Financial Frictions, Occupational Choice and Economic Inequality*

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Abstract

We develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with household data from Brazil. The theory extends Lucas (1978) by modeling heterogeneity in two skills: −working and managerial skills. Consistently with the evidence, the theory implies three occupational categories: workers, employers, and self-employed entrepreneurs. We find that the correlation between working and managerial skills (ρ) matters importantly for the distribution of earnings across occupations and for the quantitative implications of financial frictions. We also find that while most households benefit from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers in Brazil may have a vested interested in maintaining a status quo with low enforcement.

1   Introduction

A recent literature has emphasized that the misallocation of resources caused by financial frictions depress total factor productivity and, hence, output per worker (Erosa (2001),

Jeong and Townsend (2007), Amaral and Quintin (2010), Buera and Shin (2011), Buera et al. (2011), Greenwood et al. (2010)). The standard approach in the financial frictions literature and, more generally in the misallocation literature (see Guner et al. (2008), and Restuccia and Rogerson (2008)) is to calibrate the model to micro data from the United States and use the calibrated model economy to simulate policy distortions in developing countries.[1]While this approach has the advantage that the US data is readily available, it relies on the assumption that the distribution of entrepreneurial skills or plant productivities are invariant across countries or, at the very least, do not matter for the misallocation of resources induced by policy distortions or limited enforcement in the financial markets. However, there is ample evidence suggesting that the distribution of skills do vary across rich and poor countries. [2] Moreover, economic theory suggests that inequality matters for the impact of micro distortions and financial frictions (see Banerjee and Newman (1993), Galor and Zeira (1993)).

We develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with micro level data from Brazil. The theory is used to quantitatively evaluate how the impact of financial frictions on TFP and aggregate output depends on economic inequality and how, in turn, financial frictions impact on economic inequality. Our paper contributes to a seminal (mostly theoretical) literature that has emphasized the importance of the interaction between the distribution of wealth and financial frictions for the allocation of resources (see Evans and Jovanovic (1989), Greenwood and Jovanovic (1990), Banerjee and Newman (1993), Galor and Zeira (1993), Cagetti and De Nardi (2006)).

The key innovation of our theory is to extend the Lucas (1978) model in order to incorporate heterogeneity in two skills: −working and managerial skills. By modeling bivariate-skill heterogeneity, the theory can distinguish between comparative advantage in entrepreneurship (a high ratio of managerial to working skills) and absolute advantage (a high value of both skills). This distinction is necessary for the theory to be consistent with evidence on the income distribution across occupations. Moreover, by assuming that entrepreneurs can use their working and managerial skills in the operation of their businesses, the theory has the novel implication that some entrepreneurs will not hire any outside labor and be own account workers (or self-employed entrepreneurs). Building a theory that distinguishes between entrepreneurs that are employers and those that

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[1]Notable exceptions are given by Midrigan and Xu (2010) and García-Santana and Pijoan-Mas (2012)

[2]In fact, even among developed economies, recent work on international trade theory argues that the heterogeneity in the (second moments of the) skill distribution plays an important role for understanding trade patterns among similarly endowed economies (see Ohnsorge and Trefler (2007) and references in that paper).

are self-employed is relevant because there is abundant evidence that the high rates of entrepreneurship in poor countries is mostly due to the prevalence of self-employed workers (see Figure 2).

We assume a small open economy that takes as given the international interest rate. Following Buera et al. (2011), capital market imperfections are introduced by modeling an endogenous borrowing constraint that limits the amount of capital that entrepreneurs can use. We prove that in the absence of financial frictions occupational choices are driven entirely by the ratio of managerial to working skills. Employers have a comparative advantage at managing (high $\frac{z_m}{z_w}$), workers have a comparative advantage at working (low $\frac{z_m}{z_w}$), and self-employed have an intermediate skill ratio. Heterogeneity in absolute advantage implies that both at the top and bottom of the income distribution there are entrepreneurs and workers. We show that capital market imperfections distort rates of returns on skills by making the return to managerial and working skills depend on asset holdings. As a result, in the presence of financial frictions occupational choice decisions are jointly determined by the skill ratio and asset holdings. The model economy is calibrated to Brazilian household data and macro aggregates. We believe that Brazil provides a nice benchmark because it is a country that exhibits both high levels of economic inequality and of financial frictions. The correlation between managerial and working skills ρ, which we show to be crucial for the quantitative predictions of the theory, is fixed exogenously. We consider variuous values of ρ and recalibrate the rest of the parameters of the model economy for each of the values of ρ. We view the calibrated model economies as providing a different theory of Brazilian income inequality and occupational choices. We then compare the effects of financial frictions across model economies by simulating the effects of introducing perfect credit markets in each of the calibrated model economies.

We find that the output and TFP gains of improving credit market institutions are large in all economies but vary substantially across the three calibrated model economies. The output gains range from 36% to 55% and the TFP gains range from 11% to 18% as the correlation between skills decreases. The skill correlation determines the extent to which talented entrepreneurs are able to self-finance their businesses. When the correlation between these two skills is high, individuals that are talented as entrepreneurs are also talented as workers. Then, if skills are also persistent over time, young and talented individuals can work when young, build savings, and use their savings to finance their businesses when old. Thus, when managerial and working skills are highly correlated and persistent over time, the effects of financial frictions on resource allocations are less important than otherwise.

The large disparity on the quantitative effects of financial frictions across model

economies underscores the importance of testing the predictions of the calibrated model economies with household data from Brazil. To discriminate between competing model economies we use the fact that the correlation between working and managerial skills (ρ) matters importantly for the distribution of earnings across occupations (workers and entrepreneurs). When the skill correlation is high, highly skilled households have an absolute advantage at both occupations. In this case, the theory predicts that the earnings distribution of workers is shifted to the left relative to that of entrepreneurs, which is grossly at odds with the Brazilian micro data. Similarly, the economy with strongly negative correlated skills implies that the distribution of earnings of workers is shifted to the right relative to that of entrepreneurs, which is also counterfactual. We examine several other pieces of evidence and conclude that the household data from Brazil supports a moderate skill correlation of around 0.1, implying that eliminating capital market imperfections in Brazil will increase output and TFP by about 50 and 16 percent, will reduce self-employment rates and the fraction of employers by about 13 and 4 percentage points.

We find that capital market imperfections have a small effect on the Gini index of income but that they have important effects on the sources of income inequality. We divide household income between capital income and labor income, the latter computed as the sum of the returns to working and managerial skill inputs. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor income and capital income. Labor income is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions (with a Gini index of .52 versus a Gini index .56 in the latter economy). On the other hand, the Gini index of capital income is about 10 percentage points higher in the economy with imperfect credit markets. These opposing effects tend to offset each other, which accounts for the small change in the Gini index of income.

Does it matter that the distribution of factor income varies so much across economies? The answer is yes because it is symptomatic of the resource misallocation prevalent under imperfect capital markets. The low concentration of the distribution of labor income with financial frictions is due to the fact that borrowing constraints distorts rate of returns to managerial ability. Since rate of returns on managerial ability and managerial ability z_m are strongly negative correlated (with a correlation coefficient of -0.5), skillful managers tend to receive low returns on their ability. Financial frictions also generate heterogeneity in the returns to capital income. We find that the interest rate on deposits (3%) is substantially smaller than the average marginal product on capital obtained by employers (with a value of 13.2% net of depreciation). The marginal product of capital across employers features a coefficient of variation above .60. Again, this implication is

symptomatic of resources being misallocated. Since borrowing constraints imply that the returns to managerial ability are positively correlated with capital income, the correlation between capital and labor income is much higher in the economy with financial frictions than in the economy with perfect capital markets (.8 versus 0.5). Furthermore, we find that financial frictions lead to a substantial increase in the persistence of income inequality.

Since we also find that financial frictions have a sizable impact on consumption inequality, we evaluate the political economy of capital market imperfections. We assume that the economy is in steady state and that suddenly there is a once and for all reform that makes the enforcement of credit contracts perfect. Note that once the reform takes place, occupational choices and production plans are independent of the wealth distribution. This, together with the small open economy assumption, implies that all macroeconomic aggregates (capital, output, wage rate) will be immediately constant after the reform, though the distribution of wealth, consumption, and income will change for some periods until they converges to their new steady state distributions. In the reform period, competition for workers drive the wage rate to its new long run value, increasing on impact by about 40%. We compute the distribution of welfare gains of eliminating financial frictions for all individuals alive at the period that the reform takes place. We find that the average welfare gain among households alive at the period of the institutional reform is 16.5%, and the standard deviation of the distribution of welfare gains is 13.5%. While the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers. Summing up, while most households benefits from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers may have a vested interested in maintaining a status quo with low enforcement.

2 Evidence

We now document some facts on occupations and economic inequality in Brazil that guide the theory developed in this paper. The facts are based on data from the *Pesquisa Mensal de Emprego* (PME) and from the *Pesquisa de Ornamentos Familiares* (POF). The former is a monthly household employment survey, with a similar structure to the US Current Population Survey (CPS). The latter is a survey of household consumption and income. Appendix A describes how the data set used in this paper was constructed.

Income inequality Figure 1 presents data on the variance of log-income over the life cycle from the PME (similar findings arise from the POF). First, note that the variance of log-income at age 20 is 0.55, which is much higher than the value of 0.30 documented by Storesletten et al. (2005) for the United States. Thus, households in Brazil are quite heterogeneous at young ages. As in the United States, inequality in income grows during the life cycle suggesting the presence of persistent shocks to household earnings. By age 55, household log-income reaches a value of 1.01.

Occupational structure We define the occupation of a household as that of the household head. We consider two broad occupations − workers and entrepreneurs. Moreover, we further subdivide the entrepreneurial occupation in two classes − employers and own account workers (self-employed). Figure 2 uses data from the ILO to analyze occupational structure in different countries. The blue bar on Figure 2 shows the proportion of workers, the orange bar the proportion of self-employed, and the green bar the proportion of employers.[3] The evidence shows that developed countries have lower amount of entrepreneurs than developing countries, but this data pattern is driven by the lower proportion of self-employed in developed countries. The proportion of employers in the population of households is quite similar among countries. While in Brazil workers represent about 73% of households, in Germany they are about 89%. The high proportion of entrepreneurial households in Brazil is explained by self-employed households which represent about 22% of the labour force in Brazil, much lower than the 6% of self-employed households in Germany. The fraction of employers is roughly equal across these two countries (about 5%). Employers and self-employed are quite different in their average income: On average employers earn about 3 times as much as self-employed households. Moreover, self-employed earn less than the average worker.

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[3]For Canada and the United States, the ILO does not distinguish between self-employed and employers so that the orange bar is the sum of the two.

Distribution of earnings by occupation We show two graphs on the distribution of earnings by occupation: The first one shows the distribution of earnings for workers and entrepreneurs, the second one shows the earnings for entrepreneurs partitioned between self-employed and employers. Figure 3 shows that the distribution of earnings of entrepreneurs is flatter than the one of workers, having a bigger mass of people with low earnings but also a bigger mass of households with high earnings. Thus, earnings are more dispersed among entrepreneurs than workers. If we further divide the Entrepreneurs in Self-Employed and Employers we can see that the first group is the one that has more mass in the lower tail of earnings. Figure 4 shows that Self-Employment is the occupation with the lowest expected returns, while Employer is the one with the highest expected returns.

Summarizing, we draw the following lessons from the above facts:

1. Income inequality in Brazil is high relative to the US, which underscores the importance of calibrating the model to Brazilian micro data. Brazilian households are highly heterogeneous early in the life cycle and inequality grows substantially with age. These observations suggests the importance of modeling heterogeneity in fixed effects (permanent skill heterogeneity) as well as persistent shocks to skills.

2. The fact that both wages and entrepreneurial income are highly dispersed, motivates us to build a model with two dimensional skill heterogeneity.

3. It is important to build a theory that distinguishes between employers and self-employed entrepreneurs since most entrepreneurial households in Brazil are self-employed (or own-account workers) households and distribution of income differ substantially across both categories of entrepreneurs. While mean income of employers is much higher than that of self-employed households, there is substantial income heterogeneity within each of these occupational categories.

4. The variation in the rates of entrepreneurship between Brazil and rich countries is entirely explained by the high rates of self-employment in Brazil, a fact that existing theories of occupational choice cannot account for.

3   The Model

We consider a small open economy in steady state. The model features a one sector life-cycle growth model in which households are heterogeneous in two skills − working (z_w) and managerial abilities (z_m). Skills evolve stochastically over the life cycle and

there are no insurance markets to insure ability risk. Production is organized by entrepreneurs who combine managerial, capital, and labor inputs. As in Lucas (1978), entrepreneurs can only use their own managerial skills since there is no markets for managers. In each period households choose their occupation: whether to work for a wage or to operate a business and become entrepreneurs. Occupational choices are based on their comparative advantage as entrepreneurs and their access to capital. Following Buera, Kaboski, and Shin (2010), access to capital is limited by their wealth through an endogenous collateral constraint that arises because of enforcement problems. In order to match important aspects of the Brazilian micro data, the Lucas (1978) model is extended to distinguish between two types of entrepreneurial households – employers and self-employed households.

Population The economy is populated by overlapping generations, each generation consisting of a continuum of households. Households are born at age 20, retire at age 60, and die with certainty at age 75. Each households is endowed with one unit of time at every age. Before the retirement age, households decide how much of their time to allocate to working (t_w) or to managerial (t_m) activities. Households differ in working (z_w) and managerial (z_m) abilities. The logarithm of skills evolve stochastically over the life cycle according to (household i at age t)

$$\ln(z_{wit}) = \beta_w X_t + \alpha_{wi} + u_{wit},$$
$$\ln(z_{mit}) = \beta_m X_t + \alpha_{mi} + u_{mit},$$

where z_{wit} (z_{mit})denote the working (managerial) skills of household i at age t, X_t represents a quartic polynomio of age, α_{wi} and α_{mi} represent household fixed effects on working and managerial productivities, and u_{wit} and u_{mit} are life cycle shocks received at age t by household i. We assume that the fixed effects are drawn from a bi-variate normal distribution at the first period of life of the household (age 20):

$$\alpha = (\alpha_{wi}, \alpha_{mi}) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_w^2 & \rho_\alpha \sigma_w \sigma_m \\ \rho_\alpha \sigma_w \sigma_m & \sigma_m^2 \end{bmatrix}\right)$$

where ρ_α is the correlation between the two fixed effects across individuals $\rho_\alpha = corr(\alpha_{wi}, \alpha_{mi}.)$ The mean fixed effect of the distribution of working skills is normalized to 0.

The life-cycle shocks follow the stochastic process

$$u_{jit} = \rho_j u_{jit-1} + \epsilon_{jit}, \text{ for } j = w, m,$$

with $\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$ jointly drawn from a bivariate normal distribution with correlation coefficient $corr(\epsilon_{wt}, \epsilon_{mt}) = \rho_\epsilon$. We further assume that α_{ji} and u_{jit} are mutually orthogonal.

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The assumptions made imply that distribution of skills at age-t is log-normally distributed

$$
\begin{pmatrix} ln(z_{wt}) \\ ln(z_{mt}) \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{wt}^2 & \rho_{wmt}\sigma_{wt}\sigma_{mt} \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} & \sigma_{mt}^2 \end{bmatrix} \right)
$$

$$
\sigma_{wt}^2 = \sigma_{\alpha_w}^2 + \sum_{j=0}^{t-1}(\rho_w^j)^2\sigma_{\epsilon_w}^2
$$

$$
\sigma_{mt}^2 = \sigma_{\alpha_m}^2 + \sum_{j=0}^{t-1}(\rho_m^j)^2\sigma_{\epsilon_m}^2
$$

$$
\rho_{wmt}\sigma_{wt}\sigma_{mt} = cov(\alpha_w,\alpha_m) + \sum_{j=0}^{t-1}\rho_w^j\rho_m^j cov(\epsilon_w,\epsilon_m)
$$

$$
cov(\alpha_w,\alpha_m) = \rho_\alpha\sigma_{\alpha_w}\sigma_{\alpha_m}
$$

$$
cov(\epsilon_w,\epsilon_m) = \rho_{\epsilon_w\epsilon_m}\sigma_{\epsilon_w}\sigma_{\epsilon_m}
$$

Production technology Following Lucas (1978), output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. Entrepreneurs can only use their managerial input because there is no market for managers. The supply of the managerial input is equal to the product of the households' managerial ability (z_m) and the time devoted to managing a business (t_m). The output produced by a household supplying $m = z_m t_m$ units of managerial input and using k units of capital and n efficiency units of labor is:

$$
Y(m,k,n) = m^\gamma k^\nu n^\theta, \text{ where } \gamma + \nu + \theta = 1. \tag{1}
$$

The time allocation decision of entrepreneurs $(t_m \in [0,1])$ is modeled to introduced self-employment in the Lucas (1978) framework. When $0 < t_m < 1$ entrepreneurs supply *both* managerial and labor inputs to their own businesses. Specifically, the labor input supplied by entrepreneurs to their business is equal to the product of their working ability (z_w) and the time devoted to non-managerial activities $(1 - t_m)$. The total labor input used by an entrepreneur is the sum of the labor supplied by the entrepreneur $((1 - t_m)z_w)$ and the labor hired in the market (n^d) from workers outside the family:

$$
n = n^d + (1 - t_m)z_w, \tag{2}
$$

where z_w is the working ability of the household. We denote as entrepreneurs the households that choose $t_m > 0$. Entrepreneurs, in turn, are partitioned in two subgroups depending on whether they hire outside labor or not. The first subgroup is given by the employers, who are those entrepreneurs hiring labor outside the family $(n^d > 0)$. We assume that entrepreneurs that hire outside labor incur a fixed per period operating cost

of c_f.[4] The second subgroup are those entrepreneurs that only use their own household labor input ($n = (1 - t_m)z_w$ and $n^d = 0$). Workers are those households who use all their available time as workers ($t_m = 0$, obtaining labor earnings wz_w).

