Vertical Divestiture as a Competitive Strategy: 
the Case of Russian Railways Reform

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Abstract
In the formal model we analyze the cost and benefits of vertical divestiture under different assumptions about the toughness of competition in the rail industry. We found that the welfare gain from the vertical divestiture may depend on the type of regulation applied to the vertically-integrated railway monopolist (RZD) and be conditioned by the nature of downstream competition.

JEL classifications: L51, L22

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1. Introduction

The railway reform in Russia has entered the phase where crucial structural decisions are to be made. Initiated in 2001 and aimed at 1) increasing reliability, safety, and quality of railway transport; 2) cutting overall transportation costs in the economy; and 3) meeting the increasing demand for transportation, the ten-year reform plan (called ‘The program of structural reform of railway transport’) lacked the clear vision of the targeted industry structure. Nevertheless, a number of reform principles were declared to achieve stated goals, such as:

- encouraging on-track competition,
- provision of non-discriminatory access to infrastructure,
- developing tariff policy to eliminate cross-subsidies,
- ensuring the state control over infrastructure of federal railways.

The first two stages (2001-2002 and 2003-2005) of the three-stage reform plan resulted in the model of vertical access. The vertically integrated state-owned infrastructure company (Russian Railways or RZD) provided access to its infrastructure and served the downstream market. This model being a priori regulatory intensive was accompanied by the global price-cap regulation (both access charge and end-user tariff were allowed to move simultaneously in line with inflation). Several ‘unexpected’ tariff indexations and a number of reform initiatives proposed and actively lobbied by RZD indicated the instability of the industry structure creating the long-term insolvency risks.1

It has become obvious that at this stage of railway transport reform competition, regulatory and structural policies didn’t seem congruent. Different government agencies highlighted the importance to concentrate the further reform activities on one or another particular measure. Federal Antimonopoly Service (FAS) promulgated the idea that in order to foster fair competition between train operators and eliminate the RZD incentives to discriminate smaller companies three independent carriers endowed by the evenly split RZD’s wagon fleet have to be created. Federal Service for Tariffs (FST) employs an argument that scope as well as scale economies in the industry are sufficiently large and insists on closer monitoring of the RZD costs and investment activities as a precondition for the further structural changes. The RZD itself lobbies for the vertical divestiture and creation of an affiliated Cargo Company that would pay the same access charge as its rivals do, be unregulated as well and compete with them locally.2 The follow-up privatization of the Cargo Company would help to attract additional investment for the rolling stock renewal.3

The obvious contradiction between stated goals fuels the debate about the targeted industry structure. It is worth mentioning here that the importance of the provision for the non-discriminatory terms of access to the rail track and other infrastructure is commonly

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1 In an attempt to secure the investment ratings (the highest among Russian companies) the 100% state-owned company was subject to break-even price regulation leaded to compensatory tariff indexation. In spite of regular freight tariff increases, the profitability of cargo transportation decreased from 31.2% in 2003 to 21.9% in 2005.

2 Two RZD subsidiaries were established in July 2006 – “Transcontainer” and “Refservice” – operating on the niche markets for transportation of standard and refrigerated containers.

3 Morgan Stanley estimated the possible capitalization of such an unregulated monster with no social obligations as high as $6.4-8.9 billion. In two or three years the sale of 49% shares would bring around $4 billion out of $10 billion demanded by RZD for the investment in the freight wagon fleet.
admitted. Another point that is always in the center of discussions is the need to create and protect equal conditions for the integrated and independent train operators on the transportation market. RZD resorts to that rhetoric of sorts in trying to persuade regulator (FST) to allow the monopoly to set some end-user tariff freely on some local or niche transportation markets. With access charge being thoroughly regulated this point deregulation of the so called wagon component of tariff would level the playing field for the vertically integrated producer (VIP) and its downstream rivals on these particular niche markets. The corresponding methodology to detect those markets experiencing tough intra-modal as well as inter-modal competition was proposed to support this idea.

The presence of incentives for the VIP to discriminate nonintegrated operators is crucial problem for the vertical access model to survive and is clearly understood by the antimonopoly service. The adoption of Decree No. 710 of 25 November 2003 that provided rules on the provision of non-discriminatory access to infrastructure equipped the antimonopoly authority with a necessary instrument to require equal treatment for all carriers. But as Pittman (2003a, 2003b) shows institutional and sometimes political weakness of antimonopoly authorities in Russia and other transition countries can not fully guarantee the enforcement of this law and protect smaller players in the transportation market against discrimination.

Pittman (2004) examines the term discrimination’ in the context of these discussions of ‘nondiscriminatory’ access to the infrastructure. He argues that if competition regulators are unable to distinguish between discrimination that harms competition and discrimination designed only to recover fixed costs (as Ramsey pricing implies), policy makers will face a choice between large government subsidies and large welfare losses. In fact, in Russia the direct financial support of the railway transport from the state was almost absent⁴. What makes the Russian railway transport financially balanced is tariff differentiation across commodities⁵. The new tariff system introduced in 2003 (Price List #10-01) was based on the idea that higher access tariffs should be charged for transportation of goods with less elastic demand with respect to railway tariff.

The pro-competitive effect of tariff ‘perestroika’ that favored downstream entry had the form of discount from the end-user tariff for independent operators who used theirs own or leased wagons. Essentially, it was lack of tariff flexibility and inadequate investment to renew the hugely depreciated RZD’s rolling stock that created the room for competition in the market for the wagon fleet provision. As a result the state-owned railway monopolist has dramatically lost its market share in the most lucrative markets facing competition from the specialized niche operators in local geographic areas.

A set ‘defensive’ strategies for such a, in some sense, over-regulated vertically integrated producer that sells the essential facility (access to rail track) to its downstream competitors comprises of at least: 1) lobbing for frequent tariff indexation or even deregulation, 2) different methods of rivals discrimination, and 3) self-restructuring that is especially relevant in the context of railway reform in Russia. The last two strategies are not independent since under the vertical access model VIP has both the incentive and the ability to disadvantage rivals and thereby hinder industry performance. Vertical separation appears

⁴ 2007-year federal budget turned out to be the first in recent history to include 10.9 bln rubles compensation for loss-making passenger services.

