Endogenous risk in public-private partnerships

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Abstract

This study characterises the moral hazard in a public-private partnership for investment and operation of an essential facility, when market risk and synergies between building and operating activities coexist. Relying on real options techniques, we show that the firm does not properly internalise the synergies between activities when it expects to declare bankruptcy whenever the market falls significantly and continuing the operation becomes unprofitable. We find that the effort decision depends on the relative importance of the positive externality with respect to the volatility of the market. Furthermore, in contrast to Jensen and Meckling (1976), our results reveal that as the bankruptcy is endogenous, the incentive for effort is higher under equity rather than debt financing.

Keywords: Stochastic demand; public-private partnership; endogenous bankruptcy, moral hazard

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1 Introduction

Essential facilities, such as infrastructures for long-distance transportation services, involve significant investment costs. Under some circumstances, public transfers, which are ultimately financed by taxpayers, can be avoided if the government awards exclusive franchising contracts to private consortia that are willing to invest. One may think about concession contracts for highways, or Eurotunnel project and other recent private financing in railways.

The literature about public-private partnerships (hereafter, PPPs) has examined the opportunity of private involvement in essential facility projects. However, nowadays, the optimal allocation of activities between public and private sector is largely debated. In practice, European governments involve private consortia in infrastructure activities without a clear understanding of the associated long-term implications. The key feature of the debate is that building and operating essential facilities involve increasing returns technology. Competition on the market is unfeasible and market operation needs be "controlled" by public authorities.

There are two related directions in the literature about financing essential facilities, with a serious gap in between. The first one focuses on the market side. Some studies discuss how the downstream market should be regulated for the consortium to make optimal investment decisions (see Caillaud and Tirole, 2004). Others (the PPP literature) focus on the optimal contracting in the presence of market risk, which induces renegotiation. Relevant examples are given by concession contracts in highway infrastructures (see Engel, Ficher and Galetovic, 1997, 2001, 2003; Guasch et Al, 2003). It is noteworthy that, in all such studies, the risk of investment does not depend on consortium's actions.

The second domain includes works about the optimal PPP organization, in presence of synergies between different stages of an essential facility project. Suggestive studies are Hart (2003) and Bennett and Iossa (2005). Optimality depends on the importance of the synergies between building and operating activities in the presence of non-contractable effort at the infrastructure building stage.

Observe that, in practice, the governments involve private consortia in infrastructure activities for financial considerations. Private consortia make investments by using private funds and subsequently recover the initial cost from market returns. This may make private consortia performing such activities socially desirable, even in the absence of synergies between building and operating. However, in case the market falls significantly in the future, the consortium prefers to declare the bankruptcy, in which case public transfers for operation activity may materialize. Therefore, the advantage of private involvement in such projects is in that public transfers can be delayed. On the other hand, if synergies between activities exist, the consortium’s effort may reduce the risk of public subsidies since the operating flows from the
project are expected to be higher.

The arguments above suggest that the optimal organization of a PPP is exposed to market risks. Henceforth, a clear link between market and organizational perspective exists, which the literature, so far, has not precisely identified. The aim of the present study is to combine the two orientations we have been illustrating. More precisely, we examine the optimal financing of PPP projects, taking into account that both the positive synergies between building and operating activities (the organizational perspective) and the exogenous risk (the market perspective) coexist. The latter is common knowledge, whereas the former depends on the consortium’s effort.

As for the methodology, we rely upon the real option techniques. This is appropriate because the consortium has the option to reduce or not the volatility of the financial risk while exerting unobservable effort, according to the contract it receives. Moreover, the consortium has the option to declare the default of the activity whenever the market falls significantly and the project becomes unprofitable. The government has an option to delay the provision of public funds by involving a private consortium in contracting. When a sufficiently low market demand is realized and the consortium credibly threatens to abandon the activity, the government decides either to accept a contract renegotiation or to expropriate the consortium’s investment. The social consequence is the same in any of the two cases: public transfers take place in the industry ex post. The optimal decision at the evaluation stage of the project is to offer the consortium a rent that reduces the possibility of such subsidies. Thus, the option value of the government trades-off the ex-ante rent against the risk of providing higher subsidies ex post. The amount of the rent that is given up to the consortium crucially depends on the relative importance of the positive externality between the building and operating activities as compared to the (exogenous) market risk of the project.

In the context described above, we examine the optimal capital structure of the consortium that finances the project. Jensen and Meckling (1976) and a few subsequent studies argue that debt financing is always preferable to equity financing at inducing the consortium to exert optimal effort at the building stage. Our results contrast with their findings because we add the endogenous bankruptcy issue (as done by Leland, 1994) to the moral hazard problem. In this context, the default option of the consortium is lower under equity rather than debt financing. Consequently, the consortium better internalizes the positive externality in exerting effort when the project is equity rather than debt financed.

