On-the-Job Search in a Matching Model with Heterogeneous Jobs and Workers *

JUAN J. DOLADO[a], MARCEL JANSEN[b] AND JUAN F. JIMENO[c]

[a]Universidad Carlos III de Madrid, CEPR and IZA

[b]Universidad Carlos III de Madrid

[c]Banco de España, CEPR and IZA

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Abstract

This paper examines the effects of transitory skill mismatch in a matching model with heterogeneous jobs and workers. In our model, some high-educated workers may accept unskilled jobs for which they are over-qualified but are allowed to engage in on-the-job search in pursuit of a better job. We show that this feature has relevant implications for the set of potential equilibria, the unemployment rates of the different types of workers, the degree of wage inequality, and the response of the labour market to shifts in the demand and supply of skills.

Keywords: on-the-job search, skills, unemployment, wage inequality

JEL Codes: J1, J24, J41

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"An examination in Madrid to select 175 postal workers for this region yesterday gathered 15,570 candidates. Among them, 53% were college graduates ("licenciados" or "diplomados") while the required educational attainment was upper-secondary education ("graduado escolar") or an equivalent level of vocational training" (EL PAÍS, 23/03/2002).

1 Introduction

As the previous headline in a Spanish newspaper illustrates, mismatch between the skill requirements of jobs and the educational attainments of workers can be a pervasive feature in some labour markets. In this paper, we study this phenomenon in a matching model with heterogeneous jobs (skilled and unskilled) on one side and heterogeneous workers (high- and less-educated) on the other side of the market. Since the matching technology is imperfect, the high-educated workers may end up in unskilled jobs for which they are *over-qualified.* A key element in our analysis is that mismatched workers are allowed to keep the option of moving to better jobs through *on-the-job* search (henceforth, OTJ search). Skill mismatch has therefore a *transitory* nature in our economy leading to *job-to-job* (hereafter, JTJ) transitions which are shown to have relevant implications for the composition of jobs, unemployment, wages and the reaction of the labour market to shifts in the demand and supply of skills.

Labour economists have long recognised the importance of JTJ flows, but it is only recently that the literature on equilibrium unemployment has started to explore its implications systematically.[1] Our paper contributes to this stream of research by providing an analytical framework where to identify the channels through which over-qualification and OTJ search affect labour market outcomes. In particular, given long-standing concerns about the possibility that over-qualified workers may "crowd out" low-educated workers from unskilled jobs (see, e.g., Freeman, 1976 and OECD, 2001), we pay specific attention to the effects of these phenomena on the less-skilled segment

[1] Broadly speaking, the literature on OTJ search can be divided in two strands. One strand uses models in the vein of Burdett and Mortensen (1998) to explain how OTJ search may give rise to wage differentials among identical workers; see, e.g., Mortensen (2003) for an excellent overview of this literature. The second strand incorporates OTJ search in the standard matching framework to study its implications for the wage distribution, turnover and the cyclical dynamics of unemployment and vacancies; see, e.g., Pissarides (1994), Shimer (2003, 2006), Moscarini (2003) and Nagypál (2003).

of the labour market.

The starting point of our analysis is the matching model proposed by Albrecht and Vroman (2002) (henceforth, AV) who explore the consequences of skill mismatch in a setup similar to ours but where OTJ search is precluded. Like AV, we assume that firms have a choice between two types of jobs. The skilled jobs are more productive than the unskilled jobs but they require a high-educated worker, while the unskilled jobs can be performed *equally* well by all workers.[2] In the absence of OTJ search, AV show that there could be two types of equilibria: one in which high-educated workers match with both types of jobs (a *cross-skill matching* equilibrium) and another in which they refuse to take unskilled jobs (an *ex-post segmentation* equilibrium). The latter type is more likely: (i) the larger is the gap between the productivity of skilled and unskilled jobs and (ii) the higher is the share of high-educated workers in the population. Their findings suggest that shifts in the skill distribution and in the relative productivity of jobs may cause abrupt changes in unemployment rates and wages as the economy moves between the two equilibria (see also Acemoglu, 1999).

Aside from creating a more realistic model, our main goal is to analyse how the above predictions change when the option of OTJ search is taken into account. To keep the model tractable, we assume that this search is a *costless* activity for the workers. Nonetheless, the high-educated workers who meet an unskilled job may face an *opportunity cost*, since the arrival rate of future job offers drops when the match is accepted. In particular, we assume that the ratio between the arrival rates of offers to employed and unemployed job seekers takes a value between 0 and 1. In this fashion, our model nests both AV's setup (a zero arrival rate for employed job seekers) and the case in which all job seekers face equal contact rates. The remaining assumptions are borrowed from AV, including a random matching technology and the assumption that workers obtain a fixed share of the flow surplus of a match.[3] This last assumption implies that the pursuit of a better match is the only motive for OTJ search in our economy, ruling out the possibility of wage differentials between identical workers on the same type of jobs.

We obtain three main results that affect AV's conclusions. Our first result shows

[2]For simplicity, workers' skills are assumed to be perfectly correlated with their educational attaintments.

[3]For more details on the role of this assumption, see section 3.3.

that the introduction of OTJ search reduces the scope for multiple equilibria since it lowers the opportunity cost of mismatch for the high-educated workers. In fact, for a sufficiently small gap in the above-mentioned arrival rates, there is a unique type of equilibrium with cross-skill matching and OTJ search. This result rules out the drastic response to demand and supply shifts that characterises skill-mismatch models without OTJ search.

Our second result shows that transitory skill mismatch by over-qualified workers is more harmful to the prospects of less-educated workers than permanent mismatch. Here there are two channels at work. On the one hand, the introduction of OTJ search stimulates the creation of skilled jobs since the assumption that mismatched workers stay in the pool of job seekers facilitates filling these jobs. On the other hand, the quits by over-qualified workers reduce the stability of unskilled jobs, lowering the profitability of firms offering these jobs These effects result in a shift of the job distribution towards skilled jobs which, in line with the empirical evidence, induces both a fall in the job finding rate of the less-educated workers and a relatively high separation rate for unskilled jobs.

Third, we show that OTJ search widens the wage differences among the high-educated workers. As mentioned above, it reduces their opportunity cost of accepting unskilled jobs but, in return, they may suffer a pay cut. In fact, when the gap between the arrival rate of job offers for employed and unemployed job seekers is not too large, we show that mismatched workers receive a lower wage than less-educated workers. This result differs sharply from the one holding without OTJ search, where over-qualified workers get paid a higher wage, since they need to sacrifice all their outside options.

Our simulations suggest that the above effects may have important implications for the overall degree of wage inequality. For plausible parameter values we find that the total variance of the wage distribution is much larger than when JTJ transitions are ignored. Moreover, a significant fraction of this additional wage dispersion is due to wage differentials among high-educated workers. Thus, it seems worthwhile to explore whether an increase in the frequency of JTJ transitions may have contributed to the widening of the within-education and within-occupation wage dispersions that is observed notably in the U.S., but also in many other OECD countries since the 1980s (see, e.g., Katz and Autor, 1999). Moreover, our results suggest that the introduction of JTJ transitions between different jobs or occupations may help to improve the poor performance of standard search and matching models in replicating the observed variability of wages (see, Hornstein et al., 2006).

Finally, we briefly outline the connections with two related studies which also address the issue of OTJ search. Pissarides (1994) proposes the same wage-setting mechanism we use here, but in a model with homogeneous workers and heterogeneous jobs. By assuming that match productivity is growing over time (due to *learning by doing*), he is able to construct a model in which JTJ transitions only take place at short job tenures. Workers can therefore get locked into bad jobs. We ignore these tenure effects on search intensity to focus on the case where OTJ search depends on match quality and the educational attainment of the worker. In accordance with the empirical evidence (see Section 2), this generates a model in which quits are more prominent among over-qualified workers and affect the labour market position of the less-educated workers. By considering a homogeneous pool of workers, these aspects are absent in Pissarides' analysis. Closer in spirit to our work is Gautier (2002). He uses essentially the same setup for production like us. However, by construction, the wages in his model are independent of the aggregate labour market outcomes. This simplifies considerably the analysis, but it leaves out many interesting issues such as the relationship between the frequency of JTJ transitions and the degree of wage inequality that are addressed in our paper.

The rest of the paper is organised as follows. Section 2 reports some empirical evidence to motivate the analysis. Section 3 lays out the model. Section 4 describes the potential set of equilibria while Section 5 shows that the introduction of OTJ search enhances the likelihood of having a unique cross-skill matching equilibrium. Section 6 examines the comparative statics of the model in response to demand and supply shifts for the special case of equal contact rates. Section 7 presents some numerical results on the effect of these changes in a calibrated economy with realistic parameter values. Finally, Section 8 concludes. Proofs of the main propositions are gathered in two Appendices (A and B).

2 A Brief Look at the Evidence on JTJ Flows

Recent evidence suggests that JTJ transitions are an important element of the total labour market turnover. For example, regarding the U.S. labour market, Fallick and

Fleischman (2001) report that more than four million workers changed employer during an average month in the 1990s, about the same number as the workers who left the labour force from employment and more than twice the number who moved from employment to unemployment. Further, the total separation rate falls with age and is negatively correlated with the educational attainment of workers, but in relative terms the JTJ transitions account for a much larger share of total separations for college workers (50%) than for high-school dropouts (30%). Similar evidence has been provided by Nagypál (2003) who reports that around 55% of the total separations of workers with a college degree are due to JTJ flows, whereas that proportion falls to 34% for workers without a high school degree. Moreover, about 70% of the high-educated workers who undertake a JTJ transition do so for job-related reasons - as opposed to personal quits, layoffs or end of contract - whereas that fraction is below 60% for less-educated workers. Although the categorization of quits versus layoffs could be questioned - since it relies on the subjective self-report of the worker - the above evidence seems to point out that JTJ flows are a key feature in explaining separations and that OTJ search is more prevalent among high-educated workers.

