Testing for Cointegration using Induced-Order Statistics

Alvaro Escribano [1] Ana E. García [2] and Mª Teresa Santos [3]

[1] Department of Economics, Universidad Carlos III de Madrid.

[2] Department of Statistics, Universidad Rey Juan Carlos de Madrid.

[3] Department of Statistics, Universidad de Salamanca.

Abstract

In this paper we explore the usefulness of induced-order statistics in the characterization of integrated series and of cointegration relationships. We propose a non-parametric test statistic for testing the null hypothesis of two independent random walks against wide cointegrating alternatives including monotonic nonlinearities and certain types of level shifts in the cointegration relationship. We call our testing device the induced-order cointegration test (IOC), since it is constructed from the induced-order statistics of the series, and we derive its limiting distribution. This non-parametric statistic endows the test with a number of desirable properties: invariance to monotonic transformations of the series, and robustness for the presence of important parameter shifts. By Monte Carlo simulations we analyze the small sample properties of this test. Our simulation results show the robustness of the IOC test against departures from linear and constant parameter models.

Key Words: Unit roots tests, cointegration tests, nonlinearity, robustness, induced-order statistics, Engle and Granger test.

1 Introduction

Stochastic processes exhibiting cointegration will have similar long waves or similar long run behaviour in their sample paths. Granger (1981) introduced the concept of cointegration, but it was not until Engle and Granger (EG) (1987) and Johansen (1988, 1991) that this concept gained immense popularity among econometricians and applied economists. When economic variables are non-stationary cointegration which help avoid the problem of spurious regressions, see Granger and Newbold (1974) and Phillips (1987). By now it is clear how to deal with integrated and cointegrated data in a linear context, see for example Watson (1994), Johansen (1995) and Hendry (1995), but only a few recent papers have been dedicated to the simultaneous consideration of nonstationarity and nonlinearity, even

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though there is considerable consensus that these are important characteristics of many macroeconomic and financial economic relationships. Why has so little attention been devoted to this topic? The answer is clear; it is difficult to work with nonlinear time series models within a stationary and ergodic framework, and even more difficult within a nonstationary context. An introduction to the state of the art in econometrics relating nonlinearity and nonstationarity within a time series context can be found in Granger and Teräsvirta (1993), Granger (1995), Park and Phillips (2001), Bec and Rahbek (2004) and Saikkonen (2005). Granger (1995) discussed the concepts of long-range dependence in mean and extended memory that generalize the linear concept of integration, I(1), to a nonlinear framework. The main disadvantage of such definitions is that they have no Laws of Large Numbers (LLN), or Functional Central Limit Theorems (FCLT) associated with them, and it is therefore difficult to obtain estimation and inference results. On the other hand, there are interesting empirical macroeconomic applications in which nonlinearity has been found in a nonstationary context and, therefore, there is a need for those results to be justified econometrically.

Underlying the idea of cointegration is that of an equilibrium relationship (i.e. one that on average holds) between two cointegrated variables, x_t, y_t. A strict equilibrium exists when for some $\alpha \neq 0$, one has $y_t = \alpha x_t$. This unrealistic situation is replaced, in practice, by that of (linear) cointegration, in which the equilibrium error $z_t = y_t - \alpha x_t$ is different from zero. The concept of cointegration is linear since it is based on linear concepts of integration of order d, $I(d)$ see Marmol, Escribano and Aparicio (2002) for an alternative definition of cointegration using instrumental variables estimators. The usual concept of $I(d)$ is based on linear measures of dependence. Nonlinear measures of dependence, based on near epoch dependence, $\alpha - mixing$ or mutual information can be used to define a nonlinear concept of $I(d)$ and therefore nonlinear cointegration, see Aparicio and Escribano (1999) and Escribano and Mira (2002). Nonlinear error correction means that the adjustment process towards the equilibrium is nonlinear and that nonlinear cointegration refers to a nonlinear cointegration relationship. Furthermore, it should be pointed out explicitly that the same applies to the terms nonlinear error correction model and nonlinear cointegration model.

The relationship between cointegration and error correction model has been well characterized in a linear context (Granger's representation theorem), but its extension to the nonlinear context remains a challenge. Few extensions of the linear framework have been performed in the context of nonlinear error correction (NEC), see Escribano (1986, 1987b, 2004), Escribano and Mira (2002), Bec and Rahbek (2004) and Saikkonen (2005). Saikkonen (2005) also presents a Grangers's representation theorem for general nonlinear error correction models. Furthermore, the applied transformation of nonlinear vector equilibrium correction models into nonlinear vector autorregresive models may open the way for further theoretical work.

Under linear cointegration with nonlinear error correction adjustments the cointegrating errors are nonlinear. Several authors have analyzed those cases: a) cubic polynomials, Escribano (1986, 2004), b) Rational polynomials, Escribano (2004), c) threshold cointegration, Balke and Fomby (1997) d) smooth transition regression models, Granger and Teräsvirta (1993) and general nonlinear autoregressive, Saikkonen (2005). Several nonlinear cointegration models have been discussed in the literature: a) Smooth transition cointegration functions, Choi and Saikkonen (2004), other parametric nonlinear cointegration functions, Aparicio, Escribano and García (2006a,b), nonparametric cointegration, Granger and Hallman (1991) and Aparicio and Escribano (1999). General nonparametric tests have the advantage of being valid under very general conditions and the disadvantage of not providing guidelines for particular parametric modeling. In the empirical application of this paper we will consider a simultaneous case of nonlinear cointegration and nonlinear error correction, see section 4.

Although most cointegration studies rest on the assumption of a linear relationship between the variables, the possibility that these variables depend on each other through nonlinear relationships has opened up many questions for research. Since the concept of cointegration is inherently linear, some attempts have focused on extending standard definitions and on understanding how standard cointegration tests are affected by the presence of neglected nonlinearity, see Aparicio, Escribano and García (2006a). Hallman (1990a,b) and Granger and Hallman (1991) proposed the cointegrating nonlinear attractor concept for a pair of univariate integrated time series $x_t, y_t \sim I(d)$ by requiring the existence of nonlinear measurable functions $f(\cdot), g(\cdot)$ such that $f(x_t)$ and $g(y_t)$ are both $I(d), d > 0$, and $s_t = f(x_t) - g(y_t)$ is $\sim I(d')$, with $d' < d$.

