# Information and Elections 

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#### Abstract

We present a theory of electoral campaigns in an election with multiple districts. We study the relation between voters' information and campaign promises for inefficient local public good (pork) provision. We show that if voters are poorly informed, candidates promise to deliver local public goods to every district, resulting in a policy outcome that minimizes aggregate social welfare.


Keywords: Elections, information, campaigns, pork.

## Work in Progress.

Preliminary and Incomplete.

[^0]During electoral campaigns, candidates running for office make policy proposals to woo voters. Voters pay only limited attention to electoral campaigns and as a result they do not become fully informed about the policies proposed by the candidates. We study the relation between the information acquired by the voters, and the policies that the candidates announce during the campaign and execute once in office. In particular, we explain the effect of voters' information on the provision of socially inefficient particularistic public goods.

A particularistic or local public good provides a benefit only to the members of a single district or group. If the costs of provision are spread across society at large by general taxation, voters in each district want their own particularistic public good to be provided, while they prefer the public good in any other district not to be provided. Because voters enjoy the benefits of their own project fully, while they only pay a fraction of the cost of any given project, they care more about the provision of their own good than about the non-provision of the public good in any one other given district. This leads politicians to promote inefficient policies that result in the over provision of particularistic public goods. We prove that while a perfectly informed electorate is not immune to this phenomenon, the problem is exacerbated if the electorate is poorly informed.

We find the equilibrium policy proposals and electoral outcomes in an election with two office-motivated candidates, who compete for votes by proposing to implement
local public good projects that are inefficient for society, but beneficial to the district in which they are developed. Since voters care more about policies that directly affect their districts, if their attention span is limited, they naturally become better informed about these proposals than about projects in other districts that only affect them indirectly through general taxation. We solve the imperfect information game in which voters only observe the policy proposals for their own district, and form beliefs about the candidates' spending plans in other districts based on what they observe with respect to their own district. We find that results depend on these beliefs: Almost any candidate strategy can be supported in equilibrium if every voter believes that a candidate who deviates from equilibrium is a high spender who provides the local public good in all other districts.

While we prove that standard game-theoretic solution concepts refinements fail to make sharp predictions, we nevertheless argue that not all equilibria are equally plausible. Some equilibria hold only if voters hold very particular beliefs after any deviation, pessimistic beliefs that the number of projects that the candidate intends to fund in other districts is very large. Other equilibria hold for a large set of beliefs. Given that electoral campaigns can perturb voters' beliefs, we argue that the equilibrium that depend on very specific beliefs are fragile and implausible. We argue that the equilibrium that holds for the largest set of beliefs is more robust and more likely to hold in practice, and thus a better prediction. This equilibrium is unique: If local public goods are very inefficient, candidates propose not to provide any of
them, whereas if local public goods are sufficiently efficient that a minimal winning coalition of districts would prefer to provide the goods in the coalition's districts, then candidates propose to provide goods to every district, an outcome that makes every district worse off.

This "pork for everybody" result coincides with the policy outcome obtained in the seminal theory of distributive policies based on bargaining in a legislative assembly by Weingast [33], Shepsle and Weingast [31] and Weingast, Shepsle and Johnsen [34], where legislators commit to a norm of universalism by which every district gets its own inefficient project, rather than letting a minimal-winning majority distribute public goods only to the districts in this majority. An objection to this seminal theory is that it cannot explain why legislators do not embrace instead a Pareto-superior universalist norm by which no inefficient local public goods are ever provided. In fact, if legislators do not commit to any norm, Ferejohn, Fiorina and McKelvey [16] and Baron [3] show that only a minimal winning majority of districts benefit from the provision of inefficient projects.

A stream of economic theories explain targeted redistribution as the equilibrium outcome of a game in which candidates compete in elections (Lindbeck and Weibull [22]; Dixit and Londregan [11]; McKelvey and Riezman [25]; Lizzeri and Persico [23]; Chari, Jones and Marimon [9]; the survey by Persson and Tabellini [29]; and the more recent article by Roberson [30]). These theories assume that citizens are fully informed about the policy proposals made by the candidates. The assumption is unrealistic.

The empirical literature on voter behavior has conclusively established that in practice voters have a sketchy idea of these policy proposals (Campbell, Converse, Miller and Stokes [8]; Bartels [5] and Alvarez [1]).

Our contribution narrows the gap between the assumptions in the theoretical literature, and the accepted stylized facts of the empirical literature, by recognizing that voters have only partial information about candidates' policy proposals.

Ours is not the first theory of elections with voters that are not fully informed. At the opposite extreme, Grosser and Palfrey [17] assume that citizens do not know anything about the candidates. McKelvey and Riezman [25] and Muthoo and Shepsle [26] assume that citizens only know if the candidate is an incumbent or not. McKelvey and Ordeshook [24], and Baron [4] aim at an intermediate model by mixing both extreme assumptions: Some citizens are fully informed, while others are fully uninformed. ${ }^{1}$ We believe that it is more realistic to assume that voters have some but not all the information about the candidates' policy proposals, and this intermediate approach is the one we pursue.

Niou and Ordeshook [27] discuss various models of legislative bargaining. We can interpret their results on pages $255-256$ as a model in which (a) voters only observe the policy outcome in their locality, and (b) they vote naively based on this

[^1]direct evidence without making inferences about the unobserved policies in other districts, blindly reelecting incumbents who provided local public goods, even when this choice in expectation makes the voters worse off. While we assume (a), we reject (b): we model rational voters who use all the information they have to form rational expectations and to choose the alternative with the highest expected payoff.

## 1 The Model

Let there be a society partitioned into three subsets, with one representative voter $i \in\{a, b, c\}$ in each subset. We refer to these subsets as districts, but they could also be population groups of similar size divided by age or profession.

Two candidates $A$ and $B$ compete for election. Let $J \in\{A, B\}$ denote an arbitrary candidate and let $-J$ denote the other candidate, so that $\{J,-J\} \equiv\{A, B\}$.

The policy space consists on whether or not to approve each of three different projects, one per district. A strategy for each candidate consists on proposing a policy in the policy space. Let $S^{J}=S^{-J}=\{0,1\}^{3}$ be the strategy set of each candidate. Let $s^{J}=\left(s_{a}^{J}, s_{b}^{J}, s_{c}^{J}\right) \in\{0,1\}^{3}$ be a strategy by candidate $J$, where $s_{i}^{J}=1$ indicates that $J$ proposes to approve the project in district $i$ and $s_{i}^{J}=0$ indicates that $J$ proposes not to approve it. Let $s_{-i}^{J}$ denote the proposals for the other two districts, not including $i$.

Voters only observe some of the information contained in the policy proposals, as
described below, and an election is held, where each voter chooses to vote either for $A$ or $B$, or abstain. The candidate with most votes wins, and in case of a tie, each candidate wins with equal probability. We assume that the winning candidate carries out her proposal once in office; or alternatively, we assume that voters vote as if this were to be the case.

Each of the three district-specific projects brings a benefit to its district if implemented, but the costs of any implemented project are borne equally among all three districts. Voters' preferences depend on the benefit of the projects relative to their cost. We assume that projects are inefficient in the sense that no district would like to implement its own project if it had to bear its full cost. Let $\beta$ denote the ratio of the benefit/cost of each project, or, alternatively, let $\beta$ be the benefit given a normalization of the cost to one. We refer to $\beta \in\left(\frac{2}{3}, 1\right)$ as the "high benefit" case and $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$ as the "low benefit" case. ${ }^{2}$

In the high benefit case, the preference order of voter $a$ over the eight possible policy outcomes is

$$
\begin{equation*}
\{1,0,0\} \succ_{a}\{1,1,0\} \sim_{a}\{1,0,1\} \succ_{a}\{0,0,0\} \succ_{a}\{1,1,1\} \succ_{a}\{0,0,1\} \sim_{a}\{0,1,0\} \succ_{a}\{0,1,1\} . \tag{1}
\end{equation*}
$$

[^2]In the low benefit case,

$$
\begin{equation*}
\{1,0,0\} \succ_{a}\{0,0,0\} \succ_{a}\{1,1,0\} \sim_{a}\{1,0,1\} \succ_{a}\{0,0,1\} \sim_{a}\{0,1,0\} \succ_{a}\{1,1,1\} \succ_{a}\{0,1,1\} . \tag{2}
\end{equation*}
$$

An analogous preference order holds for voters $b$ and $c$.
We assume that candidates and voters are fully strategic, rational agents. Candidates are office motivated and do not care about policy outcomes. They have lexicographic preferences first for maximizing the probability of winning, and, given a fixed probability of winning, they maximize the expected margin of victory (which can be negative if the candidate loses).

We assume that each voter $i$ calculates an expected payoff for each candidate $J$, using all the information available to the voter about the policy proposal of the candidate. We refer to this calculated expected payoff as the expected payoff of candidate $J$ for voter $i$. Since voter $i$ does not know the full policy proposal $s^{J}$, then the expected payoff depends on the beliefs of the voter about $s^{J}$, and if the beliefs are not correct, the expected payoff may not be the ex-post payoff that $i$ obtains if $J$ wins the election (note however that this can only occur outside of the equilibrium path). We assume that voters vote for the candidate with the highest expected payoff, that is, voters are sequentially rational (Kreps and Wilson [20]). We assume that voters do not use weakly dominated strategies, nor iteratively weakly dominated strategies. This rules out equilibria in which all voters vote for the same candidate even though
some voters prefer the losing candidate's proposal. If the expected payoffs of both candidates coincide, voters are indifferent given their beliefs. We assume that in this case they abstain, unless abstention has been eliminated as a weakly dominated strategy.