The major contribution of the present work resides in that it provides important insights about the reasons why many governments rely upon PPPs and, at the same time, it sheds light on the drawbacks which show up in terms of implementation.
The paper is organized as follows. Section 2 presents the market, the technological context of the essential facility project and the payoff functions. In Section 3, the no bankruptcy benchmark is presented, as similar to the previous literature of PPP about the optimal organization. In Section 4, we examine the problem of the consortium that exerts all the activities in the essential facility project. The issue of equity financing versus debt financing the consortium and the implication for the social efficiency is explored in Section 5. The last Section summarizes the results and concludes.

2 The model

The government offers to a private consortium a monopoly franchising contract for building an infrastructure and operating the transportation service. The consortium derives a payoff from activity that is called the value of equity. We are interested in understanding the conditions that determine the government to involve private firms in infrastructure projects and the social efficiency outcome from the public-private partnership in an uncertain market. We assume without loss of generality that the contract is signed for an infinite time interval of operation activity. During the contract, the activity can become financially unprofitable. In that situation, either the firm is replaced or not, but the government intervenes in the sector and covers the difference between the operating costs and the market returns, so that the activity is continued. The investors (debt holder, equity holders lose the uncovered part of the investment cost if no guaranty is given initially by the government.

2.1 The characteristics of the project

The instantaneous benefit of the project is represented by a stochastic variable $y_t$ on time interval $t \in [0, \infty)$. Uncertainty is represented by a complete probability space $(\Omega, \mathcal{F}, P)$. The law of motion of this value is given by a geometric Brownian motion with drift $\alpha > 0$ and volatility $\sigma \geq 0$, so that

$$dy_t = \alpha y_t dt + \sigma y_t dz_t. \tag{1}$$

$y_0$ is the value it takes at the current date $t_0 = 0$, while $z_t$ is a simple Brownian motion defined on $(\Omega, \mathcal{F}, P)$. The flow of information on which decisions are based is expressed by the filtration $(\mathcal{F}_t)_{t \geq 0} = (\sigma \{y_t | v \leq t\})_{t \geq 0}$ generated by $(y_t)_{t \geq 0}$.

We make here two useful remarks. Firstly, the expression (1) suggests that $y_t$ is entirely given to the decision makers. In reality, the cash flow of any period is influenced by the pricing policy. We will relax this assumption at a later stage. Secondly,
the assumption that $dy_t$ follows a geometric Brownian motion well accommodates the requirement that the realizations $y_t$ are always non-negative. Indeed, $y_t$ in our notation stays for the gross benefit from the project.

Building the infrastructure requires an initial cost $I$ at $t = 0$. This amount is obtained through either debt or equity financing. In case the project is debt financed, the cost of activity during each operational period is

$$C^d(e) = \mu I + c - \delta e,$$

while in case the project is equity financed, such cost is

$$C^e(e) = c - \delta e.$$

The two costs are distinguished in that if the activity is debt financed, then an amount $\mu I$ that is to be paid to the debt holder is extracted from the market benefit during each instant of operation interval. In this way, the flow of operating cost is higher under debt rather than equity financing. We denote by $r$, such that $0 \leq r \leq 1$, the risk free interest rate in economy. $\mu$, such that $r \leq \mu \leq 1$ is the interest rate required by the debt holder.

$c - \delta e$ is the operating cost of activity. $c > 0$ is its fixed part, while $\delta > 0$ is the parameter that measures the cost reduction per unit of effort employed at the construction stage of the project. This is the positive externality between building and operating activity in the public-private partnership. Higher quality infrastructure means lower maintenance cost of operation.

As it will be shown later on, the distinction between (2) and (3) is the key feature that drives the analysis in our model, when we investigate the financial bankruptcy of the infrastructure activity.

The effort $e$ costs the consortium $\psi(e)$. We assume for simplicity that the effort can take two possible states, which are $e = 0$ and $e = 1$, so that $\psi(0) = 0$ and $\psi(1) = \psi$, where $\psi > 0$.

### 2.2 The timing

We investigate a monopoly franchising contract for a public project with uncertain benefits. The financial bankruptcy of the project arises when the operating cash flow of the project becomes negative. At this point, the government replaces or not the operator but it allocates subsidies in the sector so that the activity is not stopped. The time line of decisions in our problem is illustrated in Figure 1.
2.3 The payoffs

In case the new activity is debt financed, then the new equity payoff that it generates is written

$$E^d(y/e) = E \int_0^{\tau_d} (y_t - (\mu I + c - \delta e)) e^{-rt} dt.$$ (4)

This payoff is just the net present value, or the expected discounted net cash flows of the project. The new equity payoff is evaluated on the time interval $[0, \tau_d]$, where $\tau_d$ is the stopping time at which the bankruptcy is declared by the firm.

