On the Induced Benefits arising from Transport Infrastructure Investments in a Spatial Oligopoly

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Abstract

We investigate the welfare effects of transport cost reductions resulting from infrastructure improvements, in a heterogeneous-costs Cournot oligopoly embedded in a transport network. Transport user's benefits are compared to economy-wide effects in order to asses the magnitude and sign of so-called indirect/induced benefits, not captured in a standard transport cost-benefit analysis. Both delivered and mill pricing arrangements are analyzed. The economic environment and network structure generalize Newbery (1998) to more complex settings with congestion effects in transport. Numerical simulations considering non-marginal changes for both a single and multi demand-node economy extend the theoretical results to more general contexts. The latter allows us to associate the indirect benefits with spatial and distributional effects.

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JEL Classification. L13, L90, H54, D61, D62

1 Introduction

The economic impact of transport infrastructure investments under market failures and other distortions have been a debated topic in transport economics and related disciplines for a rather long period of time. One branch of literature can be traced back to Mohring and Harwitz (1962) and is concerned with the presence of market failures or government induced distortions in the transport sector itself. It mainly focuses on the derivation of optimal pricing and investment rules under second-best conditions such as scale economies in infrastructure provision and congestion externalities (Winston, 1985). A different branch of literature, starting with Tinbergen (1957), deals with the "correct" measure of benefits arising from transport cost reductions (e.g. a road upgrade). A

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central issue in this branch of literature is to which extent an equivalence exists between direct benefits captured in a standard cost-benefit analysis (CBA), based on surplus measures as can be derived from transport demand functions, and the benefits reflected at an aggregate economic level –e.g. in real income changes-. This comparison was originally discussed in Tinbergen's work and can be dubbed, after Jara-Diaz (1986), as the surplus *change* equivalence problem.

For the first branch of literature a consensus on the pricing and investment rules has been more or less achieved over the years (Hau, 1998). In contrast, different views co-exist and many issues remain to be studied in the second branch (Davies, 1999). One central point of concern is to which extent the use of conventional CBA, in the presence of induced benefits, can lead to over- or under-investment in infrastructure.[1] In order to give an answer to this important issue, an understanding is needed first of the economics behind the difference in benefit measures. Our aim in this paper is to contribute to this understanding, based on a partial equilibrium model of imperfect competition embedded in a transport network and featuring endogenously derived demands for transportation services.

Recent attempts to address the surplus change equivalence problem, from both theoretical and applied perspectives, are presented by Venables and Gasiorek (1997, 1999), Hussain and Westin (1998), Broecker (1998) and Norman (1998). Common characteristics in these contributions are the use of general equilibrium frameworks, the comparison of economy-wide and transport sector benefits measures, the focus on market failures at the level of transport-using sectors, and no *detailed* consideration of network issues. A parallel stream of research is present in recent contributions to the urban economics literature. In Newbery (1998), Venables (2004) and Rossi-Hansberg (2004), the presence of benefits others than those captured by a standard CBA is highlighted for partial equilibrium frameworks and monocentric urban economies. In the last two papers the focus is on commuting transport, while the first -like the previous group of literature and our paper- focuses on freight transportation. In addition, this latter line of research does not address network issues. From these contributions, our work is most related to Newbery (1998).

Our strategy to study the surplus change equivalence issue is to compare the user's benefits computed as transportation consumer's surplus (TCS) changes, measured from derived transport demand functions, with the economy-wide benefits measured as the sum of surplus changes for both final consumers and producers. This comparison is discussed in terms of indirect or "induced" benefits, not accounted for in standard CBA. It is shown that for the case of delivered pricing the induced effects are related to productive efficiency changes linked to both technological and accesibility efficiency changes at the final producer level. A different case occurs if mill pricing is assumed since then induced

[1] As will be discussed later in the paper, the incorporation of indirect benefits in CBA rules will have implications for the optimal investment rule which are barely discussed in the relevant literature.

benefits correspond to changes in productive efficiency associated only with firm-level technological efficiency changes.

