How intergenerational disagreement constrains government intervention*

Francisco M. Gonzalez
University of Calgary
(francisco.gonzalez@ucalgary.ca)

Itziar Lazkano
University of Calgary
(ilazcano@ucalgary.ca)

Sjak A. Smulders
Tilburg University
(J.A.Smulders@uvt.nl)

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PRELIMINARY

Abstract

In this paper, we explore how intergenerational disagreement constrains the calculus of second-best growth. We illustrate the contrast between two natural sources of disagreement when generations are overlapping and preferences are aggregated in a utilitarian manner. Social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations; but they tend to exhibit a bias against the present because coexisting generations are imperfectly altruistic about currently older generations. Equilibrium growth is inefficiently low when the former bias dominates. Otherwise society faces a difficult intergenerational equity problem. Ironically, altruistic generations tend to support institutions that enable commitments to lower growth, at the expense of future generations. Our analysis suggests that the ability of actual governnments to improve the welfare of future generations can be significantly constrained, even in an altruistic society.

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1 Introduction

Do we save enough for our future and our children's future? Private saving behavior suggests that individuals tend to postpone investment in the short run and would like to commit to increased investment only in the future, which is not always possible. Whether aggregate saving is also too low is not immediately clear. Individuals will support government policies and institutions to get savings decisions right. However, policy decision making creates problems of its own. First, governments have to strike a balance between the preferences of different agents, and young and old people living at the same time may have different stakes at raising savings. Second, governments today may be unable to bind future governments, and the effect of current policies on our own and our children's future may be undone by future policies. Altruism might in fact bias policies to the future, if living generations agree more easily on providing a future for the following generations than they do on the current allocation of resources among themselves.

Optimal national savings are commonly characterized as optima of some intertemporal social welfare function within the optimal growth framework developed by Ramsey (1928), Cass (1965) and Koopmans (1965). This framework relies on the assumption that social preferences are time-consistent, which ensures that the relative valuation of utility flows at different dates remains unchanged as the planning date evolves. When applied to the intergenerational context, the familiar time-consistency requirement amounts to an assumption of perfect altruism, leaving no room for intergenerational disagreement to constrain optimal growth paths.

In this paper, we explore how intergenerational disagreement constrains the calculus of second-best growth. We illustrate the contrast between two natural sources of disagreement when generations are overlapping and preferences are aggregated in a utilitarian manner. Social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations; but they tend to exhibit a bias against the present because coexisting generations are imperfectly altruistic about currently older generations. Furthermore, the properties of equilibrium growth depend significantly on the relative stregth of the two biases.

Phelps and Pollak (1968), and more recently Barro (1999) and Krusell et al. (2002), analyze equilibrium saving in the presence of imperfect altruism about future generations, where private agents suffer from a present-bias, favoring short-term consumption, under the assumption that the planner inherits the specific form of time inconsistency that afflicts private agents. Such an assumption is natural in their non-overlapping generations setting, which is the standard setting considered in other analyses of equilibrium growth with imperfect intergenerational altruism (e.g., Kohlberg, 1976, Bernheim and Ray, 1987, Ray, 1987).

We focus on intergenerational disagreement that stems from the combination of imperfect altruism about *past* generations and the overlap of generations in actual economies, which translates into disagreement between coexisting generations about the distribution of *current* aggregate consumption. It is well known that this kind of intergenerational disagreement renders plausible social welfare functions time-inconsistent, even if individuals are perfectly altruistic towards *future* generations.[1] However, today's young generations are tomorrow's old, and so it is unclear how intergenerational disagreement at each point in time translates into intertemporal allocations. In particular, neither the specific form of time-inconsistency that one may expect to afflict the planner nor its implications for (second-best) government intervention in this context are well understood. The goal of our analysis is to shed some new light into these issues.

Formally, we consider a tractable endogenous growth model with overlapping altruistic generations, and characterize Markov perfect equilibrium behavior of a sequence of short-lived utilitarian planners. We assume that each planner's objective function is a weighted sum of the utilities of generations that are currently alive. Our modeling choices reflect the facts that actual generations coexist, and democratic governments are unlikely to be immune to disagreement between current generations. Our restriction to Markov perfect equilibria captures the idea that intergenerational coordination of intertemporal choices is difficult, focusing our analysis on *discretionary* government intervention.

For simplicity, we develop our main arguments in the natural case where individuals

[1] See, e.g., Burbridge (1983), Calvo and Obstfeld (1988), Bernheim (1989), and Hori (1997).

are perfectly altruistic towards the next generation, but not about *past* generations (e.g., Barro, 1974). With overlapping generations, such a disagreement immediately translates into disagreement between current generations about the distribution of current aggregate consumption. In turn, since current generations do agree on the future distribution of aggregate consumption, there must be disagreement between current and future planners about future aggregate consumption. The simplicity of our example allows us to characterize the specific form of time-inconsistency that one may expect and its consequences for Markov perfect equilibrium plans.

We begin by showing that planners whose objective function is a weighted sum of utilities of the currently alive generations in effect have quasi-hyperbolic preferences over *aggregate consumption*, even if all individuals have standard geometric discounting. This provides a simple application where quasi-hyperbolic discounting arises naturally. Moreover, we conclude that the specific bias of individual preferences and that of the planners' can be different. We show that the planner is biased *against* the present when individuals have standard time-consistent preferences. The current planner's preferences exhibit a bias *against* present aggregate consumption, because the current young does not value the consumption of the current old, and social welfare puts positive weight on the current young. Since the current generations value future consumption equally, the current planner favors future aggregate consumption over current aggregate consumption. We also show that the direction of the planner's bias may remain unchanged even if private agents are imperfectly altruistic towards future generations, suffering from a present-bias.

The above bias in social preferences underlies the manner in which intergenerational disagreement constrains equilibrium growth. It should be noted that Markov perfect equilibria are time-consistent by construction, and the planners need to anticipate the equilibrium behavior that will actually be followed in the future. In addition, the current planner realizes that future investment will respond to future income, and so he has an incentive to invest strategically in order to manipulate future investment decisions. Our analysis focuses on the tractable class of Markov equilibria in linear strategies. When we analyze the properties of equilibrium growth, our main conclusion is that intergenerational disagreement can be conducive to growth, rather than inimical to it, as is commonly concluded from the analysis of non-overlapping generations models (e.g., Sen, 1967, Kohlberg, 1976).

Accounting for the overlapping generations demographic structure of actual economies not only gives different results, but it has implications for welfare. Strotz's (1956) seminal work, and more recently Laibson's (1997) demonstrate the general relevance that the economic agents' time-inconsistency has for the design of institutions that can cope with intertemporal disagreement by facilitating commitments. An implication of our analysis is that the availability of commitment mechanisms to cope with intergenerational disagreement migth lower growth below the second-best, benefiting current generations at the expense of future generations. Thus, rather than following the second-best, living generations would unanimously support the introduction of constitutional rules that lead to a permanent reduction in the growth rate of the economy. However, it is well known that dynamic inefficiency on the production side of the economy is ruled out in standard models of endogenous growth, such as the one we consider here (Saint Paul, 1992, King and Ferguson, 1993). Accordingly, equilibrium growth cannot be reduced without hurting future generations eventually.