The expression of $E^d(y/e)$ shows that the consortium is the only residual claimant of the stochastic benefit $y_t$ of each operating period, after the payment of debt and of operating cost. The expectation of $\tau_d$ and the effort choice $e$ have an impact on the value $E^d(y/e)$ created by the new activity. We assume for simplicity that there is no corporate tax associated to the infrastructure activity. The firm bears a disutility $\psi(e)$ of its unobservable effort so that its net payoff is written

$$V^d(y/e) = E^d(y/e) - \psi(e).$$ (5)

If the activity is equity financed, the stopping time is $\tau_s$. Then, the new equity value is written

$$E^s(y/e) = E \int_0^{\tau_s} (y_t - (c - \delta e)) e^{-rt} dt - I.$$ (6)

The difference between $E^d(y/e)$ and $E^s(y/e)$ is provided by the way the investment cost of the project is financed. In the first case, the debt holder provides the amount $I$ and then it recovers it from the new activity, in fixed payments, that augment the operating cost. In $E^s(y/e)$, the investment cost is paid by the shareholders who subsequently share the net profits from operation, that are higher without debt. Because the agents that bear the risk of investment differ between equity and debt.
financing, the stopping time \( \tau_s \) in (6) is different from \( \tau_d \), as it will be shown later on.

Suppose that if the activity is equity financed, the consortium contributes with a share \( \nu \) in the investment cost, such that \( 0 \leq \nu \leq 1 \), which is also the share of its stockholding. The other part of the cost is paid by outside equity holders. The payoff derived by the consortium from the new activity is

\[
V^e(y/e) = \nu E^e(y/e) - \psi(e).
\] (7)

(7) shows that the consortium shares the new equity value with the shareholders while it bears alone the cost of its unobservable effort.

### 3 The no bankruptcy benchmark

As the public transfers are costly, it would be ideal that the equity holders do not use their option to default the activity during the contract, so that \( \tau_d \) and \( \tau_s \) are both infinite and public transfers in the sector are avoided. This would happen if the market were not uncertain.

Let us analyze now the choice of effort under alternatives sources if financing the activity. The consortium’s payoff with debt financing and no stopping of the activity before the end of contract duration is

\[
V^d(y/e) = \left( \frac{y}{r - \alpha} - \frac{rI + c - \delta e}{r} \right) - \psi(e).
\] (8)

The effort \( e = 1 \) is chosen whenever \( \Delta V^d(y) = V^d(y/e = 1) - V^d(y/e = 0) \geq 0 \), or

\[
\frac{\delta}{r} \geq \psi.
\] (9)

Some studies have already shown that in procurement contracts for essential facilities, the agent internalizes the positive externality between initial effort and its operating cost whenever the payment for construction is linked to the resulting cost of operation. The solution adopted for this aim was that of decentralizing both activities to one entity rather than to separated agents (Hart, 2003; Bennett and Iossa, 2005). Indeed, their finding add to the other justification of such partnerships, that of financing the public activity from private sources.

However, when synergies between activities exist, the incentives for effort are different under debt/equity financing. We move in this sense to another literature about public-private partnerships, that of moral hazard under different financing alternatives (Dewatripont and Legros (2005), as based on Jensen and Meckling (1976)).
The payoff of the consortium when the public activity is equity financed is written

\[ V^s(y/e) = \nu \left( \frac{y}{r-\alpha} - \frac{c-\delta e}{r} \right) - \nu I - \psi(e), \]  

(10)

The firm exerts effort whenever \( \Delta V^s(y) = V^s(y/e = 1) - V^s(y/e = 0) \geq 0 \), or

\[ \nu \frac{\delta}{r} \geq \psi. \]  

(11)

By comparing (11) with (9) at \( \nu < 1 \), it is straightforward that if the firm is limited liability and needs to attract funds from the capital market for investing the amount \( I \), then \( e = 1 \) is less likely under equity rather than debt financing. With debt, the firm pays a fixed coupon to its creditor during each operating period while it remains the only residual claimant of the (uncertain) cash flows of the activity. By contrast, under equity financing, the firm gets only a fraction \( \nu \) of the technological advantage while it bears alone the effort \( \psi \), with direct impact on the incentive power induced by the synergies between the different stages of the public project.

We have thus presented the finding of two distinct literatures that are related to the public-private partnerships. They both ignored a central problem of such partnerships, which is that the partnership may end before the normal duration of the contract.

