On the Optimal Use of "Dirt Taxes" and Credit Subsidies in the Presence of Private Financing Constraints

(VERY PRELIMINARY)

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Motivation

- Pigou (1920): Optimal tax on goods that generate externalities should be equal to the marginal external damage arising from consumption.
  –> Straightforward implementation: Linear tax or trading of "pollution rights".

- Our starting point are the following two observations:
  - Real-world policies are often more intricate
    (e.g., non-linear grant schemes, linked to credit).
  - High (tax) financial burdens on productive activities may require additional outside finance for firms
    –> What are efficiency implications?

- We tie these two together in a model where raising finance is (endogenously) costly.

Key Insights

- Taxing externalities imposes inefficiencies arising from "financial frictions"
  - With homogeneous agents, this arises *only* due to "costly external funds"
  - With heterogeneous agents, the resulting *redistribution* exacerbates these costs
    (Heterogeneity: Arises from different adjustment/avoidance costs).

Key Insights

- There are better instruments than a linear tax or a market-based solution ("pollution rights")
  - Characterization of optimal non-linear tax.
  - Taxing externalities *and* subsidizing credit-financed avoidance.
- Both measures increase productive efficiency, as they reduce *redistribution* generated by tax on externalities.

Related Tax Literature

- "Pigou meets Mirrlees": Strand of literature that links avoidance of externalities (or collective good provision) to taxes under private information.
  - Standard case: Private information on own (labour) productivity. Utilitarian government.
  - Tax on externality leads to redistribution
    –> But there redistribution can be fully compensated through wage tax adjustment
    –> Jakobs/de Moji 2010: "Standard Pigou result"

Related Tax Literature

- Wider "dirt tax" literature:
  –> Arguments why optimal tax above/below Pigou tax
  - "Double dividend" in the presence of other distortive taxes that can then be reduced for government funding? ("Tax recycling")
  - But questioned in the literature: "Tax interaction" distortion. (Bovenberg/de Mooji 1994)

Government Intervention under Financial Frictions

- Public finance literature on entrepreneurship, including venture capital finance:
  –> Typically focused on impact of various taxes on incentives.

- Literature on extended liability / judgement proofness (e.g., Tirole 2010)
  –> Financial frictions = limited liability.

The Economy

- Economy is populated by mass one of agents: $i \in [0, 1]$. Originally endowed with no resources, but with opportunity for production.

- Timing:
  - t=0: Requires investment $I_0 + K(\cdot)$ ("externality avoidance costs").
  - t=1: Output (per agent) $x_i = 0$ or $x > 0$. And generation of externalities y_i.

Production

- Non-observable effort e at cost $c(e)$
  –$>$ Affects likelihood of positive output: $\Pr[x_i = x] = p(e)$.

- No discounting and separable utility function:
  –$>$ With final consumption w_i:

$$u_i = w_i - c(e_i) - \rho \int_{i \in I} y_i di$$

where ρ is constant marginal cost of externality on society.

Avoiding Externalities

- Costs of avoiding externalities $y \geq 0$:
  - Wlog express avoidance as: $a_i = \overline{y} - y$.
  - Given agent-specific type θ_i, costs $K(a_i, \theta_i)$.

Avoiding Externalities

- Properties: $K(0,\cdot) = 0$ and $K_1 > 0$. And "single-crossing" property

$$K_{12}(a,\theta) = \frac{dK(a,\theta)}{dad\theta} < 0,$$

which implies also $K_2 < 0$ when $a > 0$.

- Key: Type θ is private information. It does not affect "productive efficiency".

Interpretations

1. Externality y provides a sufficient statistics for a (negative) externality in production, such as emission of CO_2.
   –$>$ Straightforward to include stochastic element: E.g., avoidance a maps into distribution $G(y \mid a)$.

2. Avoidance a captures "technology choice", such as usage of energy-efficient building material.
   –$>$ As provides "sufficient statistics" for resulting externalities $\widetilde{y}$, policy can condition directly on a.
   –$>$ Then $K(a, \theta)$ captures agents' "true cost" of choice (taking into account difference in opportunity costs etc.)

The Outside Financing Problem

- Each agent must raise capital L_i
  –> E.g., without additional policy/tax: $L_i = I_0 + K(a_i, \theta)$.