⁵ This ‘differentiated’ approach to tariff regulation has implemented in Russia since 1989. Minor adjustments with respect to the cross-commodity structure was introduced.
to be more attractive as it prevents the infrastructure owner from treating different operators in different way but suffers from the potential loss from the absence of scope economy.

In our paper we address the problem of vertical divestiture\(^6\) as a competitive strategy for the VIP to deal with competition downstream. We endow the VIP with the ability to sabotage its’ rivals by raising their costs. Thus we analyze discriminatory activities of the VIP under different institutional settings and compare consumer surpluses under different assumptions about the nature of downstream competition to answer the question if vertical divestiture may be welfare improving.

2. Literature review

Vertically integrated infrastructure company may survive competition downstream and break-even even if access charge is set in accordance with Ramsey principles and is treated as indirect substitute for the regulated end-user tariff in terms of generating revenues for the whole company (see Laffont, Tirole, 1994)). As we argued elsewhere (Dementiev, 2005) when access charge and final tariff are set inflexibly relative to each other (as in Russia) those markets for transportation of high-value commodities turned out to be contested by independent operators and ‘cream-skimmed’. With no ability to change end-user tariff and access charge independently and freely the VIP may resort to a set of activities that disadvantage or sabotage downstream competitors.

The economic literature on sabotage distinguishes between cost-raising (see Beard at al., 2001) and demand reducing types of such a discriminatory activity (Mandy and Sappington, 2006). It is generally acknowledged that if raising the costs of downstream rivals is costless sabotage makes sense for the VIP (Sappington, 2006b). In Cournot settings (e.g., Economides, 1998; Sibley and Weisman, 1998) when downstream firms compete in quantities, as well as in case competition in prices \(a la\) Bertrand (e.g., Weisman, 1995; Beard et al., 2001; Kondaurova and Weisman, 2003) cost-raising sabotage brings about an increase in the profit of its downstream affiliate. This type of sabotage induces downstream competitors to produce less that would lead to a decrease in the demand for the final product. In turn the upstream profit decreases if there is some mark-up over marginal cost on the upstream market ((e.g., Weisman, 1995; Sibley and Weisman, 1998). Thus, the potential trade-off between downstream gains (measured by the consumers’ surplus) and losses from the diseconomies of scope (or vertical integration) worth considering.

This approach may be directly applied to the problem of pros and cons of vertical divestiture of infrastructure industries such as railway transport. The answer may depend on the assumptions about the nature of downstream competition. Crew et al. (2005) investigate the Cournot competition and argue that vertical divestiture can eliminate sabotage at the expense of scope economies gain. The policymakers must evaluate the potential for sabotage vs. scope economies (embodied in smaller costs of the downstream production of the VIP affiliate). If the former exceeds the latter vertical separation is preferred, otherwise integrated production is to be chosen. If the downstream firms engage in Bertrand competition (see Sappington, 2006a) this result doesn’t hold anymore: even when scope economies are high and potential for sabotage is limited the vertical separation secures higher level of expected consumers’ surplus comparing to vertical integration.

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\(^6\) The difference between vertical separation and vertical divestiture is commonly viewed as the former encompassing the latter plus prohibition of any other firm to serve the downstream market as a vertically integrated producer.
Adopting similar approach to address the question ‘whither railway reform in Russia’ we use slightly different settings. We assume that downstream competitors (the VIP affiliate, operating rival and any potential entrant) have exogenously given capacity constraint (limited wagon fleet in case of railway reform in Russia). We also assume that any two rivals can capture the whole downstream market but none of them have enough facilities to drive its rival out the market completely. Thus our paper contributes to the existing literature on the merits of vertical divestiture by considering the case when firms have pre-commitment about their production potential and compete in prices.

The reminder of the paper is organized as follows. Section 3 analyzes the institutional environment (formal and informal) of the railway reform in Russia and defines four principle regulatory schemes at the different stages of reform: past, current, and two possible future configurations. A simple theoretical framework inspired by Sappington (2006a) is built in Section 4. We depart from the original setup in our treatment of end-user tariff regulation and the nature of downstream competition. Hence, our model tries to capture the basic features of the structural and regulatory reform in Russian railway transport. The main findings are discussed in Section 5. Section 6 concludes.

3. Analytical framework

To substantiate our formal analysis we start with reporting some important stylized facts about the regulatory framework and market conditions at different stages of railway reform in Russia. We will concentrate only on freight railways.

It is the regulatory policy together with demand shocks and rolling stock depletion that turned out to influence the emergence of competition in the railway sector in the country. The list of problems and set of strategies for regulator, incumbent and competitive fringe is to be specified first to address the central question for research: under what circumstances may vertical divestiture be welfare enhancing and be in line with the optimal strategy for the incumbent that faces downstream unregulated competition and has both incentive and ability to disadvantage its competitors?

The first structural scheme to consider characterized the regulatory framework at the pre-reform stage. Despite the reform plan being officially adopted in 2001 the first structural measures were undertaken only on the second stage (2003-2005). Thus we consider the period between 1999 and 2003 as indivisible with almost no structural changes and railway end-user tariffs growing faster than average inflation.

Several features of the market structure and regulatory policy are crucial for the further analysis. Under Scheme #1 (vertical integration with end-user tariff regulation and access to infrastructure being as high as this tariff) structure of the rail tariff, though differentiated between three classes of commodities, did not make any difference between end-user tariff for shippers with no private wagons and ‘access’ charge for wagon owners. This very structure that happened to be fully inherited from the Soviet times didn’t prevent so called captive operators from entering some niche market for wagon provision. Rather than maximizing profit they tried to secure their export revenues from the risks of any delays.

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7 Frequent (though often unexpected) and substantial end-user tariff indexation turned out to be the reason for the structural reform to be delayed.
8 Private wagons of major exporters of raw materials formed the basis for competition in the market for wagon provision. Transport divisions of such companies that maintained thousands of wagons received the status of ‘operators’ only in 2003.
9 The first to enter were oil-exporting companies followed by metal producers and chemical plants.
due to poor quality and scarcity of specialized rolling stock (tank-wagons, open wagons, etc.).

