ACCOUNTING FOR UNOBSERVABLES IN PRODUCTION MODELS:
MANAGEMENT AND INEFFICIENCY

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ABSTRACT

This paper explores the role of unobserved managerial ability in production and its relationship with technical efficiency. Previous analyses of managerial ability were based in strong assumptions about its role in production or the use of proxies. We avoid these shortcomings by introducing managerial ability as an unobserved random variable in a translog production function. The resulting empirical model can be estimated as a production frontier with random coefficients.

Keywords: Managerial ability, technical efficiency, production frontier, random coefficients model, maximum simulated likelihood.

JEL codes: C5, D2
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1. INTRODUCTION

Management has always been considered an important factor of production. However, the modeling of ‘management’ is problematic because it is unobservable. For this reason it has been omitted from many production models. This may be the source of important problems because the omission of relevant variables can lead to biased estimates of the remaining parameters of the production function (Griliches, 1957). Economists have coined the term 'management bias’ to refer to this problem. The literature has suggested two strategies for avoiding this problem. Following Mundlak (1961) some authors have used covariance analysis [Massell (1987)] or similar tools, such as the within transformation, to control for the effect of time invariant management by sweeping it out of the estimating equation. Other studies have used ‘proxies’ for management [e.g. Dawson and Hubbard (1985); Mefford (1986)].

An alternative strategy is to consider management a random effect and model it as part of the stochastic element of the production function. This is the approach followed implicitly by the stochastic frontier production function literature (Aigner et al., 1977) where the stochastic structure is composed of two terms: a symmetric term, that accounts for noise, and an asymmetric term that accounts for technical efficiency (TE). In this literature, it has been common to assume that the inefficiency term is picking up (among other things) differences in the level of managerial skills.
These two strands of literature have followed parallel but independent paths. On the one hand, the ‘average production function’ literature recognizes the role of management but seldom mentions production inefficiency. On the other, the stochastic frontier literature focuses on estimating technical inefficiency but does not provide an analytical linkage between inefficiency and management.¹

In this paper we explore the analytical linkages between technical efficiency and management in the framework of a translog production model where management is treated as an unobservable fixed input. Starting from an average production function we are able to derive an economic-based stochastic production frontier and an explicit relationship between technical efficiency and management. Technical efficiency is shown to be a function of the difference between the current level of management and some notion of ‘optimal’ management. We show that the assumption of fixed management leads to a time-varying index of technical efficiency. This is a pertinent result since the estimation of models with time-invariant technical efficiency is usually justified on the assumption that management is fixed over time.

The empirical portion of the paper estimates the production function with a latent fixed input described above. The model that emerges is a type of random coefficients model. The estimation is carried out by maximum simulated likelihood. An important aspect of this estimation procedure is that it allows us to construct estimates of both current and ‘optimal’ management, as well as the associated technical efficiency.

The structure of the paper is as follows: In section 2 we develop a model in which managerial ability is treated as an unobservable input in a flexible production function. Section 3 discusses estimation issues. Empirical results based on a study of dairy farmers are presented in Section 4. Some conclusions are drawn in Section 5.

2. A PRODUCTION MODEL WITH FIXED MANAGERIAL ABILITY

¹ One example of the ambiguous notion of efficiency is the definition posed by Farrell (1957) who stated “the technical efficiency indicates the gain that can be achieved by simply ‘gingering-up’ the management.” Farrell appeared to suggest that technical inefficiency is the result of a lack of managerial ability. On the other hand, Leibenstein (1966) viewed technical inefficiency, which he called X-efficiency, as the result of a lack of motivation or effort. In this case, the solution to inefficiency calls for better organization of the work process or more motivation and supervision of employees, “all of which are commonly considered to be management functions” [Mefford (1986)].
Our starting point is a translog production function with one time varying variable input, \(x_{it}\) and managerial ability, \(m_i\), which is considered a fixed input. The flexibility of the production function relaxes ex-ante constraints in the roles of \(m_i\) and \(x_{it}\) in the production process. The translog production function can be written as:

\[
\ln y_{it} = \alpha + \beta_x \ln x_{it} + \frac{1}{2} \beta_{xx} (\ln x_{it})^2 + \beta_m m_i + \frac{1}{2} \beta_{mm} m_i^2 + \beta_{xm} \ln x_{it} m_i + v_{it}
\]

where subscripts \(i\) and \(t\) denote firms and time, respectively, and \(y_{it}\) is the single output. The inputs in the translog model are conventionally in logs, but since, by construction, managerial ability is unobservable, we have entered it in the level form, since the units of measurement, with the variable itself, are unobservable. We assume that \(v_{it}\) is a symmetric random disturbance with zero mean. Therefore, the model in corresponds to the typical ‘average production function.’ Its key feature for present purposes is the interaction of management with the variable input. Without this term, the two management terms collapse into an individual effect and the model does not differ substantially from the standard fixed or random effects production function model.