We consider several games with different information structures. In each of these games, the players are the set of candidates $\{A, B\}$ and the set of voters $\{a, b, c\}$.

Our benchmark solution concept is Pure Perfect Bayesian Equilibria that survive the iterated elimination of weakly dominated strategies. If the set of these equilibria is very large, we refine the solution concept to identify which equilibria hold when we restrict the set of admissible beliefs off the equilibrium path, as detailed below. If (and only if) there exists no equilibrium in pure strategies, we look at mixed strategy equilibria, but unless otherwise specified, an "equilibrium" means a Perfect Bayesian Equilibrium in pure strategies that survives the iterative elimination of weakly dominated strategies.

We analyze the influence of voters' imperfect information on the candidates' proposals, implemented policy, and welfare of voters, relative to the perfect information benchmark. With perfect information, Laffond, Laslier and Le Breton [21] find that abstracting from voters' strategies and letting candidates play in a tournament that represents the social preference given by a simple majority aggregation of sincere preferences, there exists a unique equilibrium, possibly in mixed strategies. In a short research note, Eguia and Nicolò [12] find this unique equilibrium in the electoral game
with strategic voters and candidates:

Claim 1 (Eguia and Nicolò [12]) There is a unique equilibrium. In the low benefit case $\beta<\frac{2}{3}$, in equilibrium zero local projects are implemented. In the high benefit case $\beta \in\left(\frac{2}{3}, 1\right)$, in equilibrium candidates use mixed strategies, in expectation each project is implemented with probability 43\%, and ex-post zero, one or two projects are implemented.

Laffond, Laslier and Le Breton find that if we abstract from voters' strategies and we let candidates play in a tournament that represents the social preference given by a simple majority aggregation of sincere voters' preferences, then there exists a unique equilibrium, possibly in mixed strategies. We compare our results under imperfect information, relative to this benchmark. We relegate some results and all proofs to an appendix.

## 2 Imperfect Information

Assume that after candidates make their policy proposals, these proposals need not become public knowledge; voters may receive only partial information and become unaware of relevant parts of the proposals.

Specifically, assume that after candidates choose policy proposals, Nature determines whether these proposals become public (and common) knowledge or not. Policy proposals become common knowledge with probability $\varepsilon \geq 0$, where $\varepsilon$ is either $\varepsilon=0$
or very small. With probability $1-\varepsilon$, each voter $i \in\{a, b, c\}$ only observes what candidates commit to do in her district, and she is completely unaware of what candidates promise in the other districts.

For each candidate $J \in\{A, B\}$, the strategy set $S^{J}$ consists of the following eight strategies.
$s_{1}$ : Propose policy $(0,0,0)$;
$s_{2}:$ Propose $(1,0,0) ;$
$s_{3}:$ Propose $(0,1,0) ;$
$s_{4}$ : Propose $(0,0,1) ;$
$s_{5}:$ Propose $(1,1,0) ;$
$s_{6}$ : Propose $(1,0,1)$;
$s_{7}$ : Propose $(0,1,1)$ and
$s_{8}:$ Propose $(1,1,1)$.

If Nature makes proposals public knowledge, voters observe $s^{A}$ and $s^{B}$ and compare the two proposals and vote for the one they prefer, according to the preference order 1 or 2 . If proposals do not become public knowledge, each voter $i$ remains unaware about what each candidate proposes in districts other than $i$. Note that voter $i$ has two information sets with respect to the strategy of candidate $J$ : Voter $i$ observes either $s_{i}^{J}=0$ or $s_{i}^{J}=1$. Since there are two candidates, $\left(s_{i}^{A}, s_{i}^{B}\right) \in\{0,1\} \times\{0,1\}$, so that voter $i$ has four information sets in which to make a decision, and three possible
actions (vote $A$, vote $B$ or abstain) in each of these sets.
Under this informational structure, in which voters have imperfect information, an equilibrium must describe strategies for voters and candidates, and beliefs for voters. The beliefs of voter $i$, denoted $\delta_{i}$, is a vector $\delta_{i}=\left(\delta_{i}^{A}, \delta_{i}^{B}\right)$, where $\delta_{i}^{J}=\left(\delta_{i}^{J}(0), \delta_{i}^{J}\left(s_{i}^{J}\right)\right)$ for each $J \in\{A, B\}$, and $\delta_{i}^{J}\left(s_{i}^{J}\right)$ is the probability distributions over $S^{J}$ that voter $i$ holds as a belief about $s^{J}$ after observing $s_{i}^{J}$. For $s_{i}^{J} \in\{0,1\}$, let $\omega_{k}^{i, J}\left(s_{i}^{J}\right)$ be the sum of weights assigned by $\delta_{i}^{J}\left(s_{i}^{J}\right)$ to the set of strategies where $J$ proposes to carry out $k$ projects in districts other than $i$.

Beliefs along the equilibrium path must be correct. Beliefs off the equilibrium path must assign all the probabilities to undominated strategies if any such strategy is consistent with the information possessed by the agent. ${ }^{3}$

The strategy pair $\left(s^{A}, s^{B}\right)$ determines the information $\left(s_{i}^{A}, s_{i}^{B}\right)$ received by each voter $i$. This information, together with beliefs $\delta_{i}$, determine the expected utility for $i$ if $A$ or $B$ wins, which in turn determines agent $i^{\prime} s$ vote and therefore, aggregating over all three agents, it determines the electoral outcome and the payoffs to $A$ and $B$. Therefore, we can express the payoffs for candidate $J$ as a function of the strategy pair $\left(s^{A}, s^{B}\right)$ and the beliefs $\delta$.

The set of Pure Perfect Bayesian Equilibria that survives the iterated elimination of weakly dominated strategies is large. While exhaustively prove in the appendix

[^3]which candidate strategy pairs can and which cannot be sustained in equilibrium, we seek sharper predictions.

We find that all equilibria that hold if $\varepsilon>0$ hold as well if $\varepsilon=0$, but the converse is not true. The equilibria that hold only for $\varepsilon=0$ and not for $\varepsilon>0$ are knife edged in the sense that they depend on a precise informational assumption. We focus on the set of equilibria that hold for any $\varepsilon \in\left[0, \frac{1}{2}\right)$; in an alternative interpretation, we refine the set of solutions with $\varepsilon=0$, including in the refined set only the equilibria that can be approached as the limit of a sequence of equilibria in games with a positive $\varepsilon \rightarrow 0$. We provide the precise set and prove in proposition 8 in the appendix. Here we merely summarize the more relevant findings.

Claim 2 If $\varepsilon>0$, in any equilibrium both candidates propose to carry out the same number of projects and the election is tied. Given any $\varepsilon \in\left[0, \frac{1}{2}\right)$, for any number of projects $k \in\{0,2,3\}$ or for any $k \in\{0,1,2,3\}$ if $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$, there exists an equilibrium in which each candidate proposes $k$ projects.

If $\varepsilon=0$, additional equilibria emerge in which candidates propose to carry out a different number of projects and one candidate wins outright. The existence of a sure loser who gives up and doesn't try to imitate the strategy of the winner makes these equilibria seem implausible in practice; their failure to hold if $\varepsilon>0$ makes them easy to dismiss on theoretical grounds as non-robust to small perturbations to the game.

All the equilibria are sequential (Kreps and Wilson [20]): At every informational
node, even those off the equilibrium path, agents maximize given their beliefs. Candidates only have one informational set: They move simultaneously at the beginning of the game. Voters have four non trivial informational sets where full information is not revealed. By assumption, at each node they vote for the candidate that maximizes the expected utility of the voter. This assumption on voters' behavior guarantees that all equilibria satisfy sequential rationality. All equilibria also trivially satisfy refinements meant for signalling games such as the intuitive criterion and divinity (Cho and Kreps [10]) because candidates are ex-ante identical and are not differentiated by type.

We show in the appendix that all the equilibria described in claim 2, which hold for zero or positive epsilon, are extensive form rationalizable (Pearce [28] and Battigali [6]) and that they satisfy not only the weak forward induction property proposed by Van Damme [32], but also a more demanding forward induction refinement that we make explicit in the appendix. ${ }^{4}$

However, despite the failure of all these refinements to discriminate any further among all the large set of remaining equilibria, we argue that not all equilibria described in claim 2 are equally plausible. Some of these equilibria require voters to have very pessimistic beliefs about candidates' strategies, and to update very nega-

[^4]tively following any deviation. Other equilibria hold for a large set of possible beliefs. For instance, assume that $\beta$ is very close to one, so that projects' inefficiency is very small. Candidates' strategy pairs $((0,0,0),(0,0,0))$ and $((1,1,1),(1,1,1))$ can both be sustained in equilibria in which voters abstain and the election is tied. However, the first equilibrium is only supported if a voter who observes a deviation is almost certain that the candidate has deviated to $(1,1,1)$. For any other belief, the voter would support the candidate who deviates. We thus find that the equilibrium with proposals $((0,0,0),(0,0,0))$ is fragile, as it depends on very specific off-equilibrium beliefs and breaks down if beliefs change. On the other hand, the equilibrium with proposals $((1,1,1),(1,1,1))$ hold for almost any belief that voters may have, so it is more robust to changes in the underlying beliefs about off-equilibrium events.

