COST-PRICES WITH VARIABLE RETURNS (*)

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ABSTRACT

In this paper a model of oligopolistic competition is presented. Firms use a mark-up in order to price but the model does not rely necessarily on the assumption of constant returns to scale. An existence theorem is proved for such an economy.

I. INTRODUCTION

There are many reasons why economist have been interested in cost-prices i.e. in prices which are proportional to average costs. On the one hand they provide a formalization of a part of the classical tradition (Smith, Ricardo, Marx) which in modern times is represented by the so-called « Cambridge School ». On the other hand they are linked with the idea that firms use a mark-up in order to decide the prices they set, and such an idea is firmely rooted in the Industrial Organization area. The failure of the Cournot model to incorporate non convexities (**), makes this approach quite relevant.

(*) This paper is a revised version of a extended essay presented at London School of Economics. I am indebted with a large number of people for adivising and suggestions. Specially, with R. Barbolla, V. Cerezo, M. Browning, J. Camio, W. Gorman, F.H. Hahn, E. Lozano, B. Murphy, R. Repullo, L. Rojo, J. Silvestre and T. Zabalza. My thanks to all of them. Financial support from Banco de España is also acknowledged. The final stages of this work were possible thanks to a grant from Fundación Empresa Pública (I.N.I.). The author wishes to acknowledge to its director, Julio Segura its help and support. The usual caveat applies.

(**) Arrow-Hahn model of monopolistic competition assumes that the set of possible productions for oligopolistic firms is strictly star-shaped. Silvestre (1978) showed how restrictive this assumption is. He pointed out that a extension of Negishi model, in which monopolistic reaction correspondence was upper-semi-continuous

In this paper I will prove that even if returns are not constant, some kind of cost-price equilibrium exists. The motivation for this work is that cost-prices and constant returns are customarily assumed together giving the impression that the first are implied by the second. Also the assumption of an unique non-produced good is made almost unvariably if cost-prices are assumed. Our work will also dispense of this assumption.

On the other hand the model presented here generalizes the so called « Non Linear Input-Output Model » (see Chander, 1983 and the references therein). In this kind of models technical coefficients are allowed to vary with the scale of production. However demand is assumed to be given, there is a unique kind of technology by firm, and there is an unique primary factor (however see Herrero, Villar, 1985). In this paper we dispense all of these assumptions: Demand is assumed to depend on prices and input-output vectors (since we allow for positive profits), there are several technologies for each level of output, and there are (possibly) many non produced goods. The first two points are particularly important since if prices depend on the scale of production the assumption that the vector of final demand is given is unpalatable, and because a quite natural explanation of the variation of coefficients can be found on the existence of many techniques. On the other hand the literature on the non linear I-O model has focussed attention on efficient (or workable) algorithms of computation. In this paper we do not provide such an algorithm, but rather we concentrate on the more basic problem of providing an existence proof for this economy.

II. THE ECONOMY AND THE MAIN ASSUMPTIONS

There are n goods. Agents are classified into two groups: competitive agents (including consumers and competitive firms) and « industrial » firms. There are m of these firms. The price of good i will be denoted by P_i. Let $P = (P_1 \dots P_n)$.

Industrial firms produce an unique output each, which in turn may be produced by different firms. Let $Y_f \subseteq \mathbb{R}_+$ be the set of possible outputs

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and convex-valued, was sufficient to stablish equilibrium, without convexity of monopolistic production sets. But as Roberts and Sonnenschein have proven there is no way one can guarantee convex valuedness and existence without severe restrictions on demand functions even when production is assumed to be costless. Notice that a Cournotian model can generate a constant mark-up if inverse demand and cost functions are assumed to be iso-elastic.

of good i for firm f. Also let us denote by $I_f \subseteq \mathbb{R}^n_+$ the set of all possible input combinations for firm f. The technology of this firm is represented by a production function f_f: $I_f \to Y_f$. Without loss of generality we will assume that firm 1 produces good 1 and firm m good n.

Competitive agents are described by an excess demand-function, i.e.

$$\forall i = 1 \dots n\,, \qquad z_{ci}\colon \prod_{f=1}^{m} Y_f \times \prod_{f=1}^{m} I_f \times Sn \to \mathbb{R}\,,$$

i.e.

$$z_{ci} = z_{ci}(P_1 \dots P_n,\, y_{11} \dots y_{mn},\, y_{11}^- \dots y_{1n}^- \dots y_{m1}^- \dots y_{mn}^-)\,;$$

being y_{fi} and $(y_{f1}^- \dots y_{fn}^-)$ elements of Y_f and I_f respectively and

$$Sn = \left\{P \in \mathbb{R}_n^+ : \sum_{i=1}^{n} P_i = 1\right\}.$$

Finally let us denote by μ_f the mark-up of firm f.

On the economy described above we will impose the following assumptions.

