Additional Condition Moment Constraints Tests

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Introduction
Objectives

Cross-Section/Short Panels.
Conditional moment models.

- Tests for additional conditional moment constraints.

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- Asymptotic null distribution.
- Asymptotic local alternative distribution.
- Simulation experiments.

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- Standardization. Asymptotically standard normal variate. Cf. chi-square distribution.

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- Replace by corresponding sets of unconditional moment restrictions. First set subset of second.
- Interpret as standard tests for additional (unconditional) moment restrictions.
- Standardization. Asymptotically standard normal variate. Cf. chi-square distribution.
- Efficient parameter estimation unnecessary.

Introduction
Outline

- Additional conditional moments. Examples.
- Restricted/unrestricted moment tests.
- Null limiting ditribution.
- Local alternative limiting distribution.
- Simulation evidence.

Some Preliminaries
Definitions

Data `i.i.d.`

General conditional moment context.

Error vector $u(z, \beta_0)$.

J_u-vector known up to p_β-vector parameters β_0.
Parameter space $\mathcal{B}$.

IV w.

Maintained hypothesis

$$E[u(z, \beta_0)|w] = 0 \text{ some } \beta_0 \in \mathcal{B}.$$

Some Preliminaries
Test Problem

Generic random vector s.
Possible w excluded or included in s.

Additional error vector $v(z, \alpha_0)$.

J_v-vector known up to p_α-vector parameters α_0.
Parameter space $\mathcal{A}$.

Null hypothesis

$$H_0 : E[v(z, \alpha_0)|s] = 0 \text{ some } \alpha_0 \in \mathcal{A}, E[u(z, \beta_0)|w] = 0.$$

Alternative hypothesis

$$H_1 : E[v(z, \alpha)|s] \neq 0 \text{ any } \alpha \in \mathcal{A}, E[u(z, \beta_0)|w] = 0.$$

Some Preliminaries
Examples

EXAMPLE 2.1 (CONDITIONAL HOMOSKEDASTICITY)

$J_u = 1$ for simplicity.
Here $s = w$.
Set

$$v(z, \alpha) = u(z, \beta)^2 - \sigma^2.$$

Thus $\alpha = (\beta, \sigma^2)$.

Null hypothesis.

$$H_0 : \sigma_0^2 = E[u(z, \beta_0)^2 | w] \text{ all } w, E[u(z, \beta_0) | w] = 0.$$

REMARK 2.1

Regression.

$$u(z, \beta) = y - \beta x.$$

Moment indicator vextor.

$$wu(z, \beta) = w(y - \beta x).$$

CUE metric. Inverse.

$$E_n[ww'(y - \beta x)^2].$$

LIML metric. Inverse.

$$\sigma^2 E_n[ww'].$$

Some Preliminaries
Examples

EXAMPLE 2.2 (INSTRUMENT VALIDITY)

$J_u = 1$ for simplicity.
Additional IV x.
Set

$$v(z, \alpha) = u(z, \beta).$$

Thus $\alpha = \beta$.

Null hypothesis.

$$H_0 : E[u(z, \beta_0)|s] = 0, E[u(z, \beta_0)|w] = 0.$$

Special cases: $s = x$; $s = (w, x)$.

REMARK 2.2

Regression.
Marginal Exogeneity.
$s = x$.
$E[y - \beta_0 x|x] = 0$. I.e., $E[y|x] = \beta_0 x$. LS β_0 consistent.
LS inefficient. Neglects maintained $E[u(z, \beta_0)|w] = 0$.

IV β_0 estimation.
Joint conditional maintained $E[y - \beta_0 x|w] = 0$ and null
$E[y - \beta_0 x|x] = 0$ moments.
At least as efficient as LS and IV using only maintained
$E[y - \beta_0 x|w] = 0$.

Control.

x control. Average effect of x on y predictable.

w control. Impact on y of w requires $E[x|w]$.
$E[y - \beta_0 x|x] = 0$ uninformative.

Effect of w on y given x requires $E[y|w, x]$.

REMARK 2.3

Regression.
Conditional Exogeneity.
$s = (w, x)$.
$E[y - \beta_0 x | w, x] = 0$. I.e., $E[y | w, x] = \beta_0 x$.
LS β_0 consistent but inefficient.

CE implies ME. More stringent than ME.

IV using null $E[y - \beta_0 x | w, x] = 0$ efficient.

Control.

w control.

Effect of w only on y same for CE and ME. I.e., $E[y|w] = E[x|w]\beta_0$.

Effect of w on y given x nil. I.e., $E[y|w, x] = \beta_0 x$.

GMM and GEL Test Statistics

Approximating Conditional Moment Restrictions

Conditional moment conditions equivalent to countable number of unconditional moment restrictions.

K positive integer. Let $q^K(s) = (q_{1K}(s), ..., q_{KK}(s))'$ K-vector of approximating functions.

Assumption: for all K for any $a(s)$ with $E[a(s)^2] < \infty$ K-vectors γ_K exist such that

$$E[(a(s) - q^K(s)'\gamma_K)^2] \to 0 \text{ as } K \to \infty.$$

REMARK 3.1: Admissible approximating functions: splines, power series and Fourier series.