When accounting for imperfect altruism about future as well as past generations, the above implication is ironic, because the very commitment mechanisms that are justified upon the grounds of intergenerational disagreement need to be introduced by current generations and so their introduction may hurt precisely those generations that is supposed to help. Of course, there are other sources of intergenerational disagreement that we have not considered here. Indeed, the problem of aggregation of heterogeneous preferences over consumption streams has received much attention (e.g., Weitzman, 2001, Caplin and Leahy, 2004, Blackorby et al., 2005, Gollier and Zeckhauser 2005, Jackson and Yariv, 2010, Zuber, 2010). For instance, Jackson and Yariv (2010) show that weighted averaging of utilities across individuals with geometric, but heterogeneous, time preferences tends to translate into time inconsistency that is necessarily characterized by a present-bias. Here, we have emphasized the relevance of heterogeneity of preferences about current consumption in an overlapping generations economy, which tends to generate a counteracting bias.

The next section presents the basic model. Section 3 illustrates the source of the bias in the planner's preferences in the context of the first-best allocation for an arbitrary planner. Section 4 analyzes the interior Markov perfect equilibrium in linear strategies and discusses the main implications of our analysis. Section 5 concludes. All proofs are found in a separate Appendix.

2 The model

Consider an economy with overlapping altruistic generations. In particular, a unit mass of individuals are born every period $t \geq 0$, each individual lives for two periods, and individuals born at date t have preferences given by

$$\begin{aligned} u_t &= u\left(c_t^y\right) + u\left(c_{t+1}^o\right) + \delta u_{t+1} \\ &= u\left(c_t^y\right) + u\left(c_{t+1}^o\right) + \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + u\left(c_{t+1+s}^o\right)\right), \end{aligned} \tag{1}$$

with $\delta \in (0,1)$, where c_t^y is the consumption of young agents at date t, and c_{t+1}^o is their consumption when old. We assume that individuals do not discount their second-period felicity, and also that felicities each period are isoelastic, with

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1, \quad \sigma > 0 \\ \ln(c) & \text{if } \sigma = 1. \end{cases} \tag{2}$$

Output is linear in the capital stock at the aggregate level, where k_t units of capital produce Ak_t units of output that become available at date $t+1$, with $A > 0$. The aggregate resources constraint in the economy is given by

$$Ak_t \geq c_t^y + c_t^o + k_{t+1} - k_t, \tag{3}$$

where we have ignored depreciation of the capital stock. We assume that $\delta(A+1) > 1$ in order to ensure positive equilibrium growth rates. We also assume that $\delta(A+1)^{\frac{1-\sigma}{\sigma}} < 1$, in order to ensure that growth is not so high that it leads to unbounded utility.

We consider a sequence of planners, each of which seeks to maximize

$$v_t = u_{t-1} + a u_t, \tag{4}$$

where $a > 0$. Note that the utility of individuals born in period t can be written as

$$u_t = u\left(c_t^y\right) + \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + \delta^{-1} u\left(c_{t+s}^o\right)\right).$$

The difference between individuals born at date $t-1$ and those born at date t, is that the latter do not care about the former. Otherwise, their preferences are time consistent: the trade-off between dates t and $t+1$ is perceived the same way by all individuals at date $t-1$ and at date t. Accordingly, the date-t planner's preferences are given by

$$v_t = u\left(c_{t-1}^y\right) + u\left(c_t^o\right) + (\delta + a) u\left(c_t^y\right) + (\delta + a) \sum_{s=1}^{\infty} \delta^s \left(u\left(c_{t+s}^y\right) + \delta^{-1} u\left(c_{t+s}^o\right)\right). \tag{5}$$

Consider the left side of (5). The planner at date t must treat the old individuals' felicity at date $t-1$ (first term) as sunk. The felicity from consumption of old individuals at date t (second term) enters with weight 1, the weight at which the planner values the old individuals' utility. However, the felicity of young individuals at date t (third term) enters not only with the direct weight on their young individuals' utility, given by a, but also indirectly because the old care about the young (with weight δ) and the planner cares about the old (with weight 1). The last term reflects that both young and old individuals care about future consumption through altruism, and the young care about their own future old-age consumption.

Inspection of (5) indicates that the planners' preferences are time inconsistent as long as $a > 0$, that is, as long as the date-t planner's preferences put any weight on the utility of currently young individuals. The weight of old-age consumption at t relative to old-age consumption at $t+1$ equals $1/(\delta + a)$, and this is smaller than the weight on the felicities of old-age consumption at $t+s$ relative to old-age consumption at $t+1+s$, which equals $1/\delta$. This stems from the fact that the currently old care about the future consumption of the currently young, but the young do not care about the currently old. However, both the currently young and currently old care about the consumption of all future generations. As a result, consumption of the currently old gets a relatively small weight.[2]

Thus, current and future planners evaluate future consumption streams differently, and so equilibrium behavior will be shaped by the underlying intergenerational conflict. Markov perfect equilibria provide a useful way to capture the consequences of such a conflict by focusing on behavior that depends solely on payoff-relevant state variables, reflecting the difficulties that current and future planners face to coordinate their behavior.

The welfare of each generation is influenced by the actions of different planners. Consequently, each planner's optimal behavior depends on its expectation of future planners' behavior. Since every planner can affect future aggregate economic conditions, equilibrium allocations depend on the interaction between current and future planners. We consider the following problem for each planner. Every period t the planner's objective is to maximize (5) subject to (2) and (3), taking as given the strategies of all other planners. Planners allocate aggregate resources among investment $(k_{t+1} - k_t)$, and consumption (c_t^y and c_t^o). A Markov strategy of the planner in period t consists of an investment policy $i^t(k_t)$, and consumption policies $c^{yt}(k_t)$ and $c^{ot}(k_t)$, which are only functions of the payoff-relevant state variable k_t. A sequence of Markov strategies for each planner $\{f_t(i^t(k_t), c^{yt}(k_t), c^{ot}(k_t))\}_{t=0}^{\infty}$ is a symmetric Markov perfect equilibrium if it is a subgame perfect equilibrium for every realization of the state variable k_t, and all planners follow the same strategy, that is, if $f_t(i^t(k_t), c^{yt}(k_t), c^{ot}(k_t)) = f(i(k_t), c^y(k_t), c^o(k_t))$, for all t. We will restrict attention to symmetric Markov perfect equilibria in linear strategies.

In order to understand the impact of time inconsistency on equilibrium behavior, it will be

[2] The time inconsistency of government's preferences would still arise in the presence of two-sided altruism (as in, e.g., Kimball, 1987). In particular, consider the following utility $u_t = u(c_t^y) + u\left(c_{t+1}^o\right) + \delta_F u_{t+1} + \delta_B u_{t-1} = \tilde{u}_t + \delta_F u_{t+1} + \delta_B u_{t-1}$. Kimball (1987) shows that this can, under certain conditions, generate the following time-consistent individual preferences: $u_t = \sum_{b=1}^{\infty}(\lambda_B)^{-b}\tilde{u}_{t-b} + \sum_{f=0}^{\infty}(\lambda_F)^f \tilde{u}_{t+f}$, where the discount factors λ_B and λ_F are functions of δ_B and δ_F. Following the same procedure as above, we see that time inconsitency of the government preferences arises if and only if $\lambda_B \neq \lambda_F$ and that the currently old get too small a weight whenever $\lambda_B > \lambda_F$. Note that the latter inequality arises whenever individuals care less about future and previous generations' felicity than they care about their own felicity, which can be assumed as the most natural assumption. This formulation would generate the same conclusions as our simpler case of one-sided altruism.

useful to consider first a benchmark problem for an arbitrary planner under the assumption that it can control future allocations.

3 Benchmark commitment solution

In order to understand the date-t planner incentives, suppose for a moment that it can control future allocations. The nature of this first-best solution is clarified by formulating the date-t planner problem recursively. We will simplify notation by avoiding time subscripts, and using primes to denote next-period values.

