firms. However, these higher wages are now exactly offset by an increase in the queue length so that unemployed workers are indifferent between jobs. In a neighbourhood around the equilibrium firms' expected wage payments are therefore independent of the investment level, and as a result firms acquire the entire marginal returns on their investments. Together with the efficiency of the entry margin this ensures that firms make efficient investments.

Proposition 6 confirms a main result of Acemoglu and Shimer (1999). They show that the equilibrium with ex ante investments is efficient if workers can direct their search to firms with different capital stocks and if the division of the returns satisfies Hosios' condition. In our case Section 1.4 illustrated that Hosios' condition is satisfied because workers are paid their actual marginal productivity. So far, this feature was obtained because we assumed that firms could not commit to a reserve bid above the value of their outside option. The next proposition relaxes this assumption and shows that the results remain unchanged as long as workers observe the distribution of reserve bids before they make their application decisions. Formally,¹⁵

¹⁵We implicitly extend the restriction on the out-of-equilibrium beliefs of firms to all

Proposition 7 *Suppose firms can credibly announce reserve bids $\pi(k)$ and unemployed workers can observe both the reserve bids and the capital stock of firms. Then, the set of equilibria coincides with the set of efficient allocations as characterised in Proposition 1 and firms announce their outside option values, so that $\pi(k^S) = J^V(k^S) = pk^S$.*

Proof. See Appendix. ■

The result that firms prefer to announce their outside option value is similar to the predictions of the competing auction models of McAfee (1993), Peters (1997), Julien, Kennes and King (2000) and Peters (2001).¹⁶ This reservation bid ensures that Hosios' condition is satisfied and maximises the welfare of workers for a given value of output. Proposition 7 extends this result to an environment in which the value of output is determined by ex ante investments, and shows that in this case efficiency is obtained if workers observe both the reservation bid and the capital choice of firms.

5.1.1 Renegotiation

Our last comment concerns renegotiation. Throughout the analysis we assumed that agents can write perfectly enforceable contracts. We imposed this condition to rule out ex post renegotiation of employment contracts. It is clear that an incentive for renegotiation exists, because in each employment relationship one of the parties is held at her reservation value. However, the assumption of perfectly enforceable contracts is unnecessarily strong. All feasible (π, k) combinations, so that firms correctly anticipate the queue length for any choice of the reserve bid. For more details see Appendix D.

¹⁶McAfee (1993) and Peters (1997) study pricing behaviour in retail markets. Julien *et al.* (2000) apply a similar setup to labour markets. However, in their model workers commit to and announce reservation wages and firms select a worker to whom they offer the job. In their model the role of workers and firms is therefore reversed. Finally, in Peters (2001) the valuation of the object is private information of the buyers.

that is needed is that agents cannot unilaterally impose renegotiation. With a proper treatment of the outside options this eliminates renegotiation in our model (e.g. Malcomson *et al.* (1993) or Malcomson (1998)). The only requirement is that the contractual payments need to satisfy the participation constraint of both agents at each moment during their relationship. When this condition is satisfied, renegotiation is a pure redistribution that will be refused by the party that is entitled to the surplus. Hence, when we allow for renegotiation by mutual consent, the timing of payments matters, but this does not change any of our efficiency results.

6 Concluding Remarks

In this paper we have shown that competition among rival applicants may prevent hold-ups in markets with frictions. The analysis built on the competing auction literature pioneered by McAfee and Preston (1993). Our main contribution is that we extend this framework to an environment in which the productivity of employment relationships depends on ex ante investments by firms. We show that this creates a scope for hold-ups. Moreover, following the suggestion of Acemoglu and Shimer (1999) we prove that the equilibrium is efficient if workers can direct their search to different jobs.

There are many interesting ways to extend the analysis. One option is to consider one-sided investments by workers. In this environment ex post bargaining may lead to over-investment rather than under-investment (Moen (1999)). The reason is that workers create a negative externality if they invest more to jump ahead of other workers in the queues. With auctions this effect should disappear. Since workers are valued efficiently, there is no difference between the private and the social marginal gains from human capital and workers should choose the optimal investment level.