The public and private interests would coincide in our model if the cash flows were not uncertain. Indeed, if \( \sigma^2 = 0 \) in (1), then the cash flow never falls in the region \( y_t \leq y^i \), where \( i = d \) or \( i = s \). In practice, the essential facility activities are risky and the contracts end whenever the financial payoffs of such projects are very low. At that time, the financial balance of the operating activity should be realized from public budget. As we will show in what follows, the use of financial literature for public-private relationships is crucial because it provides the "measure" of when the government should intervene in the sector. Also, as it forecasts the bankruptcy, the consortium decides accordingly the effort \( e \) at the construction stage of the project.

4 Endogenous bankruptcy

We investigate now the real world situation in which the activity is risky and the firm declares the bankruptcy whenever the activity becomes unprofitable. Leland (1994) shows that the firm stops the activity when \( E^d(y^d/e) = 0 \) if debt financed, or \( E^s(y^s/e) = 0 \) if equity financed.

Once the firm declares the bankruptcy, the government allocates subventions that allow for the continuation of the operating activity. The justification of the partnership in this context is not that the public transfers are avoided, but rather the fact that they are delayed. We will express first the situation in which the risk
of the project is exogenous in the problem while subsequently we move to the moral hazard issue in the public-private partnership.

4.1 Functional form of the payoffs

The explanation above shows that there are two regions on which the discussion is relevant: the region of values \( y_t > y^d \) (or \( y_t > y^s \)), where \( E^d(y_t/e) > 0 \) (or \( E^s(y_t/e) > 0 \)), and the region of values \( y_t < y^d \) (or \( y_t < y^s \)) where \( E^d(y_t/e) < 0 \) (or \( E^s(y_t/e) < 0 \)). The first one corresponds to the situation in which the activity is financed from the market while the second one corresponds to the case where the activity is financed from public subventions. The boundary \( y_t = y^d \) (or \( y_t = y^s \)) between private and public financing of the operating activity is found with the standard smooth pasting (or low contact) condition.

In Appendix A.1 we have derived the closed form expression of the payoff of the consortium for the situation in which the new activity is debt financed. This expression is

\[
V^d(y/e) = \frac{y}{r - \alpha} - \frac{\mu I + c - \delta e}{r} - \left( \frac{y}{y^d} \right)^{\beta_2} \left( \frac{y^d}{r - \alpha} - \frac{\mu I + c - \delta e}{r} \right) - \psi(e) \tag{12}
\]

\( \beta_2 \) is the negative root of the quadratic equation \( \beta (\beta - 1) \frac{\sigma^2}{2} + \alpha \beta = r \) (our notation follows the one of Dixit and Pyndick, 1994). It is essential for our subsequent analysis to remark that \( \beta_2 \) is very low when the volatility parameter \( \sigma^2 \) of the cash flow is low while it tends to zero when \( \sigma^2 \) becomes very large.

Let us explain now the various terms of (12). \( y/(r - \alpha) - (\mu I + c - \delta e)/r \) is the discounted flow of expected net market revenues, if the consortium (equity holders) do not abandon the activity before the end of the contract duration. \( y/(r - \alpha) \) are expected market revenues (that are dynamic) as evaluated at current cash flow \( y \) from service operation. \( \mu I + c - \delta e/r \) are the expected flows of (constant) operational costs, discounted with the risk-free rate of return \( r \). By exerting effort, the consortium bears the disutility \( \psi(e) \).

The remaining term in (12) is a "correction" term expressing how the financial value of the project changes when the cash flow falls at some \( y^d \) such that \( y^d < y \), at which the default takes place. After default, the public authority subsidize the activity so that the consortium saves losses of size \( y^d/(r - \alpha) - (\mu I + c - \delta e)/r < 0 \). Since the bankruptcy reduces the overall risk of the consortium, this term increases the overall value of the equity payoff by eliminating downside flows that are below a certain level. The value of this term is weighted by the discount term \( (y/y^d)^{\beta_2} \) in the space of realizations of the stochastic cash flow \( y_t \), as based on the initial value \( y_0 = y \). Differently, this term can be interpreted as the present value of one monetary value contingent on future default. Two additional remarks are useful to
be made here: $\psi(e)$ does not enter in this term because the disutility of effort is "sunk" at the moment when the consortium decides to abandon the activity. Also, the value $y^d$ that triggers the default is taken as given. Later on, we will show that $y^d$ is endogenous in the problem.

