Managerial Hedging and Portfolio Monitoring*

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Abstract

Incentive compensation induces correlation between the portfolio of managers and the cash flow of the firms they manage. This correlation exposes managers to risk and hence gives them an incentive to hedge against the poor performance of their firms. We study the agency problem between shareholders and a manager when the manager can hedge his incentive compensation using financial markets and shareholders can only imperfectly monitor the manager’s portfolio in order to keep him from hedging the risk in his compensation. We find that the optimal contract implies incentive compensation and governance provisions with the following properties: (i) the manager’s portfolio is monitored only when the firm performs poorly, (ii) the manager’s compensation is more sensitive to firm performance when monitoring is more costly or when hedging markets are more developed, and (iii) conditional on the firm’s performance, the manager’s compensation is lower when his portfolio is monitored, even if no hedging is revealed by monitoring.

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1 Introduction

The objective of incentive compensation is to induce a correlation between managers’ compensation and the cash flow of the firms they manage so as to induce them to work diligently and increase firm performance.\(^1\) But this correlation exposes managers to risk and hence gives them an incentive to trade in financial markets so as to hedge against the poor performance of their firms. In the 1990s several financial instruments have been developed which allow managers to hedge the firm specific risk in their compensation packages. Examples of such instruments include zero-cost collars, equity swaps, and basket hedges. While little data exist, off-the-record interviews with investment bankers reported in the press suggest that the market for executive hedging instruments is sizable (hundreds of millions of dollars according to the Economist (1999a)), and that most large investment banks offer such instruments (see Puri (1997), Smith (1999), and Lavelle (2001)).

Legal and financial commentators generally share the opinion that managerial hedging undermines incentives in executive pay schemes, significantly alters the executives’ effective ownership of the firm, and hence has adverse effects on performance.\(^2\) Boards and shareholders should recognize that managers might have the opportunity to hedge their incentive compensation packages and design their managers’ incentive compensation and their firm’s governance provisions accordingly. If shareholders were able to perfectly observe the managers’ transactions, they could explicitly rule out the possibility that managers trade any hedging instruments. In practice, managers’ portfolios are not publicly disclosed and they are difficult and costly to monitor. For one, disclosure rules regarding managerial transactions of hedging instruments are relatively lax,\(^3\) and only few trades are effectively disclosed to investors and shareholders.\(^4\) Moreover, financial markets have proved quite effec-

\(^1\)For evidence on the relationship between managerial incentives and firm performance see, e.g., Morck, Shleifer, and Vishny (1988), and Jensen and Murphy (1990). See Murphy (1999) for a survey on incentive compensation.

\(^2\)In the legal profession, see Easterbrook (2002), Schizer (2000), Bank (1994/5); in the financial press, see the editorial pieces in the Economist (1999a,b,c,2002), Ip (1997), Lavelle (2001), Puri (1997), and Smith (1999).

\(^3\)Since September 1994 equity swaps and similar instruments must be reported to the Securities and Exchange Commission (SEC), on Table II of Form 4; Release No. 34-34514 and Release No. 34-347260. But the back-page of Table II of Form 4 is not included in the electronic filing used by analysts; see Bolster, Chance, and Rich (1996) and Lavelle (2001). Finally, non-insiders and CEOs of non-U.S. firms are not obligated to disclose their trades. Recently, though, the Sarbanes-Oxley Act of 2002 introduced more stringent rules regarding the electronic filing of transactions involving such instruments and has substantially reduced the delay in disclosure, when disclosure is required.

\(^4\)In 1994 only 1 hedging transaction was disclosed to the SEC, Autotote’s CEO equity swap, the case studied by Bolster, Chance, and Rich (1996). The number of transaction reported in subsequent years increased to 15 transaction in 1996, 39 in 1997, and 35 in 1998 (the whole 90 transactions are studied by Bettis, Bizjak, and Lemmon (2001)), 31 transaction in 2000 (Lavelle (2001)). No evidence is yet available about the effects of the Sarbanes-Oxley Act of 2002 on disclosures.
tive in designing instruments which overcome regulation, governance provisions, and tax laws. For instance, equity swaps have been substituted with collars when swaps became subject to more stringent tax treatment (Schizer (2000)).

While costly, monitoring of managers’ portfolios can nonetheless help to align shareholders’ and managers’ objectives within an optimal incentive compensation contract. Managers are not restricted by law from trading derivatives on stocks of their own firm, but may be subject to derivative suits brought by shareholders for violation of fiduciary duty if financial transactions to hedge their incentive compensation are revealed. For transactions disclosed to the SEC, shareholders can force executives to satisfy their burden of establishing the validity of the transaction. When instead monitoring reveals evidence of breach of disclosure, action can be pursued under securities law, which is easier than under corporate law (Fox (1999)).

In this paper we study the optimal contracts when managers have access to anonymous hedging instruments in financial markets and when shareholders can monitor the portfolios of managers. Optimal contracts include incentive compensation as well as governance provisions regarding the monitoring of managers’ portfolios. Since, as we argued, managers’ portfolios are difficult to monitor we consider the case where monitoring is costly, thus less than perfect. Hence, we study executive compensation with imperfect corporate governance. Also, in accordance with the limited possi-

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5Under Section 16(c) of Securities and Exchange Act of 1934, and Rule 16c-4, managers are only prohibited from selling their firm’s stock short.

6For a discussion of the fiduciary principle and derivative suits see, e.g., Easterbrook and Fischel (1991), chapter 4, and Klausner and Litvak (2000). Of course, under Rule 10b-5 of the Securities Exchange Act of 1934, it is illegal for insiders to trade while in possession of material value-relevant information (insider trading). While there is some evidence that the observed hedging transactions of executives might in part constitute insider trading (see Bettis, Coles, and Lemmon (2000)), we concentrate in this paper on the pure hedging motives.

7Derivative suits are more easily brought against executives whose compensation contracts explicitly state trading limitations. In practice this is rarely the case; and when firms do have trading policies, they are usually not disclosed to minority shareholders; for a detailed discussion of such restrictions see Schizer (2000) and Bettis, Bizjak, and Lemmon (2001). This contractual practice could be motivated by the aim of protecting the firm against “frivolous” actions of shareholders; this is consistent with the practice of providing executives with insurance policies against such actions; see Klausner and Litvak (2000) for a discussion. Bebchuk, Fried, and Walker (2002) interpret the limited contractual restrictions of hedging instead as evidence of managerial rent extraction. See also Bebchuk and Fried (2003).

8Only for actions brought by the SEC for violations of the securities law can courts grant “any equitable relief that may be appropriate or necessary for the benefit of investors,” Sarbanes-Oxley Act of 2002, Section 305, 5. In the case of insider trading during black-out periods, e.g., it is “profit realized by a director or executive officer” that shall “be recoverable by the issuer,” Sarbanes-Oxley Act of 2002, Section 306, 2A. Sarbanes-Oxley Act of 2002 does not explicitly state any provision for hedging in violation of fiduciary duty.
bilities for legal action by shareholders discussed above, we assume that whenever hedging by a manager is detected, only payoffs that the manager would receive from this activity can be seized by the shareholders.\(^9\)

The main implication of our analysis of the optimal contract with regards to governance is that monitoring of a manager’s portfolio optimally occurs only when the performance of the firm is poor. Since for incentive reasons the manager’s compensation is low when the firm does poorly, if the manager were to hedge he would buy claims which pay off when the firm does poorly. The fact then that shareholders could seize the payoffs of managerial hedging, if detected, because it violates fiduciary duty, implies that shareholders will monitor the manager’s portfolio when such hedging positions would pay off, i.e., when the firm performs poorly.

Moreover, conditional on the firm performing poorly, the optimal compensation of the manager is lower when the manager is monitored, and hence his portfolio scrutinized, than when the manager is not monitored. This is so even if monitoring does not reveal any hedging transactions of the manager. In other words, managers strictly prefer not to be monitored at the optimal contract, despite the fact that at the optimal contract they choose not to hedge their compensation. The manager’s compensation both when he is monitored and when he is not monitored in states when the firm does poorly affects his incentive to work diligently. But the compensation when the manager is not monitored also affects his desire to hedge his compensation risk. To reduce the manager’s desire to hedge his compensation, it is thus optimal to pay him more when he is not monitored, than when he is monitored. Consequently, in our model investigations regarding the managers’ conduct are associated with reductions in their pay and benefits. This is in accord with the common perception that in practice agents who are monitored are worse off even if they did nothing wrong. The key for the result is that we assume that when the manager is monitored and hedging is detected his pay cannot be reduced (or at most can be reduced by a fixed amount).

The main implication of our analysis with regards to incentive compensation is that the higher is the cost of monitoring or the more developed the hedging markets, the steeper the incentives provided by shareholders to the manager. Thus, worse corporate governance implies that shareholders have to give managers steeper incentives.\(^10\) The intuition is as follows: when managerial hedging is costly to monitor, managers have to be induced to refrain from hedging by the structure of the compensation scheme rather than being forced to refrain by monitoring. Thus, shareholders have to make it expensive for managers to hedge. This is achieved by paying the manager more in states where the firm does well. The hedging market understands

\(^9\)We will show however that our main results carry over to the case where additional monetary penalties can be imposed on the manager when hedging is detected.

\(^10\)Similarly, La Porta, Lopez-de-Silanes, and Shleifer (1999) find that in countries with worse investor protection ownership is more concentrated.
that, given that a manager is hedging, he will work less diligently and hence states with good performance are less likely, which is reflected in the price at which the manager can sell claims contingent on such states. In short, claims contingent on good performance trade at a discount in the hedging market. Thus, an increase in the steepness of compensation decreases the present value of the manager’s compensation in the hedging market and makes it more expensive for the manager to hedge. This result provides an explanation for the following two observations: i) steeper incentive contracts and hedging instruments have appeared roughly simultaneously in the U.S. financial markets, and ii) steeper incentive contracts characterize executive pay in the U.S. and U.K. than in continental Europe, where financial markets and in particular the market for hedging instruments are less developed. This is consistent with the optimal contract characterized in our paper, which implies a steeper incentive pay scheme when hedging instruments hidden to shareholders are more widespread. This result also implies that the higher the level of monitoring as dictated by legal disclosure requirements or corporate governance rules, the less steep incentive contracts should be. Thus, the recent increase in disclosure requirements may bring a reduction in the steepness of incentive compensation and hence reduce the amount of stocks and options granted. Moreover, as the hedging markets develop, the optimal level of monitoring might well increase.

Finally we show that the managers’ incentives are also affected by the possibility of trading claims whose payoff does not depend on the firm specific risk and hence whose fluctuations are not attributable to the manager’s choice of effort. One example is the managers’ ability to borrow and lend, i.e., to trade a riskless asset. Similar considerations apply to the trade of market indices and basket hedges, where the derivative’s value is based not only on the stock price of the employer but also on a basket of correlated stocks, which allow the manager to hedge the systematic risk in his compensation. Our analysis shows that imposing restrictions also on the trade of such claims would be beneficial.

Throughout the paper we interpret managerial hedging as trades in contingent claims. But we could alternatively interpret such activity as the manager borrowing from the firm in an unobserved way and purchasing assets, such as houses. If the manager plans on repaying these loans using his bonus when the firm performance is good, but defaults on them when firm performance is bad while keeping the assets, such loans provide insurance and are a way to hedge incentive compensation. In our model, then, managers’ portfolios are optimally monitored following poor performance and the extra assets bought with the loans forgiven by the firm are seized.\footnote{Such transactions are now explicitly prohibited by Section 402 of the Sarbanes-Oxley Act of 2002.}

From the standpoint of the theory of optimal contracts, this paper introduces and studies a new class of principal agent problems, with stochastic monitoring of
the agent’s portfolio not otherwise observable. This class of problems has a wide range of applications that we do not explicitly explore in this paper. Nonetheless, we will mention here that one interesting and fitting application is to the study of credit markets where a borrower (the agent) has access to a primary lender (the principal), as well as to a secondary market for credit, and hence his total liabilities are not observable. In this context the stochastic monitoring technology represents the institution of bankruptcy, and a fundamental component of the optimal contract would consist in designing such an institution.\footnote{Bisin and Rampini (2004) study bankruptcy in a related environment, but without an explicit stochastic monitoring technology.}

We should also point out that, even though the form of the optimal contract we derive is such that the manager does not wish to engage in hedging, the presence of some hedging activity should not necessarily be viewed as evidence of a lack of optimality of the compensation contract. As discussed in Section 4, a certain amount of hedging trades could be consistent with optimality, provided the level of such trades can be effectively controlled (for example, the tax advantages of incentive compensation might imply that shareholders give managers excessive incentives while at the same time allowing partial hedging of incentive compensation). Relatedly, the disclosure requirements concerning the level of hedging imposed by the law may act as a substitute for contractual provisions against such trades (which, as discussed in Section 4, are fairly rare).

**Related literature.** In contrast to the set-up considered here, the theoretical literature on principal-agent problems has studied either the case in which the agent’s trades are perfectly observable (e.g., Prescott and Townsend (1984) and Bisin and Gottardi (2001)), or the case in which they are unobservable (see Allen (1985), Arnott and Stiglitz (1991), Kahn and Mookherjee (1998), Pauly (1974); also Admati, Pfleiderer, and Zechner (1994), Bisin and Gottardi (1999), Bisin and Guaitoli (2004), Bizer and DeMarzo (1992, 1999), Cole and Kocherlakota (2001), Park (2004)). More specifically with regard to the application to managerial incentive compensation, Jin (2002), Acharya and Bisin (2003), and Garvey and Milbourn (2003) study the case where executives can anonymously trade market indices. Garvey (1993, 1997) and Ozerturk (2003) study the case where managers can hedge (without any monitoring) in financial markets by trading a single - exclusive - contract. However, this assumes that contracts traded in the hedging market exhibit stronger enforceability properties than the compensation contract itself, which seems counterintuitive, and implies that it should be optimal to have non-zero trade in the hedging market and that the possibility of engaging in unmonitored hedging entails no efficiency loss. On the other hand, we consider the case where managers can hedge their compensation by trading (with imperfect monitoring) non exclusive contracts; our conclusions are also rather different as we find that this possibility affects the form of the optimal
compensation and entails an efficiency loss.

Costly monitoring has been introduced in the study of principal agent problems by, for instance, Townsend (1979), Gale and Hellwig (1985), and Mookherjee and Png (1989). They analyze situations where it is the realization of a privately observed state, rather than private hedging activity as in our paper, which can be monitored at a cost (costly state verification). This class of models has different implications than our analysis of portfolio monitoring. In particular, in contrast to the findings of our paper, costly state verification models imply that managers strictly prefer to be monitored at the optimal contract, as their compensation is higher when they are monitored and found to have told the truth.