Similar findings hold for Europe. For example, JTJ flows in the UK accounted for at least 40% of all separations in the 1980s (see, Pissarides, 1994). Elsewhere in Europe these flows appear to be less frequent, but in relative terms the turnover pattern is similar to that in the U.S. For example, Bachman (2006) estimates that JTJ transitions represented on average around 35 % of the monthly separation flows in Germany during 1980-2000, and that this proportion reaches 52% among high-educated workers. In line with these findings, Theodossiu and Zangadelis (2007) report cross-country evidence on JTJ transitions during the 1990s for six of the main EU economies. Although their estimates are likely to be upward biased relative to those quoted before due to their lower frequency - the estimates are based on year-to-year turnover rates in the eight available waves (1994-2001) of the European Community Household Panel (ECHP)- they report that JTJ transitions account for between 40% to 55 % of total separations in the different EU countries. Moreover, from a logit regression on the determinants of JTJ transitions, they also find that the probability of engaging in these transitions increases with workers' educational attainment.

Finally, since the focus of our study is on JTJ transitions by over-qualified workers, it is interesting to report some recent evidence provided by Eurostat (2003) about the gap in percentage points (p.p.) between the shares of workers who declare to be performing OTJ search in two groups of college graduates: (i) those who declare to be over-qualified for their current jobs, and (ii) those who declare to be appropriately matched.[4] These differentials are reported for a number of EU countries and range from 1.5 p.p. in Denmark and Finland to about 7-8 p.p. in Italy, Portugal and Spain. Thus, this evidence seemingly confirms that the pursuit of a better job is a relevant determinant of OTJ search among high-educated workers who feel over-qualified.

3 The Model

This section introduces our matching model with heterogeneous agents and OTJ search. Time is continuous and we restrict attention to steady states.

3.1 Main assumptions

The economy is populated by a continuum of heterogeneous workers with measure normalized to one and a large continuum of identical firms. All agents are risk-neutral and infinitely-lived and discount the future at the common rate r.

Production of the unique final good requires a job and a worker. We use the index $j \in \{h, l\}$ to distinguish the two types of workers in our economy: high-educated (h) and less-educated (l) workers. The fraction of less-educated workers in the population workers is denoted by $\mu \in (0, 1)$ which is assumed to be exogenously determined in our model. Likewise, there are two types of jobs that can be either filled or vacant. They are indexed by $i \in \{s, n\}$ and it is assumed that each firm can have at most one job. An unskilled job (n) can be filled by either type of worker and produces a constant flow of $y(n)$ units of output. Thus, the productivity of these jobs does not depend on the type of worker. By contrast, skilled jobs (s) can only be filled by high-educated workers, whose productivity in these jobs, $y(s)$, is larger than $y(n)$. This technology is summarized in Table 1.

[Insert Table 1]

[4] The data come from an ad hoc module carried out by Eurostat in the 2000 EU Labour Force Survey designed to collect specific information on the transition from the education system to working life in EU countries.

Our assumptions imply that high-educated workers are more productive than less-educated workers when they manage to find a skilled job. However, in a market subject to search frictions, high-educated workers may find it optimal to accept both types of jobs in equilibrium. When this occurs, using AV's terminology, we say that the equilibrium exhibits *cross-skill matching*. Conversely, when they refuse unskilled jobs, the equilibrium exhibits *ex-post segmentation*.

Finally, in our economy, the turnover of workers is partly endogeneous. A match may be dissolved when: (i) the worker decides to quit because a better job has been located, or (ii) the job is destroyed by a shock. This second source of turnover is exogenous and follows a Poisson process with arrival rate δ. The unemployed workers receive a flow payoff b from home production and leisure that satisfies the restriction that $b < y(n)$, while the firms with a vacant job incur a flow cost c until the job is filled. Since we assume free entry, firms will exhaust the rents from job creation in equilibrium.

3.2 Matching

Job seekers and firms with vacant jobs are matched together in pairs through an imperfect matching technology. Like AV, we assume that the matching process is *undirected.* However, by contrast, we allow for OTJ search by mismatched workers, while they only allow for job search during unemployment. A mismatched worker can therefore move to a better job without an intervening spell of unemployment. Below we show that this feature reduces the opportunity cost of mismatch for high-educated workers.

The total flow of random contacts between a job seeker and a firm is determined by a standard CRS meeting function:

$$m[v(n) + v(s), u(l) + u(h) + \lambda e],$$

where $u(j)$ is the mass of unemployed workers of type j , $v(i)$ is the mass of vacancies of type i, and e is the mass of employed job seekers whose relative search intensity is captured by the parameter $\lambda \in [0, 1]$.[5] The case where $\lambda = 0$ replicates AV's setup

[5]The restriction to the unit interval is natural. It implies that mismatch tends to have a cost because employed workers have a (weakly) lower contact rate than unemployed workers. In fact, in a model with endogeneous search effort, the workers would never choose a value for $\lambda > 1$ if the search costs during employment are at least as high as during unemployment.

while $\lambda = 1$ corresponds to the case in which the arrival rate of job offers is independent of the employment status of the job seeker. Finally, we assume that $m[.,.]$ is strictly increasing in both arguments and we define the effective labour market tightness by $\theta = [v(n) + v(s)]/[u(l) + u(h) + \lambda e]$. Accordingly, we can write the contact rate of a firm as $p(\theta) = m(1, \frac{1}{\theta})$, while the contact rate of a job seeker is equal to $f(\theta) = \theta p(\theta)$ during unemployment and $\lambda f(\theta) \leq f(\theta)$ during employment. The properties of $m[.,.]$ guarantee that $p'(\theta) < 0$, $f'(\theta) > 0$ and we assume that $\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} f(\theta) = \infty$ and $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} f(\theta) = 0$.

3.3 Wage determination

As mentioned earlier, our analysis focuses on one important aspect of OTJ search, namely the pursuit of a better match. However, since the work of Burdett and Mortensen (1998) it is well known that workers may use OTJ search to obtain a higher wage in the same type of job. This observation has stimulated a lot of research on the impact of OTJ search on the process of wage determination. For example, Postel-Vinay and Robin (2002) and Cahuc at al. (2006) consider extensions of the Burdett-Mortensen model in which employers are allowed to match the offer of a rival employer. In both instances, workers can exploit the outside offers from rival employers to obtain a pay rise in their current jobs. Alternatively, Shimer (2006) reconsiders the arguments of Burdett and Mortensen in a standard matching model with OTJ search and bilateral bargaining. He shows that the standard surplus-sharing rule may not be optimal in this environment because firms may find it profitable to pay a higher wage in order to reduce the probability of a quit.

Allowing for these features would complicate our model a lot. Hence, to avoid these complications, we follow Pissarides (1994) in adopting two strong simplifying assumptions regarding wage determination. The first one is that wages are set according to a linear surplus-splitting rule that entitles workers to a fraction $\beta \in (0, 1)$ of the flow rents, whereas the second one is that the wage can be revised continuously at no cost, so that long-term contracts are ruled out. Thus, even if an employed worker could start negotiations with a new employer before resigning from the current job, this would not affect the equilibrium outcome. The new employer would immediately renegotiate the wage once the worker breaks the relationship with the previous employer.[6]

[6] Notice that our assumptions also eliminate the scope for equilibria with matching wage offers. For

These assumptions lead to a wage-setting rule that looks identical to the typical Nash bargaining solution in models without OTJ search. Formally, let $U(j)$ denote the value of unemployment for a worker of type j, and $V(i)$ the value of a vacant job of type i. Similarly, let $J(i,j)$ and $W(i,j)$ denote the proceeds for the firm and the worker from a match that combines a job of type i and a worker of type j. In any match with a positive match surplus $S(i,j) \equiv W(i,j) + J(i,j) - V(i) - U(j)$, the constant wage $w(i,j)$ then satisfies the following sharing rule:

$$(1-\beta)[W(i,j) - U(j)] = \beta[J(i,j) - V(i)]. \tag{1}$$

Finally, in the rest of the analysis we assume that workers only quit their current employer if they find a better-paid job. This assumption limits the scope of JTJ transitions to the mass $e(n,h)$ of high-educated workers in unskilled jobs since these workers will leave their employer once they locate a skilled job. By contrast, appropriately matched workers will never quit because the alternative jobs would pay at most the same wage. For this reason, it is considered in the sequel that $e = e(n,h)$.[7]

3.4 Asset values

We are now in a position to define the asset value equations of workers and firms.

Let $\zeta = v(n)/[v(n) + v(s)]$ denote the share of unskilled vacancies. Then the asset value of a high-educated worker during unemployment, $U(h)$, satisfies:

$$rU(h) = b + f(\theta)\left[\zeta \max(W(n,h) - U(h), 0) + (1-\zeta)(W(s,h) - U(h))\right]. \tag{2}$$

The high-educated job seeker will accept to work in an unskilled job if this improves

example, firms with unskilled jobs could try to match the rival offers from firms with skilled jobs if $w(s,h) < y(n)$. However, the mismatched workers will reject these matching offers. They realise that the employer will renegotiate the wage back to its initial level once the worker has declined the rival offer. Thus, in equilibrium the mismatched workers will always accept rival offers from firms with skilled job as long as $w(s,h) > w(n,h)$. Below we will show that this condition always holds in our model.

[7]Alternatively, we could have assumed that the workers who engage in OTJ search incur an infinitesimally small cost $\varepsilon \simeq 0$. In this setting, the appropriately matched workers refrain from search because it reduces their lifetime income by ε, while the mismatched workers continue to search because the search costs are negligible compared to the productivity gain, $y(s) - y(n)$.

her lifetime income. The associated payoff in this case, denoted as $W(n,h)$, is given by:

$$rW(n,h) = w(n,h) + \delta[U(h) - W(n,h)] + \lambda f(\theta)(1-\zeta)[W(s,h) - W(n,h)], \tag{3}$$

where the last term on the right-hand side of (3) corresponds to the expected gain from successful OTJ search, which depends on the search-intensity parameter λ.