By analogy, the payoff of the consortium in case the project is equity financed is written

$$V^s(y/e) = \nu \left[ \left( \frac{y}{r - \alpha} - \frac{c - \delta e}{r} \right) - \left( \frac{y^s}{y^s} \right)^{\beta_2} \left( \frac{y^s}{r - \alpha} - \frac{c - \delta e}{r} \right) - I \right] - \psi(e). \quad (13)$$

### 4.2 Bankruptcy trigger

Let us take first the debt financing case. Leland (1994) shows that if the activity is risky, then the risk of investment is bear entirely by the bondholder. The consortium still bears the risk of non-positive profit from the operating activity. The debt holder chooses then the coupon payment and protective covenants rules that maximize its expected payoff and minimize the credit risk.

Let us optimize first $V^d(y/e)$. It can be easily shown that $V^d(y/e)$ decreases in $y^d$ until a certain level, while subsequently it increases. The optimal $y^d$ in (12), denoted $y^d(e)$ solves

$$y^d(e) = \frac{\beta_2}{\beta_2 - 1} \frac{\mu I + c - \delta e}{r} (r - \alpha) \quad (14)$$

The interpretation of this solution is the following. The value $y^d(e)$ that triggers the default is below the maintenance cost by the ratio $\frac{\beta_2}{\beta_2 - 1} < 1$. This decision is taken at the first passing time $\tau_d$ of the value $y^d(e)$, defined as $\tau_d \equiv \inf \{ t \geq 0 \text{ s.t. } y_t = y^d(e) \}$. At any $t > \tau_d$, the equity value remains at zero while the public budget undertake expected negative cash flows $y_t - (c + \mu I) < 0$, where $y_t < y^d(e)$.

From the maximization of (13) we derive

$$y^s(e) = \frac{\beta_2}{\beta_2 - 1} \frac{c - \delta e}{r} (r - \alpha), \quad (15)$$

with similar interpretation.

Our work builds on Leland (1994) but the issue is somehow different. The first difference is that once the project "defaults", the public authority intervenes for the continuation of the activity. Moreover, the activity requires for a huge initial investment, that remains sunk in operation. As it will be shown, this characteristic of the sector is essential for the risk transfer in the project between the public and the private sector. Moreover, the risk of the project is endogenous in the problem, when the synergies between the building and operating activities exist.
We need thus to understand under which conditions the risk of public transfers in the operating activity is low and under which conditions the firm is available to build high quality infrastructures. It is then relevant to investigate the comparative debt/equity case.

4.3 Implications

There are two crucial implications of the endogenous bankruptcy for our analysis. Firstly, \( y^d (e) > y^s (e) \), which means that the bankruptcy is more likely under debt rather than equity financing. This result is standard in corporate finance literature, but very relevant for the understanding of the risk allocation in the project of public service provision. When the activity is debt financed, the consortium transfers the risk of investment to the debt holder, by using the option to default the activity once the market income does not allow for the payment of the coupon. In order to reduce its risk, the debt holder chooses the interest rate accordingly, so that \( \mu > \tau \).

On the other hand, when the activity is equity financed, the consortium cannot transfer the risk through its bankruptcy policy, so that the activity lasts longer.

In any event, it is not clear from this analysis whether one financing source is preferable to the other in this context. A clear understanding of the optimal financial source from this perspective would require to analyze the way the risk of debt investment is shared between the public authority and the debt holder, an issue that is not in the purpose of our study.

The second implication is that \( y^i (1) < y^i (0) \), either for \( i = d \) or \( s \). This inequality shows that the risk of the project is endogenized in the problem through the effort decision of the firm. The impact of the effort choice on the social payoff is thus double. Firstly, it creates an economic value from exploiting the synergies between building and operating activities, as the previous literature has shown. Secondly, it reduces the risk of bankruptcy since the operational cash flows of the consortium are higher.

We need then to understand under which conditions the firm is available to build high quality infrastructure and consequently reduce the risk of the project.

5 Effort decision and endogenous bankruptcy

In presence of endogenous bankruptcy the risk beared by the firm is lower, with direct effect on its effort decision.

For instance, if the activity is debt financed, \( e = 1 \) is chosen whenever \( \Delta V^d \geq 0 \), or

\[
\frac{\delta}{r} \geq \psi - \frac{1}{\beta_2 - 1} \left( \frac{y}{y^d (0)} \right)^{\beta_2} \frac{c + rI}{r} \left( 1 - \left( \frac{c + rI}{c + rI - \delta} \right)^{\beta_2 - 1} \right).
\] (16)
We note first that if no uncertainty exists, then $\beta_2 \to -\infty$ and the condition (16) reduces to (9) as in the benchmark case. The comparison between (16) and (9) provides the difference between our result and the studies that characterized public-private relationships as procurement contracts. Since the consortium decides freely to abandon the activity once the cash flow is low enough and reaches the value $y^d(e) < y$, the incentive power of the synergies between the activities is weakened with respect to the benchmark situation. The additional term that is added to $\psi$ on the right hand side of (16), which is positive, expresses this idea. We can thus establish the following Proposition.