- Verifiable output: Contract with *outside investor* specifies wlog repayment R if $x_i = x$.
  –> Uniquely optimal effort:

$$p'(e^*)(x - R) - c'(e^*) = 0.$$

  –> Break-even requirement

$$p(e^*)R = L.$$

Productive Inefficiency

- Denote total expected surplus ("net of funding requirement L")

$$\omega = p(e^*)(x - R) - c(e^*).$$

–> For all $L > 0$ we have

$$\omega(L) < \omega(0) - L.$$

- In fact, with differentiability

$$\frac{d\omega(L)}{dL} < -1.$$

Properties of Surplus Function

- Extension: **Stochastic contracts** –> Investor and agent specify lottery.
  - Over different expected (!) repayments L^n so that $E[L^n] = L$.
  - Corresponds for agent to a randomization over $\omega(L^n)$.
  - Denote

$$\widehat{\omega}(L) = \max E[\omega(L^n)].$$

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- **Claim**: $\widehat{\omega}(L)$ *is strictly concave*
  –> Argument: If not at L, then contradict by using "better lottery" over (L^1, L^2) with

$$L = L^1\beta^1 + L^2\beta^2 \text{ and } \widehat{\omega}(L^1)\beta^1 + \widehat{\omega}(L^2)\beta^2 > \widehat{\omega}(L).$$

- **Simplification:** Original surplus function $\omega(L)$ already strictly concave.

Benchmark 1: Controlling Avoidance

- Suppose **utilitarian government** could observe θ_i and control y_i. Maximizes

$$E[u_i] = \int [\omega(L_i) - \rho y_i]\, di.$$

- Thus pointwise maximization of

$$s_i = \omega(L_i) - \rho y_i$$

with

$$L_i = I_0 + K(\overline{y} - y, \theta_i).$$

Benchmark 1: Controlling Avoidance

- Denote $y_{SB}(\theta)$ and $a_{SB}(\theta) = \overline{y} - y_{SB}(\theta)$.
- First-order condition:

$$-K_1(a_{SB}(\theta), \theta) \cdot \omega' \left(I_0 + K(a_{SB}(\theta), \theta)\right) = \rho.$$

–> Marginal cost of avoidance still **strictly lower** than ρ.

Benchmark 1: Controlling Avoidance

- Side observation: While always

$$\frac{da_{SB}(\theta)}{d\theta} > 0,$$

total expenditures and thus need to raise finance behave as follows:

$$\frac{dK(a_{SB}(\theta), \theta)}{d\theta} > 0 \text{ if } K_2 K_{11} > K_1 K_{12} \qquad \text{(Case 1)}$$

$$\frac{dK(a_{SB}(\theta), \theta)}{d\theta} < 0 \text{ if } K_2 K_{11} < K_1 K_{12}. \qquad \text{(Case 2)}$$

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- **Example:** *Increasing* (Case 1) if

$$K(a, \theta) = \frac{1}{2\alpha}\frac{a^2}{\theta}.$$

Benchmark 2: With Redistribution

- Government observes θ_i and controls a_i **and** can redistribute resources –> Levy an ex-ante tax T_i so that

$$L_i = K(a_i, \theta_i) + T_i$$

and

$$\int T_i di = 0.$$

- Lagrange problem:

$$\mathbf{L} = E[u_i] + \eta \int T_i di.$$

Benchmark 2: With Redistribution

- Solution:
  - Choice $a_{RD}(\theta)$ satisfies

$$\eta K_1(a_{RD}(\theta), \theta) = \rho \text{ with } \eta > 1.$$

  - Choice $T_{RD}(\theta)$ ensures that $L_i = L_{RD}(\theta) = L_{RD}$
    –> Equalization of $\omega'(L_i) = \omega'(L_{RD})$.

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    –> Equalization of $\omega'(L_i) = \omega'(L_{RD})$.

- Intuition: By concavity of $\omega(L)$, **aggregate** productivity is highest when need to raise finance is made equal at all agents!

- Implied redistribution can go to either high- or low-type agents
  –> To whoever has higher expenditure under $a_{RD}(\theta)$ ("Case 1 or 2").