Scheme #2 (vertical integration with access and end-user tariff regulation) was introduced in 2003 and dominated in the industry for the consequent three years (during the second stage of reform). The New Price List #10-01 legalized the wide spread practice of mass-scale discounts\(^{10}\) (mostly informal) off the official end-user tariff for private wagon owners. Institutionalization of *de facto* contestable market for wagon provision implied that (still) the only *de jure* carrier in the industry – RZD – would transport private wagons and charge for this 15% less (on average) comparing to the shipping the same commodity in the RZD’s wagons. It’s worth mentioning here that only most lucrative markets were subject to entry by rival operators. Obviously, captive operators (with potential cost advantages due to more favorable leasing and borrowing conditions, loading guarantees, etc.) were the first to enter the downstream market followed by independent operators. An important point is that entry decision was almost always preconditioned by the lack or bad quality of RZD’s wagon fleet available on the particular route. Thus in an attempt to capture basic features of the current stage of reform in Russia we incorporated such a capacity constrains exhibited by operators in the model we develop in the next section.

Scheme #3 (vertical integration with downstream deregulation) has been proposed by RZD as an element of its’ competitive strategy. The establishment of Cargo Company is viewed by RZD as an effective measure to fight for high value-added sectors of transportation markets with other operators. A special procedure to determine already competitive market of freight shipment was elaborated by RZD specialists to support this idea. Those markets (17 in total) with highly competitive environment (inter- and/or intra-modal) were suggested to be deregulated\(^ {11}\).

Scheme #4 (complete vertical separation with access charge set above marginal costs) means privatization of the Cargo Company and other RZD affiliates. RZD plans to retain 51% control over the Cargo Company but this idea is strongly opposed by the Federal Antimonopoly Service. The following Section 4 describes the formal model which discussed in Section 5 under different schemes listed above.

### 4. The model

The final service (transportation) is a homogenous good that can be supplied either by the vertically integrated incumbent or by its downstream competitors. Incumbent network owner provides access services that are required to produce final one. Entrants (or downstream competitors) supply a final service (operate wagons and charge their customers) in competition with the incumbent and use the single unit of the access service to the incumbent’s network as an input (essential facility).

The upstream market size is normalized to unity. Such an assumption helps us to avoid useless calculations though allows us only to make judgments in (comparative) static terms. Nevertheless, when dealing with discrete organizational changes (as in case of structural reform) this approach proves to be fruitful. The demand for final service is completely inelastic. There exists some finite price \( v \) that correspond to maximum willingness to pay.

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\(^{10}\) See Dementiev, Doronkin (2001) and OECD (2004) for the details.

\(^{11}\) In practice it is the wagon component of the end-user tariff for those who use RZD wagons that is lobbyed to be deregulated. Access charge remains under FST control. As the first step of realization of this plan two abovementioned wholly owned RZD subsidiaries with their own wagon fleets (though insufficient to capture the whole markets) started their operations on this basis.
There are no entry barriers for the downstream market. The crucial assumption to our results is the inability by any single operator to serve all the downstream market because of its limited capacity. At the same time any two operators (incumbent’s downstream affiliate, most efficient competitor and second-highest cost rival) have enough potential to supply all the market. So customers choose the operating company with lowest price until the latter fully utilizes all its capacity and then switch to the other one. More efficient competitors can displace the incumbent’s final service only together. To make the model tractable we also have to assume that if several firms charge the same price for the final service, all customers prefer to purchase from the vertically integrated infrastructure owner (VIP).

The VIP can commit some sabotage $s$ that raises his rivals’ costs symmetrically by exactly this amount $s$. This type of discrimination is assumed to be costless for the VIP but having the upper bound $\bar{s}$. Thus the incumbent’s discriminatory activity is limited (for instance by the counteraction of antimonopoly authority). Nonintegrated rivals, on the contrary, can not exercise any level of sabotage.

The incumbent’s unit cost of producing the final service is known and equal to $c'_D$. Its upstream unit cost is constant and equal to $c_u$. When the VIP supplies the downstream market, it incurs the sum of these costs less $e$ which reflects his economies of scope (or economies of vertical integration). Thus under vertical integration the VIP’s unit cost is $c_u + c'_D - e$ comparing to $c_u + c'_D$ under vertical separation.

The VIP’s downstream capacity is limited by $\alpha \in [0,1]$ the maximum share of the market which is assumed to be exogenous (when $\alpha = 1$, all the market is served by the incumbent). The rest of the market can be captured by one or another rival operator. Each rival has limited capacity equal to $(1 - \alpha)$. They can not sell it to each other or buy from the incumbent’s downstream division. The entry turns out to be profitable if there is downstream production cost efficiency. But neither the most efficient competitor nor the incumbent can serve the downstream market alone.

Since the access to infrastructure is essential facility and can not be bypassed it is subject to regulation. Under the Scheme #1 the access charge $a$ is equal to the end-user tariff $T$, while other three regulatory schemes imply that access to infrastructure is charged by the regulator at the level $a > c_u, a < T$.

There are two downstream operators with marginal costs $c^R_D$ and $c^R_D + \Delta$ ($\Delta \geq 0$). The smaller is $\Delta$ the greater is the toughness of competition. $c^R_D$ is unknown to the regulator but has density function $f(c^R_D)$ with strictly positive support on $[\underline{c}, \bar{c}]$ and cumulative distribution function $F(c^R_D)$. Such an assumption reflects the idea that regulator usually makes decisions (chooses the future industry structure) before the entry occurs.

5. Discussion
To make an assessment of relative attractiveness of different structural schemes we have to consider first which scheme is prone to sabotage and which is not.

Lemma 1.
The VIP producer will exercise the least possible level of sabotage under vertical integration with end-user tariff and access charge being effectively equal to each other (Scheme #1).
The result is quite intuitive\textsuperscript{12}. If the only way for the competitor with private wagons to enter the unoccupied niche in the downstream market is to pay access charge that is equal to the end-user tariff, the VIP will do as much as possible to attract rivals. The latter will agree to enter if his profit is non-negative ($\pi^R \geq 0$). In practice, however, this constraint may not be binding (for example, captive operators at the first stages of reform experienced negative profits, but making losses they improved the export potential and revenues of their ‘mother’ companies). One can think about negative level of sabotage that decreases rivals’ costs\textsuperscript{13}.

**Lemma 2**

The VIP may exercise some positive level of sabotage under the vertical integration with access and final service regulation (Scheme #2) but its decision to be engaged in sabotage depends on the gap between fixed end-user tariff $T$ and access tariff $a$.