**Proposition 1** A consortium running a monopoly franchising contract with uncertain cash flows expects to default the activity whenever the cash flow reduces to a certain level. Therefore, the consortium does not internalize optimally at the construction stage of the essential facility the synergies between building and operating activities, since it can limit the risk of its activity, through ex post action.

Let us explain the role of various determinants of the effort decision. Indeed, the condition $\Delta V^i \geq 0$, where $i = d$ or $s$ is influenced by

a) the initial cash flow of the project

b) the positive externality between activities

c) the volatility of the cash flow

d) the financing source of the project (debt or equity financing)

The first three determinants enumerated above influence the condition $\Delta V^i \geq 0$ for both cases $i = d$ and $i = s$. We illustrate them in the situation in which the activity is debt financed. The fourth condition will be examined subsequently in a different section.
Figure 3: $V^d(y/e)$ at $\delta_2$ and $\delta_1$ (curve 2 versus curve 1); $\delta_2 > \delta_1$

Figure 2 illustrates the payoff function of the consortium with / without effort, when the activity is debt financed, in a concrete example. As previously expressed, in case the firm exerts effort and the activity defaults, then it loses the cost $\psi$ and gains nothing from operating the infrastructure. It is interesting in this sense the intersection between the positive externality and the exogenous risk of activity. Even though the externality effect is "certain" in our model, the firm is not sure to recover the cost $\psi$ from a risky activity.

It can be easily shown that $(V^d)'(y/e = 0) < (V^d)'(y/e = 1)$. The explanation of this inequality is the following. The payoff of the firm increases in $y$, indifferently of the exerted effort, because it reflects a higher expectation of the payoff of the project. Moreover, by the fact that the firm forecasts bankruptcy, the payoff is convex in $y$, as shown by Leland (1994). On the other hand, the probability of bankruptcy is endogenized by the effort decision. It is lower at $e = 1$ with respect to the case where $e = 0$, as "measured" by the inequality $y^d(1) < y^d(0)$, so that the curve of $V^d(y/e)$ is steeper in the former case. Furthermore, due to disutility of effort, $V^d(y^d/e = 0) > V^d(y^d/e = 1)$. Therefore, there exists a threshold on the initial cash flow $y_0 = y^*$ at which the two payoffs cross, as in Figure 2. Indeed, the consortium does not exert effort if $y_0$ is such that $y_0 < y^*$. The reason why this is the case is that a low cash flow reflects a high probability of early bankruptcy. Consequently, the consortium is not available to bear the "irreversible" disutility $\psi$ which remains sunk in operation and cannot be recovered through ex post action.

We make here one more remark. If $y_0 \geq y^*$ then both the incentive for effort and the participation of the firm in the contract are ensured, if we normalize the outside opportunity to zero. By contrast, with equity financing, the participation of the consortium may not be ensured at $y_0 = y^*$.

Let us treat now the externality parameter $\delta$. It is very intuitive that the incentive of the consortium to make effort increases as $\delta$ gets higher. $\partial V^d(y/e = 1)/\partial \delta >$
Figure 4: $V^d(y/e)$ and $\sigma^2$ when $y_0 = 16$ and $\delta = 3, 5$ and 7 (curves 1, 2 and 3)

0 while $\partial V^d(y/e = 0)/\partial \delta = 0$. Also, $y^d(1)$ is lower at higher $\delta$. Consequently, the
threshold $y^*$ reduces, as shown in Figure 3 by the curve (2) as compared to (1). When the consortium benefits better from positive externality between the activities, this technological efficiency compensates for the financial risk of the project so that the incentive for $e = 1$ is easier satisfied.

We are left with the parameter $\sigma^2$, expressing the market riskiness. Leland (1994) shows that the equity value of the project with endogenous bankruptcy is increasing in the volatility of the stochastic variable. The reason is the following: the consortium benefits from upside streams of cash flows while it can limit the downside streams through ex post action. The impact of effort decision for different volatilities of the project is the following: $\partial V^d(y/e = 0)/\partial \sigma^2$ is higher than $\partial V^d(y/e = 1)/\partial \sigma^2$. If the project is more volatile, early bankruptcy is expected. In that case, the expected period on which the firm benefits from the positive externality is lower. On the other hand, at $\sigma^2 = 0$, where no bankruptcy occurs, $V^d(y/e = 1) > V^d(y/e = 0)$.

It follows that there exists a specific combination of $\delta$ and $\sigma^2$ for which the firm is indifferent between exerting effort or not, as illustrated in Figure 4. We then establish the following Proposition.