Unconditional moment indicator: $g(z, \beta) = u(z, \beta) \otimes q^K(s)$.
(Unconditional) moment conditions: $E[g(z, \beta_0)] = 0$.

$K \to \infty$.

EL, IV, GMM or GEL:

- consistent;
- asymptotically normal;
- semi-parametrically efficient.

Re-interpretation.

- Maintained hypothesis. Approximating functions $q_1^K(\cdot)$ with $s = w$.

$$E[u(z, \beta_0) \otimes q_1^K(w)] = 0, K \to \infty.$$

Dimension $J_u K$.

Re-interpretation.

- Maintained hypothesis. Approximating functions $q_1^K(\cdot)$ with $s = w$.

$$E[u(z, \beta_0) \otimes q_1^K(w)] = 0, K \to \infty.$$

Dimension $J_u K$.

- Null hypothesis. Additional approximating functions $q_0^K(s)$.

$$E[v(z, \alpha_0) \otimes q_0^K(s)] = 0, K \to \infty.$$

Dimension $J_v MK$, $M > 0$.

$$E\left[\begin{array}{c} u(z, \beta_0) \otimes q_1^K(w) \\ v(z, \alpha_0) \otimes q_0^K(s) \end{array}\right] = 0, K \to \infty.$$

Dimension $(J_u + J_v M)K$.

REMARK 4.1

Require $O(K)$ for $q_0^K(s)$ dimension.
Test statistics difference of two statistics.
Same order of magnitude dimension of $q_0^K(s)$ needs $O(K)$.
Otherwise $\max[K, \dim(q_0^K(s))]$ statistic dominates asymptotic behaviour.
Either maintained or additional moment restrictions ignored asymptotically.
N.B. Dimension of w and s independent of K.

GMM and GEL Test Statistics

Examples (Cont.)

EXAMPLE 2.1 (CONDITIONAL HOMOSKEDASTICITY CONT.)

$J_u = 1$ for simplicity.

Recall

$$v(z, \alpha) = u(z, \beta)^2 - \sigma^2.$$

Null hypothesis.

$$H_0 : \sigma_0^2 = E[u(z, \beta_0)^2 | w] \text{ all } w, E[u(z, \beta_0) | w] = 0.$$

Here $s = w$. Additional approximating functions $q_0^K(s) = q_1^K(w)$.

$$E[v(z, \alpha_0) \otimes q_1^K(w)] = 0, K \to \infty.$$

Thus $M = 1$.

GMM and GEL Test Statistics

Examples (Cont.)

EXAMPLE 2.2 (INSTRUMENT VALIDITY CONT.)

$J_u = 1$ for simplicity.
Additional IV x.
Recall

$$v(z, \alpha) = u(z, \beta).$$

Null hypothesis.

$$H_0 : E[u(z, \beta_0)|s] = 0, E[u(z, \beta_0)|w] = 0.$$

Additional approximating functions $q_0^K(s)$.

$$E[u(z, \beta_0) \otimes q_0^K(s)] = 0, K \to \infty.$$

Here MK finite positive integer.
Special cases:
$s = x$: $q_0^K(s)$ functions of x only;
$s = (w, x)$: $q_0^K(s)$ additional functions of w and x.

GMM and GEL Test Statistics

Definitions and Assumptions

$\sqrt{n}(\hat{\alpha} - \alpha_0) = O_p(1)$;

$g_i(\beta) = u(z_i, \beta) \otimes q_1^K(w_i)$, $(i = 1, ..., n)$, $\hat{g}(\beta) = \sum_{i=1}^n g_i(\beta)/n$;

$h_i(\alpha) = (u(z_i, \beta)' \otimes q_1^K(w_i)', v(z_i, \alpha)' \otimes q^K(s_i)')'$, $(i = 1, ..., n)$,
$\hat{h}(\alpha) = \sum_{i=1}^n h_i(\alpha)/n$;

$\hat{\Omega} = \sum_{i=1}^n g_i(\hat{\beta}) g_i(\hat{\beta})'/n$; $\hat{\Xi} = \sum_{i=1}^n h_i(\hat{\alpha}) h_i(\hat{\alpha})'/n$.

GMM and GEL Test Statistics
Conditional GMM Statistics

Maintained hypothesis.

$$\mathcal{T}^{g}_{GMM} = n\hat{g}(\hat{\beta})'\hat{\Omega}^{-1}\hat{g}(\hat{\beta}).$$

Null hypothesis.

$$\mathcal{T}^{h}_{GMM} = n\hat{h}(\hat{\alpha})'\hat{\Xi}^{-1}\hat{h}(\hat{\alpha}).$$

Restricted tests: incorporate maintained hypothesis $E[u(z, \beta_0)|w] = 0$.

Unrestricted tests: ignore maintained hypothesis $E[u(z, \beta_0)|w] = 0$.

Restricted GMM statistic.

Difference of GMM criterion function statistics $\mathcal{T}^h_{GMM}$ and $\mathcal{T}^g_{GMM}$.