We expect production to be monotonically increasing in \(m_i\); greater managerial ability should allow the agent to produce greater output for any amount of input. This effect will characterize the production function in if:

\[
\frac{\partial \ln y_{it}}{\partial m_i} = \beta_m + \beta_{mm} m_i + \beta_{xm} \ln x_{it} > 0
\]

The assumption of monotonicity in managerial ability implies that, for given \(x_{it}\), higher values of \(m_i\) imply higher levels of technical efficiency. The maximal output for given \(x_{it}\) (the frontier) corresponds to a maximal level of managerial input \(m_i^*\) higher than that for comparable (same \(x_{it}\)) but inefficient producers. The stochastic production frontier would then be written as:

\[
\ln y_{it}^* = \alpha + \beta_x \ln x_{it} + \frac{1}{2} \beta_{xx} (\ln x_{it})^2 + \beta_m m_i^* + \frac{1}{2} \beta_{mm} m_i^*^2 + \beta_{xm} \ln x_{it} m_i^* + v_{it}
\]

where \(y_{it}^*\) denotes efficient output.

We can now establish a link between technical efficiency and management. This follows from the definition of an output-oriented index of technical efficiency as the ratio of observed to potential output, which in log terms is:
\[
\ln TE_{it} = \ln y_{it}^* - \ln y_t = \left( \beta_m + \beta_{xm} \ln x_{it} \right) \left( m_i^* - m_i \right) + \frac{1}{2} \beta_{mm} \left( m_i^2 - m_i^* \right) \leq 0
\]

Note that when \( m_i = m_i^* \), that is when the firm is using the amount of management that defines the frontier, \( \ln TE_{it} = 0 \), and, therefore, the firm is technically efficient. Equation (4) can be rewritten as:

\[
\ln TE_{it} = \theta_i + \theta_{xi} \ln x_{it}
\]

where

\[
\theta_i = \beta_m \left( m_i - m_i^* \right) + \frac{1}{2} \beta_{mm} \left( m_i^2 - m_i^* \right)
\]

\[
\theta_{xi} = \beta_{xm} \left( m_i - m_i^* \right)
\]

Equation shows that \( TE \) has two components. One, can be modeled as a time invariant individual effect (\( \theta_i \)). But the other term, reflecting the interaction of management with input use, has to be specified as a time varying component in the production function (\( \theta_{xi} \ln x_{it} \)). Therefore, an interesting feature of expression is that the implied technical efficiency will be time varying because it depends on \( x_{it} \) even when the observed level of management and the one at the frontier are constant over time. This is so because the difference between current management and ‘efficient’ management interacts with the input level. It is important to note that only when \( \beta_{xm} = 0 \) is it possible to associate each level of technical efficiency with a level of management. This suggests that given the specification of it does not seem appropriate to model technical efficiency as a fixed effect since it depends on \( x_{it} \). This specification of the model departs from the previous literature in this field that uses panel data to estimate the production function (e.g., Schmidt and Sickles, 1984).

Another interesting feature of our measure of \( TE \) is that it is different for firms with the same level of managerial input if they use different quantities of the other inputs. The change in managerial input necessary to increase \( TE \) in a given amount differs depending on input use as well.\(^2\)

\(^2\) This idea appeared in an early paper. Hall and Winsten (1959) claimed that for similar firms (using the same amounts of inputs) more management would imply more output and therefore greater TE. In this case, there is a clear direct relationship between managerial ability and TE. However, things are less clear in the case of two firms using different inputs with the same level of TE. In this case, any increase in technical efficiency would require different increases in the levels of management for each firm.
It is interesting to analyze the effects on TE of changes in managerial ability and input use. Those effects can be seen in the following derivatives:

\[
\frac{\partial \ln TE_{it}}{\partial m_i} = \beta_{m} + \beta_{mm} m_i + \beta_{mx} \ln x_i \\
\frac{\partial \ln TE_{it}}{\partial \ln x_i} = \beta_{mx} \left( m_i - m_i^* \right)
\]

The derivative of TE with respect to managerial input corresponds exactly to the condition for monotonicity of production with respect to managerial ability shown in expression. Therefore, an increase in managerial ability increases TE given conventional inputs if the production function is monotonic in managerial ability. The derivative of TE with respect to the level of input use is negative if \( \beta_{xm} \) is positive because \( m_i \) is smaller than \( m_i^* \) by definition. Therefore, when \( \beta_{xm} \) is positive the increase in the use of conventional inputs decreases TE for a given amount of managerial ability.