We refine the set of solutions by selecting the equilibria that are supported by a largest set of beliefs.

Consider $\omega_{0}^{i, J}(k), \omega_{1}^{i, J}(k)$ and $\omega_{2}^{i, J}(k)$ where voter $i$ observes $\tilde{s}_{i}^{J}=k \in\{0,1\}$ out of the equilibrium path. Let $\rho^{i, J}(k) \equiv \omega_{1}^{i, J}(k)+2 \omega_{2}^{i, J}(k)$, so that $\rho^{i, J}(k)$ is the number of other projects that voter $i$ expects that candidate $J$ has proposed in other districts, given that $i$ observes a deviation. Informally, a high value of $\rho^{i, J}(k)$ means that $i$ holds pessimistic beliefs about $J$ following the deviation. Pessimistic beliefs make deviations fail, which in turn sustains equilibria. We seek to find which equilibria require less of such pessimism in order to hold. Given any equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)$, construct the set of values $C_{\left(s^{A}, s^{B}\right)} \subseteq[0,2]$ in the following way.

For any value $x \in C_{\left(s^{A}, s^{B}\right)}$, if the beliefs of any agent $i$ who observes any deviation are such that $i$ believes that in expectation the deviating candidates proposes at least $x$ projects in other districts, then no deviation is beneficial to a candidate. That is, $C_{\left(s^{A}, s^{B}\right)} \equiv\left\{x \in[0,2]:\left\{\delta\right.\right.$ is such that $\rho^{i, J}(k) \geq x \forall i \in\{a, b, c\}, \forall J \in\{A, B\}$, $\forall k \in\{0,1\}$ and for any out of equilibrium $\left.\tilde{s}_{i}^{J}=k\right\} \Longrightarrow\left\{\right.$ strategy pair $\left(s^{A}, s^{B}\right)$ holds in equilibrium with beliefs $\delta\}\}$. We are interested in the equilibrium with the smallest such associated set.

Definition 1 Let $S^{*}$ be the set of equilibrium candidate strategy pairs. An equilibrium candidate strategy pair $\left(\tilde{s}^{A}, \tilde{s}^{B}\right)$ is most optimistic if $C_{\left(\tilde{s}^{A}, \tilde{s}^{B}\right)} \subseteq C_{\left(s^{A}, s^{B}\right)}$ for any $\left(s^{A}, s^{B}\right) \in S^{*}$.

Each equilibrium holds for some range of off-equilibrium beliefs after each deviation. Consider each possible deviation by a candidate. Each voter who observes a deviation in her district and does not know the policy proposal in the other districts has a cutoff in $[0,2]$ such that the voter supports the deviation if the voter's expectation over the number of projects proposed in other districts is below the cutoff. For each deviation, we find the lowest common cutoff such that if all voters who observe the deviation have expectations above the cutoff, the deviation fails. We can interpret this lowest common cutoff as the minimum degree of pessimism that guarantees that this particular deviation fails to unravel the equilibrium. For each equilibrium, find the highest of all such lowest common cutoffs among all possible
deviations. That is the minimum measure of pessimism that guarantees that no deviation breaks the equilibrium. Then, find the equilibrium that requires the lowest such minimum measure of pessimism to hold. This is the most optimistic equilibrium, with the smallest set $C_{\left(\tilde{s}^{A}, \tilde{s}^{B}\right)}$. The following example shows how to calculate the cutoff for each equilibrium.

Example 1 Suppose that $\beta=\frac{5}{6}$. Consider first an equilibrium with $\left(s^{A}, s^{B}\right)=$ $((0,0,0),(0,0,0))$. Suppose voter $i$ observes the deviation $\tilde{s}_{i}^{A}=1$. Voter $i$ votes for $A$ if $\beta-\frac{1+\rho^{i, A}(1)}{3}>0$, that is, if $\rho^{i, A}(1)<\frac{3}{2}$. So the equilibrium requires voter $i$ to believe that $A$ is proposing to carry out in expectation anywhere between 1.5 and 2 projects in other districts.

Consider instead an equilibrium with $\left(s^{A}, s^{B}\right)=((1,1,1),(1,1,1))$. Suppose voter $i$ observes the deviation $\tilde{s}_{i}^{A}=0$. Voter $i$ votes for $A$ if $-\frac{\rho^{i, A}(0)}{3}>\beta-1$, that is, if $\rho^{i, A}(0)<\frac{1}{2}$. So the equilibrium requires voter $i$ to believe that $A$ is proposing to carry out in expectation anywhere between 0.5 and 2 projects in other districts. This second equilibrium requires less pessimism on the part of voters who observe a deviation and thus we find it more it more likely to hold under the Most Optimistic selection criterion.

Calculating the measure of pessimism for each of the equilibria, we obtain the following result.

Proposition 3 Assume $\varepsilon>0$. The most optimistic equilibrium is unique. If $\beta \in$ $\left(\frac{2}{3}, 1\right)$, in the most optimistic equilibrium both candidates propose to implement every project; whereas, if $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$ candidates propose the welfare maximizing policy of not implementing any project.

Since projects are inefficient, it follows that in the high benefit where the inefficiency is not too great, candidates converge on the welfare minimizing policy. On the other hand, candidates do not implement very inefficient projects: if the benefit is very low, both candidates propose the welfare maximizing policy. We can loosely interpret this result as "pork for everybody, but no bridges to nowhere."

While comparing the set of beliefs that support an equilibrium allows us to select a unique equilibrium as the most optimistic, we can also use the same intuition to provide a more complete comparison about the plausibility of different equilibria according to the sizes of the sets $C_{\left(\tilde{s}^{A}, \tilde{s}^{B}\right)}$. Our intuition is that each equilibrium is increasingly plausible in the size of its set. The most optimistic equilibrium is the most plausible, but we are unable to select it with full confidence at the expense of other equilibria; instead, we only offer an order of plausibility.

In figure 1 we show the area of $C_{s^{A}, s^{B}}$ for the Pareto efficient equilibrium $(000,000)$, the minimal winning coalition equilibrium in which each candidate proposes to carry out two projects, and the "pork for everybody" equilibrium (111, 111). Equilibria in which candidates propose to carry out one project only hold for the low benefit case,

## Lower bound of $\mathrm{C}_{\mathrm{sA}, \mathrm{sB}}$



Figure 1: Pessimism necessary to sustain different equilibria
and even then, they are supported by a smaller set of beliefs that other equilibria, so we exclude them from the figure. If $\beta$ is very low, the equilibrium with zero projects holds with many optimistic beliefs that destroy the other equilibria. If $\beta$ approaches one, the equilibrium with three projects is the one that holds even if agents have very optimistic beliefs. Thus in these extreme cases, we are fairly confident that these equilibria, and not others, are more likely to hold; whereas, if $\beta$ takes intermediate values around $\frac{2}{3}$, all three equilibria appear similarly likely to hold according to our selection criteria based on beliefs off equilibrium path.

The optimism criterion is an equilibrium refinement or at least a selection criterion based on the size of the set of beliefs that support each particular equilibrium. While in principle this criterion is applicable to any imperfect information game, we argue
that this criterion is particularly apt in the application to elections, even if it may not be as appealing in other applications. In an election, candidates conduct electoral campaigns that aim to persuade voters by changing their information and beliefs (Freedman, Franz and Goldman [15]; Huber and Arceneaux [18]). Standard game theory is adamant that in equilibrium beliefs must be correct. If so, campaigns must find it difficult to alter voters' beliefs about events along the equilibrium path. Beliefs off the equilibrium path, on the other hand, are not supported by any evidence that the voter can observe, and we therefore conjecture that the out of equilibrium beliefs are more malleable.

In a meta-game where campaigns by lobbyists or activists, or daily events can sway off-equilibrium path beliefs, an equilibrium that depends on very particular offequilibrium path beliefs will break down more easily than another equilibrium that is robust to greater variations in beliefs. We therefore argue that more optimistic equilibria are the most likely to hold.

## 3 Discussion

We have developed a theory on the provision of particularistic goods that are socially inefficient. We argue that inefficient policies arise because candidates who run for election compete for the votes of a poorly informed electorate. Citizens are better informed about government expenditures in their own district (which they favor)
than about government expenditure in other districts (which they oppose). This informational bias leads to an over provision of socially inefficient local public goods, and to a Pareto dominated outcome.

The policy outcome depends on the out of equilibrium beliefs of the voters, and a multiplicity of equilibria may hold. We proposed a selection criterion to make sharper predictions that identify the equilibria that is most likely to hold, which we argue is the equilibria that is robust to the greatest shock to out of equilibrium beliefs, which we call the most optimistic equilibrium.

We find that in a most optimistic equilibrium, public goods that provide less than 66 cts of benefit per dollar of cost are not provided, just as they are not be provided with a perfectly informed electorate. However, in a most optimistic equilibrium, public goods that are inefficient, but provide benefits of at least 67 cts on the dollar, are provided in every district. This amounts to providing pork to every constituency, as long as pork is not too inefficient. The provision of inefficient local public goods (that are not too inefficient) is therefore greater with an imperfectly informed electorate.