Nonstandardised statistic. Fixed and finite K: limiting chi-square distributed with $J_v MK$ degrees of freedom.

Standardised statistic. $K \to \infty$: limiting $N(0,1)$ distributed. Subtract mean $J_v MK$; divide by standard deviation $\sqrt{2J_v MK}$.

$$\mathcal{J} = \frac{\mathcal{T}^h_{GMM} - \mathcal{T}^g_{GMM} - J_v MK}{\sqrt{2J_v MK}}.$$

GMM and GEL Test Statistics
Conditional GEL Statistics

$\rho(v)$: concave on open interval $\mathcal{V}$ containing 0.
$\rho_j(v) = \partial^j \rho(v)/\partial v^j$, $\rho_j = \rho_j(0)$, $(j = 0, 1, 2, ...)$, $\rho_1 = \rho_2 = -1$.

GEL criteria.

Maintained hypothesis.

$$\hat{P}_n(\beta, \lambda) = \sum_{i=1}^{n} [\rho\left(\lambda' g_i(\beta)\right) - \rho_0]/n.$$

Null hypothesis.

$$\tilde{P}_n(\alpha, \eta) = \sum_{i=1}^{n} [\rho\left(\eta' h_i(\alpha)\right) - \rho_0]/n.$$

$\hat{\Lambda}_n(\beta) = \{\lambda : \lambda' g_i(\beta) \in \mathcal{V}, i = 1, ..., n\}$; $\tilde{\Lambda}_n(\alpha) = \{\eta : \eta' h_i(\alpha) \in \mathcal{V}$, $i = 1, ..., n\}$.

Lagrange multiplier estimators. Given β

$$\hat{\lambda}(\beta) = \arg \max_{\lambda \in \hat{\Lambda}_n(\beta)} \hat{P}_n(\beta, \lambda), \tilde{\eta}(\alpha) = \arg \max_{\eta \in \tilde{\Lambda}_n(\alpha)} \tilde{P}_n(\alpha, \eta).$$

Given $\hat{\beta}$

$$\hat{\lambda} = \hat{\lambda}(\hat{\beta}), \tilde{\eta} = \tilde{\eta}(\hat{\alpha}).$$

$\hat{\eta} = S_g \hat{\lambda}$.

$s(z, \alpha) = v(z, \alpha) \otimes q_0^K(s) = S_0' h(z, \alpha)$. $s_i(\alpha) = s(z_i, \alpha)$, $(i = 1, ..., n)$.

Restricted GEL LR statistic.

$$\mathcal{LR} = \frac{2n[\tilde{P}_n(\hat{\alpha},\tilde{\eta}) - \hat{P}_n(\hat{\beta},\hat{\lambda})] - J_v MK}{\sqrt{2J_v MK}}.$$

Restricted LM, score and Wald-type statistics.

$$\mathcal{LM} = \frac{n(\tilde{\eta} - \hat{\eta})'\hat{\Xi}(\tilde{\eta} - \hat{\eta}) - J_v MK}{\sqrt{2J_v MK}}.$$

$$\mathcal{S} = \frac{\sum_{i=1}^n \rho_1(\hat{\lambda}' g_i(\hat{\beta}))s_i(\hat{\alpha})' S_0' \hat{\Xi}^{-1} S_0 \sum_{i=1}^n \rho_1(\hat{\lambda}' g_i(\hat{\beta}))s_i(\hat{\alpha})/n - J_v MK}{\sqrt{2J_v MK}}.$$

$$\mathcal{W} = \frac{n\tilde{\eta}' S_0 (S_0' \hat{\Xi}^{-1} S_0)^{-1} S_0' \tilde{\eta} - J_v MK}{\sqrt{2J_v MK}}.$$

Asymptotic Null Distribution

Restricted statistics.

Theorem 4.1 $K \to \infty$, $\zeta(K)^2 K^2/n \to 0$. Then

$$\mathcal{J} \xrightarrow{d} N(0,1).$$

Theorem 4.2 $K \to \infty$, $\zeta(K)^2 K^3/n \to 0$. Then

$$\mathcal{LR}, \mathcal{LM}, \mathcal{S}, \mathcal{W} \xrightarrow{d} N(0,1)$$

and

$$\mathcal{J} - \mathcal{GEL} \xrightarrow{p} 0 \text{ where } \mathcal{GEL} = \mathcal{LR}, \mathcal{LM}, \mathcal{S}, \mathcal{W}.$$

REMARK 4.1: Asymptotic independence. Restricted GMM statistic $\mathcal{J}$ and maintained hypothesis GMM statistic

$$\mathcal{J}^g = \frac{\mathcal{T}^g_{GMM} - J_u K}{\sqrt{2 J_u K}} \xrightarrow{d} N(0,1).$$

REMARK 4.2: Similar result for restricted GEL statistics $\mathcal{LR}$, $\mathcal{LM}$, $\mathcal{S}$ and $\mathcal{W}$.

REMARK 4.3: Overall asymptotic size controllable.