A more challenging task would be to consider complementary investments by firms and workers. In that model the returns on capital depend on workers' investment in human capital, while vice versa the returns on education and training will depend on firms' investment in capital. Under ex post wage bargaining this strategic complementarity compounds the inefficiencies of one-sided investments (Acemoglu (1996) and Masters (1998)). An issue that is not yet resolved is whether job auctions or wage posting can reduce or eliminate hold-up problems in this environment.

Thirdly, one could consider the role of private information. In our model auctions are equivalent to wage posting whenever workers have perfect information about jobs. Firms could simply post the expected wage costs and hire a randomly selected candidate. This equivalence breaks down if workers are risk averse — because auctions are more risky than posting —, or if the number of agents on either side of the market is finite (Julien, Kennes and King (2001)). So far, however, the role of the information structure has not been studied in detail. It is easy to see that the information structure matters. Consider for example the case that unemployed workers observe wages, but not investments. In that case firms will opt for wage posting. This mechanism allows firms to appropriate the entire marginal returns from investments in capital, while auctions lead to hold-ups. Reversely, if workers make ex ante investments and if the resulting skill level is private information firms may prefer auctions. The reason is that auctions elicit the private information from workers. With wage posting, on the contrary, this is only feasible if firms post a menu of contracts that induces workers to self-select in different submarkets. However, the fact that these contracts need to satisfy an incentive compatibility constraint may well prevent efficiency.

Finally, one may consider the role of education as a pure screening device. In this environment education is wasteful in the sense that it does not affect productivity. Nonetheless, it may improve the allocation of ex ante hetero-

geous workers over jobs because it provides a signal about the innate ability of workers. This and other topics are on our agenda for future research.

References

Acemoglu, Daron (1996), “A Microfoundation for Social Increasing Returns in Human Capital Accumulation”, *Quarterly Journal of Economics*, 111(3), 779-804.

——— (1997), “Training and Innovation in an Imperfect Labor Market”, *Review of Economic Studies*, 64, 445-464.

Acemoglu, Daron and Robert Shimer (1999), “Holdups and Efficiency With Search Frictions”, *International Economic Review*, 40(4), 827-849.

Burdett, Kenneth, Shi and Randall Wright (2001), “Pricing With Frictions”, *Journal of Political Economy*, forthcoming.

Butters, Gerard R. (1977), “Equilibrium Distribution of Sales and Advertising Prices”, *Review of Economic Studies*, 44, 465-491.

Cole, Harold L., George J. Mailath and Andrew Postlewaith (1998), “Efficient Non-contractible Investments”, Federal Reserve Bank of Minneapolis, Staff Report 253.

Davis, Steven J. (2001), “The Quality Distribution of Jobs and the Structure of Wages in Search Equilibrium”, *NBER Working Paper*, No. 8434.

Felli, Leonardo and Kevin Roberts (2000), “Does Competition Solve the Hold-up Problem?”, mimeo, University of Oxford.

Grout, Paul (1984), "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach", *American Economic Review*, 86, 478-501.

Hosios, Arthur (1990), "On the Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies*, 57, 279-298.

Julien, Benoit, John Kennes and Ian King (2000), "Bidding for Labor", *Review of Economic Dynamics*, 3(4), 619-649.

——— (2001), "Auctions and Posted Prices in Directed Search Equilibrium", *Topics in Macroeconomics (BE Journal in Macroeconomics)*, 1(1), Article 1.

Kultti, Klaus (1998), "Efficient Allocation of Agents and the Trading Mechanisms", *Helsinki School of Economics and Business Administration, Working Paper No. W-206*.

——— (1999), "Equivalence of Auctions and Posted Prices", *Games and Economic Behavior*, 27(1), 106-113.

Laing, Derek, Theodore Palivos and Ping Wang (1995), "Learning, Matching and Growth", *Review of Economic Studies*, 62, 115-129.

MacLeod, W. Bentley and James M. Malcomson (1993), "Investments, Holdup and the Form of Market Contracts", *American Economic Review*, 83, 811-837.