This is a sharp prediction that can be easily tested in laboratory experiments, which we will conduct in future research to assess the predictive success of our proposed solution concept.

It is also worth noting that while the case with three districts suffices to convey the intuition of the results in the clearest manner, results are robust to a society $N$ with an arbitrary odd number $n$ of districts. For any $n$, we can show that with
perfect information, no local public good is provided if $\beta<\frac{n+1}{2 n}$, but if $\beta>\frac{n+1}{2 n}$ the equilibrium is in mixed strategies and in expectation each district receives its local public good with some intermediate probability. Whereas, with imperfect information, in the most optimistic equilibrium no local public good is provided if $\beta<\frac{n+1}{2 n}$ but all districts receive a local public good if $\beta>\frac{n+1}{2 n}$, which is a Pareto-dominated, aggregate welfare minimizing policy outcome. ${ }^{5}$

Our results have normative implications with regard to efforts to educate voters and obtain a more informed citizenry. We have shown that even if voters are perfectly informed about the local or group-specific issue that concerns them the most, lack of information about other issues leads to a policy failure: the overprovision of inefficient particularistic public goods, and the minimization of aggregate social welfare.

In order to reach a better policy outcome, voters must become informed not only about policies that affect their district directly, but also about national aggregates that affect them indirectly through general taxation. Therefore, the intuition that a more informed electorate induces better policy outcomes is superficially true, but must be qualified: Information drives by interest groups such as the Sierra Club (environment), the National Rifle Association (gun rights), local media or by any organization with parochial interests that educates voters on a particular policy and skews them to vote according to their information about on this issue alone may lead to an aggregate policy outcome that makes every voter worse off.

[^5]By contrast, non-partisan, non-profit organizations such as Project Vote Smart ${ }^{6}$ provide voter education, informing voters about candidates's positions on every issue without endorsing any of them. We believe that it is this kind of neutral and allencompassing voter education, and not the information campaigns driven by special interests, that will lead to an electorate that makes better choices and candidates who offer better policy proposals.

## 4 Appendix

It is useful to classify strategy pairs in classes of strategic equivalence, as follows:

$$
\begin{aligned}
& \text { Let } S_{1}=\left\{\left(s_{1}, s_{1}\right)\right\} ; S_{2}=\left\{\left(s_{1}, s_{2}\right),\left(s_{1}, s_{3}\right),\left(s_{1}, s_{4}\right),\left(s_{2}, s_{1}\right),\left(s_{3}, s_{1}\right),\left(s_{4}, s_{1}\right)\right\} ; \\
& \\
& S_{3}=\left\{\left(s_{1}, s_{5}\right),\left(s_{1}, s_{6}\right),\left(s_{1}, s_{7}\right),\left(s_{5}, s_{1}\right),\left(s_{6}, s_{1}\right),\left(s_{7}, s_{1}\right)\right\} ; S_{4}=\left\{\left(s_{1}, s_{8}\right),\left(s_{8}, s_{1}\right)\right\} ; \\
& \\
& S_{5}=\left\{\left(s_{2}, s_{2}\right),\left(s_{3}, s_{3}\right),\left(s_{4}, s_{4}\right)\right\} ; S_{6}=\left\{\left(s_{2}, s_{3}\right),\left(s_{2}, s_{4}\right),\left(s_{3}, s_{4}\right),\left(s_{3}, s_{2}\right),\left(s_{4}, s_{2}\right),\left(s_{4}, s_{3}\right)\right\} ; \\
& S_{7}=\left\{\left(s_{2}, s_{5}\right),\left(s_{2}, s_{6}\right),\left(s_{3}, s_{5}\right),\left(s_{3}, s_{7}\right),\left(s_{4}, s_{6}\right),\left(s_{4}, s_{7}\right),\left(s_{5}, s_{2}\right),\left(s_{6}, s_{2}\right),\left(s_{5}, s_{3}\right),\left(s_{7}, s_{3}\right),\left(s_{6}, s_{4}\right),\left(s_{7}, s_{4}\right.\right. \\
& \\
& S_{8}=\left\{\left(s_{2}, s_{7}\right),\left(s_{3}, s_{6}\right),\left(s_{4}, s_{5}\right),\left(s_{7}, s_{2}\right),\left(s_{6}, s_{3}\right),\left(s_{5}, s_{4}\right)\right\} ; \\
& \\
& S_{9}=\left\{\left(s_{2}, s_{8}\right),\left(s_{3}, s_{8}\right),\left(s_{4}, s_{8}\right),\left(s_{8}, s_{2}\right),\left(s_{8}, s_{3}\right),\left(s_{8}, s_{4}\right)\right\} ; \\
& \\
& S_{10}=\left\{\left(s_{5}, s_{5}\right),\left(s_{6}, s_{6}\right),\left(s_{7}, s_{7}\right)\right\} ; S_{11}=\left\{\left(s_{5}, s_{6}\right),\left(s_{5}, s_{7}\right),\left(s_{6}, s_{7}\right),\left(s_{6}, s_{5}\right),\left(s_{7}, s_{5}\right),\left(s_{7}, s_{6}\right)\right\} ; \\
& \\
& S_{12}=\left\{\left(s_{5}, s_{8}\right),\left(s_{6}, s_{8}\right),\left(s_{7}, s_{8}\right),\left(s_{8}, s_{5}\right),\left(s_{8}, s_{6}\right),\left(s_{8}, s_{7}\right)\right\} ; S_{13}=\left\{\left(s_{8}, s_{8}\right)\right\} . \\
& \text { Note that } S=S^{A} \times S^{B}=\prod_{k=1}^{13} S_{k} \text { is the set of all possible candidates' strategy }
\end{aligned}
$$

pairs. Within each class, it is without loss of generality to establish whether any one

[^6]of the elements can or cannot be supported in equilibrium.
To simplify notation, and given that voters' strategies are straightforward when full information is revealed, in all the analysis below we implicitly assume that if Nature fully reveals the policy proposals, voters vote according to their preferences and abstain when indifferent. This allows us to focus our analisys of voters on the branches of the game after Nature chooses not to reveal the full information so that voters face uncertainty. ${ }^{7}$ For each voter $i \in\{a, b, c\}$, let $s^{i}:\{0,1\} \times\{0,1\} \longrightarrow$ $\{A, B, \emptyset\}$ be a behavioral strategy for voter $i$, which is a function that maps each information set of the voter when Nature does not reveal the policy proposals fully, into an action by the voter. A complete strategy for the voter specifies $s^{i}$, and the actions to be taken when information is fully revealed. We also express $s^{i}$ as a vector $s^{i}=\left(s_{1}^{i}, \ldots, s_{4}^{i}\right)$, where $s_{k}^{i}$ is the action chosen under the $k$-th information set according to the following order: $\{(0,0),(0,1),(1,0),(1,1)\}$.

It is also useful to define $v: S \longrightarrow\{A, B, \emptyset\}^{3}$ as the list of votes by voters $\{a, b, c\}$ as a function of candidates' strategies, given some beliefs. We stress that this function is only defined given beliefs, and we will only use it after specifying beliefs, or providing a strategy pair that the voters believe is being played.

Lemma 4 Assume $\varepsilon=0$. Every strategy is undominated.

[^7]Proof. No candidate strategy is weakly dominated, because the payoffs to candidates depend on the strategies of the voters.

For the voters, consider the generic information set $\left(s_{i}^{A}, s_{i}^{B}\right)=(x, y)$ with $x, y \in$ $\{0,1\}$. If $s^{b}(x, y)=s^{c}(x, y)=\emptyset, s^{J}$ consists of proposing $(x, 0,0)$, and $s^{-J}$ consists of proposing $(y, 1,1)$, then $a$ is strictly better off voting for $J$, while if $s^{J}$ consists of $(x, 1,1)$ and $s^{-J}$ consists of $(y, 0,0)$, then $a$ is strictly better off voting for $-J$. Thus, any strategy is undominated.

Proposition 5 Assume $\varepsilon=0$ and $\beta \in\left(\frac{2}{3}, 1\right)$. An equilibrium in which candidates use the strategy pair $\left(s^{A}, s^{B}\right)$ exists if and only if $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,2,3,4,8,11,13\}$.