Unrestricted GEL statistics.

$$\mathcal{LR}^h = \frac{2n\tilde{P}_n(\hat{\alpha},\tilde{\eta}) - (J_u + J_v M)K}{\sqrt{2(J_u + J_v M)K}}, \mathcal{LM}^h = \frac{n\tilde{\eta}'\hat{\Xi}\tilde{\eta} - (J_u + J_v M)K}{\sqrt{2(J_u + J_v M)K}}$$

Unrestricted GMM statistic. Cf. $\mathcal{T}^h_{GMM}$. Score form

$$\mathcal{S}^h = \frac{n\hat{h}(\hat{\alpha})'\hat{\Xi}^{-1}\hat{h}(\hat{\alpha}) - (J_u + J_v M)K}{\sqrt{2(J_u + J_v M)K}}.$$

Remark 4.4: $\mathcal{LR}^h, \mathcal{LM}^h, \mathcal{S}^h \xrightarrow{d} N(0,1)$. Mutually asymptotically equivalent but not to restricted $\mathcal{J}$, $\mathcal{LR}$, $\mathcal{LM}$, $\mathcal{S}$ and $\mathcal{W}$.

Remark 4.5: $\mathcal{LR}^h$, $\mathcal{S}^h$ forms of GMM and GEL statistics suggested elsewhere.

Asymptotic Local Power

Local alternatives.

$$H_{1n} : E[v(z, \alpha_{n,0})|w, x] = \frac{\sqrt[4]{J_v MK}}{\sqrt{n}} \xi(w, x),$$

REMARK 5.1: $\alpha_{n,0} \to \alpha_0$; $E[u(z, \beta_{0,n})|w] \to 0$.

REMARK 5.2: Apposite for general s.

$$E[v(z, \alpha_{n,0})|s] = \frac{\sqrt[4]{J_v MK}}{\sqrt{n}} E[\xi(w, x)|s].$$

Theorem 5.1 $K \to \infty$, $\zeta(K)^2 K^2/n \to 0$. Then

$$\mathcal{J} \xrightarrow{d} N(\mu/\sqrt{2}, 1).$$

$\mu = E[\xi(w,x)'\Sigma(w,x)^{-1}\xi(w,x)]; \Sigma(w,x) = E[v(z,\alpha_0)v(z,\alpha_0)'|w,x]$.

REMARK 5.3: $K \to \infty$, $\zeta(K)^2 K^3/n \to 0$.

$$\mathcal{LR}, \mathcal{LM}, \mathcal{S}, \mathcal{W} - \mathcal{J} \xrightarrow{p} 0.$$

REMARK 5.4: Tests one-sided.

REMARK 5.5: Consistency of tests based $\mathcal{J}$, $\mathcal{LR}$, $\mathcal{LM}$, $\mathcal{S}$ and $\mathcal{W}$.

Corollary 5.1 $K \to \infty$, $\zeta(K)^2 K^2/n \to 0$. *Then*

$$\mathcal{S}^h \xrightarrow{d} N(\mu_h/\sqrt{2}, 1).$$

with

$$\mu_h = \sqrt{\frac{J_v M}{(J_u + J_v M)}}\mu.$$

REMARK 5.6: $K \to \infty$, $\zeta(K)^2 K^3/n \to 0$.

$$\mathcal{LR}^h, \mathcal{LM}^h - \mathcal{S}^h \xrightarrow{p} 0.$$

REMARK 5.7: Justifies $q_0^K(s)$ dimension linear in K.

REMARK 5.8: Choose M as small as possible.

Simulation Evidence

Regression

$$y = \beta_0 x + u,$$

Simplicity. $\beta_0 = 0$. Single parameter β_0 to ease GEL estimation.

Simulation Evidence
DGP

Covariate x. IV w.
z_x and z_w jointly N mean 0, variance 1, correlation coefficient ρ, $\rho \in (-1,1)\backslash 0$.
Set $\rho = 0.7. x = \Phi(z_x)$ and $w = \Phi(z_w)$.

Error u. Simplicity. $u = v/\sqrt{var[v]}$.

$$v = a[z_x^2 + z_w^2 - (\frac{1+\rho^2}{\rho})z_w z_x - (1-\rho^2)] + \tau(z_x - \rho z_w) + \upsilon.$$

$\upsilon \sim N(0,1)$ independent of z_x and z_w.
$var[v] = a^2(1+\rho^2)(\rho^{-1}-\rho)^2 + \tau^2(1-\rho^2) + 1$.

Simulation Evidence
Properties

- (a) maintained $E[u|w] = 0$ satisfied;

Simulation Evidence
Properties

- (a) maintained $E[u|w] = 0$ satisfied;
- (b) ME hypothesis

$$E[u|x] = \tau(1-\rho^2)\Phi^{-1}(x)/var[v].$$

Thus $E[u|x] = 0$ if $\tau = 0$ and $E[u|x] \neq 0$ if $\tau \neq 0$;

Simulation Evidence
Properties

- (a) maintained $E[u|w] = 0$ satisfied;
- (b) ME hypothesis

$$E[u|x] = \tau(1-\rho^2)\Phi^{-1}(x)/var[v].$$

Thus $E[u|x] = 0$ if $\tau = 0$ and $E[u|x] \neq 0$ if $\tau \neq 0$;

- (c) CE hypothesis,

$$\begin{aligned} E[u|w,x] &= (a[\Phi^{-1}(x)^2 + \Phi^{-1}(w)^2 - \\ &\quad (\frac{1+\rho^2}{\rho})\Phi^{-1}(x)\Phi^{-1}(w) - (1-\rho^2)] \\ &\quad +\tau[\Phi^{-1}(x) - \rho\Phi^{-1}(w)])/var[v]. \end{aligned}$$

Hence $E[u|w,x] = 0$ if $a = \tau = 0$ and $E[u|w,x] \neq 0$ if $a \neq 0$ or $\tau \neq 0$.