In the first part of the paper we employ a Cournot-Nash oligopoly model with one demand node, as in Newbery (1998), while for the second part a multi demand-node economy is embedded in a more complex network structure. Spatial and distributional effects from transport infrastructure and their relation to the magnitude and sign of indirect benefits are addressed in the latter case. Both delivered and mill pricing are discussed for the single demand node case. The network equilibrium models employed are of a partial equilibrium nature and assume that re-location of factors of production in response to infrastructure improvements is not possible. This is in contrast with general equilibrium models that feature re-location of both firms and workers, such as *new economic geography* models, that have been recently used to address the same basic topic as in this paper (e.g. Meléndez-Hidalgo et. al 2005). Furthermore, the focus here is on steady-state equilibrium models featuring interregional one-commodity networks, as opposed to network models emphasizing passenger flows (Beckmann et al 1956) or those investigating freight transportation in a dynamic context (Friesz et al. 2001). Additionally, we also allow in our network for the existence of transshipment nodes in order to better represent specific transport infrastructure elements.

Our main contribution to the literature is to combine market failures in both the transport using sectors and the transport sector itself within more general network environments, with possible applications in inland freight transportation: rails and roads. The network architecture chosen allow us to address *Braess'* paradox phenomena and to distinguish between node and link congestion effects. The existence of firm-internal congestion effects brings to discussion, in the sphere of inland transportation, recent findings for air transportation markets when carriers have market power (Brueckner 2004, Pels and Verhoef, 2004).

The rest of the paper is organized as follows. We first extend, in section two, Newbery (1998)'s model to a more complex network setting and functional forms for both preferences and technology. A distinction between mill and delivered pricing and a discussion on the effects on price and quantities after a reduction in two types of transport costs is provided. Next, we investigate Newbery's argument that, after a transport infrastructure investment, economy-wide benefits might actually decrease depending on the affected firm's (or group of firms) initial market share(s). To end this section we consider non-marginal changes under linear demands taking advantage of certain characteristics of the network game and rely on simulations for the non-linear case. Section three deals with non-marginal changes in a multi-demand node economy, employing a *spatial price equilibrium* (SPE) framework and relying completely on simulations.[2]

[2] This latter type of modeling approach has been previously used (under perfect competition) in Friesz and Jara-Diaz (1982) and Glazier and Niskanen (1991) to address the measuring of benefits under mode interactions, but has not been considered in recent contributions of the surplus change equivalence literature. Jara-Diaz (1986) employs a simple SPE to study this equivalence under imperfect competition.

2 An oligopoly embedded in a transport network: single demand-node

In this section we consider an oligopoly embedded in a transport network. The network is a directed graph, $G[L, A]$, composed of a set of nodes L and a set of arcs A. The set of nodes has $n + 2$ elements with n production (*origin*) nodes, a transshipment node and one demand (*destination*) node; the set of arcs is composed of n idiosyncratic and one common transport links (e.g. resources). Pricing is either of the *delivered* or *mill* type. Under the former pricing assumption all producers price to market (a single price) and incur in the transportation costs themselves. This pricing assumption is in contrast with recent literature dealing with congestion within an imperfectly competitive economy, where consumers travel to shopping nodes and pricing is at the factory (mill) level; consequently consumers assume the travel cost (Van Dender (2005), Proost and De Palma (2006)). We also consider this later pricing arrangement and show that under Cournot-Nash competition it is equivalent to delivered pricing in terms of equilibrium price and quantities.[3] Besides this equivalence, the composition (and incidence) of the indirect benefits differ under the two pricing arrangements. The next sub-section deals with the linear costs and general demand version of the extended model under delivered pricing and explains in more detail the economic environment.