Summarizing, entrepreneurs produce output with a production technology that combines capital, labor, and managerial inputs. The key distinguishing feature between employers and self-employed is that the latter do not hire labor outside the household and that employers pay a fixed cost in each period of business operation. They both solve a time-allocation problem regarding the fraction of their time endowment used to supply managerial versus working skills. Below, we shall characterize how entrepreneurs optimally choose the time (t_m) dedicated to the supply of managerial skills.

Capital markets We assume that the financial intermediation industry is competitive. Intermediaries take deposits from households and pay the international interest rate r. They rent capital to entrepreneurs at a rate $r + \delta$ and loan employers the fixed cost of operation c_f. Enforcement problems limit the amount of borrowing and the capital rented to entrepreneurs. Following Buera et al. (2011), entrepreneurs may renege on the contracts after production has taken place and keep a fraction $1 - \phi$ of undepreciated capital and the revenue net of labor payments ($Y(m, k, n) - wn^d + (1 - \delta)k - c_f I_{n_d > 0}$) but lose the financial assets a deposited with the intermediary. Entrepreneurs that default regain access to the financial markets the following period. The parameter $\phi \in [0, 1]$ indexes the strength of the legal institutions in the economy, with $\phi = 1$ indicating perfect financial markets and $\phi = 0$ corresponding to an economy with no credit markets. We study equilibria in which financial contracts are restricted so that there is no default in equilibrium. This occurs when the amount of capital rented is limited by the largest amount $\overline{k}(a, z_m, z_w; \phi)$ consistent with entrepreneurs choosing to abide by their financial contracts. To characterize rental limits, consider the profit maximization problem of entrepreneurs that take as given the capital k used in the business operation:

$$\pi(z_m, z_w, a; k) \equiv \max_{m, n, n_d, t_m} \{m^\gamma k^\nu n^\theta - wn^d - r(k - a) + a - \delta k - c_f I_{n_d > 0}\} \quad (3)$$

$$subject\ to$$

$$m = t_m z_m, \quad (4)$$

$$n = (1 - t_m) * z_w + n^d, \quad (5)$$

$$where\ t_m \in [0, 1], n^d \geq 0, k\ given. \quad (6)$$

The following proposition extends results in Buera et al. (2011) to characterize the rental limits $\bar{k}(a, z_m, z_w; \phi)$.

Proposition 1 *Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if*

$$\pi(z_m, z_w, a; k) \geq (1 - \phi) \max_{m,n,n_d,t_m} \{m^\gamma k^\nu n^\theta - wn^d + (1 - \delta)k - c_f I_{n_d > 0}\}$$

$$subject\ to$$

$$m = t_m z_m,$$

$$n = (1 - t_m) * z_w + n^d,$$

$$where\ t_m \in [0, 1], n^d \geq 0.$$

The upper bound on capital rental that is consistent with entrepreneurs choosing to abide by their contracts can be represented by a function $\bar{k}(a, z_m, z_w; \phi)$, which is increasing in a, z_m, z_w and ϕ.

Proof. See appendix.

The income of an entrepreneur in state (z_m, z_w, a) making optimal production decisions given prices and borrowing limits is given by

$$y^e(z_m, z_w, a) \equiv \max_k \{\pi(z_m, z_w, a; k)\} \tag{7}$$

$$subject\ to$$

$$k \leq \bar{k}(a, z_m, z_w; \phi) \tag{8}$$

The income of a household that choose to work for a wage is $y^w(z_m, z_w, a) = wz_w + ra$. Household income is the maximum between the entrepreneurial and workers income:

$$y(z_m, z_w, a) = \max\{y^e(z_m, z_w, a), y^w(z_m, z_w, a)\}. \tag{9}$$

Households maximize expected discounted lifetime utility

$$\max_{c_j, a_{j+1}} \quad E\{\sum_{j=1}^{J} \beta^j U(c_j)\}$$

$$subject\ to$$

$$c_j + a_{j+1} = y(z_{mj}, z_{wj}, a_j),$$

$$c_j, a_{j+1} \geq 0,$$

4 Time Allocation and Occupational Maps

We now study in partial equilibrium (e.g. for a fixed wage rate) how our theory can give rise to three active occupational choices: workers, self-employed, and employers. We show that when capital markets are perfect occupational choices are entirely determined by the ability ratio $\frac{z_w}{z_m}$. Individuals with a high $\frac{z_w}{z_m}$ ratio have a comparative advantage at working and choose to become workers, individuals with a low $\frac{z_w}{z_m}$ ratio have a comparative advantage at entrepreneurship and choose to become employers, and those with intermediate skill ratios prefer to be self-employed. We also characterize how tight borrowing constraints (capital market imperfections) distort occupational choices.

We start by analyzing the determinants of self-employment income. Self-employed individuals choose how much time to allocate to managerial versus working activities and how much capital to use in production. Using the linear homogeneity of the production function, the income of a self-employed individual with a units of assets who uses k units of capital can be written as

$$y_{se} = MPt_m \, t_m + MPt_w \, t_w + MPK \, k + ra - k(r + \delta),$$

where MPt_m and MPt_w denote the marginal products of managerial time and working time, respectively, and MPK represents the marginal product of capital. We are now ready to prove the following proposition:

Proposition 2 (Self-employment) *The optimal time devoted to management by self-employed entrepreneurs (entrepreneurs not hiring outside labor) is $t_m^* = \frac{\gamma}{\gamma+\theta}$. The marginal product of their time is equated across its two uses (managerial and working time) and satisfies:*

$$MPT_{se} = r_{mw} \left(z_m^\gamma z_w^\theta \right)^{\frac{1}{\gamma+\theta}},$$

where $r_{mw} = \gamma \, \nu^{\frac{\nu}{1-\nu}} \left(\frac{\theta}{\gamma} \right)^{\frac{\theta}{1-\nu}} \left(\frac{1}{r+\delta+\mu} \right)^{\frac{\nu}{1-\nu}}$ is the rate of return to the composite skill input $\left(z_m^\gamma z_w^\theta \right)^{\frac{1}{\gamma+\theta}}$ and μ is the Lagrange multiplier associated to the borrowing constraint. The income of a self-employed individual with assets a is given by

$$y_{se} = r_{mw} \left(z_m^\gamma z_w^\theta \right)^{\frac{1}{\gamma+\theta}} + \mu k + ra,$$

where $k = \overline{k}(z_m, z_w, a)$.

Proof. See appendix.

Proposition 2 establishes that self-employed individuals equate the marginal product of the time allocated to managing and to working tasks. The marginal product of the

self-employment time can be expressed as the product of the skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ and the rate of return r_{mw}. The skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ is a geometric average of the managerial and working abilities of the self-employed individual. The return to the skill composite (r_{mw}) depends on parameters of the production technology, the real interest rate (r), and the Lagrange multiplier (μ) associated to the borrowing constraint. Note that the return to the skill composite decreases with (μ). Hence, borrowing constraints generate heterogeneity in rate of returns to skills among self-employed individuals.

Since workers' income is given by

$$y_w = w z_w + r a,$$

it is immediate that $y_{se} - y_w$ is independent of asset holdings. Hence, as shown in Proposition 3, when $\mu = 0$ the decision of whether to work for a wage or to be self-employed only depends on the ability ratio $\frac{z_w}{z_m}$. On the contrary, when the borrowing constraint binds $k = \overline{k}(z_m, z_w, a)$ occupational choice decisions depend on asset holdings because they affect the rate of return to skills (e.g. the composite input $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$) and the rate of return to assets. These results are summarized in Proposition 3.

Proposition 3 (Self-employed vs Worker) *Let $R_1 \equiv e^{\frac{\theta+\gamma}{\gamma}} \frac{r_{mw}}{w}$, where r_{mw} is defined in Proposition 2. Then,*

1. *If capital markets are perfect ($\phi = 1$), working for a wage is preferred to self-employment if and only if $\frac{z_w}{z_m} < R_1$.*

2. *If capital markets are imperfect ($\phi < 1$), working for a wage is preferred to self-employment if and only if*

$$\frac{z_w}{z_m} > e^{\frac{\theta+\gamma}{\gamma}} \left[\frac{r_{mw} + \mu k / \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}}{w} \right] \equiv \overline{R}_1,$$

   where $\mu > 0$ is the Lagrange multiplier associated to the borrowing constraint and $k = \overline{k}(z_m, z_w, a)$.

When capital markets are perfect ($\phi = 1$), the occupational choice decision between working for a wage or being self-employed can be represented by a ray R_1 that goes through the origin in (z_m, z_w) space. Individuals with ability above this ray prefer to be a worker. In this case, occupational choice decisions are independent of asset holdings and maximize the marginal product of time. However, when capital markets are imperfect, occupational choice decisions depend on asset holdings and do not maximize the marginal product of time. The occupational choice between working and

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self-employment is now described by the curve $\overline{R}_1$ in (z_m, z_w) space. Note that a proportional change in both skills decreases the income ratio $\frac{y_{se}}{y_w}$ because the increase in z_m leads to a tighter borrowing constraint for a fixed asset level a, implying that the curve $\overline{R}_1$ tilts down relatively to the ray R_1 as z_m increases. An increase in assets (a) relaxes the borrowing constraint (μ decreases and r_{mw} increases) making it more likely that individuals will choose self-employment so that the position of the $\overline{R}_1$ depends on asset holdings.

We now analyze the decisions of employers. Employers choose how much of their time to allocate to managerial versus working activities and how much capital (k) and (outside) labor services (n_d) to use in production. Using the linear homogeneity of the production function the income of an employer with a units of assets can be written as

$$y_e = MPt_m\, t_m + MPt_w\, t_w + MPn_d\, n_d + MPK\, k + ra - k(r + \delta) - w\, n_d - c_f,$$

where MPt_m and MPt_w denote the marginal products of managerial time and working time, respectively, and MPK and MPn_d represent the marginal product of capital and labor services. We are now ready to prove the following proposition:

Proposition 4 (Employers)

1. *The optimal time devoted to management by employers is $t_m^* = \min\left\{\hat{t}_m, 1\right\}$, where*
   $\hat{t}_m^* \equiv \left[\frac{z_m^\gamma \overline{k}(a, z_m, z_w; \phi)^\nu \theta}{w} \left(\frac{\theta z_w}{\gamma}\right)^{\theta-1}\right]^{\frac{1}{1-\theta-\gamma}}$. *Moreover, denoting by $k^u(z_m, z_w)$ the level of capital chosen by an unconstrained entrepreneur with ability (z_m, z_w), there exist $a^*(z_m, z_w) < k^u(z_m, z_w)$ such that $t_m^* = 1$ for all $a \geq a^*(z_m, z_w)$.*

2. *The marginal product of employer's time satisfies:*

   $$MPT_e = z_m r_m \geq z_w\, w\, (\text{with strict inequality if } t_m = 1),$$

   where $r_m = \gamma\left[\left(\frac{\nu}{(r+\delta+\mu)}\right)^\nu \left(\frac{\theta}{w}\right)^\theta\right]^{\frac{1}{1-(\nu+\theta)}}$ is the rate of return to the managerial input z_m and μ is the Lagrange multiplier associated to the borrowing constraint.

3. *The income of an employer with ability (z_m, z_w) with assets a is given by*

   $$y_e = z_m r_m + \mu k + ra - c_f,$$

   where $k = \overline{k}(z_m, z_w, a)$

Proof. See appendix.

Proposition 4 states that there is a threshold level of asset holdings $a^*(z_m, z_w)$ such that for assets below this level the marginal product of entrepreneurial time is equal to

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wz_w and the time allocation problem of the employer features an interior solution in which the employer performs both managing and working activities. If asset holdings are higher than the threshold $a^*(z_m, z_w)$, then the marginal product of entrepreneurial time is higher than that as a worker and the time allocation problem exhibits a corner solution $t_m = 1$.

The marginal product of employers' time (MPT'm) can be expressed as the product of managerial skills z_m and the rate of return r_m on the employer's managerial skill. The rate of return (r_m) depends on parameters of the production technology, the real interest rate (r), and the Lagrange multiplier (μ) associated to the borrowing constraint. Note that borrowing constraints (μ) generate heterogeneity in rate of returns to skills among employers.

Proposition 4 shows that when capital markets are perfect ($\mu = 0$) then the marginal product of employer's time is proportional to her managerial ability z_m and the income difference between being an employer and being self-employed $y_e - y_{se}$ is independent of asset holdings. In this case, Proposition 5 shows that the decision of whether to be an employer or to be self-employed only depends on the ability ratio $\frac{z_w}{z_m}$, provided the fixed cost of operation faced by employers is equal to zero ($c_f = 0$). There exist a constant ratio R_2 such that individuals with a skill ratio $\frac{z_w}{z_m}$ below R_2 choose to be an employer. When the fixed cost of operation is positive, then the occupational choice decision depends on comparative advantage (skill ratio $\frac{z_w}{z_m}$) and on the absolute level of managerial ability z_m. Now, to be an employer rather than self-employed the ability vector (z_m, z_w) should satisfy $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})$. Intuitively, in the presence of fixed cost of being an employer, employers need a minimum level of managerial ability z_m in order to recoup the fixed cost of operation. The occupational choice decision between employer and self-employment is not only based on the skill ratio.

When capital markets are imperfect and borrowing constraints bind, occupational choice decisions depend on asset holdings because both the marginal product of time and the return to capital of both employed and self-employed individuals depend on their asset holdings (see Proposition 5). Intuitively, an increase in asset holdings increases the employer region in the occupational map in (z_w, z_m) relative to the self-employment ($\overline{R_2}$ in Proposition 5 shifts up). The key is that borrowing constraints tend to be tighter for employers than self-employed since employers need to operate at a larger scale.

Proposition 5 (Employer versus Self-employment) *Let $R_2 \equiv e^{\frac{\theta+\gamma}{\theta}} \frac{r_m}{r_{mw}}$, where r_{mw} and r_m are the rate of returns to the skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ and the managerial skill defined in Propositions 2 and 4, respectively. Then,*

1. If capital markets are perfect ($\phi = 1$), individuals prefer to become employers rela-

tive to self-employment when the ability ratio is such that $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})$.

2. *If capital markets are imperfect ($\phi < 1$), individuals prefer to become employers relative to self-employment when the ability ratio is such that*

$$\frac{z_w}{z_m} < e^{\frac{\theta+\gamma}{\theta}} \left[\frac{r_m + (\mu_e k_e - c_f)/z_m}{r_{mw} + (\mu_{se} k_{se})/(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}} \right] \equiv \overline{R_2},$$

where μ_e and μ_{se} are the Lagrange multipliers associated to the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production at these occupations.

Proposition 6 collects results characterizing occupational choice decisions when capital markets are perfect ($\phi = 1$). If the fixed cost of operation of employers is $c_f = 0$, occupational choices are only determined by the ability ratio $\frac{z_w}{z_m}$. Depending on parameter values (equilibrium returns to ability), the equilibrium may feature self-employed individuals or not. If equilibrium prices are such that $R_1 > R_2$, then individuals with an ability ratio $\frac{z_w}{z_m} > R_1$ work for a wage, individuals with $R_1 > \frac{z_w}{z_m} > R_2$ are self-employed, and those with $\frac{z_w}{z_m} < R_2$ are employers. A positive fixed cost of operation ($c_f > 0$), implies that employers require a minimum scale in order to operate a profitable business so that the decision to be an employer depends both on the skill ratio $R_1 > \frac{z_w}{z_m} > R_2$ and on the level of managerial ability z_m.

