On gender gaps and self-fulfilling expectations: An alternative approach based on paid-for training

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ABSTRACT

This paper presents a model of self-fulfilling expectations by firms and households which generates multiplicity of equilibria in pay and housework time allocation for ex-ante identical spouses. Multiplicity arises from statistical discrimination exerted by firms in the provision of paid-for training to workers, rather than from incentive problems in the labor market. Employers’ beliefs about differences in spouses’ reactions to housework shocks lead to symmetric (ungendered) and asymmetric (gendered) equilibria. We find that: (i) the ungendered equilibrium tends to prevail as aggregate productivity in the economy increases (regardless of the generosity of family aid policies), (ii) the ungendered equilibrium could yield higher welfare under some scenarios, and (iii) gender-neutral job subsidies are more effective that gender-targeted ones in removing the gendered equilibrium.

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Keywords: gender wage gaps, housework shares, multiple equilibria.

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1. Introduction

The recent literature on gender differences in the labour market has focused mainly on two issues. The first one deals with the mechanisms (e.g. adverse selection) that can amplify initial differences in preferences or productivity across genders leading to sizeable gender gaps in wages, labor force participation or working hours. These models allow us to understand to what extent changes in gender gaps have been due to changes in fundamentals, including differences in the pattern of endogenous accumulation of human capital by men and women.\(^1\) The second one is related to the channels explaining persistence in gender gaps even when exogenous differences across the sexes are of little importance or even absent.\(^2\) These studies tend to rely upon the existence of incentive problems in the labor market (e.g., moral hazard) leading to self-fulfilling prophecies about differences in gender roles without assuming any initial comparative advantages. Our paper falls into this second strand of the literature by considering an alternative mechanism based on statistical discrimination by firms in the provision of paid-for training to male and female workers which yields multiplicity of equilibria in gender gaps even in the absence of moral hazard.\(^3\)

The basic idea in models of gender gaps relying upon self-fulfilling prophecies is that employers’ beliefs about women’s lower attachment to the labor market lead to wage differentials in favour of men. Hence, since women expect to face a lower expected opportunity cost, they end up devoting more time to housework, validating in this way firms’ beliefs. For example, Albanesi and Olivetti (2009) propose a model where firms are subject to incentive compatible constraints due to their imperfect monitoring of workers’ effort (a moral hazard problem) and of hours of housework (an adverse selection problem). As a result, different types of labor contracts are offered to men and women. In a similar vein, Lommerud and Vagstad (2007) deal with a model where firms allocate workers to fast and slow track jobs, the former requiring a fixed investment cost. If women have traditionally exerted primary major responsibility at

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\(^3\) Following the seminal work by Arrow and Phelps, there is a large literature on statistical discrimination leading to asymmetric treatment in equilibrium of *ex ante* identical groups. In particular, our model deals with some of the issues raised earlier by Moro and Norman (2003, 2004), namely, the interaction between an informational externality and general equilibrium effects. However, whereas their cross-group externalities stem from the marginal productivity in market work of one group being affected by the size of another group (say the ratio of black and white workers in a given occupation), ours relies upon household decisions on the division of housework interacting with firms’ decision on training.
home and wages are non-contractible, they will predominantly follow a “mommy track” in equilibrium.

While incentive problems - often used to derive additional predictions on the structure of wages to be confronted with the data – provide a useful modelling device, they could be somewhat restrictive. For instance, concerning the difficulty of perfectly monitoring effort, wage gaps should be negligible for routine tasks performed by less-skilled employees for whom effort and output should be easily observable. Yet, there is evidence about substantial gender gaps in hourly wages in these categories, even after removing differences in observable characteristics and in the overall wage dispersion (see, e.g., Blau and Kahn, 2000, Bassanini and Saint Martin, 2008, and de la Rica et al. 2008).

Our paper contributes to this literature by proposing a novel mechanism which does not rely on incentive problems. We consider a two-period model in which firms initially offer paid-for training to workers, taking as given the division of housework between spouses. In the second period, spouses decide on how to split the housework. This in turn determines the division of housework in the presence of shocks affecting household tasks during the second period (e.g., unexpected need of housework or events that require parental leave, etc.). Both decisions interact because the housework decision determines whether a spouse works or quits after receiving training and the firm chooses the amount of training without observing the housework allocation. Statistical discrimination arises in this setup as a result of firms’ prior beliefs about which spouse is more likely to quit the job. Under the key assumption that wages are predetermined with respect to housework shocks, asymmetric beliefs will induce differences in the provision of training across spouses and, as a result, will lead to gender wage differences. Further, since spouses also form beliefs about their relative wages when allocating housework, this mechanism leads to self-fulfilling prophecies and therefore to a multiplicity of equilibria. Indeed, under some assumptions about the distribution of household shocks, the disutility of housework and the degree of diminishing returns to training, two types of equilibria arise: (i) an ungendered equilibrium, with a fully egalitarian division of housework and equal pay, and (ii) a gendered equilibrium, where one of the spouses earns a higher wage and devotes less time to housework.

Our model is able to generate some novel predictions about the relationship between the division of housework and the gender wage gap. First, we find that, under plausible conditions, the gendered equilibrium tends to vanish in economies with higher
aggregate productivity (i.e., where training results in larger market productivity gains). Secondly, as regards the role of policies in reducing gender gaps, we find that gender-neutral policies tend to be more effective than gender-based policies. In particular, when a *gendered* equilibrium prevails, we show that job subsidies targeted at women can backfire by shifting the economy to an even more unequal equilibrium.\(^4\) Lastly, in contrast to most existing work using incentive problems (see, e.g., the discussion in Lommerud and Vagstad, 2007), we find that welfare could be higher in the symmetric than in the asymmetric equilibrium. The converse result is often found in the literature because an asymmetric equilibrium promotes some form of “efficient specialization” in the labor market. Although this channel is also present in our model, under the rather plausible assumption that the disutility of housework is convex and minimized under an even split of these tasks between *ex-ante* identical spouses, this second effect can dominate in some scenarios (notably in *high-productivity* economies).

Finally, it is important to stress that there could be other mechanisms at play which are somewhat akin to training. For example, there is abundant evidence about the over-representation of women (with the same observable characteristics than men) in college degrees with lower market returns (despite performing better than men in high school) which has been found to explain a substantial part of the gender wage gap (see, e.g., Machin and Puhani, 2003).\(^5\) Although paid-for training differs from education (or employee-paid training) in that the former is paid by the firm and the latter by the individual, it could well be the case that women choose less demanding degrees than men anticipating firms’ reactions to their higher probability of future career interruptions. Notwithstanding, we opt to focus on paid-for training not only because it provides as a simpler device to capture the interaction between employers’ and households’ decisions, but also because there is ample empirical evidence pointing out that the intensity of on-the-job training is lower for women than for men even in full-time jobs (see, *inter alia*, Altonji and Spletzer, 1991, Barron et al., 1993, Royalty, 1996, for the US, de la Rica et al., 2008, for Spain, and Puhani and Sonderhof, 2010, for Germany).

The rest of the paper is organised as follows. Section 2 lays out the model.

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\(^4\) Moro and Norman (2003) analyze a model of racial statistical discrimination with human capital investments where they also find that affirmative action may result in higher wage inequality across racial groups, in the spirit of Coate and Loury (1993)’s seminal work on the effects of this type of policies.

\(^5\) For example, the fraction of female undergraduates in humanities degrees is much higher than in engineering or hard sciences (see Hunt, 2010).
Section 3 discusses the properties of the different equilibria. Section 4 deals with welfare analysis. Section 5 analyses the effects of using different policies to eliminate the asymmetric equilibrium. Section 6 provides empirical evidence on some of the main predictions of the model using micro-data from a time-use survey in Spain. Finally, section 7 concludes. Further data descriptions and some algebraic derivations are relegated to three Appendices.

2. Modelling gender gaps

2.1 The basic setup: A training model
To account for the joint presence of gender gaps in wages and housework, we build a model of firms’ and households’ decisions which relies on the substantial available evidence about firms providing specific paid-for training to their workers (OECD, 2003). To simplify the analysis, we ignore issues about the provision of this type of training in frictional labor markets (Acemoglu and Pischke, 1998) and simply take this fact as given.6

The basic setup is as follows. Ex ante identical men and women live for two periods (1 and 2) each of which has time length normalized to unity. Each gender represents half of the overall population whose mass is also set equal to unity. In period 1, firms are randomly matched with just one worker of either gender who is assumed to be single. The firm then invests an amount of (specific) training on the worker, $\tau$, where the cost of training is assumed to be linear and equal to $\tau$. For simplicity, we assume that workers are not paid while being trained. Finally, there is free entry of firms in period 1 until the expected profits from training workers are driven down to zero.

At the start of period 2, individuals of each gender form couples (exogenously) and decide how to split the household chores on the basis of their relative wages, where $s$ and $(1-s)$ denote the shares of housework performed by the woman and the man, respectively. These household tasks in period 2 are assumed to involve two components which require the same dedication: one which is known with certainty and another which is uncertain (stochastic). In the sequel, we will refer to the latter as a household disutility shock, $\omega$, which will play a key role in linking firms’ and households’ decisions. For analytic tractability, the size of the shock is assumed to be an i.i.d.