The paper proceeds as follows. Section 2 studies the one period case, where firms have cash flow and managers get compensated at only one point in time. Most of the intuition and main results can be obtained in this case. Section 3 extends the analysis to two periods, which introduces intertemporal considerations. We also discuss the case where managers have access only to risk free borrowing and lending and show that most of our results carry over to this case. Section 4 provides a discussion and Section 5 concludes. All proofs are in the Appendix.

2 Managerial Incentive Compensation and Portfolio Monitoring: Static Case

Our analysis will be developed in the context of a simple standard agency environment with hidden effort (see, e.g., Grossman and Hart (1983)). A (risk neutral) principal owns a production process, whose outcome is uncertain, and has to hire a (risk averse) agent to manage it. The agent’s effort level in this task is not observable and affects the probability distribution of the process’ outcome.

In this paper the principal and the agent are, respectively, the shareholders (or the board) and the manager of a firm. We study the optimal incentive compensation contract shareholders can write to align their objective with that of the manager when his effort is not observable and when \(i\) the manager can engage in trades in financial markets to hedge his risk, which may adversely affect his incentives, and \(ii\) shareholders can only imperfectly monitor the manager’s trades in financial markets.

We consider first the case where there is a single period where production and payments take place. In the following section the analysis will be extended to allow for more production and payment dates.

\(^{13}\)In addition, Winton (1995) studies costly state verification with multiple investors. Baiman and Diesnki (1980) and Dye (1986) study environments where it is the agents’ privately observed effort which can be monitored at a cost. To our knowledge, the only previous analysis of a principal-agent problem with limited observability of trades, through bankruptcy procedures, is in Bisin and Rampini (2004).
The manager and the shareholders. Let $S = \{H, L\}$, with generic element $s$, describe the possible realizations of the uncertainty. The cash flow of the firm is $y_H$ in state $H$ and $y_L$ in state $L$, with $y_H > y_L > 0$. The probability of each state $s \in S$ depends on the effort level $e \in \{a, b\}$ undertaken by the manager and is denoted $\pi_s(e)$.

The shareholders’ income coincides with the firm’s cash flow, less the compensation paid to the manager. We assume that shareholders are risk neutral (for instance because the risk of the firm is idiosyncratic and can be fully diversified by shareholders). On the other hand, the manager is risk averse. We assume he has no resources other than his ability to work and has Von Neumann-Morgenstern preferences defined over his level of consumption (equal to the compensation received) in every state as well as over his effort level:

$$\sum_{s \in \{H, L\}} \pi_s(e)u(z_s) - v(e).$$

More precisely, we require the utility index $u(.)$ to satisfy the following:

**Assumption 1** $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing, strictly concave, and $\lim_{z \rightarrow 0} u'(z) = \infty$.

The last part of the assumption implies that the manager’s compensation has to ensure him a strictly positive level of income in every state.

The term $v(e)$ in the manager’s utility function describes his disutility for effort. We assume that $v(a) > v(b) > 0$ and $\pi_H(a) > \pi_H(b)$. Thus, $a$ should be viewed as the high effort level, which entails a larger disutility but also a higher probability for state $H$, in which the firm’s cash flow is larger.

The realization of the uncertainty, that is, of $s$, is commonly observed. However, the effort undertaken by the manager is his private information and cannot be monitored. As usual, we will assume that the gains from eliciting high effort are always sufficiently big relative to its cost, $v(a) - v(b)$, so that in designing the optimal contract we face a non-trivial incentive problem. In particular, we will assume that the manager, when his compensation equals the firm’s entire cash flow, prefers to exert high effort rather than low effort even when he has the opportunity to fully hedge his risk (at prices $\pi(b)$, fair contingent on low effort):

**Assumption 2** The manager’s preferences $u(.)$ and the parameters $v(e), \pi(e)$ are such that

$$\pi_H(a)u(y_H) + \pi_L(a)u(y_L) - v(a) > u(\pi_H(b)y_H + \pi_L(b)y_L) - v(b).$$
Markets. The manager and the shareholders have access to competitive financial markets where they can trade, at the beginning of the period, claims contingent on each possible realization of the uncertainty. In particular the manager can trade any derivative contract on the firm’s cash flow, thereby hedging any incentive component of his compensation. Since the probability distribution of the firm’s cash flow depends on the manager’s effort, such derivative markets are characterized by the presence of moral hazard.

We consider here the case where the contracts traded in these markets are non exclusive, that is, the case in which a dealer trading with a manager does not know whether the manager engages in other trades in the market. This is in accordance with the flexible institutional setting of these markets: managers can trade different contracts with different investment banks, as well as construct basket hedges or simply trade using family members’ accounts. Hence, the price of these contracts cannot depend on the manager’s total portfolio (since nobody except the manager observes it), though they may vary with the sign of the transaction. It follows from the analysis in Bisin and Gottardi (1999) that equilibrium prices in the financial markets exhibit the following properties: the price of a hedging contract (that is, a purchase of insurance) is fair conditionally on low effort being exerted, i.e., it is additive with respect to state prices $p^+_s = \pi_s(b), s \in S$; the price for bets on the firm (sales of insurance) is on the other hand fair conditionally on high effort being exerted, that is, with respect to state prices $p^-_s = \pi_s(a), s \in S$. This specification of the prices reflects the fact that, at the optimal contract, if the manager hedges in the market, he will have no incentives to choose the high effort. The price will therefore take this into account, and hedging will be more costly (in particular, fair conditional on low effort). Betting on the firm’s performance, in contrast, will not induce the manager to switch from the high effort level, and hence the price for selling insurance will be low.

At these prices, a dealer trading a derivative contract on the stock of the manager’s firm makes zero-profits by perfectly hedging such risk in the market. One may think that the dealer might be able to hedge this risk at a lower cost by selling short the firm’s equity (whose market value is determined at the state prices $\pi_s(a)$). The possibility of doing so is however severely limited when the depth of the market for the firm’s equity is small, since news about such transactions quickly spreads across investors and analysts. Moreover, it would be in the manager’s interest to require

\[ \text{Moreover the financial market is arbitrage free, since the prices for purchases of state-contingent claims, } \pi_H(a) \text{ and } \pi_L(b), \text{ exceed the prices for sales, } \pi_H(b) \text{ and } \pi_L(a), \text{ for both states. In general, the viability of a competitive market in the presence of informational asymmetries requires that traders with private information (in our case, the manager) face a bid ask spread, or that a different price is charged for purchases and sales; see Bisin and Gottardi (1999).} \]

\[ \text{In Bolster, Chance, and Rich (1996)'s case study, for instance, the market discounted the performance and signaling effect of managerial hedging before the transaction was disclosed to the SEC; see also Bettis, Coles, and Lemmon (2000).} \]
the dealer not to hedge his trades by shorting the firm’s equity if such transactions are easily detected. With a deep equity market the dealer may be able to lower his hedging cost by shorting equity, but this is only possible to a limited extent, due to the lack of liquidity of short-sales markets, and only for smaller trades: the price at which managers can hedge their compensation will then still be higher than its fair price with respect to state prices $\pi_s(a)$ and will also increase in the amount of trades. Therefore, while the specification of state prices for hedging trades we consider, given by $\pi_s(b)$, should be viewed as more appropriate for situations where the depth of the market for the firm’s equity is limited, our analysis and results can be extended to other cases, where hedges are more costly than if priced at state prices $\pi_s(a)$ and less costly than if priced at $\pi_s(b)$.

Monitoring. As is well known, whether or not the agent’s trades in the market are observable by the principal plays an important role in the determination of the optimal contract between the two parties in the presence of asymmetric information. If not detected, such trades may in fact undo the incentives provided by the contract. We examine the case where a monitoring technology may be used to detect the manager’s trades in financial markets. Monitoring takes place ex post, i.e., not when trades are actually made (at the beginning of the period), but rather when the payments associated with such trades are made (at the end of the period) in a given state.

The intensity of monitoring in each state $s$ will be measured by the probability $m_s$ with which the payments due to or from the manager in state $s$ are observed. Monitoring is costly and hence will be imperfect. More precisely, we assume that the cost of exerting monitoring in each state $s$ with intensity $m_s$ is given by $\phi(\bar{m})$, where $\bar{m} = \sum_{s \in S} \pi_s(a)m_s$ and $\phi$ is a positive and increasing function of $\bar{m}$.

Furthermore, we need to specify which punishment can be inflicted on the manager if he is found to have traded in the financial markets. We assume the punishment can only take a monetary form. Given the above specification of the monitoring technology it seems natural to consider the case where punishments consist in the seizure of the payments due to the manager from his trades in the financial market. Thus, if the manager is monitored in state $s$, all the payoffs of any hedging transactions that are due to him in this state will be seized, while the manager will still have to make all the payments due from him for his hedging trades. We also discuss the case where additional penalties, e.g., an additional monetary penalty of size $k$, can be imposed on the manager and show that our main results extend to this case (see Section 2.3).

2.1 The Contracting Problem

We are now ready to describe the optimal contracting problem between the manager and the shareholders in this framework. A contract specifies the compensation due
to the manager in every contingency that is commonly observable to the parties: the firm’s cash flow realization and whether or not monitoring occurs. The contract also specifies the monitoring probabilities in each of the possible realizations of the firm’s cash flow. Finally, the contract contains a recommendation of a level of effort and of trades in financial markets.

The level of trades in financial markets can be set equal to zero without any loss of generality, since the outcome of any trade can always be replicated by appropriate changes in the net payments. In practice, of course, firms might have incentives to design compensation packages composed mostly of equity derivatives, e.g., of stock options because of their advantageous tax treatment (see Murphy (1999)), and then let the manager hedge his compensation in part in the market. In this case, the managerial hedging transaction that are observed in practice might be explicitly or implicitly part of the firms’ compensation packages.

We will first characterize the properties of the optimal compensation scheme for any given monitoring probabilities \( (m_H, m_L) \), and then discuss the determination of the optimal level of monitoring when monitoring costs are explicitly taken into account. Let then \( z^{nm}(e) = (z^{nm}_H(e), z^{nm}_L(e)) \in \mathbb{R}^2_+ \) (respectively, \( z^m(e) \in \mathbb{R}^2_+ \)) denote the payment to the manager in each state when no monitoring (respectively, monitoring) occurs and effort \( e \) is recommended. Under Assumption 2, as we will see, shareholders are always able to implement a high level of effort \( e = a \) by the manager, whatever is \( (m_H, m_L) \), and this is optimal. As a consequence, to keep the notation simpler in what follows, whenever possible, we will avoid to explicitly write the dependence of \( z \) on \( e \).

The optimal compensation contract for the manager in the presence of moral hazard and random monitoring of side trades, when monitoring occurs in the two states with probability \( m_H \) and \( m_L \), respectively, is then obtained as a solution of the following program (and prescribes a high effort level):

\[
\max_{(z^m, z^{nm}) \in \mathbb{R}^4_+} \sum_{s \in \{H, L\}} \pi_s(a) \{(1 - m_s)u(z^{nm}_s) + m_s u(z^m_s)\} - v(a) \quad (P_{MON})
\]

subject to

\[
\sum_{s \in \{H, L\}} \pi_s(a)[y_s - (m_s z^m_s + (1 - m_s) z^{nm}_s)] \geq 0, \quad (1)
\]

and

\[
\sum_{s \in \{H, L\}} \pi_s(a) \{(1 - m_s)u(z^{nm}_s) + m_s u(z^m_s)\} - v(a) \geq \sum_{s \in \{H, L\}} \pi_s(e') \left[(1 - m_s)u(z^{nm}_s - \tau_s) + m_s u(z^m_s - \max\{\tau_s, 0\})\right] - v(e') \quad (2)
\]

for all \( e' \in \{a, b\}, \left\{(\tau_H, -\tau_L) \in \mathbb{R}^2_+ : \sum_{s \in \{H, L\}} \pi_s(b) \tau_s = 0\right\}, \left\{(-\tau_H, \tau_L) \in \mathbb{R}^2_+ : \sum_{s \in \{H, L\}} \pi_s(a) \tau_s = 0\right\} \).
This program requires maximizing the manager’s utility subject to the shareholders’ participation constraint, given by (1), and the incentive compatibility constraint (2). We choose this approach rather than to maximize the shareholders expected utility subject to a participation constraint for the manager since it simplifies the analysis without affecting the results. The term appearing on the left hand side of (1) is the shareholders’ expected utility (equivalently expected net income, given the shareholders’ risk neutrality) when compensation \((z^m, z^{am})\) is paid to the manager in the various states. On the right hand side the shareholders’ reservation utility is set at zero.\(^{16}\) The participation constraint amounts to setting an upper bound on the expected payments to the manager.

Equation (2) describes the incentive constraints in our set-up, where both effort and trades in financial markets are private information of the manager. It requires the manager to be unable to achieve a higher utility level not only by choosing a different effort level \((b)\), but also by engaging in some trades \((\tau_H, \tau_L) \neq 0\). We adopt the convention that a negative value of \(\tau_s\) denotes the purchase of a claim (contingent on state \(s\)), and hence the right to receive a payment in state \(s\). Thus, in the event of monitoring, no payment is received. On the other hand, when \(\tau_s > 0\), the manager has to make a payment \(\tau_s\) whether or not monitoring occurs. Thus trades such that \(\tau_H > 0, \tau_L < 0\) correspond to the purchase of insurance and are priced at \(\pi_s(b)\), while trades such that \(\tau_H < 0, \tau_L > 0\) correspond to the sale of insurance and are priced at \(\pi_s(a)\). Note that the agent faces no restriction in his trades in the financial markets except his budget constraint; hence any self-financing trade is admissible.\(^{17}\)

Since the manager is risk averse and shareholders risk neutral, the solution of \((\mathcal{P}_{MON})\) yields the compensation scheme with minimal risk that is compatible with incentives. The tightness of the incentives, and hence the specific form of the compensation, depends, as we will see, on the values of \((m_H, m_L)\).

### 2.2 The Optimal Contract

We provide here a characterization of the solution of the optimal contracting problem described in the previous section. We first determine in which of the states (i.e., for which realizations of the firm’s cash flow) monitoring should optimally occur. Next, we characterize the manager’s optimal compensation scheme.

\(^{16}\)This is without loss of generality since cash flows can always be redefined to be net of a fixed payment to shareholders.

\(^{17}\)With the above formulation of the trading constraints in the primary markets managers never choose to engage in side trades. Hence there is no need to specify what happens to the payments seized from them since no payments are ever seized.
2.2.1 When should monitoring occur?

Our first result shows that the optimal contract does not depend on the monitoring probability in the high state, $m_H$.\textsuperscript{18}

**Proposition 1** The optimal contract (that is, the solution of ($\mathcal{P}_{MON}$)) is independent of $m_H$.

From this it follows that, if monitoring is costly, as we assume, it should never occur in state $H$, but only in state $L$, that is, when the realized cash flow of the firm is low. Thus we can always set $m_H = 0$ and, to simplify the notation, $m \equiv m_L$. We will henceforth consider the contracting problem as a function of $m$.