Proposition 2 The consortium’s incentive to internalize the synergies between activities increases in externality parameter and reduces in the volatility parameter of the project. Therefore, the effort decision of the consortium depends on the relative importance of the synergies between the infrastructure activities with respect to the uncertainty of the cash flow of the project.

For instance, in Figure 4, if $\delta = 3$ then $V^d(y/e = 1)$ is lower than $V^d(y/e = 0)$ even without uncertainty. If $\delta = 5$ then the firm exerts effort just in case the
uncertainty is very low. If \( \delta = 7 \) then the incentive for effort with respect to the uncertainty parameter is relaxed considerably.

### 6 Effort decision under equity versus debt financing

Let us characterize now the effort decision of the consortium when the activity is either debt or equity financed. The key inequality of this part of our analysis is \( y^e \prec y^d \). Indeed, under equity financing, the risk of investment cannot be transferred to the debt holder. Once the investment is sunk, the firm cannot recover it through ex post action.

Assume first that \( \nu = 1 \). From the equations of the payoffs, we can say that \( V^d(y/e) > V^s(y/e) \). This inequality is determined by the fact that under debt, the firm does not bear the risk of investment in the project. On the other hand, because the risk is endogenous in the project, \( V^d(y/e = 1) - V^s(y/e = 1) < V^d(y/e = 0) - V^s(y/e = 0) \). Written differently, \( \Delta V^d(y) \prec \Delta V^s(y) \). Indeed, when the firm exerts effort, the risk of investment reduces so that the advantage of risk transfer when the firm chooses debt rather than equity is also lower. This inequality has a crucial significance for policy implication, which is expressed in the following proposition and illustrated in Figure 5.

**Proposition 3** In case the activity of a consortium engaged in a public-private partnership is not risky, the power of the incentive to build high quality infrastructure is higher under debt rather than equity financing. By contrast, in the presence of market risk, equity financing is socially preferable.

In Figure 5 the firm exerts effort whenever \( \Delta V^i(y) \geq 0 \), \( i = d \) or \( s \). We see that the curve of \( \Delta V^d(y) \) is below the one of \( \Delta V^s(y) \), whenever \( \nu = 1 \). We distinguish there three relevant regions. For \( y_0 \) very small, the consortium does not exert effort. For the values \( y_0 \) that are between the intersects of the curves \( \Delta V^s(y) \) at \( \nu = 1 \) and \( \Delta V^d(y) \) with the horizontal axe of values \( y \), the source of financing matters. Indeed, this is the region in which the firm should be entirely financed from its own funds for a high quality infrastructure to be built. In the last region, the firm exerts effort in any of the two cases. The cash flow of the project is expected to be sufficiently high so that the probability of bankruptcy is very low.

If the firm is limited liability, then there exists a specific share \( \nu \) at which \( \Delta V^d(y) = \Delta V^s(y) \) and \( \Delta V^d(y) \geq \Delta V^s(y) \) at any \( \nu \) on the interval \( [\nu, \infty) \). This share takes in Figure 5 the value \( \nu = 0.85 \). However, as evident in the figure, whenever the firm does not finance entirely the project by itself, it is less likely that high quality infrastructure would be built.
7 Summary of results and conclusion

Our results can be summarized as follows. Involving private consortia in public projects maybe socially desirable if the private sector alone can finance the project. However, the synergies between the activities are not well internalized in the programme of the private consortium since the firm forecasts bankruptcy. The consortium has incentive to exert optimal effort in case the externality in cost reduction by exerting unobservable effort is high enough with respect to the market risk.

The source of financing is crucial in this perspective. The public authorities should always give priority to consortia that have strong financial health and can finance the project by themselves rather than by debt issuance. This result is in contradiction with the previous literature of moral hazard in public-private partnerships. The difference is given by the fact that the probability of bankruptcy is endogenized in the consortium’s programme.

A simple intuition of the endogenous bankruptcy is also that as the renegotiation is very likely in such contracts, firms who "sunk" their funds in the activity are weaker than smaller and limited liability firms in threatening to quit the activity. We have not included the renegotiation issue in our analysis.

In case firms with own funds to be used in the activity are not available on the market, so that the activity is debt financed, then the public authority should undertake at least partially the risk of the debt from the creditor, if it wants to induce optimal effort to the firm. Indeed, it is evident in our model that as long as $\mu$ is closer to $r$, then the incentive for effort is higher. If high quality infrastructures are to be built, the government should guaranty the debt with public resources so that $\mu = r$. 

Figure 5: Incentive for effort under debt/equity financing
We note the impact of the moral hazard issue of the public-private partnership in the framework of Engels Ficher and Galetovic (1997) of a flexible contract duration and fixed market revenue. Such a contract would limit the upside stream of the returns and reduce incentives for effort. The firm would be available to build a good infrastructure just in case the volatility of the project would be very low.