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6 Our results would not be affected if the firm pays for only part of the training and the worker pays for the rest. What is crucial is simply that the employer engages in statistical discrimination when choosing how much training the firm finances.
random variable with a uniform distribution, i.e., $ω \sim U[0, ε]$, where $ε \leq 1$ given the unit-time length of period 2. Hence, when a household receives a shock of size $ω$, the woman will bear a cost of $ω_f = sω$ and the man of $ω_m = (1 - s)ω$, implying that the support for the shock is $[0, ε_f]$ for females and $[0, ε_m]$ for males, with $ε_f \equiv sε$ and $ε_m \equiv (1 - s)ε$. Notice that these two upper bounds for the size of the shock, which may differ in equilibrium, are taken as given by firms.

At the beginning of period 2, before the household shock is realized, each trained worker receives a predetermined wage offer, $W_i$, with $i = f, m$, which the firm chooses to maximize expected profits. We assume that the monetary loss $ω_i$ induced by the shock is only borne by the individual if he/she is employed. Accordingly, monetary utility in period 2 becomes $W_i - ω_i$ if the worker remains in the firm, and zero otherwise. Alternatively, whenever $W_i < ω_i$ the trained worker will quit the job before production starts in period 2 whereas, if $W_i \geq ω_i$, production takes place and wages are subsequently paid by firms.

Output per worker depends on the level of training in period 1 and is denoted as $a(τ_i)$. In particular, the production technology is assumed to be $a(τ_i) = βτ_i^{α/2}$, where $β > 0$ is a shift factor capturing the productivity level in the economy (say, TFP) and $0 < α < 1$, so that $a(τ_i)$ is increasing and strongly concave.

The timing of decisions can be graphically represented as follows:

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  t=1                       t=2
|------------------|-------------------------------------------------------|
↑                 ↑                        ↑                        ↑
Training       Wage offer      Household decision      Disutility shock      Production
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Firms move first by determining workers’ training investment in period 1 and the corresponding wage at the beginning of period 2, taking as given the household’s allocation of housework in period 2. A key feature of the model is that the firm will not internalise the response of the household since wages are predetermined. Thus, it cannot affect the time allocation decision of the spouses once the household shock is realized. At the start of period 2, before the shock occurs and taking as given the wages offered by firms, spouses choose the division of housework regarding the certain and uncertain
component of housework. Accordingly, workers will always get trained in period 1 and they will not quit in period 2 insofar as the value of staying with the firm is higher than the outside option. Next, the disutility shock is realized, participation decisions made, and production takes place at the predetermined wage by those who remain in employment. If the worker quits no wage is paid.

2.2 Firms’ decisions

To solve for the wage and the amount of paid-for training, we proceed by backward induction, first considering decisions in period 2 and later in period 1. Under the assumption that the wage offer is made by the firm before the disutility shock \( \omega \) is realized, firms will choose the wage \( W_i \) to maximize expected gross profit in period 2, \( \Pi_i \), taking into account that the worker may quit after being trained. This leads to the following optimization problem:

\[
\max_{W_i} \Pi_i = \max_{W_i} \int_0^{W_i} [a(\tau) - W_i] d\omega = \max_{W_i} \frac{a(\tau)W_i - W_i^2}{\epsilon_i}, \quad i = f, m
\]

where the integral in the middle term of (1) captures the firm’s expected gross profit when the worker does not quit. Hence, the first-order condition (hereafter, f.o.c.) with respect to the wage implies that the wage paid in equilibrium, \( W^*_i \), satisfies:

\[
W^*_i(\tau) = \frac{a(\tau)}{2}.
\]

which, in turn, implies the following expected gross profit in period 2:

\[
\Pi(\tau_i) = \left[ a(\tau) - \frac{a(\tau)}{2} \right] \frac{W^*_i}{\epsilon_i} = \frac{a(\tau)^2}{4\epsilon_i},
\]

where the term \( W^*_i / \epsilon_i \) captures the probability of not quitting, i.e., \( \Pr(\omega \leq W_i) \).

Once the firm’s decision on the wage (conditional on training) has been established in (2), let us go back to period 1 when the firm chooses the level of training. Since we are assuming free entry, the zero-profit condition pins down the equilibrium

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7 This is just the average of the worker’s productivity and the outside wage, which is assumed to be zero. The weight \( \frac{1}{2} \) in the wage is due to the choice of the uniform distribution. Alternative distributions will give rise to a weighted average with unequal weights.
level of training in period 1, $\tau^*$, which is given by:\(^8\)

$$\Pi(\tau^*) - \tau^* = 0. \quad (4)$$

Hence, under the functional form assumed for $a(\tau)$, $\tau^*$ is chosen to be:

$$\tau^*_i = \left( \frac{\beta^2}{4\epsilon_i} \right)^{\frac{1}{\alpha}}. \quad (5)$$

which, when replaced into (2), yields the optimal wage:

$$W^*_i = \left( \frac{\beta^2}{4\epsilon_i^{\alpha}} \right)^{\frac{1}{1-\alpha}}. \quad (6)$$

Equations (5) and (6) imply that, as the size of the disutility shock becomes larger (i.e., as $\epsilon_i$ increases), workers face a higher probability of quitting in period 2 and, since this reduces expected profits, firms respond by lowering the amount of training and therefore wages. Note that our assumption that $0 < \alpha < 1$ plays a crucial role in this result. If $\alpha \geq 1$ (i.e., if there were weak diminishing returns in training) then the firm would respond to a higher probability of quitting by increasing the amount of training, raising the wage to offset the higher expected value of the shock. Our assumption of strong diminishing returns to training prevents this rather counterintuitive outcome.

From (6), the probability of working ($P^*_i = \Pr(\omega_i \leq W^*_i) = W^*_i / \epsilon_i$) and the expected wage ($P^*_i W^*_i = W^*_i^2 / \epsilon_i$) are given respectively by:

$$P^*_i = \left( \frac{\beta^2}{4\epsilon_i^{\alpha}} \right)^{\frac{1}{2(1-\alpha)}}. \quad (7)$$

$$P^*_i W^*_i = \left( \frac{\beta^2}{4\epsilon_i^{\alpha/2}} \right)^{\frac{1}{1-\alpha}}. \quad (8)$$

As before, a larger value of $\epsilon_i$ results in lower participation and a lower expected wage since $\alpha \in (0, 1)$.

Further, the following assumption is needed:

**Assumption 1:** Let $b_i \equiv (\beta^2 / 4\epsilon_i)^{1/1-\alpha}$ and $b_f \equiv (\beta^2 / 4\epsilon_f)^{1/1-\alpha}$. We suppose that $b_i$ verifies $b_i \leq \epsilon_i$ for $i = m, f$ and that $b_i^{1-\alpha/\alpha} < 1/2$.

\(^8\) Alternatively, we could have allowed for positive profits and have the firm maximize profits with respect to training. Equilibrium training would be lower, but all the comparative statics would remain unchanged.
The inequality \( b_i \leq \varepsilon_i \) ensures that the unit length of period 2 is not exceeded, namely, that \( P^*_i \leq 1 \) for both types of agents. Notice that, since this inequality can be rewritten as \( \beta^2 \leq 4\varepsilon_i^{1-\alpha} \), it requires the productivity parameter (\( \beta \)) to be not too large relative to the size of the household shock; otherwise, the resulting wage would be sufficiently high (relative to the individual shock) to lead to a corner solution for participation in the labor market. As will be seen later, the second inequality, \( b_i^{1-\alpha/\alpha} < 1/2 \), ensures that an interior solution exists in the household decision problem. Since this inequality is equivalent to \( \beta^2 \leq \varepsilon 2^{2-\alpha} \), it again requires that \( \beta \) is not too large.

3. Household division of labor and multiplicity of equilibria

3.1 Household division of labor

The next step is to endogenize the household’s housework allocation decision at the beginning of period 2 (once couples have been formed). We assume that there is a household good to be produced by the spouses, and that this good provides a fixed utility level denoted by \( \bar{u} \). As mentioned earlier, the production of this good involves two utility costs. Part of the cost is perfectly known in advance, while the remaining component is uncertain (stochastic) and depends on the uniformly distributed disutility shock. To provide a specific example, suppose that the household good consists of raising children. Children have to be collected from school and ferried to their after-school activities every day, imposing a (known) utility cost to the parent in charge of this task, irrespective of whether he/she is employed or not. Additionally, there are shocks, such as a child falling sick and needing to stay home with a carer. The latter impose an opportunity cost only if the parent is working since they imply a reduction in the (monetary) utility derived from market work.