Linear Tax

- Tax rule:

$$\tau(y) = \tau_0 + \tau y,$$

which must satisfy

$$\tau_0 + \tau \int y_i di = 0.$$

- Resulting need to raise finance for type θ and choice y:

$$L(y, \theta) = I_0 + K(\overline{y} - y, \theta) + \tau(y),$$

using $a = \overline{y} - y$.

Outcome with Linear Tax

- Given tax, clearly optimal choice $a^*(\theta)$:

$$K_1(a^*(\theta), \theta) = \tau.$$

−> Implies that

$$\frac{dL(\theta)}{d\theta} = K_2(a^*(\theta), \theta) < 0 \text{ when } a^*(\theta) < 0.$$

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- **Optimal tax rate:**

$$\tau \left(- \int_\Theta \omega'(L(\theta)) dF(\theta) \right) = \rho - \frac{\int_\Theta \omega'(L(\theta)) \left[y^*(\theta) - \int_\Theta y^*(\theta') dF(\theta') \right] dF(\theta)}{\int_\Theta \frac{dy^*(\theta)}{d\tau} dF(\theta)}$$

where the last term is strictly positive.

Outcome with Linear Tax

- Short-hand: Optimal tax rate

$$\tau \left(- \int_{\Theta} \omega'(L(\theta)) dF(\theta) \right) = \rho - D \text{ with } D > 0.$$

- Thus two reasons for why $\tau < \rho$:
  - LHS-multiplier >1: "Average costs of outside financing".
  - RHS "subtraction": Loss of productive efficiency due to "redistribution".

Outcome with Linear Tax

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- Thus two reasons for why $\tau<\rho$:
  - LHS-multiplier $>$1: "Average costs of outside financing"
  - RHS "subtraction": Loss of productive efficiency due to "redistribution".
- **Redistribution**
  - always to high-type agents;
  - and thus always to agents who have *less* expenditures $K(\cdot)$ and thus *less* additional need for outside funding $L(\cdot)$.

Alternative Implementation

- Pollution capacity Y allocated uniformly over agents ($Y_i = Y$).
  –$>$ Generates uniform trading price

$$K_1(a^*(\theta), \theta) = \tau$$

together with

$$\int a^*(\theta) dF(\theta) = \overline{y} - Y.$$

- For each θ we have thus additional financing needs

$$\tau(y^*(\theta) - Y).$$

Comparison to Second-Best Benchmark

- There, no redistribution but direct control of $a_{SB}(\theta)$.
- If total expenditures are increasing with type under second-best benchmark, then
  - high types have strictly higher levels of avoidance under (redistributive) linear tax;
  - low types have strictly lower levels.
- Opposite prediction when $K(a_{SB}(\theta), \theta)$ is decreasing.

Nonlinear Taxes

- General $\tau(y)$.
  –> Can then *no longer* be implemented by allocating and trading "pollution rights".

Nonlinear Taxes

- General $\tau(y)$.
  –> Can then *no longer* be implemented by allocating and trading "pollution rights".

- **Mechanism design approach:** Specify $y(\theta)$ and $T(\theta)$.
  –> Stipulate wlog that loan size $L(\cdot)$ exactly equal to required funds
  - On-equilibrium ("truthtelling"):

$$L(y(\theta), T(\theta), \theta) = I_0 + K(\overline{y} - y(\theta), \theta) + T(\theta).$$

  - Off-equilibrium:

$$L(y(\widehat{\theta}), T(\widehat{\theta}), \theta) = I_0 + K(\overline{y} - y(\widehat{\theta}), \theta) + T(\widehat{\theta}).$$

- Restriction to continuous differentiable solutions
  –> Apply optimal control techniques.

Incentive Compatibility

- Local truthtelling condition:

$$\left.\frac{d\omega\left(L(y(\widehat{\theta}),\theta)\right)}{d\widehat{\theta}}\right|_{\widehat{\theta}=\theta} = T'(\theta) - y'(\theta)K_1(\overline{y} - y(\theta),\theta) = 0.$$

- With $u(\theta) = \omega\left(L(y(\theta), T(\theta), \theta)\right)$, likewise

$$\begin{aligned}\frac{du(\theta)}{d\theta} &= \left.\frac{\partial\omega\left(L(y(\widehat{\theta}), T(\widehat{\theta}),\theta)\right)}{\partial\theta}\right|_{\widehat{\theta}=\theta} \\ &= \omega'(\cdot)K_2(\overline{y} - y(\theta),\theta) > 0 \text{ when } y(\theta) > 0.\end{aligned}$$

Incentive Compatibility

- "First-order approach": Assume that local incentive compatibility already implies global incentive compatibility
  $->$ Requires that $y(\theta)$ is nondecreasing.