In this section we have shown that the model in raises interesting issues about the relationship between fixed management and technical efficiency. In particular, it shows that TE is not necessarily a fixed effect, rather TE can vary over time and that the relationship between TE and managerial ability depends on the amount of managerial ability and conventional inputs. In the next section we discuss the estimation of this model in more general terms, with more than one observed input.

### 3. ESTIMATION ISSUES

Model cannot be directly estimated because the individual level of management is unobservable. Previous authors have dealt with this problem by introducing a proxy for management in a cost function. [See Dawson and Hubbard (1985), Alvarez and Arias (2003)]. Since the use of a proxy introduces new ambiguities (measurement error) into the model, we will employ a more direct approach to the problem which takes advantage of the panel nature of the data set we will analyze. We will now translate the model in the preceding section into an empirically estimable form that includes managerial ability as an input.

#### 3.1. Stochastic Frontier Model with Fixed Management
We consider production with \( K \) inputs, \( x^1, \ldots, x^K \). As before, let \( m^*_i \) denote the level of management that defines the frontier and \( m_i \) the actual management input for firm \( i \). We continue to employ the translog form. The production model will then be

\[
\ln y_{it} = \ln y^*_i - u_{it} = \alpha + \sum_{k=1}^{K} \beta_k \ln x_{ikt} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{ikt} \ln x_{ilt} + \beta_{m} m^*_i + \frac{1}{2} \beta_{mm} m^2_i + \sum_{l=1}^{K} \beta_{ml} \ln x_{ilt} m^*_i + v_{it} - u_{it}
\]

where

\[
u_{it} = \ln y^*_i - \ln y_{it} = \left( \beta_{m} + \sum_{k=1}^{K} \beta_{km} \ln x_{ikt} \right) \left( m^*_i - m_i \right) + \frac{1}{2} \beta_{mm} \left( m^2_i - m^2_i \right) \geq 0.
\]

In (7), \( u_{it} \) now corresponds to the standard definition of technical inefficiency, so that \( TE_{it} = \exp(-u_{it}) \). The assumption that production is monotonically increasing in managerial input enters the definition of the production frontier. The amount of \( m^*_i \) produces the maximum output given inputs. The resulting production frontier is the same in spirit as the familiar stochastic frontier model save for the time variation in the ‘inefficiency term’ and the presence of the unobservable input \( (m^*_i) \) in the frontier production function.

For estimation, a critical assumption is the orthogonality of \( u_{it} \) and the input levels in \( x_{ikt} \). While \( \ln x_{ikt} \) does appear in the time varying part of \( u_{it} \), we assume that it does not influence \( m^*_i - m_i \). Thus, \( u_{it} \) is of the form \( u_{it} = \sum_{k=1}^{K} \left( m^*_i - m_i \right) g(x_{ikt}) \), and each term \( \left( m^*_i - m_i \right) g(x_{ikt}) \) will, by virtue of the presence of the freely varying \( (m^*_i - m_i) \) be uncorrelated with \( x_{ikt} \). We note, this is essentially the argument of Zellner et al. (1966), who argued that in a production function, the input levels would be uncorrelated with the deviation of output from the optimal output, even though they would obviously be correlated with the actual output, itself.

Although involves an unobservable variable \( m^*_i \), we can translate it into an empirically estimable form. The result follows from the fact that the unobservable can be seen as a ‘random effect’ in a panel data model. For that purpose, we rewrite as follows:

\[
\ln y_{it} = \left( \alpha + \beta_{m} m^*_i \right) + \frac{1}{2} \beta_{mm} m^2_i + \sum_{k=1}^{K} \left( \beta_k + \beta_{km} \right) \ln x_{ikt} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{ikt} \ln x_{ilt} + v_{it} - u_{it}
\]
In this form, the model takes the appearance of a random coefficients stochastic frontier model [See Tsionas (2002), Greene (2003)] although it differs from more familiar random coefficients models in several respects. First, the random component of each random parameter is the same, $m^*_i$. Second, the square of the random component appears in the model. Despite these complications the model is estimable.

The estimation of the model has two important requirements that remain to be considered. First, the random coefficients model requires as an identification condition that the random components of the coefficients be uncorrelated with the explanatory variables. The random component of the coefficients in our model is the level of management that defines the frontier ($m^*_i$), which likely is correlated with at least some of the inputs. (Note this is a different issue from the deviation of $m_i$ from $m^*_i$ which we argued above is uncorrelated with the inputs). In order to avoid this problem, we take the approach suggested for random effect models by Chamberlain (1984) (and borrow from early work by Mundlak) and specify $m^*_i = \tau \ln x_i + w_i$ where $\ln x_i$ is the vector of the means of the logs of inputs, $\tau$ is a vector of parameters to be estimated (a constant term will not be identified) and $w_i$ is a random term that follows a standard normal distribution and which we assume is uncorrelated with the inputs.