References


Dewatripont, M. and Legros, P. (2005), Public-private partnerships: contract design and risk transfer, mimeo


Nombela G., Rus de G. (2003), Flexible term contracts for road franchising, Working paper, UCTC, No. 660

Appendix

A.1 Functional form of the payoffs

Let us take any payoff \( F(y_t) \) based on the stochastic process (1); any such value satisfies the standard Ito’s lemma

\[
\alpha y_t F_y dt + \frac{1}{2} \sigma^2 y^2 F_{yy} - rF + A_0 = 0. \tag{17}
\]

The general solution of (17) at \( y_t = y \) is written

\[
F(y) = A_0 + A_1 y^{\beta_1} + A_2 y^{\beta_2}. \tag{18}
\]
\( \beta_1 > 1 \) and \( \beta_2 < 0 \) are known constants (see Dixit and Pyndick, 1994), while \( A_0, A_1, A_2 \) are determined by the boundary conditions of (17). The remaining terms show how the payoff function \( F(y) \) changes when \textit{ex post} actions are taken at some \( y_t > y \) or \( y_t < y \). In our case, \textit{ex post} actions are relevant just when the demand is below the initial one, in the region \( y_t < y \). Thus, if \( y \to \infty, F(y) = A_0 \). We must set in this case \( A_0 = 0 \), so that

\[
F(y) = A_0 + A_2 y^{\beta_2}
\]  

(19)

We express in what follows the concrete functional form of \( F(y) \) for both the consortium’s payoff and for the social one. We find for each case the specific value of the constants \( A_0 \) and \( A_2 \).

**The payoff of the consortium**

By the properties of the geometric Brownian motion (1), it implies that the expected discounted flow of the demand from operating infinitely the infrastructure is \( y/(r - \alpha) \). If the expected drift \( \alpha \) of the demand is high and \( r - \alpha \) reduces, higher market revenues are expected. The consortium bears instantaneous costs \( C(e) \) and initial disutility \( \psi(e) \). Thus, \( A_0 = \frac{y}{r - \alpha} - \frac{C(e)}{r} - \psi(e) \). It follows that the payoff of the consortium is written as

\[
V(y) = \frac{y}{r - \alpha} - \frac{C(e)}{r} - \psi(e) + A_2 y^{\beta_2}.
\]

\( A_2 \) is found from the remaining boundary condition of (14). Indeed, when the demand reaches a certain \( y_b < y \), the consortium renounces to finance the operation activity of infrastructure. We denoted by \( \tau \) in the main text the expected period when \( y_t = y_b \). We can thus write that at any \( t > \tau \), the value of the project is \( V_b \), such that \( V_b = -\psi(e) \). Indeed, once the activity defaults, the consortium losses the sunk cost \( \psi(e) \) but it does not bear losses from operation. The absolute and relative values of \( V(y) \) between the regions \( y < y_b \) and \( y > y_b \) must be equal at \( y = y_b \). Therefore,

\[
\frac{y_b}{r - \alpha} - \frac{C(e)}{r} + s + A_2 y_b^{\beta_2} = 0
\]

\[
\frac{y_b}{r - \alpha} + \beta_2 A_2 y_b^{\beta_2} = 0
\]

From the first equation, we can write

\[
A_2 = y_b^{-\beta_2} \left( \frac{C(e)}{r} - \frac{s}{r} - \frac{y_b}{r - \alpha} \right),
\]
so that the closed form functional form of the payoff \( V(y) \) defined in the region \( y > y_b \) is

\[
V(y) = \frac{y}{r - \alpha} - \frac{C(e)}{r} - \psi(e) - \left( \frac{y}{y_b} \right)^{\beta_2} \left( \frac{y_b}{r - \alpha} - \frac{C(e)}{r} \right),
\]

as in the main text.

**A.2 Numerical example**

In Table 1, the default parameters of the stochastic variable are \( \alpha = 0.01, \sigma^2 = 0.175 \). The other parameters are \( r = 0.04, I = 250, c = 10, \delta = 5 \) and \( \psi = 100 \).

In Table 2, the volatility parameter \( \sigma^2 \) varies and the payoffs are calculated at the initial state \( y_0 = 16 \).
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Note: \( y \) values range from 2.2 to 16, and the table shows payoffs for different values of \( \delta \) with \( V^d(y/e=1) \) and \( V^d(y/e=0) \) for each \( y \) value.
Table 2: Sensitivity of the payoffs in volatility parameter, at $y_0=1$

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For $N = 5$, $N = 7$, $a_2$ and $N = 3$, the payoffs are slightly different, showing the impact of volatility on the payoffs.