It is important to mention here that if the VIP is the second efficient firm in the industry (having marginal costs just between the two rivals), he can easily guarantee that all his capacity is fully utilized and the respective share of the downstream market is captured. Under the assumption of inelastic demand he might be indifferent whether to use sabotage or not because his price is always fixed at some level $T$. More important this assumption leads us to a situation when raising rivals costs has now effect on the rivals output (it the end-user price charged by independent rival is smaller than $\nu$.

Taking into account the expected gain from sabotage the VIP may want to decrease the probability of being the least efficient in the industry. Thus the VIP would prefer to exercise some positive (if not maximum) level of sabotage under Scheme #2.

**Lemma 3**

Under the vertical integration with access and final service regulation (Scheme #2) two levels of prices may constitute the market equilibrium if rivals are not very cost efficient:

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<th>incubment</th>
<th>lc.retailer</th>
<th>slc.retailer</th>
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<tr>
<td>if $c^R_D \in \left[ c, \hat{c} \right]$</td>
<td>$T$</td>
<td>$a + c^R_D + \Delta + s$</td>
<td>-</td>
</tr>
<tr>
<td>if $c^R_D \in \left[ c - \Delta, \hat{c} \right]$</td>
<td>$T$</td>
<td>$a + c^R_D + \Delta + s$</td>
<td>-</td>
</tr>
<tr>
<td>if $c^R_D \in \left[ \hat{c}, c - \Delta \right]$</td>
<td>-</td>
<td>$T$</td>
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where $\hat{c} = c_s + c^L_D - e - a - s$

The market will be fully served by two operators with limited capacities. The equilibrium prices will depend on the relationship between rivals’ marginal costs and the level of regulated tariff. The double price equilibrium occurs in the situation when sabotage is limited and end-user tariff is not very high with respect to rivals’ marginal costs. When the VIP is displaced by the non-integrated rivals there will be the unique price in the downstream

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\textsuperscript{12} For the formal proof of this and all the following lemmas and propositions see the Appendix.

\textsuperscript{13} The evidence of such a practice is abundant (granting the most attractive routes, providing ‘special’ discounts, etc.)
market that is equal to the regulated tariff $T$ (or slightly below it to prevent the VIP’s ‘come back’ to the market).

**Lemma 4**

The VIP will exercise the maximum level of sabotage $\bar{s}$ under vertical integration without regulation (Scheme #3).

This result reproduces those of Sappington (2006a).

**Lemma 5**

Under the vertical integration without regulation (Scheme #3) there will be a unique equilibrium price set at the level of marginal costs of that player who has not entered the market.

$$
\begin{align*}
\text{VIP} & : a + c_D^R + \Delta + \bar{s} \\
\text{lc.retailer} & : a + c_D^R + \Delta + \bar{s} \\
\text{slc.retailer} & : -
\end{align*}
$$

The price is unique because the market is contested by the threat of entry by another firm. Since we assumed limited capacities this price would be higher comparing to pure Bertrand case with no limits from the supply side. The reason is obvious: the market can not be served by the most efficient competitor and the less efficient one enters the market bidding equilibrium price up.

**Lemma 6.**

The VIP will refrain from sabotage under vertical separation (Scheme #4).

**Lemma 7.**

Under the vertical separation (Scheme #4) the equilibrium price downstream will be equal to the marginal costs of the potential entrant who contests the downstream market.

$$
\begin{align*}
\text{former monopolist} & : a + c_D^R + \Delta \\
\text{lc.retailer} & : a + c_D^R + \Delta \\
\text{slc.retailer} & : -
\end{align*}
$$

Note that neither scope economies no sabotage parameters influence the equilibrium prices.

Having derived equilibrium prices for different structural schemes we now turn to the direct comparisons of the welfare effects of the reform measured as the difference between the two consumers’ surpluses realized under the corresponding stages of the reform. We begin with an assessment of relative attractiveness of vertical separation in comparison to vertical integration with no downstream regulation. Specifically, we estimate the incremental expected consumer’s surplus as $D_{4-3} = ECS_s^4 - ECS_i^3$. 


Proposition 1.

The incremental expected consumer’s surplus under vertical separation relative vs. vertical integration without regulation (Scheme #4 vs. Scheme #3) decreases as the costs of the downstream rivals become more similar, i.e. \( \frac{dD_{i+3}}{d\Delta} > 0 \)

This proposition tells us that as the toughness of competition increases (relative proximity of the rivals’ costs becomes more pronounced and \( \Delta \to 0 \)) the consumer’s gain from separating the VIP (i.e. moving from vertical integration to vertical separation) decreases. This result contradicts to those of Sappington (2006a) derived for the case of standard Bertrand competition with no capacity constraints.

In the context of railway reform in Russia it would imply that in the presence of tough competition in the downstream market supplied by (or contested by) the unregulated VIP complete vertical separation should not be considered as a necessary precondition to guarantee higher welfare gain. In terms of reform measures this result may have the following possible interpretation: the RZD strategy to create wholly owned subsidiary operating downstream is worth considering together with the estimation of toughness of competition on particular niche markets.

Proposition 2.

The incremental expected consumer’s surplus under vertical separation relative to vertical integration without regulation (Scheme #4 vs. Scheme #3) decreases as the level of sabotage engaged by the VIP decreases (expected consumer’s surplus under integration increases), i.e. \( \frac{dD_{i+3}}{ds} > 0 \).

The natural interpretation of this proposition is the following. When regulator effectively controls the VIP ability to exercise sabotage consumer’s expected gain from the vertical separation will be less pronounced.

Now we turn to the direct assessment of the welfare gains from the vertical separation of the incumbent VIP producer which is regulated both downstream and upstream (i.e. end-user tariffs and access charge is determined by the regulator). To make the welfare comparison possible we still continue to assume that \( a > c_u \). In addition we need to assume, that \( T = c_u + c_{D} - e \), i.e. the end-user tariff set by the regulator for the VIP doesn’t allow the latter any (economic) profit to earn. One possible extension of the model (left for the further analysis) might be the introduction of some regulatory mark-up above marginal costs.

Proposition 3.

The incremental expected consumer’s surplus under vertical separation relative to vertical integration with access and end-user regulation (Scheme #4 vs. Scheme #2) increases as the costs of retail rivals become more similar, i.e. \( \frac{dD_{i+2}}{d\Delta} < 0 \)

This result holds when \( \Delta - \hat{c} < 0 \), or the cost advantage of the most efficient downstream rival comparing to the second one is small (implying tougher competition). Proposition 3
claims that the expected benefits of the complete vertical divestiture increases as the inter-rival competition downstream increases (\( \Delta \) approaches zero).