The second issue concerns the stochastic specification of $u_{ir}$. The estimation by maximum likelihood of the model in requires a distributional assumption about $u_{ir}$. Greene (1980) shows that the choice of this assumption may affect the results of the estimation. In this paper, we model the distribution of $u_{ir}$ as half normal, which produces a random coefficients stochastic frontier model in the spirit of Aigner et al. (1977). The impact that the specific assumption has on the empirical results is an interesting question that has been left for future research.
3.2. Estimation of the Random Coefficients Stochastic Frontier Model

This section will describe the method used to estimate the parameters of the random coefficients stochastic frontier model (RCSFM). A crucial aspect of the parameterization of the model is that each occurrence in of frontier management, as \( m_i^* \), \( m_i^{\tau^2} \), and \( \log x_{it} m_i^* \) appears attached to a free parameter, and there is a free constant term in the model. As such, aside from the regression parameters, \( \tau \), in \( m_i^* = \tau \ln x_i + w_i \), even though there is no free constant term in \( m_i^* \), neither location parameter, \( \mu_w = E[w_i] \), nor scale parameter, \( \sigma_w^2 = \text{Var}[w_i] = E[w_i^2] - E[w_i]^2 \) are identified from observable data in the context of the model. We assume, then, with no loss of generality, that \( w_i \) has mean zero and variance 1. Normality is not strictly necessary, but normalization of both moments is. No free parameters of the distribution of \( w_i \) are estimable.

From , we define

\[
\varepsilon_{it} = (v_{it} - u_{it})
\]

In what follows, it is useful to note explicitly that \( \varepsilon_{it} \) will be conditioned on \( m_i^* \). The conditional density for a single observation in the half normal stochastic frontier model is

\[
f(\varepsilon_{it} | m_i^*) = \frac{2}{\sigma} \phi\left( \frac{\varepsilon_{it} | m_i^*}{\sigma} \right) \Phi\left( -\lambda \varepsilon_{it} | m_i^* \right)
\]

where \( \phi(z) \) and \( \Phi(z) \) denote the density and CDF of the standard normal variable, respectively [See Aigner et al., (1977)]. The joint density for \( T \) observations on firm \( i \) is

\[
f(\varepsilon_{i1}, \ldots, \varepsilon_{iT} | m_i^*) = \prod_{t=1}^{T} f(\varepsilon_{it} | m_i^*).
\]

This is the contribution to the conditional likelihood for firm \( i \), \( L_i | m_i^* \). The unconditional contribution to the likelihood function is

\[
L_i = \int_{m_i^*} \prod_{t=1}^{T} f(\varepsilon_{it} | m_i^*) g(m_i^*) \, dm_i^*
\]

where \( g(m_i^*) \) is the marginal density of \( m_i^* \). Consistent with the preceding discussion, there are no new parameters in this density –we have assumed that \( m_i^* \) has a standard normal distribution. The log likelihood is

\[
\log L(\delta) = \sum_{i=1}^{N} \log L_i(\delta)
\]
where we use $\delta$ to denote the full vector of parameters in the model. The maximum likelihood estimates of the parameters are obtained by maximizing with respect to $\delta$. Since the integral in will not have a closed form, it is not possible to maximize directly. We will use the method of maximum simulated likelihood, instead. [See Train (2003), Greene (2003a) and Gourieroux and Monfort (1996) for discussion].

4. EMPIRICAL APPLICATION

Our empirical application uses data from a balanced panel of 247 dairy farms located in Northern Spain. We have data on these farms for a period of six years (1993-1998). Since the farms are specialized in milk production we consider only one output. The variables used in the estimation of the production frontier are described in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Variables Used in Production Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Milk  Milk production (liters)</td>
</tr>
<tr>
<td>Cows  Number of milking cows</td>
</tr>
<tr>
<td>Labor Number of man-equivalent units</td>
</tr>
<tr>
<td>Land  Hectares of land devoted to pasture and crops</td>
</tr>
<tr>
<td>Feed  Kilograms of feedstuffs fed to dairy cows</td>
</tr>
</tbody>
</table>

We wish to explore the empirical consequences of restrictions on the role of management in the production function. For that purpose, we first estimate a conventional (pooled) stochastic production frontier with four inputs and including time-effects (this is equivalent to estimating equation without considering management). The results of the estimation of the production frontier can be seen in the first column of estimates in Table 2. The explanatory variables in the original data were divided by their geometric mean. By doing this, the first order coefficients can be interpreted as output elasticities evaluated at the geometric mean of the sample. They are positive and significantly different from zero at conventional levels of significance.