2.1 The Basic Model: Delivered Pricing

In a Cournot oligopoly with n firms and a homogeneous good, let demand be denoted by Q and $s(Q)$ $(= \int_0^Q p(s)ds)$ define the gross benefit (*surplus*) accruing to consumers. The inverse market demand function corresponds to,

$$s'(Q) = p(Q) = p\left(\sum_{i=1}^{n} q_i\right)$$

where q_i denotes firm i level of production, with $i \in N = \{1, 2, ..., n\}$. Standard assumptions on $p(Q)$ include twice continous differentiability, with $p'(Q) < 0$ (as long as $p(Q) > 0$). Consumer surplus is given by $s(Q) - Qp(Q)$. There is only one demand node while production takes place in n different sites, each of them located at a certain distance from the *central market place* and linked to it by transport facilities. Transport costs are incurred by each firm in two segments of a network (e.g road, railroad). The first segment is specific to each firm (*idiosyncratic*): the i-th firm incurs a transport cost per-unit of output that is given by t_i. The second segment is common to all firms and has a transport cost per-unit of t_c. The connection node of the idiosyncratic and common segments is a transshipment node meant to represent infrastructure such as rail yards (Harker, 1986), or the entrance to a ring-road in the outskirts of a city.[4]

[3] This equivalency does not hold under Bertrand competition with homogeneous goods, with or without congestion effects in transport.

[4] The common link cost could also be interpreted as the cost incurred "in" the transshipment node. This is the case, for example, in recent models of air traffic when carriers have market power and

Marginal (and average) production costs are assumed constant for each firm, but differ accross firms.[5] The i-th firm sets output q_i to maximize *operational* profits (net of fixed costs, F_i, if any)[6], taking as given other firms output. This is equivalent to the following n-person game in normal form:

$$\Gamma = \langle N, Z_i, \pi_i \rangle, \tag{1}$$

where $Z_i = [0, \infty)$ is firm i's production set, and

$$\pi_i = (p(Q) - c_i - t_c - t_i)q_i - F_i, \qquad \mathbf{q} = (q_1, ..., q_n) \in \mathbf{Z} = \Pi_{i \in N} Z_i$$

is the payoff function for firm i. An equilibrium for this game is a vector of outputs where each q_i is the best response to $\mathbf{q}_{-1} = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_n)$. Additionally, assuming that all firms produce a strictly positive quantity in equilibrium (*interior equilibria*), this vector is defined by the simultaneous solution to the first-order conditions (f.o.c) for all firms. For the i-th firm the f.o.c. are:

$$p(Q) + \frac{dp(Q)}{dq_i} q_i - c_i - t_c - t_i = 0, \qquad i = 1, ..., n \tag{2}$$

which under Cournot-Nash competition reduces to:[7]

$$p(Q) + p'(Q)q_i = c_i + t_c + t_i \tag{3}$$

Let $\eta = -\frac{p}{Q}\frac{dQ}{dp}$ denote the market demand elasticity and $s_i = \frac{q_i}{Q}$ the market share for firm i in equilibrium, then

Lemma 1 *The i-th firm Cournot percentage mark-up (or Lerner index) is given by,*

$$\frac{s_i}{\eta} = \frac{p - (c_i + t_c + t_i)}{p} \tag{4}$$

Proof. Lemma 1 follows from substituting η and s_i in (3) and rearranging. ■

The equilibrium mark-up (i.e. *price-cost margin*) for firm i reflects the economic inefficiency (*allocative and productive*) arising under Cournot competition[8] and depends

congestion effects are present. (Pels and Verhoef, 2004). This network structure will allow us to bring to discussion in section 2.2.3 *Braess paradox* phenomena in parallel to Venables (1999) but in the context of freight transportation.

[5] Later in the paper we consider more general functional forms for the production costs.

[6] We are assuming here myopic profit maximization and for the moment neglect the problem of potential entry. Fixed costs are relevant in the case entry is allowed and can also be interpreted as a cost to enter remote markets (Melitz, 2003).

[7] In our setting $\frac{dp(Q)}{dq_i} = \frac{dp(Q)}{dQ}$, since we are only considering Cournot competition. Second order conditions are given by $2p'(Q) + p''(Q)q_i > 0$.