Figure 1 illustrates the case of a cointegrating nonlinear attractor obtained by simulating a nonlinearly related pair of random walks with i.i.d. Gaussian errors.

Aparicio and Granger (1995), Aparicio and Escribano (1998), Aparicio, Escribano and García (2006a,b), proposed the use of first differences of ranges and induced-order statistics to characterize cointegrating relationships. In this paper we analyze the properties of a new nonparametric test based on induced order statistics that is robust to monotonic nonlinear transformations and structural changes. In this paper we derive its asymptotic distribution and show, by Monte Carlo simulations, that the Engle and Granger (1987) test (EG test) fails dramatically in the presence of nonlinearities and structural breaks. We show that the well-known lack of robustness that affects most of the available tests of non-cointegration, (EG test, Johansen (1991)) does not affect our IOC test. The small sample properties of the IOC nonparametric test are analyzed by Monte Carlo simulations. In particular, we test the null hypothesis of non-cointegration against the alternative hypothesis of a nonlinear error correction (NEC) and/or nonlinear cointegration. Furthermore, we are able to show that the IOC test for non-cointegration is robust for nonlinearities and structural breaks. One important advantage of

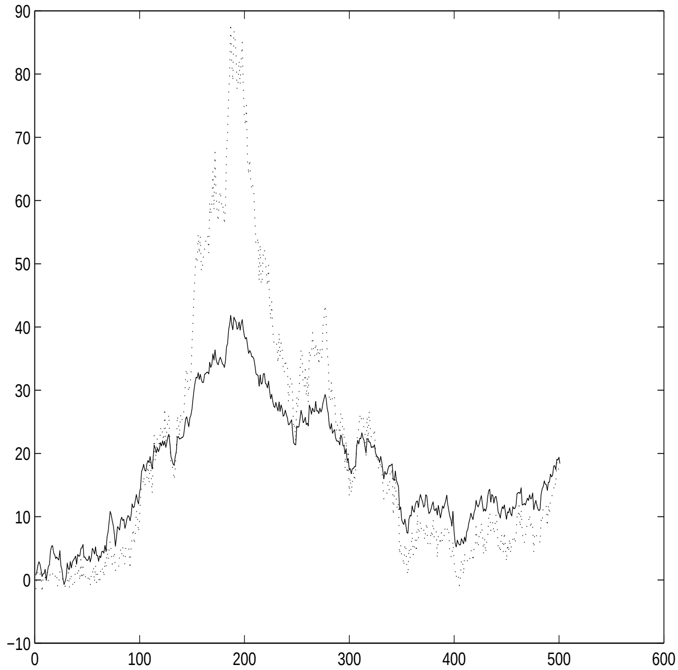

Figure 1: A cointegrating nonlinear attractor obtained with a factorial model. The upper series was generated as $x_t = w_t + \epsilon_t$, while the lower one corresponds to $y_t = g(w_t) + \xi_t$, where $g(\cdot)$ represents a third-order polynomial of its argument random walk variable w_t, and ϵ_t are normally distributed.

this IOC approach is that it does not require prior estimation of the unknown (perhaps nonlinear) cointegrating relationship.

The structure of the paper is as follows. In section 2 we introduce the IOC test, we derive its asymptotic distribution and study the consistency of the test. Section 3 analyses its power performances against different alternative hypotheses. In section 4 we apply our IOC test to a nonlinear cointegration empirical example based on economic time series and compare the results with those obtained by means of standard non-cointegration tests. Finally, after the concluding remarks of section 5, the proofs of the main theoretical results are included in sections 6 to 8.

2 Characterizing cointegration with induced-order statistics

For the time series sample of size n, say $x_1, \cdots, x_n$, the order statistics of x_t are given by the sequence $x_{1,n} \leq \cdots \leq x_{n,n}$ obtained after a permutation of the indexes $1, \cdots, n$ such that $x_{i,n} \leq x_{i+j,n}, \forall j > 0$. Related to order statistics are the so-called induced-order statistics (Bhattacharya, 1984). The induced Y-order statistics based on the ordering of x_t are defined as $\hat{y}_{i,n} = y_j$ if $x_{i,n} = x_j$; in general, notice that $\hat{y}_{i,n} \neq y_{i,n}$. Since for any order-preserving transformation such as monotonic nonlinear functions it follows that the order statistics are invariant, and this property was used by Hallman (1990a) to increase the robustness of the Dickey Fuller unit root test (DF) against monotonic nonlinear departures from the linear cointegration assumption. However, classical unit-root regression theory cannot be applied when the variables have discrete probability distributions, as in the case of rank variables. In fact, Breitung and Gouriéroux (1997) showed that the asymptotic null distribution is different in this case. Rank induced-order statistics should be useful for testing the existence of any sort of prominent low-frequency comovements, even in the more general cases of fractionally integrated time series and long-run relationships containing monotonic nonlinearities. However, they offer additional advantages, such as the possibility of constructing test statistics without nuisance parameters in their null distribution.

Let $\{P_x^{(n)}, P_y^{(n)}\}$ be sequences of $n \times n$ stochastic permutation matrices see Horn and Johnson (1990), defined as:

$$P_x^{(n)} X = X_{(0)} \tag{1}$$

$$P_y^{(n)} Y = Y_{(0)}, \tag{2}$$

where $Z = (z_1, \cdots, z_n)'$ and $Z_{(0)} = (z_{1,n}, \cdots, z_{n,n})'$, $Z = X, Y$. The vector of induced Y-order statistics (induced by the ordering of X). $\hat{Y} = (\hat{y}_{1,n}, \cdots, \hat{y}_{n,n})'$ is obtained as

$$\hat{Y} = P_x^{(n)} Y. \tag{3}$$

Now, since $P_y^{(n)}$ is a permutation matrix, it is invertible and $P^{-1} = P'$, where P' denotes the transpose of P. Therefore, we can form the vector:

$$\tilde{Y} = (P_y^{(n)})^{-1} P_x^{(n)} Y. \tag{4}$$

Notice that for any order-preserving transformations, say $g(\cdot)$, we have

$$\{g(X)\}_{(0)} = g(X_{(0)}). \tag{5}$$

It follows that the order statistics of y_t induced by the ordering of $s_t = g(x_t)$ will not change. That is, induced-order statistics are robust for monotonic nonlinearities in the DGPs of the series.