The rest of the asset value equations of workers are standard (see Pissarides, 2000):

$$rU(l) = b + f(\theta)\zeta[W(n,l) - U(l)] \tag{4}$$

$$rW(n,l) = w(n,l) + \delta[U(l) - W(n,l)] \tag{5}$$

$$rW(s,h) = w(s,h) + \delta[U(h) - W(s,h)]. \tag{6}$$

Next, to define the asset value equations of vacant jobs, we denote the share of unemployed job seekers by $\psi = [u(l)+u(h)]/[u(l)+u(h)+\lambda e(n,h)]$. Similarly, we let $\phi = u(l)/[u(l)+u(h)]$ denote the share of less-educated workers in the pool of unemployed. Accordingly, we can write the asset value equation for an unskilled vacancy, $V(n)$, as :

$$rV(n) = -c + \psi p(\theta)\left[\phi(J(n,l) - V(n)) + (1-\phi)\max(J(n,h) - V(n), 0)\right], \tag{7}$$

while the corresponding expression for a skilled vacancy, $V(s)$, satisfies:

$$rV(s) = -c + (1-\psi\phi)p(\theta)[J(s,h) - V(s)]. \tag{8}$$

Finally, the asset values of filled jobs verify:

$$rJ(s,h) = y(s) - w(s,h) + \delta[V(s) - J(s,h)] \tag{9}$$

$$rJ(n,l) = y(n) - w(n,l) + \delta[V(n) - J(n,l)] \tag{10}$$

$$rJ(n,h) = y(n) - w(n,h) + (\delta + \lambda f(\theta)(1-\zeta))[V(n) - J(n,h)]. \tag{11}$$

The fact that the separation rate in (11) is larger than δ just reflects that mismatched workers will leave their employer when they find a skilled job. In the next sections, we will analyze how this feature affects the decisions of firms and workers in equilibrium.

4 Equilibria

In this section we proceed to define the set of equilibria. Since we are primarily interested in the equilibria with cross-skill matching and OTJ search, we shall initially assume that the surplus of filling an unskilled job with a high-educated worker is non-negative, namely $S(n, h) \geq 0$. The alternative case of ex-post segmentation, where $S(n, h) < 0$, will be discussed at the end of this section.

It is useful to start with the derivation of the equilibrium surplus expressions. From (5), (10) and the free-entry condition for unskilled jobs, $V(n) = 0$, it follows that $S(n, l)$ satisfies:

$$(r + \delta)S(n, l) = y(n) - rU(l). \tag{12}$$

Together with (1), this implies that the wage of less-educated workers, $w(n, l)$, is given by:

$$w(n, l) = rU(l) + \beta[y(n) - rU(l)]. \tag{13}$$

Likewise, regarding high-educated workers in skilled jobs, the corresponding expressions are:

$$(r + \delta)S(s, h) = y(s) - rU(h) \tag{14}$$

$$w(s, h) = rU(h) + \beta[y(s) - rU(h)], \tag{15}$$

where we have used (1), (6), (9) plus the free-entry condition for skilled jobs, $V(s) = 0$. The above solutions for the appropriately matched workers are standard, while we obtain the following less conventional solutions for over-qualified workers:[8]

[8] As mentioned earlier, since OTJ search is costless, we only have to analyze the matching decisions of the high-educated workers.

$$[r+\delta+\lambda f(\theta)(1-\zeta)]S(n,h)=y(n)-rU(h)+f(\theta)\lambda(1-\zeta)\beta S(s,h), \tag{16}$$

$$w(n,h)=rU(h)+\beta[y(n)-rU(h)]-f(\theta)\lambda(1-\zeta)\beta(1-\beta)S(s,h). \tag{17}$$

Comparison of (16) with (12) reveals two important differences. First, the output generated by a mismatched worker is discounted at a higher rate than the output of a low-educated worker. Second, the value of $S(n,h)$ includes the *expected* gains from OTJ search which amount to $f(\theta)\lambda\beta(1-\zeta)S(s,h)$. Since the *actual* gains from OTJ search will accrue to the worker and not to the firm, mismatched workers compensate their employers by accepting a wage reduction given by $f(\theta)\lambda(1-\zeta)\beta(1-\beta)S(s,h)$.

Next, to obtain the reservation values of the two types of workers, we rewrite (2) and (4) as:

$$rU(l)=b+f(\theta)\zeta\beta S(n,l) \tag{18}$$

$$rU(h)=b+f(\theta)\beta\left[\zeta S(n,h)+(1-\zeta)S(s,h)\right]. \tag{19}$$

Using (12), (14) and (16) this yields the following expressions:

$$rU(l)=\frac{(r+\delta)b+f(\theta)\beta\zeta y(n)}{r+\delta+f(\theta)\beta\zeta} \tag{20}$$

$$rU(h)=\frac{(r+\delta)\alpha_1 b+f(\theta)\beta\left[\zeta(r+\delta)y(n)+(1-\zeta)\alpha_2 y(s)\right]}{\alpha_2\alpha_3+(r+\delta)f(\theta)\beta\zeta(1-\lambda)}, \tag{21}$$

where $\alpha_1=r+\delta+f(\theta)\lambda(1-\zeta)$, $\alpha_2=r+\delta+f(\theta)\lambda(1-\zeta+\beta\zeta)$ and $\alpha_3=r+\delta+f(\theta)(1-\zeta)\beta$ are discount factors.

Finally, inserting the previous expressions for $S(i,j)$ and $rU(j)$ plus the wage rule (1) into both (7) and (8), we can write the two zero-profit conditions $V(n)=0$ and $V(s)=0$, respectively, as:

$$\frac{c}{(1-\beta)p(\theta)\psi}=\left[\frac{\phi[y(n)-b]}{r+\delta+f(\theta)\zeta\beta}+\frac{(1-\phi)\{\alpha_3\left[y(n)-b\right]-f(\theta)\beta(1-\lambda)(1-\zeta)\left[y(s)-b\right]\}}{\alpha_2\alpha_3+(r+\delta)f(\theta)\beta\zeta(1-\lambda)}\right], \tag{22}$$

$$\frac{c}{(1-\beta)p(\theta)(1-\psi\phi)}=\left[\frac{\alpha_1\left[y(s)-b\right]+f(\theta)\beta\zeta[y(s)-y(n)]}{\alpha_2\alpha_3+(r+\delta)f(\theta)\beta\zeta(1-\lambda)}\right]. \tag{23}$$

Equations (22) and (23) constitute the first two equilibrium relationships of the model. The remaining ones arise from to the steady state flow conditions for $u(l)$, $u(h)$ and $e(n,h)$. Denoting the total mass of unemployed workers by $u \equiv u(h) + u(l)$, we can express these conditions as follows:

$$\zeta f(\theta)\phi u = \delta(\mu - \phi u) \tag{24}$$

$$f(\theta)(1-\phi)u = \delta[1 - \mu - (1-\phi)u] \tag{25}$$

$$\zeta f(\theta)(1-\phi)u = [\delta + f(\theta)\lambda(1-\zeta)]e(n,h). \tag{26}$$

All together, these five conditions lead to the following definition of a cross-skill matching equilibrium when OTJ search is present:

Definition 1 *A steady-state equilibrium with cross-skill matching and OTJ search consists of a set of value functions for $W(i,j), J(i,j), V(i), U(j)$ and $S(i,j)$ that satisfy (2)-(11), (12), (14) and (16) plus a vector $\{u,\theta,\phi,\zeta,\psi\}$ such that*

1. *All matches produce a non-negative surplus for the equilibrium values of $\{\theta,\zeta\}$.*
2. *The vector $\{u,\theta,\phi,\zeta,\psi\}$ solves the free entry conditions (22) and (23) plus the steady state conditions (24) to (26).*

Our last task in this section is to describe the necessary conditions that define an equilibrium with ex-post segmentation. As mentioned earlier, this type of equilibrium arises in AV's model ($\lambda = 0$) when high-educated workers make up a relatively large share of the population and/or when the productivity gap between jobs is relatively large. In the next section, we will show that these conditions still hold in our model with OTJ search when λ is relatively small but positive.

Formally, when high-educated workers refuse to work in unskilled jobs ($S(n,h) < 0$), the solution for $rU(h)$ in (21) reduces to:

$$rU(h) = \frac{(r+\delta)b + f(\theta)(1-\zeta)\beta y(s)}{r + \delta + f(\theta)(1-\zeta)\beta},$$

while the solution for $rU(l)$ still satisfies (20). Replacing these solutions into (12) and (14), we find that:

$$S(n,l) = \frac{y(n) - b}{r + \delta + f(\theta)\zeta\beta}; \qquad S(s,h) = \frac{y(s) - b}{r + \delta + f(\theta)(1-\zeta)\beta}$$

As a result, an ex-post segmentation equilibrium can be defined as follows:

Definition 2 *A steady state equilibrium with ex-post segmentation can be summarized by a vector* $\{\theta, \phi, \zeta, u\}$ *that generates an asset value for high-educated workers* $rU(h) > y(n)$ *and solves the four equilibrium conditions:*

$$\frac{c}{(1-\beta)p(\theta)} = \phi \frac{[y(n) - b]}{r + \delta + f(\theta)\beta\zeta} \tag{27}$$

$$\frac{c}{(1-\beta)p(\theta)} = (1-\phi)\frac{[y(s) - b]}{r + \delta + f(\theta)\beta(1-\zeta)} \tag{28}$$

$$f(\theta)\zeta\phi u = \delta(\mu - \phi u) \tag{29}$$

$$f(\theta)(1-\zeta)(1-\phi)u = \delta(1 - \mu - (1-\phi)u) \tag{30}$$

Obviously, JTJ transitions are precluded in this segregated equilibrium since the mass of mismatched workers is equal to zero.