[8] Allocative inefficiency since consumers are buying at a price exceeding marginal costs of production. Productive inefficiency arise as production is not generated at the lowest cost.

directly on the firm market share and inversely on the elasticity of demand (i.e. a constant). One can rank firms according to their *overall* unitary costs, without loss of generality, so that $(c_i + t_i) \leq (c_{i+1} + t_{i+1})$ for any $q_i > 0$ and for $i = 1, ..., (n-1)$. A classification of firms follows with firm 1 as the most "*overall*" efficient firm while firm n will correspondly be the least efficient one. Using the definition of s_i leads to the following relation:

$$s_1 \geq ... \geq s_{n-1} \geq s_n > 0$$

Consequently, what determines the distribution of market shares across firms in the model with delivered pricing (still without congestion effects) is not only the transport costs each firm has to incur to deliver its production to the central market place, t_i and t_c (e.g. accesibility), as in Newbery (1998), but also its own technological efficiency, c_i (e.g. productivity). The latter is at a great extent controlled by the firm while the former is usually not.[9] One could make different assumptions on the magnitude and sign of the correlation between transport and production costs for a particular industry, or a set of firms from different industries, and in turn link the previous ranking to this correlation.[10] In the rest of the paper we do not assume a specific sign or value for it. For the delivered pricing case we assume the extreme case where firms have their own fleet of trucks, so transport costs for their shipments are totally internalized.

Lemma 2 *The Cournot-Nash equilibrium price, p, corresponds to,*

$$p = \frac{\overline{c} + t_c + \overline{t}}{1 - \frac{1}{n\eta}} \tag{5}$$

with $\overline{c} = \frac{1}{n}\sum_i^n c_i, \overline{t} = \frac{1}{n}\sum_i^n t_i$.

Proof. Lemma 2 follows from taking the sum over the n firms in (4). ■

The equilibrium price depends only on the sum and not on the distribution of both production and idiosyncratic transport costs.[11]

Lemma 3 *The Cournot-Nash industry output, Q, is the solution of:*

$$np + p'Q = n\overline{c} + nt_c + n\overline{t} \tag{6}$$

[9] Firms can control transport cost only at a limited extent for example with their location decisions. The major proportion of drivers of these costs are outside the firm control (e.g. competition in the transport markets services, level of infrastructure, govermental policy). Besides that, we are assuming that while firms incur the delivery cost themselves they have a certain range of maneuvere over these costs (or they even have their own fleet).

[10] On one hand, Newbery (1998) comments on the possibility of this correlation being negative. On the other hand, recent evidence provided by both theoretical and empirical work on trade with heteregenous firms suggests an opposite sign.

[11] This is a result already noticed, for the case of technological differences, in Bergstrom and Varian (1985).

and the Cournot-Nash equilibrium firm i production and market share are given by:

$$q_i = Qs_i = Q\left(\frac{1}{n} - \frac{(c_i - \bar{c} + t_i - \bar{t})\eta}{p}\right) \tag{7}$$

Proof. *See Appendix.* ■

First thing to notice in (7) is that in the absence of congestion effects and for a given total output, the distribution of output across firms does not depend directly on the common transport cost. The dependency is only indirect through the effect of this cost on the level of total output and price. Secondly, in equilibrium, firm size reflects cost efficiency. The market share of the i-th firm depends directly both on its "technological" productivity (i.e. the inverse of its marginal cost) and its "accesibility" to the market (i.e. the inverse of transport costs incurred). Output of firm i corresponds to the lower bound unweighted average output per firm arising in an homogeneous cost industry, $\frac{1}{n}$, adjusted by a factor reflecting demand and costs conditions. The term $\frac{\eta}{p}$ is inversely related to market power, as already shown in (4). Firm i output depends both on the differential accessibility (as in Newbery, 1998) and the differential productivity. Relatively "overall" inefficient (*below the average*) firms will produce less than efficient ones (*above the average*). Overall efficiency can be decomposed in technological and accesibility efficiency, then changes in it can be explained by changes in those two components. Since we are assuming here that firms incur the transport costs, this difference is less relevant here than in a mill pricing arrangement. A given firm can be efficient in one dimension but not efficient in the other, and viceversa, but what matters for market outcomes under delivered pricing is the overall efficiency. As will be emphasized in the rest of the paper the difference between direct benefits and economy-wide benefits hinges on the efficiency effects triggered by transport cost reductions (both of common and idiosyncratic kind).