If x_t and y_t are cointegrated $\tilde{Y}$ should be close to Y, therefore $\hat{Y}$ and $Y_{(0)}$ should move together in the long run. However if x_t and y_t are not cointegrated the behaviour of $\tilde{Y}$ and Y should be different since $X_{(0)}$ and $Y_{(0)}$ are very different. Therefore if y_t and x_t are cointegrated the permutations $P_y^{(n)}$ and $P_x^{(n)}$ should be identical in the long run. The intuition of this result can be seen in Figure 2. In particular in Figure 2 (a), we plot two independent random walks of y_t and x_t, Figure 2 (b), plots the corresponding induced order statistics $\widehat{Y} = P_x^{(n)} Y$ and $Y_{(0)} = P_y^{(n)} Y$. It is clear that there is no relationship between the two series of Figure 2 (b). On the contrary, when the series are cointegrated, in Figure 2 (c), the crossplot of both induced ordered series is around the 45 degree line (slope equal to 1) as in Figure 2 (d). Cointegrated series preserve the induced order in the long run.

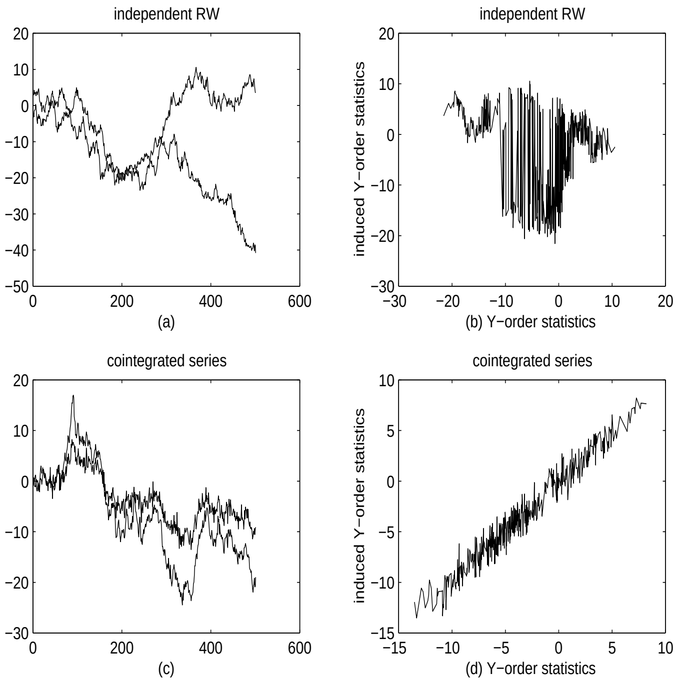

Figure 2: (a) Two independent random walks, (b) the corresponding induced order statistics $\widehat{Y} = P_x^{(n)}Y$ in the y axis and the order statistics $Y_{(0)} = P_y^{(n)}Y$ in the x axis, (c) cointegrated series, and (d) the corresponding induced order statistics $\widehat{Y} = P_x^{(n)}Y$ in the y axis and $Y_{(0)} = P_y^{(n)}Y$ in the x axis.

In what follows we suggest testing the discrepancy between the induced ordered series based on the empirical distribution. We propose an alternative statistical measure of cointegration based on induced-order statistics to compare the orderings of the two time series y_t and $\hat{y}_t$. The corresponding testing device will be referred to as the induced-order cointegration test (IOC). We shall now consider the K1 statistic (Kolmogorov-Smirnov type statistics) defined below for testing the null hypothesis of two independent random walks

$$K1 = \sup_{j=1,n} |\hat{F}_Y^{(n)}(\hat{y}_{l_s(j),n}) - \hat{F}_Y^{(n)}(y_{j,n})|, \tag{6}$$

where $\hat{F}_Y^{(n)}(y)$ is the empirical distribution function obtained from a sample of length n of y_t, that is, $\hat{F}_Y^{(n)}(y) = n^{-1}\sum_{t=1}^{n} 1(y_t \leq y)$, where $1(\cdot)$ denotes the indicator function; s takes the sign of the cointegration parameter (which can be directly obtained from the scatter plot or simple regression of the two series), and $l_s(j)$ is equal to j or to $n-j$, depending on whether $s = 1$ or $s = -1$, respectively.

Smaller values of K1 suggest that two independent random walks should be rejected, whereas large values indicate that two independent random walks hold. Therefore, we can consider the left tail of the distribution of K1 to discriminate between non-cointegrated and cointegrated series.

It is also easy to show that K1 is robust against the order-preserving transformations of the variables, such as monotonic nonlinearities. If the actual cointegration relationship is nonlinear this test provides no guidelines for selection of particular parametric forms.

Our statistic is simply a particular distance measure between the sequences of order statistic $\hat{y}_{l_s(j),n}$ and $y_{j,n}$. Transforming these sequences with the empirical distribution function of y_t, say $\hat{F}_Y^{(n)}$, renders the statistic K1 unaffected by the variance of y_t, although it is still dependent on the signal-to-noise ratio in the relationship between x_t and y_t.

In Figure 3 we plot four different cases: (a) independent random walks, (b) quadratic polynomial atractor (nonlinear cointegration), (c) lineal cointegrated series and (d) linear cointegrated with structural breaks. In each case Figure 3 shows the discrepancy between the sequences $\hat{F}_Y^{(n)}(y_{j,n})$ and $\hat{F}_Y^{(n)}(\hat{y}_{l_s(j),n})$. K1 is merely a measure of the variability of the sequence $\hat{F}_Y^{(n)}(\hat{y}_{l_s(j),n})$ around the diagonal line, represented by the sequence $\hat{F}_Y^{(n)}(y_{j,n})$. In the plots, the straight diagonal line corresponds to the sequence $\hat{F}_Y^{(n)}(y_{j,n})$, while the superimposed series corresponds to $\hat{F}_Y^{(n)}(\hat{y}_{l_s(j),n})$, where s stands for the sign of the cointegration parameter.

We observe that when the series are cointegrated (linearly, nonlinearly and with structural breaks) the crossplot of the empirical distributions are centered around the 45 degree line. Deviations are only transitory, while when the series are non cointegrated the discrepancies are permanent and most of the observations are off the 45 degree line.