Proof. For each strategy pair class, we find whether an element of the class can be sustained in equilibrium:
$S_{1}:$ Voter strategy $s^{i}=(\emptyset, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega_{0}^{i, J}(0)=$ 1 and $\omega_{2}^{i, J}(1)=1$ for any voter $i$ and any candidate $J$ make the election tied and if candidate $J$ deviates to any $s^{J} \neq s_{1}$, then $J$ loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.
$S_{2}$ : Assume without loss of generality that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{2}\right)$. In equilibrium, $v\left(s_{1}, s_{2}\right)=(B, A, A)$. Let $\omega_{2}^{a, B}(0)=1$ and $\omega_{2}^{b, A}(0)=\omega_{2}^{c, A}(0)=1$. Then candidate
$A$ cannot win the election by deviating and candidate $B$ cannot increase her vote margin by deviating.
$S_{3}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{5}\right)$. In equilibrium, $v\left(s_{1}, s_{5}\right)=(B, B, A)$. Suppose $\delta$ is such that $\omega_{2}^{a, A}(1)=\omega_{2}^{b, A}(1)=\omega_{2}^{c, B}(1)=1$. Then neither candidate can improve her electoral outcome by deviating.
$S_{4}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{8}\right)$. Given beliefs such that $\omega_{2}^{i, B}(0)=1$ for all $i \in\{a, b, c\}$, in equilibrium every voter votes for $A$ and continues to vote for $A$ after any deviation by $B$.
$S_{5}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$. Given $\left(s_{2}, s_{2}\right)$, every voter $i$ abstains. If $A$ deviates to $s^{A}=s_{6}, v\left(s_{6}, s_{2}\right)=(\emptyset, \emptyset, A)$.
$S_{6}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$. Given $\left(s_{2}, s_{3}\right), v\left(s_{2}, s_{3}\right)=(A, B, \emptyset)$. If $A$ deviates to $s^{A}=s_{6}$, only voter $c$ observes the deviation, so $v\left(s_{6}, s_{2}\right)=(A, B, A)$.
$S_{7}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{5}\right)$. Given $\left(s_{2}, s_{5}\right), v\left(s_{2}, s_{5}\right)=(A, B, A)$. If $B$ deviates to $s^{B}=s_{7}$, only voter $c$ observes the deviation, so $v\left(s_{2}, s_{7}\right)=(A, B, B)$. $S_{8}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{7}\right)$. Beliefs such that $\omega_{2}^{a, B}(1)=\omega_{2}^{b, A}(1)=$ $\omega_{2}^{c, A}(1)=1$ support an equilibrium in which $v\left(s_{2}, s_{7}\right)=(A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.
$S_{9}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{8}\right)$. Given $\left(s_{2}, s_{8}\right), v\left(s_{2}, s_{8}\right)=(A, B, B)$.
If candidate $A$ deviates to $s_{8}$, voters $b$ and $c$ either vote for candidate $A$ or abstains, depending upon their beliefs, and therefore candidate $A$ wins the election (voter $a^{\prime} s$ beliefs do not change).
$S_{10}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{5}\right)$. Given $\left(s_{5}, s_{5}\right)$, all voters abstain. If candidate $A$ deviates and proposes $s_{8}$, beliefs of voters $a$ and $b$ are unaffected, while voter $c$ votes for candidate $A$ for all possible beliefs over candidate $A^{\prime} s$ strategy. Therefore candidate $A$ wins the election.
$S_{11}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Beliefs such that $\omega_{2}^{a, A}(0)=\omega_{2}^{a, B}(0)=$ $\omega_{2}^{b, B}(1)=\omega_{2}^{c, A}(1)=1$ support an equilibrium in which $v\left(s_{5}, s_{6}\right)=(\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.
$S_{12}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{8}\right)$. Given $\left(s_{5}, s_{8}\right), v\left(s_{5}, s_{6}\right)=(A, A, B)$.
If $A$ deviates to $s_{8}$, voter $c$ either votes for $A$ as well, or abstains, hence $A$ is better off deviating.
$S_{13}$ : Voter strategy $s^{i}=(\emptyset, B, A, \emptyset)$ for each voter $i$ and beliefs such that $\omega_{2}^{i, J}(0)=$ $\omega_{2}^{i, J}(1)=1$ for any voter $i$ and any candidate $J$ make the election tied, and if candidate $J$ deviates to any strategy $s^{J} \neq s_{8}, J$ loses the election.

Proposition 6 Assume $\varepsilon=0$ and $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair $\left(s^{A}, s^{B}\right)$ exists if and only if $\left(s^{A}, s^{B}\right) \notin S_{10} \cup S_{12}$.

Proof. First note that $\left(s^{A}, s^{B}\right) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{5}\right)$. Given $\left(s_{5}, s_{5}\right)$, all voters abstain. If candidate $J$ deviates and proposes $s_{8}$ voters $a$ and $b$ beliefs are unaffected, while voter $c$ votes for candidate $J$ for all possible beliefs over candidate $J$ strategy. Therefore candidate $J$ wins the election.

Similarly, $\left(s^{A}, s^{B}\right) \in S_{12}$ cannot be supported in equilibrium. Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{8}\right)$. Given $\left(s_{5}, s_{8}\right)$, voters $a$ and $b$ vote $A$, while voter $c$ votes $B$. Suppose $A$ deviates to $s_{8}$. Voters $a$ and $b$ do not observe the deviation, and continue to vote $A$, while voter $c$ now abstains or votes for $A$ depending on her beliefs. Hence now $A$ wins the election by a greater margin.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal $s_{i}^{J}$, then $\omega_{2}^{i, J}\left(1-s_{i}^{J}\right)=1$ for both $J \in\{A, B\}$ and for $i \in\{a, b, c\}$. These are most pessimistic beliefs that a voter can have regarding candidates' strategy when she observes a deviation. We list the equilibrium electoral outcome for each class of candidates' strategies. Voters' strategies follow straightforwardly from their beliefs. Note that given the pessimistic beliefs a candidate who deviates cannot increase the number of votes she gets in any of the cases below.
$S_{1}:$ All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.
$S_{2}$ : Assume without loss of generality that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{2}\right)$. In equilibrium, $v\left(s_{1}, s_{2}\right)=(A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.
$S_{3}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{5}\right)$. In equilibrium, every voter votes for $A$ and continues to vote for $A$ after any deviation by $B$.
$S_{4}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{8}\right)$. In equilibrium every voter votes for
$A$ and continues to vote for $A$ after any deviation by $B$.
$S_{5}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$. Given $\left(s_{2}, s_{2}\right)$, every voter abstains, and votes against any candidate who deviates.
$S_{6}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$. Given $\left(s_{2}, s_{3}\right), v\left(s_{2}, s_{3}\right)=(A, B, \emptyset)$. Every voter votes against any deviating candidate.
$S_{7}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{5}\right)$. Given $\left(s_{2}, s_{5}\right), v\left(s_{2}, s_{5}\right)=(A, B, A)$.
Voters $a$ and $c$ do not vote for $B$ after any deviation by $B$, and voter $b$ does not vote for $A$ after any deviation.
$S_{8}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{7}\right)$. Given $\left(s_{2}, s_{7}\right), v\left(s_{2}, s_{7}\right)=(A, B, B)$, and given any deviation by $J$, no voter changes her vote from voting for $-J$ to abstention or voting for $J$.
$S_{9}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{8}\right)$. All three voters vote for $A$ and continue to do so after any deviation by $B$.
$S_{10}$ : Not an equilibrium as shown above.
$S_{11}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Given $\left(s_{5}, s_{6}\right), v\left(s_{5}, s_{g}\right)=(\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.
$S_{12}$ : Not an equilibrium as shown above.
$S_{13}$ : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

Lemma 7 Assume $\varepsilon>0$. In any equilibrium, each candidate wins with equal proba-
bility.

Proof. Consider any strategy profile such that candidate $J$ wins with probability less than $\frac{1}{2}$. Since in equilibrium voters hold correct beliefs, the probability that $J$ wins conditional on full information being revealed, or not revealed, is less than $\frac{1}{2}$ in each case. In pure strategies, the probability of victory is in the set $\left\{0, \frac{1}{2}, 1\right\}$ so if it is less than $\frac{1}{2}$, it is zero. Deviating to $s^{J}=s^{-J}$, candidate $J$ ties the election if full information is revealed, so the probability of winning is at least $\frac{\varepsilon}{2}$.

Proposition 8 Assume $\varepsilon \in\left(0, \frac{1}{2}\right)$. For any $\beta \in\left(\frac{2}{3}, 1\right)$, an equilibrium in which candidates use the strategy pair $\left(s^{A}, s^{B}\right)$ exists if and only if $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,11,13\}$.

For any $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$, an equilibrium in which candidates use the strategy pair $\left(s^{A}, s^{B}\right)$ exists if and only if $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,5,6,11,13\}$.