5 Equilibrium configurations

In this section we provide a complete characterization of the possible equilibrium configurations. Our goal is to prove that the introduction of OTJ search ($0 < \lambda \leq 1$) enhances the likelihood of having an equilibrium with cross-skill matching. To do so, it is useful to recall that the existence of this equilibrium is guaranteed under two conditions: (i) firms must be willing to provide both types of jobs, and (ii) the high-educated workers must be willing to accept employment in unskilled job. To guarantee condition (i), it is sufficient to rule out the corner solution in which firms exclusively offer unskilled jobs. The following result shows that this requirement places a lower-bound on the share of high-educated workers and on the productivity differential between skilled and unskilled jobs:

Proposition 1. *A sufficient condition for firms to offer both skilled and unskilled jobs is that*

$$\frac{y(s)-y(n)}{y(n)-b} > \frac{\mu(r+\delta)}{(1-\mu)[r+\delta+f(\theta^*)\beta]} \tag{31}$$

where θ^* is the labour market tightness associated with a single job-type distribution.
Proof. See Appendix A. ■

Next, in order to guarantee condition (ii), we need to ensure that high-educated workers and firms with unskilled jobs are willing to match, i.e., $S(n,h) > 0$. In Appendix A we show that this leads to the following condition:

Proposition 2. *A necessary condition for a cross-skill matching equilibrium to exist is that,*

$$\frac{y(s)-y(n)}{y(n)-b} < \frac{r+\delta+f(\theta)\beta\lambda}{f(\theta)\beta(1-\lambda)(1-\zeta)} \tag{32}$$

for the equilibrium values of θ and ζ in this type of equilibrium,

In general, condition (32) can only be verified *a fortiori* once the equilibrium values of θ and ζ have been determined. However, there is an exception. Since the right-hand side of (32) approaches infinity as λ tends to 1, the above inequality is always verified in an economy where employed and unemployed job seekers face equal contact rates, i.e., $\lambda = 1$. Hence, the following result holds:

Corollary 1. *With the same search intensity for all job seekers ($\lambda = 1$), there always exists a cross-matching equilibrium when condition (31) is satisfied.*

To gain some intuition for this benchmark result, it is useful to consider the expression for $S(n,h)$ that is obtained after replacing $U(h)$ in (16) by (19):

$$S(n,h) = \frac{y(n)-[b+f(\theta)(1-\lambda)(1-\zeta)\beta S(s,h)]}{r+\delta+f(\theta)[\lambda(1-\zeta+\beta\zeta]}. \tag{33}$$

From the numerator of (33) it follows that $S(n,h) > 0$ when $y(n)$ is larger than the bracketed term which measures the opportunity cost of a high-educated worker accepting an unskilled job. When OTJ search is ruled out ($\lambda = 0$), this opportunity cost is simply the expected return of a high-educated worker under ex-post segmentation. The effect of allowing for OTJ search is to reduce this opportunity cost. In fact, when $\lambda = 1$, the only component of the opportunity cost is b since mismatched and unemployed job seekers face the same contact rates. Given our assumption that $y(n) > b$, this immediately implies that $S(n, h)$ is positive.

Proposition 2 provided the necessary condition for the existence of a cross-skill matching equilibrium when firms offer both types of jobs. The next proposition goes one step further by providing a sufficient condition in terms of λ that rules out ex-post segmentation. The proof is again based on the idea that a rise in λ lowers the opportunity cost of mismatch for high-educated workers:

Proposition 3. *For any economy that satisfies (31) there exists a value* $\overline{\lambda} \in [0, 1)$ *such that the equilibrium always exhibits cross-skill matching for any* $\lambda \in (\overline{\lambda}, 1]$.

Proof. See Appendix A. ■

The threshold value $\overline{\lambda}$ is defined the lowest value of λ at which a high-educated worker and a firm with an unskilled job can deviate from an ex-post segmentation equilibrium without incurring a loss. Thus, the equilibrium *always* exhibits cross-skill matching for $\lambda > \overline{\lambda}$.

Notice, however, that the above argument does not rule out the existence of a cross-skill matching equilibrium at lower values than $\overline{\lambda}$. By definition, an *individual* firm-worker pair will incur a loss if they deviate from an ex-post segmentation equilibrium for any $\lambda < \overline{\lambda}$. Yet, if *all* high-educated workers would collectively start to accept unskilled jobs, firms would react by increasing the proportion of unskilled jobs in the economy. This shift in the job distribution would make skilled jobs more scarce and, hence, for the same value of λ, all high-educated workers may now find it optimal to accept unskilled jobs. Thus, for some intermediate values of λ the equilibrium may exhibit either cross-skill matching or ex-post segmentation. For future purposes, we denote the lower-bound of this interval as $\underline{\lambda}$ so that multiple equilibria are possible in the range $[\underline{\lambda}, \overline{\lambda}]$.

Finally, using similar arguments, it is easy to prove that the introduction of OTJ search never leads to the destruction of a cross-skill matching equilibrium:

Proposition 4. *Consider an economy that generates a cross-skill matching equilibrium when* $\lambda = 0$. *The same economy will have a cross-skill matching equilibrium with OTJ search for any* $\lambda > 0$.

Proof. See Appendix A. ■

Summing up, the results in Propositions (1) to (4) imply that OTJ search unambiguously narrows the scope for equilibria with ex-post segmentation, leading to the following two alternative equilibrium configurations:

1. The economy always exhibits cross-skill matching.

2. The economy exhibits cross-skill matching for any $\lambda > \overline{\lambda}$ and ex-post segmentation for $\lambda < \underline{\lambda}$, while it may exhibit both types of equilibria for the intermediate range of values $\lambda \in [\underline{\lambda}, \overline{\lambda}]$.

Finally, from the discussion in Appendix A, it becomes clear that the values of the above-mentioned thresholds $\underline{\lambda}$ and $\overline{\lambda}$ depend positively on the share of skilled jobs, $1-\zeta$, and on the value of $S(s,h)$. Hence, in line with AV, we find that the likelihood of having an ex-post segmentation equilibrium increases both with the productivity gap between skilled and unskilled jobs and the share of high-educated workers.

6 Equal contact rates

Our previous analysis has shown that the introduction of OTJ search tends to narrow the set of equilibria. The aim of this section is to show that it also has interesting implications for the distribution of wages and the response of the economy to shocks. To illustrate these effects, we restrict the analysis to an economy with equal contact rates ($\lambda = 1$), though our previous arguments imply that the results below also hold for values of λ sufficiently close to unity (see Section 7.3).

6.1 Wages

Our first objective is to show that the introduction of OTJ search raises the degree of wage inequality in the economy. To derive the distribution of wages in an economy with $\lambda = 1$, it is useful to start from the following Lemma:

Lemma 1. *In any economy with equal contact rates that satisfies (31),* $0 < S(n,h) < S(n,l)$.

Proof. See Appendix A. ■

The insight for this result (see equation A. 14 in Appendix A) is that mismatched workers produce the same increment in output as less-educated workers do, namely,

$y(n) - b$. Yet, for the former, this "flow surplus" is discounted at a higher rate due to the possibility of a quit. Hence, in principle, firms would prefer to hire a more stable less-educated worker. Given our surplus-sharing rule, the mismatched workers therefore have to accept a lower wage than the less-educated workers, as shown in the next result:

Proposition 5. *In any economy with equal contact rates that satisfies (31),* $w(n,h) < w(n,l) < w(s,h)$.

Proof. See Appendix A. ■

The above result differs sharply from the one obtained with $\lambda = 0$. In the latter case, the over-qualified workers receive a higher wage than the less-educated workers since they have to sacrifice their entire reservation wage, $rU(h)$, when they accept an unskilled job. When $\lambda = 1$, by contrast, mismatched workers only need to receive compensation for $rU(h) - f(\theta)(1-\zeta)\beta S(s,h) = b + f(\theta)\zeta\beta S(n,h)$. From (18), this value is smaller than $rU(l)$ implying that mismatched workers receive a lower wage than the less-educated workers. Thus, the fact that OTJ search reduces the opportunity cost of mismatched workers also shows up in the bargained wages. This interesting source of within-group wage inequality is ignored in conventional matching models. Furthermore, for future purposes, it is important to notice that $w(n,h)$ does not depend directly on $y(s)$. Below we show that this feature has important implications for the response of the labour market to shifts in the relative productivity of skilled jobs. However, before examining these comparative statics results, we need to establish the conditions that guarantee uniqueness of the equilibrium.

6.2 Uniqueness

To obtain a set of conditions that rule out the possibility of multiple cross-skill matching equilibria, we solve the flow conditions (24) to (26) for u, ζ and ψ in terms of θ and ϕ. Substituting the resulting expressions into the two free-entry conditions (22) and (23) yields a system of two equations in two unknowns where conditions for uniqueness can be derived.

The only complication is that we have three types of job seekers. In effect, from the perspective of firms, a change in the value of θ induces two effects: a standard *congestion* effect, as $p'(\theta) < 0$, and a novel *composition* effect, as the fraction of unemployed less-educated workers in the mass of job seekers, ψ, tends to fall with higher values of θ. Thus, à *priori* it is unclear how a change in θ affects the matching rate of skilled jobs, $(1-\phi\psi)p(\theta)$. From the perspective of high-educated workers, a similar ambiguity arises since $f'(\theta)>0$ and the share of skilled jobs, $1-\zeta$, falls for higher values of θ. Nonetheless, for relatively large values of μ, it can be shown that these composition effects are small compared to the changes in $f(\theta)$ and $p(\theta)$.

Using this feature we are able to show that the equilibrium is unique if: (i) μ is at least 0.5, so that $\partial[p(\theta)(1-\theta\psi)]/\partial\theta<0$ and $\partial[f(\theta)(1-\zeta)]>0$,[9] (ii) workers obtain at least half the surplus of any match ($\beta \geq 0.5$), and (iii) the productivity differential between skilled and unskilled jobs, $y(s)-y(n)$, is sufficiently large (see Appendix B for details).

The unique equilibrium is illustrated in Figure 1. The free entry locus of unskilled jobs, $V(n)=0$, is upward sloping since firms with unskilled jobs prefer to hire less-educated workers. Thus, a rise in tightness, θ, needs to be compensated by a rise in the proportion of less-educated workers in the total mass of unemployed, ϕ. Conversely, the free entry locus for skilled jobs, $V(s)=0$, is downward sloping because these jobs can only be performed by high-educated workers so that a lower value of ϕ (a larger fraction of unemployed high-educated workers) is needed when θ increases. Therefore, both loci can cross at most once.

[Insert Figure 1]

6.3 Responses to shifts in demand and supply of skills

As shown above, the pool of job seekers contains a mass of mismatched workers who temporarily accept a job below their qualifications in return for a lower wage than equally productive less-educated workers. In this section we show how this feature alters the response of the labour market to a rise in the productivity of skilled jobs and/or the share of high-educated workers. Following the existing literature, we refer to these changes as *skill-biased technological change* (SBTC) and *skill upgrading* (SU), respectively, and throughout the analysis we assume that the conditions for uniqueness hold.

[9] In our numerical simulations these derivatives are always negative for any $\mu \geq 0.5$.

6.3.1 Skill-biased technological change

The effects of SBTC are illustrated in Figure 2. The increase in $y(s)$ raises the profits of skilled jobs while the profits of unskilled jobs are unaffected because $y(s)$ drops out of equation (22) when $\lambda = 1$. Hence, the main effect of SBTC is an increase in the mass of skilled vacancies, $v(s)$. As a result, the $V(s) = 0$ locus shifts upwards along the $V(n) = 0$ locus, leading to a rise of θ and ϕ.