Proposition 6 (Occupational maps when capital markets are perfect ($\phi = 1$) *Assume that $\phi = 1$. Let $R_1 \equiv e^{\frac{\theta+\gamma}{\gamma}} \frac{r_{mw}}{w}$ and $R_2 \equiv e^{-\frac{\theta+\gamma}{\theta}} \frac{r_m}{r_{mw}}$, where r_{mw} and r_m are the rate of returns to the skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ and the managerial skill defined in Propositions 2 and 4, respectively.*

1. *If there are no fixed cost of operation of being an employer ($c_f = 0$), then the optimal occupational choice is the one that maximizes the marginal product of time and is only determined by the skill ratio ($\frac{z_w}{z_m}$) as follows:*

    (a) *If equilibrium prices are such that $R_1 > R_2$, then individuals with an ability ratio $\frac{z_w}{z_m} > R_1$ work for a wage, individuals with $R_1 > \frac{z_w}{z_m} > R_2$ are self-employed, and those with $\frac{z_w}{z_m} < R_2$ are employers.*

    (b) *If equilibrium prices are such that $R_1 < R_2$, there is no self-employed individuals in equilibrium. Individuals with a skill ratio such that $z_w w < z_m r_m$ choose to become employers. Otherwise, they choose to work for a wage.*

2. *If employers incurred a positive fixed cost of operation ($c_f > 0$), the decision to be an employer depends on the skill ratio ($\frac{z_w}{z_m}$) and on the absolute level of managerial*

ability (z_m). Individuals prefer to become employers relative to self-employment when the ability ratio is such that $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})$.

Summarizing, we have developed a theory with three occupational choices and characterize occupational decisions. The theory implies that, in the absence of capital market imperfections, the skill ratio $\frac{z_w}{z_m}$ drives occupational choices: Workers have a high $\frac{z_w}{z_m}$ ratio, employers a low $\frac{z_w}{z_m}$ ratio, and the self-employed have an intermediate skill ratio. Capital market imperfections distort returns to skill and, thus, occupational choices. A tight borrowing constraint depresses the rate of return to the managerial ability of employers and the return to the composite skill input supplied by self-employed individuals. It also increases the rate of return to capital faced by entrepreneurs. As a result, asset holdings matter importantly for occupational choice decisions in the presence of financial frictions.

5 Quantitative Analysis

In this section, we calibrate the model economy and show that the impact of financial frictions crucially depend on the correlation between managerial and working skills. To assess these effects, our calibration strategy assumes that the stochastic process on skills is such that the correlation between fixed effects and the correlation between the innovations to the autoregressive bivariate process on skills take the same value

$$\rho = corr(\alpha_{wi}, \alpha_{mi}) = corr(\epsilon_{wt}, \epsilon_{mt}). \tag{10}$$

We consider various values for the correlation between managerial and working skills (ρ). For each value of ρ, we calibrate the rest of the parameters of the model economy to Brazilian data on occupational structure and income inequality. Below, we present results for our preferred value of $\rho = 0.1$ and for two economies with a low and a high correlation of managerial and working skills ($\rho = -0.8$ and $\rho = 0.8$, respectively). We then evaluate the impact of financial frictions across the calibrated model economies.

5.1 Calibration

We partition the parameters in the model economy in two. The first group includes the parameters that are set using estimates from other studies in the literature. The second group consists of all the parameters that are calibrated by simulating the model economy.

Parameters set exogenously The model period is set to an year. The international interest rate is set at 3%. The utility function is assumed to be of the CES type:

$$u(c) = \frac{c^{1-\sigma}}{1 - \sigma}$$

with $\sigma = 1.5$. The parameters of the production function are set to standard values in the literature: $\gamma = .2, \nu = .3, \theta = .5$ (see Guner et al. (2008), Buera et al. (2011)). The annual depreciation rate is set to at $\delta = 0.06$.

Calibrated parameters We present results for three calibrated model economies that differ on the correlation between working and managerial skills ($\rho \in \{-0.8, 0.1, 0.8\}$). For ease of exposition, below we list the parameters to be calibrated together with a corresponding target that helps identify each parameter. Nonetheless, it is important to keep in mind that the calibration is a multidimensional mapping in which all parameters and calibration targets are inter-related.

1. The discount factor β is chosen so that the capital to income ratio in the steady is equal to 2.4, which is consistent with the capital to income ratio in Brazil (see Júnior et al. (2004)).

2. Enforcement of credit contracts ϕ to match a credit to GDP ratio of 43% in Brazil[5].

3. The coefficients on the quartic polynomio on age determining how the two working and managerial sills vary with age are set so that the age-profile of mean earnings for workers and entrepreneurs are roughly consistent with the data.

4. Following Storesletten et al. (2005), the parameters determining the stochastic process on working ability such as the variance of fixed effects $\sigma^2_{\alpha_w}, \sigma^2_{\alpha_m}$, persistence of autoregressive process ρ_w, and the variance of the innovation to working ability over the life-cycle $\sigma^2_{\epsilon_w}$ to match the age profile of the variance of log wages.

5. There are various parameters determining the stochastic process on managerial ability (i) the variance of fixed effect on managerial skills $\sigma^2_{\alpha_m}$; (ii) variance of innovations to managerial abilities ($\sigma^2_{\epsilon_m}$) ; and (iii) the persistence of the auto-regressive process on managerial ability (ρ_m). To pin down these parameters, we target: (i) the proportion of entrepreneurs and workers in the population of households (32% versus 68%); (ii) the variance of entrepreneurial log-earnings (1.06); and (iii) the persistence of being an employer between two consecutive years (68%).

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[5]We use the average Private Credit/GDP from 2003 until to 2010 from the World Development Indicators from the World Bank

6. The fixed cost of operation of employers c_f is set to match the fraction of employers among entrepreneurs (one fourth).

Discretization of shocks To solve the model numerically, we first find a finite state approximation of the following bivariate process describing the life-cycle shocks to skills

$$u_t = Au_{t-1} + \epsilon_t,$$

where u_t is a 2×1 vector, A is a 2×2 matrix, and ϵ is a 2×1 vector with mean 0 and variance-covariance matrix $\Sigma = E(\epsilon\epsilon^{'})$. Using that Σ is a symmetric matrix, we can express it as follows:

$$\Sigma = Q\Lambda Q^{'}, \tag{11}$$

where Λ is a diagonal matrix (with the eigenvalues of Σ in the diagonal) and Q is the matrix of eigenvectors of Σ as columns. The bivariate process can be expresed

$$\widehat{u}_t = \widehat{A}\widehat{u}_{t-1} + \widehat{\epsilon}_t,$$

where $\widehat{u}_t = Qu_t, \widehat{A} = Q^{'}AQ$, and $\widehat{\epsilon}_t = Q\epsilon_t$. The key to this transformation is that $\widehat{\epsilon}_t$ has a diagonal variance-covariance matrix: $E(\widehat{\epsilon}_t\widehat{\epsilon}_t^{'}) = Q^{'}\Sigma Q = \Lambda$. We then approximate $\widehat{u}_t$ with a Markov chain with states given by a matrix $\widehat{U}_t$ with dimension 2×100. Then the states of the Markov chain which approximate u_t are given by the matrix $U = Q\widehat{U}_t$ with dimension 2×100. Because of our life-cycle environment, the variance of shocks grow with age. To deal with this feature, we allow the support of the shocks and the Markov chain to change with age. The Markov chain is allowed to vary with age so that the finite state approximation of the autoregressive bivariate process matches the unconditional variance of the continuous bivariate shock process at each age.

Regarding fixed effects, the bivariate normal distribution is discreticized with 3 values for working skill and 5 values for managerial skills. As a result, there are 15 pairs of fixed effects. At each age, there are 1500 possible pairs of skills (z_w, z_m).

5.2 Calibration results

We now discuss how the three calibrated model economies match the calibration targets. The values of the calibrated parameters are reported in Table 1.

Table 2 shows that the model economies match reasonably well the targets for the credit to GDP ratio of 43% and the capital to income ratio of 2.4. Figure 5 compares the variance of log-earnings of workers in the model economy with the Brazilian data. The model economies are consistent with the fact that there is a large amount of inequality

early in the life cycle and that inequality grows substantially with age over the life cycle. In all the calibrated model economies the stochastic process on working skills is characterized by a high persistence (ρ_w close to 1), which is needed to match the linear age-profile of the variance of log wages in the Brazilian data. This is consistent with the findings of Storesletten, Telmer, and Yaron (2005) for the US. Relatively to previous findings for the US economy, the calibration requires a large variance of individuals fixed effects (σ_{α_w}) to match the high inequality of wages at age 20 in Brazil.

In all the calibrations, the variances of fixed effects and of the innovations of managerial skills are much larger than the corresponding variances of working skills (see Table 1). This is necessary for the model economy to be consistent with the large variance of entrepreneurial earnings in the Brazilian data. The variances of entrepreneurial shocks − both for innovations and fixed effects − are larger when entrepreneurial and working skills are negatively correlated. This is due to the self-selection of households into occupations. When skills are negatively correlated, households with a low realization of managerial skills are likely to have a high realization of the working-ability shock and choose to become workers. It follows that the distribution of managerial skills among entrepreneurs has a much smaller variance than the population distribution of managerial skills. As a result, the calibration requires an increase in the variance of managerial shocks in order to match the target for the variance of entrepreneurial log-earnings

Table 3 compares the fraction of households that are workers, self-employed, and employers in the calibrate model economies and in Brazil. The model economies match quite closely the fractions of workers (68%), self-employed (24%), and employers (8%) in the data. The calibration targeted the fact that about 70% of the employers in Brazil at a given point in time are still employers one year after (see Table 3).

Table 4 reports predictions of the model economies on occupational transitions. First, consistently with the data, the model economies predict that being a worker is quite persistent. About 94% of the total population of workers in Brazil in a given year are still workers in the next year. Moreover, both in the model economies and in the data, entrepreneurs are less likely to remain in their occupation than workers. Moreover, among the entrepreneurial class, employers are less likely to remain in their occupation than the self-employed. The persistence of employers in Brazil is 70% and the persistence of self-employed is 78%. The calibrated model economies match the patterns on the persistence of occupational choices remarkably well (see Table 4). We remind the reader that the calibration only targeted the persistence of being an employer between two consecutive years. In all the calibrated model economies the persistence of entrepreneurial shocks is high (ρ_m in the range of 0.75 to .0.85), but less than the persistence of shocks on working ability ($\rho_w = 0.98$)

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Summing up, while the fit of the data is not perfect, we believe that the calibrated model economies provide a reasonable account of earnings inequality and occupational choices in Brazil.

5.3 Financial frictions and misallocation: The role of bivariate skill heterogeneity.

We now show that the quantitative implications of financial frictions vary substantially across the three calibrated model economies. We perform three experiments to assess the impact of financial frictions in the calibrated model economies. The first experiment computes equilibrium under the assumption of perfect credit-enforcement institutions ($\phi = 1$) in order to evaluate the effects of credit market institutions on equilibrium aggregates. The second experiment evaluates the impact of financial frictions when there is no heterogeneity in working ability across individuals. This is done by first eliminating heterogeneity in labor productivity in the calibrated model economies and then simulating the impact of removing financial frictions in the resulting economies with no wage heterogeneity. The idea of this experiment is to isolate the importance of modeling heterogeneity in two skills for our quantitative results. The third experiment assesses the impact of financial frictions when individuals are heterogeneous in two skills but entrepreneurs can only allocate their time to managing. In this economy, by construction, entrepreneurs cannot be own account workers (self-employed). Table 5 reports the results.

The first experiment reveals that the output gains of removing financial frictions are large but vary substantially across the three calibrated model economies. The output gains range from 36% to 55% as the correlation between skills decreases from 0.8 to -0.8. Hence, financial frictions have much lower effects on output per worker when skills are (strongly) positively correlated. The second experiment provides insights about why the correlation of skills matters importantly for the misallocation of resources caused by financial frictions. Recall that, for each of the calibrated model economies, the second experiment evaluates the output gain of eliminating financial frictions in the absence of heterogeneity in working skills. We find that the output gain in the economy with strongly positively correlated skills ($\rho = 0.8$) would be about 4 percentage points higher if there were no heterogeneity in working skills. When the correlation of skills is positive, heterogeneity in wages mitigate the impact of financial frictions because talented entrepreneurs can work for a high wage and accumulate savings. On the other hand, the misallocation of resources induced by financial frictions is enhanced by heterogeneity in wages when skills are negatively correlated. The output gain in the economy with

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strongly negatively correlated skills ($\rho = -0.8$) would be about 8 percentage points lower if there were no heterogeneity in working skills. When skills are negatively correlated, individuals with high entrepreneurial skills have a harder time, on average, building up savings relative to other individuals with lower entrepreneurial talents. Finally, heterogeneity in wages is not important for the impact of financial frictions when the correlation between skills is moderate ($\rho = 0.1$).

To further understand how the skill correlation matters for the impact of financial frictions, we analyze how financial frictions distort the rate of returns to the various production inputs. Recall that when capital market are perfect the rate of return of all productive inputs are equalized across production units. However, rates of return do vary across production units under financial frictions (see Section 4). We now show that the skill correlation parameter ρ matters importantly for the variation in rate of returns caused by financial frictions. Table 6 compares the variation in rates of returns among employers and self-employed individuals in the calibrated model economies. The standard deviation of the marginal product of capital among employers is twice as large in the economy with $\rho = -0.8$ than in the economy with $\rho = 0.8$ (.14 versus .07). The variation in rates of returns to capital reflects the variation in the tightness of the borrowing constraint across entrepreneurs.[6] The results in Table 6 show that when skills are positively correlated there is less heterogeneity in rates of return on capital across entrepreneurs than when skills are negatively correlated. Intuitively, the financing problems faced by talented entrepreneurs are less severe when entrepreneurs are also talented workers. This is because households with high working skills can rapidly accumulate savings and alleviate the financial constraints that limit the operation of their businesses. On the other hand, when skills are negatively correlated borrowing constraints are tighter because talented entrepreneurs find it more difficult to accumulate savings.

Borrowing constraints also generate heterogeneity on the rate of return to the managerial input among employers (r_m) and on the rate of return on the self-employment composite(r_{mw}). Both of these returns decrease with the tightness of financial constraints (see Section 4). Table 6 shows that the variation in rates of return to the managerial input among employers is about twice as large in the economy with $\rho = -0.8$ than in the economy with $\rho = 0.8$ (0.31 versus 0.17.) Moreover, while in all economies the return to the managerial skill is negatively correlated with the level of managerial ability, this correlation is the lowest in the economy with $\rho = -0.8$ (about -0.63). When skills are strongly negatively correlated, the tight borrowing constraints faced by entrepreneurs

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[6]Recall that the marginal product of capital can be expressed as $MPK = r + \delta + \mu$, where μ represents the Lagrange multiplier associated to the borrowing constraint (see Section 4).

with high managerial skills imply that they obtain a lower return to their skills than less able entrepreneurs. In this case financial frictions generate a strong comparative advantage at entrepreneurship for households with lower managerial talent but higher working ability, reducing the average entrepreneurial ability, and total factor productivity. Table 5 shows that changes in TFP associated with the elimination of financial frictions range from 11% to 18%, with the largest (lowest) increase attained in the economy with strongly negative (positive) correlation of skills.

A key innovation of our theory is that it allows entrepreneurs to choose what fraction of their time they allocate to managing versus working. This assumption allows our theory to be consistent with the fact that self-employment is quite important in poor countries. The results from the third experiment indicate that allowing for self-employment matters for the quantitative impact of financial frictions. We find that in all the calibrated model economies the output gains due to the elimination of financial frictions increase substantially in the absence of self-employment (about 8 percentage points). Self-employment allows individuals with no significant comparative advantage at either entrepreneurship or working to diminish the negative effects of financial frictions on their earnings. By being self-employed, they can supply both of their skills to their business operation.

5.4 Discriminating between economies with different correlation of skills (ρ)

We have shown that the impact of financial frictions vary substantially across the three calibrated model economies. Then, in order to assess the impact of financial frictions in the Brazilian economy it is important to use Brazilian data to test the predictions of the calibrated model economies.