First, consider the static intergenerational allocation of consumption every period from the viewpoint of the date-t planner. At date t the optimal intergenerational allocation of consumption solves:

$$\max_{c_y, c_o} \left\{ u\left(c_y\right) + (\delta + a)^{-1} u\left(c_o\right) \right\} \tag{6}$$

subject to $c_y + c_o \leq c,$ with $c_y, c_o \geq 0,$

and so the young's share of aggregate consumption is given by

$$\tau_c \equiv \frac{c_y}{c} = 1 - \frac{c_o}{c} = \frac{1}{1 + (\delta + a)^{-1/\sigma}}, \tag{7}$$

where $c = c_y + c_o$.

In contrast, from date $t+1$ onwards, the date-t planner would choose intergenerational consumption allocations every period differently than future planners would actually do. Instead, the date-t planner's optimal allocation would solve the static problem:

$$\max_{c_y, c_o} \left\{ u\left(c_y\right) + \delta^{-1} u\left(c_o\right) \right\} \tag{8}$$

subject to $c_y + c_o \leq c,$ with $c_y, c_o \geq 0.$

Accordingly, the young's share of aggregate consumption at every future date would be

$$\overline{\tau}_c \equiv \frac{c_y}{c} = 1 - \frac{c_o}{c} = \frac{1}{1+\delta^{-1/\sigma}}, \tag{9}$$

where $c = c_y + c_o$. It is easy to see that $\tau_c > \overline{\tau}_c$. The date-$t$ planner prefers to allocate a larger share of aggregate consumption to the current young than the share he would like to allocate to the young in every future period.

Now consider the date-t planner's investment problem. Taking into account the optimal allocations of aggregate consumption every period, the relevant preferences for the date-t planner can be expressed in terms of aggregate consumption levels as

$$\widetilde{v}_t = q\left(\tau_c, a\right) u\left(c_t\right) + \sum_{s=1}^{\infty} \delta^s q\left(\overline{\tau}_c, 0\right) u\left(c_{t+s}\right), \tag{10}$$

where

$$q\left(\tau, a\right) = \tau^{1-\sigma} + \left(\delta + a\right)^{-1}\left(1-\tau\right)^{1-\sigma}. \tag{11}$$

Note that $\widetilde{v}_t$ is simply a positive linear transformation of the preferences for the date-t planner given in equation (5). The representation of social welfare in equation (10) reveals the key to understanding the planners' problem. It shows that the planners' preferences are time inconsistent whenever $q\left(\tau_c, a\right) \neq q\left(\overline{\tau}_c, 0\right)$, which in turn is the case if and only if $a > 0$. Even though all individuals have standard time-consistent, geometric preferences, planners in effect have time-inconsistent, quasi-geometric preferences (over aggregate consumption streams) of the form used by Phelps and Pollak (1968), Laibson (1997) and Krusell, Kuruşçu and Smith (2002). In our model, it is the overlapping generations demographic structure and the fact that current generations care insufficiently about previous generations that imply that the planners' preferences, which aggregate the preferences of the generations currently alive, are quasi-geometric. Furthermore, it can be easily seen that $q\left(\tau_c, a\right) < q\left(\overline{\tau}_c, 0\right)$, and thus, planners have an "excessive" incentive to postpone current consumption. This will be the source of our main results below.

From date $t+1$ onwards, the date-t planner has time consistent preferences, and so it would solve the problem:

$$W(k) = \max_{0 \le k' \le Ak} \{ q(\overline{\tau}_c, 0) \, u(Ak - k' + k) + \delta W(k') \}, \tag{12}$$

where $q(\tau, a)$ is given by equation (11) and $\overline{\tau}_c$ is given by (9). It is easy to verify that the corresponding first-order condition equates the marginal disutility from additional investment and the marginal value of additional capital next period:

$$-q(\overline{\tau}_c, 0) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial W(k')}{\partial k'}. \tag{13}$$

Furthermore, since preferences are time consistent from date $t+1$ onwards, the solution to the above problem satisfies the familiar envelope condition

$$\frac{\partial W(k)}{\partial k} = q(\overline{\tau}_c, 0) \frac{\partial u(c)}{\partial c} \frac{\partial c}{\partial k} \tag{14}$$

every period. Combining (14), evaluated one period ahead, and (13), it is easy to see that the intertemporal allocation of aggregate consumption satisfies the familiar Euler equation

$$\frac{\partial u(c) / \partial c}{\delta (\partial u(c') / \partial c')} = \frac{-\partial c' / \partial k'}{\partial c / \partial k'},$$

which equates the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation. Taking derivatives and noting that consumption and capital grow at the common rate $\overline{g}_c$, it can be verified that the solution to the above standard dynamic programming problem implies that investment from date $t+1$ onwards is given by $k' - k = \overline{g}_c k$, where

$$1 + \overline{g}_c = [\delta (A + 1)]^{1/\sigma}. \tag{15}$$

Finally, the investment problem at date t can be formulated as:

$$W_0(k) = \max_{0 \le k' \le Ak} \{ q(\tau_c, a) \, u(Ak - k' + k) + \delta W(k') \}, \tag{16}$$

where $q(\tau, a)$ is given by equation (11) and τ_c is given by (7). As before, the corresponding first-order condition equates the marginal disutility incurred from additional investment and the marginal value of additional capital next period:

$$-q\left(\tau_c, a\right) \frac{\partial u\left(c\right)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial W\left(k'\right)}{\partial k'}. \tag{17}$$

Noting that the growth rate from date $t+1$ onwards is the constant $\overline{g}_c$, as given by (15), and noting that

$$\delta W\left(k'\right) = \sum_{s=1}^{\infty} \delta^s q\left(\overline{\tau}_c, 0\right) u\left(c_{t+s}\right),$$

it can be verified that

$$W\left(k'\right) = \text{constant} + \left(\frac{q\left(\overline{\tau}_c, 0\right)}{1-\delta\left(1+\overline{g}_c\right)^{1-\sigma}}\right) u\left(c'\right),$$

and so we have

$$\frac{\partial W\left(k'\right)}{\partial k'} = \left(\frac{q\left(\overline{\tau}_c, 0\right)}{1-\delta\left(1+\overline{g}_c\right)^{1-\sigma}}\right) \frac{\partial u\left(c'\right)}{\partial c'} \frac{\partial c'}{\partial k'}. \tag{18}$$

Combining (17) and (18), it is easy to see that time inconsistency influences the date-t planner's allocation of aggregate consumption at date t by introducing a wedge between the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation:

$$\frac{\partial u\left(c\right)/\partial c}{\delta\left(\partial u\left(c'\right)/\partial c'\right)} = \left(\frac{q\left(\overline{\tau}_c, 0\right)/q\left(\tau_c, a\right)}{1-\delta\left(1+\overline{g}_c\right)^{1-\sigma}}\right) \frac{-\partial c'/\partial k'}{\partial c/\partial k'}.$$

The magnitude of the wedge takes into account that $c_y = \tau_c c$ and $c'_y = \overline{\tau}_c c'$, through the term $q\left(\overline{\tau}_c, 0\right)/q\left(\tau_c, a\right)$, and applies an effective discount rate equal $\delta\left(1+\overline{g}_c\right)^{1-\sigma}$ to $\left(\frac{\overline{\tau}_c}{\tau_c}\right)^{-\sigma}$, because the young's share of aggregate consumption in all future periods is equal to $\overline{\tau}_c$ rather than τ_c. To interpret the wedge, note that the ratio $q\left(\overline{\tau}_c, 0\right)/q\left(\tau_c, a\right)$ specifies the relative weight placed on $u\left(c_{t+1}\right)$ rather than $u\left(c_t\right)$ by the social welfare function in equation (10).