Proof. We prove the high benefit case first.
$S_{1}$ : As shown in the proof of proposition 5 , if full information is not revealed, a deviation by $J$ causes $J$ to lose the election. If $\varepsilon<\frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss.
$S_{2}$ : Assume without loss of generality that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{2}\right)$. Given $\left(s_{1}, s_{2}\right)$, $v\left(s_{1}, s_{2}\right)=(B, A, A)$. By Lemma 7, this cannot occur in equilibrium.
$S_{3}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{5}\right)$. Given $\left(s_{1}, s_{5}\right), v\left(s_{1}, s_{5}\right)=(B, B, A)$.
Ruled out by Lemma 7.
$S_{4}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{8}\right)$. Given beliefs such that $\omega_{2}^{i, B}(0)=1$ for all $i \in\{a, b, c\}$, every voter votes for $A$. Ruled out by Lemma 7 .
$S_{5}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$. Given $\left(s_{2}, s_{2}\right)$, every voter $i$ abstains. If $A$ deviates to $s^{A}=s_{6}$, and full information is not revealed, only voter $c$ observes the deviation, so $v\left(s_{6}, s_{2}\right)=(\emptyset, \emptyset, A)$ and $A$ wins the election. Hence by deviating, $A$ wins with probability at least $1-\varepsilon>\frac{1}{2}$.
$S_{6}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$. Given $\left(s_{2}, s_{3}\right), v\left(s_{2}, s_{3}\right)=(A, B, \emptyset)$. If $A$ deviates to $s^{A}=s_{6}$ and full information is not revealed, only voter $c$ observes the deviation, so $v\left(s_{6}, s_{2}\right)=(A, B, A)$ and $A$ wins the election. Hence by deviating, $A$ wins with probability at least $1-\varepsilon>\frac{1}{2}$.
$S_{7}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{5}\right)$. Given $\left(s_{2}, s_{5}\right), v\left(s_{2}, s_{5}\right)=(A, B, A)$. Ruled out by Lemma 7.
$S_{8}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{7}\right)$. Given $\left(s_{2}, s_{7}\right), v\left(s_{2}, s_{7}\right)=(A, B, B)$.
Ruled out by Lemma 7.
$S_{9}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{8}\right)$. Given $\left(s_{2}, s_{8}\right), v\left(s_{2}, s_{8}\right)=(A, B, B)$.
Ruled out by Lemma 7.
$S_{10}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{5}\right)$. Given $\left(s_{5}, s_{5}\right)$, all voters abstain. If candidate $A$ deviates to $s^{A}=s_{8}$ and full information is not revealed, only voter $c$ observes the deviation and $v\left(s_{8}, s_{5}\right)=(\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins with probability at least $1-\varepsilon>\frac{1}{2}$.
$S_{11}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Given $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$, beliefs such
that $\omega_{2}^{i, J}\left(1-s_{i}^{J}\right)=1$ for each $i \in\{a, b, c\}$ and $J \in\{A, B\}$ support an equilibrium in which $v\left(s_{5}, s_{6}\right)=(\emptyset, A, B)$ and each candidate wins with equal probability. It suffices to check that $A$ has no incentives to deviate. If $A$ deviates to $s^{A} \in\left\{s_{1}, s_{2}, s_{3}, s_{4}, s_{6}, s_{7}\right\}$ and full information is not revealed, $A$ loses the election. If $A$ deviates to $s^{A}=s_{8}$ and full information is not revealed, the election is tied, but if full information is revealed, $A$ loses the election. In any case, after a deviation $A$ wins the election with probability less than $\frac{1}{2}$.
$S_{12}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{8}\right)$. Given $\left(s_{5}, s_{8}\right), v\left(s_{5}, s_{6}\right)=(A, A, B)$.
Ruled out by Lemma 7.
$S_{13}$ : Voter strategy $s^{i}=(\emptyset, B, A, \emptyset)$ for each voter $i$ and beliefs such that $\omega_{2}^{i, J}(0)=$ $\omega_{2}^{i, J}(1)=1$ for any voter $i$ and any candidate $J$ make the election tied, and if candidate $J$ deviates to any strategy $s^{J} \neq s_{8}$ and full information is not revealed, $J$ loses the election.

Next we prove the low benefit case. To sustain equilibria, assume that offequilibrium path beliefs in cases $S_{1}, S_{5}, S_{6}, S_{11}$ and $S_{13}$ are such that given the equilibrium proposal $s_{i}^{J}$, $\omega_{2}^{i, J}\left(1-s_{i}^{J}\right)=1$ for each $i \in\{a, b, c\}$ and $J \in\{A, B\}$. That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.
$S_{1}$ : All voters abstain and the election is tied. If candidate $J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election.
$S_{2}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{2}\right)$. Given $\left(s_{1}, s_{2}\right), v\left(s_{1}, s_{2}\right)=(B, A, A)$.
Ruled out by Lemma 7.
$S_{3}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{5}\right)$. Given $\left(s_{1}, s_{5}\right), v\left(s_{1}, s_{5}\right)=(A, A, A)$.
Ruled out by Lemma 7.
$S_{4}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{8}\right)$. Given $\left(s_{1}, s_{5}\right), v\left(s_{1}, s_{5}\right)=(A, A, A)$.
Ruled out by Lemma 7.
$S_{5}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$. Given $\left(s_{2}, s_{2}\right)$, every voter abstains.
If $J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election.
$S_{6}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$. Given $\left(s_{2}, s_{3}\right), v\left(s_{2}, s_{3}\right)=(A, B, \emptyset)$. If
$J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election.
$S_{7}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{5}\right)$. Given $\left(s_{2}, s_{5}\right), v\left(s_{2}, s_{5}\right)=(A, B, A)$.
Ruled out by Lemma 7 .
$S_{8}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{7}\right)$. Given $\left(s_{2}, s_{7}\right), v\left(s_{2}, s_{7}\right)=(A, B, B)$.
Ruled out by Lemma 7.
$S_{9}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{8}\right)$. All three voters vote for $A$. Ruled out by Lemma 7 .
$S_{10}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{5}\right)$. Given $\left(s_{5}, s_{5}\right)$, all voters abstain. If candidate $A$ deviates to $s^{A}=s_{8}$ and full information is not revealed, only voter $c$ observes the deviation and $v\left(s_{8}, s_{5}\right)=(\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins
with probability at least $1-\varepsilon>\frac{1}{2}$.
$S_{11}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Given $\left(s_{5}, s_{6}\right), v\left(s_{5}, s_{6}\right)=(\emptyset, A, B)$. It suffices to check that $A$ has no incentives to deviate. If $A$ deviates to $s^{A} \in$ $\left\{s_{1}, s_{2}, s_{3}, s_{4}, s_{6}, s_{7}\right\}$ and full information is not revealed, $A$ loses the election. If $A$ deviates to $s^{A}=s_{8}$ and full information is not revealed, the election is tied, but if full information is revealed, $A$ loses the election. In any case, after a deviation $A$ wins the election with probability less than $\frac{1}{2}$.
$S_{12}$ : Assume w.l.o.g. that $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{8}\right)$. Given $\left(s_{5}, s_{8}\right), v\left(s_{5}, s_{6}\right)=(A, A, B)$.
Ruled out by Lemma 7 .
$S_{13}$ : All voters abstain and the election is tied. If candidate $J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election.

We show that all the equilibria in proposition 8 are extensive form rationalizable and satisfy a forward induction refinement. First, we show that they are extensive form rationalizable, in the sense of Pearce [28] definitions 9 and 10, and Battigalli [6] definitions 1 and 2. We follow Battigalli's choice of notation.

Claim 9 Assume $\varepsilon \in\left(0, \frac{1}{2}\right)$. For any $\beta \in\left(\frac{2}{3}, 1\right)$, candidates' strategy pairs $\left(s^{A}, s^{B}\right) \in$ $S_{k}$ for some $k \in\{1,11,13\}$ are extensive form rationalizable. For any $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$, candidates' strategy pairs $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,5,6,11,13\}$ are extensive form rationalizable.

Proof. Assume that for an arbitrary natural number $k, R(k)$ contains all candidate pure strategies and the voter strategy $s^{i}(x, x)=\emptyset s^{i}(x, 1-x)=A$ and $s^{i}(1-x, x)=B$ for $x \in\{0,1\}$ and $i \in\{a, b, c\}$.

We claim that $R(k+1)$ contains all candidate pure strategies. Consider an arbitrary candidate strategy $\hat{s}=\left(\hat{s}_{a}, \hat{s}_{b}, \hat{s}_{c}\right)$. Let $c^{-A}$ assign probability 1 to candidate $B$ choosing $s^{B}=\hat{s}$, and each voter $i \in\{a, b, c\}$ choosing strategy $s^{i}$. Then the unique best response for $A$ is to choose $s^{A}=\hat{s}$.

We claim that $R(k+1)$ contains the voter strategy $s^{i}$ defined above for each $i \in\{a, b, c\}$. Suppose each voter $i$ form conjectures $c^{-i}$ that $s^{J}=\left(\hat{s}_{i}, \hat{s}_{i}, \hat{s}_{i}\right)$ and if this conjecture proves false because $s_{i}^{J}=1-\hat{s}_{i}$, then the new conjecture is $s_{-i}^{J}=$ $(1,1)$. Under this conjecture, it is a best response to abstain if voter $i$ observes the same proposal from both candidates, and to vote for candidate $J$ with $s_{i}^{J}=\hat{s}_{i}$ over candidate $-J$ who proposes $s_{i}^{J}=1-\hat{s}_{i}$ when voter $i$ observes different proposals.

Since $R(0)$ contains all strategies, by induction, $R(n)$ contains all candidates' strategy pairs, including the ones in the claim.

With respect to forward induction, given a candidate strategy pair $\left(s^{A}, s^{B}\right)$ and beliefs $\delta$, let $u^{J}\left(s^{J}, s^{-J}, \delta\right)$ be the expected utility of candidate $J$.

Let $\Delta_{4} \equiv\left\{x \in[0,1]^{4}: \sum_{i=1}^{4} x_{i}=1\right\}$. Then $\Delta=\left(\Delta_{4}\right)^{4}$ is the set of all feasible beliefs a voter may have.