It is worth comparing this conclusion with Proposition 1. Different assumptions about the nature of regulatory schemes led as to the opposite conclusions which are discussed in the concluding section.

**Proposition 4.**

The incremental expected consumer’s surplus under vertical separation relative to vertical integration with access and end-user regulation (Scheme #4 vs. Scheme #2) increases as the maximum level of sabotage engaged by the VIP supplier increases, i.e. \( \frac{dD^{+2}}{ds} > 0 \)

Again this proposition provides as with an argument in favor of strict control over the possible discriminatory activities of the VIP. The weaker is the monitoring and antimonopoly law enforcement in the industry the higher is attractiveness of the structural separation as a reform strategy.

**6. Conclusions**

The primary purpose of the paper was to contribute to the discussion of pros and cons of the vertical separation in the context of Russian railways reform. The possibility of vertical divestiture of the RZD posed a challenge for the reformers: how to deal with the incentives of the vertically integrated monopolist to discriminate its downstream competitors? The RZD proposed a number of initiatives concerning the future industry structure, namely 1) to establish the wholly owned subsidiary to operate downstream and compete with its’ rivals, 2) to deregulate the end-user tariff for such a subsidiary and guarantee the same access conditions as its rivals have, 3) to privatize it partially but keep the control (51% of shares would be held by RZD). On the contrary, Federal Antimonopoly Service insists of vertical divestiture as a necessary instrument to prevent RZD affiliate from the discriminatory activities.

In the paper we developed the model to address these very issues. As can be seen from Proposition 2 and 4 the vertical divestiture of RZD becomes less attractive in terms of welfare gain accrued downstream when antimonopoly authority is able to effectively control any non-price discrimination activities by the RZD. This is true for the vertical divestiture of both downstream regulated and downstream unregulated vertically integrated producer.

The other two propositions imply that in case of contestable downstream market with relatively tough competition welfare gain from the vertical divestiture may depend on the type of regulation applied to the vertically integrated producer. Once unregulated but not fully privatized (Scheme #3) the welfare gain from the consequent complete separation (Scheme #4) would be less pronounced the tougher is the downstream competition. On the contrary, this tougher downstream competition would favor the complete vertical divestiture of the monopolist which is subject to end-user and access charge regulation (Scheme #2). Put it differently, if the cost difference between downstream rivals is not very much pronounced (the inter-rival competition in that market is intense and increasing) the complete vertical divestiture becomes more attractive relative to the situation, when the vertically integrated producer is regulated both upstream and downstream.
One should be very cautious when interpreting our findings since the only welfare measure we used in the model was the consumer’s surplus. More detailed analysis based on the total surplus estimation (weighted sum of consumer’s and producer’s surpluses) is to be considered as a proxy for the social objective function.
Appendix

Proof of lemma 1

The profit of the VIP depends on whether downstream rival agrees to operate in the market. The profit of retail supplier will be equal to \( \pi^r = (P-T-c^r_D-s)(1-\alpha) \).

The profit of the incumbent supplier will depend on whether retailer’s profit is positive or negative:

\[
\begin{align*}
\pi^l &= (T-c^l_D+e-c_u)\alpha+(T-c_u)(1-\alpha) \quad \text{if } \pi^R \geq 0 \\
\pi^l &= (T-c^l_D+e-c_u)\alpha \quad \text{if } \pi^R < 0
\end{align*}
\]

Downstream rival enter if \( \pi^R \geq 0 \), or \( P-T-c^R_D-s \geq 0 \). Consequently, \( P \geq T + c^R_D + s \)

If \( P = T \) then we get \( c^R_D + s \leq 0 \Rightarrow s \leq -c^R_D \).

Proof of lemma 2

Under Scheme #2 the end-user price and access charge are set by regulator at a certain levels \( T \) and \( a \) respectively. The pair \( (T,a) \) is rigid both upwards and downwards.

\[
\begin{align*}
\pi^l &= (T-c^l_D+e-c_u)\alpha+(a-c_u)(1-\alpha) \quad \text{if } T \leq a + c^R_D + s \\
\pi^l &= (T-c^l_D+e-c_u)\alpha+(a-c_u)(1-\alpha) \quad \text{if } a + c^R_D + s < T \leq a + c^R_D + \Delta + s \\
\pi^l &= (a-c_u) \quad \text{if } a + c^R_D + \Delta + s < T
\end{align*}
\]

Since the incumbent monopolist is strictly regulated and doesn't incur any costs of sabotage the strategic variable \( s \) has no direct effect on the incumbent’s profit. If downstream activity turns out to be more profitable for the incumbent, namely if \( T-a > c^R_D - e \), the firm will be engaged in sabotage just to guarantee that condition \( T \leq a + c^R_D + s \) is satisfied. The level of sabotage will not necessarily be at maximum in this case. It’s important to note here that starting from a certain level of sabotage incumbent becomes indifferent between raising rivals costs further and stop doing this since it’s profit remains unaffected.

Proof of lemma 3

Marginal costs of the firms are the following:

- \( c_u + c^l_D - e \) - Incumbent’s marginal costs
- \( a + c^R_D + s \) - Least-cost retailer’s marginal costs
- \( a + c^R_D + \Delta + s \) - Marginal costs of retailer with the second lowest costs.

Let’s denote \( \hat{c} = c_u + c^l_D - e - a - \bar{s} \).