It is now easy to verify that the solution to the current problem (16) implies that investment at date t is given by $k' - k = g_c k$, with

$$1 + g_c = \frac{A+1}{1 + \left(\frac{q(\tau_c, a)}{q(\overline{\tau}_c, 0)} \frac{\delta^{-1} - (1+\overline{g}_c)^{1-\sigma}}{(A - \overline{g}_c)^{1-\sigma}} \right)^{1/\sigma}}, \tag{19}$$

where $q(\tau, a)$ is given by equation (11) and τ_c, $\overline{\tau}_c$ and $\overline{g}_c$ are given by (7), (9), and (15), respectively. In the Appendix we show that $g_c > \overline{g}_c$, for all $\sigma > 0$. Thus, if the date-t planner could commit future allocations, it would choose a current growth rate that is larger than the growth rate it would dictate to future generations. This is because the current planner cares more about the future old than it does about the current old generation, and so $\frac{q(\tau_c, a)}{q(\overline{\tau}_c, 0)} < 1$, for $a > 0$. In turn, this occurs because the current planner puts positive weight on the current young, but the current young does not care about the current old. Indeed, it can be verified that the right side of (19) is equal to $1 + \overline{g}_c$ if and only if $a = 0$ (see Appendix). It should be noted that our assumption that the current young do not place any weight at all on the current old is made for simplicity. The essential feature of the above problem is that the current young do not place *sufficient* weight on the current old.[3]

The following proposition summarizes our discussion so far.

Proposition 1 *If the date-t planner could precommit future allocations, optimal allocations would be given by $i(k) = gk$, $c^y(k) = \tau(A - g)k$, and $c^o(k) = (1-\tau)(A-g)k$, with*

$$(\tau, g) = \begin{cases} (\tau_c, g_c) & \text{in the first period} \\ (\overline{\tau}_c, \overline{g}_c) & \text{in every future period,} \end{cases}$$

where τ_c and $\overline{\tau}_c$ are given by (7) and (9), respectively, with $\tau_c > \overline{\tau}_c$, and g_c and $\overline{g}_c$ are given by (19) and (15), respectively, with $g_c > \overline{g}_c$.

Of course, the problem with the above solution is that it is time inconsistent. Accordingly, each planner needs to take into account that future planners will deviate from the allocation that the current planner would dictate if it could control future allocations. In the following section we consider equilibrium behavior when current planners recognize that future allocations will be chosen optimally by future planners.

[3] See Hori (1997), and foonote 1.

4 Markov perfect equilibrium

Now each planner recognizes that every future planner will choose the same optimal intergenerational allocation of consumption each period as the one chosen in the current period by the current planner. This is the allocation that solves the above problem (6) and so the young's share of aggregate consumption is now given by

$$\tau^* \equiv \frac{c_y}{c} = 1 - \frac{c_o}{c} = \frac{1}{1 + (\delta + a)^{-1/\sigma}}. \tag{20}$$

every period. Of course, $\tau^* = \tau_c$.

Now suppose that the current planner anticipates that every future planner follows the linear investment policy $i' = \widehat{g}k'$, with $\delta\left(1 + \widehat{g}\right)^{1-\sigma} < 1$. Then, the current investment decision solves the following problem:

$$V_0\left(k\right) = \max_{0 \leq k' \leq Ak} \left\{ q\left(\tau^*, a\right) u\left(Ak - k' + k\right) + \delta V\left(k'\right) \right\}, \tag{21}$$

where

$$V\left(k\right) = q\left(\tau^*, 0\right) u\left(Ak - \left(1 + \widehat{g}\right) k + k\right) + \delta V\left(\left(1 + \widehat{g}\right) k\right), \tag{22}$$

where $q\left(\tau, a\right)$ is given by equation (11), and τ^* is given by equation (20). An investment policy $i\left(k\right) = gk$ that is part of a symmetric Markov perfect equilibrium must be such that $g = \widehat{g}$.

In order to appreciate the role of commitment problems, it will be useful to examine the first-order condition of the above problem. To that end, first note that the first-order condition with respect to k' at date t is given by

$$-q\left(\tau^*, a\right) \frac{\partial u\left(c\right)}{\partial c} \frac{\partial c}{\partial k'} = \delta \frac{\partial V\left(k'\right)}{\partial k'}. \tag{23}$$

Solving the recursion in equation (22) it can be verified that

$$V\left(k'\right) = \text{constant} + \left(\frac{q\left(\tau^*, 0\right)}{1 - \delta\left(1 + \widehat{g}\right)^{1-\sigma}} \right) u\left(c'\right),$$

and so we have

$$\frac{\partial V\left(k'\right)}{\partial k'}=\left(\frac{q\left(\tau^{*},0\right)}{1-\delta\left(1+\widehat{g}\right)^{1-\sigma}}\right)\frac{\partial u\left(c'\right)}{\partial c'}\frac{\partial c'}{\partial k'}. \tag{24}$$

As shown above, the first-order condition at date t if the date-t planner could control future allocations would be the same as (23), except that V is replaced by W. The main difference lies in the marginal effect of current investment on the value function next period. The difference may be understood as follows. Each planner recognizes that a marginal increase in current investment results in extra income next period that will in turn influence investment next period. Since current and future planners disagree about future investment decisions, current planners have an incentive to manipulate future investment decisions via current investment.

In contrast, if the current planner could control future allocations, time consistency of the date-t planner's preferences from date $t+1$ onwards would ensure that the familiar envelope condition holds, which ensures that the above effect of current on future investment can be ignored when making current investment decisions. In turn, this guarantees that investment in every future period will be given by $\overline{g}_c k$. The difference between (24) and (18) lies in that, in the Markov perfect equilibrium, the current planner anticipates intergenerational disagreement in every future period.

Let us return to the characterization of the Markov perfect equilibrium. Combining (23) and (24) we have

$$\frac{\partial u\left(c\right)/\partial c}{\delta\left(\partial u\left(c'\right)/\partial c'\right)}=\left(\frac{q\left(\tau^{*},0\right)/q\left(\tau^{*},a\right)}{1-\delta\left(1+\widehat{g}\right)^{1-\sigma}}\right)\frac{-\partial c'/\partial k'}{\partial c/\partial k'}. \tag{25}$$

Although the intertemporal allocation of aggregate consumption in the Markov perfect equilibrium is time consistent by construction, there is a wedge between the marginal rate of substitution between current and next-period consumption and the corresponding marginal rate of transformation. The magnitude of the wedge takes into account that the young's share of aggregate consumption every period is equal to $\tau^{*}=\tau_c>\overline{\tau}_c$ rather than $\overline{\tau}_c$, and also anticipates that investment in all future periods is given by $i'=\widehat{g}k'$.