Table 2. Estimated Frontier Models

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3 We include a summary of the main features of the simulation in the appendix.

4 All models were estimated using LIMDEP 8.0 (Greene, 2002).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Param.</th>
<th>Stochastic Frontier</th>
<th>Random Parameters Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Means of random parameters</td>
<td>Management x inputs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_k$</td>
<td>$\beta_{km}$</td>
</tr>
<tr>
<td>Cows</td>
<td>$\beta_1$</td>
<td>0.6106 (0.0223)**</td>
<td>0.6597 (0.0113)**</td>
</tr>
<tr>
<td>Labor</td>
<td>$\beta_2$</td>
<td>0.0254 (0.0128)**</td>
<td>0.0406 (0.0081)**</td>
</tr>
<tr>
<td>Land</td>
<td>$\beta_3$</td>
<td>0.0239 (0.0146)**</td>
<td>0.0251 (0.0013)*</td>
</tr>
<tr>
<td>Feed</td>
<td>$\beta_4$</td>
<td>0.4393 (0.0121)**</td>
<td>0.3090 (0.0060)**</td>
</tr>
<tr>
<td>Constant</td>
<td>$\alpha$</td>
<td>11.719 (0.0128)**</td>
<td>11.6761 (0.0041)**</td>
</tr>
<tr>
<td>Management</td>
<td>$\beta_m$</td>
<td>-0.1102 (0.0014)**</td>
<td>-0.0165 (0.0017)**</td>
</tr>
<tr>
<td>Management $\times$ Management</td>
<td>$\beta_{mm}$</td>
<td>-0.1102 (0.0014)**</td>
<td>-0.0165 (0.0017)**</td>
</tr>
<tr>
<td>Cows $\times$ Cows</td>
<td>$\beta_{11}$</td>
<td>0.5556 (0.1158)**</td>
<td>0.1517 (0.0479)**</td>
</tr>
<tr>
<td>Labor $\times$ Labor</td>
<td>$\beta_{22}$</td>
<td>-0.1042 (0.0457)**</td>
<td>-0.0453 (0.0173)**</td>
</tr>
<tr>
<td>Land $\times$ Land</td>
<td>$\beta_{33}$</td>
<td>0.2116 (0.0371)**</td>
<td>-0.1202 (0.0334)**</td>
</tr>
<tr>
<td>Feed $\times$ Feed</td>
<td>$\beta_{44}$</td>
<td>-0.01150 (0.0964)</td>
<td>0.1016 (0.0151)**</td>
</tr>
<tr>
<td>Cows $\times$ Labor</td>
<td>$\beta_{12}$</td>
<td>-0.1078 (0.0507)**</td>
<td>-0.0172 (0.0209)</td>
</tr>
<tr>
<td>Cows $\times$ Land</td>
<td>$\beta_{13}$</td>
<td>-0.3390 (0.0638)**</td>
<td>0.0919 (0.0241)**</td>
</tr>
<tr>
<td>Cows $\times$ Feed</td>
<td>$\beta_{14}$</td>
<td>0.2230 (0.0753)**</td>
<td>-0.0826 (0.0258)**</td>
</tr>
<tr>
<td>Labor $\times$ Land</td>
<td>$\beta_{23}$</td>
<td>0.0694 (0.0263)**</td>
<td>0.0019 (0.0148)</td>
</tr>
<tr>
<td>Labor $\times$ Feed</td>
<td>$\beta_{24}$</td>
<td>-0.0027 (0.0500)</td>
<td>-0.0178 (0.0108)</td>
</tr>
<tr>
<td>Land $\times$ Feed</td>
<td>$\beta_{34}$</td>
<td>-0.0695 (0.0411)</td>
<td>-0.0073 (0.0143)</td>
</tr>
<tr>
<td>Year 1993</td>
<td>$\delta_{33}$</td>
<td>-0.0363 (0.0120)**</td>
<td>-0.0907 (0.0044)**</td>
</tr>
<tr>
<td>Year 1994</td>
<td>$\delta_{34}$</td>
<td>-0.0198 (0.0126)**</td>
<td>-0.0598 (0.0044)**</td>
</tr>
<tr>
<td>Year 1995</td>
<td>$\delta_{35}$</td>
<td>-0.0013 (0.0117)</td>
<td>-0.0326 (0.0044)**</td>
</tr>
<tr>
<td>Year 1996</td>
<td>$\delta_{36}$</td>
<td>-0.0067 (0.0121)</td>
<td>-0.0228 (0.0043)**</td>
</tr>
<tr>
<td>Year 1997</td>
<td>$\delta_{37}$</td>
<td>-0.0110 (0.0125)</td>
<td>-0.0188 (0.0044)**</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>1.8657 (0.1521)**</td>
<td>1.1969 (0.0465)**</td>
</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td>0.1932 (0.0061)**</td>
<td>0.0961 (0.0013)**</td>
</tr>
<tr>
<td>Log L</td>
<td></td>
<td>860.649</td>
<td>1406.474</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
We now estimate the random coefficients model specified in equation following the estimation procedure described previously. The results, which were obtained using 1,000 Halton draws in each replication, can be seen at the right in Table 2. The means for the random parameters remain positive and significant at the geometric mean of the sample. The sum of these coefficients is close to 1.0, implying a constant returns to scale technology. As expected, these means differ slightly from the coefficients of the conventional stochastic frontier. The coefficients of management ($\beta_m$, $\beta_{mm}$ and $\beta_{km}$) are significantly different from zero at conventional levels of significance. This can be interpreted as evidence in favor of the random coefficients model with respect to the conventional stochastic frontier approach since the coefficients of the production function change with the level of management of the farm. The coefficients of the interactions of management with inputs ($\beta_{km}$) have a positive sign for cows and land and a negative sign for feed and labor. Finally, the negative sign of management squared can be reasonably interpreted as evidence that management has a positive but decreasing effect on production. The coefficients of management at the frontier on input means ($\tau$) are all positive with the exception of the coefficient of feed. This result seems intuitive, as using more feed (cp) can be seen as a way of compensating for management mistakes due to a lack of managerial ability.