I) $\forall i = 1, 2 \dots m$ we have that

a) $f_i(\,)$ is strictly quasiconcave and continuous. $f_i(0) = 0$.

b) If $y_{fi} \in Y_f$, then $\forall y'_{fi}, y_{fi} > y'_{fi} \geqq 0$, $y'_{fi} \in Y_f$ (free disposal).

c) Y_f is a compact set (an interval) (*).

II) *a*) $z_{ci}(\,)$ is single valued, continuous and homogeneous of degree zero in $P_1 \dots P_n$, $\forall i = 1 \dots n$.

b) $\sum_{i=1}^{n} P_i\left(z_{ci} + \sum_f y_{fi}^- - \sum_f y_{fi}\right) = 0$ (Walras Law).

Under I) and II) we can prove the following.

1) Let

$$C_f^*(P, y_{fi}^0) = \inf\left\{\sum_{j=1}^{n} P_j\, y_{fj}^- : f_f(y_{f1}^- \dots y_{fn}^-) = y_{fi}^0 \in Y_f\right\}. \tag{1}$$

(*) This assumption can be replaced by the assumption that the set of feasible allocations is bounded at cost of some complications.

Then, $C_f^*(\,)$ is a well defined function and it is continuous over $P_1 \ldots P_n, y_{fi}$.

2) Let $y_{fi}^{*-} = y_{fi}^{-}(P, y_{fi}^0)$ be the i-th input demand function for f, ($y_{fi}^{-}(\,)$ is derived from (1)). Then $y_{fi}^{-}(\,)$ is a single valued and continuous function $\forall i = 1 \ldots n$.

For a proof of 1) and 2) the reader can consult Arrow-Hahn Theorem 9. Also notice that because our boundedness assumption on Y_f, y_{fi}^{-} is also continuous when some price is zero.

III) Let $ac_f(P, y_{fi}) = C_f^*(P, y_{fi})/y_{fi}$. Then $ac_f(\,)$ is a continuous and well defined function of its arguments at $y_{fi} = 0$.

This last assnmption forbids some kind of increasing returns if for instance $f_i(\,)$ were of Cobb-Douglas type.

III. EQUILIBRIUM

Definition 1: An equilibrium relative to μ_f, $f = 1 \ldots m$, is a

$$\{P_i y_{fi}, y_{fi}^{-}, z_{ci}\}_{i=1 \ldots n}^{f=1 \ldots m}$$

such that $\forall i = 1 \ldots n$, $\forall f = 1 \ldots m$:

(i) $y_{fi} \in Y_f$,

(ii) $y_{fj}^{-} = y_{fj}^{-}(P, y_{fi})$, $j = 1 \ldots n$,

(iii) $z_{ci}(P_1 \ldots P_n, y_{11} \ldots y_{mn}, y_{11}^{-} \ldots y_{mn}^{-}) + \sum_{f=1}^{m} y_{fi}^{-}(\,) - \sum_{f=1}^{m} y_{fi} \leqq 0$,

(iv) $P_i y_{fi} \geqq (1 + \mu_f) C_f^*(P_1 \ldots P_n, y_{fi}$ if $y_{fi} > 0$. If $P_i > ac_f(P, y_{fi}) \cdot$ $\cdot (1 + \mu_f)$ then, $y_{fi} = \sup Y_f$. If $P_i < ac_f(P, y_{fi})(1 + \mu_f), y_{fi} = 0$.

Our notion of equilibrium requires that *a*) all markets clear and that *b*) if production is positive total revenue is no less than total cost. Moreover if price exceeds average cost, the corresponding output will be on the upper boundary of Y_f. This accounts for goods for which demand exceeds the available supply at the current prices. Therefore suppliers of this good earn an extra profit over the level of the « target » mark-up (this phenomenon is completely analogous to Ricardo's theory of rent). Finally those goods for which cost are not covered are not produced.

Theorem 1: Under I), II), III) there exist an equilibrium relative to μ_i, $i = 1 \dots m$, for the economy described above.

Proof: Define

$$\left.\begin{aligned} z_i() &\equiv z_{oi}() - \sum y_{fi}^-() + \sum y_{fi} = z_i \\ P_i() &\equiv P_i \end{aligned}\right\} \quad i = 1 \dots n\,,$$

$$F_f() \equiv ac_f(P, y_{fi})(1 + \mu_f) = F_f \qquad f = 1 \dots m\,.$$

Let $\tilde{z}_i = z_i(P, y_{11} \dots y_{mn}, y_{11}^-() \dots y_{mn}^-()) \equiv \tilde{z}_i(P, y_{11} \dots y_{mn})$, say. Then it is clear that $F_f()$, P_i, $\tilde{z}_i()$ define a mapping T_1,

$$T_1\colon Sn \times \prod_{f=1}^{m} Y_f \to Sn \times \mathbb{R}^{n+m}\,.$$

Because of continuity of T_1 the image of T_1 lies in a compact, convex set $K \subseteq \mathbb{R}^{n+m} \times S_n$.