In line with our assumption of ex-ante identical individuals, we consider a unitary household, so that there is full-income sharing and no bargaining between members. Hence, on the basis of their expected wages, the spouses decide jointly on how to split the responsibility for production of the household good by choosing the fraction \( s \in [0,1] \) of the household chores allocated to the wife and \( 1-s \in [0,1] \) to the husband which maximizes the household’s utility function given by:

\footnote{Note that the random shock need not be solely related to the presence of children in the household. Other examples could be the need to stay at home to undertake unexpected house repair, etc.}
\[ V^H = \bar{u} + Y(s) - [c(s) + c(1-s)], \tag{9} \]

where \(c(s)\) and \(c(1-s)\) are the disutility costs of housework incurred by the woman and the man, respectively.\(^{10}\) These costs are assumed to be increasing and convex in the amount of housework undertaken, i.e. \(c' > 0\) and \(c'' > 0\). Expected net household income, \(Y(s)\), is a function of the allocation of housework, given by:

\[
Y(s) = \int_0^{w_s/(1-s)} W_m - (1-s)\omega \frac{d\omega}{\epsilon} + \int_0^{w_f/s} W_f - s\omega \frac{d\omega}{\epsilon} \tag{10}
\]

where this expression also captures the fact that a woman (man) will quit the firm if \(W_f < s\omega\) (respectively, \(W_m < (1-s)\omega\)) and that the monetary loss induced by the shock is only borne by the individual if he/she is employed.

Hence, because couples decide the division of housework before \(\omega\) is realized, they choose \(s\) to maximize (9) subject to (10). Actual income and hence actual utility will however differ across households depending on the realization of the shock. A key assumption is that there is no change in \(s\) after the shock takes place. Under this assumption, the f.o.c. and the s.o.c. for a maximum yield:

\[
\frac{\partial V^H}{\partial s} = Y'(s) - c'(s) + c'(1-s) = 0, \tag{11}
\]

\[
\frac{\partial^2 V^H}{\partial s^2} = Y''(s) - c''(s) - c''(1-s) < 0 \tag{12}
\]

To understand the determinants of the household decision, it is convenient to consider separately the two components (besides \(\bar{u}\)) in \(V^H\): expected income and the disutility of housework. As regards the former, integration of (10) yields:

\[
Y(s) = \frac{1}{2\epsilon} \left[ \frac{W_m^2}{(1-s)} + \frac{W_f^2}{s} \right]. \tag{13}
\]

It is straightforward to verify that \(Y(s)\) is a U-shaped function of \(s\) implying that expected household income is maximized when the household member with the highest wage undertakes a sufficiently low amount of housework to never withdraw from the labor market.\(^{11}\) This is the traditional division of labour emphasized by Becker (1985, 10)

\(^{10}\) The assumption we are making here is that time not spent in the labour market is not necessarily leisure since it may be devoted to household production. See Rapoport, Sofer and Solaz (2010) on the implications of such a formulation.

\(^{11}\) For example, the man would have the highest wage when \(1-s = W_m/\epsilon\). Hence, expected household income would be given by \(Y = W_m + 0.5 W_f^2/(\epsilon - W_m)\), where the first term is the (certain) male income and the second term is the expected female income.
1991) as a result of comparative advantage. In our alternative setup, however, the
intuition is simply that the expected income loss is largest when both spouses bear some
of the cost because a large shock may imply that both individuals quit employment.
Hence, there is a participation effect which leads to specialization since expected
household income is maximized when the household member with the lower wage bears
most of the shock since this ensures full labor market participation of at least one of
them.

The second element of $V^H$ is the convex cost of housework by each of the
spouses- the last bracketed term in (9) - which has an equalizing effect as total disutility
is minimized when housework is evenly split between husband and wife ($s = 0.5$). This
result is due to the symmetry in the way in which we model the costs of housework (i.e.,
no comparative advantage of either gender in home production), together with our
assumption of convex costs.

Thus, the choice of $s$ is driven by the trade-off between full specialization and
equal share of housework. In order to examine how these two forces interact, we
adopt in the sequel a specific functional form for the cost function that yields
analytically tractable solutions. Yet, in Appendix 1 we show that the same
characterization of equilibria applies to any cost function satisfying our previous
assumptions. Our assumed cost function for the woman takes the form
$c(s) = 0.5 \ s/(1-s)$, which is increasing and convex in her share of housework, $s$, and
tends to infinity when she bears the entire burden, i.e. as $s \rightarrow 1$. Likewise, the husband's
cost function is given by $c(1-s) = 0.5 \ (1-s)/s$, which now tends to infinity if he bears
the entire burden, i.e., as $(1-s) \rightarrow 1$. The key feature of these cost functions is that they
tend to infinity when one of the household members fully specializes in housework, i.e.,
as $s=0$ or 1. As a result, they will lead to interior solutions where marketplace and
housework activities are always combined. We adopt this assumption to mimic the
evidence drawn from time-use surveys in developed countries where strictly positive
housework shares are reported by both partners (see Section 6).

Using these cost functions, the expected utility of the household becomes:

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12 The importance of the convexity assumption in the disutility function can be seen by inspecting the
s.o.c. in (12). Since income is a convex function of $s$, the only way in which $\partial^2 V^H / \partial s^2$ can be negative is
if $c''(s)$ is positive. That is, with concave costs of housework, there would be no interior maximum and.
the overall disutility would be minimized when one agent performs all housework, reinforcing the
tendency toward specialisation of household members induced by income maximization.
\[ V^H = \bar{u} + \frac{1}{2\epsilon} \left[ \frac{W_m^2}{(1-s)} + \frac{W_f^2}{s} \right] - \frac{1}{2} \left[ \frac{1-s}{s} + \frac{s}{1-s} \right]. \] (14)

Maximizing with respect to \( s \) yields the f. o. c.:

\[ \frac{1}{2\epsilon} \left[ \frac{W_m^2}{(1-s)^2} - \frac{W_f^2}{s^2} \right] + \frac{1}{2} \left[ \frac{1}{s^2} - \frac{1}{(1-s)^2} \right] = 0, \] (15)

which implies that the equilibrium share of housework, denoted by \( s^* \), is determined by equating the marginal rates of substitution between marketplace and household work:

\[ \left( \frac{1-s^*}{s^*} \right)^2 = \frac{1}{\epsilon} \frac{W_m^2}{W_f^2}. \] (16)

It can be easily shown that, for \( s^* \) to be a maximum, we require \( W_f^2/\epsilon < 1 \) which, using (6), is ensured by Assumption 1. As a result, \( ds^*/dW_f < 0 \) and \( ds^*/dW_m > 0 \), implying that a higher female (male) wage leads to a reduction (increase) of the female housework share. Moreover, when wages are equalised, i.e., \( W_f = W_m \), then \( s^* = 1 - s^* = 0.5 \). Note that this result is due to the symmetry assumption in the way in which we model the costs of housework (i.e., no comparative advantage of either gender) which leads to total cost minimization when housework is evenly split. Lastly, since the household takes wages as given, the household’s decision in (16) leads to the following proposition:

**Proposition 1:** Under Assumption 1, for given relative wages, an increase in the support of the household disutility shock (\( \epsilon \)) decreases (increases) \( s^* \) whenever \( W_m > W_f \) (\( W_m < W_f \)).

The intuition for this result stems again from the first bracketed term in (14): the higher is the upper bound of the shock, the lower is expected income and, as a result, the spouses will prefer to share housework more evenly in order to maximize \( V^H \). However, as will be shown below, the effect of \( \epsilon \) on \( s \) will change its sign once we combine households’ and firms’ decisions. This different effect of \( \epsilon \) on \( s \) in the two setups will become one of the testable implications of the model in the empirical section.
3.2 Multiplicity of equilibria

Firms’ and households’ decisions are given by equations (6) and (16). In equilibrium, expectations are fulfilled and hence the values of wages and housework shares are jointly determined as the solution of the following system of equations:

\[ W_f = \left( \frac{\beta^2}{4\epsilon^\alpha s^\alpha} \right)^{\frac{1}{1-\alpha}}, \]  
\[ W_m = \left( \frac{\beta^2}{4\epsilon^\alpha (1-s)^\alpha} \right)^{\frac{1}{1-\alpha}}, \]  
\[ \left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-W_m^2}{1-W_f^2}, \]  

(E.1) \hspace{1cm} (E.2) \hspace{1cm} (E.3)

To analyse the equilibrium configurations, it is useful to substitute (E.1) and (E.2) into (E.3), so that the f.o.c. in (16) can be rewritten as:

\[ \left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-b_1(1-s^*)^{\frac{\alpha}{1-\alpha}}}{1-b_1(s^*)^{\frac{\alpha}{1-\alpha}}}, \]

(E.17)

where we had defined \( b_1 = (\beta^2 / 4\epsilon)^{1/(1-\alpha)} \). From (E.1) and (E.2), we can also define the gender wage and participation gaps as follows:

\[ w = \frac{W_m}{W_f} = \left( \frac{s}{1-s} \right)^{\frac{\alpha}{2(1-\alpha)}}, \]
\[ p = \frac{P_m}{P_f} = \left( \frac{s}{1-s} \right)^{\frac{2-\alpha}{2(1-\alpha)}}. \]

(E.18)

To solve for \( s \) in (17), it is convenient to think of the LHS and RHS of (17) as the functions \( f(s) \) and \( g(s) \), respectively, whose intersection results in the equilibrium allocation of the housework share, \( s^* \). On the one hand, \( f(s) \) (which is a monotonically increasing transformation of the disutility cost of housework for men) is decreasing and convex with a vertical asymptote at \( s=0 \), such that \( f(0) = 0 \) and \( f(0.5) = 1 \). On the other, under Assumption 1, \( g(s) \) is increasing in the range \( s \in 0, b_1(1-\alpha)/\alpha \) and decreasing when \( s \in \left( b_1^{-\alpha/\alpha}, 1 \right) \), with two vertical asymptotes at \( s = b_1^{-\alpha/\alpha} \) and \( s = 1 \), such that \( g(0) = 0 \), \( g(0.5) = 1 \) and \( g\left( 1-b_1^{-\alpha/\alpha} \right) = 0 \). Lastly, \( g(s) \) has an inflection point
within the range \( s \in \left[ b_1^{1-\alpha} \frac{1}{\alpha} , 1-b_1^{1-\alpha} \right] \). Notice that the non-monotonicity of \( g(.) \) is due to the fact that expected household income is a U-shaped function of \( s \): when men bear a high share (\( s < 0.5 \)) expected income is higher the lower is \( s \), but when women bear a higher share (i.e. \( s > 0.5 \)) expected income is increasing in \( s \) and maximized when there is full specialisation.