The K1 statistic has been widely used both to test whether two random samples come from the same parent distribution and as a measure of distance between two probability distributions. Here we used it in a different sense, to obtain a discrepancy measure between the relative orders of x_t and y_t or, equivalently, of y_t and the induced order series $\widehat{y}_t$.

We shall now derive the limit behavior of the null distribution of K1 by using standard asymptotic theory for $I(1)$ processes. We shall first focus on deriving the asymptotic of the order and induced-order statistics for $I(1)$ time series. We shall invoke the continuous mapping theorem (CMT) to obtain the asymptotic of our test.

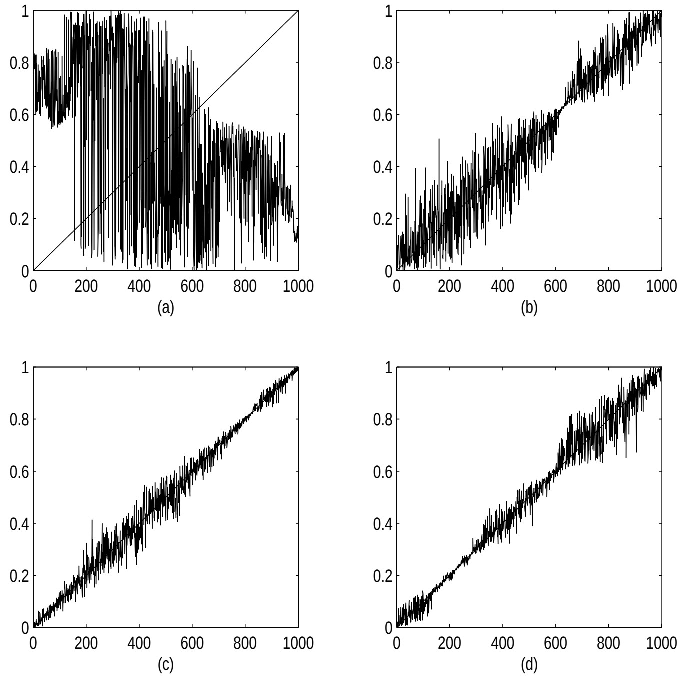

Figure 3: Illustration of the proposed nonparametric cointegration measures for pairs of: (a) independent random walks, (b) series related through a quadratic atractor, (c) cointegrated series, and (d) linear cointegration with structural changes generated from equation (16).

Theorem 2.1 *Let $\hat{y}_{i,n}$ be the i^{th} order statistic of y_t induced by the ordering of x_t, where $x_t \sim I(1)$. Then,*

$$\frac{\hat{y}_{i,n}}{n^{1/2}} \Rightarrow \int_0^1 W_y(r)\mathbf{1}\left(\int_0^1 \mathbf{1}\left(W_x(s) < W_x(r)\right) ds = l\right) dr = \int_0^1 W_y(r) G_x^{(l)}(r) dr,$$

where $G_x^{(l)}(r)$, see Breitung and Gouriéroux (1997), represents a random process given by

$$G_x^{(l)}(r) = \mathbf{1}\left(\int_0^1 \mathbf{1}\left(W_x(s) < W_x(r)\right) ds = l\right), \tag{7}$$

$W_x(\cdot)$ and $W_y(\cdot)$ are the corresponding Brownian motion process associated to x_t and y_t respectively, and "$\Rightarrow$" denotes convergence in distribution as $n \to \infty$.

Proof. See Appendix 1 ■

Corollary 2.2 *Under the hypothesis of independent random walks, $\frac{\hat{y}_{i,n}}{n^{1/2}}$ converges to zero in probability.*

Proof. See Appendix 2 ■

Let $F_y^{(n)}$ represent the empirical distribution function of y_t, and let us recall that the rank of x_i in the sample $\{x_1, ..., x_n\}$ is defined as $R_x(x_i) = \sum_{j=1}^{n} 1(x_j < x_i)$, such that $x_{i,n} = R_x^{-1}(i) = x_{\pi(i,n)}$, with $\pi(.)$ denoting a stochastic permutation applied to the indexes of a sample of size n in order to have the observations ordered. We can then define the induced-order statistics of y_t as $\hat{y}_{i,n} = y_{\pi(i,n)}$.

Theorem 2.3 *Let $x_t = \sum_{i=1}^{n} \epsilon_i$, and $y_t = \sum_{i=1}^{n} \varepsilon_i$, where $\{\epsilon_i\}_{i\geq1}$, and $\{\varepsilon_i\}_{i\geq1}$ are continuous i.i.d. random variables with bounded and symmetric pdf, zero means, and finite variances. Then,*

i) $K1 \Longrightarrow \sup_{l\in(0,n)} \left| \int_0^1 1(W_y(s) < W_y(q(l))ds - l \right|.$

ii) If x_t and y_t are cointegrated, then we have $K1 \underset{n\to\infty}{\longrightarrow} 0$ and here the IOC test is consistent against this sort of alternative.

Proof. See Appendix 3 ■

3 Small-sample performance of the IOC test: Monte Carlo Simulations

In this section we provide simulation evidence in small samples such that the nonparametric test statistic K1 is useful for testing our null hypothesis of two independent random walks. Let the data generation process (DGP) be the following independent random walks:

$$DGP : H_0 \qquad \Delta y_t = w_{1t} \tag{8}$$

$$\Delta x_t = w_{2t}, \tag{9}$$

where w_{1t} and w_{2t} are standard normal distributions and mutually independent.

In Table 1 we estimate the quintals of the empirical distribution of the *IOC* test statistic, under H_0, for different significance values ν and different samples sizes ,n, computed by using 50,000 replications of independent random walks with i.i.d. Gaussian errors. Figure 4 shows the corresponding empirical density of K1 estimated by kernel smoothing, using the Epanechnikov kernel for $1,000$ replications for different sample sizes.

Table 1: Critical values of the K1 test statistics.