Given an equilibrium candidate strategy pair $\left(s^{A}, s^{B}\right)$ and beliefs $\delta$, let $\Delta_{\delta}$ be the set of beliefs that coincide with $\delta$ along the equilibrium path, that is, $\Delta_{\delta} \equiv\{\tilde{\delta} \in \Delta$ :
$\tilde{\delta}_{i}^{J}\left(s_{i}^{J}\right)=\delta_{i}^{J}\left(s_{i}^{J}\right) \forall i \in\{a, b, c\}, \forall J \in\{A, B\}$.

Axiom 10 (Forward Induction) Given an equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)$ and beliefs $\delta$, off equilibrium path beliefs satisfy forward induction if, given any deviation $\tilde{s}_{i}^{J}$ :

If there exist beliefs $\tilde{\delta} \in \Delta_{\delta}$ and a strategy $\tilde{s}^{J}$ consistent with $\tilde{s}_{i}^{J}$ such that $u^{J}\left(\tilde{s}^{J}, s^{-J}, \tilde{\delta}\right)>$ $u^{J}\left(s^{J}, s^{-J}, \delta\right)$, then $\delta_{i}^{J}\left(\tilde{s}_{i}^{J}\right)$ assigns probability zero to any $\hat{s}^{J}$ such that $\left\{u^{J}\left(\hat{s}^{J}, s^{-J}, \hat{\delta}\right) \leq\right.$ $u^{J}\left(s^{J}, s^{-J}, \delta\right)$ for any $\left.\hat{\delta} \in \Delta_{\delta}\right\}$.

In words, off equilibrium path beliefs satisfy forward induction if given any voter $i$ who observes a deviation by candidate $J$, voter $i$ does not believe the candidate to have deviated to a strategy that cannot possibly bring any benefit to $J$ regardless of what voters believe off equilibrium; on the contrary, the voter must believe that the candidate has deviated to some other strategy (if one is available) that can deliver an advantage to the candidate if voters have the off-equilibrium beliefs that the candidate would like them to have. The intuition is that deviations are not interpreted as accidental mistakes; rather, voters believe that a candidate must have deviated for a reason, the reason being the hope that the deviation changes beliefs in such a way that brings a benefit to the candidate.

Proposition 11 Assume $\varepsilon \in\left(0, \frac{1}{2}\right)$. For any $\beta \in\left(\frac{2}{3}, 1\right)$, an equilibrium with candidates's strategy pair $\left(s^{A}, s^{B}\right)$ and beliefs that satisfy forward induction exists if and only if $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,11,13\}$.

For any $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$, an equilibrium with candidates's strategy pair $\left(s^{A}, s^{B}\right)$ and beliefs that satisfy formward induction exists if and only if $\left(s^{A}, s^{B}\right) \in S_{k}$ for some $k \in\{1,5,6,11,13\}$.

Proof. To sustain equilibria, assume that off-equilibrium path beliefs in all cases are such that given the equilibrium proposal $s_{i}^{J}, \omega_{2}^{i, J}\left(1-s_{i}^{J}\right)=1$ for each $i \in\{a, b, c\}$ and $J \in\{A, B\}$.We denote by $\delta$ the equilibrium beliefs. The proof strategy is to show for each equilibrium in proposition 8 and each possible deviation by a candidate, that the equilibrium beliefs off the equilibrium path satisfy the Forward Induction axiom. We prove the high benefit case first.
$S_{1}$ : Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that, if full information is not revealed, for any $i \in\{a, b, c\}$ and any $J \in\{A, B\}$, when voter $i$ observes $\tilde{s}_{i}^{J}=1$, voter $i$ assigns probability one to candidate $J$ carrying out only the project in district $i$ and no other project. Then, $u^{J}\left(s_{8}, s_{1}, \tilde{\delta}\right)>u^{J}\left(s_{1}, s_{1}, \delta\right)$, that is, if voters had beliefs $\tilde{\delta}$ the deviation $s_{8}$ would be profitable. Thus, if voter $i$ observes a deviation $\tilde{s}_{i}^{J}=1$, it is consistent with forward induction for voter $i$ to believe that $\tilde{s}^{J}=s_{8}$ with probability one.
$S_{11}$ : Assume w.l.og. $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{i, A}(0)=1$ for $i \in\{a, b\}$ and $\tilde{\omega}_{0}^{c, A}(1)=1$. Under beliefs $\tilde{\delta}$ if candidate $A$ deviates to $s_{7}$ then $A$ wins the election when full information is not revealed because voter $b$ does not observe the deviation (and therefore votes for $A$ ) and voter $c$ votes for
candidate $A$. Thus, if voter $a$ observes a deviation $\tilde{s}_{a}^{A}=0$, it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{A}=s_{7}$ with probability one. If candidate $A$ deviates to $s_{6}$, under beliefs $\tilde{\delta}, A$ wins the election when full information is not revealed because voters $b$ and $c$ vote for candidate $A$. Thus, if voter $b$ observes a deviation $\tilde{s}_{b}^{A}=0$, it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{A}=s_{6}$ with probability one. If candidate $A$ deviates to $s_{8}$, under beliefs $\tilde{\delta}, A$ wins the election when full information is not revealed because voters $b$ and $c$ vote for candidate $A$. Thus, if voter $c$ observes a deviation $\tilde{s}_{c}^{A}=1$, it is consistent with forward induction for voter $c$ to believe that $\tilde{s}^{A}=s_{8}$ with probability one. The same argument made for voter $a^{\prime} s$ beliefs on candidate $A^{\prime} s$ deviation holds for candidate $B^{\prime} s$ deviation $\tilde{s}_{a}^{B}=0$; the argument made for voter $b^{\prime} s$ beliefs on candidate $A^{\prime} s$ deviation holds for voter $c^{\prime} s$ beliefs on candidate $B^{\prime} s$ deviation $\tilde{s}_{c}^{B}=0$, and, finally, the argument made for voter $c^{\prime} s$ beliefs on candidate $A^{\prime} s$ deviation holds for voter $b^{\prime} s$ beliefs on candidate $B^{\prime} s$ deviation $\tilde{s}_{b}^{B}=1$.
$S_{13}:$ Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{i, J}(0)=1$ for $i \in\{a, b, c\}$ and $J \in\{A, B\}$. Under beliefs $\tilde{\delta}$ if candidate $J$ deviates to $s_{7}, s_{6}$ or $s_{5}, J$ wins the election when full information is not revealed because the voters who do not observe the deviation abstain and voter $i$ who observes $\tilde{s}_{i}^{J}=0$ votes for $J$. Thus, if voter $i$ observes a deviation $\tilde{s}_{i}^{J}=0$, it is consistent with forward induction for voter $i$ to believe that candidate $J$ is carrying out the project in the other two districts.