2.2.2 Optimal compensation

In this section we characterize the optimal compensation scheme $z(m) = (z_H(m), z_L^m(m), z_L^m(m))$ for any $m$, $0 \leq m \leq 1$. We consider first two benchmark cases: (i) perfect observability of trades/perfect monitoring ($m = 1$); (ii) non-observability of trades/no monitoring ($m = 0$).

If monitoring takes place with probability $m = 1$, trades are perfectly observable. In this case the manager is unable to profit from any trade in the financial market (since their proceeds will be seized with certainty). We can support then the incentive efficient (or second best) contract $(z_H^*, z_L^*)$, which is the solution of

$$\max_{(z_H, z_L) \in \mathbb{R}_+^2} \sum_{s \in \{H, L\}} \pi_s(a)u(z_s) - v(a)$$

subject to

$$\sum_{s \in \{H, L\}} \pi_s(a)[y_s - z_s] \geq 0,$$  \hspace{1cm} (3)

and

$$\sum_{s \in \{H, L\}} \pi_s(a)u(z_s) - v(a) \geq \sum_{s \in \{H, L\}} \pi_s(b)u(z_s) - v(b),$$  \hspace{1cm} (4)

where in the incentive compatibility constraint (4) we are only checking for deviations concerning the effort level, and the compensation only depends on the realized state.\textsuperscript{19} The solution of $\mathcal{P}_{SB}$ is given by the values of $z_H^*, z_L^*$ satisfying (3) and (4) as equalities.\textsuperscript{20}

\textsuperscript{18}This is not necessarily true if other forms of punishment than the seizure of the payments due for side trades were allowed. But see the discussion of alternative punishments below.

\textsuperscript{19}When there is no uncertainty over monitoring, i.e., when $m = 1$ or $m = 0$, the participation constraint (1) simplifies as in (3).

\textsuperscript{20}Under our assumption that preferences are separable in consumption and effort, it is known, see, e.g., Bennardo and Chiappori (2003), that at any incentive efficient allocation the participation constraint binds.
On the other hand, if $m = 0$, shareholders are unable to do any monitoring of the manager’s trades. Thus the manager can always engage in trades in financial markets without any risk of being detected. It is easy to see that in this case the best the manager can do by trading in the market is to fully insure (at the price $\pi(b)$) against the fluctuations in his income (and in that case he would switch to low effort). Under Assumption 2 the high level of effort can still be implemented in this case; the optimal compensation scheme is then the one that makes the manager just indifferent between making such trades and not making them (incentive compatibility), i.e.,

$$
\pi_H(a)u(z_H) + \pi_L(a)u(z_{nm}^L) - v(a) = u(\pi_H(b)z_H + \pi_L(b)z_{nm}^L) - v(b) \tag{5}
$$

and satisfies the participation constraint (3) as equality.\footnote{For sufficient conditions implying that the participation constraint binds in this case, see Lemma 3 in the Appendix.} We will denote by $(z_H(0), z_{nm}^L(0))$ the solution of (3), (5) describing the optimal compensation scheme when $m = 0$. The incentive constraint is now clearly more restrictive and we can easily show that the optimal compensation is characterized by a higher level of risk than when trades are fully observable (i.e., at the second best $(z_H^*, z_L^*)$ the manager’s compensation is less steep).\footnote{Garvey (1993) studies a similar problem with continuous effort choice.}

**Proposition 2** Comparing the optimal compensation scheme with no monitoring and with full monitoring, we have $z_H(0) > z_H^* > z_L^* > z_{nm}^L(0)$.

From Proposition 2 we get so:

$$
z_H(0) - z_{nm}^L(0) > z_H^* - z_L^*.
$$

Since $(z_H(0), z_{nm}^L(0))$ and $(z_H^*, z_L^*)$ are characterized, as we said, by the same expected value of the payments to the manager, we conclude that the variance of the manager’s compensation is higher with zero than with full monitoring of his trades.

We proceed now to the characterization of the optimal compensation scheme for any given intermediate value of $m \in (0, 1)$. When $m = 1$, as we saw, both the incentive and the participation constraints hold as equality at an optimum so that, since there are only two states, the optimal compensation in each state is simply obtained by solving these constraints. In fact, we can show that, whatever $m$ is, at an optimum contract the incentive constraint still holds as equality (Lemma 2 in the Appendix) and provide some sufficient conditions for the participation constraint to also bind (Lemma 3 in the Appendix). We will assume in what follows that the participation constraint binds.

To characterize the level of steepness that is required in the manager’s compensation to satisfy incentive compatibility, we have to determine the maximum utility
the manager can attain, for any given compensation $z$, by switching to low effort and hedging his risk in the market. This is the maximal value of the term on the right hand side of the inequality in the incentive compatibility condition (2). As argued in the proof of Proposition 1 (since at the optimal compensation scheme the manager can never gain by selling insurance and maintaining a high effort level), it suffices to look at trades involving the purchase of insurance; thus, we have to consider the problem:

$$\max_{(\tau_H, -\tau_L) \in \mathbb{R}_+^2} \pi_H(b)u(z_H - \tau_H) + \pi_L(b)[mu(z^m_L) + (1 - m)u(z^{nm}_L - \tau_L)] - v(b)$$

such that $\sum_{s \in \{H, L\}} \pi_s(b)\tau_s = 0$.

Its first order conditions are:

$$u'(z_H - \tau_H) \geq (1 - m)u'(z^{nm}_L)$$

$$\tau_H \geq 0.$$  \hspace{1cm} (6)

Therefore, if

$$u'(z_H) < (1 - m)u'(z^{nm}_L)$$

(i.e., if $z_H$ is considerably larger than $z^{nm}_L$), the maximal utility (by deviating to low effort) is attained with a non-zero level of trade in the market, while if

$$u'(z_H) \geq (1 - m)u'(z^{nm}_L)$$  \hspace{1cm} (7)

the manager prefers not to engage in trades in the market.

On this basis we can easily show that if the probability of monitoring $m$ is sufficiently high (though less than 1), the optimal contract is the same as the one with perfect observability of trades ($m = 1$):

**Proposition 3** Let $m^* \equiv 1 - u'(z^*_H)/u'(z^*_L) < 1$. Then, for any $m \geq m^*$, the second best contract $z^*_H, z^*_L$ can be implemented (satisfies (2)) and hence constitutes the optimal compensation scheme: $z_H(m) = z^*_H$ and $z^{nm}_L(m) = z^m_L(m) = z^*_L$.

To better understand this finding, notice that by trading in the market the manager can freely transfer income from state $H$ to state $L$ when no monitoring occurs (he is obviously unable to transfer income to state $L$ when monitoring occurs since all the proceeds from any trade will be seized). The relative price at which such a transfer can occur is $\frac{\pi_L(b)}{\pi_H(b)}$ while the odds of these states are $\frac{\pi_L(b)(1-m)}{\pi_H(b)}$. Thus monitoring implies that the manager can hedge (some of) his risk but at a price which is less than fair. When $m$ is sufficiently close to 1, the cost of hedging becomes so high that the manager prefers not to do any of it.
For any \( m < m^* \) the second best contract is not implementable: the manager can in fact attain a higher utility by switching to low effort and making non-zero trades in the market than by exerting high effort. To sustain incentives the optimal compensation scheme will hence have to depart from \( z^* \), but in which direction? A first answer is provided by the following:

**Proposition 4** For any \( m < m^* \) the optimal compensation scheme \( z(m) \) is such that:

\[
z_{nm}^L(m) > z_m^L(m)
\]

and, if the manager were to deviate to low effort, he would choose to buy insurance, \( \tau_H > 0 \),

This result shows that, when the manager wishes to engage in side trades, it is optimal to condition his compensation on whether or not monitoring occurs. To gain some intuition on this, notice first that the contract must provide incentives to exert high effort: the compensation in the high state has to be sufficiently higher than the compensation in the low state. But the contract must also provide incentives not to engage in trades in the market. Such trades, as we said, allow the manager to transfer income from the high state to the low state when monitoring does not occur. Hence the possibility to engage in these trades will be more valuable to the manager the larger is the difference between his income in these two states, \( z_H \) and \( z_{nm}^L \). On the other hand, his compensation in the low state when monitoring does occur, \( z_m^L \), plays no role for this. As a consequence, by setting \( z_{nm}^L \) relatively high we can enhance the manager’s incentives not to engage in side trades and can sustain his incentive to exert high effort with a sufficiently low level of \( z_m^L \).

**Remark 1** It is interesting to point out that the property \( z_{nm}^L(m) > z_m^L(m) \) we find is in contrast to the finding in the costly state verification literature that the agent is rewarded if he is monitored and did tell the truth (see in particular Lemma 2 in Mookherjee and Png (1989)). In our model, when the agent is monitored his compensation is low even if he did nothing wrong. The critical difference is that in our model there is a link between the compensation of the manager when he is monitored and found not to have engaged in hedging trades, given by \( z_{nm}^L \), and the compensation when he is monitored and did engage in such trades, which is \( z_m^L - \max\{\tau_L, 0\} \). Reducing \( z_{nm}^L \) relative to \( z_{nm}^L \) increases the cost in utility terms of the seizure of the payoffs from the hedging trades and thus increases the penalty for hedging. Furthermore, increasing \( z_{nm}^L \) relative to \( z_m^L \) reduces the benefits of hedging since the agent would enjoy these in state \( L \) when he is not monitored in which case he would consume \( z_{nm}^L - \tau_L \). In the standard costly state verification model in contrast there is no link between what the agent gets paid when he is monitored and announced the cash flow truthfully and what he is paid when he is monitored and found to have
understated the cash flow. Mookherjee and Png (1989) for example assume that the agent is paid 0 in that case. Without a link between the compensation when a deviation is detected and when monitoring occurs and no deviation is detected, it is then optimal to reward the agent when he is monitored and no deviation occurred. His compensation in that state affects only the objective and the left hand side of the incentive compatibility constraint, whereas the compensation when he is not monitored also affects the right hand side of the incentive compatibility constraint, i.e., the agent’s incentives to understate cash flow. The analysis of alternative specifications of penalties in the next Section provides additional discussion of this point.

Also, while it is often observed that, to exert monitoring after the agent’s action has been made, the principal has to credibly commit to do so ex ante, in our set-up this problem may not arise. This is because at an optimum the compensation paid to the agent/manager is lower when monitoring is exerted, and this may prove a sufficient incentive for the principal to indeed be willing to monitor, if the cost of doing so is not too high.23

Example 1 Consider the case in which the manager has logarithmic preferences, i.e., \( u(z_s) = \ln z_s \). In this case, we can explicitly compute the level of trade \( \tau_H \) the manager would choose if he were to undertake low effort when his compensation is \( z \):

\[
\tau_H = \max \left\{ \frac{(1-m)z_H - z_{nm}}{(1-m)+\frac{\pi_H(b)}{\pi_L(b)}}, 0 \right\}
\]

Note that \( \tau_H \) varies linearly with \( z \) and is larger the larger the difference between \( z_H \) and \( z_{nm} \) (i.e., the larger the gains from insurance).

Consider then the following parameter values: \( y_H = \frac{5}{4}, y_L = \frac{1}{4}, \pi_H(a) = \frac{3}{4}, \pi_H(b) = \frac{1}{4}, v(a) = \frac{1}{4}, \) and \( v(b) = 0 \). The manager’s optimal compensation for different values of \( m \) are reported in Panel A of Table 1 and in Figure 1. The optimal compensation with perfect observability \((z_H^*, z_L^*)\) (dotted) lies between the optimal compensation with no monitoring \((z_H(0), z_L(0))\) (dashed), and thus the compensation contract is steeper without monitoring (see Proposition 2). The solid line graphs the compensation contract \((z_H(m), z_{nm}^*(m), z_{nm}^*(m))\) as a function of \( m \). When the monitoring probability exceeds \( m^* \approx 39\% \), the compensation schedule is as if hedging were perfectly observable (see Proposition 3). Moreover, the manager’s utility increases monotonically as \( m \) is increased from 0 to \( m^* \). Also, the steepness in the manager’s compensation decreases as \( m \) rises; in particular the compensation in the good state \( H \) goes down while the one in the bad state \( L \) when monitoring occurs goes up. On the other hand, in this example the compensation in state \( L \) when no monitoring occurs varies non-monotonically with \( m \): as \( m \to 0 \), \( z_{nm}^*(m) \to z_L^*(0) < z_L^* \), but for \( m \) close to but less than \( m^* \), \( z_{nm}^*(m) \) is even higher than the second best level \( z_L^* \). Here, the effect that higher \( z_{nm}^*(m) \) reduces the incentives to hedge dominates. Finally, for all \( m < m^* \), \( z_{nm}^* \) is strictly greater than \( z_L^* \) (which is optimal as we argued since it reduces the manager’s incentive to engage in hedging activity; see Proposition 4).

\( ^{23} \)We owe this observation to Sonje Reiche.
We have studied so far the optimal contracting problem for given monitoring probability \( m \). By introducing the consideration of monitoring costs the optimal intensity of monitoring can also be determined.

Let \( V(m) \) denote the manager’s expected utility at the optimal contract \( z(m) \), obtained as a solution of \((P_{MON})\) for given \( m \). We can show that this value is increasing in \( m \):

**Proposition 5** \( V(m) \) is strictly increasing in \( m \), for \( m < m^* \).

The optimal level of \( m \) is then obtained as the solution of the following problem:

\[
\max_m V(m) - \phi(\pi_L(a)m).
\]

In fact, assuming the cost function \( \phi(\cdot) \) is not only increasing but also sufficiently convex, the optimal level of \( m \) is uniquely determined.

### 2.3 Alternative Specification of Penalties

So far we have restricted attention to environments where the only penalty is the seizure of payoffs of side trades which the manager is due to receive. While this specification is consistent with the limited disclosure requirements and with the limited case law, as we argued in the Introduction, harsher penalties would clearly be valuable. In this section we extend our analysis to consider an alternative specification in which a reduction in the pay to the manager can be imposed when he is monitored and caught hedging.

Suppose, more specifically, that the manager’s pay can be reduced by a given amount \( k \) if managerial hedging is detected, so that he consumes \( z_s^m - k - \max\{\tau_s, 0\} \) in that case. It turns out that all of our results still obtain in this case, which we show in part within the set-up of the example considered earlier numerically and in part more generally.