Finally, equation (7) represents also the *derived transportation services demand*, generated by a particular final producer. The equivalence between final and transport demands is possible here because there is an unique region demanding the good and there is no production of that good in the consumption node (see Gasiorek and Venables, 1999). A more general approach defining the derived transport demand should be based on the "optimal" cost function for the final good producer (Friedlaender and Spady, 1980).[12]

The model presented is a generalization of the one employed in Newbery (1998) and its main purpose was to be more specific about the mechanisms behind total productivity changes after an infrastructure improvement. We will refer to this version later in the paper in order to compare the more general results that are going to be discussed in the following sections. In the next sub-section a discussion of the mill pricing alternative

[12] In a multi demand-node economy the derived transport demand will correspond to the net sum of freight inflow and outflow as shown in Friesz et. al. (1991). On the other hand, if there is a single demand and single production node the derived transport demand can be expressed in terms of underlying cost and final demand functions, as shown in Jara-Diaz (1986).

is presented and it is shown that under Cournot-Nash competition the equilibrium conditions (and comparative statics) are equivalent to the case with delivered pricing. An important difference arises concerning the incidence of the direct effects and composition of indirect benefits.

2.2 The Basic Model: Mill Pricing

Until now we had assumed that the producers take account directly of the transport cost when deciding which level of production they are going to supply. This is the delivered pricing assumption where consumers face a final price (possibly including unitary transport costs) and producers are due to deliver the product to the central market. Under congested traffic this assumption will imply that perfect internalization of congestion externalities will be the case for the idiosyncratic links while only partial internalization will apply for the common resource. We will consider in more detail later in the paper the implications of delivered pricing under congestion externalities.

An alternative to delivered pricing strategy corresponds to the case where firms decide only on the price at the factory level and consumers face directly the transport cost either by travelling to production sites (*shopping travel*) or paying a carrier for the delivery then facing the shipping cost from the factory site to where they are located, in our case the central market place. We analize this alternative pricing arrangment because it has implications for the composition of induced benefits. The analysis here follows closely van Dender (2005) for the case of Cournot competiton but in contrast to his paper we do not consider congestion at this stage. He discusses the case of an homogeneous oligopoly where each firm is linked to a group of consumers -sharing the same location- through idiosyncratic links. We extend this to our network structure.

The oligopoly is composed of n firms that supply a homogeneous good implying that consumer demand is, as before, given by the sum of firm outputs, $Q = \sum q_i$. Each firm's marginal production costs is given by c_i (constant) and each firm charges a (possibly) different price P_i.[13] Firm i profit function is then given by $\pi_i = (P_i - c_i)q_i$. As discussed above, consumers incur the access cost either by travelling to the firm or paying a (competitive) carrier for delivery. We focus on the latter interpretation in the rest of the paper. Under the same network structure as in the delivered pricing case the generalized price, g, is the sum of transport costs and (factory) price. In line with the previous section we consider only interior solutions, then the generalized price is:

$$g = t_i + t_c + P_i, \qquad\qquad i = 1, ..., n \tag{8}$$

Consumer demand, $Q(g) = \sum q_i$, declines with the generalized price, with $Q' < 0$. The inverse demand function under these conditions is denoted by $G(Q)$ (equivalent to $p(Q)$ in the previous subsection), with $G' < 0$. The firm maximizes profits while the

[13] This price should not be confused with the lower case p under delivered pricing.

consumer equilibrium (equality of generalized prices across locations) holds simultaneously. Following van Dender (2005) we can derive the pricing rule for firm k maximizing the following problem with respect to $P_k, q_k, P_j, j \neq k$,

$$\vartheta = (P_k - c_k)q_k + \sum_{i=1}^{n} \lambda_i (G - P_i - t_i - t_c)$$

giving as first-order conditions,

$$\begin{aligned} q_k &= \lambda_k \\ P_k &= c_k - G' \sum_{i=1}^{n} \lambda_i \\ \lambda_j &= 0, \ \forall j \neq k \end{aligned}$$

solving the system gives the pricing rule per-firm:

$$P_k = c_k - q_k G', \qquad k = 1, ..., n \tag{9}$$

and the optimal quantity for firm k is given by[14],

$$q_k = \frac{P_k - c_k}{-G'}, \qquad k = 1, ..., n \tag{10}$$

where after multiplying both sides of (10) by $\frac{G}{Q}$, replacing P_k from (8) and taking the demand elasticity as $\eta = -\frac{G'Q}{G}$, gives the Cournot (*percentage*) mark-up with mill pricing as,

$$\frac{s_k}{\eta} = \frac{G - t_k - t_c - c_k}{G}, \qquad k = 1, ..., n$$

which is equivalent to (4), valid when delivered pricing is in place. Additionally, the pricing rule (9) is also equivalent to the one holding with delivered pricing, (3), after replacing in it the equilibrium generalized cost, yielding,

$$G - G'q_k = t_k + t_c + c_k, \qquad k = 1, ..., n$$

This is an important result for at least two reasons. First, it makes easier the analysis since under certain assumptions (i.e. interior solutions) we can focus on one type of pricing policy but are still able to interpret the results under both types of pricing assumptions. Second, as discussed in van Dender (2005) and Engel. et. al (1999), the Cournot case under congested conditions (i.e. capacity constraints), no matter the pricing scheme, holds as a lower (*less competitive*) bound for homogeneous-good imperfect competition games in the sense that it can be shown that the applied mark-up and congestion internalization is greater than the one applied under Bertrand competition.[15] In the rest of the paper we refer to the mill pricing case when appropiate but the main discussion is conducted under delivered pricing.

[14] A paralell with the computation of market shares and ranking of firms for the delivered price case can be made here.

[15] As shown also in van Dender (2005), the mark-up under Bertrand competition disappear when

2.3 Comparative Statics: Equilibrium Quantities and Price

Total productive efficiency (and its components) are affected in different ways depending on which type of transport cost is changing (e.g. idiosyncratic or common).[16] In the next section, we will investigate in more detail these effects in the presence of congestion. In this subsection we explore the differences in magnitude and sign of the efficiency changes arising from a change in t_c, as compared to a change in one of the idiosyncratic costs, t_i. A comparison of economy-wide benefits and direct benefits and their connection with efficiency changes is also discussed. We start with an analysis of the effects of changes in the common link transport cost. Define $\Theta(Q) = \frac{p''Q}{p'}$ as the elasticity of the slope of the inverse demand with respect to Q (Seade (1980), Kimmel (1992)).[17]

Proposition 1 *After a (marginal) change in t_c the equilibrium total output, firm i output and price change according to:*

$$\frac{\partial Q}{\partial t_c} = \frac{n}{p'\gamma}, \quad \frac{\partial q_i}{\partial t_c} = \frac{\Theta(1 - s_i n) + 1}{p'\gamma}, \quad \frac{\partial p}{\partial t_c} = p'\frac{\partial Q}{\partial t_c} = \frac{n}{\gamma} \tag{11}$$

where $\gamma = \Theta + n + 1$.

Proof. See Appendix. ■

When $\Theta > 0$ then γ is positive, while for $\Theta < 0$, γ is positive only if $\Theta < n + 1$.[18] The level of Θ is in general a function of Q. Assuming $\gamma > 0$,[19] from the first term in (11) is clear that an improvement in the common resource will lead to an increase in output. The sign of the second term in (11) depends on the sign of Θ. If $\Theta < 0$ (resp. $\Theta > 0$), it will be negative for those firms with $s_i < \frac{1}{n}$(resp. $> \frac{1}{n}$), that is, for less overall efficient (resp. *more overall efficient)* firms. This implies that the reduction in the common resource transport cost, redistributes total output across firms. Only for the case of linear demand ($\Theta = 0$) its sign is unambigously negative, implying no redistribution of output and no change in overall efficiency -in total costs-, for a given price. On the other hand, if total output is heavily shifted to small -less overall-efficient

mixed traffic (oligopolic and competitive) is considered, while under Cournot competition the mark-up still holds but with an adjustment (negative) arising from the competitive traffic. This leads to think that considering mixed traffic in our context will destroy the equivalency we are relying on, despite the adjustment with mixed traffic in the Cournot case is usually small.