It is now straightforward to write the above Euler equation as

$$\left(\frac{k'}{Ak - k' + k}\right)^{\sigma} = \frac{q(\tau^*, 0)(A - \widehat{g})^{1-\sigma}}{q(\tau^*, a)\left(\delta^{-1} - (1 + \widehat{g})^{1-\sigma}\right)},$$

which describes the best response k' to the anticipation of $\widehat{g}$, for given k. Clearly, the best response to any given $\widehat{g}$ is linear in k. Consequently, we obtain the best-response mapping

$$1 + g = \frac{A + 1}{1 + \left(\frac{q(\tau^*, a)}{q(\tau^*, 0)} \frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A - \widehat{g})^{1-\sigma}}\right)^{1/\sigma}} \equiv 1 + B(\tau^*, \tau^*, \widehat{g}). \tag{26}$$

The best response function $g = B(\tau, \tau', \widehat{g})$ characterizes the best investment response by a planner that allocates a share τ of current consumption to the current young and anticipates that future planners will allocate a share τ' of consumption to the young and invest according to $i' = \widehat{g}k'$. Note that the structure of the best response function in equation (26) is identical to that in equation (19). In particular, the above commitment solution has $g_c = B(\tau_c, \overline{\tau}_c, \overline{g}_c)$, whereas a symmetric Markov perfect equilibrium has $g^* = B(\tau^*, \tau^*, g^*)$.

Proposition 2 *(i) There exists a unique symmetric, interior, Markov perfect equilibrium in linear strategies. The equilibrium is characterized by $i(k) = g^*k$, $c^y(k) = \tau^*(A - g^*)k$, and $c^o(k) = (1 - \tau^*)(A - g^*)k$, where τ^* is given by (20), and $g^* = B(\tau^*, \tau^*, g^*) \in (\overline{g}_c, A)$, where B is given by (26) and.$\overline{g}_c$ is given by (15). (ii) For all $\widehat{g} \in (\overline{g}_c, A)$, $\partial B(\tau^*, \tau^*, \widehat{g})/\partial \widehat{g} \geq 0$ if and only if $\sigma \geq 1$, with equality if and only if $\sigma = 1$.*

Part (i) characterizes the unique symmetric, interior Markov perfect equilibrium in linear strategies. Part (ii) provides additional insight into the role of commitment problems. Note that the disagreement between planners about investment decisions takes the particular form that date-$(t + 1)$ planner invests too much from the viewpoint of date-t planner. The best response mapping (26) indicates how current planners will attempt to manipulate investment next period. Part (ii) of the proposition says that locally around the equilibrium current and next-period investments are "strategic" complements if $\sigma > 1$ and "strategic" substitutes if $\sigma < 1$. The panels in Figure 1 plot the different types of best-responses. A detailed analysis is found in the Appendix.

Figure 1: best investment responses

(1) $\sigma < 1$ (2) $\sigma = 1$ (3) $\sigma > 1$

Panel (1) shows that $B(\tau^*, \tau^*, g)$ increases at first, peaking at $\overline{g}_c$, and then decreases, when $\sigma < 1$. Panel (2) shows that the best response is flat when $\sigma = 1$. In this case, $g^* = B(\tau^*, \tau^*, g^*)$ has a closed-form solution and the equilibrium growth rate is given by

$$1 + g^* = \frac{A+1}{1 + \left(\frac{\delta+a+1}{\delta+a}\right)\left(\frac{1-\delta}{1+\delta}\right)}. \tag{27}$$

Panel (3) in the above figure illustrates that $B(\tau^*, \tau^*, g)$ decreases at first, reaching a minimum at $\overline{g}_c$, and then increases, when $\sigma > 1$.

The role of the elasticity of intertemporal substitution, given by $1/\sigma$, is particularly interesting in the present context. With respect to a generation's lifetime, higher values of σ indicate greater aversion to differences in consumption over the life cycle. However, since individuals are altruistic, higher values of σ also indicate greater aversion to unequal consumption across generations. With balanced growth, the higher the value of σ, the less individuals are willing to tolerate larger positive, or smaller negative, growth rates. Indeed, in the Appendix we show that the equilibrium growth rate decreases as individuals are less willing to substitute consumption intertemporally. This is as expected. The role of σ when comparing growth rates with and without commitment is more interesting, as illustrated in the following proposition.

Proposition 3 *(i) $g^* > \overline{g}_c$ for all $\sigma > 0$, with $g^* > g_c$ if and only if $\sigma > 1$. (ii) Suppose that $\sigma > 1$. Then,*

$$\lim_{\delta \to 1} g^* > \overline{g}_c, \quad a > 0,$$

$$\lim_{a\to 0} g^* = \overline{g}_c, \quad 0 < \delta \leq 1,$$

$$\lim_{a\to\infty} g^* = A, \quad 0 < \delta \leq 1.$$

This proposition compares long-run growth in the Markov perfect equilibrium and the commitment solution. The key feature of the equilibrium is that the economy's growth rate is permanently higher than $\overline{g}_c$. Part (i) of the proposition also states that a property of the equilibrium is that $g^* > g_c$ if and only if $\sigma > 1$. Thus, if the elasticity of intertemporal substitution is sufficiently low ($\sigma > 1$), the equilibrium growth rate would exceed even the current growth rate that an arbitrary planner would prefer under the assumption that future allocations can be precommitted. To see why, first note that inspection of (26) and (19) immediately shows that $g_c = g^*$ if $\sigma = 1$. Moreover, note that future planners weight future consumption too little relative to the current planner, that is, $q(\tau^*, a) < q(\tau^*, 0)$. With $\sigma > 1$ (higher inequality aversion) income effects of the poor dominate, hence the current planner has an incentive to favor future consumption, which it can do by growing relatively faster.

Part (i) of the above proposition is a striking result for two reasons. First, $\overline{g}_c$ is the first-best growth rate from the viewpoint of the old, and it is also the growth rate that every young generation, and every planner, would dictate on every future generation, if they could do so. In this sense, commitment problems lead to equilibrium growth that is too high, relative to the preferences of all generations. Second, the private return to investment is lower than the social return to investment. The latter is given by the constant marginal product of capital A, whereas the former is given by $A - \partial(g^* k)/\partial k = A - g^*$. Although this has implications for efficiency, as we discuss below, the fact that $g^* > \overline{g}_c$ is driven by the time inconsistency of the planners' preferences, which implies that each planner perceives the next planner to invest too much, as explained above.

Part (ii) of the proposition illustrates sharply the equilibrium implications of the time inconsistency of planners' preferences. Even in the limit as the discount rate on future generations approaches zero, the equilibrium growth rate is strictly higher than $\overline{g}_c$. Note that as $\delta \to 1$ the preferences of the young, the old and the planner become aligned every period. Yet, as long as the planner puts weight on the current young the time inconsistency of the planners' preferences creates a non-trivial problem, which does not disappear as δ approaches 1.

Furthermore, as the weight the planner puts on the current old becomes negligible, the equilibrium growth rate becomes arbitrarily close to A (for $\sigma > 1$), and so the savings rate approaches 1. In this sense, the equilibrium growth rate is arbitrarily higher than the growth rate that is preferred by all generations.

Recall that we have maintained the assumption that $\delta(A+1) > 1$ in order to ensure that $\overline{g}_c > 0$. This implies that the equilibrium growth rate is always positive, since we have shown that $g^* > \overline{g}_c$. We have also maintained the assumption that $\delta(A+1)^{\frac{1-\sigma}{\sigma}} < 1$, in order to ensure that $\overline{g}_c < A$. With logarithmic utility, equilibrium growth is given by (27). Thus, g^* approaches A, and so $\frac{c_y+c_o}{k}$ approaches 0, as δ goes to 1. If $\sigma < 1$, the equilibrium investment rate becomes negligible as δ approaches $(A+1)^{\frac{\sigma-1}{\sigma}} < 1$. In principle this can be reconciled with the evidence if intergenerational altruism is sufficiently low. Even so, it is unclear why high social discounting would be a problem in this case. In particular, under the (unpalatable) assumption that $\sigma \leq 1$, and under the conventional utilitarian approach that is being followed here, one would need to accept that individuals, and planners, would willingly sacrifice their lifetime consumption for the benefit of future generations, if only they were sufficiently altruistic. Most of us would agree that there is something wrong with this. Moreover, upon reflection, the problem is that most of us, we believe, have a stronger preference for intergenerational equity that is implied by $\sigma \leq 1$. While this is a simple proxy for individuals' preferences for intergenerational equity, it should be recognized that the conventional focus on social discounting, here captured by δ, is simplistic, and it does not capture well the relevant concern, namely intergenerational equity. Similarly, it should be noted that the problems we are discussing here are not specific to low values of σ. They arise for any fixed value of σ.