To show that monitoring in the low state only is optimal, we take the unconditional monitoring probability, say \( \bar{m} \), as given, and assume that the monitoring probability in the two states is chosen optimally subject to the constraint that

\[
\pi_H(a) m_H + \pi_L(a) m_L \leq \bar{m}.
\]

We find, within the set-up of Example 1, that it is still optimal to set \( m_H = 0 \) (and hence \( m_L = \bar{m}/\pi_L(a) \)). The intuition is as follows. Since compensation in state \( L \)

\[24\] Yet another possible specification would include penalties imposed on the investment banks offering derivative hedging contracts to managers. In practice, though, legitimate reasons for the managers to hedge might exist, and requiring investment banks to monitor the managers’ motivations for trading may then not be optimal.
is lower than in state $H$, the penalty $k$ is larger in utility terms in state $L$ and thus monitoring occurs in state $L$ only. Moreover, since managerial hedging pays off in state $L$ and such payoffs can be seized, this is another reason why the manager’s portfolio is monitored in state $L$ (indeed, this is the intuition for Proposition 1).

**Example 1 (Continued)** Consider the same environment of Example 1 but assume that monetary penalties of size $k$ can be imposed for hedging (in addition to the seizure of all payoffs of hedging activity). When $k = 0$ we obtain then the situation of Example 1 as a special case (thus the dotted, dash-dotted, and solid line are as in Figure 1). In the numerical computation of this example, we allow monitoring to take place in both states with $m_H, m_L$ chosen subject to the constraint that $\pi_H(a)m_H + \pi_L(a)m_L \leq \bar{m}$. We find that $m_H = 0$, i.e., that monitoring occurs in state $L$ only and hence that $m_H = 0$ is still optimal, even if $k$ is positive.

In Figure 2 the optimal compensation is then again plotted as a function of $m \equiv m_L = \bar{m}/\pi_L(a)$, for three values of $k$: 0 (which, as we argued, corresponds to the case discussed previously), 0.02 (dash-dotted line), and 0.05 (bold dotted line). Consider the optimal compensation for $k = 0.05$, which is the bold dotted line in the figure. First, note that the compensation is only graphed for $m$ less than approximately 14%. When $m$ is higher than that, the compensation contract is equivalent to the case of perfect observability. In the previous case (i.e., with $k = 0$), this only occurs for $m > m^* \approx 39\%$, i.e., much higher levels of monitoring were required for the compensation contract to be equivalent to perfect observability. The additional penalty imposed by $k > 0$ clearly improves matters. When $m$ is less than 14%, the compensation contract varies with $m$ in a similar fashion as before (with $k = 0$), but the difference between $z_{nm}^L$ and $z_{m}^L$, which is again positive, is in fact larger: the compensation when the manager is monitored is reduced further (when $k > 0$) since this gives the monetary penalty $k$ additional bite. Note also that at $m \approx 14\%$ there is a discontinuity in the compensation, which jumps to the perfect observability contract; this is due to the fact that there is a penalty of fixed size here.

With $k = 0.02$, a monitoring probability of at least 21% is required for the manager’s access to hedging markets not to affect the compensation contract (i.e., for the second best contract to be implementable). Otherwise the results are comparable.

In the Example we have also seen that, except for the fact that the minimum level of monitoring needed to implement the second best contract is now lower and the compensation is discontinuous at that point, the optimal compensation with $k > 0$ exhibits similar features to those found when $k = 0$, in particular the property $z_{nm}^L \geq z_{m}^L$ is still valid. We can show that this property has general validity:

**Lemma 1** If an additional penalty in the form of a salary reduction of size $k$ is imposed when hedging is detected, the optimal contract is such that $z_{nm}^L \geq z_{m}^L$, with strict inequality when the optimal deviation is characterized by $\tau_H > 0$. 

18
Moreover, this result - as well as the previous findings - remains valid even if we assume that the payoffs of managerial hedging cannot be seized (in which case the only penalty for hedging is a reduction of salary of size $k$, so that the manager would get $z_L^n - \tau_L - k$ in state $L$ when monitored and hedging is detected). As already argued in Remark 1, what is essential for the result is that there is a link between what the manager gets paid when he is monitored and did nothing wrong and what he gets paid when hedging is detected. In the presence of such link, paying the manager more when he is not monitored reduces the benefits of hedging and paying him less when he is monitored increases the penalty in utility terms if caught having traded.

On the other hand, if we were to consider the case where the penalty consists in reducing the compensation of the manager down to a minimum level $K$, independently of what the compensation promised to the manager in state $L$ was (analogously to Mookherjee and Png (1989)), our results could be overturned.

### 3 Managerial Compensation and Portfolio Monitoring: Intertemporal Case

This section extends the analysis of the contracting problem to an intertemporal framework, where there is output (and consumption) at two possible dates, date 0 and date 1. The firm produces a deterministic cash flow at date 0, given by $y_0 > 0$, and a random cash flow at date 1, again taking values $y_H$ and $y_L$ with probability dependent on the manager’s effort level. The manager and shareholders have a common discount factor, equal to one. The manager’s preferences over his income at date 0 and date 1 in every possible state are:

$$u(z_0) + \sum_{s \in \{H, L\}} \pi_s(e)u(z_s) - v(e).$$

The interest of the extension lies in the fact that, in this set-up, the optimal incentive contract has implications regarding the optimal distribution of the manager’s compensation over time. We find that, relative to the case where managers cannot hedge, their compensation is shifted from date 0 to date 1. In fact, as shown by Rogerson (1985), in an intertemporal agency problem with hidden action, when no side trades are possible, at the optimal contract the time profile of the compensation is distorted in favor of the initial period - i.e., exhibits front loading - as this allows to improve incentives; as a consequence, the agent would want to save (if he had the option to do so). When the manager has access to hedging markets, shareholders face some limitations in the extent by which they can distort the time profile of the manager’s compensation. The characterization of the optimal contract parallels otherwise the one in the case without date 0 consumption: monitoring occurs in state $L$, the manager bears more risk, and his compensation in state $L$ is higher when he

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In their case $K = 0$, but their problem is still not trivial since they assume $u(0) = 0 > -\infty$. 

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19
is not monitored than when he is monitored.\footnote{Park (2004) considers a similar environment in which the agent’s date 0 consumption and savings decision is not observable and is taken prior to contracting. He concludes that only low effort is implementable.}

In an intertemporal framework we have to distinguish between the case in which the manager can make side trades in a complete set of contingent claims, so that he is free to borrow and lend as well as to insure against any possible fluctuation in his compensation, and the case in which the manager’s side trades are restricted to risk free borrowing and lending. We examine both cases in turn.

### 3.1 Hedging Incentive Compensation with Contingent Claims

Suppose the manager (and shareholders) have access to financial markets where, at date 0, claims contingent on any state \( s \in S \) can be traded. As in the previous section, markets are anonymous and competitive: agents face a given unit price, which may differ for purchases and sales, at which they are free to choose the level of their trades.

Equilibrium prices are the same as before: for purchases of claims contingent on the \( L \) state and sales of claims contingent on the \( H \) state (corresponding to hedging trades) they are fair conditional on low effort, \( p_L^+ = \pi_L(b) \), \( p_H^+ = \pi_H(b) \), while for sales of claims contingent on \( L \) and purchases of claims contingent on \( H \) (that correspond to betting on the firm) they are fair conditional on high effort, \( p_L^- = \pi_L(a) \), \( p_H^- = \pi_H(a) \).

Under Assumption 2 the optimal compensation scheme again implements high effort and we will show that, at the above prices, the manager does not wish to engage in trades in the financial market.

Note that the above expressions for the equilibrium prices now also implies that the riskless rate at which the manager can borrow between date 0 and 1 is \( 1/(p_H^+ + p_L^-) - 1 = 1/(\pi_H(b) + \pi_L(a)) - 1 > 0 \), while the riskless rate at which he can lend is \( 1/(p_H^- + p_L^+) - 1 = 1/(\pi_H(a) + \pi_L(b)) - 1 < 0 \). Thus there is a positive spread not only for the trade of each contingent claim, but also for the trade of a claim with a riskless payoff.

In what follows, we will focus our attention on the case where monitoring only takes place at date 1, not at date 0. This is primarily for simplicity and will make the comparison with the results for the one period model easier. In this case, we are able to show (see Lemma 4 in the Appendix) a result analogous to Proposition 1, i.e., that exerting monitoring only in state \( L \) is optimal. This obviously does not mean that if monitoring could also be exerted at date 0, this would necessarily be redundant. However, the substance of our results would not be affected if monitoring at date 0 were allowed and, moreover, monitoring in state \( L \) is most effective since, as we will show, the manager’s compensation is lowest in that state and hence monetary
penalties (seizing hedging payoffs or additional penalties as in Section 2.3) have the largest effect on utility.

We will show that the optimal compensation scheme for the manager in this two-period framework, when monitoring of side trades is random and occurs only in state $L$, with probability $m$, is obtained as the solution of the following problem:

$$
\max_{[z_0(m), z_H(m), z_L^m(m), z_L^m(m)] \in \mathbb{R}_+^4} [u(z_0) + \pi_H(a)u(z_H) + \pi_L(a) \left\{ (1-m)u(z_L^m) + mu(z_L^m) \right\} - v(a)]
$$

subject to

$$(y_0 - z_0) + \pi_H(a)(y_H - z_H) + \pi_L(a)\{y_L - (mz_L^m + (1-m)z_L^m)\} \geq 0 \tag{8}$$

and

$$u(z_0) + \pi_H(a)u(z_H) + \pi_L(a) \left\{ (1-m)u(z_L^m) + mu(z_L^m) \right\} - v(a) \geq u(z_0 - \tau_0) + \pi_H(e')u(z_H - \tau_H) + \pi_L(e') \left\{ (1-m)u(z_L^m - \tau_L) + mu(z_L^m - \max\{\tau_L,0\}) \right\} - v(e'), \tag{9}$$

for all $e' \in \{a, b\}$ and $(\tau_0, \tau_H, \tau_L) \in \mathcal{T}(b)$, where $\mathcal{T}(b) \equiv \{\tau_0, (\tau_H, -\tau_L) \in \mathbb{R} \times \mathbb{R}_+^2 : \tau_0 + \pi_H(b)\tau_H + \pi_L(b)\tau_L = 0\}$ is the set of trades in financial markets that are budget feasible and are restricted to be only purchases of insurance, i.e., sales of $H$ claims and purchases of $L$ claims.$^{27}$

In problem $\mathcal{P}_{MON}^0$ we imposed two additional restrictions on the contracting problem: we required monitoring to take place only in state $L$, not in $H$, and required trades to lie in $\mathcal{T}(b)$. Lemma 4 in the Appendix shows that neither of these restrictions is binding and hence that a solution of problem $\mathcal{P}_{MON}^0$ indeed gives the optimal compensation scheme when the manager is free to choose both to sell as well as to purchase insurance in the market for contingent claims at the prices $p^+$ and $p^-$ described above, and when monitoring occurs in both states at date 1.

Let $Z(m) \equiv [z_0(m), z_H(m), z_L^m(m), z_L^m(m)]$ denote the solution of problem $\mathcal{P}_{MON}^0$. By the previous argument this defines the optimal compensation paid to the manager in each date and in every contingency. Whenever it is possible without generating confusion, the dependence on $m$ will be omitted.

In what follows we will examine how different levels of ability to monitor the manager’s trades of contingent claims affect the optimal contract. The focus will be primarily on the distribution of the compensation over time (between date 0 and 1); the effects on the steepness of the compensation (its variability between the $H$ and the $L$ state) are - qualitatively - similar to the one found in the previous section, as we will see.

$^{27}$These are the trades for which prices are given by $\pi(b)$, i.e., are fair conditional on low effort being exerted.
To characterize the optimal contract it is useful, as in the previous section, to begin with the two extreme cases where there is no monitoring, i.e., $m = 0$, and where there is perfect monitoring in state $L$, i.e., $m = 1$. Note that, since we ruled out by assumption the possibility of exerting monitoring at date 0, the case $m = 1$ no longer corresponds to the second best (incentive efficient) contract, but rather to the contract obtained as the solution of the following program:

$$\max_{[z_0, z_H, z_L] \in \mathbb{R}^3} u(z_0) + \pi_H(a) u(z_H) + \pi_L(a) u(z_L) - v(a) \quad (P_{SBc}^0)$$

subject to

$$(y_0 - z_0) + \pi_H(a)(y_H - z_H) + \pi_L(a)(y_L - z_L) \geq 0$$

and

$$(1 + \pi_H(b)) u \left( \frac{1}{1 + \pi_H(b)} z_0 + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H \right) + \pi_L(b) u(z_L) - v(b) \geq u(z_0) + \pi_H(a) u(z_H) + \pi_L(a) u(z_L) - v(a) \quad (10)$$

Since trades in state $L$ are fully monitored, and payoffs seized, the manager will never engage in such trades: $\tau_L \equiv 0$ (hence $z_{nm}^L = z_{nm}^L \equiv z_L$). On the other hand, the manager will now still be able to sell, unmonitored, claims contingent on $H$, and will then optimally use this opportunity to perfectly smooth his income between state $H$ and date 0, as in (10) above. Let us denote a solution of problem $P_{SBc}^0$ by $Z^+ \equiv [z_0^+, z_H^+, z_L^+]$ and the income at date 0 and in state $H$ under the optimal deviation by $z_d^+ \equiv \frac{1}{1 + \pi_H(b)} z_0^+ + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H^+$.

We can show (all results are formally stated and proved in the Appendix) that the optimal compensation with no monitoring $Z(0)$ is characterized by perfect intertemporal smoothing ($u'(z_0(0)) = \pi_H(a) u'(z_H(0)) + \pi_L(a) u'(z_L^m(0))$), while the one with full monitoring (in state $L$) is distorted in favor of the initial period, i.e., exhibits front loading: $u'(z_0^+) < \pi_H(a) u'(z_H^+) + \pi_L(a) u'(z_L^+)$. As mentioned earlier, the latter property (i.e., the presence of front loading) was established by Rogerson (1985) for the case where no side trades are possible. Our result shows that this is also true when side trades are restricted to take place only in some markets, those for the $H$ claims.

Moreover, as in the static case, incentives are steeper, and the compensation in state $H$ larger, with no monitoring compared to the case of full monitoring and, if $u''' > 0$, the compensation at date 0 is lower with no monitoring (as we argued in this case there is no front loading) than with full monitoring.\footnote{It is possible to show that exactly the same properties established in Proposition 6 in the Appendix hold when the optimal compensation scheme with no monitoring, $Z(0)$, is compared to the optimal compensation scheme with full monitoring in all markets (also at date 0), i.e., to the incentive efficient (second best) contract $Z^*$. The proof is similar and is hence omitted.}
Consider then the case of imperfect monitoring, \( m \in (0, 1) \). We find again that, as long as the probability of monitoring \( m \) is sufficiently high, the optimal contract with imperfect monitoring is the same as with full monitoring (in state \( L \)). When the probability of monitoring is not sufficiently high (so that the optimal contract with full monitoring is no longer implementable), the optimal compensation scheme is such that the compensation is higher in state \( L \) in the event of no monitoring than when monitoring occurs and, if the manager were to trade in the financial markets, he would choose to buy insurance, \( \tau_L < 0 \). Also, for all \( m \) we have \( z_{H}(m) > z_{0}(m) > z_{nm}^{H}(m) \).