[Insert Figure 2]

The increase in θ reduces the unemployment rate of high-educated workers -labeled by $\widetilde{u}(h)$ $(= u(h)/(1-\mu) = \delta/[\delta + f(\theta)])$- while we cannot draw definite conclusions about the unemployment rate of less-educated workers- denoted as $\widetilde{u}(l)$ $(= u(l)/\mu = \delta/[\delta + \zeta f(\theta)])$. The reason is that the rise in $v(s)$ causes a fall in the share of unskilled vacancies, ζ. These results can be summarized as follows:

Proposition 6. *In a unique cross-skill matching equilibrium with* $\lambda = 1$*, SBTC increases* θ *and* ϕ*, and reduces* $\widetilde{u}(h)$*, while its effect on* $\widetilde{u}(l)$ *is ambiguous.*

Once more, this comparative statics result differs from the one derived by AV. In effect, for the case of cross-skill matching, they show that SBTC raises $\widetilde{u}(l)$ while it has no effect on $\widetilde{u}(h)$. Hence, the JTJ flows of high-educated workers increase (reduce) the sensitivity of $\widetilde{u}(h)$ ($\widetilde{u}(l)$) to changes in $y(s)$.[10] The stronger response of $\widetilde{u}(h)$ is due to the fact that skilled jobs can attract both unemployed and employed job seekers. The supply of these jobs is therefore more elastic when $\lambda = 1$ than when $\lambda = 0$. Yet, at the same time, the JTJ flows also insulate the profits of unskilled jobs from the effects of SBTC because $w(n, h)$ and $V(n)$ do not directly depend on $y(s)$. Thus, under cross-skill matching the drop in unskilled vacancies turns out to be smaller than in a model without JTJ transitions. Moreover, as explained before, the JTJ transitions prevent a possible shift to ex-post segmentation that may be accompanied by a rise in the unemployment rates of both types of workers as shown in AV (see Section 7.2 for a numerical example).

[10] This result is unrelated to our assumption of a common value for b and c. In particular, we would obtain the same result if the unemployment income of high-educated workers and the flow cost of skilled vacancies are indexed to $y(s)$. In our model, technological change is only neutral when $y(s)$, $y(n)$, b and c all grow at the same rate. A shock to the relative productivity of workers can therefore move the equilibrium to a different balanced growth rate in which the unemployment rate of high-educated workers is permanently lower than before.

6.3.2 Skill-upgrading

An increase in the share of high-educated workers leads to a similar shift in the relative demand for workers as SBTC, because a rise in $(1-\mu)$ makes it easier for firms to fill a skilled job. In this case, however, it is *a priori* ambiguous how the unemployment rate of high-educated workers will respond since it depends on the relative size of the shifts in the demand and the supply of these workers.

The ambiguous response of the labour market to SU is illustrated in Figure 3. The reduction in μ shifts downwards both free-entry loci. Hence, we obtain a fall in ϕ, whereas θ, and therefore $\widetilde{u}(h)$ and $\widetilde{u}(h)$, may go up or down. In sum:

Proposition 7. *In a unique cross-skill matching equilibrium with $\lambda = 1$, SU increases ϕ while its effects on θ, $\widetilde{u}(l)$* and *$\widetilde{u}(h)$ are ambiguous.*

[Insert Figure 3]

Despite the ambiguity, it is evident that a shift in the skill distribution should provoke a stronger reaction of labour demand in our economy than in an economy without OTJ search since mismatched workers do not drop out of the pool of job seekers. Given AV's finding that $\partial\theta/\partial\mu = 0$ under cross-skill matching this suggests that SU may actually lead to a fall in the unemployment rate of the high-educated workers. In the next section we will present some simulations in which this is indeed the case.

7 Numerical solutions

In this section we report the results of some illustrative numerical simulations. Our aim is to gauge the quantitative importance of the JTJ flows and their impact on the distribution of jobs, wages and employment for different values of λ. Furthermore, at the end of this section, we discuss to what extent the model is able to explain some of the stylized facts regarding unemployment, wage inequality and JTJ flows in Europe and the U.S.

7.1 The benchmark economy

Following Petrongolo and Pissarides (2001), we assume a standard Cobb-Douglas meeting function with an elasticity of 0.5, i.e. $f(\theta) = \sqrt{\theta}$. Time is measured in quarters and the rest of the parameter values are given by $\beta = 0.5$, $r = 0.01$, $c = 0.5$, $\delta = 0.1$, $b = 0.1$, $y(s) = 1.5$, $\mu = 0.75$, plus a normalized value $y(n) = 1$. This parameter configuration is similar to the one used by AV.

[Insert Table 2]

The first column of Table 2 presents the labour market outcomes for our benchmark economy with OTJ search and $\lambda = 1$ while, for comparative purposes, the second column reports results for $\lambda = 0$. For the chosen parameters unique cross-skill matching equilibria are obtained in both instances. The first difference to highlight is that the proportion of skilled jobs, $1 - \zeta$, is much higher with OTJ search (33%) than without OTJ (11%). This is so since firms are more willing to open skilled jobs in an economy where mismatched workers remain in the pool of job seekers. Consequently, transitory mismatch yields a higher unemployment rate of less-educated workers (10.9%) than permanent mismatch (8.3%). At the equilibrium value of θ with $\lambda = 1$ ($\lambda = 0$), a less- educated worker exits unemployment at a rate $\zeta f(\theta) = 0.817$ (1.101) while a high-educated worker does so at a rate $f(\theta) = 1.219$ (1.236). Thus, with equal contact rates, the job finding rate is 50% higher for the latter type of workers whereas it is only 12% higher if OTJ search is ignored. Given a job destruction rate of 10% this leads to a 3.3 percentage-point higher unemployment rates for less-educated workers when $\lambda = 1$, as opposed to only 0.8 percentage points when $\lambda = 0$. Thus, OTJ search leads to predictions about the differential in unemployment rates by educational attainment that are more consistent with the available evidence in most OECD countries.

The second difference worth stressing is that the share of mismatched workers among the high-educated ones is much lower with $\lambda = 1$ ($0.156 = 0.039/0.25$) than with $\lambda = 0$ ($0.824 = 0.206/0.25$). This again adds further realism to our model. Interestingly, when $\lambda = 1$, JTJ transitions account for almost for 35% of all separations by high-educated workers, i.e., a proportion which is in line with those reported in Section 2. [11]

[11] The proportion of JTJ transitions in total separations of high-educated workers is computed as the ratio between the flow of JTJ transitions in any small time interval dt ($f(\theta)(1 - \zeta)e(n, h)dt$) and

Finally, to quantify the effect of these JTJ transitions on wage dispersion, we report four useful statistics. As a proxy for the degree of between-group wage inequality, we compute the ratio between the average wage of high-educated workers and the wage of less-educated workers. Likewise, the within-education wage inequality is measured by the ratio between the average wage of high-educated workers and their wage in unskilled jobs. Finally, to control for the relative size of the two groups, we also compute the total variance of the wage distribution which is further decomposed into a permanent component due to between-group wage differences and a transitory component due to within-group wage differences. These statistics are reported at the lower panels of Table 2. They show that our benchmark model with $\lambda = 1$ yields considerably higher wage dispersion than the alternative model without OTJ search. In the latter case, the skill premium is less than 7% while the former generates 35.9%. Even more striking is the difference in the degree of within-group wage inequality. Since mismatched workers earn less than less-educated workers, our model can easily explain a gap of 60% between the mean and the lowest wage of high-educated workers, while this gap is reduced to 2% with $\lambda = 0$. A similar picture emerges when we look at the overall variance of the wage distribution which is about fifteen times larger when $\lambda = 1$, and a substantial part of this additional variance can be attributed to the wage dispersion among high-educated workers.

7.2 Comparative statics

Our next objective is to gauge how important are JTJ transitions in affecting the response of the labour market to shifts in the relative productivity of skilled jobs. To simulate the effects of SBTC, we raise the value of $y(s)$ from its benchmark value of 1.5 to a value of 2. The results are summarized in Table 3 which, for brevity, reports the values of a subset of key variables which include the labour market tightness, the unemployment rates of both types of workers, the share of OTJ seekers, and two measures of wage dispersion.

In our benchmark economy with equal contact rates (column 1) the increase in $y(s)$ leads to a fall of 0.6 p.p. in $\widetilde{u}(h)$ and a rise of 0.3 p.p. in $\widetilde{u}(l)$ compared to

the total flow of separations by this type of workers in the same time interval $(f(\theta)(1-\zeta)e(n,h) + \delta(1-\mu-u(h))dt$. Inserting into this ratio the outcomes reported in the first column of Table 2, yields a value of 0.35.

the results reported in Table 2. Thus, the changes in both unemployment rates are small relative to the changes in productivity. By contrast, the share of high-educated job seekers drops by as much as 6 p.p. while the degree of between and within-group wage inequality increase by, 35.4% and 34.2%, respectively. Hence, the bulk of the adjustment takes place via a change in wages and an increase in the share of skilled jobs. Much more striking results are obtained when $\lambda = 0$. In this case the strong increase in the outside option of high-educated workers, $U(h)$, induces a shift to an equilibrium with ex-post segmentation. As a result, both $\widetilde{u}(h)$ and $\widetilde{u}(l)$ jump up by 6.9 and 3.6 p.p., respectively. This drastic response to SBTC contrasts with the gradual changes experienced by the unemployment rates when OTJ search is accounted for.