The correlation between skills have important effects on the earnings distribution across occupations. When ρ is sufficiently high, the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ becomes positive. In this case, a high skill ratio z_{mt}/z_{wt} is also associated with high values of z_{mt} and z_{wt} so that households that have a *comparative advantage* at managing (high z_{mt}/z_{wt}) also have an *absolute advantage* in both skills. When entrepreneurs have an absolute advantage in both occupations, highly skilled workers tend to have a comparative advantage at managing and choose the entrepreneurial occupation. Low skill workers do not have a comparative advantage at managing and choose to work for a wage. As a result, the earnings distribution among entrepreneurs is shifted to the right relative to the earnings distribution among workers and earnings inequality between occupations is large. On the other hand, when the correlation between $ln(z_{mt}/z_{wt})$ and

$ln(z_{wt})$ is negative households in one occupation tend to be better at that occupation than households choosing the other occupation. Earnings inequality across occupations is not as large as in the absolute advantage case.

It is easy to show that the skill ratio and the working skill are jointly log-normally distributed for each age t:

$$\begin{pmatrix} ln(z_{mt}/z_{wt}) \\ ln(z_{wt}) \end{pmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{mt}^2 + \sigma_{wt}^2 - 2\rho_{wmt}\sigma_{wt}\sigma_{mt} & \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_{wt}^2 \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_{wt}^2 & \sigma_{wt}^2 \end{bmatrix} \right)$$

The absolute advantage case arises when the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ is positive, which holds if and only if

$$\rho_{wmt} > \frac{\sigma_{wt}}{\sigma_{mt}}$$

Thus, the correlation of skills have to be sufficiently strong for the absolute advantage case to hold. Figure 6 graphs the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ for the three calibrated model economies. The economy with $\rho = 0.80$ is the only one with a positive correlation. In this economy, households with high managerial ability tend to have an absolute advantage in skills (have higher managerial and working skills). The economy with $\rho = 0.10$ exhibits a correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ of roughly -0.30.

Figure 9 shows that the calibrated model economies differ importantly in the distribution of earnings by occupation (workers versus entrepreneurs). The economy with strongly correlated shocks $\rho = 0.80$ is grossly at odd with the brazilian data: It counterfactually predicts that the distribution of earnings of workers is shifted to the left relative to that of entrepreneurs. The economy with strongly negative correlated shocks $\rho = -0.80$ is also at odds with the brazilian data since it implies that the distribution of earnings of workers is shifted to the right relative to that of entrepreneurs. On the other hand, the economy with $\rho = 0.10$ fits the brazilian evidence on the earnings distribution across occupations reasonably well. Indeed, the ratio of median earnings between workers and entrepreneurs is 1.0 both in Brazil and in the model economy with $\rho = 0.1$. This statistic takes a value of 0.7 in the economy with $\rho = 0.8$ and a value of 1.3 when $\rho = -0.8$.

Figure 10 compares the distribution of earnings in the calibrated model economies and Brazil when the population is divided in three occupational groups (workers, self-employed, and employers). The economy with strongly correlated shocks counterfactually predicts that the earnings distribution of self-employed individuals is shifted to the right relative to that of workers. As ρ decreases, and hence the absolute advantage of entrepreneurs disapears, the distribution of earnings of self-employed individuals shifts

to the left. As a result, consistently with the evidence, the economies with $\rho = 0.10$ and -0.80 exhibit a distribution of earnings of self-employed households that is shifted to the left relative to that of workers. Overall, the economy with $\rho = .10$ is the one that fits the evidence best. Relative to the data, the economy with a high negative skill correlation implies that the self-employed individuals have too low earnings relative to workers.

The correlation between skills also matter for the persistence of earnings over time. Intuitively, earnings are less volatile when skills are positively correlated than negatively correlated. To compare the persistence of earnings across the calibrated model economies, for each economy we simulate artificial data and run the following regression:

$$log(y_{t,j}) = \alpha_j + \beta log(y_{t-1,j}) + b_2 age_t + b_3 age_t^2, \tag{12}$$

where y_{tj} represents the income of individual j at age t, α_j is an individual fixed effect, and β measures the persistence of log-income. Table 7 presents the estimates of β for the model economies. The persistent of log-income increases from 0.73 to 0.84 as ρ increases from -0.8 to 0.8. Unfortunately, we do not have panel data from Brazil to estimate the persistence of income in Brazil. Nonetheless, we can use consumption data from Brazilian households to test the predictions of the theory. The idea is that consumption theory implies that permanent income is a key determinant of consumption decisions. Hence, the higher the persistence of income the higher should be the cross-sectional correlation between consumption and income across households (e.g. the correlation between consumption and income at a given date t). Table 7 shows that the correlation between consumption and income varies widely across the calibrated model economies: from 0.24 when $\rho = -0.8$, to .79 when $\rho = 0.1$, and up to 0.85 when $\rho = 0.8$. This correlation is 0.71 in the Brazilian data.

Altogether, the Brazilian evidence reviewed suggests that the correlation between managerial and working skills is moderately positive $\rho = 0.1$ Hence, from now on, we set the model economy with $\rho = 0.1$ as our baseline economy.

5.5   Other implications of financial frictions

We further evaluate the effects of introducing perfect capital markets in the baseline economy ($\rho = 0.1$). We focus on how capital markets impact on occupational choices, resource allocation, aggregate output, and the distribution of income in the economy.

5.5.1 Capital markets and occupational maps.

We now analyze how capital market imperfections impact on occupational choices. Figure 7 draws the occupational map for an economy with perfect enforcement. As shown in Proposition 6, when capital markets are perfect occupational choices are determined by the $\frac{z_m}{z_w}$ skill ratio. When capital markets are imperfect, occupational choices are determined by the skill ratio and asset holdings since borrowing constraints affect the returns to skills and assets. Figure 8 represents graphically, for two fixed asset levels, how occupation varies across individuals that differ on (z_m, z_w). In Panel a, the level of assets is fixed at the median income and in Panel b it is fixed at the mean income. A comparison of the occupational maps, reveal that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

The occupational maps suggest that capital market imperfections affect importantly occupation choice decisions. Indeed, they have important consequences on the occupational structure in the economy: The fraction of entrepreneurs in the baseline economy is twice the one in the economy with perfect enforcement (33% versus 18%). The reduction in self-employment rates accounts for most of the decrease in the number of entrepreneurs: While self-employment rates drop from 24% to 11%, the fraction of employers drops by about 2 percentage points. Altogether, the theory is consistent with key stylized facts on changes in the occupational structure with economic development. The theory is consistent with the evidence that most of the changes in rates of entrepreneurship across rich and poor countries is due to changes in the self-employment rate. The theory is also consistent with the fact that the fraction of workers in the labor force tends to increase with economic development: It increases from 68% in the baseline economy to 82% with perfect capital markets.

5.5.2 Capital markets and rate of returns to production inputs.

In Section 4 we analyzed how, for each occupation, income can be decomposed in terms of the various inputs supplied and their rate of returns. We now discuss how capital market imperfections affect the rate of returns faced by heterogeneous individuals.

As in Section 4, we distinguished between the following inputs and rates of return: (i) wage rate w per unit of labor services, (ii) marginal product of capital $MPK = r + \delta + \mu$, (iii) return r_m on managerial input, (iv) return on self-employment composite ability r_{mw}, (v) return on financial assets r. The Lagrange multiplier μ in the borrowing constraint varies across individuals with different characteristics $(z_m, z_w, assets)$ and affect rates of returns of various inputs differently. While the marginal product of capital

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increases with μ, the returns on the managerial input (r_m) and the self-employment composite (r_{mw}) decrease with μ. Moreover, when capital markets are perfect $(\phi = 1)$ there is no variation in rate of returns across individuals.

Table 6 compares statistics on rates of returns across economies and offer some clues about why capital market imperfections have a profound impact in the occupational structure. There are two key changes in rate of returns that account for the low self-employment rate in the economy with perfect capital markets (11% relative to 24% in the baseline economy) . First, the wage rate is about 50% higher in the economy with perfect capital markets. Second, the marginal product of capital of self-employed is 0.09 relative to the 0.11 value in the baseline economy. The return to the self-employment ability-composite varies across economies, but the change is small. Altogether, the higher wage rate and the decrease in the return to capital account for the decrease in self-employment rates and for the increase in the fraction of workers in the population.

Table 6 also shows an interesting pattern on the returns to managerial input. The mean return is about the same in both economies, with a value of roughly .30. Now, while the returns to the managerial input does not vary across employers in the economy with perfect capital markets, they exhibit a coefficient of variation of roughly 65% in the baseline economy. Hence, capital market imperfections lead to substantial variation in the equilibrium returns to managerial ability attain by employers. Moreover, returns tend to be low when managerial ability is high and high when managerial ability is low, with a correlation coefficient between r_m and z_m of -0.5. The reason is that talented managers tend to face tight borrowing constraints so that a high value of μ depresses the rate of return on managerial ability (recall that r_m decreases with μ). The fact that employers face tight borrowing constraints in the baseline economy is reflected in the high average value of the marginal product of capital among employers (0.19 in comparison to 0.09 in the economy with perfect capital markets).

5.5.3 Capital market imperfections and misallocation of resources.

Capital market imperfections have sizable effects on aggregate output: Output per person is about 50% higher in the economy with perfect enforcement than in the baseline economy. Enforcement problems limit the capital that entrepreneurs can use and lead to a low capital to output ratio (2.5 in the baseline economy compared to 3.7 with perfect enforcement). Enforcement problems also limit the efficient allocation of productive resources. Total factor productivity is about 16% higher when capital markets are perfect.

The low TFP in the baseline economy is due to the misallocation of productive

resources caused by capital market imperfections. The misallocation of resources is due to distortions in both the intensive margin and extensive margin of entrepreneurship. The misallocation along the intensive margin is due to distortions in the allocation of capital and labor inputs across active entrepreneurs. The misallocation along he extensive margin is due to distortions in the selection (number and productivity) of entrepreneurs. To evaluate the importance of the intensive, we compute the output gain of reallocating capital in order to equate the marginal product of capital across entrepreneurs in the baseline economy. We find that the output gain due to reallocation of capital is 11%. Thus, the intensive margin accounts for about 70% of the gains in TFP.

We now express the change in aggregate output of removing capital market imperfections in terms of the changes in the value added by the different productive inputs. By adding output over all production units we express aggregate output as follows:

$$
\begin{aligned}
Y &= \sum_j \left(MPT_e^j + MPT_{se}^j + (r + \mu^j + \delta)k^j + MPL^j n_j^d \right) \\
&= \frac{\sum_j MPT_e^j}{N_e} N_e + \frac{\sum_j MPT_{se}^j}{N_{se}} N_{se} + \frac{\sum_j (r + \mu^j + \delta)k^j}{K} K + MPL\, N_w \\
&= \overline{MPT_e}\, N_e + \overline{MPT_{se}}\, N_{se} + \overline{MPK}\, K + MPL\, N_w
\end{aligned}
$$

The change in output between the economy with perfect capital markets and the baseline economy can be decomposed in terms of the value added by the different inputs:

$$
\frac{\triangle Y}{\triangle Y} = \frac{\triangle(\overline{MPT_e}\, N_e)}{\triangle Y} + \frac{\triangle(\overline{MPT_{se}}\, N_{se})}{\triangle Y} + \frac{\triangle(\overline{MPK}\, K)}{\triangle Y} + \frac{\triangle(MPL\, N_w)}{\triangle Y}
$$

$$
1 = 0.23 - 0.10 + .54 + .33
$$

We find that the contribution of the managerial input of employers accounts by 23% of the output change, which is remarkable given the low fraction of employers in the labor force. The decrease in self-employment rate accounts for a decrease in the contribution of the composite ability input supplied by self-employed individuals. The value added by capital and by workers account for 33% and 54% of the increase in output.

5.5.4 Capital markets and the distribution of income.

We find that capital market imperfections have a small effect on the Gini index of income but that they have important effects on the sources of income inequality. The Gini index of income in the baseline economy is slightly higher than the one of the economy with perfect enforcement (.53 versus .52).

Financial frictions have important effects on the sources of income inequality. To illustrate this point, we analyze how financial frictions affect the distribution of capital versus non-capital income. Capital income is computed as $ra + \mu k$. Non-capital income is computed as the sum of labor income, managerial rents, and self-employment rents. Abusing terminology from now on we refer to non-capital income as labor income. Table 8 compares the Gini indexes of capital and labor income across economies. We find that capital income is much more unevenly distributed than labor income both in the baseline economy and in the perfect capital market economies. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor and capital income. Labor income is more evenly distributed in the baseline economy than in the economy with perfect enforcement, with a Gini index of .52 in the former economy and of 56 in the latter economy). On the other hand, the Gini index of capital income is about 10 percentage points higher in the baseline economy. The opposite effects of capital market imperfections on the distributions of capital income and labor income offset each other and account for the small change in the Gini index of income.

The fact that the distribution of factor income varies so much across economies is symptomatic of the resource misallocation prevalent under imperfect capital markets. The low concentration of the distribution of labor income in the baseline economy is due to the fact that borrowing constraints distorts rate of returns to managerial ability (recall that $\mu > 0$ reduces r_m). Moreover, in the baseline economy returns to managerial ability r_m and managerial ability z_m are strongly negative correlated, with a correlation coefficient of -0.5. Thus, skillful managers tend to receive low returns to their ability. On the other hand, the correlation coefficient between these two variables is zero in the economy with perfect capital markets, as there is no heterogeneity in rate of returns to ability.

Capital income is highly unequal in the baseline economy because there is substantial heterogeneity in the returns to capital. The interest rate on deposits (3%) is substantially smaller than the average marginal product on capital obtained by employers (13.2% net of depreciation). Moreover, the marginal product of capital across employers varies importantly and its distribution features a coefficient of variation above .60. Again, this fact is symptomatic of resource being inefficiently allocated.

The presence of borrowing constraints imply that the returns to managerial ability are positively correlated with capital income. Hence, the correlation between capital and labor income is equal to .80 in the baseline economy, which is much larger than the .50 value in the economy with perfect capital markets. In the latter, the positive correlation between capital and labor income is due to the fact that highly able people tend to hold more capital than low ability people but not to rate of return differentials.

5.5.5 Capital markets and the persistence of income.

To evaluate the effect of imperfect capital markets on the persistence of income, we compare the estimates of β from running the regression (12) in the baseline economy and in an economy with perfect capital markets ($\phi = 1$). We find that removing financial frictions in the baseline economy reduces the estimated value of β from 0.81 to 0.74. Income is more persistent in the baseline economy because assets are positively correlated with rate of returns and because assets matter for occupational choices. On the other hand, when there is perfect enforcement assets do not affect rates of returns and occupational choices and the persistence of income is only driven by the persistence of shocks and asset holdings.

5.5.6 Capital markets and the distribution of consumption.

Financial frictions have an heterogeneous impact across households. To assess the distributive impact of financial frictions, Table 9 compares consumption inequality in the baseline model economies with that in an economy with perfect enforcement of credit contracts $\phi = 1$. We find that financial frictions have important effects on the distribution of consumption. We find that the Gini coefficient of consumption is 2 percentage points lower in the economy with perfect enforcement of credit contracts ($\phi = 1$). It is interesting that financial frictions have opposite effects on the inequality at the top and the bottom of the consumption distribution. The ratio of consumption between the 10th percentile and the 50th percentile of the consumption distribution is equal to 0.29 in the baseline model economy. This ratio increases to 0.33 in the economy with $\phi = 1$. On the other hand, the consumption ratio between households at the 90th and 50th percentile increases from 3.3 to 3.44. Hence, relative to the perfect credit economy, the baseline model economy has more inequality at the bottom of the consumption distribution but less inequality at the top. The first effect is more important than the latter effect so that overall consumption inequality, as measured by the Gini index, is higher in the baseline economy than in the $\phi = 1$ economy.

5.6 The Political Economy of Capital Market Imperfections

Having shown that financial frictions have an heterogeneous impact on households, it is interesting to analyze the political economy of financial frictions. It is clear that financial frictions are likely to decrease welfare for most households in the economy. Nonetheless, it is still possible that some households may gain from financial frictions. Essentially,

financial frictions depress the labor demand of entrepreneurs by limiting the amount of capital that entrepreneurs can use and by distorting the selection of entrepreneurs. By decreasing the equilibrium wage rate, wealthy entrepreneurs who do not need much external financing to operate their businesses at an efficient scale may benefit from financial frictions.