In summary, management plays a complex role in production far from the simplistic approach implied by a production frontier with fixed coefficients. Therefore, the fixed coefficients frontier is not a good instrument to analyze firm behavior when management is unobservable. This result is important in a number of settings. For example, unobserved management is a key factor in explaining firm size and growth (Jovanovic, 1982). Another example is farm policy, where the level of management is important to assess the effects of increasing the size of farms (Dawson and Hubbard, 1985; Alvarez and Arias, 2003). The efficiency levels computed in the model with fixed management can be computed according to Jondrow et al.’s (1982) prescription. Neglecting the presence of the management variable for the moment, the computation is

$$E[u_i | \varepsilon_i, m_i^*] = \frac{\sigma \lambda}{(1 + \lambda^2)} \left[ \Phi(-\varepsilon_{ii} | m_i^* \lambda / \sigma) - \frac{(\varepsilon_{ii} | m_i^* \lambda)}{\sigma} \right]$$

where $\varepsilon_i$ is defined in . Like the other quantities which involve $m_i^*$, this must be simulated. We will compute this from the conditional mean, given the other data for farm $i$. The value
of \( m_i^* \) can be computed from the conditional distribution of \( m_i^* \) given the data on farm \( i \) using Bayes theorem as follows: Let \( y_i \) denote the vector of logs of the outputs for farm \( i \) for the six years. Let \( X_i \) denote the other data (inputs and year dummy variables, linear and quadratic terms in logs) for farm \( i \). The conditional distribution of \( m_i^* \) given \( y_i \) is

\[
f(m_i^* \mid y_i, X_i) = \frac{f(y_i \mid m_i^*, X_i)g(m_i^*)}{f(y_i \mid X_i)}
\]

\[
= \frac{f(y_i \mid m_i^*, X_i)g(m_i^*)}{\int_{m^*_i} f(y_i \mid m_i^*, X_i)g(m_i^*)dm_i^*} \tag{15}
\]

The denominator is the contribution of farm \( i \) to the likelihood function for the sample (not the log likelihood) in equation. Thus, we can estimate \( m_i^* \) for farm \( i \) as the conditional mean from this distribution. This would be

\[
E(m_i^* \mid y_i, X_i) = \frac{\int_{m_i^*} m_i^* f(y_i \mid m_i^*, X_i)g(m_i^*)dm_i^*}{\int_{m_i^*} f(y_i \mid m_i^*, X_i)g(m_i^*)dm_i^*} \tag{16}
\]

Like the likelihood, itself, this quantity cannot be computed directly, as the integrals will not have a closed form. But, they can be simulated, in the same fashion. Thus, the simulation based estimator of \( m_i^* \) is

\[
\hat{E}(m_i^* \mid y_i, X_i) = \frac{(1/R)\sum_{r=1}^{R} m_i^* \hat{f}(y_i \mid m_i^*, X_i)}{(1/R)\sum_{r=1}^{R} \hat{f}(y_i \mid m_i^*, X_i)} \tag{17}
\]