Finally, Table 4 reports similar results for the case of SU, which is captured by an increase in the share of high-educated workers, $1-\mu$, from 25% to 50%. Again we have chosen a parameter configuration such that the new equilibrium with $\lambda = 0$ exhibits ex-post segmentation. When comparing the results with those in Table 2, we find a growing gap between $\widetilde{u}(h)$ and $\widetilde{u}(l)$ alongside a widening of the between- and within group wage inequality. The only qualitative difference with the case of SBTC is in the evolution of the share of high-educated job seekers. While SBTC led to a reduction in this share, we now observe a strong increase, from 34% to 49%. Interestingly, the increase in the fraction of high-educated workers gives rise to a small reduction in $\widetilde{u}(h)$. Thus, as anticipated in Section 5, our model exhibits cohort-size effects. The intuition for these effects is that the JTJ flows make labour demand so responsive to supply shifts that high-educated workers face a lower risk of unemployment as they become more abundant in the population.[12]

[Insert Tables 3 and 4]

7.3 Unequal contact rates

The two limiting cases analyzed above are useful for analytical purposes, but a realistic value of λ probably lies somewhere in between of 0 and 1 (see Christensen *et al.,* 2005*).* To analyze this case we compute the labour market outcomes for a range of λ that goes from 0.2 to our benchmark value of 1. Although not reported here to save space (but

[12] Shimer (2001) uses a similar argument to explain the fall in the unemployment rate of young workers when the baby-boom generation entered the U.S. labor market.

available upon request), our main findings are that the unemployment rates show little variation over this range relative to the rates displayed in Table 2 with $\lambda = 1$. The bulk of the adjustment takes place through a shift in the composition of employment and the pool of job seekers. In particular, since mismatched workers move quicker to skilled jobs at higher values for λ, we obtain a gradual monotonic reduction in the share of high-educated job seekers together with a rise in the share of separations of high-educated workers that are due to JTJ transitions. For example, at $\lambda = 0.2$ this last share is equal to 0.22, compared to 0.35 in the benchmark model with $\lambda = 1$. Furthermore, as λ increases, there is a strong rise in the degree of wage inequality, both between groups and within the cohort of high-educated workers. The evolution of the wage of mismatched workers is key for this result. As depicted in Figure 4, at the starting value of $\lambda = 0.2$, $w(n, h) > w(n, l)$ as in AV, but beyond a threshold value of $\lambda = 0.42$, this order is reverted so that $w(n, h) < w(n, l)$. Thus, this example illustrates that the result obtained in Proposition 5 with $\lambda = 1$ still holds for positive values of λ below unity.

[Insert Figure 4]

7.4 Europe vs. U.S.

As discussed in Section 2, JTJ transitions explain roughly a similar share (between 40% and 50%) of the separations in the EU and the U.S. Further, two other well-known stylized facts are that while the unemployment rate is higher in Europe, wage inequality is higher in the U.S.

In this last section, we explore how our parameter choice in the benchmark model could be modified to account simultaneously for these three stylized facts. To capture the lower unemployment and higher wage inequality in the U.S, we assume that the U.S. labour market has a higher matching efficiency than the European one, so that its meeting function changes from $\sqrt{\theta}$ to $z\sqrt{\theta}$ with $z > 1$. In our model, this change would lead to lower unemployment rates for both types of workers and a rise in wage inequality, replicating the evidence for the U.S. However, a logical consequence of this higher matching efficiency would be a fall in the share of mismatched workers which would reduce the proportion of the total separations of high-educated workers that JTJ flows represents. Hence we need to consider an additional parameter change to capture the similarity of these shares. One plausible assumption is that the higher flexibility of the U.S. labour market also results in a larger value for λ than in Europe. From our previous simulations we know that a rise in λ leads to more quits and more wage inequality, without a drastic change in the unemployment rates. Thus, we should be able to account for all the stylized facts by assuming higher values of λ and z in the U.S. than in Europe.

Table 5 presents an example in which we compare the labour outcomes for an economy with $z = 1.25$ and $\lambda = 1$ (U.S.) and another where $z = 1$ and $\lambda = 0.5$ (EU). Inspection of the results shows that the first economy generates more wage inequality and lower unemployment rates while the ratio between the JTJ transitions and the total flow of separations of the high-educated workers is almost identical (31%) in both economies.

[Insert Table 5]

8 Conclusions

OTJ search by over-qualified workers is a prominent feature in labour markets. In this paper we analyse how this phenomenon affects the structure of employment and wages in an economy where high- and less-educated workers compete for unskilled jobs. From a policy perspective, two results stand out. First, transitory mismatch is more harmful for the labour market position of less-educated workers than permanent mismatch. It induces a shift in the job distribution towards skilled jobs and it lowers the overall stability of unskilled jobs. At the same time, however, we show that it also reduces the sensitivity of the profits in unskilled jobs to changes in the upper-segment of the labour market. As a result, shifts in the demand and supply of higher skills have a milder impact on the unemployment rate of the less-educated workers than what is predicted by models where OTJ search is ignored.

Our analysis focuses on a single motive for JTJ transitions, namely the pursuit of a better match. A logical extension would be to consider alternative wage-setting mechanisms that allow for wage dispersion among identical workers in the same type of job. This extension would make JTJ transitions more frequent since workers may try to use OTJ search to obtain a pay rise. However, this extension will not affect qualitatively the main conclusions reached here, except those on wage dispersion.

A more challenging extension would be to explore the efficiency properties of the JTJ transitions. The mobility decisions of workers are based on a comparison between actual and future wages. There is clearly no reason why these decisions should be efficient because the workers ignore both the negative effects on their incumbent employers and the positive effect on future employers. In addition, the employed job seekers congest the market for unemployed job seekers and their higher quit rate discourages the creation of unskilled jobs. Since a utilitarian social planner would take all these effects into account, it would be interesting to analyse under which conditions the planner prefers more or less frequent JTJ transitions than in the decentralised economy.

Another avenue for future research would be to analyse the response of the economy to aggregate productivity shocks. In an economy with costly OTJ search this could give rise to pro-cyclical fluctuations in the intensity of OTJ search as mismatched workers search more intensively during booms. An interesting aspect of such an economy is that the overall match quality changes over the cycle. From the viewpoint of less-educated workers, a recession is therefore a period of low job creation and intense competition with high-educated workers, while booms are periods of high job creation and a gradual release of jobs that were previously occupied by high-educated workers.

Finally, as argued in the Introduction, Hornstein et al. (2006) have documented that actual residual wage inequality in the U.S. is twenty times larger than the one predicted by a large class of search and matching models. They also claim that the introduction of OTJ search only leads to a modest improvement. Our results seem to suggest that this conclusion could be driven by the fact that Hornstein et al. only consider OTJ search within narrowly defined markets. However, as shown in this paper, high-educated workers are typically willing to accept a wide range of jobs. It would therefore be interesting to analyze whether the introduction of OTJ search and mismatch can improve the empirical performance of calibrated search and matching models.

Appendix A: Proofs

Proof of Proposition 1

Consider an equilibrium in which firms offer exclusively unskilled jobs ($\zeta = 1$). Then, the asset value of vacancies would be given by:

$$rV(n) = -c + p(\theta)(1-\beta)\frac{y(n) - rU}{r+\delta} = 0, \qquad \text{(A.1)}$$

where

$$rU = b + f(\theta)\beta\left(\frac{y(n) - rU}{r+\delta}\right) \qquad \text{(A.2)}$$

is the identical outside-option value of both types of workers. Let θ^* denote the unique value of the labour market tightness in this ex-post segmentation equilibrium that solves (A.1) given (A.2). To rule out an equilibrium of this type, it must hold that::

$$rV(s) = -c + p(\theta^*)(1-\beta)(1-\mu)\frac{y(s) - rU}{r+\delta} > 0, \qquad \text{(A.3)}$$

namely, a deviant firm can make positive profits by opening a skilled job. Comparing (A.1) and (A.3), it follows that this condition leads to the requirement that:

$$(1-\mu)\left[y(s) - rU\right] > y(n) - rU. \qquad \text{(A.4)}$$

Finally, solving for rU in (A.2) and replacing it into (A.4), yields (31).■

Proof of Proposition 2

In a cross-skill matching equilibrium, all three possible types of matches need to generate a positive surplus. First, to show that $S(n,l) > 0$ and $S(s,h) > 0$, we proceed as follows. First, substituting (20) into the right-hand side of (12) yields:

$$S(n,l) = \frac{y(n) - b}{r + \delta + f(\theta)\zeta\beta}, \qquad \text{(A.5)}$$

while the solution for $S(s,h)$ is obtained by substituting (21) into (14)

$$S(s,h) = \frac{\alpha_1[y(s) - b] + f(\theta)\beta\zeta[y(s) - y(n)]}{\alpha_2\alpha_3 + (r+\delta)f(\theta)\beta\zeta(1-\lambda)}, \qquad \text{(A.6)}$$

where both surplus expressions are positive because $b < y(n) < y(s)$.

Next, the expression for $S(n,h)$ can be obtained from (16) by using (14) and replacing $S(s,h)$ by (A.6). After some algebraic manipulations, it becomes:

$$S(n,h) = \frac{\alpha_3[y(n)-b] - f(\theta)\beta(1-\zeta)(1-\lambda)[y(s)-b]}{\alpha_2\alpha_3 + (r+\delta)f(\theta)\beta\zeta(1-\lambda)}. \tag{A.7}$$

The proof is completed by noticing that condition (32) is equivalent to $S(n,h) \geq 0$, namely, $\alpha_3[y(n)-b] \geq f(\theta)\beta(1-\zeta)(1-\lambda)[y(s)-b]$.■

Proof of Proposition 3

Consider an economy with a unique ex-post segmentation equilibrium for $\lambda = 0$, and let (θ^e, ζ^e) denote the associated equilibrium values for θ and ζ. In this economy, the expected asset value of an unemployed high-educated worker is:

$$rU(h) = b + f(\theta^e)(1-\zeta^e)\beta S(s,h), \tag{A.8}$$

where $S(s,h) = [y(s)-b]/[r+\delta+f(\theta^e)(1-\zeta^e)\beta]$. Since, high-educated workers refuse to work in unskilled jobs in this case, it must be that $S(n,h) < 0$ which, in turn, requires that $rU(h) > y(n)$.

Now consider the same economy but with $\lambda > 0$. In this case, we can derive the minimum wage at which a high-educated worker would be willing to accept an unskilled job. Denote this wage by $\underline{w}$. If $\underline{w} < y(n)$, then a high-educated worker and a firm with an unskilled job can both obtain a gain if they accept to match and fix some wage $w \in (\underline{w}, y(n))$. In such a case, the equilibrium with ex-post segmentation would cease to exist. Below we show that there always exists some value of $\lambda < 1$ for which this is the case.