The intersections of \( f(s) \) and \( g(s) \) are depicted in Figure 1 where the vertical axis represents the inverse of the wage gap in the LHS of (18), so that a value above below 0.5 implies \( W_f < W_m \). As can be seen, there are three values of \( s \) that satisfy equation (17). In one of them, \( s_1^* = 0.5 \), while in the other two we have \( s_2^* \in \left( 0.5, 1-b_1^{1-\alpha} \right) \) and \( s_3^* \in \left( 0, b_1^{1-\alpha} \right) \). Corner solutions are ruled out by our assumption that disutility becomes infinite under complete specialization in housework.

**FIGURE 1**: Gendered and ungendered equilibria

![Gendered and ungendered equilibria](image)

Due to our assumption of *ex-ante* symmetry across genders, two possible asymmetric equilibria exist: one in which women bear a greater housework share and get a lower wage (G), and another in which the same outcomes apply to men (G’). The two equilibria are actually symmetric on the sense that \( s_3^* = 1-s_2^* \). In the sequel, we will solely focus on case where women carry out a disproportionate share of the household
chores, so that 

\[ s \in \left( \frac{b_1^{1-a}}{a}, 1 \right) = S \]

becomes the permitted domain of \( g(s) \). This assumption therefore restricts the analysis to two possible interior equilibria, labelled respectively as the *gendered* equilibrium (denoted by G), where \( s^*_G > 0.5 \), and the *ungendered* equilibrium (denoted by U) where \( s^*_U = 0.5 \). Likewise, the gender wage gaps in these two equilibria are labelled as \( w^*_G \) and \( w^*_U \). The following result summarises this discussion:

**Proposition 2:** Under Assumption 1 and with \( s \in S \), there are two equilibrium solutions for the female share of housework and the wage gap: (i) an ungendered solution with \( s^*_U = 0.5 \) and \( w^*_U = 1 \) and (ii) a gendered solution with \( s^*_G \in (0.5, 1 - b_1^{1-a}/a) \) and \( w^*_G > 1 \).

To gauge how different these equilibria could be, let us consider the following numerical example. Using the parameter values \( \alpha = 0.5 \), \( \epsilon = 1 \), and \( \beta = 2^{3/4} \), which jointly satisfy Assumption 1, the roots of equation (17) become \( s^*_G = 0.7236 \) and \( s^*_U = 0.5 \). The G-equilibrium leads to a wage gap in favour of men of 62%, illustrating that the differences in the outcomes of the two equilibria can be very large.

3.3 The effect of changes in aggregate productivity level on equilibria

Inspection of (17) and Figure 1 indicates that the system (E.1)-(E.3) may exhibit a unique equilibrium. Indeed, the existence of multiple equilibria crucially depends on the size of the \( b_1 \) parameter. In effect, as \( b_1 \) increases, the range \( s \in \left( b_1^{1-a}/a, 1 - b_1^{1-a}/a \right) \) shrinks and, as a result, \( g(s) \) becomes steeper. This shifts the G-equilibrium to the left, leading to a more even division of housework and a lower wage gap.

As depicted in Figure 2, there will be a unique U-equilibrium for sufficiently high values of \( b_1 \). Since \( b_1 = \left( \beta^2 / 4 \epsilon \right)^{2/3} \), changes in its value will be driven by changes

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13 The equilibrium in which males undertake a greater share of housework than women is, in terms of our model, as likely as the one in which women do a greater share. However, such a situation is not found in the data, probably because past patterns of comparative advantage had led women to specialise in home production, making it difficult for an economy to jump to an equilibrium with the opposite pattern of specialisation. Properly ruling out this equilibrium would require modelling the way in which beliefs about gender roles are formed, which is beyond the scope of this paper.
in $\beta$ and $\varepsilon$, with $\partial b_1 / \partial \beta > 0$ and $\partial b_1 / \partial \varepsilon < 0$.

**FIGURE 2**: The effect of an increase in $b_1$ on gender gaps

The previous discussion can be summarised in the following proposition:

**Proposition 3**: Under Assumption 1 and $s \in S$, it holds that:

(i) An increase in the aggregate productivity parameter $\beta$ leads to lower equilibrium gender gaps. Moreover, economies with a sufficiently high value of $\beta$ will exhibit a unique ungendered equilibrium.

(ii) An increase in the upper bound $\varepsilon$ of the household disutility shock increases the equilibrium gender gaps.

To understand the intuition behind Proposition 3, note that the only effect of an increase in $\beta$ is to raise wages. The reason why productivity matters is that it leads to an income effect. Recall the trade-off faced by a household between expected income and housework disutility: the former effect implies that income is higher with full specialization ($s=1$), while the latter effect induces an even allocation of housework ($s=0.5$). When wages are low ($\beta$ is small), the household is less willing to forgo expected income in order to reduce the utility cost. Hence, if firms offer different wages, housework will be unevenly allocated. By contrast, when wages are high ($\beta$ is large), the opposite holds, leading to a lower $s^*_G$. If wages are sufficiently high, the disutility effect dominates, making the housework division more and more equal across
genders.\(^\text{14}\) Yet, if \(s\) is (close to) 0.5, then firms will pay similar wages to men and women. Hence the G-equilibrium cannot exist.

Concerning the effect of \(\varepsilon\), notice that it increases unambiguously the equilibrium gender gaps, in sharp contrast to the opposite result obtained in (16) about the household decision for given wages, where a larger value of \(\varepsilon\) lowered the gaps in the gendered equilibrium. The intuition for this contrasting result is that an increase in \(\varepsilon\) operates in the same way as a decrease in \(\beta\) in the full solution of the model, and thus it leads to a higher gender gap.

In sum, productivity plays a crucial role in determining the equilibrium gender gaps in wages and time allocation. Interestingly, this result by itself suggests that, abstracting from the differences in the generosity of family-aid policies, rich countries/regions could exhibit the lower gender gaps than poor countries/regions solely due to their higher aggregate productivity.

4. Welfare analysis

In order to compare welfare in the two equilibria, let us consider the unitary-household problem faced by a planner who, conditioning on the firm’s optimal training and wage decisions, is allowed to choose the allocation of housework internalizing its effect on training and therefore on wages. Since firms make zero expected profits due to the free-entry assumption, aggregate welfare in this economy, \(V^w\), is simply equal to the welfare of the representative household where, in contrast to (9), the choice of \(s\) is now allowed to affect wages through the amount of training provided by firms. Substituting (E.1) and (E.2) into (14), yields the following unitary-household’s welfare function:

\[
V^w(s) = \bar{u} + \frac{1}{2} \left[ b_1 (1-s)^{1-\alpha} + b_1 s^{1-\alpha} \right] - \frac{1}{2} \left[ \frac{1-s}{s} + \frac{s}{1-s} \right], \quad (19)
\]

We can now examine which of the two equilibria results in a higher level of welfare by substituting (17) into (19). This yields:

\[
V^w(s^*) = 1 + \frac{1 - b_1 s^{\frac{\alpha}{1-\alpha}}}{2(s^*)^2}. \quad (20)
\]

Then, differentiation of (20) implies that:

\(^{14}\) To see this simply let \(b_1 \to \infty\) in equation (17), which makes its RHS equal to 1, implying that \(s=0.5\) for all wages.
\[
\frac{dV^W(s^*)}{ds^*} = \frac{1}{2s^*} \left[ 2 - \frac{2 - \alpha}{1 - \alpha} b_s s^\alpha \right],
\]

which may be positive or negative depending on the sign of the bracketed term. Hence, it is ambiguous whether welfare in the unitary-household problem is higher in the G-equilibrium or in the U-equilibrium, the reason being once more the trade-off between full specialisation and equal sharing of housework.

As before, the level of productivity $\beta$ is a key parameter determining which effect dominates. Since $b_1$ is increasing in $\beta$, $s^*_G$ will decrease with the productivity level. Hence, $dV^W(s^*)/ds^* < 0$ for sufficiently high values of $\beta$. This implies higher welfare in the U-equilibrium since $s^*_G > s^*_U = 0.5$.