**Example 2** Modify the environment of Example 1 by introducing date 0 consumption and a date 0 endowment of \( y_0 = 1/4 \). The values of the optimal compensation in this case are reported in Panel B of Table 1 and in Figure 3. In this example, \( m^+ \approx 35\% \) so that this monitoring intensity is sufficient to get managers to refrain from hedging their compensation in state \( L \). Furthermore, note that while the compensation with perfect monitoring in state \( L \) only, \( Z^+ \), and the compensation with perfect monitoring in both states, \( Z^* \), do not coincide, they are almost indistinguishable; this suggests that the manager’s main concern is to insure against his low income in state \( L \) at date 1. Once this is prevented by monitoring in that state, the compensation contract looks almost identical to the optimal compensation when there is perfect observability of trades. Also, note that the manager’s compensation at date 0, \( z_0(m) \), increases as \( m \) increases: the higher \( m \) is, the more front loading of the compensation is possible. The other aspects of the characterization parallel the ones of Example 1.

### 3.2 Hedging Incentive Compensation with Hidden Borrowing and Lending

We turn our attention next to the case where the manager has no access to markets for contingent claims, but only to markets where a riskless asset is traded, or equivalently there can only be hidden borrowing and lending.\(^{29}\) Markets are again anonymous and competitive: agents face a given unit price at which they are free to choose the level of their trades. Since there are no informational asymmetries concerning the payoff of such claims, their price at equilibrium will be the same for sales and purchases and equal to the common discount factor, \( p = 1 \). As in the previous section, we consider the case where monitoring takes place only at date 1. We will also assume that monitoring only takes place in state \( L \). Indeed, numerical computations suggest that this is again optimal. The intuition is as follows: if the manager were to save using the riskless asset, these savings could be seized when he is monitored. But having the savings seized is more of a penalty when output and hence his compensation is

\(^{29}\)This is the case which is most studied in the literature; see, e.g., Allen (1985) and Cole and Kocherlakota (2001).
The optimal compensation scheme with hidden borrowing and lending (in a riskless asset) and random monitoring is then obtained as solution of the maximization of the manager’s utility

$$\max_{[z_0(m), z_H(m), z_{nm}^L(m), z_L^m(m)] \in \mathbb{R}^4_+} u(z_0) + \pi_H(a)u(z_H) + \pi_L(a) \{(1 - m)u(z_{nm}^m) + mu(z_L^m)\} - v(a)$$

subject to the same participation constraint as in the previous Section, (8), and the following new expression for the incentive compatibility constraint:

$$u(z_0) + \pi_H(a)u(z_H) + \pi_L(a) \{(1 - m)u(z_{nm}^m) + mu(z_L^m)\} - v(a) \geq u(z_0 - \tau_0) + \pi_H(b)u(z_H - \tau) + \pi_L(b) \{(1 - m)u(z_{nm}^m - \tau) + mu(z_L^m - \max\{\tau, 0\})\} - v(b)$$

for all $\tau_0, \tau \in \mathbb{R}^2$ such that $\tau_0 + \tau = 0$. Let $Z^f(m)$ denote its solution.

Note that, when $m = 0$, $\mathcal{P}_{MON}^{0,f}$ is the “classic” problem yielding the optimal contract with hidden savings. On the other hand, when $m = 1$, its solution is given by the second best contract $Z^*$, i.e., by the optimal contract with no side trades (with $m = 1$ the manager can in fact only use side trades to transfer income, at a price equal to 1, from date 0 to state $H$ at date 1 and from both states at date 1 to date 0 - i.e., to borrow - and it is possible to verify that at the second best contract the manager does not wish to engage in such trades).

As mentioned in the previous section, we know from Rogerson (1985) that at the second best contract $Z^*$ in a two period framework the manager’s income is distorted in favor of the first period: $u'(z_0^*) < \pi_H(a)u'(z_H^*) + \pi_L(a)u'(z_L^*)$. Hence if the manager can engage in hidden trades in a risk free asset the optimal contract would be different, $Z^* \neq Z^f(0)$, and characterized by a lower payment at the initial date, $z_0^f(0) < z_0^*$. In the Appendix we show that, in addition, all the properties of the optimal contract established in the previous section for the case in which the manager could hedge using a complete set of contingent claims remain valid when he is restricted to side trades in a risk free asset.

**Example 3** Consider again the same set-up of Example 2. The levels of the optimal compensation for the case where side trades are restricted to risk free borrowing and lending are reported in Panel C of Table 1 and in Figure 4. The results are qualitatively similar to what was found in Example 2 for the case where the manager can

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30This intuition also suggests that the same is true if additional monetary penalties (of size $k$) can be imposed when hedging is detected, and numerical computations confirm that.

31In the previous Section (see Lemma 6 in the Appendix) we established the same property for the optimal contract $Z^+$ when there is full monitoring, but only in state $L$, of trades in contingent claims.
use contingent claims to hedge his compensation. However, since here the scope for hedging is more limited, the manager’s compensation is less distorted. Indeed, we find that $m_f \approx 30\%$, which means that a lower monitoring probability is sufficient for the manager’s compensation to be identical to the compensation he would get with perfect observability. (In the case of hedging with contingent claims we had $m^+ \approx 35\%$, and even for $m > m^+$ the optimal compensation contract was not identical to the one under perfect observability.) Finally, note that the main distortion when $m$ is low is that compensation is shifted from date 0 to state $H$.

4 Discussion

At least since the 1990s managers have had access to financial instruments which allow them to hedge the firm specific risk in their compensation packages. Moreover, until recently, regulation has been ineffective in requiring managers to promptly disclose these financial transactions to shareholders and other investors. The analysis of these facts by legal and financial commentators has generally come to the conclusion that managerial hedging undermines incentives in executive pay schemes, significantly alters the executives’ effective ownership of the firm, and hence has adverse effects on performance. But is this necessarily the case?

Boards and shareholders should have recognized the managers’ ability to hedge their incentive compensation packages as well, and should have designed their managers’ incentive schemes accordingly. If so, we show in this paper, they should monitor their managers’ portfolio, scrutinize their financial transactions, and possibly bring derivative suits for violation of fiduciary duty when they observe transactions by the managers which hedge the risk of incentive compensation beyond the amount mutually understood to be acceptable. In light of this, it is then somewhat puzzling, that executive pay contracts rarely state explicitly the form and amount of hedging that managers are allowed to engage in.

One possible explanation is that corporate governance is severely ineffective and boards collude with executives to extract rents at the expense of shareholders, as argued by Bebchuck and Fried (2003). In this case incentive pay schemes should not be expected to restrict the managers’ hedging ability in financial markets. The fact however that shareholders keep giving stocks and options to managers in their compensation package poses a challenge to this view in our opinion.

Another explanation, more in line with the approach followed in this paper, is possible: the managers’ hedging transactions are allowed to a limited extent because firms have incentives to design excessively risky compensation packages, e.g., provide compensation largely in the form of stock options, due to their advantageous tax treatment. In this case, at the optimal contract, some managerial hedging should be observed but this does not imply a violation of fiduciary duty by managers and
as a consequence no legal action against them by shareholders should be observed.\textsuperscript{32} Furthermore, the fact that monitoring of managerial hedging trades is costly and difficult, together with the presence of (albeit limited) disclosure requirements imposed by laws or regulations are consistent with the limited evidence of monitoring activities.

5 Conclusion

Our analysis of the optimal compensation contract when managers can hedge the risk in their compensation and monitoring these hedging trades is costly and hence imperfect shows that monitoring of managers’ portfolios optimally occurs when firm performance is poor. Increased scrutiny of managers’ affairs when a firm does poorly may hence be optimal rather than being an attempt by shareholders to expropriate managers ex post. Moreover, we find that, conditional on the firm’s performance, the manager’s compensation is lower when his portfolio is monitored, even if no hedging is revealed by monitoring; hence managers may be worse off, i.e., their pay reduced, when their affairs are scrutinized even if they have done nothing wrong. Finally, we show that when monitoring is more costly or hedging markets are more easily accessible, shareholders provide managers with steeper incentives. This result may provide an explanation for the fact that in cross sections, countries where hedging markets are more developed, such as the US or the UK, also have steeper incentive compensation, and that over time incentive compensation became steeper as hedging markets developed. Additional empirical predictions of our model are that the recent increase in disclosure requirements may result in a reduction of incentive compensation (and hence of payments in the form of stocks and options) and that in industries where managerial hedging is easier managerial incentive compensation should be steeper.

\textsuperscript{32}In such an event, explicit mention in the contract of allowances for hedging might expose the firm to legal action by the tax authority.
Appendix: Proofs

Appendix A: Proofs for Section 2 - The Static Case

Proof of Proposition 1. Let \((z^m, z^{nm})\) be the optimal contract (i.e., a solution of \(P_{MON}\)) when monitoring is exerted both in \(H\) and \(L\). Such a contract as we said always implements the high effort level, hence we must have:

\[
(\pi_H(a) - \pi_H(b)) \left[(1 - m_H)u(z_H^m) + m_Hu(z_H^m) - (1 - m_L)u(z_L^{nm}) - m_Lu(z_L^m)\right] \geq v(a) - v(b).
\]

Any transaction in the financial market such that \(\tau_H < 0, \tau_L > 0\) (i.e., a sale of insurance) increases the manager’s income in state \(H\) (when no monitoring occurs) and lowers it in state \(L\) (whether or not monitoring occurs); as a consequence, the above inequality remains valid, so that the agent still prefers to exert a high effort level.

The optimality of \((z^m, z^{nm})\) then implies that the manager cannot attain a higher level of utility by engaging in such trades. Since the manager would keep exerting high effort, his trades would have no adverse effect on the shareholders’ utility; therefore, if such trades increase the manager’s utility we would have a contradiction to the optimality of \((z^m, z^{nm})\).

We have thus shown that, if \((z^m, z^{nm})\) is the solution of \(P_{MON}\) (when monitoring is exerted both in \(H\) and \(L\)), the manager never wants to engage in trades in the financial market that entail a sale of insurance, or the incentive compatibility constraint (2) never binds with \((-\tau_H, \tau_L) \in \mathbb{R}_+^2\) such that \(\sum_{s \in \{H, L\}} \pi_s(a)\tau_s = 0\). This implies that monitoring is not needed to discourage trades consisting in the sale of insurance. It leaves us with only one possible role for monitoring in state \(H\), that of introducing some randomness in the manager’s compensation in state \(H\), which may vary according to whether or not monitoring occurs: \(z_H^{nm} \neq z_H^m\). However, from the concavity of \(u(.)\) it follows that a pure randomization of the manager’s compensation, i.e., not motivated by incentives, is never optimal. \(\square\)

Proof of Proposition 2. From the form of the incentive compatibility constraint given in (5) for the case \(m = 0\) and the strict concavity of \(u(.)\) we get\(^{33}\)

\[
\pi_H(a)u(z_H) + \pi_L(a)u(z_L^{nm}) - v(a) > \pi_H(b)u(z_H) + \pi_L(b)u(z_L^{nm}) - v(b).
\]

But then

\[
u(z_H) - u(z_L^{nm}) > \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*),
\]

which implies \(z_H(0) > z_H^* > z_L^* > z_L^{nm}(0)\), since both \((z_H(0), z_L^{nm}(0))\) and \((z_H^*, z_L^*)\) have the same expected value (as they both satisfy (3) as an equality). \(\square\)

Lemma 2 At the optimal compensation scheme the incentive constraint (2) always holds as an equality, for all \(m\).

\(^{33}\)Note that, for (5) to be satisfied we must have \(z_H(0) > z_L^{nm}(0)\), hence the strict inequality sign.
Proof of Lemma 2. To induce the manager to exert high effort his compensation, as we argued, cannot be flat. If (2) were holding as an inequality, it would still be satisfied if we consider a small change in the compensation that keeps the expected value constant and brings closer the payments in the $H$ and the $L$ state. This would still satisfy (1) and increase the manager’s utility. A contradiction.

Lemma 3 Suppose that the manager’s preferences are such that $u(z) = \frac{z^{1-\sigma}}{1-\sigma}$ with $0 < \sigma < 1$ or $u(z) = \ln(z)$. Then, at the optimal compensation scheme, the participation constraint (1) holds as an equality, for all $m$.

Proof of Lemma 3. Suppose $u(z) = \frac{z^{1-\sigma}}{1-\sigma}$ with $0 < \sigma < 1$ and let $z = (z_H, z_L^{nm}, z_L^m)$ be the optimal compensation. In the light of Proposition 1, the incentive compatibility constraint (2), evaluated at $\lambda z$, can be written as follows:

$$\lambda^{1-\sigma}[\pi_H(a)u(z_H) + \pi_L(a)\{1 - m\}u(z_L^{nm}) + mu(z_L^m)] - (\pi_H(b)u(z_H - \tau_H) + \pi_L(b)\{1 - m\}u(z_L^{nm} - \tau_L) + mu(z_L^m - \max\{\tau_L, 0\})] \geq v(a) - v(b),$$

for all budget feasible $\tau_H, \tau_L$. Hence, since $z$ is incentive compatible, so is $\lambda z$ for all $\lambda > 1$. Evidently, $\lambda z$ is preferable to $z$, for all $\lambda > 1$. Since $z$ is optimal, $\lambda z$ must then violate the participation constraint (1), for all $\lambda > 1$, which implies that (1) must hold as equality for $z$.

Proceeding similarly for $u(z) = \ln(z)$ we find that in that case the set of incentive compatible compensation schemes is a convex cone (if $z$ satisfies (2) so does $\lambda z$ for all $\lambda > 0$).

Proof of Proposition 3. For all $m \geq m^\ast$, by construction we have:

$$u'(z_H^*) \geq (1 - m)u'(z_L^*).$$

Condition (7) is thus satisfied when $z_H = z_H^*, z_L^{nm} = z_L^*$, so that the manager does not wish to make any trade when he switches to low effort: $\tau_H = \tau_L = 0$. Since high effort was sustainable at $z_H^*, z_L^*$ with $m = 1$, it will also be for all $m \geq m^\ast$.

Proof of Proposition 4. Fix $m$ and omit for simplicity to write the optimal compensation as a function of $m$. We first show that the optimal level of trades in the market (obtained from (6)) is characterized by $\tau_H > 0$.