$\nu \setminus$ n	100	250	500	1000
0.01	0.3564	0.3665	0.3812	0.3726
0.025	0.4059	0.4382	0.4591	0.4346
0.05	0.4653	0.4980	0.4830	0.4915
0.10	0.5248	0.5697	0.5649	0.5794
0.90	0.9505	0.9681	0.9721	0.9780
0.95	0.9703	0.9801	0.9860	0.9870

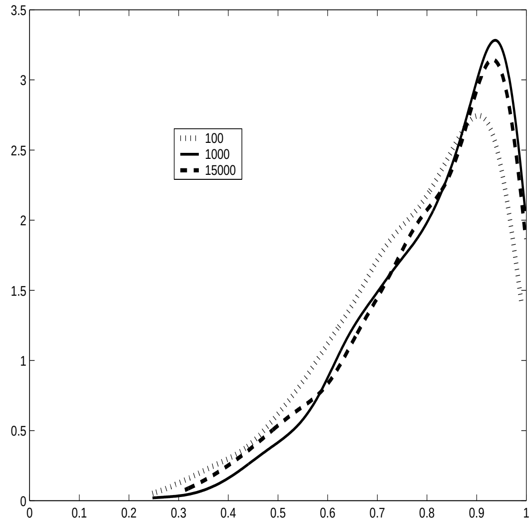

Figure 4: Plot of the empirical density of K1 under the null hypothesis for different sample sizes $n = 100, 1000$ and 15000.

3.1 Size and Power of the IOC test: Monte Carlo simulation

We shall analyze the power of the IOC test using the 5% left tail critical value, based on 10,000 replications of the Monte Carlo experiment. The DGP under the alternative hypothesis of cointegration is generated by a bivariate vector error correction model with weakly exogenous variables for the cointegrating parameter vector. Consider the following restricted VAR model for the (y_t, x_t) vector, which is generated by

$$\begin{bmatrix} \Delta y_t \\ \Delta x_t \end{bmatrix} = \begin{bmatrix} c \\ 0 \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} (1, -\alpha) \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix}, \tag{10}$$

where w_t is standard Normal.

3.1.1 Linear cointegration

The alternative hypothesis is a standard linear error correction model (ECM):

$$DGP : H_{1a} \qquad \Delta y_t = c + b(y_{t-1} - \alpha x_{t-1}) + w_{1t} \tag{11}$$

$$\Delta x_t = w_{2t}, \tag{12}$$

where $\alpha = 1$ and $c = 0$. We shall study the power of different cointegration tests: IOC and EG test for different parameter values of the parameter b. We set $b = 0$ (no cointegration), $b = -0.05$, -0.5, -0.75 (cointegration). This DGP follows the parameterization used by Kremers et al. (1992) , and Arranz and Escribano (2001) . The results are in Table 2.

Table 2: Empirical Size and Power of the IOC and EG test based on H_{1a} at 5% significance level for different values of $b = 0$ (no cointegration), $b = -0.05$, -0.5, -0.75 (cointegration) and different sample sizes $n = 100, 250, 500$.

$b \backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
0	0.05	0.051	0.051	0.052	0.05	0.05
-0.05	0.43	0.1450	0.51	0.59	0.6	0.98
-0.5	0.65	1	0.85	1	0.9	1
-0.75	0.7	1	0.9	1	0.91	1

Since the EG test is based on estimating the true error correction model (DGP), the EG test for non-cointegration is more powerful than IOC. However, IOC is more powerful for small values of b; see, for example, $b = -0.05$ in Table 2.

3.1.2 Nonlinear cointegration

Escribano (1986, 2004) analyzed error correction models in nonlinear contexts where the cointegration relationship is linear or nonlinear and the equilibrium correction term could also be linear or nonlinear. Alternative representation theorems for nonlinear error correction (NEC) models based on general concepts of I(1) and I(0) were introduced by Escribano (1986, 1987b), Escribano and Mira (2002), Bec and Rahbek (2004) and Saikkonen (2005). The main advantage of those definitions is that they have associated Laws of Large Numbers and Central Limit Theorems to derive the asymptotic properties of the estimators of the error correction models parameters. Escribano (1986, 2004) proposed a methodology for implementing parametric and nonparametric error correction models. Using the data bases of Friedman and Schwartz (1982) and Ericsson et al. (1997), extended until the year 2000, he implemented this methodology to estimate a nonlinear money demand in the U.K. from 1878 to 2000. In section 4 we will apply our non-cointegration IOC test to this data set. Within the class of parametric models he discusses cubic polynomial (and rational polynomial) error correction models, see also Hendry and Ericsson (1991). Nonlinearities can eliminate most of the power of the usual non-cointegration test (EG test), as will be seen in the following Monte Carlo simulations.

3.1.3 Power of IOC against a Nonlinear cointegrating relationship

Let us now consider that the DGP under the alternative hypothesis is given by the following linear error correction model (ECM), with a nonlinear cointegration relationship.

H_{1b}: The alternative hypothesis is a linear ECM, but with nonlinear cointegration,

$$DGP : H_{1b} \qquad \Delta y_t = c + b(y_{t-1} - g(x_{t-1}, \alpha)) + w_{1t} \tag{13}$$

$$\Delta x_t = w_{2t}, \tag{14}$$

where w_{1t} and w_{2t} are standard normal distributions and mutually independent errors, with $\alpha = 1$. Let the nonlinear cointegration relationship be given by the polynomial cointegration term $g(z_{t-1}, \alpha) = z_{t-1}^j$. Based on 10,000 replications of the Monte Carlo experiment, we now analyze the power of the *IOC* test at 5% significance level for different values of b, and we compare the results with those of the EG test, whose results are shown in Table 3.

Table 3: Empirical power of IOC and EG based on H_{1b} with $g = z_{t-1}^{j}$ at 5% significance level for different values of b and different sample sizes n.

$b = -0.05$

$j\backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
2	0.83	0.08	0.94	0.02	0.98	0.01
3	0.94	0.1	1	0.03	1	0.008
4	0.92	0.08	1	0.01	1	0.006

$b = -0.5$

$j\backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
2	0.88	0.09	0.96	0.04	1	0.01
3	0.9	0.1	1	0.06	1	0.05
4	0.88	0.069	1	0.027	1	0.019

$b = -0.75$

$j\backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
2	0.9	0.1	0.95	0.07	0.98	0.07
3	0.9	0.17	1	0.14	1	0.16
4	1	0.23	1	0.234	1	0.275

As can be seen from Table 3, when the ECM is linear but with a polynomial cointegration function, the IOC test is much more powerful than EG in all cases. In particular, we observe that the highest power is obtained for $j = 3$, followed by $j = 4$ and $j = 2$ respectively. The intuitive explanation given by Escribano (2004) is as follows: cubic polynomials are very flexible and can approximate different level shifts. Furthermore, the error correction term in this case can be equilibrium-correcting (stable nonlinear adjustment) and hence not a deviating error adjustment term, as may happen with the quadratic polynomial. We shall now consider other functional forms. Let the nonlinear cointegrating function be $g(z_{t-1}, \alpha) = \exp(z_{t-1}/100)$. The empirical powers of the IOC test and EG test for different values of b are given in Table 4.