Next we prove the low benefit case.
$S_{1}$ :Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that for each voter $i \in\{a, b, c\}$, if full information is not revealed, beliefs $\tilde{\delta}_{i}^{J}(1)$ assigns probability one to candidate $J$ carrying out only the project in district $i$ and no other project. Then voter $i$ with beliefs $\tilde{\delta}_{i}$ votes for candidate $J$ and $u^{J}\left(s_{8}, s_{1}, \tilde{\delta}\right)>u^{J}\left(s_{1}, s_{1}, \delta\right)$. Thus, if voter $i$ observes a deviation $\tilde{s}_{i}^{A}=1$, it is consistent with forward induction for voter $i$ to believe that $s^{A}=s_{8}$ with probability one.
$S_{5}$ : Assume w.l.o.g. $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$. Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{a, J}(0)=1$ and $\tilde{\omega}_{0}^{i, J}(1)=1$ for $i \in\{b, c\}$ and $J \in\{A, B\}$. Under beliefs $\tilde{\delta}$ if candidate $J$ deviates to $s_{7}$, then $J$ wins the election when full information is not revealed because voter $b$ and $c$ vote for candidate $J$. Thus, if voter $a$ observes a deviation $\tilde{s}_{a}^{J}=0$, it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{J}=s_{7}$ with probability one. If candidate $J$ deviates to $s_{8}$, under beliefs $\tilde{\delta}, J$ wins the election when full information is not revealed because voters $b$ and $c$ vote for candidate $J$. Thus, if voter $i \in\{b, c\}$ observes a deviation $\tilde{s}_{i}^{J}=1$, it is consistent with forward induction for voter $i$ to believe that $\tilde{s}^{J}=s_{8}$ with probability one.
$S_{6}$ : Assume w.l.o.g. $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$. Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{a, A}(0)=1$ and $\tilde{\omega}_{0}^{i, A}(1)=1$ for $i=b, c, \tilde{\omega}_{0}^{b, B}(0)=1$ and $\tilde{\omega}_{0}^{i, B}(1)=1$ for $i=a, c$. Under beliefs $\tilde{\delta}$ if candidate $A$ deviates to $s_{7}$, then $A$ wins the election when full information is not revealed because voters $a$ and $c$ vote for candidate $A$. Thus, if voter $a$ observes a deviation $\tilde{s}_{a}^{A}=0$, it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{A}=s_{7}$ with probability one. If candidate $A$ deviates to $s_{8}$ then,
under beliefs $\tilde{\delta}, A$ wins the election when full information is not revealed because voters $a$ (who do not observe the deviation) and voter $c$ vote for $A$. Thus, if voter $i \in\{b, c\}$ observes a deviation $\tilde{s}_{i}^{A}=1$, it is consistent with forward induction for voter $i$ to believe that $\tilde{s}^{A}=s_{8}$ with probability one. Under beliefs $\tilde{\delta}$ if candidate $B$ deviates to $s_{6}$, then $B$ wins the election when full information is not revealed because voters $b$ and $c$ vote for candidate $B$. Thus, if voter $b$ observes a deviation $\tilde{s}_{b}^{B}=0$, it is consistent with forward induction for voter $b$ to believe that $\tilde{s}^{B}=s_{6}$ with probability one. If candidate $B$ deviates to $s_{8}$ then, under beliefs $\tilde{\delta}, B$ wins the election when full information is not revealed because voters $b$ (who do not observe the deviation) and voter $c$ vote for $B$. Thus, if voter $i=a, c$ observes a deviation $\tilde{s}_{i}^{B}=1$, it is consistent with forward induction for voter $i$ to believe that $\tilde{s}^{B}=s_{8}$ with probability one.
$S_{11}$ : Assume w.l.og. $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$. Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{a, J}(0)=1$ for $J \in\{A, B\}, \tilde{\omega}_{0}^{b, A}(0)=\tilde{\omega}_{0}^{b, B}(1)=1$ and $\tilde{\omega}_{0}^{c, A}(1)=\tilde{\omega}_{0}^{c, B}(0)=1$. Under beliefs $\tilde{\delta}$ if candidate $A$ deviates to $s_{7}$, then $A$ wins the election when full information is not revealed because voters $a$ and $b$ vote for candidate $A$. Thus, if voter $a$ observes a deviation $\tilde{s}_{a}^{A}=0$, it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{A}=s_{7}$ with probability one. If candidate $A$ deviates to $s_{6}$ then, under beliefs $\tilde{\delta}$, $A$ wins the election when full information is not revealed because voters $b$ and $c$ vote for candidate $A$. Thus if voter $b$ observes a deviation $\tilde{s}_{b}^{A}=0$ it is consistent with forward induction for voter $a$ to believe that $\tilde{s}^{B}=s_{6}$ with probability one. If candidate $A$ deviates to $s_{8}$, under beliefs $\tilde{\delta}$, $A$ wins the election when full
information is not revealed because voters $b$ and $c$ vote for candidate $A$. Thus if voter $c$ observes a deviation $\tilde{s}_{c}^{A}=1$ it is consistent with forward induction for voter $c$ to believe that $\tilde{s}^{A}=s_{8}$ with probability one. The same arguments hold for voters' beliefs on candidate $B^{\prime} s$ : the same argument holds for voter $a^{\prime} s$ beliefs while the argument made for voter $b$ holds for voter $c^{\prime} s$ beliefs and viceversa.
$S_{13}:$ Consider alternative beliefs $\tilde{\delta} \in \Delta_{\delta}$ such that $\tilde{\omega}_{0}^{i, J}(0)=1$ for $i \in\{a, b, c\}$ and $J \in\{A, B\}$. Under beliefs $\tilde{\delta}$ if candidate $J$ deviates to $s_{7}, s_{6}$ or $s_{5}, J$ wins the election when full information is not revealed because the voters who do not observe the deviation abstain and voter $i$ who observes $\tilde{s}_{i}^{J}=0$ votes for $J$. Thus, if voter $i$ observes a deviation $\tilde{s}_{i}^{J}=0$, it is consistent with forward induction for voter $i$ to believe that candidate $J$ is carrying out the project in the other two districts.

Proof of proposition 3.
Proof. Assume first that $\beta \in\left(\frac{2}{3}, 1\right)$. We find the degree of pessimism necessary to sustain each of the three classes of equilibria identified in Proposition 8.
$S_{1}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{1}\right)$ holds with any beliefs such that $\rho^{i, J}(1) \geq 3 \beta-1$. Whereas, if $\rho^{i, J}(1)<3 \beta-1$, candidate $J$ can deviate to $s_{2}$ and win the election. Thus, $C_{\left(s_{1}, s_{1}\right)}=[3 \beta-1,2]$.
$S_{11}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$ holds with any beliefs such that $\rho^{i, J}\left(1-s_{i}^{J}\right)>1$. Whereas, if $\rho^{c, A}(1) \leq 1$, candidate $A$ can deviate to $s_{8}$ so that if full information is not revealed, $a$ abstains, $b$ votes for $A$, and $c$ abstains or votes for $A$. Thus, $C_{\left(s_{5}, s_{6}\right)}=[1,2]$.
$S_{13}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{8}, s_{8}\right)$ holds with any beliefs such that $\rho^{i, J}(0) \geq 3(1-\beta)$. Whereas, if $\rho^{i, J}(1)<3(1-\beta)$, candidate $J$ can deviate to $s_{1}$ and win the election. Thus, $C_{\left(s_{8}, s_{8}\right)}=[3(1-\beta), 2]$.

Since $3(1-\beta)<1<3 \beta-1$ for any $\beta \in\left(\frac{2}{3}, 1\right)$, it follows $C_{\left(s_{1}, s_{1}\right)} \subseteq C_{\left(s_{5}, s_{6}\right)} \subset C_{\left(s_{8}, s_{8}\right)}$ and $\left(s_{8}, s_{8}\right)$ is the candidates' strategy profile in the most optimistic equilibrium.

Assume next that $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right)$. We find the degree of pessimism necessary to sustain each of the five classes of equilibria identified in Proposition 8.
$S_{1}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{1}, s_{1}\right)$ holds with any beliefs such that $\rho^{i, J}(1) \geq 3 \beta-1$. Whereas, if $\rho^{i, J}(1)<3 \beta-1$, candidate $J$ can deviate to $s_{2}$ and win the election. Thus, $C_{\left(s_{1}, s_{1}\right)}=[3 \beta-1,2]$.
$S_{5}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{2}\right)$ holds with any beliefs such that $\rho^{i, J}(1) \geq 3 \beta$. Whereas, if $\rho^{b, J}(1)<3 \beta$, candidate $J$ can deviate to $s_{5}$ and win the election. Thus, $C_{\left(s_{2}, s_{2}\right)}=[3 \beta, 2]$.
$S_{6}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{2}, s_{3}\right)$ holds with any beliefs such that $\rho^{i, J}(1) \geq 3 \beta$. Whereas, if $\rho^{c, J}(1)<3 \beta$, candidate $A$ can deviate to $s_{6}$ and win the election. Thus, $C_{\left(s_{2}, s_{3}\right)}=[3 \beta, 2]$.
$S_{11}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{5}, s_{6}\right)$ holds with any beliefs such that $\rho^{i, J}\left(1-s_{i}^{J}\right)>1$. Whereas, if $\rho^{c, A}(1) \leq 1$, candidate $A$ can deviate to $s_{8}$ so that if full information is not revealed, $a$ abstains, $b$ votes for $A$, and $c$ abstains or votes for $A$. Thus, $C_{\left(s_{5}, s_{6}\right)}=[1,2]$.
$S_{13}$ : An equilibrium with candidate strategy pair $\left(s^{A}, s^{B}\right)=\left(s_{8}, s_{8}\right)$ holds with
any beliefs such that $\rho^{i, J}(0) \geq 3(1-\beta)$. Whereas, if $\rho^{i, J}(1)<3(1-\beta)$, candidate $J$ can deviate to $s_{1}$ and win the election. Thus, $C_{\left(s_{8}, s_{8}\right)}=[3(1-\beta), 2]$.

Since $3 \beta-1<\min \{3 \beta, 1,3(1-\beta)\}$ for any $\beta \in\left(\frac{1}{3}, \frac{2}{3}\right), C_{\left(s_{2}, s_{2}\right)} \cup C_{\left(s_{2}, s_{3}\right)} \cup C_{\left(s_{5}, s_{6}\right)} \cup$ $C_{\left(s_{8}, s_{8}\right)} \subset C_{\left(s_{1}, s_{1}\right)}$ and $\left(s_{1}, s_{1}\right)$ is the candidates' strategy profile in the most optimistic equilibrium.

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[^1]:    ${ }^{1}$ Feddersen and Pesendorfer [13] and [14] study an electorate in which some voters are poorly informed about the state of the world and Banks [2] and Callander and Wilkie [7] let candidates execute different policies once in office than those announced during the campaigns. These models are more distantly related because they do not deal with distributive policies, and they assume that voters are perfectly informed about the candidates' proposals.

[^2]:    ${ }^{2}$ If $\beta<1 / 3$, the problem is trivial, since no district wants to implement any project.

[^3]:    ${ }^{3}$ If no undominated strategy is consistent with the information possessed by the agent, then we let the agent hold any beliefs over the entire strategy set.

[^4]:    ${ }^{4}$ We also check that we obtain the same set of equilibria if we change the utility function of candidates, to make candidates care not about margin of victory, but about policy outcomes such as aggregate social welfare, or the utility of a particular voter. The -largely redundant- proofs are available from the authors.

[^5]:    ${ }^{5}$ These results are (OR ARE SOON GOING TO BE) available from the authors.

[^6]:    ${ }^{6}$ Named Best Political Science website in 1998 by the American Political Science Association.

[^7]:    ${ }^{7}$ We stress that this simplifies notation, but does not change our behavioral assumption that voters are fully strategic and rational: In the branches of the game with full information the voters' decision problem can be solved by simple domination arguments, and we directly anticipate and impose the outcome that follows from the unique undominated solution.