Malcomson, James M. (1998), "New Developments in the Study of Contracts in Labour Markets", Chapter 3? in Orly Ashenfelter and David Card (eds.): *Handbook of Labor Economics*, Vol. 3b, North-Holland Publishers, Amsterdam.

Masters, Adrian M. (1998), “Efficiency of Investment in Human and Physical Capital in a Model of Bilateral Search and Bargaining”, *International Economic Review*, 39, 477-493.

McAfee, R. Preston (1993), “Mechanism Design by Competing Sellers”, *Econometrica*, 61, 1281-1312.

Moen, Espen R. (1996), “Human Capital Investments and Market Imperfections”, *SNF Oslo, Working Paper No. 1996-8*.

——— (1997), “Competitive Search Equilibrium”, *Journal of Political Economy*, 105, 385-411.

——— (1999), “Education, Ranking and Competition for Jobs”, *Journal of Labor Economics*, 17(4), 694-723.

Mortensen, Dale T. (1982), “Property Rights and Efficiency in Mating, Racing, and Related Games”, *American Economic Review*, 72, 968-979.

Peters, Michael (1997), “A Competitive Distribution of Auctions”, *Review of Economic Studies*, Vol. 64, 97-123.

——— (2001), “Surplus Extraction and Competition”, *Review of Economic Studies*, 68, 613-631.

Pissarides, Christopher A. (1984), “Search Intensity, Job Advertising and Efficiency”, *Journal of Labor Economics*, 2, 128-143.

Riley, John G. and William F. Samuelson, “Optimal Auctions”, *American Economic Review*, 71, 381-392.

Shimer, Robert (1996), “Essays in Search Theory”, PhD dissertation, Massachusetts Institute of Technology.

——— (1999), “Job Auctions”, mimeo, Princeton University.

7 Appendix

Proof of Proposition 1

(i) *Characterisation of efficient stationary allocations*

Consider the current value Lagrangean \mathcal{L}^C associated with eqns. (2)-(3):

$$\begin{aligned} \mathcal{L}^C = & \sum_{t=0}^{\infty} \beta^t \left\{ u(t)\mu(q(t))Y(k(t)) - \frac{u(t)}{q(t)} (1 - \beta(1 - s))pk(t) \right. \\ & \left. - \xi(t)[u(t+1) - u(t) - s(1 - u(t)) + (1 - s)\mu(q(t))u(t)] \right\} \end{aligned} \quad (24)$$

where $\xi(t)$ is the (undiscounted) shadow value of an unemployed worker in period t and $Y(k(t))$ satisfies (4). Since we are interested in steady states, we henceforth suppress all time indices. The first-order conditions for k , q and u can then be written as:

$$u \left[\mu(q)(1 - s)\beta \left[\frac{f'(k)}{1 - \beta(1 - s)} - p \right] - \frac{1}{q}(1 - \beta(1 - s))p \right] \leq 0, \quad (25)$$

$$u \left[\mu'(q)Y(k) + \frac{1}{q^2}(1 - \beta(1 - s))pk - \xi(1 - s)\mu'(q) \right] \leq 0, \quad (26)$$

$$\beta^t \left[\mu(q)Y(k) - \frac{1}{q}(1 - \beta(1 - s))pk + \xi(1 - s)(1 - \mu(q)) \right] - \beta^{t-1}\xi \leq 0, \quad (27)$$

Our task is to show that the solution to equations (25)-(27) can be characterised by the solution of equation (5) in the main text. First of all, notice that the first order conditions for k and q are linear in u . For any given value for ξ , eqns. (25) and (26) can therefore be solved for the optimal choice of k and q , independent of the value of u . Denote these optimal choices by $k(\xi)$ and $q(\xi)$. These variables correspond of the solution to the following static optimization problem:

$$\max_{\{k, q\}} \left(\mu(q)Y(k) - \xi(1 - s)\mu(q) - \frac{1}{q}(1 - \beta(1 - s))pk \right) \quad (28)$$

Equation (28) is a recursion that defines the shadow value of workers; each unemployed worker is hired with probability $\mu(q)$ and with probability $(1-s)$ this match yields net-output $f(k)/(1-\beta(1-s))$ in the next period. From this we need to deduce the implicit cost of hiring labour ξ and capital pk . Moreover, while a worker is unemployed the planner sustains $1/q$ vacancies for her, and the flow cost of these vacancies is given by the last term on the right.