Now let us consider the efficiency properties of the equilibrium. First, it should be noted that the Markov perfect equilibrium in our context is Pareto inefficient, simply because the private and the social return to investment are different. A Pareto improvement would result from investing optimally from the viewpoint of the currently young generation at the socially optimal rate of return, without changing the allocation for any other generation. This is in contrast with the common perception that perfect altruism about the following generation must lead to Pareto efficiency (Streufert, 1993). This is the case in the non-overlapping generations models studied in the literature, because it leads to time-consistent preferences. However, with time-inconsistent preferences, as is the case here, the private return to investment is necessarily lower than the social return, because the incentive to manipulate future investment does not disappear.

Now consider the possibility of dynamic inefficiency. In principle, allocations can be dynamically inefficient on the production side and/or the consumption side of the economy. We say that an investment allocation is dynamically efficient if there is no alternative allocation that provides more aggregate consumption in one period and at least the same consumption in every other period. We say that consumption allocations are dynamically efficient if there is no alternative allocation of aggregate consumption across generations that provides higher utility for one generation and at least the same utility for any other generation. The following result follow from standard arguments (Saint Paul, 1992).

Proposition 4 *Equilibrium investment is dynamically efficient.*

In the Appendix we show that investment is dynamically efficient if the growth rate is lower than the *social* return to investment, that is, if $A \geq g^*$. Then, the proposition follows from the aggregate resources constraint: $Ak_t \geq c_t + k_{t+1} - k_t$

In contrast, note that consumption is dynamically efficient if the growth rate is lower than the *private* return to investment. Since the private return to investment is given by $A - (1 + g^*)$, equilibrium consumption is dynamically efficient if and only if $1 + g^* \leq (1/2)(A + 1)$. It can be shown that there is a number $\overline{\delta}(a, \sigma) \in (0, 1)$ such that consumption is dynamically inefficient if and only if $\delta > \overline{\delta}(a, \sigma)$.

Our analysis so far implies that the commitment solution and the Markov perfect equilibrium are not Pareto ranked. This has implications for the individuals' incentive to design institutions to enable commitments that resolve intergenerational disagreements. The main implication is easiest to see in the log utility case, but it should be clear that it follows more generally.

Proposition 5 *Assume that $\sigma = 1$. Consider the Markov perfect equilibrium, and suppose that the date-t planner implements institutions to commit all future allocations optimally. Current generations are made better off, but there is a time $T \geq t$ such that all generations born after date T are made worse off, from the perspective of young age as well as old age.*

The above proposition illustrates the possibility that individuals (and the planner) would support institutions to enable commitments to lower the growth rate of the economy permanently. With log utility, the shift from the Markov perfect equilibrium to the commitment solution does not affect the first period consumption allocation, and it leads to the first best allocation for both generations after the current period. Hence, both current generations must be better off. If $\sigma \neq 1$, the first period allocations change as well, and one needs to consider several possible cases, but it is not difficult to find conditions where the current generations are better off under the planner's commitment solution than they are under the Markov perfect equilibrium.

The fact that an infinite number of generations must be made worse off by the move (at date t) from the equilibrium allocation to the commitment solution follows trivially from the fact that the wealth differential between the two allocations grows without bound and the original equilibrium allocation has lower growth at all times. Furthermore, it is easy to verify that this is so even in the limit as individuals do not discount the future.

Finally, consider briefly the corresponding Ramsey-Cass-Koopmans problem, which amounts to the case where the social welfare function is given by the utility of the currently old agents. Interestingly, the solution to the Ramsey-Cass-Koopmans problem is Pareto inefficient. A marginal transfer of consumption from the current old to the current young leaves the old indifferent but is strictly preferred by the young. This presents a problem for the argument that a "solution" to the problem is to have the planner "choose" a time-consistent social welfare function at the outset (e.g., Strotz, 1956, Calvo and Obstfeld, 1988). The problem is that this requires a commitment technology.

4.1 Imperfect altruism about future generations

The most natural departure from the perfect altruism assumption made in the Ramsey-Cass-Koopmans optimal growth model is simply that individuals are imperfectly altruistic about future generations. Indeed this is the standard assumption in the literature on equilibrium growth with imperfect intergenerational altruism that originated with Phelps and Pollak's (1968) seminal work, although imperfect altruism has been formalized in several different ways (compare e.g., Sen, 1967, Phelps and Pollak, 1968, Kohlberg, 1976). Arguably, this is also precisely what those who justify government intervention to target social investments on the grounds of intergenerational disagreement usually have in mind (e.g., Sen, 1967, Kohlberg, 1976). With respect to this, Strotz's (1956) seminal work, and more recently Laibson's (1997) demonstrate the general relevance that the economic agents' time-inconsistency has for the design of institutions that can cope with intertemporal disagreement by facilitating commitments.

We can easily extend our analysis so far to allow for the fact that individuals themselves may be imperfectly altruistic about future as well as past generations. Specifically, the following analysis allows for individuals having quasi-hyperbolic discounting, with a present-bias, as in Phelps and Pollak (1968). Conversely, it extends Phelps and Pollak's (1968) work to account for overlapping generations.

Consider the above model, but assume that individuals born at date t have preferences

$$u_t(\beta) = u\left(c_t^y\right) + \beta\left[u\left(c_{t+1}^o\right) + \sum_{s=1}^{\infty}\delta^s\left(u\left(c_{t+s}^y\right) + u\left(c_{t+1+s}^o\right)\right)\right], \tag{28}$$

whose discount structure implies that consumption streams are discounted according to the sequence of discount factors $1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots$. In this section, we assume that $\beta \leq 1$. When $\beta = 1$, discounting is geometric, and so individuals have standard, time-consistent preferences. This is the case we analyze above. When $\beta < 1$, individuals have quasi-hyperbolic discounting, with a present-bias, in the sense that the *rate* at which utility flows at date $t+2$ are discounted falls between date t and date $t+1$.