Table 4: Empirical power of IOC and EG based on H_{1b} with $g = exp(z_{t-1}/100)$ at 5% significance level for different values of b and different sample sizes n.

$b\backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
-0.05	0.43	0.05	0.52	0.05	0.57	0.05
-0.5	0.65	0.07	0.85	0.09	0.9	0.1
-0.75	0.7	0.1	0.9	0.11	1	0.15

The power of the EG test is very low since this linear procedure misspecifies the estimation of the cointegrating vector by assuming that it is linear. In contrast, the power of the IOC test is very high. Similar results are obtained when the nonlinear cointegrating function is $g(z_{t-1}, \alpha) = \log(z_{t-1} + 100)$.

3.1.4 Linear cointegration with structural changes in the cointegrating vector

There is a large body of literature concerning the effects of cointegration testing in the presence of structural changes, see Escribano (1987a). In what follows we wish to simulate a case based on the DGP of Arranz and Escribano (2001) to evaluate the power of IOC in this context when the break point is in the middle of the sample.

The alternative hypothesis is a linear error correction and cointegration model in the presence of a structural change in the cointegrating vector

$$DGP : H_{1c} \qquad \Delta x_t = w_{1t} \qquad (15)$$

$$\Delta y_t = c + b[y_{t-1} - (c_1 D_{1t-1} x_{t-1} + \alpha x_{t-1})] + w_{2t}, \qquad (16)$$

where w_{1t} , w_{2t} are standard, and mutually independent normal errors, where c_1 measures the change in the cointegrating vector. We shall consider the values, $\alpha = 1$, $c_1 = 2$ and, we shall understand that the structural break is created by the artificial dummy variable D_{1t}, defined by:

$$D_{1t} = \begin{cases} 1, & t \geq \frac{n}{2} \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

Based on 10,000 replications of the Monte Carlo simulations, we obtain the empirical power of the IOC and EG tests, see Table 5.

Table 5: Empirical power of IOC and EG based on H_{1c} with $\alpha = 1$, $c_1 = 2$ at 5% significance level for different values of b and different sample sizes n.

$b\backslash n$	100		250		500	
	IOC	EG	IOC	EG	IOC	EG
-0.05	0.7	0.15	0.8	0.16	0.95	0.17
-0.5	0.81	0.23	1	0.25	1	0.25
-0.75	0.8	0.25	1	0.26	1	0.29

Table 5 shows that the IOC is more powerful than the EG test in rejecting the null hypothesis of noncointegration in the presence of a structural break in the cointegrating vector. The power of IOC is really good when the parameter of the error correction adjustment, $b = -0.5$ or higher. For very low adjustment parameter values, $b = -0.05$, the empirical power of IOC is over 70% and much higher than EG-test, which is lower than 20%.

4 Empirical Application

4.1 Analysis of the UK money demand (1878-2000)

Escribano (1986, 2004) found a cointegration relationship between the logarithmic (logs) transformation of velocity of the circulation of money (V) and short run interest rates in nominal terms (RNA) from annual observations taken from 1878 to 2000 based on nonlinear error correction adjustments. Those error correction models consider several nonlinear parametric and nonparametric (smoothing splines) alternatives. Among the parametric nonlinear adjustments considered are cubic polynomials and rational polynomials of the cointegrating errors obtained from the nonlinear cointegration relationship between velocity of circulation of money and short run interest rates. Therefore, standard unit root test will not work well since they are not robust to these types of nonlinearities. This cointegration relationship is nonlinear (exponential) since the interest rate (RNA) is without logs while velocity of circulation of money is in logs (V), see Figure 5.

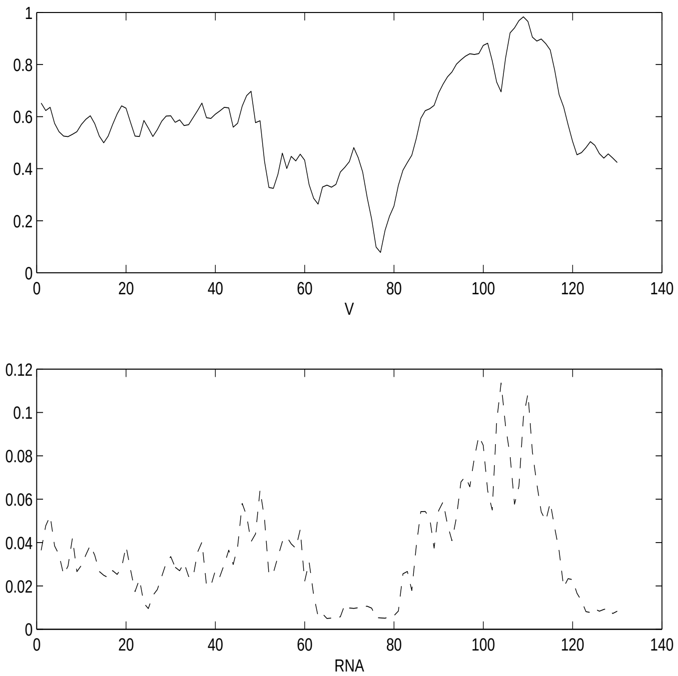

Figure 5: Log of the velocity of the circulation of money (V) and short run interest rates in nominal terms (RNA).

When we test the null hypothesis of non-cointegration between V and RNA based on the residuals of the EG test statistic we obtain a value of -1.7, and therefore we cannot reject the null hypothesis of non-cointegration at the 5% significance level. This result contradicts the ECM test carried out in the nonlinear error correction model, see Escribano (2004).

However, applying our IOC test statistic , we obtain $K1 = 0.2$, which is smaller than the 5% critical value, for $n = 130$ (0.47). Therefore, using our induced order cointegration test IOC, we are able to reject the null hypothesis of non-cointegration.