Empirical size

Sample sizes $n = 200, 500, 1000$ and 3000.
Nominal size 0.05.

Empirical power

Sample sizes $n = 200$ and 500.

Two designs:
a varies and $\tau = 0$, i.e., ME holds but CE does not unless $a = 0$;
$a = 0$ and τ varies, i.e., both ME and CE do not hold unless $\tau = 0$.

5000 replications.

Simulation Evidence
Choice of the Number of Instruments

Require $K^4/n \to 0$.

Donald, Imbens and Newey (2009) method. Choice $K = 2$.
Explore $K = 2$ and $K = 3$ or 5.

REMARK 6.1: Alternatively information criteria such as AIC or BIC.

ME $E[u|x] = 0$: $K^{\text{M}} = [\text{A}_{\text{M}} K]$. Choices $\text{A}_{\text{M}} = 1$ or 1.5.

CE $E[u|w, x] = 0$: $K^{\text{C}} = [(\text{A}_{\text{C}} K)^{1/2}]$. Choices $\text{A}_{\text{C}} = 2$ or 4.5.

REMARK 6.2: A_{M} and A_{C} mimic M.

Numbers of instruments.

ME.

		$A_M = 1$		$A_M = 1.5$
K	K^M	Total Number of Instruments	K^M	Total Number of Instruments
2	2	3	3	4
3	3	5	4	6
5	5	9	7	11

CE.

		$A_C = 2$		$A_C = 4.5$
K	K^C	Total Number of Instruments	K^C	Total Number of Instruments
2	2	4	3	8
3	2	5	3	9
5	3	11	4	17

Simulation Evidence
Empirical Size

Nominal size approximated relatively more closely by empirical size

- (a) the non-standardised tests;
- (b) tests based on efficient estimators;
- (c) the score-type statistic $\bar{\mathcal{S}}_l^i(k)$ robust to estimation effects.

Wald versions $\mathcal{W}_l^i(j)$, $\bar{\mathcal{W}}_l^i(j)$ poor empirical size properties.

Results: $K = 2, 5$; $K = 3$ similar $K = 2$.

ME $E[u|x] = 0$

$K = 2, 5$; $\mathrm{A_M} = 1, 1.5$.

Summary.

- Unrestricted $\mathcal{LR}_{\mathrm{EL}}^{\text{DIN-M}}(\mathrm{EL_M})$, $\mathcal{LR}_{\mathrm{ET}}^{\text{DIN-M}}(\mathrm{ET_M})$, $\mathcal{LM}_{\mathrm{EL}}^{\text{DIN-M}}(\mathrm{EL_M})$, $\mathcal{LM}_{\mathrm{ET}}^{\text{DIN-M}}(\mathrm{ET_M})$ size distortions for $n = 200, 500$.

ME $E[u|x] = 0$

$K = 2, 5$; $A_M = 1, 1.5$.

Summary.

- Unrestricted $\mathcal{LR}_{EL}^{\text{DIN-M}}(\text{EL}_M)$, $\mathcal{LR}_{ET}^{\text{DIN-M}}(\text{ET}_M)$, $\mathcal{LM}_{EL}^{\text{DIN-M}}(\text{EL}_M)$, $\mathcal{LM}_{ET}^{\text{DIN-M}}(\text{ET}_M)$ size distortions for $n = 200, 500$.
- Unrestricted $\mathcal{J}^{\text{DIN-M}}(\text{GMM}_M)$, $\mathcal{S}_{CUE}^{\text{DIN-M}}(\text{CUE}_M)$, $\mathcal{S}_{EL}^{\text{DIN-M}}(\text{EL}_M)$, $\mathcal{S}_{ET}^{\text{DIN-M}}(\text{ET}_M)$ satisfactory.

ME $E[u|x] = 0$

$K = 2, 5$; $\mathrm{A}_{\mathrm{M}} = 1, 1.5$.

Summary.

- Unrestricted $\mathcal{LR}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL}_{\mathrm{M}})$, $\mathcal{LR}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET}_{\mathrm{M}})$, $\mathcal{LM}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL}_{\mathrm{M}})$, $\mathcal{LM}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET}_{\mathrm{M}})$ size distortions for $n = 200, 500$.
- Unrestricted $\mathcal{J}^{\mathrm{DIN\text{-}M}}(\mathrm{GMM}_{\mathrm{M}})$, $\mathcal{S}_{\mathrm{CUE}}^{\mathrm{DIN\text{-}M}}(\mathrm{CUE}_{\mathrm{M}})$, $\mathcal{S}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL}_{\mathrm{M}})$, $\mathcal{S}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET}_{\mathrm{M}})$ satisfactory.
- Given n. Deterioration as K increases for fixed A_{M}; as A_{M} increases for fixed K.