It should be noted that the specification of individual preferences in equation (28) assumes that the quasi-hyperbolic discounting structure applies equally to all future utility flows, rather than applying only to the future generations' utility flows. This assumption implies that individuals' relative valuation of utility flows for young and old agents at a given date does not evolve with the passage of time. If it did, there would be an additional source of disagreement, referring to the distribution of valuations between generations at a point in time. Accordingly, equation (28) can be written as

$$u_t(\beta) = u(c_t^y) + \beta \sum_{s=1}^{\infty} \delta^s \left(u(c_{t+s}^y) + \delta^{-1} u\left(c_{t+s}^o\right)\right).$$

It should be noted that quasi-hyperbolic discounting implies preference reversals, and so the relevant preferences for the old in the social welfare function should be their current preferences. Accordingly, we assume that the objective function of the date-t planner is

$$v_t(\beta) = u_{t-1}^t(\beta) + a u_t(\beta), \tag{29}$$

where $a > 0$, and where $u_{t-1}^t(\beta)$ is the utility of the currently old from the viewpoint of date t:

$$u_{t-1}^t(\beta) = u(c_t^o) + \delta \left(u(c_t^y) + \beta \sum_{s=1}^{\infty} \delta^s \left(u(c_{t+s}^y) + \delta^{-1} u\left(c_{t+s}^o\right)\right)\right). \tag{30}$$

Hence, we have

$$v_t(\beta) = u(c_t^o) + (\delta + a) u(c_t^y) + (\delta + a) \beta \sum_{s=1}^{\infty} \delta^s \left(u(c_{t+s}^y) + \delta^{-1} u\left(c_{t+s}^o\right)\right). \tag{31}$$

Equivalently, one can think of the planner as maximizing

$$\widetilde{v}_t(\beta) = u(c_t^y) + (\delta + a)^{-1} u(c_t^o) + \beta \sum_{s=1}^{\infty} \delta^s \left(u(c_{t+s}^y) + \delta^{-1} u\left(c_{t+s}^o\right)\right). \tag{32}$$

It is easy to verify that our previous analysis goes through essentially unchanged. If the date-t planner could precommit future allocations, optimal allocations would be given by

$i(k) = gk$, $c^y(k) = \tau(A-g)k$, and $c^o(k) = (1-\tau)(A-g)k$, with

$$(\tau, g) = \begin{cases} (\tau_c, g_c(\beta)) & \text{in the first period} \\ (\overline{\tau}_c, \overline{g}_c) & \text{in every future period,} \end{cases}$$

where τ_c and $\overline{\tau}_c$ are given by (7) and (9), respectively, with $\tau_c > \overline{\tau}_c$, $\overline{g}_c$ is given by (15), and $g_c(\beta)$ solves

$$1 + g_c(\beta) = \frac{A+1}{1 + \left(\frac{q(\tau_c, a)}{\beta q(\overline{\tau}_c, 0)} \frac{\delta^{-1} - (1+\overline{g}_c)^{1-\sigma}}{(A-\overline{g}_c)^{1-\sigma}}\right)^{1/\sigma}}. \tag{33}$$

Using the definition of $q(\tau, a)$ in equation (11), one can verify that $g_c(\beta) > \overline{g}_c$ if and only if $\beta \in \left(\left(\frac{\overline{\tau}_c}{\tau_c}\right)^\sigma, 1\right]$.

Similarly, there is a unique symmetric, interior, Markov perfect equilibrium in linear strategies. The equilibrium is characterized by $i(k) = g^* k$, $c^y(k) = \tau^*(A - g^*)k$, and $c^o(k) = (1-\tau^*)(A-g^*)k$, where τ^* is given by (20), $\overline{g}_c$ is given by (15), and $g^* = B_\beta(\tau^*, \tau^*, g^*)$, where B is given by

$$1 + g = \frac{A+1}{1 + \left(\frac{q(\tau^*, a)}{\beta q(\tau^*, 0)} \frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}} \equiv 1 + B_\beta(\tau^*, \tau^*, \widehat{g}). \tag{34}$$

Finally, using the definition of $q(\tau, a)$ in equation (11), one can verify that $g^* > \overline{g}_c$ if and only if $\beta \in (b, 1]$, where

$$b = \frac{1}{1 + (1-\tau^*)a/\delta} \in (0,1). \tag{35}$$

The main implication of the analysis in this section is that the effect of commitment depends significantly on whether or not the individuals' present-bias dominates the bias against the present that results from aggregation of preferences in the social welfare function. As in Proposition 5, it is easiest to illustrate the main implication of our analysis for the case of log utility, although it holds more generally.

Proposition 6 *Assume that $\sigma = 1$. Consider the Markov perfect equilibrium, and suppose that the date-t planner implements institutions to commit all future allocations optimally.*

Current generations are made better off at date t.

(i) If $\beta < b$, there is a time $T \geq t$ such that all generations born after date T are made better off, from the perspective of young age as well as old age.

(ii) If $\beta = b$, all generations born after date t are made better off, from the perspective of young age as well as old age, if and only if $\delta > \overline{\delta}$, for some $\overline{\delta} \in (0, 1)$.

(iii) If $\beta > b$, there is a time $\widehat{T} \geq t$ such that all generations born after date $\widehat{T}$ are made worse off, from the perspective of young age as well as old age.

The statements in the proposition follow trivially from the fact that the wealth differential between the equilibrium allocation and the commitment solution implemented at date t grows without bound. Hence, whether future generations are made eventually worse off or better off by the move (at date t) to the commitment solution depends only on whether the new growth path lies above or below the original path, which is determined by whether $\beta \leq b$ or $\beta > b$. It should also be noted that, for the case where $\beta \leq b$, it is easy to construct numerical examples where $T = t$, for δ sufficiently hign, so all current and future generations are made better off. However, whether or not some generations are made worse off depends in general on the configuration of parameter values for β, δ, and a.

The bottom line of our analysis is that intergenerational disagreement can imply that the incentive of individuals, and governments, are such that institutions that enable commitments to cope with intergenerational disagreement will tend to favor the introducing generations at the expense of future generations. This result, we believe, suggests more generally, that the ability of actual governnments to improve the welfare of future generations is seriously constrained, even in an altruistic society.

5 Conclusion

In this paper, we explore how intergenerational disagreement constrains the calculus of second-best growth. We illustrate the contrast between two natural sources of disagreement when generations are overlapping and preferences are aggregated in a utilitarian manner.

Social preferences tend to exhibit a present-bias because generations are imperfectly altruistic about future generations; but they tend to exhibit a bias against the present because coexisting generations are imperfectly altruistic about currently older generations. Equilibrium growth is inefficiently low when the former bias dominates. Otherwise society faces a difficult intergenerational equity problem. Ironically, altruistic generations tend to support institutions that enable commitments to lower growth, at the expense of future generations. This is so even with perfect altruism about future generations and even without discounting.

We believe that our analysis can be fruitfully extended to analyze government intervention to target social investments more generally, including climate change mitigation policies. Our analysis suggests that the ability of actual governnments to improve the welfare of future generations is seriously constrained, even in an altruistic society.

Appendix

Proof of Proposition 1

All parts of the proposition are proven in the main text, except for the inequality $g_c > \bar{g}_c$. To prove this, we first define the following function:

$$1 + \tilde{B}(\hat{g}, Q) = \frac{A+1}{1 + Q\left(\frac{\delta^{-1} - (1+\widehat{g})^{1-\sigma}}{(A-\widehat{g})^{1-\sigma}}\right)^{1/\sigma}} \tag{36}$$

Using (10), we can easily check that $\tilde{B}(\bar{g}_c, 1) = \bar{g}_c$.From (19) it follows that $\tilde{B}(\bar{g}_c, Q_c) = g_c$,where we define

$$Q_c = \left(\frac{q\,(\tau_c, a)}{q\,(\overline{\tau}_c, 0)}\right)^{1/\sigma} = \frac{\overline{\tau}_c}{\tau_c} < 1,$$

where the second equality and the inequality follow from (7) and (9). Since $\partial\tilde{B}(\hat{g}, Q)/\partial Q < 0$, we can combine results as follows $\tilde{B}(\bar{g}_c, Q_c) = g_c > \tilde{B}(\bar{g}_c, 1) = \bar{g}_c$. This proves Proposition 1. **QED**

Proof of Proposition 2

We rewrite the best response (26) as $B(\hat{\tau}, \hat{\tau}, \hat{g}) = \tilde{B}(\hat{g}, Q^*)$,where we define

$$Q^* \equiv \left(\frac{q\,(\tau^*, a)}{q\,(\tau^*, 0)}\right)^{1/\sigma} = \left(\frac{1}{1 + (1 - \tau^*)a/\delta}\right)^{1/\sigma} < 1.$$

which implies the following characteristics:

$$sign\frac{\partial\tilde{B}(\hat{g}, Q)}{\partial\hat{g}} = sign(\sigma - 1)\left[(1 + \widehat{g})^{\sigma} - \delta(A + 1)\right]$$

Hence, for given Q the best response function $B(.)$ has a global minimum (maximum) at $\widehat{g} = \bar{g}_c$ if $\sigma < 1$ (if $\sigma > 1$) and is flat at $\widehat{g} = \bar{g}_c$ if $\sigma = 1$. This proves Part (ii) of the proposition.