[16] As will be shown later in the paper the results also depend on the technological conditions and preferences applying for the economy.

[17] Θ measures the degree of concavity of the inverse demand function (Frevier and Linnemer (2005)). If demand is linear $\Theta = 0$, whereas if it is concave (resp. convex), then $\Theta > 0$ (< 0). To ensure the existence of an equilibrium Θ cannot be too negative. It is usually assumed that $\Theta \geq -2$ for the existence and uniqueness of an equilibrium. Additionally $p' + q_i p''$ is also the slope of the residual (perceived) marginal revenue which is negative when firm outputs are assumed strategic substitutes.

[18] Under the standard assumption that $\Theta \geq -2$, γ will be in general positive for an oligopoly ($n \geq 2$).

[19] Two special cases where Θ is constant arise from a specific type of inverse demand functions. If $p = a - bQ^\sigma$, then $\Theta = \sigma - 1$. Notice that if $\sigma < 0, b < 0$ and $a = 0$ an isoelastic function arises, $p = -bQ^\sigma$. For $\sigma = 1$ the linear case holds.

firms- from large -more efficient firms-, an improvement in the common resource may lead to a decrease in total efficiency (an increase in total costs). This possibility occurs only when the demand is convex ($\Theta < 0$). The final composition of the increase (or decrease) in total productive efficiency, in terms of accessibility and technological efficiency, will depend on the correlation between transport and production unitary costs. If both transport and production unitary costs are the same for all firms ($c_i = \widetilde{c}; t_i = \widetilde{t}$), or the demand is linear ($\Theta = 0$) the effect on firm i-th quantity reduces to a constant. Then no reallocation of output is generated implying no effect on productive efficiency.[20] Finally, the effect on the equilibrium price is given by the third term in (11), which is in general (for $\gamma > 0$) positive, implying a increase in consumer surplus. It is also the case that the effect on prices is less than 1, then the *passthrough* of the decrease in transport costs in not reflected completely in prices, meaning that part of the efficiency gains are kept as increased profits, that is, allocative inefficiency.We now turn to the analysis of the effects of a change in t_k, the idiosyncratic transport cost for a single firm k, on price and quantities.

Proposition 2 *After a (marginal) change in t_c the equilibrium total output, firm i output and price change according to,*

$$\begin{aligned} \frac{\partial Q}{\partial t_k} &= \frac{1}{p'\gamma}, \\ \frac{\partial q_i}{\partial t_k} &= \frac{n + \sum_{i \neq k} \frac{q_i p''}{p'}}{p'\gamma}, \qquad i = k, \ \frac{\partial q_i}{\partial t_k} = \frac{-(1 + \frac{q_i p''}{p'})}{p'\gamma}, \ i \neq k, \\ \frac{\partial p}{\partial t_k} &= p' \frac{\partial Q}{\partial t_k} = \frac{1}{\gamma} \end{aligned} \tag{12}$$

Proof. See Appendix. ■

The sign of the two components in the second line of (12) depends on the sign of the denomonator, which under the standard extra (and usual) assumption that $p' + p'' q_i < 0$,[21] is negative for the first and positive for the second. Under these conditions an idiosyncratic link improvement increases output for both the directly affected firm and the aggregate economy, and decreases the individual outputs of all other firms.[22] This is in contrast with the previous case where depending on the parameters, output increases for some firms and decreases for the rest, or could even increase for all firms. In the present case, overall efficiency can increase or decrease depending on the magnitude of the positive effect on output that the firm directly affected by the infrastructure improvement experiences, as compared to the effect on the rest of firms. The effect on the equilibrium price for this case is also positive, implying a increase in consumer welfare, and given by

[20] In fact, only allocative inefficiency is present in this case.

[21] This meaning that the residual (perceived) marginal revenue has a negative slope, which implies outputs strategic substitutability.