This finding differs from the results in models that rely on incentive problems, where it has been generally found that specialization results in higher welfare. The difference lies in both the symmetry in preferences and the fact that we assume an increasing and convex cost of housework for both spouses. Moreover, our analysis has the implication that the nature of the efficient equilibrium may change over time as the productivity parameter grows. Initially, when $\beta$ is low, specialization delivers higher welfare. Yet, as productivity grows, the opportunity cost of sharing housework falls and the U-equilibrium becomes more efficient.

5. Policies

5.1 Affirmative actions

We next discuss which kind of gender policies could shift the economy from the G-equilibrium to the U-equilibrium. The literature on this issue has focused on two specific policies: affirmative action and subsidised family aid. In our setup, affirmative action would take the form of a law that prevents firms from engaging in statistical discrimination and offering differential training to men and women. Since men and women receive now the same amount of training, (2) implies that they also receive identical wages leading to equal sharing of housework tasks. Hence, the only possible

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15 See Lommerud and Vagstad (2007) for a discussion of welfare in this type of models. In the statistical discrimination literature, however, there are examples where discrimination leads to lower welfare. For example, this is so in Coate and Loury (1993) because the discriminated group invests less than optimally in human capital. This is also the case in the racial discrimination model with exogenous posted wages proposed by Lang at al. (2005).

16 For alternative analyses of how exogenous changes in productivity affect gender differences in the labour market, see Olivetti (2006) and Albanesi and Olivetti (2009).
equilibrium is $s = 0.5$, implying that it is optimal for the firm to offer the same amount of training to the household partners. In other words, since the reason for the existence of the U-equilibrium is a coordination problem, affirmative action will coordinate firms and households on the U-equilibrium in which firms would choose not to differentiate between genders even if they could.

There is an extensive debate on the effects of affirmative action policies. As discussed earlier, Coate and Loury (1993) show that an exogenous increase in the hiring probability reduces the educational effort of the minority. However, Moro and Norman (2003) find that this result depends crucially on assuming that the marginal product of labor is constant for each type of workers. By contrast, when the marginal products depend on the relative supply of the two groups, equilibrium effects imply that the wage changes resulting from affirmative action policies may induce minority workers to increase, rather than decrease, their educational investment. Our analysis illustrates how, even in the absence of these externalities, targeted policies towards statistically discriminated groups can have different effects.

One problem with affirmative action policies is that they may be difficult to implement in many instances. This would be the case if the training that individuals receive is imperfectly observed by the policymaker or when there are differences in the ‘quality’ of training provided by firms. In this case, the standard tool left to affect the equilibrium is subsidized family aid whose effects are analyzed in the next subsections.

### 5.2 Subsidised family aid

#### 5.2.1 Gender-based vs. Gender-neutral family aid

Consider the introduction of government-funded family aid subsidy. To start with, suppose that it is targeted on working women and that this subsidy, $\kappa$, is proportional to the female wage in period 2. Thus, women will receive an income equal to $W_f \cdot (1 + \kappa)$, where $0 < \kappa < 1$, so that they will not quit in period 2 if $W_f \cdot (1 + \kappa) - \omega \geq 0$, whereas men will work if $W_m - \omega \geq 0$. For the time being, we neglect the issue of how the targeted subsidy is financed which will be addressed at the end of this section.

The analysis of firms’ decisions, taking household decisions as given, is the same as in section 2.2, but this time with the upper limit of the integral for women in (1) changed from $W_f$ to $W_f \cdot (1 + \kappa)$. This yields the following optimal amount of training and wage chosen by the firm:
\[
\tau_j^{\kappa} = \left( \frac{(1 + \kappa)\beta^2}{4\varepsilon_f} \right)^{\frac{1}{2}}, \quad W_j^{\kappa} = \frac{\beta}{2} (\tau_j^{\kappa})^{\alpha/2},
\]

(21)

where the superscript \( \kappa \) is used to denote the equilibrium values under subsidies. Male workers are offered the training level and wage derived in (5) and (6). Note that the total income of women in period 2, \( Y_f^{\kappa} \), is now given by:

\[
Y_f^{\kappa} = (1 + \kappa)W_f^{\kappa} = \frac{\beta(1 + \kappa)}{2} (\tau_f^{\kappa})^{\alpha/2}.
\]

(22)

Not surprisingly, women fare better in the labour market when they are subsidised to stay in the job since \( \tau_f^{\kappa} > \tau_f^* \) and \( W_f^{\kappa} > W_f^* \),\(^{17}\) despite the fact that, for \( \kappa < (\varepsilon_f - \varepsilon_m) / \varepsilon_m \) (i.e. if the subsidy is not too large), they will still receive less training and lower wages than men, that is, \( \tau_f^{\kappa} < \tau_m^* \) and \( W_f^{\kappa} < W_m^* \).\(^{18}\)

Expressions (21) and (22) imply that the corresponding participation and wage gaps would be lower than without subsidies. However, this result changes once the division of housework is endogenized. In this case, each household chooses \( s \) to maximize the expected net utility given by:

\[
V^{\kappa s} = \bar{r} - 2 + \frac{1}{2\varepsilon} \left[ \frac{W_m^2}{(1-s)} + \frac{Y_f^{\kappa^2}}{s} \right] - \frac{1}{2} \left[ \frac{1-s}{s} + \frac{s}{1-s} \right].
\]

(23)

The resulting f. o. c., once we have substituted for wages, yields the new equilibrium relationship:

\[
\left( \frac{1-s^{\kappa}}{s^{\kappa}} \right)^2 = \frac{1-b_1(1-s^{\kappa})^{\alpha/\alpha}}{1-b_2s^{\kappa^{1-\alpha}}},
\]

(24)

where \( b_2 = b_1(1+\kappa)^{2-\alpha} > b_1 \). The LHS of equation (24) is the same as in (17), while the RHS tilts upwards and takes a value greater than 1 when \( s=0.5 \). The new equilibrium is depicted in Figure 3.

\(^{17}\) They may even get higher gross wages than men if the subsidy is sufficiently large but we ignore this possibility in the sequel.

\(^{18}\) In equilibrium, since \( \varepsilon_f = s\varepsilon \) and \( \varepsilon_m = (1-s)\varepsilon \), this condition becomes \( \kappa < (2s-1)/(1-s) \).

The following proposition summarizes the main result of this section.

**Proposition 4:** Under Assumption 1 and with \( s \in S \), a wage subsidy targeted to female workers leads to a gendered equilibrium with \( s^* \in (0.5, 1) \), so that gender gaps in wages and housework are larger than in the absence of the subsidy.

The remarkable feature of (24) is that, with the subsidy in place, the U-equilibrium with \( s = 0.5 \) no longer exists. In other words, a gender-based subsidy policy only yields the G-equilibrium since the asymmetry in income induced by the subsidy prevents a symmetric equilibrium. In effect, suppose that households set \( s = 0.5 \). Then women have a lower probability of quitting than men (the combination of the same shock plus the subsidy) which implies that firms will offer more training and a higher gross wage to them. Yet, if female wages are different from men’s, then \( s = 0.5 \) cannot be a solution to the household’s problem. Hence, the U-equilibrium no longer exists. Moreover, it can be easily shown that the new G-equilibrium in Figure 3 lies to the right of the initial one in Figure 1, leading to a higher equilibrium value of \( s \). Thus, gender-based job subsidies can backfire. For example, using our previous choice of parameter values and \( \kappa = 0.1 \), yields \( s^* = 0.7299 > s^*_G = 0.7236 \).

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To show this, denote the LHS of (24) as a function of \( s \) by \( g_\kappa(s) \). Then differentiating \( g_\kappa(s) \) with respect to \( s \) in a neighbourhood of its crossing with the LHS of (24), given by \( f(s) \), one gets that \( g_\kappa'(s) \) becomes more negative (steeper) for a higher value of \( \kappa \). Hence, since \( f(s) \) is the same as in the case where \( \kappa = 0 \), the new gendered equilibrium must be to the right of the equilibrium without subsidies.
The intuition behind this seemingly puzzling result relies once again the trade-off faced by the household. Because the subsidy increases the probability of female labor participation, the unitary household can now afford to raise the probability of male participation by reducing men’s housework share. This result shares the spirit of the analysis of affirmative action policies in Coate and Loury (1993) where it is argued that an exogenous increase in the hiring probability faced by a minority would reduce their educational effort and hence increase the educational gap. Similarly, in our framework the exogenous increase in the probability of participation of women reduces their commitment to the labor market.

By contrast, consider now an alternative policy which offers the same subsidy to men and women. Following the same reasoning as above, this would yield the equilibrium relationship:

\[
\left(1 - s^*\right)^2 = \frac{1 - b_2 (1 - s^*)^{1-\alpha}}{1 - b_2 s^*^{1-\alpha}},
\]

which again will narrow the range of values of \(s \in S\) for which the RHS of (25) is positive, since \(b_2 > b_1\). The first implication is that the subsidy shifts the G-equilibrium to the left, reducing the value of \(s^*_G\). Moreover, if the subsidy is high enough (i.e. for sufficiently large values of \(b_2\)), equation (25) will yield a unique U-equilibrium, as depicted in Figure 5. Once more, the above-mentioned trade-off faced by the household explains this result. The subsidy effectively increases expected income and hence reduces the opportunity cost of sharing housework. If the increase in income is large enough, the household will simply minimize the disutility associated with housework and choose an even allocation of domestic chores. Interestingly, this reasoning in favour of neutral-gender subsidies also echoes some of Saint-Paul (2007)’s arguments against gender-based taxation.