Note, \( \hat{f} \) denotes the portion of the likelihood function for farm \( i \), evaluated at the parameter estimates and the current draw of \( m_i^* \). Draws on \( m_i^* \) come from the standard normal distribution and, as before, are generated using Halton sequences. With these estimates in hand, estimated inefficiencies for the farms are produced using . An estimate of the unobserved \( m_i \) can be obtained by substituting the estimate of \( m_i^* \) in the definition of \( u_o \) in
and solving for $m_i$. The resulting expression is:

$$m_i = \frac{-\left[ \beta_m + \sum_{k=1}^{K} \beta_{km} (\ln x_{kit}) \right]}{\beta_{mm}}$$

$$\pm \sqrt{\left[ \beta_m + \sum_{k=1}^{K} \beta_{km} \ln x_{kit} \right]^2 - 2\beta_{mm} \left( u_{it} - \left[ \beta_m + \sum_{k=1}^{K} \beta_{km} \ln x_{kit} \right] m_i^\ast - \frac{1}{2} \beta_{mm} m_i^{\ast 2} \right)}$$

Table 3 presents descriptive statistics for the estimates of $u_{it}$ based on produced from the basic stochastic frontier model and from the random coefficients stochastic frontier model, first without accounting for the effect of management and then from the random parameters model that does include it. The distribution for the latter set of estimates has a much smaller mean and a much tighter distribution. Figure 1 below suggests the same pattern. This suggests that not accounting for the effect of management on production has somewhat inflated the estimates of $u_{it}$. One might view this as a decomposition of the inefficiency into two parts, one explicitly accounted for by the management effect and the other apparently due to other unexplained factors.

Table 3. Descriptive Statistics for Estimated Inefficiencies and Estimated Management

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Inefficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Management</td>
<td>0.1352</td>
<td>0.0794</td>
<td>0.0107</td>
<td>0.4747</td>
</tr>
<tr>
<td>With Management</td>
<td>0.0576</td>
<td>0.0252</td>
<td>0.0113</td>
<td>0.2609</td>
</tr>
<tr>
<td>Estimated Management</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current management, $m_i$</td>
<td>-0.5024</td>
<td>1.1761</td>
<td>-4.6250</td>
<td>2.8767</td>
</tr>
<tr>
<td>Frontier management, $m_i^\ast$</td>
<td>0.0029</td>
<td>1.2251</td>
<td>-3.7545</td>
<td>3.1970</td>
</tr>
</tbody>
</table>

In the quadratic formula for estimating $m_i$, notice that $m_i$ depends on variables which vary over time, since it is a function of the data and of $u_{it}$. In order to obtain a time invariant $m_i$, we compute this at the time varying data, and take the farm means of the estimated $m_i$’s. Also, note that the $u_{it}$ that appears in $u_{it}$ is not the estimator of $u_{it}$ computed in that we use to do the computation. As such, the time invariance is not preserved in our estimate – and we use the farm average as an estimate of the underlying quantity.
Table 3 also shows descriptive statistics for the estimates of $m_i$ and $m_i^*$. As expected, $m_i$ and $m_i^*$ are positively correlated, as can be seen in Figure 2, which shows the farm mean of the time varying estimated values for $m_i$. The diagonal line drawn in the figure shows that current management $m_i$ is always less than the level of management at the frontier, $m_i^*$. As expected, the relationship between $m_i$ and technical inefficiency is negative. The relationship is shown in Figure 3, where the time varying estimate is plotted against the time invariant $\bar{m}_i$. The relationship is weak ($R^2$ is only 0.012) but statistically significant.  

5. CONCLUSIONS

This paper explores the relationship between fixed managerial ability and technical efficiency. For that purpose, fixed managerial ability is introduced in the model as an unobservable input. The interaction between the unobservable input and conventional inputs creates a great deal of difficulty in estimating the resulting model. However, the model can be cast as a stochastic frontier with random coefficients under certain assumptions about the relationship between managerial ability and conventional inputs.

We illustrate the feasibility of the proposed estimation procedure using a sample of dairy farms in Northern Spain. The mean of the random coefficients are of the same magnitude as the coefficients of a standard stochastic frontier, but managerial ability is found to affect the value of some random input coefficients.

The fixed coefficients frontier provides a measure of technical efficiency that can be related with different levels of management depending on the circumstances of the farm (input use and output production). The feasibility of estimating both the farm current level management and the level of management that defines the production frontier is a clear advantage of the empirical model developed in the paper.

The empirical model developed in the paper can be useful in analyzing firm policy issues since management is considered a key variable in assessing the effects of these policies. In fact, the model avoids both the simplistic assumptions about management implied by fixed coefficients frontiers and the econometric problems associated with the use of proxies.

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$^6$ The estimated regression equation is $\hat{u}_i = 0.0576 (0.00065) - 0.00227 (0.00053) \hat{m}_i^*$ (estimated standard errors in parentheses).
References


Figure 1. Kernel Density Estimates for Inefficiency Estimates.
Figure 2. Plot of Current Management (MEANMI) vs. Management at the frontier (MANAGE)

Figure 3. Plot of Estimated Inefficiency vs. Management at the frontier
APPENDIX.