In order to use (28) to solve for the equilibrium value of k and q , we need to show that ξ is independent of u . This follows from eqn. (27). Solving this equation yields:

$$\xi = \beta \left(\frac{\beta(1-s)\eta(q) \left(\frac{f(k)}{1-\beta(1-s)} - pk \right) - (1-\beta(1-s))pk}{(1-\beta(1-s))q + \beta(1-s)\eta(q)} \right), \quad (29)$$

which does not depend on u . The same is therefore true for $k(\xi)$ and $q(\xi)$.

Finally, substituting (29) in (28) yields eqn. (5) in the main text. The solution to this static optimisation problem thus characterises the efficient (steady state) capital investment and queue length. Moreover, the associated unemployment rate u can be found by substituting the efficient queue length q in a steady state version of (3). This yields:

$$u = \frac{s}{(1-s)\mu(q) + s} \quad (30)$$

(ii) *Existence*

The existence of an interior solution is established using the Fundamental Theorem of Calculus. First let $V(k, q)$ denote the value of maximand (5) which is continuous on $(k, q) \in (0, \infty)^2$. The following results are immediate: $\lim_{q \rightarrow 0} V(k, q) < 0$; $\lim_{q \rightarrow \infty} V(k, q) = 0$; $\lim_{k \rightarrow 0} V(k, q) = 0$; $\lim_{k \rightarrow \infty} V(k, q) < 0$;

Extremal values of k and/or q can therefore never be a solution as long as we can show that there exist positive and finite values of k and q for which $V(k, q)$ is strictly positive. Following Acemoglu and Shimer (1999) this can be established using the Fundamental Theorem of Calculus. Fix any positive value of q such that $\eta(q) > 0$ and define the value of k^q by:

$$\frac{\beta(1-s)\eta(q)}{1-\beta(1-s)+\beta(1-s)\eta(q)} f'(k^q) = (1-\beta(1-s))p \quad (31)$$

Such a strictly positive value for k^q always exists, since I assumed that $\lim_{k \rightarrow 0} f'(k) = \infty$. Then by the fundamental theorem of calculus, it follows that $V(q, k^q)$ is equal to:

$$\begin{aligned} \int_0^{k^q} \frac{(1-s)\beta\eta(q)f(\kappa) - (1-\beta(1-s))(1-\beta(1-s)+\beta(1-s)\eta(q))p\kappa}{(1-\beta(1-s))q + \beta(1-s)\eta(q)} d\kappa &> \\ \int_0^{k^q} \frac{(1-s)\beta\eta(q)f(k^q) - (1-\beta(1-s))(1-\beta(1-s)+\beta(1-s)\eta(q))pk^q}{(1-\beta(1-s))q + \beta(1-s)\eta(q)} dk^q &= 0 \end{aligned} \quad (32)$$

which is strictly positive. The maximum is therefore an interior extremum and the Kuhn-Tucker conditions are necessary conditions. In addition, the payoffs in the original infinite horizon problem are simply equal to the discounted sum of the constant per period payoffs. These payoffs are therefore also well-defined and since q is strictly positive and finite, the steady state unemployment rate is strictly positive. Equations (28)-(30) therefore hold with equality and a solution to the original dynamic programme exists and coincides with the solution to (5) in the main text.

(iii) *Homogeneous submarkets*

As mentioned in the main text, objective function (5) is not jointly concave in k and q . As a result, equations (6) and (7) may have more than one solution. Not all of these solutions need to correspond to a maximum. Nonetheless, there may be more than one (k^S, q^S) combination that yield the same maximal welfare. The planner may therefore decide to open several submarkets. However, within each submarket only firms of one type will be allowed to operate. To see this, notice that eqn. (6) defines k^S as a strictly increasing (and concave) function of q , while conversely, (7) defines q^S as a strictly increasing (and concave) function of k . Hence, there is a positive correlation between the equilibrium values of k^S and q^S and to each value of k^S corresponds a unique value of q^S , and vice versa. Submarkets therefore consist of at most one type of job and submarkets with more capital-intensive jobs have a higher queue length. Finally, the result that a submarket contains at most one type of job, confirms that our formulation of the planner's problem is not restrictive. ■