Three cases are possible:

\[
a + c^R_D + \Delta + \bar{s} \geq a + c^R_D + s \geq c_u + c^l_D - e \Rightarrow c^R_D \in \left[ \hat{c}, \bar{c} \right]
\]

The incumbent supplier has the lowest costs and he will first serve the market at price \( T \). The most efficient rival will enter after him. But he is free in setting the price, so he will set it at the level of marginal costs of the rival with the second lowest costs: \( P = a + c^R_D + \Delta + \bar{s} \)
\[ a + c^R_D + \Delta + \bar{s} \geq c_u + c^I_D - e > a + c^R_D + \bar{s} \Rightarrow c^R_D \in \left[ c^\Delta c, \right. \right. \]

Both incumbent supplier and the least-cost retail operator will serve the market: incumbent at price \( T \); retailer at price \( a + c^R_D + \Delta + \bar{s} \). The difference is that retailer will now be the first who serves the market.

\[ c_u + c^I_D - e \geq a + c^R_D + \Delta + \bar{s} > a + c^R_D + \bar{s} \Rightarrow c^R_D \in \left[ c^\Delta c, \right. \right. \]

The incumbent supplier won’t operate at all, since his costs are the largest ones. In spite of the maximum level of sabotage this situation may also happen, but its probability reduces with the increase in sabotage (see lemma 1.1). Two rivals will capture the market. They know that they can rise the price up to \( T \) in order to gain more profit. Therefore, the price will be unique for the whole market and will be equal to \( T = c_u + c^I_D - e \).

Thus, the equilibrium prices in the market will be as following:

- **VIP**
  - if \( c^R_D \in [c^\Delta c, c] \)
    - \( c_u + c^I_D - e \)
    - \( a + c^R_D + \Delta + \bar{s} \)
    - -
  - if \( c^R_D \in [c^\Delta c, c^\Delta c] \)
    - \( c_u + c^I_D - e \)
    - \( a + c^R_D + \Delta + \bar{s} \)
    - -
  - if \( c^R_D \in [c^\Delta c, c^\Delta c] \)
    - -
    - \( c_u + c^I_D - e \)
    - \( c_u + c^I_D - e \)

**Proof of Lemma 4**

When the rival is more efficient then the VIP the profit of the latter will be:

\[ \pi^I = (a - c_u)(1 - \alpha) + \alpha(P - c_u - c^I_D + e). \]

When VIP is more efficient he will serve the market first and will set his price equal to the marginal costs of the retailer having the profit

\[ \pi^I = (a - c_u)(1 - \alpha) + \alpha(a + c^R_D + s - c_u - c^I_D + e). \]

As can be seen, there is a positive relationship between sabotage and incumbent’s profit: the higher the sabotage the higher the profit level \( \Rightarrow \) it is optimal to exercise the maximum level of sabotage \( \bar{s} \).

**Proof of lemma 5**

When the VIP has the lowest costs in the industry and he will be the first to serve the market.

1. \( a + c^R_D + \Delta + \bar{s} \geq a + c^R_D + \bar{s} \geq c_u + c^I_D - e \Rightarrow c^R_D \in [c^\Delta c, c] \)

The most efficient rival will enter after him. Under Scheme #3 they both are free in setting the price, so they will set it at the level of marginal costs of the rival with the second lowest costs \( a + c^R_D + \Delta + \bar{s} \).
2. \( a + c^R_D + \Delta + \bar{s} \geq c_u + c^I_D - e > a + c^R_D + \bar{s} \Rightarrow c^R_D \in \left[ \hat{c} - \Delta, \hat{c} \right] \)

Both the VIP and the least-cost downstream rival will operate at price \( a + c^R_D + \Delta + \bar{s} \). The difference is that retailer will now be the first who serves the market.

3. \( c_u + c^I_D - e \geq a + c^R_D + \Delta + s > a + c^R_D + \bar{s} \Rightarrow c^R_D \in \left[ \hat{c} - \Delta, \hat{c} \right] \)

The incumbent supplier won’t operate at all. Retailers will raise the price up to the marginal costs of the incumbent supplier in order to gain more profit. The price will be equal to \( c_u + c^I_D - e \)

Thus, the equilibrium prices in the market will be equal:

<table>
<thead>
<tr>
<th>VIP</th>
<th>least cost rival</th>
<th>second lc.rival</th>
</tr>
</thead>
<tbody>
<tr>
<td>if ( c^R_D \in \left[ \hat{c}, \bar{c} \right] )</td>
<td>( a + c^R_D + \Delta + \bar{s} )</td>
<td>( a + c^R_D + \Delta + s )</td>
</tr>
<tr>
<td>if ( c^R_D \in \left[ \hat{c} - \Delta, \hat{c} \right] )</td>
<td>( a + c^R_D + \Delta + \bar{s} )</td>
<td>( a + c^R_D + \Delta + s )</td>
</tr>
<tr>
<td>if ( c^R_D \in \left[ \hat{c}, \bar{c} - \Delta \right] )</td>
<td>-</td>
<td>( c_u + c^I_D - e )</td>
</tr>
</tbody>
</table>

Proof of lemma 6.

It is evident that under vertical separation \( s=0 \), since sabotage never increases equilibrium sales and therefore never increases incumbent’s profit.

Proof of lemma 7.

Under the vertical separation marginal costs of the firms will be different:

- \( a + c^I_D \) - Incumbent’s marginal costs. There are several differences with - incumbent’s marginal costs under integration: no economies of scope e, the unit price on the downstream market is now the same for all firms and equal to a.

- \( a + c^R_D \) - Least-cost rival’s marginal costs

- \( a + c^R_D + \Delta \) - Marginal costs of rival with the second lowest costs.

Three cases are possible:

1. \( a + c^R_D + \Delta \geq a + c^R_D \geq a + c^I_D \Rightarrow c^R_D \in \left[ c^I_D, \bar{c} \right] \)

The former monopolist has the lowest costs and he will first serve the market. The most efficient rival will enter after him. They both are free in setting the price, so they will set it at the level of marginal costs of the rival with the second lowest costs \( a + c^R_D + \Delta \).

2. \( a + c^R_D + \Delta \geq a + c^I_D > a + c^R_D \Rightarrow c^R_D \in \left[ c^I_D - \Delta, c^I_D \right] \)

Both the former monopolist and the least-cost rival operator will serve the market at price \( a + c^R_D + \Delta \). The competitor now serves the market first.
3. \( a + c_D^l - e \geq a + c_D^R + \Delta > a + c_D^R \Rightarrow c_D^R \in [c, c_D^l - \Delta] \)

The former monopolist will not operate at all. Independent operators will raise the price up to his marginal costs in order to gain more profit. The price will be equal to \( a + c_D^l \).