We compute the distribution of welfare gains of eliminating financial frictions in the baseline economy ($\rho = 0.10$). We assume that the calibrated model economy is in steady state and that suddenly and unexpectedly there is a once and for all institutional reform that increases ϕ to 1. Note that after the reform occupational choice decisions do not depend on asset holdings. As a result, some wealthy but untalented managers will cease to operate their businesses and other poor but talented managers will start operating businesses. The distribution of entrepreneurs will change on impact and, given our small open economy assumption, international capital flows will ensure that all entrepreneurs will operate at an efficient scale. The marginal product of capital will be equated across entrepreneurs and will be equal to the international interest rate plus the depreciation rate of capital. Competition for workers will drive the wage rate to its new long run value, which increases on impact by about 40%. While the distribution of wealth, consumption, and income may change for some periods after the reform, all macroeconomic aggregates (capital, GDP, wage rate) will be constant after the initial period of the reform. Since there are no transitional dynamics in the macroeconomic aggregates, we can then compute the distribution of welfare gains for all individuals alive at the moment of the reform as follows:

1. Simulate the distribution of households across states $s = (age, assets, z_m, z_w)$ from the initial steady state prior to the reform.

2. For each household in state s, compute the permanent consumption compensation in the original steady state that will let the household attain the same utility as in the perfect credit economy. Denoting by $V^{baseline}(s)$ the discounted lifetime utility of a household in the baseline economy, and $V^{\phi=1}(s)$ the value function in the perfect enforcement economy, the consumption compensation $\lambda(s)$ is computed as follows:

$$\lambda(s) = \left(\frac{V^{\phi=1}(s)}{V^{baseline}(s)} \right)^{\frac{1}{1-\sigma_c}} - 1,$$

where σ_c denotes the curvature of the period utility function in consumption ($\sigma_c = 1.5$). Households with $\lambda(s) > 0$ gain from the elimination of enforcement problems. Households with $\lambda(s) < 0$ see their welfare decrease with the reform of financial market institutions.

We find that the average welfare gain among households alive at the period of the institutional reform is 16.5%. The standard deviation of the distribution of welfare gains is 13.5%. Figure 11 shows the distribution of welfare gains across the population and documents that there is substantial heterogeneity. While the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Who are the households that lose with the reform?

Figure 12, 13, 14, 15, show the age, asset, and managerial-skill distributions among those who oppose and support the reform. We find that households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers.

Employers are a positive selection from the population distribution of managerial skills. Then , the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers support the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of higher managerial ability than those who oppose it. The reason is that high ability employers are more likely to be borrowing constrained than low ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower skill employers and force them to change their occupation status: About 46% of the entrepreneurs that oppose to the reform and would have been employers had the reform not taken place, do not hire any labor after the reform (most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

Summing up, while most households benefits from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Obviously, entrepreneurs as a group will benefit even more by forming a cartel in order to restrict labor demand and depress the wage rate. This achieves the goal of depressing the wage rate but without distorting the capital markets. However, this cartel agreement is not incentive compatible as each entrepreneur will have incentives to violate the group agreement and hire labor. Impor-

tantly, limited enforcement is an incentive feasible mechanism that leads to a depress wage rate. Our theory thus suggests that employers may have a vested interested in maintaining a status quo with low enforcement.

Table 1: Calibrated Parameters

	$\rho_{w,m}$		
	-0.80	0.1	0.8
ρ_w	0.98	0.98	0.98
ρ_m	0.8	0.78	0.85
$\sigma^2_{\alpha,w}$	0.37	0.38	0.41
$\sigma^2_{\alpha,m}$	0.64	1.59	0.77
σ^2_w	0.10	0.03	0.02
σ^2_m	2.16	0.99	0.54
c_f	0.11	0.10	0.06
ϕ	0.24	0.23	0.22
β	1.10	0.995	0.985

Table 2: Calibration Results-Model Aggregates

	Data	$\rho_{w,m} = -0.8$	$\rho_{w,m} = 0.1$	$\rho_{w,m} = 0.8$
K/Y	2.4	2.2	2.3	2.4
Credit/GDP	43%	42%	42%	45%
Var Log(Earn)-Entrepreneurs	1.1	1.4	1.2	1.0

Table 3: Calibration Results-Occupational Structure

Fraction	Data	$\rho_{w,m} = -0.8$	$\rho_{w,m} = 0.1$	$\rho_{w,m} = 0.8$
Workers	68%	68%	67%	68%
Self-Employed	24%	23%	24%	24%
Employers	8%	9%	9%	8%
Emp to Emp	70%	63%	68%	72%

Table 4: Performance of the Model-Transitions

Transitions	Data	$\rho_{w,m} = -0.8$	$\rho_{w,m} = 0.1$	$\rho_{w,m} = 0.8$
W to W	94%	90%	90%	90%
SE to W	5%	9%	9%	9%
E to W	1%	1%	1%	1%
W to SE	14%	29%	28%	28%
SE to SE	78%	60%	64%	67%
E to SE	8%	11%	8%	6%
W to E	8%	8%	6%	5%
SE to E	22%	29%	26%	23%
E to E	70%	63%	68%	72%

In the table above we use W for Workers, SE for Self-Employed and E for Employers.

Table 5: Changes in Output-From benchmark to $\phi = 1$

	ρ		
	-0.8	0.1	0.8
Output Change (%)- Baseline	55	48	37
Output Change (%)- No Labor Heterogeneity	47	48	41
Output Change (%)- No Self-Employed	63	54	44

Table 6: Statistics on returns for different occupations and TFP gains

	$\rho_{w,m} = -0.8$	$\rho_{w,m} = 0.1$	$\rho_{w,m} = 0.8$
TFP gains (%)	18	15	11
Employers			
Std. Dev. MPK_e	0.14	0.10	0.07
Std. Dev. c_e	0.31	0.19	0.17
$corr(c_e, z_m)$	-0.63	-0.55	-0.49
Self-Employed			
Std. Dev. MPK_{se}	0.09	0.06	0.04
Std. Dev. c_{se}	0.06	0.07	0.07
$corr(c_{se}, z_m)$	-0.38	-0.30	-0.34

For $\phi = 1$ the standard deviation is 0 for all variables

c_e is the return to managerial input for employers

c_{se} is the return to the composite input for self-employed

MPK_e, MPK_{se} are the marginal product of capital for employers and self-employed respectively

Table 7: Implications of ρ for earnings and consumption

	Data	$\rho_{w,m} = -0.8$	$\rho_{w,m} = 0.1$	$\rho_{w,m} = 0.8$
Ratio Median Earnings Worker to Entrepreneur	1.0	1.3	1.0	0.7
$corr(earnings_t, earnings_{t-1})$	N.A.	0.73	0.81	0.84
$corr(earnings_t, consumption_t)$	0.71	0.24	0.79	0.85

Table 8: Gini Inex-Labor and Capital Income

	Gini Index	
	$\phi = 0.23$	$\phi = 1$
Labor Income	0.52	0.56
Capital Income	0.67	0.59

Table 9: Consumption Inequality and Financial Frictions

	$\phi = 0.23$	$\phi = 1$
Gini	0.50	0.48
p90/p10	11.5	10.33
p90/p50	3.29	3.44
p10/p50	0.29	0.33

Figure 1: Variance of Log Earnings in Brazil
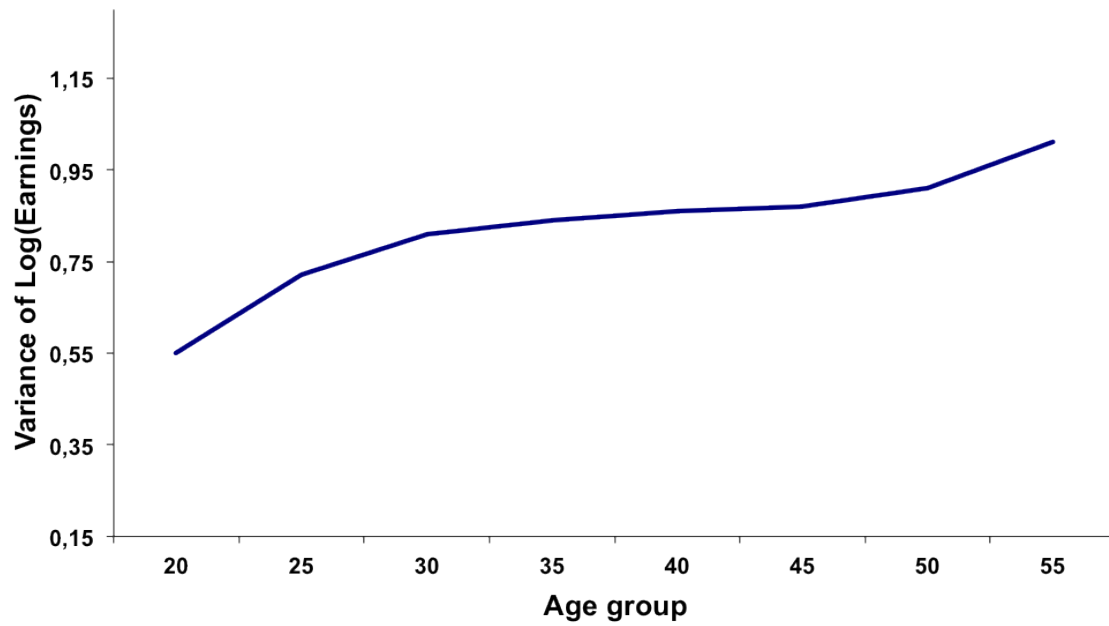
Source: Author's Elaboration based on PME 2003-2010
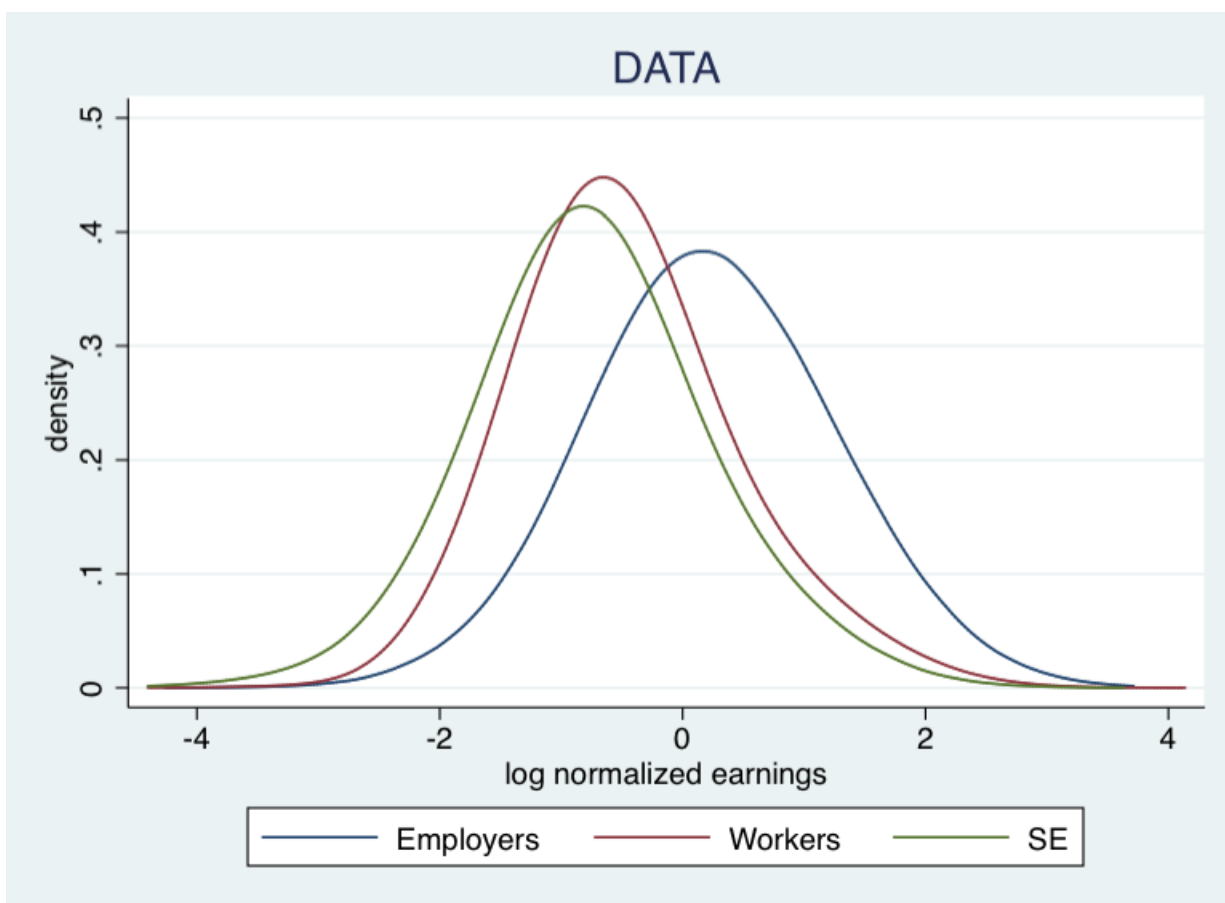
Figure 2: Occupational Structure across Countries

■ EMPLOYERS	■ SELF-EMPLOYED	■ WORKERS
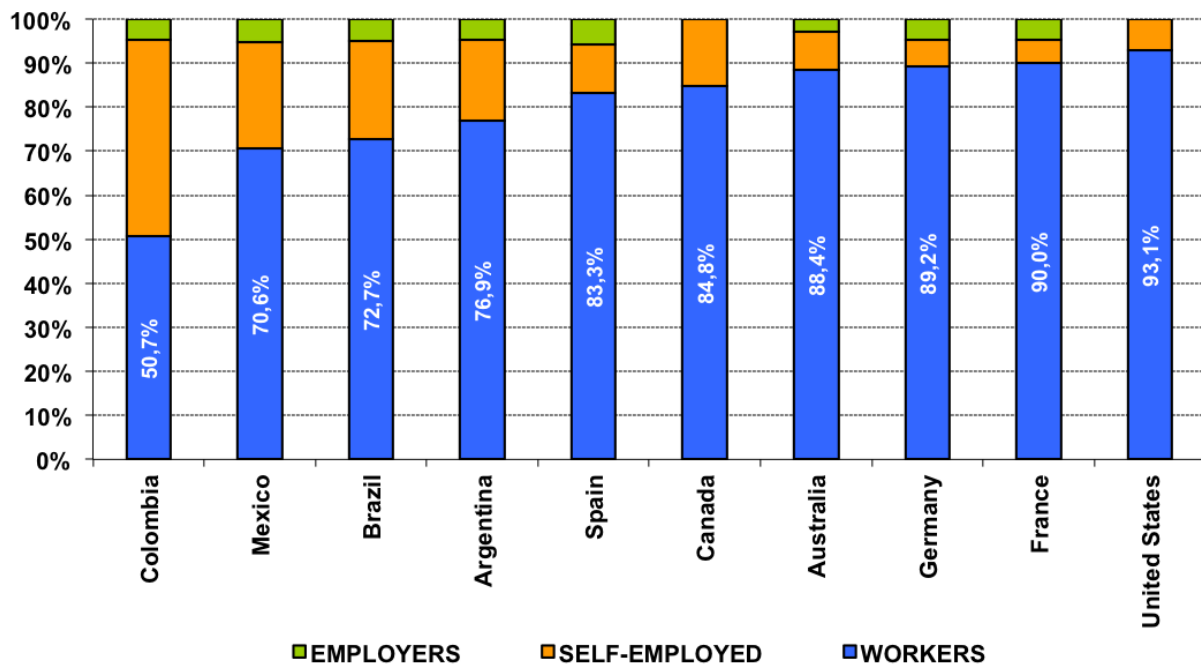
Source: Author's Elaboration based on ILO 2008