Proof of Proposition 4

(i) *Existence*

First, consider the graph of eqn. (15). Since the first term on the left is increasing in q and $f(k)$ is strictly concave, this graph is upward sloping in (q, k) space. Moreover, the left-hand side of (15) is continuously differentiable with respect to q and k on the whole domain $[0, \infty]^2$. The conditions of the implicit function theorem are therefore satisfied and eqn. (15) implicitly defines a function $K^R(q)$ that is monotonically increasing, continuous and differentiable with $K(0) = 0$ and $\lim_{q \rightarrow \infty} K^R(q) = \bar{k}$, where $0 < \bar{k} < \infty$ is implicitly defined by:

$$\beta(1-s) \frac{f'(\bar{k})}{1-\beta(1-s)} = p. \quad (33)$$

Next, dividing (17) by (15) and rearranging terms we arrive at the following

expression:

$$\frac{\beta(1-s)e^{-q}}{1-\beta(1-s)+\beta(1-s)(1-e^{-q}-qe^{-q})} = \frac{1-\epsilon(k)}{\epsilon(k)} \quad (34)$$

where $\epsilon(k) = f'(k)k/f(k)$ is the elasticity of the production function.

The left-hand side of (34) is a decreasing function of q that maps $[0, \infty]$ onto $[0, \frac{\beta(1-s)}{1-\beta(1-s)}]$. Hence, for all values of k such that $\epsilon(k) > 1-\beta(1-s)$, condition (34) has a solution. Denote this solution by $Q^R(k)$. Two results follow immediately. First, given Condition 1, $\lim_{k \rightarrow 0} Q^R(k)$ exists and is strictly positive and finite. Second, due to the smoothness of the production function, $Q^R(k)$ has a closed graph. Given these features $Q^R(k)$ intersects $K^R(q)$ at least once on the interior of $(0, \infty)^2$. Such an intersection is a solution to equations (5.8) and (5.10) and is therefore an equilibrium.

(ii) Uniqueness

The equilibrium is unique if $\epsilon(k)$ is non-increasing. First, if $\epsilon(k)$ is constant and Condition 1 is satisfied, then $Q^R(k)$ defines a vertical line in (q, k) space with $0 < q < \infty$ which intersects $K^R(q)$ once at a level of $k < \bar{k}$. In contrast, if $\epsilon(k)$ is monotonically decreasing, then the graph of $Q^R(k)$ is strictly decreasing and has a vertical intercept at $\tilde{k} > 0$ which is defined by:

$$\frac{1}{1-\beta(1-s)} = \frac{f(\tilde{k})}{f'(\tilde{k})\tilde{k}} \quad (A13)$$

Again, therefore $K^R(q)$ and $Q^R(k)$ will intersect once on the interior. In all other cases $Q^R(k)$ and $K^R(q)$ intersect at least once, but the equilibrium need not be unique. ■

Proof of Proposition 6

Write the Lagrangean associated with programme (P1) in Lemma 4 as:

$$\mathcal{L} = \mathcal{A} \frac{f(k)}{1-\beta} + \lambda \left(\mathcal{B} \frac{f(k)}{1-\beta(1-s)} - kp \right), \quad (35)$$

where:

$$\mathcal{A} = \frac{\beta(1-s)e^{-q}}{1-\beta(1-s) + \beta(1-s)(1-qe^{-q})}$$

$$\mathcal{B} = \frac{\beta(1-s)(1-e^{-q} - qe^{-q})}{1-\beta(1-s) + \beta(1-s)(1-qe^{-q})}$$