### 5.2.2 Financing of the subsidy

We next consider the financing of the subsidy. One possibility would be to finance the wage subsidy by taxing men. However, our analysis in the previous section indicates that such policy would introduce an asymmetry in the RHS of (25) eliminating therefore the U-equilibrium. In other words, taxing men would imply that the only equilibrium that exists is one with unequal wages and an unequal division of labor.
An alternative mode of financing the subsidy is to tax firms. Specifically, we suppose that they are taxed for their training expenditures in period 1 at a proportional rate $t$. Under a balanced budget, this implies that $t(\tau_f + \tau_m) = \kappa(W_f + W_mP_m)$. In this tax-subsidy scheme, denoted by the superscript $TS$, participation is given by $P_i^{TS} = (1 + \kappa)W_i^{T^*/\varepsilon_i}$, and firms offer the wage $W_i^{TS^*}(\tau_i) = a(\tau_i)/2$, implying that gross expected profits become now:

$$\Pi(\tau_i) = \frac{a(\tau_i)^2}{4\varepsilon_i}(1 + \kappa), \quad (26)$$

while the zero-profit condition for firms yields:

$$\Pi(\tau_i) - (1 + t)\tau_i = 0. \quad (27)$$

Noticing that we can write $\Pi(\tau_i) = P_iW_i$, this condition is simply equivalent to $\tau_i(1 + t) = P_iW_i$, which can be replaced into the budget constraint to obtain the equilibrium relation between the tax and the subsidy rates, i.e., $t = \kappa/(1 - \kappa)$. The zero-profit condition, together with this value of $t$, yields the optimal level of training:

$$\tau_i^{TS^*} = \left[\frac{\beta^2(1 + \kappa)}{4\varepsilon_i(1 + t)}\right]^{1/a} = \left[\frac{\beta^2(1 - \kappa^2)}{4\varepsilon_i}\right]^{1/a}. \quad (28)$$

Equation (28) implies lower training and wages than without subsidies as a result of the labor tax paid by firms. Participation, given by $P_i^{TS^*} = (1 + \kappa)W_i^{TS^*/\varepsilon_i}$, may be higher or lower than under laissez-faire due to the opposite effects of the subsidy and the lower wage. The former tends to increase participation while the latter tends to reduce it.

As regards the household decision on $s$, a similar argument as before yields the following f.o.c.:

$$\left(1 - s^{TS^*}\right)^2 = \frac{1 - b_3(1 - s^{TS^*})^{-a}}{1 - b_3s^{TS^*{-a}}} \quad (29)$$

where $b_3 \equiv b_1h(\kappa)$ with $h(\kappa) \equiv [(1 - \kappa)^a(1 + \kappa)]^{1/a}$. Then, $h(0) = 1$ and $h'(\kappa) > 0$ if and only if $\kappa < (1 - \alpha)/(1 + \alpha)$. Thus, for not too high values of $\kappa$, $h(\kappa)$ is increasing and therefore $b_3 > b_1$. Hence, this tax-subsidy scheme makes the $g(s)$ function steeper,

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$^20$ We have also examined the case where the tax is lump-sum in the first period. This case yields similar results though the calculations are somewhat more cumbersome.
implying that the equilibrium value of $s$ will decrease and, potentially, a unique U-equilibrium could be achieved. Indeed, for the G-equilibrium to disappear, we need that there is a unique intersection, which will be the case if $1 - b_3^{3-a}/a \leq 0.5$, that is, if $(1 + \kappa)(1 - \kappa)^a \geq 2^{3-2a} \epsilon / \beta^2$. Hence, the following result holds.

**Proposition 5:** Under Assumption 1 and with $s \in S$, if $\kappa$ is not too large, i.e., $(1 + \kappa)(1 - \kappa)^a \geq 2^{3-2a} \epsilon / \beta^2$, an equal wage subsidy to male and female workers financed through a proportional tax on training expenditures by firms in period 1 will reduce gender gaps and may even lead to an ungendered equilibrium with $s^{TS^*} = 0.5$.

The intuition for this result relies on the two conflicting effects affecting participation: a direct effect from the subsidy which tends to increase participation, and an indirect one, operating through the reduction in training induced by the tax paid by firms, which tends to reduce participation. The condition $(1 - \alpha)/(1 + \alpha) > \kappa$ is easy to interpret since, from (29), a low value of $\alpha$ implies a low elasticity of training with respect to the subsidy. This means that the wage does not decrease by much, implying that the direct effect dominates and leads to higher expected income for any given division of housework. As in section 3.3, a higher income implies that couples can afford to reduce the disutility of housework, thereby choosing an even split.

6. **Some empirical micro evidence**

6.1. **Data and descriptive statistics**

In this section we use data from a time use survey to test some relevant implications of our model regarding households’ decision on housework allocation both for given wages and in the full solution of the model. The main reason for neglecting the joint estimation of these decisions with firms’ decisions on paid-for training is that we are not aware of the availability of either time-use surveys reporting information on workers’ paid-for training or firms’ databases on training reporting time use of their workers.

In particular, we use micro data drawn from the Spanish Time Use Survey (STUS) which is part of the Multinational Time Use Survey (MTUS). The latter contains harmonized information on the use of time by households living in a variety of European countries. Specifically, it reports data on how much time each individual
devotes to a wide range of activities (41 in total) on a representative day. For each 10-minute interval (and during 24 hours), respondents are required to keep a diary recording which are their primary and secondary activities during this period of time. These are coded according to a list provided in Table A1 in Appendix.1. Housework time is defined as the number of minutes reported in the diary that each individual devotes to categories AV7 (housework) as primary activity. Likewise, this definition can be extended to include time devoted to childcare (housework plus childcare), in which case AV7 and AV11 are lumped together. The spouses’ shares of household work are therefore computed for each of these two definitions.

In addition, MTUS provides information on basic demographic and labor-market characteristics of the respondents, including wages. However, one important shortcoming of MTUS is the lack of information on the availability of family-aid subsidies, domestic service and the region of residence of the households, all of which are relevant variables in our analysis. Fortunately, some information about these missing variables can be retrieved from a larger questionnaire from which the Spanish harmonized data for MTUS is drawn, which was made available to us. It is for this reason that we limit our empirical evidence to STUS.

To mimic our modelling assumptions, we restrict our sample to two-earner couples with full-time jobs, who belong to the 25-64 age bracket and report complete information on housework share, wages and the remaining controls. This gives rise to a sample of 2915 couples. Further, as will be discussed further below, to mimic our assumption of ex-ante identical individuals we will use age and education differences between spouses as additional controls in the two regression models. In this way, our results can be interpreted as comparing spouses with the same age and educational attainment.

In Table 1 we present descriptive statistics about net hourly wages (computed as the ratio between reported net monthly wages and (four times) weekly working hours) and some demographics (age, education levels and presence of children) in STUS.

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21 By focusing exclusively on full-time workers, hourly wages (times the number of fixed working hours) correspond to the income each spouse brings to the household. Otherwise, if their supply of working hours was freely chosen, like in part-time jobs, labour income would become endogenous.
As can be observed, the average hourly wage gap is 22 log-points. Men tend to be two years older than women whereas the latter exhibit slightly higher educational attainments. The fraction of households with no children is 57% reflecting the low fertility rate in Spain at the beginning of the 2000s. Lastly, 26% of the Spanish households in our sample have domestic service and 4% report receiving some form of family-aid subsidies.

Table 2 reports the female shares ($s$) using the two above-mentioned definitions of household work, which are equal to 80% and 76%, respectively. By age and education, the shares are lower for younger and more educated women.
6.2. Testable implications

Our empirical application focuses on the estimation of equations (16) and (17), describing how spouses allocate housework time for given wages and in the full solution of the model, respectively. To obtain an estimable regression models, we log-linearize (16) and (17) around a generic (possibly gendered) equilibrium value. As shown in Appendix 3, these approximations yield:

\[ \ln \left( \frac{s}{1-s} \right) = \theta_0 + \theta_1 (\ln W_m - \ln W_f) + \theta_2 \ln W_f + \theta_3 \ln \varepsilon, \]  

(30)

for (16),\(^{22}\) and:

\[ \ln \left( \frac{s}{1-s} \right) = \phi_0 + \phi_1 \ln \beta + \phi_2 \ln \varepsilon, \]  

(31)

for (17), where the logit transformation of the dependent variable in both equations is always feasible since all individuals in our sample report values of \( s \) different from either 0 or 1. Further, under the plausible assumption that wages are taken as parametric by the household in (16), both equations can be consistently estimated by OLS (with heteroskedasticity-robust standard errors).