Suppose instead that $(z_H, z_L^{nm}, z_L^m)$ are such that $\tau_H = \tau_L = 0$ at the optimal contract. Thus, $z(m)$ satisfies $u'(z_H) \geq (1 - m)u'(z_L^{nm})$ and

$$\pi_H(a)u(z_H) + \pi_L(a)[mu(z_L^m) + (1 - m)u(z_L^{nm})] - v(a) \geq \pi_H(b)u(z_H) + \pi_L(b)[mu(z_L^m) + (1 - m)u(z_L^{nm})] - v(b)$$

as well as the participation constraint. We will first argue that $z_L^{nm} = z_L^m$. To see this assume the opposite and notice that there exists a perturbation $(dz_L^{nm}, dz_L^m)$ such that

$$mu'(z_L^m)dz_L^m + (1 - m)u'(z_L^{nm})dz_L^{nm} = 0,$$
i.e., keeping the expected utility in the low state the same, which relaxes the participation constraint since \(mdz_L^m + (1 - m)dz_L^{nm} < 0\), a contradiction. Note that, by the envelope theorem, we need not consider if the manager hedges his compensation due to the marginal change. Also, notice that the value function is differentiable at the point where \(\tau\) reaches zero, since the right hand derivative and left hand derivative coincide at that point. We use this fact throughout the proofs. Now, since \(m < m^*\), we know that

\[
u'(z_H) \geq (1 - m)\nu'(z_L^{nm}) > \frac{u'(z_H^*)}{u'(z_L^*)}u'(z_L^{nm})
\]

or \(\frac{u'(z_H)}{u'(z_L^{nm})} > \frac{u'(z_H^*)}{u'(z_L^*)}\). But since the two compensation schemes have the same expected value and do not coincide, we conclude that \(z_H^* > z_H\) and \(z_L^{nm} > z_L^*\). (For suppose otherwise, i.e., \(z_H > z_H^*\) which implies \(z_L^{nm} < z_L^*\). Then, \(u'(z_H) < u'(z_H^*)\). But above inequality then requires that \(u'(z_L^{nm}) < u'(z_L^*)\) which contradicts \(z_L^{nm} < z_L^*\).) But this contradicts the (second best) optimality of the compensation \((z_H^*, z_L^*)\).

We now turn to prove the other (and main) statement of the proposition. Suppose \(z_L^{nm} \leq z_L^m\). Consider then an infinitesimal change in the compensation \((dz_H, dz_L^{nm}, dz_L^m)\), with \(dz_H = 0\), \(dz_L^{nm} > 0 > dz_L^m\), leaving unchanged the manager’s expected utility (and hence the term on the left hand side of incentive compatibility constraint (2)):

\[\pi_L(a)\{(1 - m)\nu'(z_L^{nm})dz_L^{nm} + mu'(z_L^m)dz_L^m\} = 0,\]

Thus

\[(1 - m)dz_L^{nm} = \frac{u'(z_L^{nm})}{u'(z_L^m)}m(-dz_L^m) \leq m(-dz_L^m).\]

As a consequence the participation constraint (1) still holds since the effect on it of the change in \(z\) is

\[\pi_L(a)\{(1 - m)dz_L^{nm} + mdz_L^m\} \leq 0,\]

with the inequality being strict if \(z_L^{nm} < z_L^m\).

Finally, the effect of the change on the value of the term on the right hand side of the incentive constraint (2) is

\[\pi_L(b)\{(1 - m)\nu'(z_L^{nm} - \tau_L)dz_L^{nm} + mu'(z_L^m)dz_L^m\} < 0,\]

where the strict inequality follows from the fact, shown in (i), that \(\tau_L < 0\). Thus, the change allows to keep the manager’s utility unchanged while making the incentive constraint slack, a contradiction. □

**Proof of Proposition 5.** Fix \(m < m^*\) and drop it as an argument of the compensation for simplicity. Consider a perturbation \(dm > 0\) and \((dz_H, dz_L^{nm}, dz_L^m)\) with \(dz_H = dz_L^{nm} = 0\) and which satisfies the participation constraint, i.e.,

\[\pi_L(a)\{dm(z_L^m - z_L^{nm}) + mdz_L^m\} \leq 0\]
and thus $0 < d z^m_L \leq \frac{z^m_L - z^m_L}{m} dm$.

The effect of this perturbation on the objective and the left hand side of the incentive constraint (2) is

$$\pi_L(a) \{ (u(z^m_L) - u(z^m_L)) dm + m u'(z^m_L) dz^m_L \}.$$

If $d z^m_L = \frac{z^m_L - z^m_L}{m} dm$, this effect is

$$\pi_L(a) \{ u'(z^m_L)(z^m_L - z^m_L) - (u(z^m_L) - u(z^m_L)) \} dm > 0$$

due to the concavity of $u$. Thus, there is a $dz^m_L$ such that $dz^m_L < \frac{z^m_L - z^m_L}{m} dm$ and such that the effect on the objective equals zero. The effect on the right hand side of (2) evaluated at such a value of $dz^m_L$ is

$$\pi_L(b) \{ (u(z^m_L) - u(z^m_L - \tau_L)) dm + m u'(z^m_L) dz^m_L \} < 0$$

since $u(z^m_L - \tau_L) - u(z^m_L) > u(z^m_L) - u(z^m_L)$. By continuity, there is a $dz^m_L$ such that the effect on the objective and the left hand side of the incentive constraint is strictly positive while the effect on the right hand side is strictly negative. Such a perturbation is incentive compatible and a strict improvement. □

Proof of Lemma 1. The proof proceeds exactly as the proof of Proposition 4 under the assumption that monitoring occurs in state $L$ only. We will hence only sketch the proof here. Suppose, by contradiction, that $z^m_L \leq z^m_L$ and consider a change in compensation such that $dz^m_L > 0$ and $dz^m_L$, leaving the manager’s expected utility (and hence the term on the left hand side of incentive compatibility constraint (2)) unchanged:

$$\pi_L(a) \{ (1 - m) u'(z^m_L) dz^m_L + m u'(z^m_L) dz^m_L \} = 0.$$

Since $u'(z^m_L) \leq u'(z^m_L)$ the participation constraint (1) still holds. The effect of the change on the value of the term on the right hand side of the incentive constraint (2) is

$$\pi_L(b) \{ (1 - m) u'(z^m_L - \tau_L) dz^m_L + m u'(z^m_L - \tau_L) dz^m_L \} \leq 0,$$

with strict inequality when $\tau_L < 0$. Thus, the change allows to keep the manager’s utility unchanged while making the incentive constraint slack, a contradiction. □

Appendix B: Formal Statements and Proofs for Section 3.1 - The Intertemporal Case with Contingent Claims

Let $T$ denote the set of all budget feasible trades in financial markets, given by $(\tau_0, \tau_H, -\tau_L) \in \mathbb{R}^3$ such that

$$\tau_0 + \pi_H(b) \max\{\tau_H, 0\} + \pi_L(b) \min\{\tau_L, 0\} + \pi_H(a) \min\{\tau_H, 0\} + \pi_L(a) \max\{\tau_L, 0\} = 0.$$
Lemma 4 The compensation scheme obtained as solution of problem $\mathcal{P}^0_{\text{MON}}$ is also a solution of the same problem when (i) monitoring in state $H$ is also allowed; (ii) the set of admissible trades $T(b)$ is replaced by the larger set $\mathcal{T}$.

Proof of Lemma 4. We will rely on the characterization of $\mathcal{P}^0_{\text{MON}}$ obtained in this Appendix. In Proposition 8, we conclude that $z_L^m \leq z_H^m < z_0 < z_H$ no matter what $m$ is.

Let us consider purchases of claims on $H$ ($d\tau_H < 0$ and $d\tau_0 > 0$) at price $\pi_H(a)$ such that $d\tau_0 + \pi_H(a)d\tau_H = 0$ starting from a zero deviation. Suppose the manager puts in high effort $(a)$. The benefit of the purchase would be $-\pi_H(a)u'(z_H)d\tau_H$ and the cost $-u'(z_0)d\tau_0$ and thus the net benefit $-\pi_H(a)(u'(z_H) - u'(z_0))d\tau_H < 0$. Suppose the manager puts in low effort $(b)$. The benefit would then be $-\pi_H(b)u'(z_H)d\tau_H$ and the cost would be unchanged. Thus, the net benefit would be $-(\pi_H(b)u'(z_H) - \pi_H(a)u'(z_0))d\tau_H < 0$. Hence, the manager would never buy claims on state $H$ at price $\pi_H(a)$.

Let us consider sales of claims on $L$ ($d\tau_L > 0$ and $d\tau_0 < 0$) at price $\pi_L(a)$ such that $d\tau_0 + \pi_L(a)d\tau_L = 0$ starting from a zero deviation. Suppose the manager puts in high effort $(a)$. The benefit of the purchase would be $-u'(z_0)d\tau_0$ and the cost $-\pi_L(a)((1 - m)u'(z_L^m) + mu'(z_L^m))d\tau_L$ and thus the net benefit $\pi_L(a)(u'(z_0) - ((1 - m)u'(z_L^m) + mu'(z_L^m)))d\tau_L < 0$. Suppose the manager puts in low effort $(b)$. The benefit would then be unchanged and the cost would be $-\pi_L(b)((1 - m)u'(z_L^m) + mu'(z_L^m))d\tau_L$. Thus, the net benefit would be $(\pi_L(a)u'(z_0) - \pi_L(b)((1 - m)u'(z_L^m) + mu'(z_L^m)))d\tau_L < 0$. Hence, the manager would never sell claims on state $L$ at price $\pi_L(a)$ either.

Notice that any trade involving either purchases of $H$ claims ($d\tau_H < 0$) or sales of $L$ claims ($d\tau_L > 0$) or both (with $d\tau_0 \leq 0$) could hence be improved upon and thus the manager would never consider such trades.

In sum, given these prices, the manager would not want to sell claims on state $L$ or purchase claims on state $H$. But then, given our assumption about the form of penalties, monitoring in state $H$ is irrelevant. □

We establish first a preliminary result on the properties of the solutions of problem $\mathcal{P}^0_{\text{MON}}$, analogous to what we found in the previous section (Lemmas 2 and 3):

Lemma 5 At an optimal compensation scheme, $u(z_H) > (1 - m)u(z_L^m) + mu(z_L^m)$ and the incentive compatibility constraint (9) always holds as equality, for all $m$. Moreover, a sufficient condition for the participation constraint (8) to also hold as equality is that $u(z) = \frac{1}{1 - \sigma} \cdot z$ with $0 < \sigma < 1$ or $u(z) = \ln(z)$.

Proof of Lemma 5. The inequality $u(z_H) > (1 - m)u(z_L^m) + mu(z_L^m)$ is clearly needed to support high effort with a zero level of side trades; with non-zero trades in financial markets it must also hold, a fortiori. Suppose next that (9) were not binding. Then the manager’s utility could be increased by lowering the utility of the payment in state $H$ and increasing the one in state $L$, while keeping unchanged the total expected payment, a contradiction.
The proof of the second claim follows the proof of Lemma 3 quite closely and is hence omitted. □

Next we provide a comparison of the case with no monitoring, i.e., $m = 0$, and with perfect monitoring in state $L$, i.e., $m = 1$, which is the problem denoted $P^0_{SBc}$ in the text.

**Lemma 6** (i) The optimal contract with zero monitoring, $Z(0)$ is such that $z_H(0) > z_0(0) > z_L^m(0)$ and $u'(z_0(0)) = \pi_H(a)u'(z_H(0)) + \pi_L(a)u'(z_L^m(0))$. (ii) The optimal contract with perfect monitoring, $m = 1$, is given by the compensation scheme $Z^+$ solving problem $P^0_{SBc}$, and is such that $z_H^+ > z_0^+ > z_L^+$ and $u'(z_0^+) < \pi_H(a)u'(z_H^+) + \pi_L(a)u'(z_L^+)$.

**Proof of Lemma 6.** (i): When $m = 0$, (9) can be written as:

$$u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L^m) - v(a) \geq 2u \left( \frac{1}{2}z_0 + \frac{\pi_H(b)}{2}z_H + \frac{\pi_L(b)}{2}z_L \right) - v(b),$$

where the term on the right hand side reflects the fact that, with no monitoring, the best the manager can do by trading in the market is to perfectly smooth his income across time and the two states. \(^{34}\) The first order conditions for problem $P^0_{MON}$ when $m = 0$, can then be written as:

$$u'(z_0) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}u'(\bar{z}_d)$$

$$u'(z_H) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}\pi_H(b)u'(\bar{z}_d)$$

$$u'(z_L^m) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}\pi_L(b)u'(\bar{z}_d)$$

where $\mu$ and $\lambda$ are the Lagrange multipliers associated with the constraints (8) and (9) and $\bar{z}_d \equiv \frac{1}{2}z_0(0) + \frac{\pi_H(b)}{2}z_H(0) + \frac{\pi_L(b)}{2}z_L(0)$. Since $\frac{\pi_L(b)}{\pi_L(a)} > 1 > \frac{\pi_H(b)}{\pi_H(a)}$, from the equations in (12) we get $z_H(0) > z_0(0) > z_L^m(0)$. Furthermore, $u'(z_0(0)) = \pi_H(a)u'(z_H(0)) + \pi_L(a)u'(z_L^m(0))$.

(ii): Consider the first order conditions for problem $P^0_{SBc}$:

$$u'(z_0) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}u'(\bar{z}_d^+),$$

$$u'(z_H) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}\pi_H(b)u'(\bar{z}_d^+)$$

$$u'(z_L) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}\pi_L(b)u'(\bar{z}_L)$$

where $\mu$ and $\lambda$ are the multipliers associated with the two constraints of $P^0_{SBc}$ and $\bar{z}_d^+$ is as defined earlier. Hence we have $z_H^+ > z_0^+$ and, since by construction $\bar{z}_d^+ \in (z_L^+, z_H^+)$, $z_H^+ > z_0^+ > z_L^m(0)$.

\(^{34}\)Since, as we show below, $z_H > z_0 > z_L^m$, the smoothing of income requires selling $H$ claims and buying $L$ claims; it will then take place at prices $\pi(b)$.
\( \bar{z}_d^+ > z_0^+ \). Furthermore, from the first equation in (13) we obtain \( \mu = u'(z_0^+) + \lambda(u'(z_0^+) - u'(\bar{z}_d^+)) > u'(z_0^+) \), and from the third one \( \mu = u'(z_L^+) + \lambda u'(z_L^+)(1 - \pi_L(0)) < u'(z_L^+) \); thus \( z_0^+ > z_L^+ \). Finally, summing the last two equations in (13), multiplied by \( \pi_H(a) \) and \( \pi_L(a) \), and using the first equation, we get:

\[
\pi_H(a)u'(z_H^+) + \pi_L(a)u'(z_L^+) = u'(z_0^+) + \frac{\lambda}{1 + \lambda}\pi_L(b)(u'(z_L^+)) > u'(z_0^+),
\]

where the last inequality follows from the fact that \( u'(z_L^+) > u'(\bar{z}_d^+) \) \( \Box \)

**Proposition 6** Comparing the optimal compensation schemes in an intertemporal framework with full and with no monitoring, if the participation constraint binds in both cases, we have: \( z_H(0) - z_L^{nm}(0) > z_H^+ - z_L^+ \), \( z_H(0) > z_H^+ \), and, if \( u'' > 0 \), then \( z_0^+ > z_0(0) \).