Figure 3: Distribution of Earnings in Brazil by Occupation-I
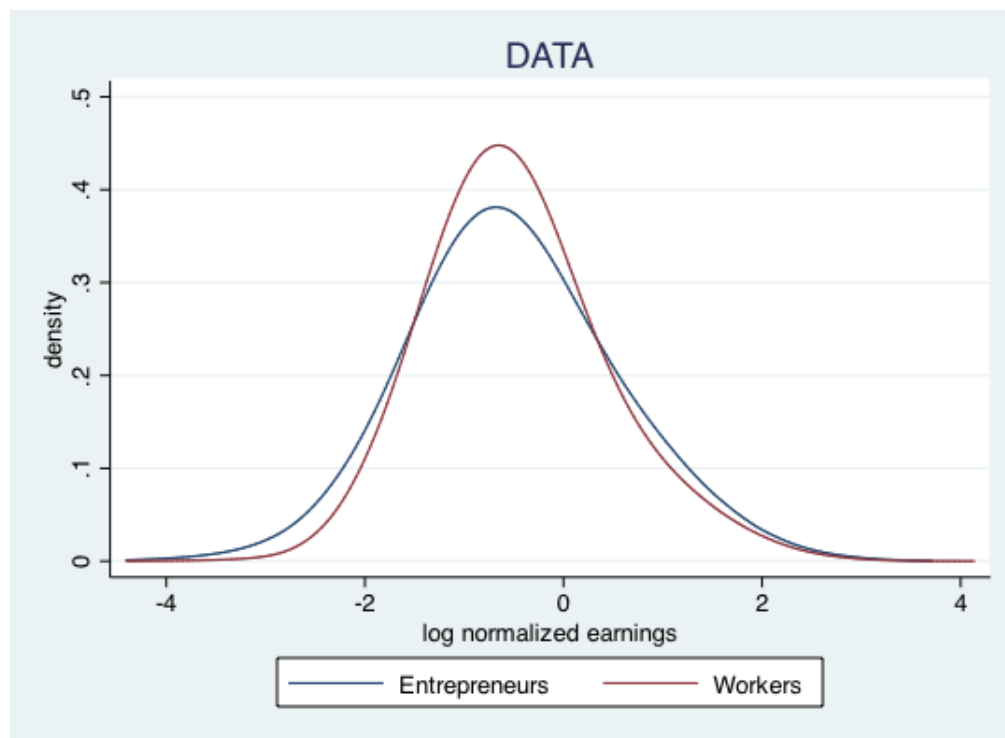
Source: Author's Elaboration based on PME 2003-2010

Figure 4: Distribution of Earnings in Brazil by Occupation-II

Source: Author's Elaboration based on PME 2003-2010

Figure 5: Variance of Log(Earnings)-Model vs Data
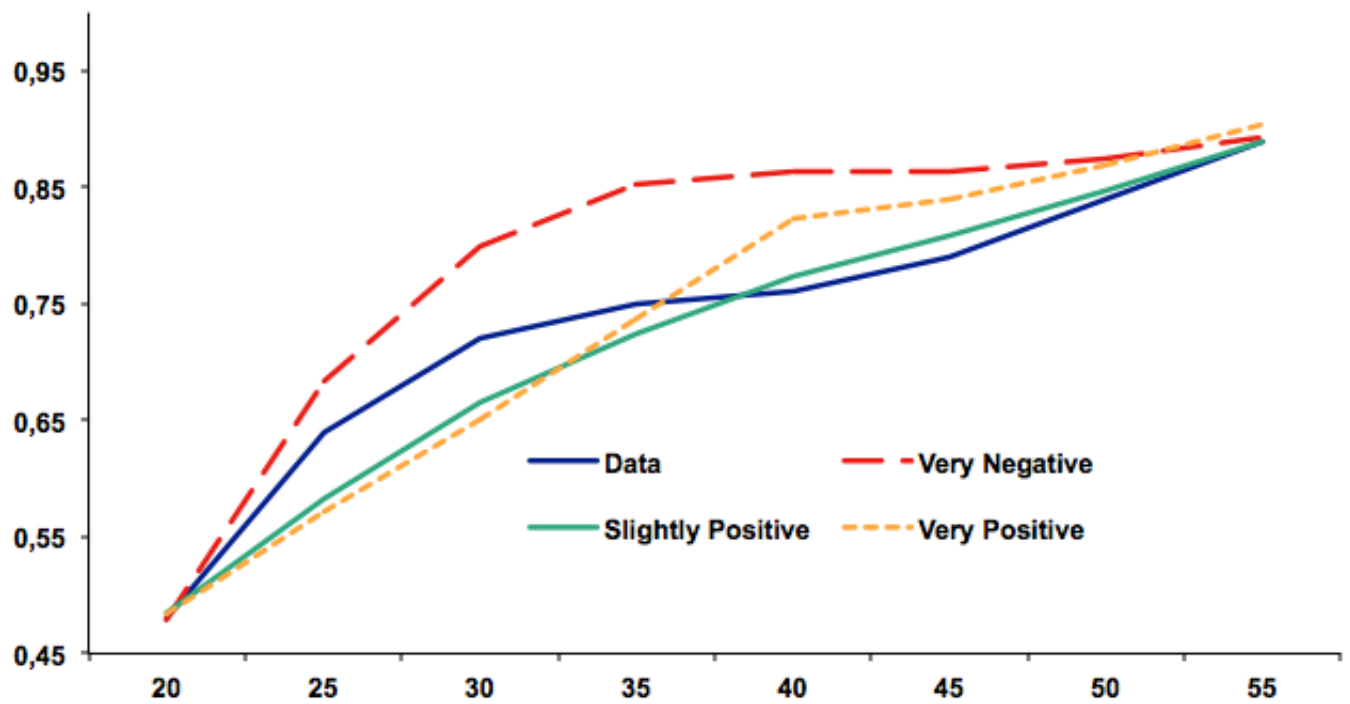
Source: Author's Elaboration based on PME 2003-2010

Figure 6: Correlation between $\frac{z_m}{z_w}$ and z_w for different $\rho_{w,m}$
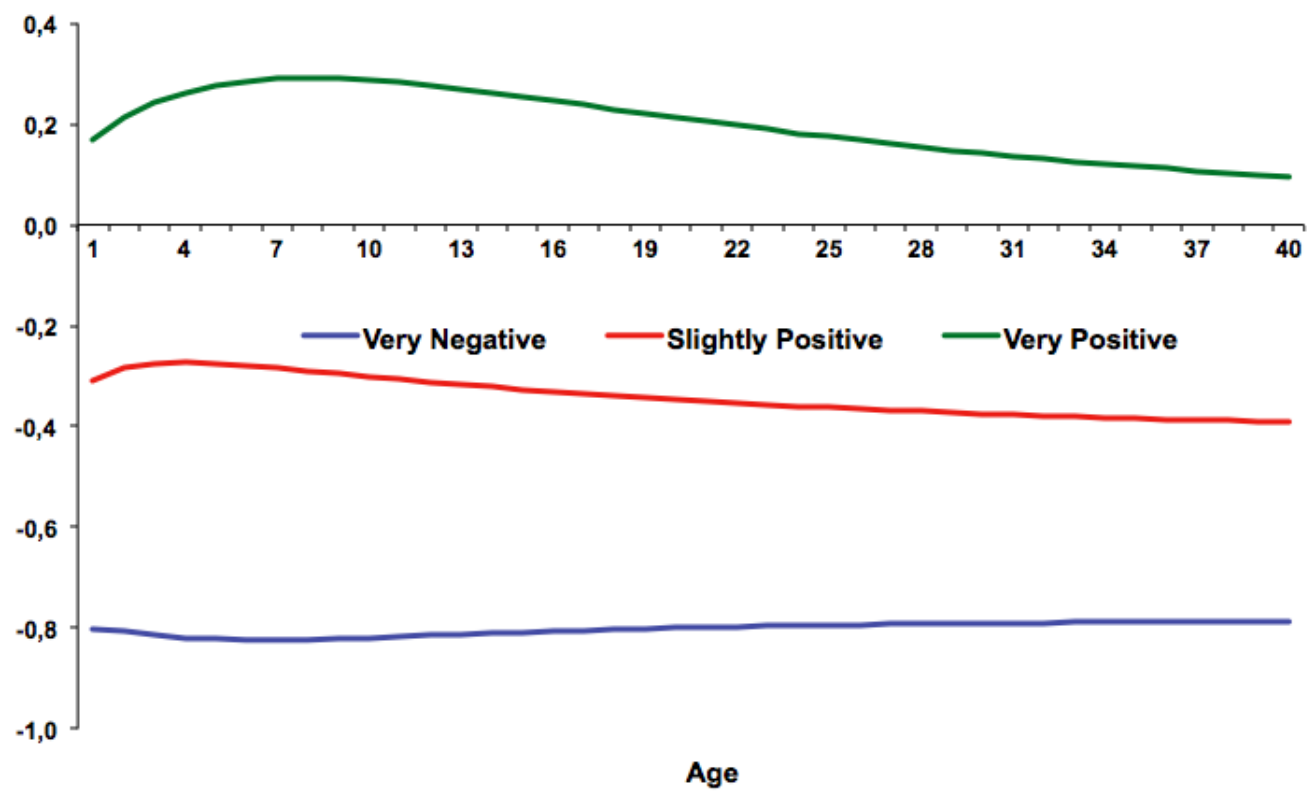
Figure 7: Occupational Map-$\phi = 1$
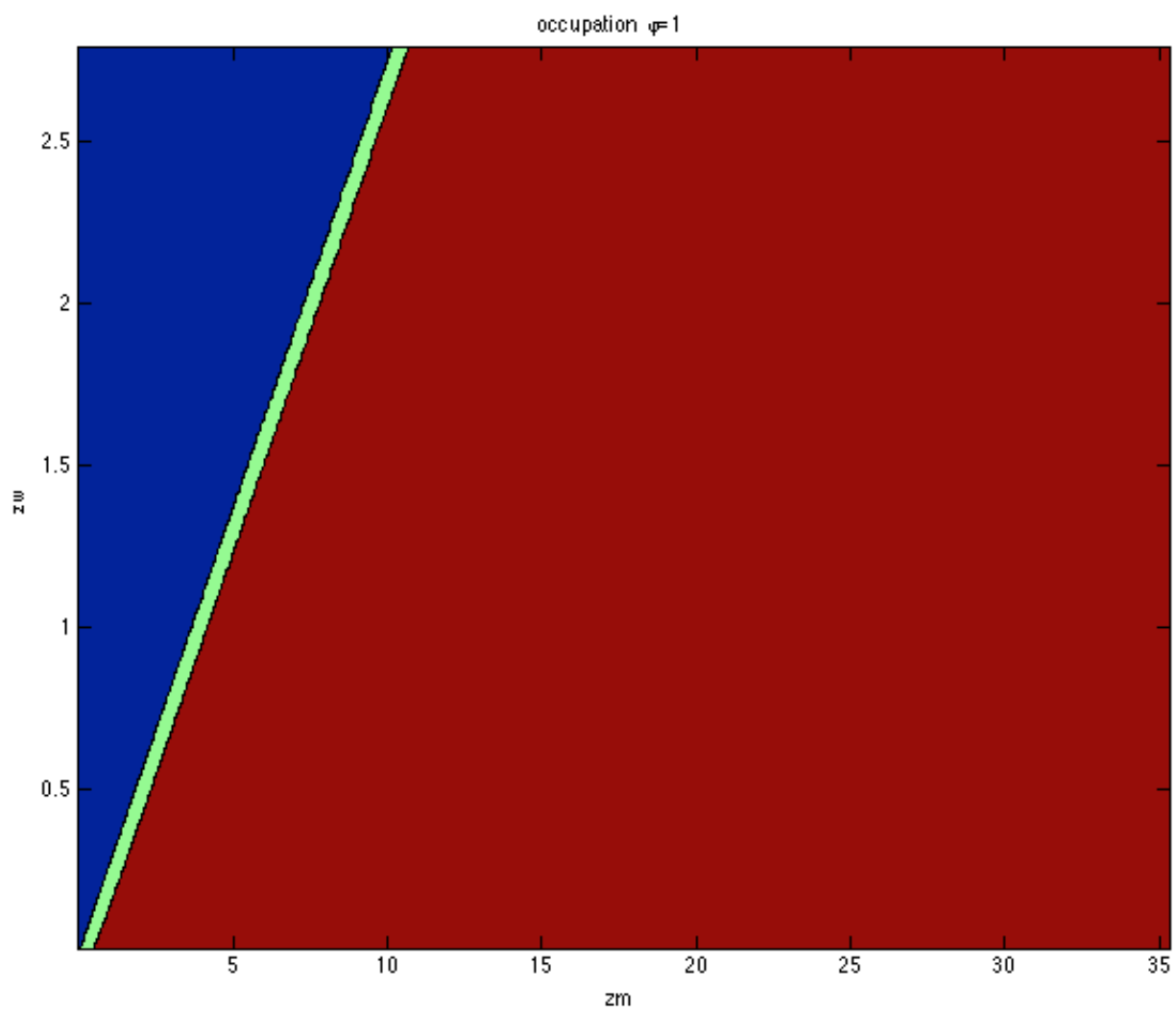
occupation $\varphi = 1$