5 Conclusions

Cointegration is an important property of many economic variables but in order to find convincing empirical evidence we usually need to extend the linear framework by allowing some time-varying parameter models or by considering nonlinear relationships. Standard cointegration tests, such as the

EG test, are not robust for nonlinearities or for certain time-varying parameter models (level shifts, structural breaks, etc). In this paper, we have presented a model-free methodology that allows testing for the presence of cointegration in time series, and that is robust for the presence of monotonic nonlinearities and structural changes. These properties are very important because, on the one hand, standard cointegration tests are tailor-made to a specific parametric linear models for the individual series under the null hypothesis and, on the other, because in many applications one does not really know the transformation of the series that can linearize their relationship. For this, we propose an alternative testing device based on our induced-order test statistic from the series which has the advantage of not requiring prior estimation of the cointegration parameter, thereby leading to null distributions that are free of nuisance parameters. On the other hand it provides no clear guidelines for particular parametric modelling.

Extensions of our nonparametric approach to more than one cointegration vector, to larger multivariate systems or to linear trends are out of the scope of this paper and are left for future research.

6 Appendix 1

Definition 6.1 *A time series x_t is said to be $I(0)$ if the process X_n defined in the unit interval by*

$$X_n(\xi) = \sum_{t=1}^{[n\xi]} (\frac{x_t - E[x_t]}{\sigma_{x,n}}), \quad 0 < \xi \leq 1,$$

where $\sigma^2_{x,n} = Var\left[\sum_{t=1}^{n} x_t\right]$, denotes the long-run variance of X_n, converges weakly to a standard Brownian motion, W_x, as $n \to \infty$.

Let $x_{i,n}$ denote the i^{th} order statistic of an $I(1)$; that is, one that after the first differences becomes $I(0)$ following the definition by Davidson (1994) given before. We can write:

$$x_{i,n} = \sum_{k=1}^{n} x_k \mathbf{1}(\sum_{i=1}^{n} 1(x_j < x_k) = i).$$

We can easily obtain

$$\begin{aligned} \frac{x_{i,n}}{n^{1/2}} &\Rightarrow \int_0^1 W_x(r) \mathbf{1} \left(\int_0^1 \mathbf{1} \left(W_x(s) < W_x(r) \right) ds = l \right) dr = \\ &= \int_0^1 W_x(r) G_x^{(l)}(r) dr. \end{aligned}$$

where "$\Rightarrow$" denotes convergence in distribution as $n \to \infty$. Following Breitung and Gouriéroux (1997) , $G_x^{(l)}(r)$ represents a random process given by

$$\begin{aligned} G_x^{(l)}(r) &= \mathbf{1}\left(\int_0^1 \mathbf{1}\left(W_x(s) < W_x(r)\right) ds = l\right) = \\ &= \mathbf{1}\left(\int_0^1 \mathbf{1}\left(W_x(r-s) > 0\right) ds + \int_r^1 \mathbf{1}\left(W_x(s-r) > 0\right) ds = l\right) = \\ &= \mathbf{1}\left(rA_1 + (1-r)A_2 = l\right), \end{aligned}$$

with A_1, A_2 denoting two independent random variables with an arcsin distribution. Note that $\int_0^1 \mathbf{1}\left(W_x(s) < W_x(r)\right) ds$ represents the occupation time of the set $(-\infty, W_x(r))$ by the Brownian motion, W_x, and $G_x^{(l)}(r)$ takes the value 1 whenever this occupation time equals $l \in (0,1)$ and 0 otherwise. Therefore, when $x_t \sim I(1)$ then $\frac{x_{i,n}}{n^{1/2}}$ converges weakly to a stochastic process indexed by l. In contrast, if $x_t \sim I(0)$ then it will converge to zero in probability.

Similarly, by allowing $\hat{y}_{i,n}$ to represent the i^{th} order statistic of y_t induced by the ordering of x_t, we can write:

$$\hat{y}_{i,n} = \sum_{k=1}^{n} y_k \mathbf{1}(\sum_{i=1}^{n} 1(x_j < x_k) = i),$$

and we obtain:

$$\begin{aligned} \frac{\hat{y}_{i,n}}{n^{1/2}} &\Rightarrow \int_0^1 W_y(r)\mathbf{1}\left(\int_0^1 \mathbf{1}\left(W_x(s) < W_x(r)\right) ds = l\right) dr = \\ &= \int_0^1 W_y(r) G_x^{(l)}(r) dr, \end{aligned}$$

with l and $G_x^{(l)}(r)$ given as before. Note that if x_t and y_t are independent then the limiting process in the previous equation vanishes to zero in probability. We can even work out the nature of these limiting processes a little more. Let us focus on the behavior of $\int_0^1 W_y(r) G_x^{(l)}(r) dr$ under cointegration, and let $S_x^{(l)}$ be the stochastic set given by

$$S_x^{(l)} = \{r \in (0,1) : \; rA_1 + (1-r)A_2 = l\},$$

which for each possible value of the pair of random variables (A_1, A_2), say (a_1, a_2), will be formed by the single point:

$$r_l = \frac{l - a_2}{a_1 - a_2}.$$

In fact, $S_x^{(l)}$ is a random variable, and we may write:

$$\int_0^1 W_y(r) G_x^{(l)}(r) dr = \int_{S_x^{(l)}} W_y(r) dr,$$

which gives $\int_{S_x^{(l)}} W_y(r)dr = W_y(\frac{l-A_1}{A_1-A_2})$ if $\frac{l-A_1}{A_1-A_2}$ belongs to the interval $(0,1)$ and 0 otherwise. Thus, we obtain a limiting non-Gaussian doubly stochastic process with positive probability mass at zero. Indeed, the Brownian motion process, W_y, is indexed by the process on the unit interval $q_x(l) = \frac{l-A_1}{A_1-A_2}$ indexed by l.

7 Appendix 2

Let us compute the mean and the variance of $\int_{S_x^{(l)}} W_y(r)dr$. First, for the mean, let us notice that

$$\begin{aligned} E\left[\int_0^1 W_y(r)G_x^{(l)}(r)dr\right] &= E_M\left[E\left(W_y(\frac{l-A_1}{A_1-A_2})\backslash A_1 = a_1, A_2 = a_2\right)\right] = \\ &= \int\int_M V_l(a_1,a_2)f_{A_1}(a_1)f_{A_2}(a_2)da_1da_2, \end{aligned}$$

where M represents the set given by those pairs of values (a_1, a_2) such that either $a_1 < l$ and $a_2 < 2\ a_1 - l$, or $a_1 > l$ and $a_2 > 2\ a_1 - l$, and $f_A(.)$ stands for the probability density of the random variable A, and

$$V_l(a_1,a_2) = E\left[W_y(\frac{l-A_1}{A_1-A_2})\backslash A_1 = a_1, A_2 = a_2\right].$$

However, it is clear that $V_l(a_1,a_2) = 0$, such that $E\left[\int_0^1 W_y(r)G_x^{(l)}(r)dr\right] = 0$.