**Proof of Proposition 6.** Comparing (11) and (10), and noting that for all \( z_0, z_H, z_L^{nm} \) we have

\[
2u \left( \frac{1}{2}z_0 + \frac{\pi_H(b)}{2} z_H + \frac{\pi_L(b)}{2} z_L^{nm} \right) - v(b) \geq (1 + \pi_H(b))u \left( \frac{1}{1 + \pi_H(b)} z_0 + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H \right) + \pi_L(b)u(z_L^{nm}) - v(b);
\]

we see that the feasible set of problem \( P_{MON}^0 \) when \( m = 0 \) is clearly contained in the feasible set of problem \( P_{SBc}^0 \). As a consequence, the solution \( Z(0) \) of the first problem is also an admissible solution of the second, \( P_{SBc}^0 \). However, it is not the optimal solution of such problem since, as we saw in Lemma 6, \( z_L^{nm}(0) \) is strictly smaller than both \( z_H(0) \) and \( z_0(0) \). So the inequality in (14) is strict, or the incentive compatibility constraint of \( P_{SBc}^0 \) is slack at \( Z(0) \). Hence the manager, by choosing the optimal deviation when \( m = 0 \), must get a higher utility when his compensation is given by \( z^+ \) rather than by \( \bar{Z}(0) \):

\[
(1 + \pi_H(b))u \left( \frac{1}{1 + \pi_H(b)} z_0 + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H \right) + \pi_L(b)u(z_L^+) > (1 + \pi_H(b))u \left( \frac{1}{1 + \pi_H(b)} z_0(0) + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H(0) \right) + \pi_L(b)u(z_L^{nm}(0)).
\]

Define the expected cost of the manager's compensation \( z = (z_0, z_H, z_L) \), when he exerts effort \( e \), as \( PV^e(z) = z_0 + \pi_H(e)z_H + \pi_L(e)z_L \). Notice that \( PV^b(z) = PV^a(z) - (\pi_H(a) - \pi_H(b))(z_H - z_L) \). Under the assumption that the participation constraint is binding both at the solution of \( P_{SBc}^0 \) and of \( P_{MON}^0 \), the expected cost under effort \( a \) is the same at the solutions of the two problems: \( PV^a(z^+) = PV^a(Z(0)) \). Suppose the first claim in the Proposition does not hold, i.e., \( z_H^+ - z_L^+ \geq z_H(0) - z_L^{nm}(0) \). Then from the above expressions we must have \( PV^b(Z(0)) \geq PV^b(z^+) \) and the validity of (15) requires:

\[
\frac{1}{1 + \pi_H(b)} z_0(0) + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H(0) > \frac{1}{1 + \pi_H(b)} z_0^+ + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H^+ = z_L^+ > z_L^{nm}(0), \tag{16}
\]
since otherwise a lottery with (weakly) lower expected value would never be preferred.
The last inequality in (16) above in turn implies, under the assumed condition \( z_H^+ - z_L^+ \geq z_H(0) - z_L^{nm}(0) \), that \( z_H^+ > z_H(0) \). Hence from (16) we get \( z_0^+ < z_0(0) \), and so, recalling the properties established in Lemma 6:

\[
\pi_H(a)u'(z_H^+) + \pi_L(a)u'(z_L^+) > u'(z_0^+) > u'(z_0(0)) = \pi_H(a)u'(z_H(0)) + \pi_L(a)u'(z_L^{nm}(0)).
\]  

(17)

But this contradicts our previous finding that \( z_H(0) < z_H^+ \) and \( z_L^{nm}(0) < z_L^+ \). Thus, we must have \( z_H^+ - z_L^+ < z_H(0) - z_L^{nm}(0) \).

By the same argument, \((z_H(0), z_L^{nm}(0)) \not\leq (z_H^+, z_L^+)\). Suppose this was not true, i.e., \((z_H(0), z_L^{nm}(0)) \leq (z_H^+, z_L^+)\). Since \( PV^a(z^+) = PV^a(Z(0)) \), we have \( z_0^+ \leq z_0(0) \). Thus again \( u'(z_0^+) \geq u'(z_0(0)) \), which together with the properties established in Lemma 6 leads to a contradiction. Thus, \((z_H(0), z_L^{nm}(0)) \not\leq (z_H^+, z_L^+)\).

Combining this property with the fact that, as shown above, \( z_H^+ - z_L^+ < z_H(0) - z_L^{nm}(0) \), we must have \( z_H(0) > z_H^+ \).

To prove the last claim of the Proposition we also proceed by contradiction: suppose \( u'' > 0 \) and \( z_0^+ \leq z_0(0) \). From the property \( u'(z_0(0)) = \pi_H(a)u'(z_H(0)) + \pi_L(a)u'(z_L^{nm}(0)) \) established in Lemma 6, we get \( z_0(0) < \pi_H(a)z_H(0) + \pi_L(a)z_L^{nm}(0) \). Moreover, given the properties \( z_H(0) > z_H^+ \) and \( PV^a(z^+) = PV^a(Z(0)) \) shown above, if \( z_0^+ \leq z_0(0) \) the following must hold: \( z_L^+ > z_L^{nm}(0) \) and \( \pi_H(a)z_H^+ + \pi_L(a)z_L^+ \geq \pi_H(a)z_H(0) + \pi_L(a)z_L^{nm}(0) \). As a consequence, since \( u' \) is decreasing and convex, and the lottery \((z_H(0), z_L^{nm}(0))\) has higher variance and lower mean than the lottery \((z_H^+, z_L^+)\), we must have

\[
\pi_H(a)u'(z_H(0)) + \pi_L(a)u'(z_L^{nm}(0)) > \pi_H(a)u'(z_H^+) + \pi_L(a)u'(z_L^+).
\]

This inequality in turn implies, using the relationships established in Lemma 6, that \( z_0^+ > z_0(0) \), i.e., a contradiction. \( \Box \)

**Remark 2** It is possible to show that exactly the same properties as those established in Proposition 6 hold when the optimal compensation scheme with no monitoring, \( Z(0) \), is compared to the optimal compensation scheme with full monitoring in all markets (i.e., also at date 0), given by the incentive efficient contract \( Z^* \).

We consider then the case of imperfect monitoring: \( m \in (0,1) \).

**Proposition 7** Let \( m^+ \equiv 1 - u'(z_d^+)/u'(z_L^+) \). Then, \( m^+ < 1 \) and, for any \( m \geq m^+ \), the optimal contract with perfect monitoring, \( Z^+ \), solves \( P^0_{MON} \).

**Proof of Proposition 7.** First, as shown in Lemma 6, \( z_H^+ > z_0^+ > z_L^+ \). By construction we have then \( z_H^+ > z_d^+ > z_L^+ \), so that \( m^+ < 1 \).

Consider then the optimal deviation in problem \( P^0_{MON} \) (i.e., the best trades the manager can do in the financial market when switching to low effort), for given \( m \):

\[
\max_{\tau \in T(b)} u(z_0 - \tau_0) + \pi_H(b)u(z_H - \tau_H) + \pi_L(b)\{(1-m)u(z_L^{nm} - \tau_L) + mu(z_L^{nm} - \max\{\tau_L, 0\})\} - v(b).
\]
The first order conditions for the above problem are

\[ u'(z_0 - \tau_0) \leq u'(z_{H - \tau_H}) \]

\[ u'(z_0 - \tau_0) \geq (1 - m)u'(z_{Lm}^m - \tau_L), \]

with equalities if, respectively \( \tau_H > 0, \tau_L < 0 \). We will show that, when \( m \geq m^+ \) these conditions are satisfied at \( z = [z_0^+, z_H^+, z_L^+, z_0^+] \) with \( \tau_L = 0 \). Since, as we already noticed, \( z_H^+ > z_0^+ \), when \( \tau_L = 0 \) the optimal choice of the trades in the other markets \( \tau_0, \tau_H \) is at a level such that \( z_0 - \tau_0 = z_H - \tau_H = \hat{z}_d^+ \). Substituting these values in the first order conditions above, the first one is trivially satisfied while the second one has the following expression:

\[ u'(\hat{z}_d^+) \geq (1 - m)u'(z_L^+), \]

always satisfied for \( m^+ \leq m \).

Thus, when \( m \geq m^+ \) the manager does not wish to trade in the market for \( L \) claims. As a consequence, since \( z^+ \) constitutes the optimal contract when the manager cannot engage in such trades in the \( L \) market \( (m = 1) \), it is also the optimal choice when \( m \geq m^+ \).

**Proposition 8** For any \( m < m^+ \) the optimal compensation scheme \( Z(m) \) is such that (i) if the manager were to deviate, he would choose \( \tau_L < 0 \), and (ii) \( z_H(m) > z_0(m) > z_L^m(m) > z_L^m(m). \) For \( m \geq m^+ \), \( z_H(m) > z_0(m) > z_L^m(m) = z_L^m(m). \)

**Proof of Proposition 8.** (i) Notice that if the manager were to choose \( \tau_L = 0 \), then we know from the first order conditions of \( \mathcal{P}_{MON}^0 \) that \( z_L^m(m) = z_L^m(m) \). Moreover, the first order conditions of \( \mathcal{P}_{MON}^0 \) and \( \mathcal{P}_{SBc}^0 \) would coincide except for the additional constraint in \( \mathcal{P}_{MON}^0 \) that \( u'(z_0 - \tau_0) \geq (1 - m)u'(z_L^m(m)). \) But, generically, \( Z^+ \) does not satisfy this additional constraint, a contradiction.

(ii) We first show that \( z_L^m(m) > z_L^m(m) \) for \( m < m^+ \). The proof follows very similar lines to that of claim (ii) of Proposition 4. Suppose \( z_L^m \leq z_L^m \). Consider the perturbation \( dz = (dz_0, dz_H, dz_L^m, dz_L^m) \) with \( dz_0 = dz_H = 0 \) and \( dz_L^m > 0 > dz_L^m \) such that its effect on the objective and the left hand side of the incentive constraint is

\[ \pi_L(a)\{(1 - m)u'(z_L^m)dz_L^m + mu'(z_L^m)dz_L^m\} = 0. \]

This perturbation satisfies the participation constraint since \( \pi_L(a)\{(1 - m)dz_L^m + m dz_L^m\} \leq 0 \). The effect on the right hand side of the incentive constraint is

\[ \pi_L(b)\{(1 - m)u'(z_L^m - \tau_L)dz_L^m + mu'(z_L^m)dz_L^m\} \leq 0. \]

with strict inequality, by claim (i) of this Proposition, if \( m < m^+ \). Thus, the perturbation renders the incentive constraint slack, while the manager’s utility is unchanged, a contradiction.

Next we show that \( z_H > z_0 > z_L^m \) for \( m < m^+ \). By claim (i) of this Proposition \( \tau_L < 0 \). The first order condition of the optimal deviation then implies \( (1 - m)u'(z_L^m - \tau_L) = 35 \)
\( u'(z_0 - \tau_0) \). But then, using the envelope theorem and the first order conditions of the maximization problem, we have

\[
u'(z_{L}^{nm}) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_L(b)}{\pi_L(a)} u'(z_{L}^{nm} - \tau_L) \]

\[
= \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_L(b)}{\pi_L(a)} \frac{1}{1-m} u'(z_0 - \tau_0) \]

\[
> \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} u'(z_0 - \tau_0) = u'(z_0).
\]

Hence, \( z_{L}^{nm} < z_0 \). Moreover, again using the first order condition of the optimal deviation \( u'(z_0 - \tau_0) \leq u'(z_H - \tau_H) \). If the inequality is strict, \( \tau_H = 0 \) which implies that \( \tau_0 > 0 \) and, in turn, \( z_0 > z_H \). But this is not possible since otherwise the perturbation \( dz_H > 0 > dz_0 \) such that \( u'(z_0)d_0 + \pi_H(a)u'(z_H)dz_H = 0 \) would be feasible \( (dz_0 + \pi_H(a)dz_H = 0) \) and would relax the incentive constraint (the effect on the right hand side, again using the envelope theorem, is \( u'(z_0)d_0 + \pi_H(b)u'(z_H)dz_H < 0 \)). Thus, the first order condition holds with equality, and we can use the first order conditions of the maximization problem to conclude that

\[
u'(z_H) = \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_H(b)}{\pi_H(a)} u'(z_H - \tau_H)
\]

\[
< \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} u'(z_0 - \tau_0) = u'(z_0).
\]

Thus, \( z_H > z_0 > z_{L}^{nm} > z_{L}^{m} \) for \( m < m^+ \).

Finally, if \( m \geq m^+ \), then by Proposition 7, \( Z(m) \) satisfies \( z_0 = z_0^+, z_H = z_H^+, z_{nm}^m = z_{L}^+ \), and \( z_{L}^m = z_{L}^+ \) and, from Lemma 5, \( z_H^+ > z_{0}^+ > z_{L}^+ \). □

Appendix C: Formal Statements and Proofs for Section 3.2 - The Intertemporal Case with Borrowing and Lending

**Proposition 9** If the participation constraint binds, we have: \( z_H^f(0) - z_{nm}^{m,f}(0) > z_H^* - z_L^* \), \( z_H^f(0) > z_H^* \), and, if \( u'' > 0 \), then \( z_0^* > z_H^f(0) \).