Now let us choose y_{fi} and P respectively by means of the following maximization programs.

$$(a)\ \text{To}\ \operatorname{Max} \sum_{j=1}^{n} \sum_{f=1}^{m} y_{fj}(P_j - F_f) \qquad y_{fi} \in Y_f\,,\ f = 1 \dots m\,.$$

For given P_j, F_f, $j = 1 \dots n$, $f = 1 \dots m$.

$$(b)\ \text{To}\ \operatorname{Max} \sum_{i=1}^{n} P_i \tilde{z}_i \qquad P \in Sn\,.$$

For given $\tilde{z}_i$.

Because Sn and Y_f are compact and convex (Y_f is convex because the free disposal condition I b)) the above maximizations define a upper-semi-continuous and convex valued mapping T_2,

$$T_2\colon K \to Sn \times \prod_{f=1}^{m} Y_f\,.$$

Then if we denote by

$$T = \{T_1, T_2\}\,, \qquad T\colon K \times Sn \times \prod Y_f \to K \times Sn \times \prod Y_f\,.$$

All the conditions of Kakutani fixed point theorem are satisfied, hence there exist a fixed point for T. Now we will prove that such a fixed point is an equilibrium.

Obviously conditions (i) and (ii) are satisfied. If $F_f > P_j$ then $y_{fj} = 0$. Then because I a) and the Definition of $C_j^*()$ we have that (iv) holds.

If $F_j < P_j$ then $y_{fj} = \sup Y_j$. Hence (iv) also holds. If $F_j = P_j$ (iv) is satisfied.

Also a standard argument in General Equilibrium will show that maximization *b*) above plus Walras Law implies that in the Fixed Point we have that (iii) is satisfied.

The following example shows the necessity of Assumption III).

Example: Let $n = 2$, $m = 1$. Because of II *b*) we focus on the market for good 1. $C_1 = B + cy_{11}$ and $z_1 = A/P_1$ for $P_1 \in [\varepsilon, \infty)$, $\varepsilon > 0$ and $f(P_1)$ for $P_1 \in [0, \varepsilon]$ (see fig. 1 for the exact form of $f(\,)$). $Y_1 = [0, A/\varepsilon]$ and $\mu_1 = 0$.

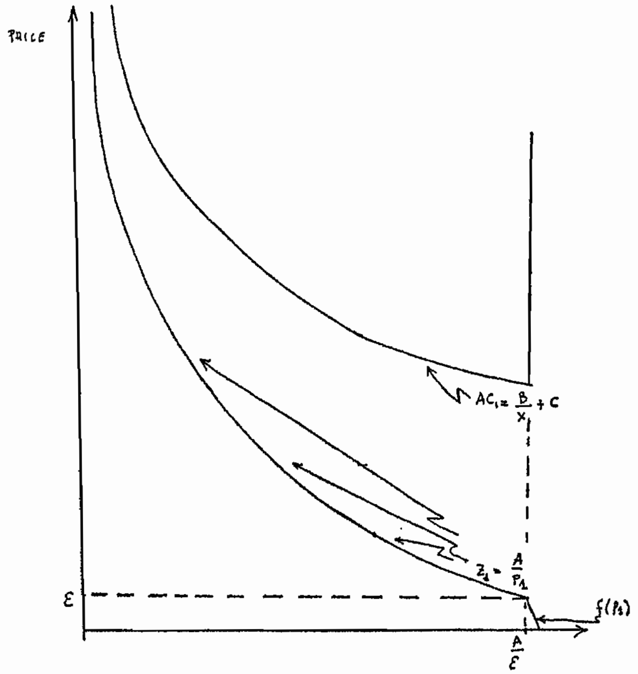

Figure 1.

Let us assume that $B > A$. For an equilibrium to exist we should have either $P_1 = A/x = B/x + C$ (x is the production—and the excess demand—in equilibrium), in which case $x = (A - B)/C < 0$ or $x = 0$, but P_1 is infinite in this second case.

IV. FINAL COMMENTS

We have proved that, contrarily to Bliss opinions (see Bliss, 1975), cost prices are not necessarily linked with constant returns and one non produced factor. Our approach, tough, relies on strong assumptions on the technology. First it is assumed that the production function $f_i(\,)$ is (strictly) quasi concave. If not, the input demand correspondence $y^*_{f_i}(\,)$ will be not convex valued an equilibrium may not exist (the convex valuedness of such correspondence is assumed directly by Silvestre (1977)). Second the average cost function is assumed to be continuous for $y_{f_i} = 0$ and this rules out some specific functional forms of $f_i(\,)$ at least if increasing returns to scale were assumed. We feel, however, that these two points are mainly technical and that our main task is fulfilled. Finally it must be remainded that we did not offer any theory of how mark-ups are determined. An interesting extension of our work would consists in an integration of alternative theories about mark-ups (or rates of interest) in our framework.

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