Figure 8: Occupational Maps Benchmark Economy

occupation φ=0.23-Median Asset

occupation φ=0.23-Mean Asset
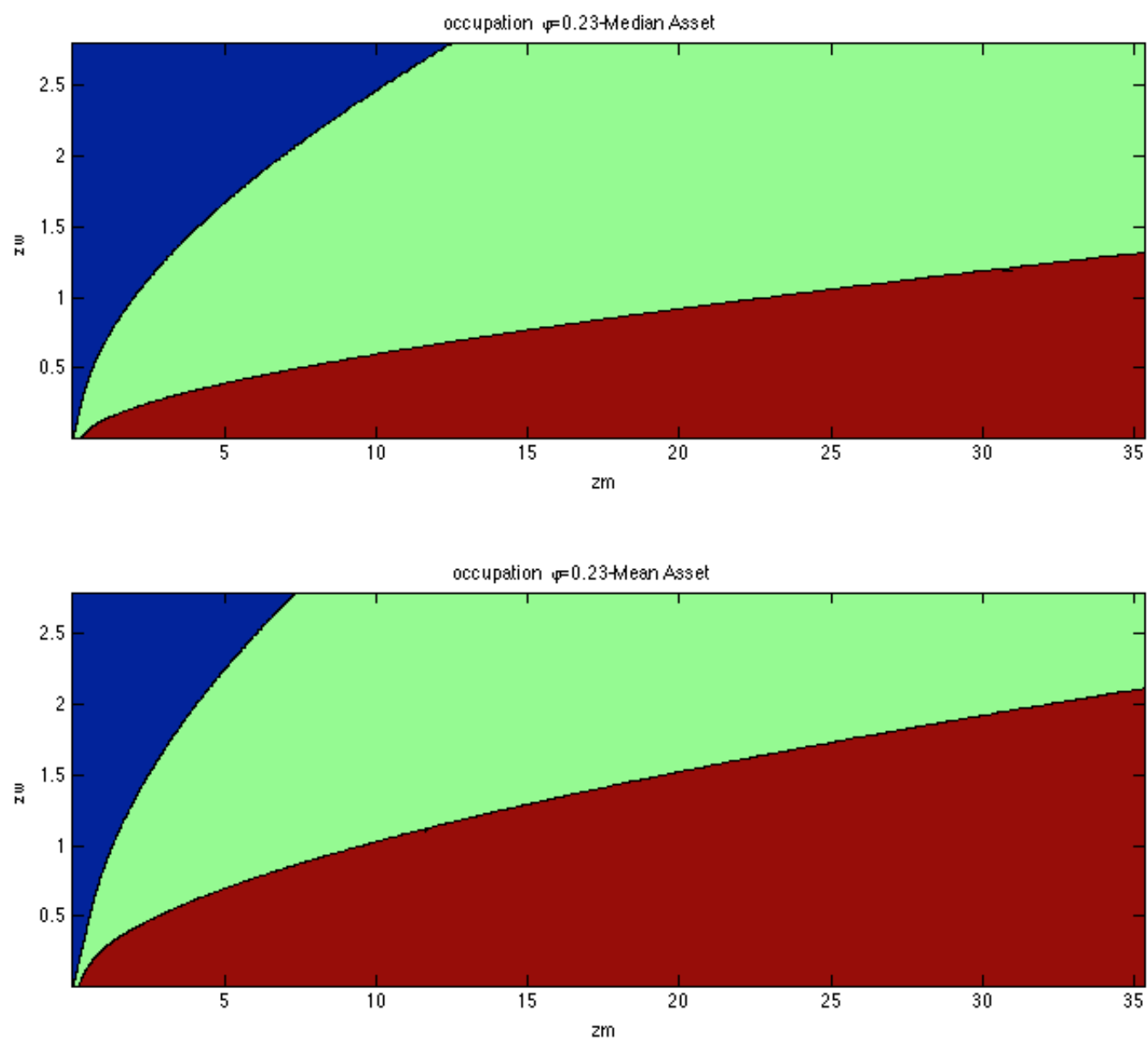
Figure 9: Distribution of Earnings-Data vs Model I
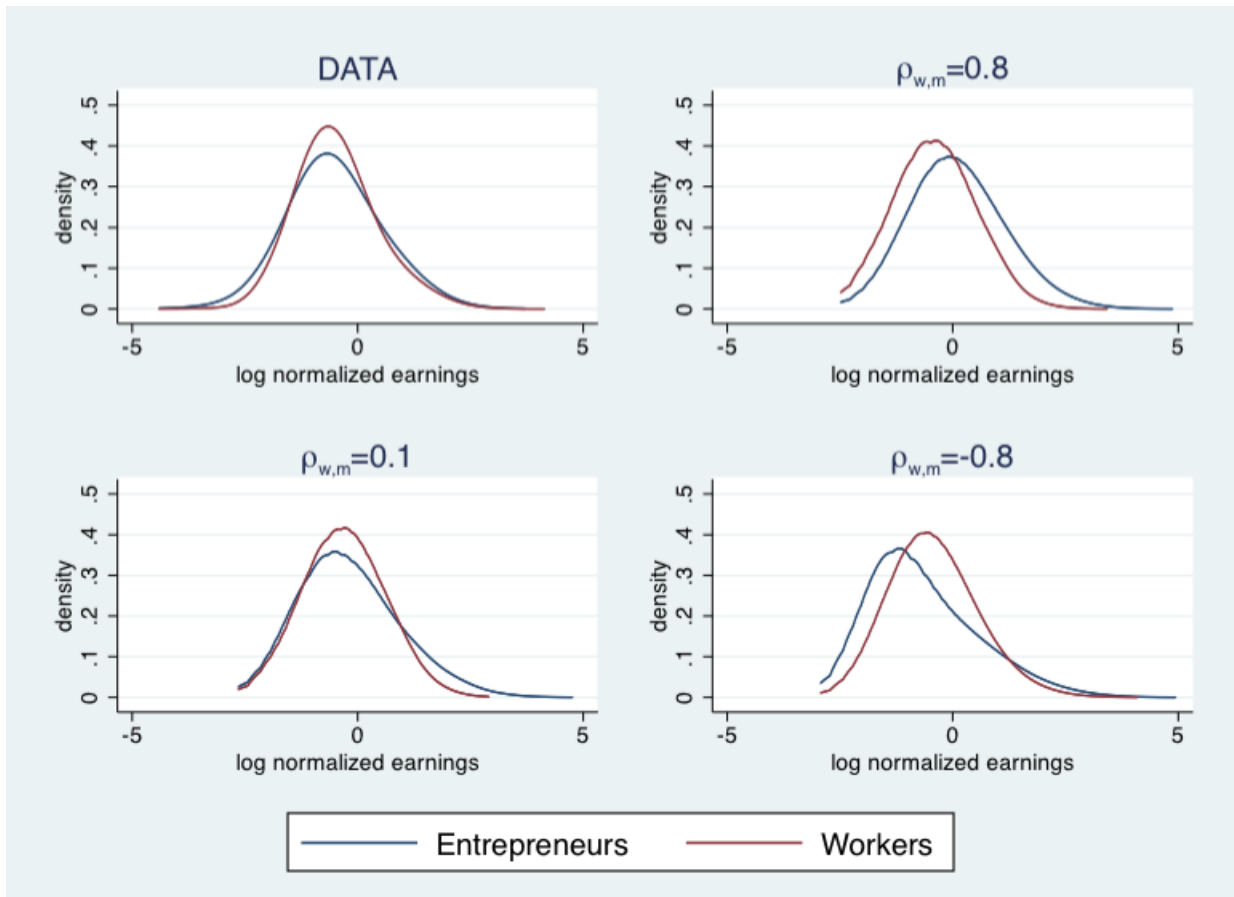
Figure 10: Distribution of Earnings-Data vs Model II
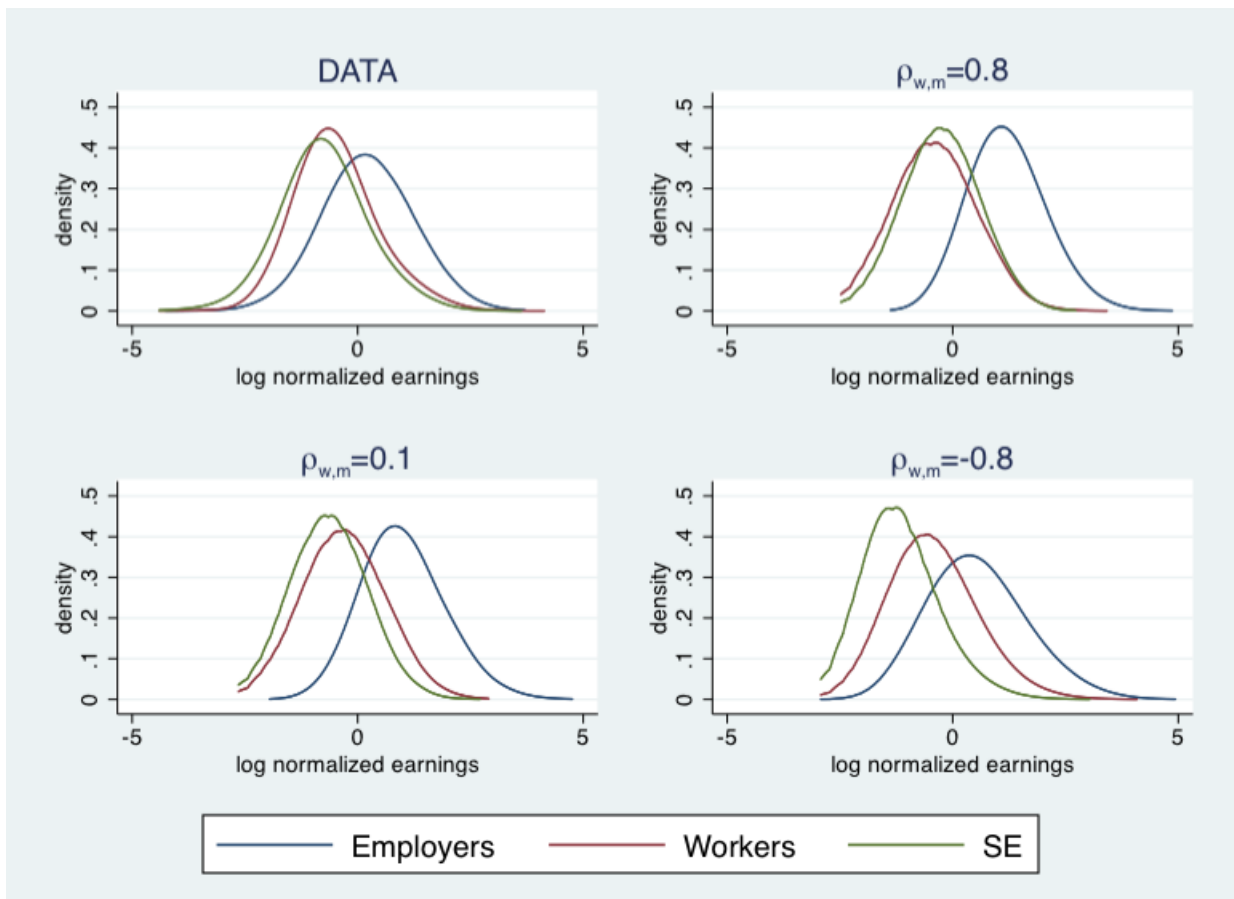
Figure 11: Welfare Gains from Financial Reform
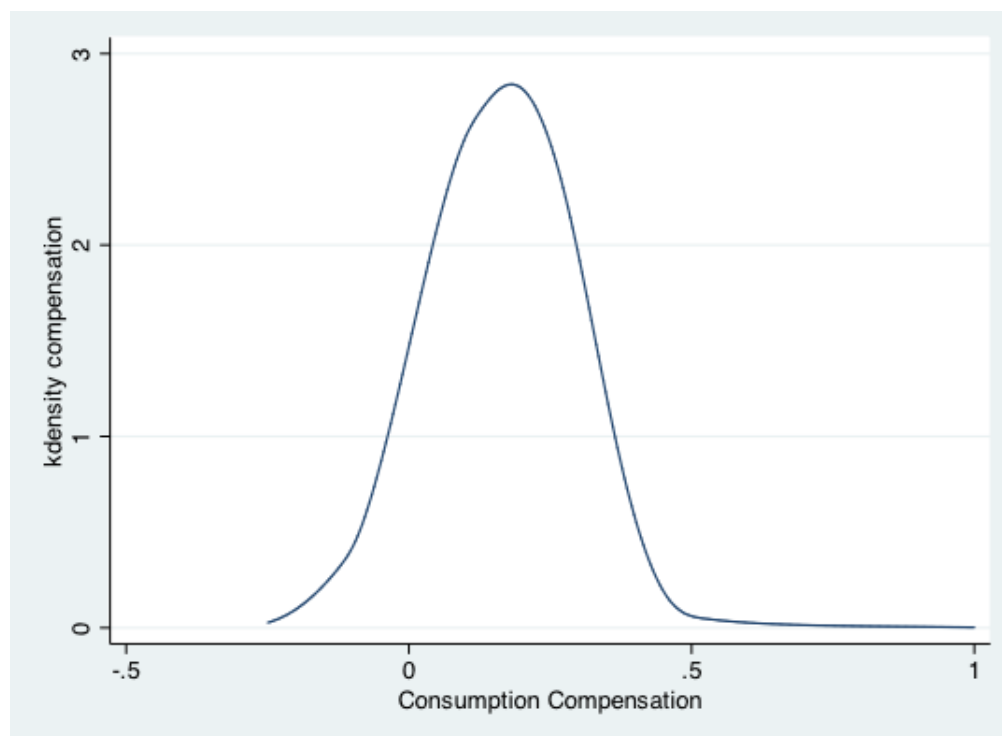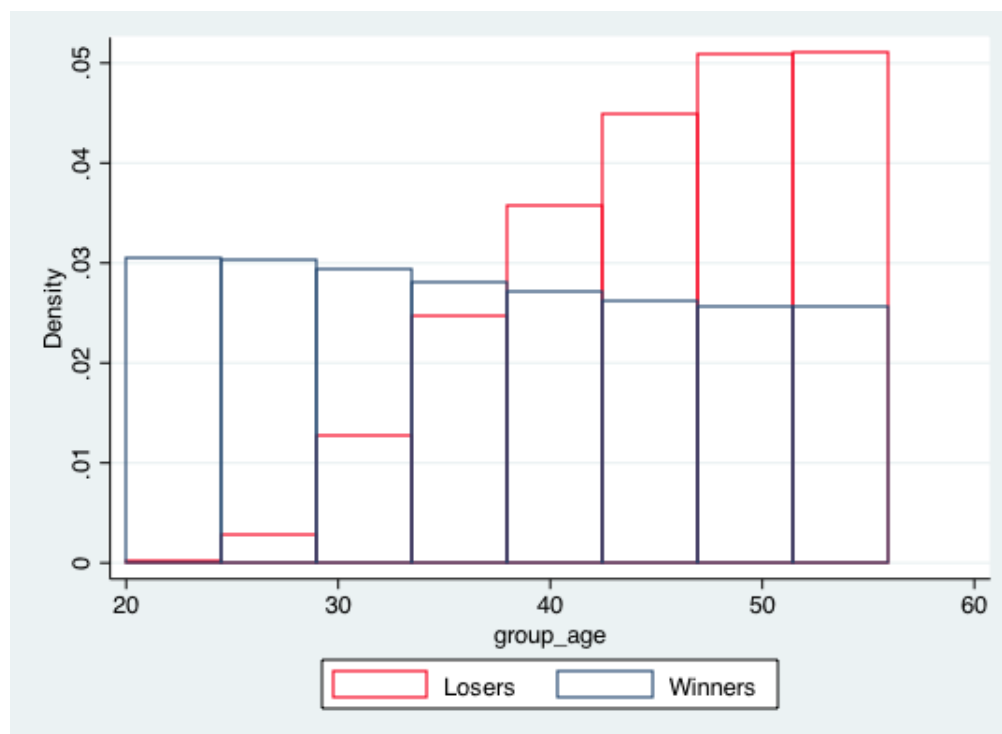

Figure 12: Age Distribution of Winners and Losers from the Reform
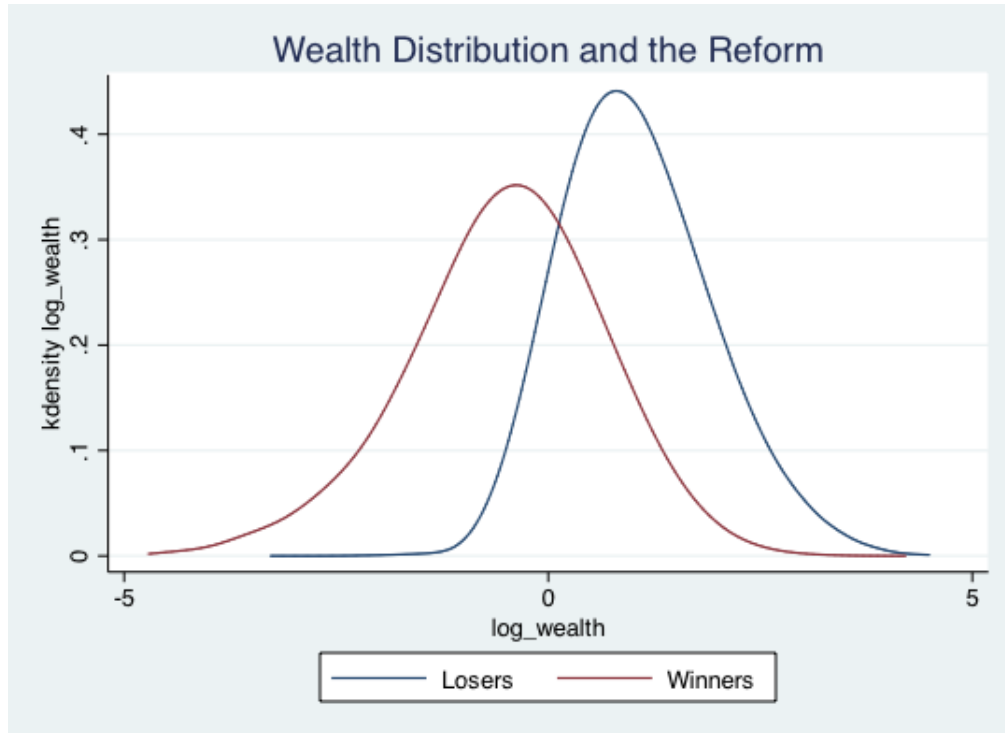
Figure 13: Wealth Distribution and the Reform

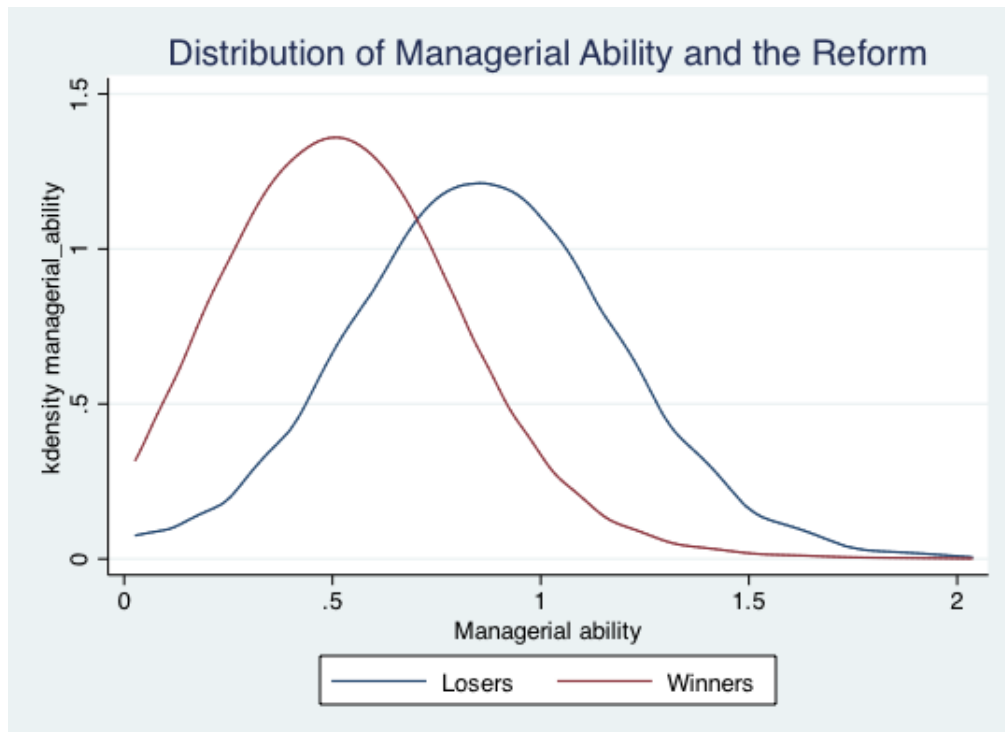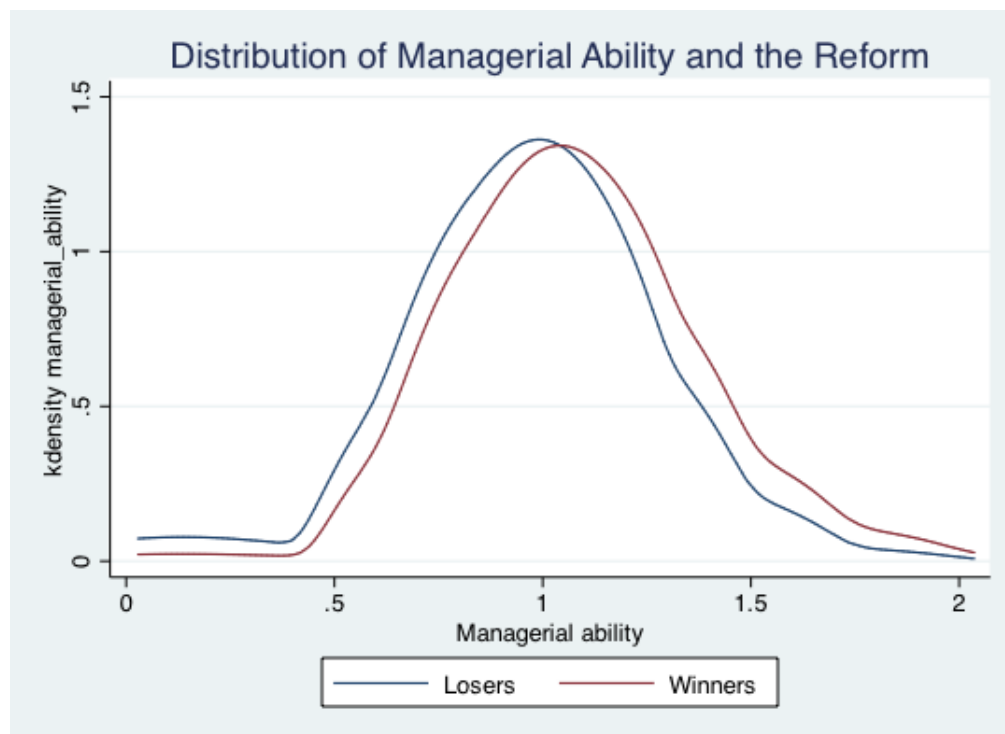Figure 14: Distribution of Managerial Ability and the Reform