The first set of testable implications relates to the signs and relative sizes of some of

\(^{22}\) Notice that \( \ln \beta \) does appear in (30) since we are conditioning on given wages and it is only through wages that productivity affects housework shares.
the parameters in (30) and (31). As shown in Appendix 3, the coefficients on the wage gap and on the female wage in (30) satisfy the following restrictions: $\theta_1 > \theta_2 > 0$ as long as $W_m > W_f$. Accordingly, the impact of the male wage (given by $\theta_1$) on the relative share is positive, i.e., a rise in $W_m$ increases $s$, whereas the corresponding impact of the female wage (given by $\theta_2 - \theta_1$) is negative, i.e., a rise in $W_f$ decreases $s$. Moreover, insofar as there is a unitary-household decision model with equal weights, $\theta_2$ is predicted to be smaller in economies where gender gaps are closer to the ungendered equilibrium. Interestingly, if men were to have larger bargaining power than women, we also show in Appendix 3 that that the restriction $\theta_1 > \theta_2$ need not hold. Thus, testing whether $\theta_1 > \theta_2$ can serve as a first-pass check on whether asymmetric bargaining plays a relevant role in explaining the gap. Next, according to Propositions 1 and 3, the coefficient on $\ln \epsilon$ should be negative for given wages and positive in the full-solution model, that is, $\theta_3 < 0$ in (30) and $\phi_3 > 0$ in (31). Finally, according to Proposition 3, another key implication of the model is that higher aggregate productivity reduces the gender gap, so that in the full-solution model $\phi_3 > 0$.

A second testable prediction relates to comparing the sizes of the estimated coefficients in (30) and (31) for the two above-mentioned definitions of household work. Since it is plausible that disutility shocks are likely to be more frequent in households with children, we would expect the estimated coefficients to be more sizeable for the definition of housework that contains childcare.

Before discussing the results, there are two issues to be addressed. First, given that our model assumes individuals’ characteristics to be ex-ante identical (including education), we add a set of additional controls to (30) and (31) which includes the age gap between husband and wife plus two dummy variables capturing differences in educational attainments. Secondly, there is the issue of how to measure $\ln \beta$ in (31) and $\ln \epsilon$ in (30) and (31). As regards $\ln \beta$, we proxy aggregate productivity by means of a dummy variable of whether the household lives in a high-productivity region (living in a low productivity region is the reference category). Using indexes of regional labor productivity in 2002-03, this indicator (Dummy rich region) takes a value of 1 for

23 These dummy variables are defined as follow: (i) both spouses have a college degree (High-ed.), and (ii) both spouses have less than a college degree (Low-ed.). The reference category is only one of the spouses has a college degree (Mixed-ed.).
couples living in one of the five regions with the highest GDP per employee (Balearic Islands, Cataluña, Madrid, Navarra and the Basque Country) out of the seventeen regions in which Spain is divided. With regard to \( \ln \epsilon \), individual heterogeneity in the upper bound of the household shock is captured by children age status (household without children are the reference category) plus a dummy variable on the availability of domestic help (Dummy domestic service). Finally, we also include an indicator variable on whether the household receives family-aid subsidies (Dummy family aid) which, lacking information on its nature, we interpret it as being non-gender targeted.

6.3. Results

OLS estimates of the coefficients in specifications (30) and (31) are presented in the first two and the last two columns of Table 3, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Eq. (30) (for given wages)</th>
<th>Eq. (31) (full sol.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Housework Duties</td>
<td>Housework &amp; Childcare</td>
</tr>
<tr>
<td>Log. Wage Gap</td>
<td>0.24***</td>
<td>0.33***</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Log. Fem Wage</td>
<td>0.09***</td>
<td>0.17***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Children&lt;5 yrs.</td>
<td>-0.23***</td>
<td>-0.28***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Children&gt;5 yrs.</td>
<td>-0.09</td>
<td>-0.15***</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Dummy rich regions</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy domestic service</td>
<td>-0.09**</td>
<td>-0.11***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Dummy family aid</td>
<td>-0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>N.obs. (couples)</td>
<td>2915</td>
<td>2915</td>
</tr>
</tbody>
</table>

Note: OLS estimates with heteroskedasticity-robust standard errors. *, **, *** mean significantly different from zero at 10%, 5% and 1% levels, respectively. Additional controls are the age gap (H-W) and dummy variables (High-ed., Low-ed.) for differences in education levels between spouses.

Regarding the first set of predictions, the estimates reported for (30) show that there is highly significant response of the relative housework share with respect to the
female wage, pointing out to the existence of a gendered equilibrium. Moreover, the finding that the estimated coefficient on the wage gap ($\theta_1$) is significantly larger than the coefficient on the female wage ($\theta_2$) yields some support for the symmetric household decision model. Further, in agreement with Proposition 3 and 4, the estimated coefficients on the dummy variables of having children are negative in (30), when wages are taken as given, whereas they are positive (though much less statistically significant) in (31), when the full-solution model is considered. Yet, the estimated coefficient on the domestic service dummy variable, which is also negative and significant in (30), does not become positive in (31) although it is insignificant. Another important finding is that the coefficient on the rich region dummy, capturing aggregate productivity, is negative and highly significant in (31), as predicted by our model. Finally, the evidence on family-aid effects is inconclusive since the coefficient on this dummy is statistically insignificant. One possible reason for this inconclusive finding is that family-aid in Spain is often means-tested and is hence capturing the fact that the household is low income. Since family-aid would tend to reduce the female share but lower income tends to increase it, the two offsetting effects may cancel out.

As regards the second testable implication, we find that, as expected, the estimated impacts of the different covariates tend to be larger for the extended definition of housework.

Additional specifications of (30) have been tried without altering the previous main qualitative results. For example, neither the inclusion of quadratic terms in the partners’ wages (to account for previous evidence on the existence of a convex effect in the impact of relative earnings on the relative housework due to social norms; see, e.g., Bittman et al., 2003) - nor interactions of the education dummies and the age gap with the wages led to statistically significant coefficients on those terms.

7. Conclusions

We have proposed a model of self-fulfilling prophecies in which statistical discrimination results in both wage and housework time differences across ex ante...
generically identical individuals, except for gender. In contrast to a large strand of this literature, our model does not rely on either moral hazard due to unobservable effort, efficiency wages in some sectors or adverse selection problems. In our setup, employers would provide identical training to *ex ante* equally-able men and women in the absence of uncertainty. However, under uncertainty, they form different expectations about the burden of the housework shocks that each spouse would face. If firms believe that, say, women are more likely to quit than men when shocks arise, they will offer them less training leading to a gender wage gap. Conversely, when couples make decisions about the division of housework, women would undertake more housework than men if they expect their wages to be lower, validating in this way both sets of beliefs.

The model gives rise to two types of equilibria - *gendered* and *ungendered* - leading to several novel policy implications which are harder to obtain in other type of models: (i) higher aggregate productivity tends to eliminate the gendered equilibrium; (ii) welfare in the symmetric equilibrium can be greater than in the asymmetric one; (iii) a gender-targeted policy (e.g., wage subsidies targeted to married women) may not only fail to achieve a symmetric equilibrium but could also worsen the prior gender gap, and (iv) a gender-neutral subsidy (i.e., targeted to both members of the couple) could be more efficient in achieving an ungendered equilibrium, and that such policy works better in more productive economies.

Empirical evidence using micro data from a time-use survey for Spain yields some support to of our main theoretical predictions concerning the relationship between the hourly wages and the sharing of housework of two full-time earner couples. However, more empirical work is clearly needed in order to test other implications, notably the effect of alternative tax-subsidy policies whose effects cannot be identified with the datasets at hand.

Finally, our model could also be used to interpret the time profiles of gender wage gaps. In our set-up, men and women have the same (zero) wage when entering the labor market, but differential training implies faster wage growth for males. This result is in line with recent evidence that shows that there are no gender wage differences at entry level but a gap appears shortly afterwards and grows up to at least age 40-45 (see Manning and Swaffield, 2008). This literature has stressed the role played by on-the-job learning and out-of-employment spells in explaining lower wage growth for women, but our paper suggests that firms’ statistical discrimination in training may be another relevant mechanism. This issue remains in our future research agenda.
Appendix 1: The household’s decision under a general cost function

Let us rewrite the f.o.c. for the household as given by equation (11) as \( Y'(s^*) = C'(s^*) \), where \( C(s) \equiv c(s) + c(1-s) \) is the total disutility incurred by the household, and its first derivative is \( C'(s) = c'(s) - c'(1-s) \). The function \( Y'(s) \) is characterized by \( Y'(0) = \infty = -Y'(1) \), and \( Y''(s) > 0 \), implying that \( Y'(s) \) is an increasing function that goes from minus infinity to plus infinity.