Control Problem

- Take as state variable

$$L(\theta) = I_0 + K(\overline{y} - y(\theta), \theta) + T(\theta).$$

–> From incentive compatibility and with $u(\theta) = \omega(L(\theta))$ must satisfy

$$\begin{aligned} \frac{du(\theta)}{d\theta} &= \frac{\partial u(\theta)}{\partial \theta} = \omega'(\cdot)K_2(\overline{y} - y(\theta), \theta) \text{ and} \\ \frac{du(\theta)}{d\theta} &= \omega'(\cdot)\frac{dL(\theta)}{d\theta} \end{aligned}$$

so that

$$\frac{dL(\theta)}{d\theta} = K_2(\overline{y} - y(\theta), \theta) < 0.$$

Control Problem

- Substitute pointwise for

$$T(\theta) = L(\theta) - [I_0 + K(\overline{y} - y(\theta), \theta)]$$

–> Single control variable $y(\theta)$.

Control Problem

- The objective is thus to maximize

$$\int_{\Theta} [\omega(L(\theta)) - \rho y(\theta)] \, dF(\theta)$$

subject to the "law of motion" $\frac{dL(\theta)}{d\theta} = K_2(\overline{y} - y(\theta), \theta)$ and the budget balance condition

$$\int_{\Theta} [L(\theta) - K(\overline{y} - y(\theta), \theta) - I_0] \, dF(\theta) = 0.$$

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$$\int_{\Theta} [L(\theta) - K(\overline{y} - y(\theta), \theta) - I_0]\, dF(\theta) = 0.$$

- Hamiltonian

$$\begin{aligned} H &= [\omega(L(\theta)) - \rho y(\theta)]\, f(\theta) \\ &\quad + \eta\, [L(\theta) - K(\overline{y} - y(\theta), \theta) - I_0]\, f(\theta) \\ &\quad + \lambda(\theta) K_2(\overline{y} - y(\theta), \theta). \end{aligned}$$

Characterization

- First-order condition for $y(\theta)$

$$f(\theta)\left[-\rho + \eta K_1(\overline{y} - y(\theta), \theta)\right] - \lambda(\theta) K_{12}(\overline{y} - y(\theta), \theta) = 0.$$

- Costate

$$\frac{\partial H}{\partial L} = -\lambda'(\theta) \Leftrightarrow f(\theta)\left[\omega'(L(\theta)) + \eta\right] = -\lambda'(\theta).$$

- **Implication:** As $L(\theta)$ is decreasing and $\omega(\cdot)$ concave
  –$>$ $\omega'(L(\theta)) + \eta < 0$ for low and $\omega'(L(\theta)) + \eta > 0$ for high types.
  –$>$ $\lambda'(\theta)$ first positive, then negative
  –$>$ $\lambda(\theta)$ is hump-shaped.

Characterization

- With transversality conditions given by

$$\begin{aligned} \lim_{\theta \to \overline{\theta}} \lambda(\theta) &= 0, \\ \lim_{\theta \to \underline{\theta}} \lambda(\theta) &= 0. \end{aligned}$$

–> Substitute to obtain "marginal social cost of spending more"

$$\eta = -\int_{\underline{\theta}}^{\overline{\theta}} \omega'(L(\vartheta))dF(\vartheta) > 1.$$

- And thus $\lambda(\theta)$ is hump-shaped, with zero at the boundaries.

Characterization

- Rearrange first-order condition for $y(\theta)$

$$\begin{aligned} \eta K_1(\overline{y} - y(\theta), \theta) &= \rho + \frac{\lambda(\theta)}{f(\theta)} K_{12}(\overline{y} - y(\theta), \theta) \\ &< \rho. \end{aligned}$$

- As with linear tax, **two reasons** for why $< \rho$!