Regarding the variance of $\int_{S_x^{(l)}} W_y(r)dr$, from the previous result we have:

$$\begin{aligned} Var\left[\int_{S_x^{(l)}} W_y(r)dr\right] &= E\left[\int_0^1 \left(W_y(r)G_x^{(l)}(r)dr\right)^2\right] = \\ &= E_M\left[E\left[W_y^2(\frac{l-A_1}{A_1-A_2})\backslash A_1 = a_1, A_2 = a_2\right]\right] = \\ &= \int\int_M \frac{l-a_1}{a_1-a_2} f_{A_1}(a_1)f_{A_2}(a_2)da_1da_2 = \\ &= \frac{4}{\pi^2}\int_0^l\int_0^{2a_1-l} \frac{l-a_1}{a_1-a_2}\left(1-a_1^2\right)^{-1/2}\left(1-a_2^2\right)^{-1/2} + \\ &\quad + \frac{4}{\pi^2}\int_l^1\int_{2a_1-l}^1 \frac{l-a_1}{a_1-a_2}\left(1-a_1^2\right)^{-1/2}\left(1-a_2^2\right)^{-1/2}. \end{aligned}$$

Here, we have used the fact that $Var\left[W_y(r)\right] = r$. Note that if x_t and y_t were independent we would obtain

$$\begin{aligned} Var\left[\int_{S_x^{(l)}} W_y(r)dr\right] &= E\left[\left(\int_0^1 W_y(r)G_x^{(l)}(r)dr\right)\left(\int_0^1 W_y(r')G_x^{(l)}(r')dr'\right)\right] = \\ &= \int_0^1\int_0^1 E\left[W_y(r)W_y(r')\right]E\left[G_x^{(l)}(r)G_x^{(l)}(r')\right]drdr' = \\ &= \int_0^1\int_0^1 \min(r,r')E\left[G_x^{(l)}(r)G_x^{(l)}(r')\right]drdr'. \end{aligned}$$

However,

$$E\left[G_x^{(l)}(r)G_x^{(l)}(r')\right] = E\left[\mathbf{1}\left(rA_1 + (1-r)A_2 = l\right)\mathbf{1}\left(r'A_1 + (1-r')A_2 = l\right)\right],$$

which is equal to zero unless $r = r'$, and hence:

$$Var\left[\int_{S_x^{(l)}} W_y(r)dr\right] = \int_0^1 rE\left[G_x^{(l)}(r)\right]^2 dr.$$

Finally, we remark that

$$\begin{aligned} E\left[G_x^{(l)}(r)\right]^2 &= P\left(G_x^{(l)}(r) = 1\right) = \\ &= P\left(\mathbf{1}\left(rA_1 + (1-r)A_2 = l\right) = 1\right) = \\ &= P\left(rA_1 + (1-r)A_2 = l\right) = \\ &= 0, \end{aligned}$$

since $rA_1 + (1-r)A_2$ is a continuous random variable for each r. Thus,

$$Var\left[\int_{S_x^{(l)}} W_y(r)dr\right] = 0.$$

This result entails that under the hypothesis of independent random walks $\frac{\hat{y}_{i,n}}{n^{1/2}}$ must converge to zero in the quadratic mean, and therefore also in probability. However, when the series are cointegrated $\frac{\hat{y}_{i,n}}{n^{1/2}}$ will converge weakly to a double stochastic process obtained from a Brownian motion process.

8 Appendix 3

We have

$$\begin{aligned} F_y^{(n)}(\hat{y}_{i,n}) &= n^{-1}R_y(\hat{y}_{i,n}) = \\ &= n^{-1}\pi^*(i,n), \end{aligned}$$

where $\pi^*(\cdot)$ denotes a different stochastic permutation of the indexes. Therefore:

$$\begin{aligned} K1 &= \sup_{i=1,n}\left|F_y^{(n)}(\hat{y}_{i,n}) - F_y^{(n)}(y_{i,n})\right| = \\ &= n^{-1}\max_{1\le i\le n}\left|\pi^*(i,n) - i\right|. \end{aligned}$$

Now, if the series x_t and y_t are cointegrated (either linearly or monotonically nonlinearly) we would expect $\pi^*(i,n) - i$ to remain close to zero, $\forall i$, in the range $1 \le i \le n$. Following Breitung and

Gouriéroux (1997), we have:

$$\begin{aligned} n^{-1}R_y(\hat{y}_{i,n}) &= n^{-1}\sum_{t=1}^{n} 1(y_t < y_{\pi(i,n)}) = \\ &= n^{-1}\sum_{t=1}^{n} 1(n^{-1/2}y_t < n^{-1/2}y_{\pi(i,n)}) = \\ &= \sum_{t=1}^{n} 1(n^{-1/2}y_{\left[\frac{t}{n}n\right]} < n^{-1/2}y_{\left[\frac{\pi(i,n)}{n}n\right]}) \left[\frac{t}{n} - \frac{t-1}{n}\right] \\ &\Longrightarrow \int_0^1 1(W_y(s) < W_y(q(l))ds, \end{aligned}$$

where $q(l) = \dfrac{\pi(i,n)}{n} = n^{-1}\pi(nl,n)$ and $l = \frac{i}{n}$. Under cointegration $\hat{y}_{i,n} = y_{i,n} + \zeta_i$, where ζ_i is $I(0)$. It follows that $n^{-1}R_y(\hat{y}_{i,n}) \xrightarrow{p} l$, such that $K1 \xrightarrow{p} 0$. In contrast, for non-cointegrated series, by the CMT

$$K1 \Longrightarrow \sup_{l\in(0,n)} \left| \int_0^1 1(W_y(s) < W_y(q(l))ds - l \right|,$$

which represents a random variable taking strictly positive values with probability one.

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