ME $E[u|x] = 0$

$K = 2, 5$; $\mathrm{A_M} = 1, 1.5$.

Summary.

- Unrestricted $\mathcal{LR}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL_M})$, $\mathcal{LR}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET_M})$, $\mathcal{LM}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL_M})$, $\mathcal{LM}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET_M})$ size distortions for $n = 200, 500$.
- Unrestricted $\mathcal{J}^{\mathrm{DIN\text{-}M}}(\mathrm{GMM_M})$, $\mathcal{S}_{\mathrm{CUE}}^{\mathrm{DIN\text{-}M}}(\mathrm{CUE_M})$, $\mathcal{S}_{\mathrm{EL}}^{\mathrm{DIN\text{-}M}}(\mathrm{EL_M})$, $\mathcal{S}_{\mathrm{ET}}^{\mathrm{DIN\text{-}M}}(\mathrm{ET_M})$ satisfactory.
- Given n. Deterioration as K increases for fixed $\mathrm{A_M}$; as $\mathrm{A_M}$ increases for fixed K.
- Restricted test statistics. Similar conclusions.

ME $E[u|x] = 0$

$K = 2, 5$; $\mathrm{A_M} = 1, 1.5$.

Summary.

- Unrestricted $\mathcal{LR}_{\mathrm{EL}}^{\text{DIN-M}}(\mathrm{EL_M})$, $\mathcal{LR}_{\mathrm{ET}}^{\text{DIN-M}}(\mathrm{ET_M})$, $\mathcal{LM}_{\mathrm{EL}}^{\text{DIN-M}}(\mathrm{EL_M})$, $\mathcal{LM}_{\mathrm{ET}}^{\text{DIN-M}}(\mathrm{ET_M})$ size distortions for $n = 200, 500$.
- Unrestricted $\mathcal{J}^{\text{DIN-M}}(\mathrm{GMM_M})$, $\mathcal{S}_{\mathrm{CUE}}^{\text{DIN-M}}(\mathrm{CUE_M})$, $\mathcal{S}_{\mathrm{EL}}^{\text{DIN-M}}(\mathrm{EL_M})$, $\mathcal{S}_{\mathrm{ET}}^{\text{DIN-M}}(\mathrm{ET_M})$ satisfactory.
- Given n. Deterioration as K increases for fixed $\mathrm{A_M}$; as $\mathrm{A_M}$ increases for fixed K.
- Restricted test statistics. Similar conclusions.
- Restricted $\mathcal{J}^{\mathrm{M}}(\mathrm{GMM_M}, \mathrm{GMM_{MA}})$, $\mathcal{LR}_{\mathrm{CUE}}^{\mathrm{M}}(\mathrm{CUE_M}, \mathrm{CUE_{MA}})$, $\bar{\mathcal{S}}_{\mathrm{EL}}^{\mathrm{M}}(\mathrm{EL_M})$, $\bar{\mathcal{S}}_{\mathrm{ET}}^{\mathrm{M}}(\mathrm{ET_M})$ most satisfactory.

CE $E[u|w,x] = 0$

$K = 2, 5$; $\mathrm{A}_{\mathrm{C}} = 2, 4.5$.

Summary

- General conclusions quite similar to ME.

CE $E[u|w,x] = 0$

$K = 2, 5$; $\mathrm{A_C} = 2, 4.5$.

Summary

- General conclusions quite similar to ME.
- Overall performance worse for larger $K = 5$ and $\mathrm{A_M} = 4.5$.

CE $E[u|w,x] = 0$

$K = 2, 5$; $A_C = 2, 4.5$.

Summary

- General conclusions quite similar to ME.
- Overall performance worse for larger $K = 5$ and $A_M = 4.5$.
- Unrestricted $\mathcal{J}^{\text{DIN-C}}(\text{GMM}_C)$, $\mathcal{S}^{\text{DIN-C}}_{\text{CUE}}(\text{CUE}_C)$, $\mathcal{S}^{\text{DIN-C}}_{\text{EL}}(\text{EL}_C)$, $\mathcal{S}^{\text{DIN-C}}_{\text{ET}}(\text{ET}_C)$ satisfactory.

CE $E[u|w,x] = 0$

$K = 2, 5$; $\text{A}_{\text{C}} = 2, 4.5$.