Thus, the equilibrium prices in the market will be:

<table>
<thead>
<tr>
<th></th>
<th>former VIP</th>
<th>least cost rival</th>
<th>second least cost rival</th>
</tr>
</thead>
<tbody>
<tr>
<td>if ( c_D^R \in [c_D^l, c] )</td>
<td>( a + c_D^R + \Delta )</td>
<td>( a + c_D^R + \Delta )</td>
<td>-</td>
</tr>
<tr>
<td>if ( c_D^R \in [c_D^l - \Delta, c_D^l] )</td>
<td>( a + c_D^R + \Delta )</td>
<td>( a + c_D^R + \Delta )</td>
<td>-</td>
</tr>
<tr>
<td>if ( c_D^R \in [c, c_D^l - \Delta] )</td>
<td>-</td>
<td>( a + c_D^l )</td>
<td>( a + c_D^l )</td>
</tr>
</tbody>
</table>

**Proof of proposition 1.**

Expected consumer surplus under vertical separation (Scheme #4) is:

\[
ECS^4_s = v - \int_{c}^{\xi} (a + c_D^R + \Delta) dF(c_D^R) \quad - \quad \int_{c}^{\xi} (a + c_D^R + \Delta) dF(c_D^R) - \int_{c}^{\xi} (a + c_D^l) dF(c_D^R) = \\
= v - \int_{c}^{\xi} (a + c_D^R + \Delta) dF(c_D^R) - \int_{c}^{\xi} (a + c_D^l) dF(c_D^R) = v - a - \int_{c}^{\xi} (c_D^R + \Delta) dF(c_D^R) - \int_{c}^{\xi} (c_D^l) dF(c_D^R) \\
(2)
\]

Expected consumer surplus under vertical integration without regulation (Scheme #3) is:

\[
ECS^3_I = v - \int_{c}^{\xi} (a + c_D^R + \Delta + \bar{s}) dF(c_D^R) - \int_{c}^{\xi} (a + c_D^R + \Delta + \bar{s}) dF(c_D^R) - \int_{c}^{\xi} (c_a + c_D^l - e) dF(c_D^R) = \\
= v - \int_{c}^{\xi} (a + c_D^R + \Delta + \bar{s}) dF(c_D^R) - \int_{c}^{\xi} (c_a + c_D^l - e) dF(c_D^R) \\
(3)
\]

Applying (2) and (3) we can find the difference between the two surpluses:
\[ D_{4-3} = E\text{CS}_s^4 - E\text{CS}_i^2 = v - a - \int(\hat{c}^R_D + \Delta)dF(\hat{c}^R_D) - \int(\hat{c}^L_D)dF(\hat{c}^R_D) - v + \int(\hat{c}^L_D + \Delta + s)dF(\hat{c}^R_D) + \int(\hat{c}^L_D + c_D^L - e)dF(\hat{c}^R_D) - \Delta F(\hat{c}^L_D + \Delta) - \int(c_D^L + \Delta + s)dF(\hat{c}^R_D) - \int(c_D^L + \Delta + s)dF(\hat{c}^R_D) + (a + \Delta + s + \Delta)F(\hat{c}^L_D - F(\hat{c}^L_D)) = -a + (a + s)F(\hat{c}^L_D - F(\hat{c}^L_D)) = -a + (a + s)[1 - F(\hat{c}^L_D)] = -a + a + s - (a + s)F(\hat{c}^L_D) + (c_u - e)[F(\hat{c}^L_D)] = s - (a + s)F(\hat{c}^L_D)] + (c_u - e)[F(\hat{c}^L_D)] = \bar{s} - s - (a + s)\hat{c}(\hat{c}^L_D - F(\hat{c}^L_D))\]

Now we can find the increment of the difference when \(\Delta\) changes. This is the partial derivative of \(D_{4-3}\) with respect to \(\Delta\):

\[ \frac{dD_{4-3}}{d\Delta} = (c_u - e - a - s)[f(\hat{c}^L_D)](-1) = (a + s + e - c_u)[f(\hat{c}^L_D)] \]

\([f(\hat{c}^L_D)] > 0; \ a \geq c_u\), since the incumbent supplier cannot sell the product to the retailers on the upstream market at the lower price then he has to pay himself. Therefore, \(\frac{dD_{4-3}}{d\Delta} > 0\).

**Proof of proposition 2.**

Using the expression for \(D_{4-3}\) and differentiating it with respect to \(\bar{s}\) we get:

\[ \frac{dD_{4-3}}{d\bar{s}} = -1 - F(\hat{c}^L_D) + (c_u - e - a - \bar{s})[f(\hat{c}^L_D)](-1) = -1 - F(\hat{c}^L_D) + (e + a + \bar{s} - c_u)[f(\hat{c}^L_D)]. \]

\(1 - F(\hat{c}^L_D) > 0\) and \(e + a + s - c_u > 0 \Rightarrow \frac{dD_{4-3}}{d\bar{s}} > 0\).

One may also look at the effect of scale economies and differentiate \(D_{4-3}\) with respect to \(e:\)

\[ \frac{dD_{4-3}}{de} = -F(\hat{c}^L_D) + (c_u - e - a - \bar{s})[f(\hat{c}^L_D)](-1) = -F(\hat{c}^L_D) + (e + a + \bar{s} - c_u)[f(\hat{c}^L_D)] \]

The sign is ambiguous, though when \(F(\hat{c}^L_D) \in [0,1]\) is sufficiently small comparing with \((e + a + s - c_u)[f(\hat{c}^L_D)]\), \(\frac{dD_{4-3}}{de} > 0\).

**Proof of proposition 3.**

The expected consumer’s surplus under vertical integration with downstream regulation (Scheme #2) is
\[ ECS_i^2 = v - \alpha \int \hat{c} TdF(c_D^R) - \int (a + c_D^R + \Delta + s) dF(c_D^R) - \alpha \int \hat{c} TdF(c_D^R) - \]
\[ - (1 - \alpha) \int (a + c_D^R + \Delta + s) dF(c_D^R) = v - \alpha \int \hat{c} TdF(c_D^R) - \int \hat{c} TdF(c_D^R) - \]
\[ - (1 - \alpha) \int (a + c_D^R + \Delta + s) dF(c_D^R) \]

(4)