Let $m_{ir}^*$ denote the $r$-th random draw from the standard normal population of $m_{ir}^*$ in a sample of $R$ such draws. Then, the contribution to the simulated likelihood function for firm $i$ is

$$L_i^S = \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T} f(\varepsilon_{it} | m_{ir}^*)$$

The simulated log likelihood that is maximized is

$$\log L_i^S (\delta) = \sum_{i=1}^{N} \log \left( \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T} f(\varepsilon_{it} | m_{ir}^*) \right)$$

This function is smooth and twice continuously differentiable in the parameters. (For conditions under which maximization of the simulated log likelihood produces an estimator with the same asymptotic properties as the true MLE, see Gourieroux and Monfort (1996), Train (2003) and Greene (2003)). The derivatives of the simulated log likelihood are obtained as follows:

$$\frac{\partial \log L_i^S}{\partial \delta} = \frac{\partial L_i^S / \partial \delta}{L_i^S} = \frac{\sum_{r=1}^{R} \left[ \prod_{t=1}^{T} f(\varepsilon_{it} | m_{ir}^*) \right] \sum_{r=1}^{R} \left[ \partial \log f(\varepsilon_{it} | m_{ir}^*) / \partial \delta \right]}{\sqrt{\sum_{r=1}^{R} \left[ \prod_{t=1}^{T} f(\varepsilon_{it} | m_{ir}^*) \right]}}$$

$$= \sum_{r=1}^{R} \omega_i \sum_{t=1}^{T} g_{it}(m_{ir}^*)$$

$$= \sum_{r=1}^{R} \omega_i^* g_i(m_{ir}^*)$$

where $\omega_i^*$ is a set of nonnegative weights that sum over $r$ to one for each $i$ by construction and $g_i(m_{ir}^*)$ is the vector of derivatives, $\partial \log f(\varepsilon_{it} | m_{ir}^*) / \partial \delta$. Let

$$x_{ir}^* = [1, \ldots, \ln x_{itk}, \ldots, 1^{1}, \ln x_{itk} \ln x_{itl}, \ldots, m_{ir}^*, 1^{1} m_{ir}^*, 2, \ldots, 1^{1} 2 \ln x_{itk} m_{ir}^*]$$

so that $\varepsilon_{it} | m_{ir}^* = y_{it} - \beta x_{i}(m_{ir}^*) = \varepsilon_{it}^*$. For convenience, let $h_{it}^* = \phi(-\lambda \varepsilon_{it}^* / \sigma ) \Phi(-\lambda \varepsilon_{it}^* / \sigma )$. The required first derivatives are

$$\frac{\partial \log f(\varepsilon_{it}^*)}{\partial \beta} = \left[ \begin{array}{c} \frac{\sigma}{\sigma} \frac{\lambda}{\lambda} \end{array} \right]$$

$$= \left[ \begin{array}{c} \left( \frac{\varepsilon_{it}^*}{\sigma} \right) + h_{it}^* \lambda \right] x_{it}^*$$

$$= \left( \frac{\varepsilon_{it}^*}{\sigma} \right) - 1 + \left( \frac{\varepsilon_{it}^*}{\sigma} \right) \lambda h_{it}^*$$

$$= -\varepsilon_{it}^* h_{it}^*$$

(22)
The integrals in and its derivatives are approximated by obtaining a sufficient number of draws from the population generating \( m_i^* \). The law of large numbers is invoked to infer that the sample averages will converge to the underlying integral. Random draws from the population are sufficient for this process, but not necessary. What is essential is coverage of the range of variation of \( m_i^* \), not randomness of the draws. The method of Halton sequences [see Bhat (1999), for example] is used to provide much more efficient coverage of the range, and in turn, much faster estimation than the method of random simulation (See, as well, Train (2003, pp. 224-238) for a discussion of Halton sequences). Thus, \( m_{ir}^* \) is the \( r \)th element of the Halton sequence for individual \( i \). The elements of the Halton sequence, \( H_i^r \) are spread over the unit interval, \((0,1)\). The draw of \( m_{ir}^* \) is obtained by the inverse probability transform. Thus, 
\[
m_{ir}^* = \Phi^{-1}(H_i^r).
\]
The estimated standard errors of the parameter estimators are computed by using the BHHH estimator, as before, with simulation used for the derivative vectors. Since we are only integrating over a single dimension, the gain in efficiency, if this application is like others, is on the order of ten fold - that is, the same results are obtained with only about one tenth the number of draws needed. We have used 1,000 draws in our estimation, which would correspond to several thousand draws were they produced with a random number generator instead.