Summary

- General conclusions quite similar to ME.
- Overall performance worse for larger $K = 5$ and $\text{A}_{\text{M}} = 4.5$.
- Unrestricted $\mathcal{J}^{\text{DIN-C}}(\text{GMM}_{\text{C}})$, $\mathcal{S}^{\text{DIN-C}}_{\text{CUE}}(\text{CUE}_{\text{C}})$, $\mathcal{S}^{\text{DIN-C}}_{\text{EL}}(\text{EL}_{\text{C}})$, $\mathcal{S}^{\text{DIN-C}}_{\text{ET}}(\text{ET}_{\text{C}})$ satisfactory.
- Restricted GMM $\mathcal{J}^{\text{C}}(\text{GMM}_{\text{C}},\text{GMM}_{\text{MA}})$, $\mathcal{LR}^{\text{C}}_{\text{CUE}}(\text{CUE}_{\text{C}},\text{CUE}_{\text{MA}})$, $\bar{\mathcal{S}}^{\text{C}}_{\text{EL}}(\text{EL}_{\text{MA}})$, $\bar{\mathcal{S}}^{\text{C}}_{\text{ET}}(\text{ET}_{\text{MA}})$ most satisfactory.

Simulation Evidence
Empirical Size-Adjusted Power

$K = 2$.

Size-adjusted power declines for larger $K = 5$.

Power increases substantially with n.

$\tau = 0$

ME $E[u|x] = 0$ holds. CE $E[u|w,x] = 0$ fails unless $a = 0$.

ME tests

Power closely approximates nominal size.

CE tests

Unrestricted tests.

Small a. $\mathcal{LM}_{\text{EL}}^{\text{DIN-C}}(\text{EL}_{\text{C}})$, $\mathcal{LM}_{\text{ET}}^{\text{DIN-C}}(\text{ET}_{\text{C}})$ maximum power. $\mathcal{LR}_{\text{EL}}^{\text{DIN-C}}(\text{EL}_{\text{C}})$, $\mathcal{LR}_{\text{ET}}^{\text{DIN-C}}(\text{ET}_{\text{C}})$ slightly less powerful.

Differences less for larger a and for larger $n = 500$.

REMARK 6.3: Display least satisfactory empirical size.

Power similar for others. $\mathcal{S}_{\text{EL}}^{\text{DIN-C}}(\text{EL}_{\text{C}})$, $\mathcal{S}_{\text{ET}}^{\text{DIN-C}}(\text{ET}_{\text{C}})$ marginally superior.

Increase A_{C} increases power contrary to theory. Reversed for larger $K = 5$.

Restricted tests

Small a. $\mathcal{LM}_{\text{EL}}^{\text{C}}(\text{EL}_{\text{C}},\text{EL}_{\text{MA}})$, $\mathcal{LM}_{\text{ET}}^{\text{C}}(\text{ET}_{\text{C}},\text{ET}_{\text{MA}})$, $\mathcal{LR}_{\text{EL}}^{\text{C}}(\text{EL}_{\text{C}},\text{EL}_{\text{MA}})$, $\mathcal{LR}_{\text{ET}}^{\text{C}}(\text{ET}_{\text{C}},\text{ET}_{\text{MA}})$ dominate.

Ameliorated for larger a and n.

REMARK 6.4: Empirical and nominal size differences quite large for $n = 200$.

Relatively little power difference for others. $\bar{\mathcal{S}}_{\text{EL}}^{\text{C}}(\text{EL}_{\text{MA}})$, $\bar{\mathcal{S}}_{\text{ET}}^{\text{C}}(\text{ET}_{\text{MA}})$ marginally superior.

Increase A_{C} increases power contrary to theory. Reversed for larger $K = 5$.

Incorporation of maintained $E[u|w] = 0$ improves power.

$a = 0$

ME TESTS

Power differences less for larger τ and n.

Restricted tests more powerful than unrestricted.

Power decreases with increased $\mathrm{A_M}$ in line with theory.

Unrestricted tests

Small τ. Small $n = 200$. Differences in power relatively small. $\mathcal{LM}_{\text{EL}}^{\text{DIN-M}}(\text{EL}_{\text{M}})$, $\mathcal{LM}_{\text{ET}}^{\text{DIN-M}}(\text{ET}_{\text{M}})$ power somewhat less.

Restricted tests

Small τ. Small $n = 200$. All except $\mathcal{LM}_{\text{EL}}^{\text{M}}(\text{EL}_{\text{M}},\text{EL}_{\text{MA}})$, $\mathcal{LM}_{\text{ET}}^{\text{M}}(\text{ET}_{\text{M}},\text{ET}_{\text{MA}})$ similar empirical power.

Power differences less for larger τ and n.

Restricted tests more powerful than unrestricted.

CE TESTS

Power decreases with increases in A_C as expected from theory.

Restricted tests display higher power than unrestricted.

Unrestricted tests

Power mostly similar except for $\mathcal{LM}^{\text{DIN-C}}_{\text{EL}}(\text{EL}_\text{C})$, $\mathcal{LM}^{\text{DIN-C}}_{\text{ET}}(\text{ET}_\text{C})$ tests especially for smaller τ and smaller $n = 200$.

Restricted tests

$\mathcal{J}^{\text{C}}(\text{GMM}_\text{C}, \text{GMM}_\text{MA})$, $\mathcal{LR}^{\text{C}}_{\text{CUE}}(\text{CUE}_\text{C}, \text{CUE}_\text{MA})$, $\bar{\mathcal{S}}^{\text{C}}_{\text{EL}}(\text{EL}_\text{MA})$, $\bar{\mathcal{S}}^{\text{C}}_{\text{ET}}(\text{ET}_\text{MA})$ dominate.