Expected consumer surplus under vertical separation has already been derived in (2). Applying (2) and (4) we can find the difference in consumer’s surpluses:

\[ D_{4-2} = ECS_S^4 - ECS_i^2 = v - a - \int \hat{c} (c_D^R + \Delta) dF(c_D^R) - \int (c_D^R) dF(c_D^R) - v + \alpha \int \hat{c} TdF(c_D^R) + \int \hat{c} TdF(c_D^R) + \]
\[ + (1 - \alpha) \int (a + c_D^R + \Delta + s) dF(c_D^R) = -a - \Delta[F(\hat{c}) - F(\hat{c} + \Delta)] - \int (c_D^R) dF(c_D^R) - c_D^I[F(\hat{c} + \Delta) - F(\hat{c})] + \]
\[ + \alpha T[F(\hat{c}) - F(\hat{c} + \Delta)] + T[F(\hat{c}) - F(\hat{c} + \Delta)] + (1 - \alpha)(a + \Delta + s)[F(\hat{c}) - F(\hat{c} + \Delta)] + (1 - \alpha) \int (c_D^R) dF(c_D^R) = \]
\[ = -a + (T - c_D^I)[F(\hat{c} + \Delta) - F(\hat{c})] + (1 - \alpha)(a + \Delta + s)\Delta + \alpha T][F(\hat{c}) - F(\hat{c} + \Delta)] - \alpha \int (c_D^R) dF(c_D^R) = \]
\[ = -a + (T - c_D^I)[F(\hat{c} + \Delta) + (1 - \alpha)(a + \Delta + s) - \alpha \Delta + \alpha T][1 - F(\hat{c} + \Delta)] - \alpha \int (c_D^R) dF(c_D^R) = \]
\[ = -a + (T - c_D^I)[F(\hat{c} + \Delta) + (1 - \alpha)(a + s) - \alpha \Delta + \alpha T] - [(1 - \alpha)(a + \Delta + s) - \alpha \Delta + \alpha T][F(\hat{c} + \Delta)] - \]
\[ - \alpha \int (c_D^R) dF(c_D^R) = [T - c_D^I - (1 - \alpha)(a + \Delta + s) + \alpha \Delta - \alpha T][F(\hat{c} + \Delta)] + (1 - \alpha)(a + \Delta + s) - \alpha \Delta + \alpha T - a] - \]
\[ - \alpha \int (c_D^R) dF(c_D^R) = [(1 - \alpha)\Delta - \alpha T - \alpha a] + [\alpha \Delta - c_D^I + (1 - \alpha)(T - a - \Delta)] - \]
\[ - \alpha \int (c_D^R) dF(c_D^R) = [T - c_D^I - (1 - \alpha)(a + s) + \alpha \Delta - \alpha T][F(\hat{c} + \Delta)] + (1 - \alpha)(a + \Delta + s) - \alpha \Delta + \alpha T - a] - \]
\[ - \alpha \int (c_D^R) dF(c_D^R); \]

Now we can find the increment of the difference when \( \Delta \) changes. This is the partial derivative of \( D_{4-2} \) with respect to \( \Delta \):
\[
\frac{dD_{4-2}}{d\Delta} = -\alpha + \alpha[F(\hat{c} - \Delta)] + 1 \times [f(\hat{c} - \Delta)][\alpha\Delta - c'_{D} + (1 - \alpha)(T - a - \bar{s})] +
\]
\[
+ \alpha(\hat{c} - \Delta)[f(\hat{c} - \Delta)](-1) = -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][\alpha\Delta - c'_{D} + (1 - \alpha)(T - a - \bar{s}) - \alpha(\hat{c} - \Delta)] =
\]
\[
= -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][2\alpha\Delta - c'_{D} + (1 - \alpha)(T - a - \bar{s}) - \alpha\hat{c}]
\]

Substituting \(\hat{c} = c_{u} + c'_{D} - e - a - \bar{s}\) and \(T = c_{u} + c'_{D} - e\) into the expression above we get:
\[
\frac{dD_{4-2}}{d\Delta} = -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][2\alpha\Delta - c'_{D} + (1 - \alpha)(c_{u} + c'_{D} - e - a - \bar{s}) - \alpha c_{u} - \alpha c'_{D} + \alpha e + \alpha a + \alpha s] =
\]
\[
= -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][2\alpha\Delta - 2\alpha c_{D} + (1 - 2\alpha)(c_{u} - e - a - \bar{s})] =
\]
\[
= -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][2\alpha\Delta - 2\alpha c'_{D} + (1 - 2\alpha)(c - c'_{D})] =
\]
\[
= -\alpha + \alpha[F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][2\alpha\Delta + c - c'_{D} - 2\alpha c] = \alpha[F(\hat{c} - \Delta)] - 1 + [f(\hat{c} - \Delta)][2\alpha(\Delta - c) + (c - c'_{D})]
\]

\[
F(\hat{c} - \Delta) - 1 < 0; \Delta - \hat{c} < 0 \text{ and } c - c'_{D} < 0 \Rightarrow \frac{dD_{4-2}}{d\Delta} < 0.
\]

Strictly speaking this result holds if the end-user tariff \(T\) is set by the regulator at the level equal to the VIP’s marginal costs. This assumption makes the direct comparison between the two schemes possible.

**Proof of proposition 4.**

Using the expression for \(D_{4-2}\) and differentiating it with respect to \(\bar{s}\) we get:
\[
\frac{dD_{4-2}}{d\bar{s}} = (1 - \alpha) - (1 - \alpha)[F(\hat{c} - \Delta)] + [\alpha\Delta - c'_{D} + (1 - \alpha)(T - a - \bar{s})][f(\hat{c} - \Delta)](-1) =
\]
\[
= (1 - \alpha)[1 - [F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][-\alpha\Delta + c'_{D} - (1 - \alpha)(T - a - \bar{s})] =
\]
\[
= (1 - \alpha)[1 - [F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][-\alpha\Delta + c'_{D} + (\alpha - 1)(c_{u} + c'_{D} - e - a - \bar{s})] =
\]
\[
= (1 - \alpha)[1 - [F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][-\alpha\Delta + c'_{D} + (\alpha - 1)\hat{c}] =
\]
\[
= (1 - \alpha)[1 - [F(\hat{c} - \Delta)] + [f(\hat{c} - \Delta)][\alpha(\hat{c} - \Delta) + (c'_{D} - \hat{c})] =
\]
\[
1 - [F(\hat{c} - \Delta)] > 0; \hat{c} - \Delta > 0 \text{ and } c'_{D} - \hat{c} > 0 \Rightarrow \frac{dD_{4-2}}{d\bar{s}} > 0.
\]
References