From (26), it follows that the equilibrium growth rate is the solution g^* that satisfies $\tilde{B}(g^*, Q^*) = g^*$.To show existence of a unique fixed point g^*, evaluate $g = \tilde{B}(\hat{g}, Q)$ at $\hat{g} = g$ and rewrite it as

$$g = \tilde{B}(g, Q) \Longleftrightarrow \delta^{-1} Q^{\sigma}(1+g)^{\sigma} + (1-Q^{\sigma})(1+g) - (A+1) = 0 \tag{37}$$

As long as $Q \leq 1$, the LHS is increasing in g, is negative when $g = -1$, and positive when $g = A$. Hence, there is exactly one fixed point, $g^* = \tilde{g}(Q^*) < A$. Applying the implicit function theorem to (37), we find $\tilde{g}'(Q) < 0$ iff $1 > \delta\,(1+\tilde{g}(Q))^{1-\sigma}$. For $\sigma \leq 1$, the latter inequality holds since $g \leq A$ and we assume $1 < (A+1)^{1-\sigma}\delta$. For $\sigma > 1$, first note that $\tilde{g}(1) = \bar{g}_c$ and $1 > \delta\,(1+\tilde{g}(1))^{1-\sigma}$, so that $\tilde{g}'(1) < 0$; hence for all $Q \leq 1$ the inequality $1 > \delta\,(1+\tilde{g}(Q))^{1-\sigma}$ holds a fortiori. Since $Q^* < 1$, we have $\tilde{g}(Q^*) = g^* > \tilde{g}(1) = \bar{g}_c$. We conclude $A > g^* > \bar{g}_c$, which proves Part (i) of the proposition. **QED**

Proof of Proposition 3

Part (i) From the definitions in the previous two proofs, we have:

$$\frac{Q_c}{Q^*} = \left(\frac{\tau_c^{1-\sigma} + \delta^{-1}(1-\tau_c)^{1-\sigma}}{\bar{\tau}_c^{1-\sigma} + \delta^{-1}(1-\bar{\tau}_c)^{1-\sigma}} \right)^{1/\sigma}$$

Since the RHS is increasing (decreasing) in τ_c for $\sigma > 1$ ($\sigma < 1$) and $\tau_c > (1+\delta^{-1/\sigma})^{-1} = \bar{\tau}_c$, we find:

$$\sigma \gtreqless 1 \Leftrightarrow Q_c \gtreqless Q^*$$

If $\sigma > 1$, $\tilde{B}(g^*, Q^*) = g^* > \tilde{B}(g^*, Q_c) > \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from $\partial \tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from $g^* > \bar{g}_c$ and $\partial \tilde{B}(g, Q)/\partial g > 0$ for $g > \bar{g}_c$.

If $\sigma < 1$, $\tilde{B}(g^*, Q^*) = g^* < \tilde{B}(g^*, Q_c) < \tilde{B}(\bar{g}_c, Q_c) = g_c$, where the first inequality follows from $\partial \tilde{B}(\hat{g}, Q)/\partial Q < 0$, and the second one from $g^* > \bar{g}_c$ and $\partial \tilde{B}(g, Q)/\partial g < 0$ for $g > \bar{g}_c$.

This proves $g^* > g_c$ in Part (i).

For $\delta \to 1$, $g^* > \bar{g}_c$, see proposition 2, which was already proven irrespective of the value of δ.

For vanishing a, we have $\lim_{a \to 0} Q^* = 1$, so that $\lim_{a \to 0} g^* = \lim_{a \to 0} \tilde{g}(Q^*) = \tilde{g}(1) = \bar{g}_c$.

For infinite a, we apply l'Hopital's rule to find $\lim_{a \to \infty} Q^*$. If $\sigma > 1$, $\lim_{a \to \infty} Q^* = 0$, and $\lim_{a \to \infty} g^* = \tilde{g}(0) = A$, where the last equality follows from substituting $Q = 0$ into the implicit function (37). For $\sigma < 1$, $\lim_{a \to \infty} Q^* = 0$ and $\lim_{a \to \infty} g^* = \tilde{g}(1) = \bar{g}_c$. **QED**

Proof of Proposition 4

The proof replicates the argument in Saint Paul (1992). We show that equilibrium investment is dynamically efficient if $g^* < A$. Then, the proposition follows from the aggregate resources constraint: $Ak_t \geq c_t + k_{t+1} - k_t$.

Consider an allocation $\left\{\widetilde{k}_t\right\}$ with $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. Since $\widetilde{k}_{t+1} = (A+1)\widetilde{k}_t - \widetilde{c}_t$ and $k_{t+1} = (A+1)k_t - c_t$, for $t \geq s$, it must be that $k_{t+1} - \widetilde{k}_{t+1} \geq (A+1)\left(k_t - \widetilde{k}_t\right)$, for $t \geq s$. In turn this implies that $k_{s+T} - \widetilde{k}_{s+T} \geq (A+1)^T\left(k_s - \widetilde{k}_s\right)$, and thus

$$\widetilde{k}_{s+T} \leq (1+g^*)^T k_s - (A+1)^T\left(k_s - \widetilde{k}_s\right),$$

for any $T \geq 1$. Clearly, if $g^* < A$, the right side of the inequality becomes negative for T sufficiently large, contradicting the hypothesis that there is a feasible deviation $\widetilde{k}_s < k_s$, for some s, with $\widetilde{c}_t \geq c_t$ for $t \geq s$. This concludes the proof. **QED**

Proof of Proposition 5

The proposition is a special case of Proposition 6. The proof is found below. **QED**

Proof of Proposition 6

Current generations are made better off at date t because the date-t allocation does not change and the new allocation thereafter is their first-best allocation. (i) If $\beta < b$, then $\bar{g}_c > g^*$. Since the difference in wealth between the two allocations grows without bound, and wealth is strictly higher under the commitment solution after date t, all generations born after some date $T > t$ must be made better off from the viewpoint of their birthdate. Every old generation is better off at all dates under the commitment solution, because that is their first-best allocation.

(ii) If $\beta = b$, then $\bar{g}_c = g^*$. Thus, the two aggregate consumption allocations are identical. For a given path of aggregate consumption, it is easy to verify that the move from allocating a share τ^* of aggregate consumption to the young every period to allocating a share $\overline{\tau}_c$ is profitable for an arbitrary young generation born after date t if and only if $\delta > \overline{\delta}$, for some $\overline{\delta} \in (0,1)$.

(iii) If $\beta > b$, then $\bar{g}_c < g^*$. Since the difference in wealth between the two allocations grows without bound, and wealth is strictly lower under the commitment solution after date

t, all generations born after some date $T > t$ must be made worse off from the viewpoint of their birthdate as well as old age. This concludes the proof. **QED**

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