Also, \( Y'(\tilde{s}) = 0 \), with \( \tilde{s} \equiv \left[ 1 + W_m/W_f \right]^{-1} \) which is less than 0.5 whenever \( W_m > W_f \) and equal to ½ if \( W_m = W_f \). This implies that \( Y'(0.5) > 0 \) insofar as there is a wage gap in favour of men, while for equal wages \( Y'(0.5) = 0 \). Also, it can be shown that \( Y'(s) \) is first concave and then convex. Turning now to \( C'(s) \), we have \( C'(0.5) = 0 \) and \( C''(s) = c''(s) + c''(1-s) > 0 \). Hence, \( C'(s) \) is an increasing function that takes the value 0 at \( s=0.5 \). Lastly, we have that \( C''(s) = c''(s) - c''(1-s) = -C''(1-s) \), implying that if \( C'(s) \) is concave for \( s=x \), then it will be convex for \( s=1-x \), and vice versa.

Assuming a solution to the household problem exists, and since both \( Y'(s) \) and \( C'(s) \) are increasing, the functions may cross once or several times. Consider first the case of a wage gap in favour of men. Then, all the solutions for the f.o.c. that yield \( s^* < 0.5 \) cannot be a global maximum. To see this consider a solution given by \( s^* = x_1 < 0.5 \), so that household utility is given by \( V^H(x_1) = Y(x_1) - \left[ c(x_1) + c(1-x_1) \right] \). Now consider the utility achieved under a division of labor where \( s = x_2 = 1-x_1 > 0.5 \), so that \( V^H(x_2) = Y(x_2) - \left[ c(1-x_1) + c(x_1) \right] \). The disutility term is the same in both cases, but, as long as \( W_m > W_f \), income is higher when females do a higher share of housework, i.e. \( Y(x_2) > Y(x_1) \) implying that \( V^H(x_2) > V^H(x_1) \). Thus, \( s^* = x_1 \) cannot maximize household utility and any solution to the household problem must yield a value of \( s^* > 0.5 \). It is possible, however, that there exists more than one solution satisfying \( s^* > 0.5 \). In this case, only one of them is a global maximum, and that is the allocation chosen by the household.

In the case of equal wages, it holds that \( Y'(0.5) = C'(0.5) = 0 \), implying that an equal division of housework is a solution to the f.o.c. Whether this is a maximum or a minimum depends on the relative concavity of the functions \( Y(s) \) and \( C(s) \) and hence on specific functional forms.
## Appendix 2

Table A1 – List of Activities coded in the Multinomial Time Use Survey

<table>
<thead>
<tr>
<th>MTUS Variable Name</th>
<th>Variable Label</th>
<th>MTUS Variable Name</th>
<th>Variable Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV1</td>
<td>Paid work</td>
<td>AV21</td>
<td>Walking</td>
</tr>
<tr>
<td>AV2</td>
<td>Paid work at home</td>
<td>AV22</td>
<td>Religious activities</td>
</tr>
<tr>
<td>AV3</td>
<td>Paid work, second job</td>
<td>AV23</td>
<td>Civic activities</td>
</tr>
<tr>
<td>AV4</td>
<td>School, classes</td>
<td>AV24</td>
<td>Cinema or theatre</td>
</tr>
<tr>
<td>AV5</td>
<td>Travel to/from work</td>
<td>AV25</td>
<td>Dances or parties</td>
</tr>
<tr>
<td>AV6</td>
<td>Cook, wash up</td>
<td>AV26</td>
<td>Social clubs</td>
</tr>
<tr>
<td>AV7</td>
<td>Housework</td>
<td>AV27</td>
<td>Pubs</td>
</tr>
<tr>
<td>AV8</td>
<td>Odd jobs</td>
<td>AV28</td>
<td>Restaurants</td>
</tr>
<tr>
<td>AV9</td>
<td>Gardening</td>
<td>AV29</td>
<td>Visit friends at their homes</td>
</tr>
<tr>
<td>AV10</td>
<td>Shopping</td>
<td>AV30</td>
<td>Listen to radio</td>
</tr>
<tr>
<td>AV11</td>
<td>Childcare</td>
<td>AV31</td>
<td>Watch television or video</td>
</tr>
<tr>
<td>AV12</td>
<td>Domestic travel</td>
<td>AV32</td>
<td>Listen to records, tapes, cds</td>
</tr>
<tr>
<td>AV13</td>
<td>Dress/personal care</td>
<td>AV33</td>
<td>Study, homework</td>
</tr>
<tr>
<td>AV14</td>
<td>Consume personal services</td>
<td>AV34</td>
<td>Read books</td>
</tr>
<tr>
<td>AV15</td>
<td>Meals and snacks</td>
<td>AV35</td>
<td>Read papers, magazines</td>
</tr>
<tr>
<td>AV16</td>
<td>Sleep</td>
<td>AV36</td>
<td>Relax</td>
</tr>
<tr>
<td>AV17</td>
<td>Free time travel</td>
<td>AV37</td>
<td>Conversation</td>
</tr>
<tr>
<td>AV18</td>
<td>Excursions</td>
<td>AV38</td>
<td>Entertain friends at home</td>
</tr>
<tr>
<td>AV19</td>
<td>Active sports participation</td>
<td>AV39</td>
<td>Knit, sew</td>
</tr>
<tr>
<td>AV20</td>
<td>Passive sports participation</td>
<td>AV40</td>
<td>Other leisure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AV41</td>
<td>Unclassified or missing activities</td>
</tr>
</tbody>
</table>

The two housework share definitions used in the empirical analysis are:

1) AV7: Housework, which includes the following activities: Washing clothes, hanging washing out to dry, bringing it in, Ironing clothes, Making, changing beds, Making, changing beds, Dusting, hovering, vacuum cleaning, general tidying, Outdoor cleaning, Other manual domestic work, Housework elsewhere unspecified, Putting shopping away.

2) AV7+AV11, where AV11 includes the following activities: Feeding and food preparation for babies and children, Washing, changing babies and children, Putting children and babies to bed or getting them up, Babysitting (i.e. other people’s children), Other care of babies, Medical care of babies and children, Reading to, or playing with babies and children, Helping children with homework, Supervising children, Care of children and babies elsewhere unspecified.
Appendix 3: Log-linearization of household’s time allocation decision

To log-linearize a function \( f(X) \), with \( X > 0 \), around a reference value \( \bar{X} \), recall that

\[
f(X) \approx f(\bar{X})[1+\eta \cdot x],
\]

where \( x \equiv \ln X - \ln \bar{X} \) and \( \eta \equiv [(\partial f(\bar{X})/\partial X) \cdot (\bar{X}/f(\bar{X}))]. \) Let us consider the inverse of (16):

\[
\left( \frac{s}{1-s} \right)^2 = \frac{1-a_f}{1-a_m}, \tag{A.1}
\]

where \( a_i = W_i^2/\varepsilon, (i = f, m). \) Then, using the previous result, log-linearization of (A.1) around the reference values \((s^*/1-s^*)\) and \(a_i^*\) yields

\[
\ln \left( \frac{s}{1-s} \right) = 0.5 \left( \frac{a_m^* - a_f^*}{1-a_m^*} \right) (\ln a_m - \ln a_f) + \frac{a_m^* - a_f^*}{(1-a_m^*)(1-a_f^*)} \ln a_f \tag{A.2}
\]

Since \( \ln a_i = 2 \ln W_i - \ln \varepsilon \), we get equation (30) in the main text where

\[
\theta_1 = \frac{a_m^*}{(1-a_m^*)} > 0, \quad \theta_2 = \frac{(a_m^* - a_f^*)}{(1-a_f^*)(1-a_m^*)} > 0, \quad \text{and} \quad \theta_3 = -0.5 \theta_2 < 0.
\]

Under Assumption 1 (i.e., \( a_i^* < 1 \)), and with \( s \in S \) so that \( a_m^* > a_f^* \), it can be easily checked that \( \theta_1 > \theta_2 > 0 \) since \( \theta_1/\theta_2 = [(a_m^* - a_m^*a_f^*)/(a_m^* - a_f^*)] > 1. \) Further, since \( \theta_2 \) is proportional to \( (a_m^* - a_f^*) \), it should be smaller for countries with gender gaps closer to the ungendered equilibrium, where \( a_m^* = a_f^* \). Using (E.1) and (E.2), to replace wages in (16) in terms of \( \ln \varepsilon \) and \( \ln \beta \), a similar procedure can be used to obtain equation (31).

Lastly, let us consider the case where the bargaining weights of the spouses in the household’s expected utility are different. For simplicity, let us assume that, instead of being 0.5 as in (14), the male and female weights now become \( 0.5(1+\eta) \) and \( 0.5(1-\eta) \), respectively, with \( 0 < \eta < 1 \). Calling \( \xi = (1+\eta)/(1-\eta) > 1 \), the corresponding f.o.c. becomes in this case:

\[
\left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-\xi (W_f^2/\varepsilon) a_m}{\xi - (W_f^2/\varepsilon)} \tag{A.3}
\]

which can be log-linearized to yield:

\[
\ln \left( \frac{s}{1-s} \right) = 0.5 \left( \frac{a_m^* \xi}{1-a_m^* \xi} \right) (\ln a_m - \ln a_f) + \frac{a_m^* \xi^2 - a_f^*}{(1-a_m^* \xi)(\xi - a_f^*)} \ln a_f \tag{A.3}
\]

It can be checked that, in contrast to the symmetric bargaining case, \( \eta = 0 \), the coefficient on the female wage need not be be smaller than the coefficient on the wage gap.
References