**Proof of Proposition 9.** Consider the optimal deviation in problem \( P_{MON}^{m,f} \) when \( m = 0 \):

\[
\max_{\tau_0, \tau \in \mathbb{R}^2: \tau_0 + \tau = 0} u(z_0 - \tau_0 + \pi_H(b)u(z_H - \tau) + \tau_L(b)u(z_{L}^{nm} - \tau) - v(b) \geq u(z_0) + \pi_H(b)u(z_H) + \pi_L(b)u(z_{L}^{nm}) - v(b).
\]

This implies that \( u(z_H) - u(z_{L}^{nm}) \geq \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} \). At the second best contract \( Z^* \), as already mentioned in Section 3.2, Rogerson (1985) showed that \( u'(z_0^*) < \pi_H(a)u'(z_H^*) + \pi_L(a)u'(z_L^*) \);
moreover, the incentive compatibility constraint holds as equality, so that 
\[ \pi_H^*(a) - \pi_H^*(b) = u(z_H^*) - u(z_L^*) \], and hence \( z_H^* > z_L^* \). Therefore, we also have
\[ u'(z_H^*) < \pi_H(a)u'(z_H^*) + \pi_L(a)u'(z_L^*) < \pi_H(b)u'(z_H^*) + \pi_L(b)u'(z_L^*) \]
which implies that \( Z^* \) is not an admissible solution of \( \mathcal{P}_{MON}^{0,f} \), since at that compensation contract the agent would like to save and would then be able to achieve a higher utility by engaging in side trades. Thus \( Z^f(0) \neq Z^* \). Furthermore, we have
\[ u(z_H^f) - u(z_{L^{nm,f}}(0)) \geq \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*) \]
Suppose \( (z_H^f(0), z_{L^{nm,f}}(0)) \leq (z_H^*, z_L^*) \). From the participation constraint we get then \( z_0^f(0) \geq z_0^* \) and, using (19), \( u'(z_0^f(0)) < \pi_H(b)u'(z_H^f(0)) + \pi_L(b)u'(z_{L^{nm,f}}(0)) \), which implies \( \tau_0 > 0 > \tau \). Consider \( dz = (dz_0, dz_H, dz_{L^{nm,f}}) \) with \( dz_0 < 0 < dz_H = dz_{L^{nm,f}} \) such that the change in the value of the objective function of \( \mathcal{P}_{MON}^{0,f} \) (and hence of the term on the left hand side of the incentive constraint) is
\[ u'(z_0^f(0))dz_0 + (\pi_H(a)u'(z_H^f(0)) + \pi_L(a)u'(z_{L^{nm,f}}(0))dz_H = 0. \]
Since \( u'(z_0^f(0)) < \pi_H(a)u'(z_H^f(0)) + \pi_L(a)u'(z_{L^{nm,f}}(0)) \) (which again follows from (19)), we have \( dz_0 + dz_H < 0 \), i.e., the participation constraint is still satisfied. Using the first order conditions for the optimal level of side trades from \( Z^f(0) \) in \( \mathcal{P}_{MON}^{0,f} \), \( u'(z_0^f(0) - \tau_0) = \pi_H(b)u'(z_H^f(0) - \tau) + \pi_L(b)u'(z_{L^{nm,f}}(0) - \tau) \), we find
\[ u'(z_0^f(0) - \tau_0)dz_0 + (\pi_H(b)u'(z_H^f(0) - \tau) + \pi_L(b)u'(z_{L^{nm,f}}(0) - \tau)dz_H < 0, \]
i.e., the perturbation \( dz \) also allows to relax the incentive compatibility constraint, which contradicts the optimality of \( Z^f(0) \). Thus we must have \( (z_H^f(0), z_{L^{nm,f}}(0)) \not\leq (z_H^*, z_L^*) \).

Suppose \( z_H^f(0) \leq z_H^* \) and hence \( z_{L^{nm,f}}(0) > z_L^* \). But this contradicts equation (20) above. As a consequence we must have \( z_H^f(0) > z_H^* \) and, using (20) and the concavity of \( u(.) \), \( z_H^f(0) - z_{L^{nm,f}}(0) > z_H^* - z_L^* \), as stated in the Proposition.

It remains then to show that, if \( u'' > 0 \), \( z_0^f(0) < z_0^* \). Suppose not, i.e., \( z_0^f(0) \geq z_0^* \). This implies, using the participation constraint, that \( z_{L^{nm,f}}(0) < z_L^* \) and \( \pi_H(a)z_H^f(0) + \pi_L(a)z_{L^{nm,f}}(0) \leq \pi_H(a)z_H^* + \pi_L(a)z_L^* \). But then, noting that the previous inequality can also be written as \( z_H^f(0) - z_H^* > z_{L^{nm,f}}(0) - z_L^* \), we also have \( \pi_H(b)z_H^f(0) + \pi_L(b)z_{L^{nm,f}}(0) \leq \pi_H(b)z_H^* + \pi_L(b)z_L^* \). If \( u'' > 0 \), so that \( u' \) is decreasing and convex, it follows that
\[ \pi_H(e)u'(z_H^f(0)) + \pi_L(e)u'(z_{L^{nm,f}}(0)) > \pi_H(e)u'(z_H^*) + \pi_L(e)u'(z_L^*) > u'(z_0^*) \]
for \( e \in \{a, b\} \) (where we again used (19)). This inequality again implies that the same perturbation \( dz \) considered earlier, which does not affect the value of the objective function, also satisfies the participation constraint: \( dz_0 + dz_H < 0 \). By the same argument as above, using the first order conditions for the optimal level of side trades we find that

37
such perturbation decreases the value of the term on the right hand side of the incentive compatibility constraint:

$$u'(z_0^f(0) - \tau_0)dz_0 + (\pi_H(b)u'(z_H^f(0) - \tau) + \pi_L(b)u'(z_L^{nm,f}(0) - \tau))dz_H < 0,$$

a contradiction. Thus, $$z_0^f(0) < z_0^*.$$ □

**Proposition 10** There exists $$m^f \in (0, 1)$$ such that for all $$m \geq m^f$$, the optimal compensation scheme obtained from problem $$\mathcal{P}_{MON}^{0,f}$$ is given by the second best contract, $$Z^*$$, and at this contract the optimal deviation is characterized by $$\tau_0 = \tau = 0$$.

**Proof of Proposition 10.** Consider the first order conditions for the optimal level of side trades at a solution $$Z^f(m)$$ of problem $$\mathcal{P}_{MON}^{0,f}(m)$$. If $$\tau \leq 0$$ we have

$$u'(z_0^f(m) - \tau_0) \geq \pi_H(b)u'(z_H^f(m) - \tau) + \pi_L(b)(1 - m)u'(z_L^{nm,f}(m) - \tau)$$

while, if $$\tau > 0$$,

$$u'(z_0^f(m) - \tau_0) = \pi_H(b)u'(z_H^f(m) - \tau) + \pi_L(b)\{((1 - m)u'(z_L^{nm,f}(m) - \tau) + mu'(z_L^{m,f}(m) - \tau))\}.$$

Evaluating these conditions at $$z = [z_0^*, z_H^*, z_L^*, z_L^{m,f}]$$, when $$\tau > 0$$ we have

$$u'(z_0^* - \tau_0) = \pi_H(b)u'(z_H^* - \tau) + \pi_L(b)u'(z_L^* - \tau)$$

which, since $$\tau > 0$$ implies $$\tau_0 < 0$$, contradicts (19). Thus, we must have $$\tau \leq 0$$. Let $$m^f$$ be such that

$$u'(z_0^*) = \pi_H(b)u'(z_H^*) + \pi_L(b)(1 - m^f)u'(z_L^*);$$

note that, since from (19) it follows that $$\pi_H(b)u'(z_H^*) < u'(z_0^*) < \pi_H(b)u'(z_H^*) + \pi_L(b)u'(z_L^*)$$, we have $$0 < m^f < 1$$. For all $$m \geq m^f$$, by construction the first order conditions for the optimal level of side trades hold at $$Z^*$$ with $$\tau = 0$$, hence $$Z^*$$ is an admissible solution and hence the optimal solution of $$\mathcal{P}_{MON}^{0,f}(m)$$. □

**Proposition 11** For $$m < m^f$$, the optimal compensation contract $$Z^f(m)$$ is different from the second best, $$Z^*$$, and such that $$z_L^{nm}(m) \geq z_L^m(m)$$; at such contract, the optimal deviation is characterized by $$\tau < 0$$.

**Proof of Proposition 11.** To prove the first claim, suppose the optimal level of side trades is such that $$\tau > 0$$. Then the first order condition are:

$$u'(z_0^f(0) - \tau_0) = \pi_H(b)u'(z_H^f - \tau) + \pi_L(b)\{(1 - m)u'(z_L^{nm,f} - \tau) + mu'(z_L^{m,f} - \tau)\} \quad (21)$$

and, since $$\tau > 0$$ implies $$\tau_0 < 0$$, $$u'(z_0^f) > \pi_H(b)u'(z_H^f) + \pi_L(b)\{(1 - m)u'(z_L^{nm,f}) + mu'(z_L^{m,f})\}.$$

Consider the perturbation $$dz_0 > 0 > dz_H = dz_L^{nm} = dz_L^m \equiv dz_1$$ such that $$dz_0 + dz_1 = 0$$. Notice that the first order conditions of $$\mathcal{P}_{MON}^{0,f}(m)$$ imply that

$$\frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}(\pi_H(b)u'(z_H^f - \tau) + \pi_L(b)\{(1 - m)u'(z_L^{nm,f} - \tau) + mu'(z_L^{m,f} - \tau)\}) = u'(z_0^f),$$
where $\mu$ and $\lambda$ are the Lagrange multipliers associated with the constraints of problem $\mathcal{P}_{MON}^{0,f}(m)$, and the last equality follows from (21) together with the first order condition with respect to $z_0^f$. As a consequence, the effect of the perturbation $dz_0, dz_1$ on the value of the objective function of $\mathcal{P}_{MON}^{0,f}$ and of the term on the left hand side of the incentive constraint is

$$u'(z_0^f)dz_0 + [\pi_H(a)u'(z_H^*) + \pi_L(a)\{(1-m)u'(z_m^m) + mu'(z_L^m)]}dz_1 = 0,$$

Also, its effect on the value of the term on the right hand side of the incentive compatibility constraint is

$$u'(z_0 - \tau_0)dz_0 + (\pi_H(b)u'(z_H - \tau) + \pi_L(b)\{(1-m)u'(z_m^m - \tau) + mu'(z_L^m - \tau)]}dz_1 = u'(z_0 - \tau_0)(dz_0 + dz_1) = 0.$$

Thus, the perturbation is admissible and does not decrease the value of the objective function. Hence, whenever the optimal deviation is characterized by $\tau > 0$ we can always find an alternative solution, with higher $z_0$ and lower $z_H, z_L^m, z_L^m$ at which the optimal deviation is $\tau \leq 0$.

Next, suppose that $m < m^f$ but $\tau = 0$. First, note that when $m < m^f$, $Z^f(m) \neq Z^*$ since $u'(z_0^f) < \pi_H(b)u'(z_H^*) + \pi_L(b)(1-m)u'(z_L^*)$, so that the manager would save at $Z^*$. Moreover, using the first order conditions of problem $\mathcal{P}_{MON}^{0,f}$ at $\tau = 0$, we have $z_L^m = z_L^m \equiv z_L$. Next, note that given $\tau = 0$, the incentive compatibility constraint implies

$$u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L) - v(a) = u(z_0) + \pi_H(b)u(z_H) + \pi_L(b)u(z_L) - v(b)$$

and hence

$$u(z_H) - u(z_L) = \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*), \quad (22)$$

where the second equality uses the incentive compatibility constraint of the second best problem. Now, there are two cases to consider: on the one hand, if $z_H > z_L^*$, then using (22) $z_L > z_L^*$ and, using the participation constraint, $z_0 < z_0^*$; on the other hand, if $z_H < z_L^*$, then $z_L < z_L^*$ and $z_0 > z_0^*$. The first order conditions of $\mathcal{P}_{MON}^{0,f}$ imply

$$\pi_H(a)\frac{1}{u'(z_H)} + \pi_L(a)\frac{1}{u'(z_L)} = \frac{1}{u'(z_0)},$$

and $Z^*$ satisfies an equivalent equation. But then $(z_H, z_L) > (z_H^*, z_L^*)$ and $z_0 < z_0^*$ would imply

$$\pi_H(a)\frac{1}{u'(z_H)} + \pi_L(a)\frac{1}{u'(z_L)} > \pi_H(a)\frac{1}{u'(z_H^*)} + \pi_L(a)\frac{1}{u'(z_L^*)} = \frac{1}{u'(z_0^*)} > \frac{1}{u'(z_0)},$$

a contradiction. When $(z_H, z_L) < (z_H^*, z_L^*)$ and $z_0 > z_0^*$, both inequalities are reversed, again a contradiction. We conclude that $\tau < 0$.

The proof that $z_L^m(m) \geq z_L^m(m)$ is identical to the proof of the corresponding claim in Proposition 8. \qed
References


Table 1: Managerial Compensation with Portfolio Monitoring

Panel A: Optimal Compensation: One Period Case

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Panel B: Optimal Compensation: Two Period Case

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Panel C: Optimal Compensation: Two Period Case with Hidden Risk Free Borrowing and Lending Only

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Figure 1: Managerial Compensation with Portfolio Monitoring: One Period Case.

Compensation as a function of monitoring probability $m$. Top three lines plot $z_H(m)$ (solid), $z_H^k$ (dotted), and $z_H(0)$ (dashed). Bottom four lines plot $z_L^m(m)$ and $z_L^m(m)$ (both solid), $z_L^k$ (dotted), and $z_L^m(0)$ (dashed).
Figure 2: Managerial Compensation with Portfolio Monitoring: One Period Case with Alternative Specification of Penalties.

Compensation as a function of monitoring probability $m$. Top five lines plot $z_H(m)$ (solid), $z_H^+(dotted)$, $z_H(0)$ (dashed), $z_H(m|k=0.02)$ (dash-dotted), and $z_H(m|k=0.05)$ (bold dotted). Bottom eight lines plot $z_L^m(m)$, $z_L^m(m)$ (both solid), $z_L^+(dotted)$, $z_L^m(0)$ (dashed), $z_L^m(m|k=0.02)$ and $z_L^m(m|k=0.02)$ (both dash-dotted), and $z_L^m(m|k=0.05)$ and $z_L^m(m|k=0.05)$ (both bold dotted).
Figure 3: Managerial Compensation with Portfolio Monitoring: Two Period Case.

Compensation as a function of monitoring probability $m$. Top four lines plot $z_H(m)$ (solid), $z_H^*$ (dotted), $z_H(0)$ (dashed), and $z_H^+$ (dash-dotted). Middle three lines plot $z_0(m)$ (solid), $z_0^*$ (dotted), $z_0(0)$ (dashed), and $z_0^+$ (dash-dotted). Bottom four lines plot $z_L^{nm}(m)$ and $z_L^m(m)$ (both solid), $z_L^*$ (dotted), $z_L^{nm}(0)$ (dashed), and $z_L^+$ (dash-dotted).
Figure 4: Managerial Compensation with Portfolio Monitoring: Hidden Risk Free Borrowing and Lending Only

Compensation as a function of monitoring probability $m$. Top three lines plot $z_H(m)$ (solid), $z_H^*$ (dotted), and $z_H(0)$ (dashed). Middle three lines plot $z_0(m)$ (solid), $z_0^*$ (dotted), and $z_0(0)$ (dashed). Bottom four lines plot $z_{nm}(m)$ and $z_{m}(m)$ (both solid), $z_{L}^*$ (dotted), and $z_{nm}(0)$ (dashed).