The associated F.O.C.'s for k and q are given by:

$$\mathcal{A} \frac{f'(k)}{1-\beta} + \lambda \left(\mathcal{B} \frac{f'(k)}{1-\beta(1-s)} - p \right) \leq 0 \quad (36)$$

$$\frac{\partial}{\partial q(k)} \left(\mathcal{A} \frac{f(k)}{1-\beta} + \lambda \frac{\partial}{\partial q(k)} \left(\mathcal{B} \frac{f(k)}{1-\beta(1-s)} \right) \right) \leq 0 \quad (37)$$

Suppose that the above maximisation problem has an internal solution. We can then solve for λ from (37). This yields:

$$\lambda = \frac{1-\beta(1-s) + \beta(1-s)(1-e^{-q})}{(1-\beta(1-s))q + \beta(1-s)(1-e^{-q})} \frac{1-\beta(1-s)}{1-\beta} \quad (38)$$

Next, substituting (38) into (36) we obtain:

$$\frac{\beta(1-s)(1-e^{-q})}{1-\beta(1-s) + \beta(1-s)(1-e^{-q})} \left(\frac{f'(k)}{1-\beta(1-s)} \right) = p \quad (39)$$

This expression coincides with (6). Hence, the investment margin is efficient. The efficiency of the entry margin was shown already in Section 4. The equilibrium allocation therefore coincides with an efficient allocation and Proposition 1 shows that such an allocation exists. ■

Proof of Proposition 7

Let π denote the reserve bid of a firm. The value of a vacant firm with capital intensity k that posts a reserve bid π can then be expressed as:

$$J^V(k, \pi) = \beta(1 - s) \left(e^{-q} J^V(k, \pi) + qe^{-q}\pi + (1 - e^{-q} - qe^{-q}) (S(k) - J^U) \right), \quad (40)$$

while the expected income of a worker who applies at this firm satisfies:

$$J^U(k, \pi) = \beta \left((1 - e^{-q}) J^U + e^{-q} \left((1 - s) (S(k) - \pi) + sJ^U \right) \right), \quad (41)$$

where we have suppressed the dependence of q on k and q . The difference with eqns. (12) and (18) is that a single applicant now receives $S(k) - \pi$ instead of $S(k) - J^V(k)$. By announcing a reserve bid $\pi > J^V(k)$ the firm can therefore reduce the extent of ex post rent-sharing. However, this needs to be offset against the drop in the expected number of applicants q .

In order to solve for the equilibrium reserve bid, we need to extend the restriction on the out-of-equilibrium beliefs of firms, so that firms have consistent beliefs about q for all relevant values of k and π . Given this restriction the logic of Lemma 6.1 applies and any combination (k, π) that is observed in equilibrium must maximise the welfare of a representative worker subject to the zero profit condition of firms. A simple manipulation of eqns. (40) and (41) then yields the following constrained optimisation problem:

$$\max_{\{k, \pi, q\}} \frac{\beta(1-s)e^{-q}}{1 - \beta(1-s) + \beta(1-s)e^{-q}} \left(\frac{f(k) - \beta(1-s)\pi}{1 - \beta} \right)$$

subject to: (P2)

$$\frac{\beta(1-s)}{1 - \beta(1-s)e^{-q}} \left(qe^{-q}\pi + (1 - e^{-q} - qe^{-q}) \left(\frac{f(k) - (1-\beta)J^U}{1 - \beta(1-s)} \right) \right) - pk \geq 0$$

Assume for a moment that the above programme has an internal solution. The first-order conditions for π and q can then be solved for the optimal reserve bid π as a function of k and q . This yields:

$$\pi = \frac{\beta(1-s)(1 - e^{-q} - qe^{-q})}{1 - \beta(1-s) + \beta(1-s)(1 - qe^{-q})} \left(\frac{f(k)}{1 - \beta(1-s)} \right). \quad (42)$$

By virtue of eqn. (20) firms therefore prefer to announce their true outside option value. Next, substitute (42) into the objective function and the constraint of (P2). This yields the constrained optimisation programme (P1) that is characterised in Lemma 5 and from Proposition 6 we know that this problem has an internal solution that coincides with an efficient allocation $(k^S, q^S) \in (0, \infty)^2$. In equilibrium the reserve bid of firms is therefore given by $\pi(k^S) = J^V(k^S) = pk^S$ and the resulting allocation is constrained efficient. ■