Formally, let $W_n(w)$ denote the lifetime income of a deviant high-educated worker who accepts an arbitrary wage w to work in an unskilled job. Since mismatched workers will quit when they find a skilled job, the asset value equation for $W_n(w)$ satisfies:

$$rW_n(w) = w + \delta\left[W_n(w) - U(h)\right] + \lambda f(\theta^e)(1-\zeta^e)\beta S(s,h), \tag{A.9}$$

which is strictly increasing in w. Combining (A.8) and (A.9), we find that:

$$W_n(w) - U(h) = \frac{w - b - f(\theta^e)(1-\lambda)(1-\zeta^e)\beta S(s,h)}{r+\delta}. \tag{A.10}$$

Likewise, the asset value of a firm with an unskilled job that offers a high-educated worker a wage w satisfies:

$$J_n(w) = \frac{y(n) - w}{r + \delta + \lambda f(\theta^e)(1 - \zeta^e)}. \tag{A.11}$$

Now, we can define $\underline{w}$ implicitly by the following condition :

$$W_n(\underline{w}) = U(h). \tag{A.12}$$

From (A.10) it follows that the solution is given by:

$$\underline{w} = b + f(\theta^e)(1 - \lambda)(1 - \zeta^e)\beta S(s, h), \tag{A.13}$$

that is, firms need to pay high-educated workers at least their opportunity cost. Moreover, (A.11) implies that a firm with an unskilled job would be willing to offer this minimum acceptable wage as long as $y(n) - \underline{w} \geq 0$. Notice that for $\lambda = 0$, $\underline{w} = rU(h)$ and $J_n(\underline{w}) < 0$ while for $\lambda = 1$, $\underline{w} = b$ and $J_n(\underline{w}) > 0$. In other words, if workers cannot perform OTJ search, the equilibrium with ex-post segmentation is well defined because a firm with an unskilled job would make negative profits if it were to pay a high-educated worker her opportunity cost $\underline{w} = rU(h)$. On the contrary, when $\lambda = 1$, a mismatched worker and an unemployed job seeker have the same chances to match with a skilled job. Thus, the worker will be willing to accept this job provided that $w \geq b$. Since $y(s) > b$, a firm with an unskilled job can therefore make a high-educated worker an offer $w \in (b, y(n))$ such that the worker and the firm are strictly better off when they deviate.

From here, it follows that, for any pair $(\theta^e, \zeta^e) \in (0, \infty) \times (0, 1)$, there exists a $\overline{\lambda} \in (0, 1)$ such that $J_n(\underline{w}) > 0$ for any $\lambda > \overline{\lambda}$. For given values of θ and ζ, the right-hand side of (A.13) defines $\underline{w}$ as a continuously decreasing function of λ that maps $[0, 1]$ onto $[b, rU(h)]$. Thus, since $b < y(n) < rU(h)$, there exists a unique value $\lambda \in (0, 1)$, denoted by $\overline{\lambda}$, such that $y(n) - \underline{w} = 0$ while $J_n(\underline{w}) > 0$ for all $\lambda > \overline{\lambda}$.$\blacksquare$

Proof of Proposition 4

Once more it holds that $\underline{w} = rU(h)$ for $\lambda = 0$ but this time we have that $rU(h) < y(n)$ because the equilibrium exhibits cross-skill matching. Thus, since $\underline{w}$ is decreasing in λ, it must be that $y(n) - \underline{w} > 0$ for any $\lambda > 0$. Consequently, in this case, a high-educated worker and a firm with an unskilled job incur a loss if they deviate from the equilibrium.$\blacksquare$

Proof of Lemma 1

The surplus expressions for the general case of $\lambda \in [0,1]$ are provided in (22). For $\lambda = 1$, these expressions satisfy the following condition:

$$S(n,h) = \frac{y(n) - b}{r + \delta + f(\theta)\left[1 - \zeta + \beta\zeta\right]} \leq \frac{y(n) - b}{r + \delta + f(\theta)\beta\zeta} = S(n,l), \tag{A.14}$$

with a strict inequality when $\zeta < 1$. Notice that the assumption $y(n) > b$ ensures that both $S(n,h)$ and $S(n,l)$ are strictly positive for finite values of θ.■

Proof of Proposition 5

The second inequality follows from the assumption that $y(s) > y(n)$, so that $U(h) > U(l)$ when $\zeta < 1$. Since $w(s,h) = rU(h) + \beta\left[y(s) - rU(h)\right]$ and $w(n,l) = rU(l) + \beta\left[y(n) - b\right]$ this implies that $w(s,h) > w(n,l)$. To obtain the first inequality, we insert (19) into (17). Hence:

$$w(n,h) = b + \beta[y(n) - b] + f(\theta)\beta(1-\beta)\zeta S(n,h).$$

Similarly, after replacing $U(l)$ in (13) by (18), we can rewrite the expression for $w(n,l)$ as:

$$w(n,l) = b + \beta[y(n) - b] + f(\theta)\beta(1-\beta)\zeta S(n,l)$$

From the above expressions, it holds that:

$$w(n,h) - w(n,l) = f(\theta)\beta(1-\beta)\zeta[S(n,h) - S(n,l)] < 0$$

where the last inequality follows from Lemma 1.■

Appendix B: Uniqueness

To prove uniqueness it is convenient to rewrite the equilibrium conditions in the following way. First, we solve (24) and (25) for u and ζ in terms of θ and ϕ, yielding:

$$u(\theta,\phi) = \frac{\delta}{\delta + f(\theta)}\frac{1-\mu}{1-\phi}, \tag{A.15}$$

$$\zeta(\theta,\phi) = \frac{(1-\phi)f(\theta)\mu + \delta(\mu - \phi)}{f(\theta)\phi(1-\mu)}. \tag{A.16}$$

Inspection of (A.16) shows that $\partial\zeta(.)/\partial\phi < 0$ and $\partial\zeta(.)/\partial\theta > 0$ (as $\phi > \mu$). Next, our definition of ψ implies that $\psi/(1-\psi) = u/e(n,h)$. Thus, combining (26) and (A.16) allows us to express ψ in terms of θ and ϕ, namely:

$$\psi(\theta,\phi) = \frac{\delta + f(\theta)(1-\zeta(\theta,\phi))}{\delta + f(\theta)(1-\zeta(\theta,\phi)) + f(\theta)\zeta(\theta,\phi)(1-\phi)} \tag{A.17}$$

with $\partial\psi(.)/\partial\theta < 0$ and $\partial\psi(.)/\partial\phi > 0$. The next step is to substitute (A.16) and (A.17) into the two free-entry conditions (22) and (23). Evaluating the resulting expressions at $\lambda = 1$, this yields the following system of two equations in two unknowns (θ and ϕ):

$$p(\theta)\psi(\theta,\phi)\left[\frac{\phi}{r+\delta+f(\theta)\zeta(\theta,\phi)\beta} + \frac{(1-\phi)}{\alpha_2}\right] = \frac{c}{(1-\beta)[y(n)-b]}, \tag{34}$$

$$\frac{p(\theta)(1-\psi(\theta,\phi)\phi)}{\alpha_3}\left[R - \frac{f(\theta)\beta\zeta(\theta,\phi)}{\alpha_2}\right] = \frac{c}{(1-\beta)[y(n)-b]}, \tag{A.19}$$

where $R = [y(s)-b]/[y(n)-b] > 1$.

In implicit form we shall refer to (A.18) and (A.19) as $V_N(\theta,\phi) = 0$ and $V_S(\theta,\phi) = 0$, respectively. Our aim is to show that these two loci intersect at most once under the following set of conditions: (1) There is a sufficiently large majority of low of low-educated workers, so $\mu \geq 0.5$ (2) Workers obtain at least one half of the surplus and so $\beta \geq 0.5$ and (3) R is sufficiently large. The first condition is needed to guarantee that the composition effects are small so that $\partial p(\theta)(1-\psi\phi)/\partial\theta) < 0$ while $\partial f(\theta)(1-\zeta) > 0$ which, in turn, imply that $\partial\alpha_2/\partial\theta$ and $\partial\alpha_3/\partial\theta$ are both positive.

Skilled jobs: To show that the locus associated with $V_S(\theta,\phi) = 0$ has a negative slope, we need to prove that:

$$\left.\frac{d\phi}{d\theta}\right|_{V_S=0} = -\frac{\partial V_S/\partial\theta}{\partial V_S/\partial\phi} < 0.$$

First, notice that the numerator can be written as follows:

$$\begin{aligned}
\frac{\partial V_S}{\partial \theta} &= \frac{1}{\alpha_3} \cdot \left[R - \frac{\beta \zeta f(\theta)}{\alpha_2} \right] \cdot \frac{\partial [p(\theta)(1 - \psi\phi)]}{\partial \theta} \\
&\quad - \frac{p(\theta)(1 - \psi\phi)}{(\alpha_3)^2} \cdot \left[R - \frac{\beta \zeta f(\theta)}{\alpha_2} \right] \cdot \frac{\partial \alpha_3}{\partial \theta} \\
&\quad - \frac{p(\theta)(1 - \psi\phi)}{\alpha_2 \alpha_3} \cdot \left[\frac{r + \delta}{\alpha_2} \right] \cdot \beta \zeta \frac{\partial f(\theta)}{\partial \theta} \\
&\quad - \frac{p(\theta)(1 - \psi\phi)}{\alpha_3} \cdot \beta f(\theta) \cdot \frac{\partial}{\partial \zeta} \left[\frac{\zeta}{\alpha_2} \right] \cdot \frac{\partial \zeta}{\partial \theta}.
\end{aligned}$$

Given our assumptions, all four terms are negative. Thus, $\partial V_S / \partial \theta < 0$.

Next, the expression for the partial derivative $\partial V_S / \partial \phi$ is given by:

$$\begin{aligned}
\frac{\partial V_S}{\partial \phi} &= -\frac{p(\theta)}{\alpha_3} \left[R - \frac{\beta \zeta f(\theta)}{\alpha_2} \right] \cdot \left(\psi + \phi \frac{\partial \psi}{\partial \phi} \right) \\
&\quad + \frac{p(\theta)(1 - \phi\psi)}{(\alpha_3)^2} \cdot \left[R - \frac{\beta \zeta f(\theta)}{\alpha_2} - \frac{\alpha_3}{(\alpha_2)^2} (r + \delta + f(\theta)) \right] f(\theta) \beta \frac{\partial \zeta}{\partial \phi}.
\end{aligned}$$

In principle, the sign of this expression is ambiguous since the last term between brackets contains a positive and two negative terms. Nonetheless, using the feature that $\alpha_2 - \alpha_3 > 0$, one can show that $\partial V_S / \partial \phi$ is unambiguously negative when the term

$$\left[R - \frac{r + \delta + f(\theta)[1 + \beta\zeta]}{r + \delta + f(\theta)[1 - \zeta + \beta\zeta} \right],$$

is positive. This sufficient condition requires that R is sufficiently larger than 1 which is guaranteed by condition (3). Thus, since $\partial V_S / \partial \phi$ and $\partial V_S / \partial \theta$ are both negative, the curve $V_S = 0$ has a negative slope.