Characterization

- First-order condition for $y(\theta)$

$$\eta K_1(\overline{y} - y(\theta), \theta) = \rho + \frac{\lambda(\theta)}{f(\theta)} K_{12}(\overline{y} - y(\theta), \theta)$$

- **Reason 1:** $\eta > 1$
  –> Average marginal costs of higher avoidance, arising from financial imperfection.

Characterization

- First-order condition for $y(\theta)$

$$\eta K_1(\overline{y} - y(\theta), \theta) = \rho + \frac{\lambda(\theta)}{f(\theta)} K_{12}(\overline{y} - y(\theta), \theta)$$

- **Reason 1:** $\eta > 1$
  –> Average marginal costs of higher avoidance, arising from financial imperfection.
- **Reason 2:** $\lambda(\theta) > 0$ –> Reduced average productive inefficiency due to redistribution.
  - $\lambda(\theta)$ is marginal increase in welfare resulting from a marginal shift of required financing from types below θ to types above θ
    –> Dampens redistribution!
  - No one benefitting from redistribution at $\underline{\theta}$, no one contributing at $\overline{\theta}$
    –> Thus $\lambda(\underline{\theta}) = \lambda(\overline{\theta}) = 0$.

Optimal Nonlinear Tax $\tau(y)$

- Note

$$T'(\theta) = y'(\theta)K_1(\cdot) \leq 0.$$

–> Still transfer to high-type agents!

Optimal Nonlinear Tax $\tau(y)$

- Now with $\tau(y) = T(\theta(y))$: Marginal tax on externality

$$
\begin{aligned}
\tau'(y) &= T'(\theta)\frac{d\theta}{dy} = \frac{T'(\theta)}{y'(\theta)} \\
&= K_1(\overline{y} - y, \theta) \\
&= \frac{1}{\eta}\left[\rho + \frac{\lambda(\theta)}{f(\theta)} K_{12}(\overline{y} - y, \theta)\right]
\end{aligned}
$$

where we use $\theta = \theta(y) = y^{-1}(y(\theta))$.

Optimal Nonlinear Tax $\tau(y)$

- This yields

$$\eta\tau''(y) = -\frac{\lambda(\theta)}{f(\theta)}K_{112} + \frac{d\theta}{dy}\left[\frac{\lambda(\theta)}{f(\theta)}K_{122} + K_{12}\frac{d}{d\theta}\left[\frac{\lambda(\theta)}{f(\theta)}\right]\right].$$

"Non-linearity"

- Take

$$y_l = y(\overline{\theta}) < y_h = y(\underline{\theta}).$$

- Evaluated at lowest and highest generated externality:

$$\eta\tau''(y_l) = \frac{d\theta}{dy}K_{12}\frac{\lambda'(\overline{\theta})}{f(\overline{\theta})} < 0 \text{ I.e., marginal tax decreases for very low } y.$$
$$\eta\tau''(y_h) = \frac{d\theta}{dy}K_{12}\frac{\lambda'(\underline{\theta})}{f(\underline{\theta})} > 0 \text{ I.e., marginal tax increases for very high } y.$$

- I.e., the marginal tax is *highest at the very low and the very high end*
–> "First units" and "last units" of avoidance are rewarded most.

Discussion: Taxes on Output?

- E.g., tax on positive outcome $z(\theta) < x$. Gives then rise to expected probability of success $p(\theta)$ and expected output tax $Z(\theta) = z(\theta)p(\theta)$
  –> Modified resource constraint

$$\int [T(\theta) + Z(\theta)]\, dF(\theta) = 0.$$

Discussion: Taxes on Output?

- E.g., tax on positive outcome $z(\theta) < x$. Gives then rise to expected probability of success $p(\theta)$ and expected output tax $Z(\theta) = z(\theta)p(\theta)$
–> Modified resource constraint

$$\int [T(\theta) + Z(\theta)]\, dF(\theta) = 0.$$

- **Alternative implementation**:
–> Suppose that instead up-front redistribution according to

$$\widetilde{T}(\theta) = T(\theta) + Z(\theta).$$

  – Requires to raise finance by $Z(\theta)$.

  – Leads to same $p(\theta)$.

Loan-Based Grants

- Idea: Tax externality y *and* provide grant when, together with avoidance a, a firm raises credit L.
  –> Tax $\tau(y)$ together with loan-based grant $g(L)$.