Both unrestricted and restricted tests for CE appear more powerful than corresponding tests for ME when ME violated.

Simulation Evidence
Summary

- Non-standardised tests empirical size better approximates nominal size.

Simulation Evidence
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- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.

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- Power declines with increases in A_M or A_C.

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- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in A_M or A_C.
- ME $E[u|x] = 0$ Statistics

Simulation Evidence
Summary

- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in A_M or A_C.
- ME $E[u|x] = 0$ Statistics
    - Distributions of restricted $\mathcal{J}^{M}(\text{GMM}_{M},\text{GMM}_{MA})$, $\mathcal{LR}^{M}_{CUE}(\text{CUE}_{M},\text{CUE}_{MA})$, $\bar{\mathcal{S}}^{M}_{EL}(\text{EL}_{M})$ and $\bar{\mathcal{S}}^{M}_{ET}(\text{ET}_{M})$ most closely approximate nominal size.

Simulation Evidence

Summary

- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in $\mathrm{A_M}$ or $\mathrm{A_C}$.
- ME $E[u|x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^{\mathrm{M}}(\mathrm{GMM_M}, \mathrm{GMM_{MA}})$, $\mathcal{LR}^{\mathrm{M}}_{\mathrm{CUE}}(\mathrm{CUE_M}, \mathrm{CUE_{MA}})$, $\bar{\mathcal{S}}^{\mathrm{M}}_{\mathrm{EL}}(\mathrm{EL_M})$ and $\bar{\mathcal{S}}^{\mathrm{M}}_{\mathrm{ET}}(\mathrm{ET_M})$ most closely approximate nominal size.
  - Restricted CE tests dominate ME tests in terms of size-adjusted power.

Simulation Evidence

Summary

- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in A_M or A_C.
- ME $E[u|x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^{M}(\text{GMM}_{M},\text{GMM}_{MA})$, $\mathcal{LR}^{M}_{CUE}(\text{CUE}_{M},\text{CUE}_{MA})$, $\bar{\mathcal{S}}^{M}_{EL}(\text{EL}_{M})$ and $\bar{\mathcal{S}}^{M}_{ET}(\text{ET}_{M})$ most closely approximate nominal size.
  - Restricted CE tests dominate ME tests in terms of size-adjusted power.
- CE $E[u|w,x] = 0$ Statistics

Simulation Evidence
Summary

- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in $\mathrm{A_M}$ or $\mathrm{A_C}$.
- ME $E[u|x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^{\mathrm{M}}(\mathrm{GMM_M},\mathrm{GMM_{MA}})$, $\mathcal{LR}^{\mathrm{M}}_{\mathrm{CUE}}(\mathrm{CUE_M},\mathrm{CUE_{MA}})$, $\bar{\mathcal{S}}^{\mathrm{M}}_{\mathrm{EL}}(\mathrm{EL_M})$ and $\bar{\mathcal{S}}^{\mathrm{M}}_{\mathrm{ET}}(\mathrm{ET_M})$ most closely approximate nominal size.
  - Restricted CE tests dominate ME tests in terms of size-adjusted power.
- CE $E[u|w, x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^{\mathrm{C}}(\mathrm{GMM_C},\mathrm{GMM_{MA}})$, $\mathcal{LR}^{\mathrm{C}}_{\mathrm{CUE}}(\mathrm{CUE_C},\mathrm{CUE_{MA}})$,$\bar{\mathcal{S}}^{\mathrm{C}}_{\mathrm{EL}}(\mathrm{EL_{MA}})$ and $\bar{\mathcal{S}}^{\mathrm{C}}_{\mathrm{ET}}(\mathrm{ET_{MA}})$ empirical size closest to nominal 0.05.

Simulation Evidence
Summary

- Non-standardised tests empirical size better approximates nominal size.
- Restricted tests dominate unrestricted statistics.
- Power declines with increases in A_M or A_C.
- ME $E[u|x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^M(GMM_M, GMM_{MA})$, $\mathcal{LR}^M_{CUE}(CUE_M, CUE_{MA})$, $\bar{\mathcal{S}}^M_{EL}(EL_M)$ and $\bar{\mathcal{S}}^M_{ET}(ET_M)$ most closely approximate nominal size.
  - Restricted CE tests dominate ME tests in terms of size-adjusted power.
- CE $E[u|w, x] = 0$ Statistics
  - Distributions of restricted $\mathcal{J}^C(GMM_C, GMM_{MA})$, $\mathcal{LR}^C_{CUE}(CUE_C, CUE_{MA})$, $\bar{\mathcal{S}}^C_{EL}(EL_{MA})$ and $\bar{\mathcal{S}}^C_{ET}(ET_{MA})$ empirical size closest to nominal 0.05.
  - Restricted $\bar{\mathcal{S}}^C_{EL}(EL_{MA})$ and $\bar{\mathcal{S}}^C_{ET}(ET_{MA})$ size-adjusted power marginally superior to $\mathcal{J}^C(GMM_C, GMM_{MA})$ and $\mathcal{LR}^C_{CUE}(CUE_C, CUE_{MA})$.