Unskilled jobs: To show that the $V_N(\theta, \phi) = 0$ locus has a positive slope, it is sufficient to show that:

$$\left. \frac{d\phi}{d\theta} \right|_{V_N = 0} = -\frac{\partial V_N / \partial \theta}{\partial V_N / \partial \phi} > 0.$$

The numerator of this expression is given by:

$$
\begin{aligned}
\frac{\partial V_N}{\partial \theta} \quad = \quad & \left[\frac{\phi}{r+\delta+f(\theta)\zeta\beta} + \frac{1-\phi}{\alpha_2}\right] \cdot \left[\psi \frac{\partial [p(\theta)]}{\partial \theta} + p(\theta)\frac{\partial \psi}{\partial \theta}\right] \\
& - p(\theta)\psi \cdot \left[\frac{\beta\phi}{[r+\delta+f(\theta)\zeta\beta]^2}\right] \cdot \left[\zeta \frac{\partial f(\theta)}{\partial \theta} + f(\theta)\frac{\partial \zeta}{\partial \theta}\right] \\
& - p(\theta)\psi \cdot \frac{(1-\phi)}{(\alpha_2)^2} \cdot \left[(1-\zeta+\beta\zeta)\frac{\partial f(\theta)}{\partial \theta} - (1-\beta)f(\theta)\frac{\partial \zeta}{\partial \theta}\right].
\end{aligned}
$$

In equilibrium $\phi > 1-\phi$ because $\phi > \mu$ and $\mu \geq 0.5$. Moreover, $\alpha_2 > r+\delta+f(\theta)\zeta\beta$ and, by condition (2), $\beta \geq 0.5$. Using these results, it can be easily shown that the above expression has a negative sign. Hence, $\partial V_N/\partial\theta < 0$.

Finally, the derivative $\partial V_N/\partial \phi$ is given by:

$$
\begin{aligned}
\frac{\partial V_N}{\partial \phi} \quad = \quad & p(\theta) \cdot \left[\frac{\phi}{r+\delta+f(\theta)\zeta\beta} + \frac{1-\phi}{\alpha_2}\right] \cdot \frac{\partial \psi}{\partial \phi} \\
& + p(\theta)\psi \cdot \left[\frac{1}{r+\delta+f(\theta)\zeta\beta} - \frac{1}{\alpha_2}\right] \\
& - p(\theta)\psi \cdot \frac{\phi\beta f(\theta)}{[r+\delta+f(\theta)\zeta\beta]^2} \cdot \frac{\partial \zeta}{\partial \phi} \\
& + p(\theta)\psi \frac{(1-\phi)}{[\alpha_2]^2} \cdot f(\theta)(1-\beta)\frac{\partial \zeta}{\partial \phi}.
\end{aligned}
$$

The first three terms of this expression are positive, while the fourth term is negative. Nonetheless, since $\beta \geq 0.5$, $\phi > 1-\phi$ and $\alpha_2 > r+\delta+f(\theta)\zeta\beta$ the last term is smaller in absolute value than the third term. Hence, the overall expression for $\partial V_N/\partial\phi$ is positive. The latter implies that the locus $V_N = 0$ is upward sloping, since $\partial V_N/\partial\theta < 0$. Consequently, the two free entry loci can intersect at most once.■

Appendix C: Figures and tables

Table 1: Match productivity per unit of time

Workers / Jobs	Unskilled	Skilled
l-type	$y(n)$	0
h-type	$y(n)$	$y(s)(> y(n))$

Table 2: labour Market Outcomes in the Benchmark Model

Variables	With OTJ search ($\lambda = 1$)	W/O OTJ search ($\lambda = 0$)
θ	1.486	1.528
ζ	0.671	0.891
ϕ	0.806	0.770
ψ	0.765	1
u	0.101	0.081
$e(n, h)$	0.031	0.206
$\widetilde{u}(h)$	0.076	0.075
$\widetilde{u}(l)$	0.109	0.083
$w(n, l)$	0.905	0.920
$w(n, h)$	0.750	0.955
$w(s, h)$	1.310	1.210
JTJ/ total separations	0.349	0
Wage Inequality		
Between	1.359	1.065
Within	1.640	1.026
Variance of Wages		
Total	0.033	0.002
High-educated	0.039	0.006

Table 3: The effects of SBTC ($y(s) = 2$)

Variables	OTJ	W/o OTJ[a]
θ	1.762	1.786
$1 - \phi\psi$	0.321	0.287
$\widetilde{u}(h)$	0.070	0.144
$\widetilde{u}(l)$	0.112	0.119
JTJ/total separations	0.335	0
Wage inequality		
Between	1.854	1.944
Within	2.317	0

Table 4: The effects of SU ($\mu = 0.5$)

Variables	OTJ	W/o OTJ[a]
θ	1.657	1.672
$1 - \phi\psi$	0.494	0.208
$\widetilde{u}(h)$	0.072	0.120
$\widetilde{u}(l)$	0.136	0.152
JTJ/total separations	0.300	0
Wage inequality		
Between	1.484	1.534
Within	1.923	0

(a)The reported figures correspond to the unique equilibrium with ex-post segmentation.

Table 5: *The U.S. vs. Europe*

Variables	Economy I (EU) $f(\theta)=\sqrt{\theta}$ & $\lambda=0.5$	Economy II (U.S.) $f(\theta)=1.25\sqrt{\theta}$ & $\lambda=1$
θ	1.511	1.570
$1-\phi\psi$	0.462	0.377
$\widetilde{u}(h)$	0.075	0.060
$\widetilde{u}(l)$	0.108	0.088
JTJ/total separations	0.309	0.308
Wage Inequality		
Between	1.342	1.400
Within	1.377	1.716

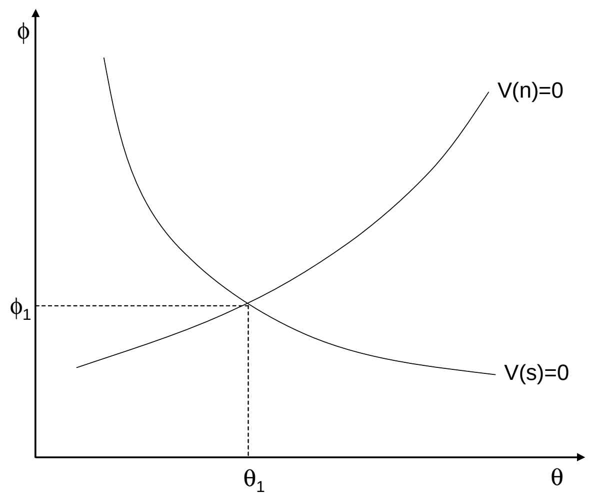

Figure 1: The unique cross-skill matching equilibrium

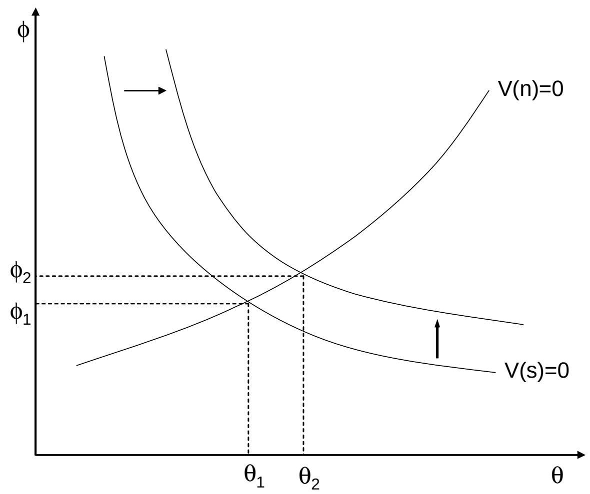

Figure 2: The effects of skill-biased technological change

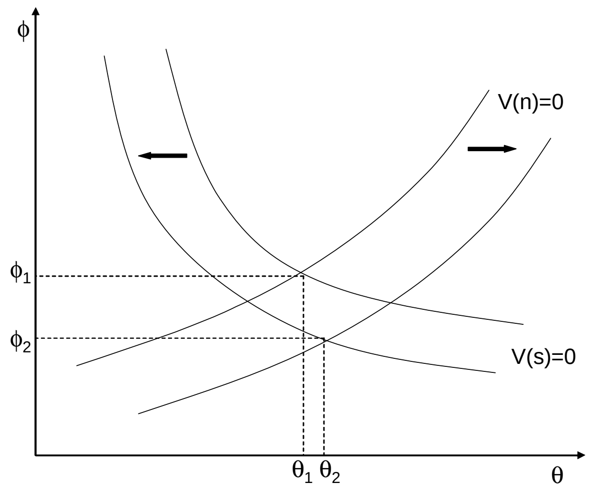

Figure 3: The effects of skill-upgrading

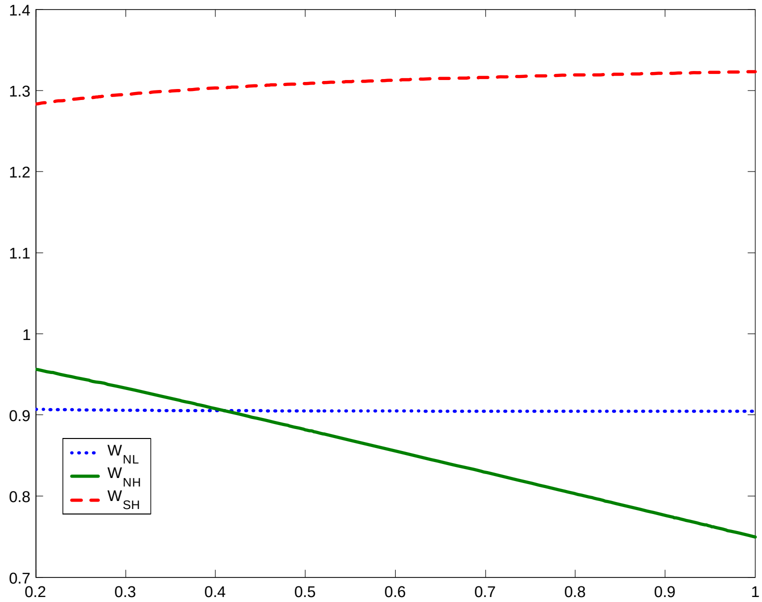

Figure 4: The equilibrium wages with unequal contact rates

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