- But still relevance of private information: Raise L to obtain grant even though not needed
  –> Benefit: Additional grant / subsidy.
  –> Cost: Higher-than-necessary inefficiency from outside financing!

- Question: Can loan-based grants be used as an additional instrument? Implications for optimal policy?

Linear Tax with Subsidized Loan

- Incentives to reduce externalities given by linear tax τ
  –> Leads to choice $y^*(\theta)$.

- In addition, to counteract the implied redistribution, government specifies a tax $t(\theta)$
  –> Without additional "instrument", $t(\theta) = \tau_0$, so that again

$$T(\theta) = y^*(\theta)\tau + \tau_0.$$

- Then, recall that using $K_1(\cdot) = \tau$:

$$\frac{du(\theta)}{d\theta} = \omega'(L(\theta))\frac{dL(\theta)}{d\theta} = \omega'(L(\theta))K_2(a^*(\theta), \theta).$$

Linear Tax with Subsidized Loan

- Additional instrument: Minimum loan size is observable.
  –> Stipulate $L(\theta)$ together with $t(\theta)$ ("grant", will be decreasing).

Linear Tax with Subsidized Loan

- As $L(\theta)$ will be strictly increasing, need to consider (only) downward deviations –> I.e., mimic $\widehat{\theta} \leq \theta$ and derive utility

$$u(\theta, \widehat{\theta}) = \omega(L(\widehat{\theta})) + \left[L(\widehat{\theta}) - I_0 - K(\overline{y} - y^*(\theta), \theta) - \tau y^*(\theta) - t(\widehat{\theta}) \right].$$

Linear Tax with Subsidized Loan

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- When $t(\theta)$ with $t'(\theta) < 0$ shall be made as steep as possible, then we have

$$\frac{du(\theta)}{d\theta} = \frac{\partial u(\theta)}{\partial \theta} = -K_2(a^*(\theta), \theta)$$

and

$$t'(\theta) = -K_2(a^*(\theta), \theta)\frac{1 - \omega'(L(\theta))}{\omega'(L(\theta))} < 0.$$

- Intuition: Costly to pretend to need a higher loan!

Nonlinear Tax Plus Loan-based Grant

- Again mechanism-design approach: Next to $y(\theta)$ and $T(\theta)$ specify a *minimum* loan size $L(\theta)$.

- Agents could secretly raise higher outside finance, but will not arise in equilibrium.

- Key constraint: As $L(\theta)$ will be strictly increasing, need to consider (only) downward deviations
  –> I.e., mimic $\widehat{\theta} \leq \theta$ and derive utility

$$u(\theta, \widehat{\theta}) = \omega\left(L(\widehat{\theta})\right) + K(\overline{y} - y(\widehat{\theta}), \widehat{\theta}) - K(\overline{y} - y(\widehat{\theta}), \theta).$$

  –> Thus marginal benefits from deviating are again given by $K_2(\cdot)$!

Nonlinear Tax Plus Loan-based Grant

- In summary: Change to nonlinear tax without loan-based grant is "law-of-motion" for state variable $L(\theta)$ (Recall $u(\theta) = \omega\left(L(\theta)\right)$!)

$$L'(\theta) = -\frac{K_2(\overline{y} - y(\theta), \theta)}{\omega'(L(\theta))}.$$

- And first-order condition for $y(\theta)$

$$\eta K_1(\overline{y} - y(\theta), \theta) = \rho + \lambda(\theta)\frac{K_{12}(\overline{y} - y(\theta), \theta)}{\omega'(L(\theta))}.$$

Concluding Remarks

- Objective: Analyze optimal policy towards externalities in light of two constraints
  - Need to raise outside finance to avoid externalities, which is "costly" due to financial imperfections (agency problem);
  - Marginal avoidance costs are private information (vis-á-vis policymaker).
- Interaction of the two problems: Tax on externality leads to redistribution of resources, which leads to reduced aggregate efficiency.
  –> Utilitarian government would want to redistribute.

Concluding Remarks

- Finding 1: Optimal linear tax strictly smaller than "first-best" Pigou tax.
  –> Two reasons!
- Finding 2: Higher efficiency with nonlinear tax.
- Finding 3: Higher efficiency with tax on externalities *plus* loan-based grants.

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