Figure 15: Distribution of Managerial Ability and the Reform among Employers

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6   Appendix A

Pesquisa Mensal de Emprego   From this survey we have data for the years 2003 until 2010. The PME is a monthly household survey covering the metropolitan areas of six Brazilian regions: Rio de Janeiro, São Paulo, Porto Alegre, Belo Horizonte, Recife and Salvador. Each individual is followed for three months, left out of the sample the next eight months and interviewed again the following 4 months. We take the first and fifth interview of each individual for the years 2003 until 2010. In this way we keep two observation of each individual, which corresponds to the same month of consecutive years. We keep only household where the head is male and he is older than twenty and younger than sixty years old. The earnings of the household are the sum of the earnings of all members. In order to make the earnings comparable we deflect them with the corresponding month Consumer Price Index (CPI) and we divide them by the number of adults equivalents in the house. In addition, we only keep individuals who are employed in both periods of the survey. In the final data set we have 131,056 households with data for earnings. Individual households age is defined as the age of the household head. We use 5 years bin, centered at the age of interested, in order to

compute statistics by age. To do the transition matrix of employment we consider the individual data. The variable of earnings that we consider is a constructed variable, which includes the earnings effectively perceived by the individual in the month from all the works done.

Pesquisa de Ornamentos Familiares   The POF is a Consumption-Income survey done every five or six years. We use data from the last wave, 2008-2009. We consider households where the main earner is a male, older than twenty and younger than sixty years old. We end up with 44,930 observations. Our income variable includes: income from work, Transfers, Income from rents, other and Asset Variation. Our measure of consumption includes: food, housing, clothing, transport, health and personal care, education, recreation and culture, smoking, personal services and other current expenses. We normalize household income and consumption by dividing them by the number of adults equivalents in the house.

7   Appendix B

Proof of Proposition 1. Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if

$$\max_{m,n,n_d,t_e} \{m^\gamma k^\nu n^\theta - wn^d - r(k-a) + a - \delta k - c_f I_{n_d>0}\} \geq$$
$$\max_{m,n,n_d,t_m} \{m^\gamma k^\nu n^\theta - wn^d + (1-\delta)k - c_f I_{n_d>0}\}$$

which is equivalent to

$$(1+r)a \geq \phi\left[\frac{1-\phi+r+\delta+\delta\phi}{\phi}k - \phi\max_{m,n,n_d,t_e}\{m^\gamma k^\nu n^\theta - wn^d - c_f I_{n_d>0}\}\right]$$

Following arguments in Buera et al. (2011), the set of enforceable levels of capital rentals is characterized by a simple set of rental limits. Two cases are relevant. If the max in the RHS is attained with $n_d = 0$, the set of enforceable levels of capital is $[0, \bar{k}(a, z_m, z_w; \phi)]$ where $\bar{k}(a, z_m, z_w; \phi)$, where $\bar{k}(a, z_m, z_w; \phi)$ is given by unique root of the equation

$$(1+r)a = \phi\left[\frac{1-\phi+r+\delta+\delta\phi}{\phi}k - \phi\max_{m,n,n_d,t_e}\{m^\gamma k^\nu n^\theta - wn^d - c_f I_{n_d>0}\}\right]$$

If the max in the RHS is attained with $n_d > 0$, then there are two positive roots of the above equation and the set of enforceable levels of capital rental is $[\underline{k}(a, z_m, z_w; \phi), \bar{k}(a, z_m, z_w; \phi)]$, where $\underline{k}(a, z_m, z_w; \phi)$ represents the smallest root. Nonetheless, the optimal production plan of the entrepreneur coincides with the solution to the individual problem subject to

the simpler limit $k \leq \overline{k}(a, z_m, z_w; \phi)$. It can also be shown that $\overline{k}(a, z_m, z_w; \phi)$ is strictly increasing in a, z_m, ϕ and weakly (strictly) increasing in z_w (if $n_d = 0$).

Proof of Proposition 2. The optimal production plan of self-employed individuals solve

$$\pi^{se} = (z_m t_m)^\gamma k^\nu (z_w(1 - t_m))^\theta - (r + \delta)k + (1 + r)a + \mu_k(\overline{k} - k)$$

where $\gamma + \nu + \theta = 1$. The FOC imply:

$$\{t_m\} \quad z_m^\gamma k^\nu z_w^\theta [\gamma t_m^{\gamma-1}(1 - t_m)^\theta - t_m^\gamma \theta (1 - t_m)^{\theta-1}] = 0 \Rightarrow t_m^* = \frac{\gamma}{\gamma + \theta}$$

$$\{k\} \quad (z_m t_m)^\gamma \nu k^{\nu-1}(z_w(1 - t_m))^\theta - r - \delta - \mu_k = 0 \Rightarrow k = \left[\frac{(z_m t_m)^\gamma \nu (z_w(1 - t_m))^\theta}{r + \delta + \mu_k}\right]^{\frac{1}{1-\nu}}.$$

Note that the first FOC equates the marginal product of entrepreneurial time at managing and worker. Combining the FOC we obtain that the marginal product of entrepreneurial time satisfies:

$$
\begin{aligned}
MPT_{se} \quad &= \quad \gamma z_m^\gamma (t_m^*)^{\gamma-1} k^{*\nu} (z_w(1 - t_m^*))^\theta \\
&= \quad r_{mw} \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}, \\
\text{where} \quad r_{mw} &= \gamma \nu^{\frac{\nu}{1-\nu}} \left(\frac{\gamma\theta}{(\gamma+\theta)^2}\right)^{\frac{\theta}{1-\nu}} \left(\frac{1}{r + \delta + \mu}\right)^{\frac{\nu}{1-\nu}}.
\end{aligned}
$$

Income of self-employed individuals can then be written as

$$
\begin{aligned}
y_{se} \quad &= \quad MPt_m\, t_m + MPt_w\, t_w + MPK\, k + ra - k(r + \delta), \\
y_{se} \quad &= \quad MPT_{se} \times 1 + (r + \mu + \delta)k + ra - k(r + \delta), \\
y_{se} \quad &= \quad r_{mw} \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}} + \mu k + ra.
\end{aligned}
$$

Proof of Proposition 3. An individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if

$$log(z_w w + ra) < log\left[\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\theta+\gamma}} r_{mw} + \mu k + ra\right],$$

which holds when the skill ratio satisfies

$$\frac{z_w}{z_m} < e^{\frac{\theta+\gamma}{\gamma}} \frac{r_{mw} + \mu k / \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\theta+\gamma}}}{w}.$$

If capital markets are perfect ($\phi = 1$), the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and the individual prefers to be self-employed rather than work for a wage if and only if

$$\frac{z_w}{z_m} < e^{\frac{\theta+\gamma}{\gamma}} \frac{r_{mw}}{w} \equiv R_1.$$

Proof of Proposition 4. The optimal production plan of employers solves

$$\pi(z_m, z_w, a) = Max_{t_m, t_w, n_d, k}(z_m t_m)^\gamma k^\nu (n_d + z_w t_w)^\theta - wn_d - (r+\delta)k + (1+r)a$$

$$k \leq \bar{k}$$

$$t_m + t_w = 1,$$

$$t_w \geq 0.$$

The non-negativity constraint on t_w ensures that managerial time cannot be bigger than 1. Associate the multiplier μ_k to the borrowing constraint, μ_t to the time constraint, and μ_{tw} to the non-negative constraint on the working time. The FOC of the problem imply

$$MPK = (z_m t_m)^\gamma \nu k^{\nu-1}(n_d + z_w t_w)^\theta = r + \delta + \mu_k,$$

$$MPn_d = (z_m t_m)^\gamma k^\nu \theta (n_d + z_w t_w)^{\theta-1} = w,$$

$$MPt_m = z_m \gamma (z_m t_m)^{\gamma-1} k^\nu (n_d + z_w t_w)^\theta = \mu_t,$$

$$MPt_w = (z_m t_m)^\gamma \theta k^\nu (n_d + z_w t_w))^{\theta-1} z_w = \mu_t - \mu_{tw},$$

where we have assumed that parameters are such that it is optimal to hire outside labor $(n_d > 0)$. Combining the FOC we obtain:

$$wz_w = MPt_w \leq MPt_m, \text{with equality only if } t_w > 0.$$

We divide the analysis in two steps.

Step 1: We first show that if the borrowing constraint does not bind $(\mu_k = 0)$, then the entrepreneur allocate all his time to managerial tasks $(t_w = 0, t_m = 1)$. Assume that $\mu_k = 0$ and let $L \equiv n_d + z_w(1 - t_m)$. Furthermore, to find a contradiction assume that $t_w > 0$. Then, $\mu_{tw} = 0$ implies $MPt_m = MPt_w$ so that

$$z_m \gamma L = t_m z_m \theta z_w \rightarrow t_m = \frac{\gamma L}{\theta z_w}. \tag{13}$$

Combining the FOC for MPK and MPn_d, gives

$$(z_m t_m)^\gamma \left(\frac{w \nu L}{(r+\delta)\theta}\right)^\nu \theta L^{\theta-1} = w. \tag{14}$$

Combining (13)-(14) gives

$$L^{\gamma+\theta+\nu-1} \left(\frac{z_m \gamma}{\theta z_m}\right)^\gamma \left(\frac{w\nu}{\theta(r+\delta)}\right)^\nu \theta = w, \tag{15}$$

which is false in general given that $\gamma + \theta + \nu - 1 = 0$. We conclude that if the borrowing constraint does not bind, then an employer optimally choose to devote all his time to managerial tasks.

Step 2: Assume that the borrowing constraint binds ($k = \bar{k}$). We now show that there exists a threshold level of assets $a^*(z_m, z_w)$ such that the optimal production plan features $t_w > 0$ if $a < a^*(z_m, z_w)$ and $t_w = 0$ if $a > a^*(z_m, z_w)$. Thus, if the borrowing constraint is not too tight, employers allocate all their time to managerial activities. We now find conditions for which $t_m < 1 (or, equivalently, t_w > 0)$. Note that $t_m < 1$ only if $\mu_{tw} = 0$. In this case, the marginal product of entrepreneurial time is equated across the two uses of time. From the FOC it can be obtained that

$$MPtw = MPtm \Rightarrow L = \frac{\theta z_w t_m}{\gamma}.$$

Plugging L into the FOC with respect to labor demand and solving for t_m gives an expression for the optimal fraction of time dedicated to managerial tasks:

$$t_m = \left[\frac{\theta z_m^\gamma \bar{k}^\nu}{w} \left(\frac{\gamma}{z_w \theta} \right)^{1-\theta} \right]^{\frac{1}{1-\gamma-\theta}}.$$

Note that $t_m < 1$ iff

$$\bar{k}(a, z_m, z_w) < k^*(z_m, z_w) \equiv \left[\frac{w}{\theta z_m^\gamma} \left(\frac{z_w \theta}{\gamma} \right)^{1-\theta} \right]^{\frac{1}{\nu}}.$$

Since $\bar{k}(a, z_m, z_w)$ is increasing in a, the inverse of this function can be used to define a threshold level of assets $a^*(z_m, z_w)$ such that $t_m < 1$ if and only if assets are below this threshold. Otherwise, $t_m = 1$.

Step 3: Compute the marginal product of employers time. From Step 1 and 2, when assets are below $a^*(z_m, z_w)$ we have $MPtm = MPtw = wz_w$. On the other hand, when assets are above $a^*(z_m, z_w)$, $t_m = 1$ and $MPtm > MPtw$. To obtain an expression for MPtm note that the FOC with respect to capital and outside labor imply:

$$k = \frac{w\nu}{(r + \delta + \mu_k)\theta} n_d$$

$$n_d = \left(\frac{\theta z_m^\gamma}{w} \left[\frac{w\nu}{(r + \delta \mu_k)\theta} \right]^\nu \right)^{\frac{1}{1-(\nu+\theta)}}$$

Plugging k and n_d into $MPtm = \gamma z_m^\gamma k^\nu n_d^\theta$ gives

$$MPT_e = z_m \gamma \left[\left(\frac{\nu}{(r + \delta + \mu)} \right)^\nu \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1-(\nu+\theta)}}$$

Proof of Proposition 5. An individual with ability (z_m, z_w) and assets a prefers being an employer rather than self-employment if and only if

$$log \left[\left(z_m^\gamma z_w^\theta \right)^{\frac{1}{\theta+\gamma}} r_{mw} + \mu_{se} k_{se} + ra \right] < log \left[z_m r_m + \mu_e k_e + ra \right],$$

where μ_e and μ_{se} are the Lagrange multipliers associated to the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production at these occupations. This inequality holds when the ability ratio is such that

$$\frac{z_w}{z_m} < e^{\frac{\theta+\gamma}{\theta}} \left[\frac{r_m + (\mu_e k_e - c_f)/z_m}{r_{mw} + (\mu_{se} k_{se})/(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}} \right] \equiv \overline{R_2},$$

If capital markets are perfect ($\phi = 1$), the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and the individual prefers to be an employer rather than be self-employed if and only if

$$\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}}), \quad \text{where } R_2 \equiv e^{\frac{\theta+\gamma}{\theta}} \frac{r_m}{r_{mw}}.$$

Proof of Proposition 6. When $\phi = 1$ the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and occupational choice decisions are independent of asset holdings and maximize the marginal product of time. Proposition 4 established that an individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_1 \equiv e^{\frac{\theta+\gamma}{\gamma}} \frac{r_{mw}}{w}.$$

Assuming that the fixed cost of operation $c_f = 0$, then Proposition 5 implies that an individual with ability (z_m, z_w) prefers to be entrepreneur instead of self-employed if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_2 \equiv e^{-\frac{\theta}{\theta+\gamma}} \frac{r_m}{r_{mw}}.$$

If parameters are such that $R_2 < R_1$, then self-employment is dominated by either being an employer or a worker. The optimal occupational choice is to be an employers if and only if $\frac{z_w}{z_m} > \frac{w}{r_m}$. Otherwise, the optimal occupational choice is to work for a wage. When employers incur a fixed cost of operation, being an employer is preferred to being self-employed if and only if

$$log \left(z_m^\gamma z_w^\theta \right)^{\frac{1}{1-\nu}} r_{mw} < log \left(z_m r_m - c_f \right),$$

which holds when $\frac{z_w}{z_m} < e^{-\frac{\theta}{\theta+\gamma}} \frac{(r_m - c_f/z_m)}{r_{mw}}$.