[22] In the presence of congestion in the common link these effects would bring to discussion Braess paradox phenomena as in Venables (1999). We discuss this later in the paper.

the third line in (12). In contrast with the previous case, the magnitude of the effect on prices and consumer welfare is smaller $(n-1)$ times. Additionally, the effect on price is again less than one.

2.4 Economy-wide Effects, Direct and Indirect Benefits

In order to asses the effects of infrastructure improvements we look now at *economy-wide* impacts. Social welfare (incorporating capital expenses) is our base for measuring economy-wide effects and corresponds to the sum of consumers, producers and government (network operator) surplus. Economy-wide effects are defined as the change in social welfare after an infrastructure improvement. They are first decomposed into economy-wide *benefits* and capital expenses changes. In turn, economy-wide *benefits* are going to be classified either as of direct or indirect *(induced)* nature.[23].The economy-wide *benefits* in turn include the usual input cost-savings benefits but also the result of product market strategic interactions across firms realized as productive efficiency changes.[24] These latter effects, as before, are what we identified as "induced" effects and can be positive or negative.[25]

The magnitude and sign of the indirect effects is assesed from the perspective of a social planner who controls the parameters characterizing the capacity of the transport network and also possibly charge for the use of this infrastructure.[26] The components of the change in total social welfare, W, after an infrastructure investment are classified either as indirect effects, I (productive efficiency changes), direct benefits, D, or capacity cost changes K (capital cost per-unit). In our setting a reduction in either t_k -the capacity related parameter in firm's k idiosyncratic link- or t_c -the common resource capacity- are investigated. In turn, the following relationship holds between all these terms,

$$\begin{aligned} EWE &= -\frac{\partial W}{\partial z} \\ &= I + D + K_z \\ &= EWB + K_z \end{aligned} \tag{13}$$

where z corresponds to the capacity-related parameter in a particular link. After an infrastructure improvement the net (*total*) economy-wide effect (EWE) will correspond

[23] This last classification is in parallel with the linear case discussed in section 2.

[24] A similar approach is taken in Newbery (1998), "the question asked is how much larger the total social benefits are compared with the apparent benefits, or more precisely, what is the ratio of the non-transport to the transport benefits". In our context the apparent benefits correspond to the transport benefits while the non-transport benefits are what we named induced effects.

[25] At this point is also important to notice that in the presence of congestion externalities the induced effects can be determined not only by strategic interactions across firms in the product marker but also in the transport market.

[26] Is important to notice at this point that we will not be maximizing welfare. The focus is on the composition of the change in social welfare after an infrastructure improvement, and in particular the classification of its components either as direct or indirect benefits. Despite this, we will be able to relate our results with first and second best rule for capacity expansion.

to the difference between economy-wide benefits (EWB) and the change in capital cost expenses (K_z).[27] Our main interest is on the magnitude and size of I as compared to D, then we focus on EWB, but will later refer to (13) when necessary (for example, to relate our results on the indirect benefits to *first* and *second*-best rules for capacity expansion). To be consistent with previous literature we would like to interpret our results in terms of a multiplier, m, by which direct transport benefits should be scaled to obtain total EWB. That is,

$$EWB = I + D = mD$$

This multiplier in turn implies a ratio, r, between indirect and direct benefits, which is our "index" of induced benefits not accounted for in conventional CBA,

$$m = 1 + \frac{I}{D} = 1 + r \tag{14}$$

with r, as shown in (14), as the ratio of indirect to direct benefits.

Social welfare changes consist of the sum of changes in industry profits and consumer surplus.[28] The latter can be anticipated from the definition of consumer welfare given at the beginning of this section, and depends directly on the effect of the infrastructure improvement on the equilibrium price. The change in profits depends -for a marginal change in transport costs- on the effects in quantities. Total social welfare is given by,

$$W = CS + \sum \pi_i = \int_0^Q p(s)ds - \sum (c_i + t_c + t_i)q_i$$

Then the economy-wide benefits (EWB) due to a reduction in a transport cost of either the common or the k-th idiosyncratic link type are equal to the (negative of) change in W ($\frac{\partial W}{\partial t_{c(k)}}$), given respectively by,

$$\begin{aligned} EWB_{dt_c} &= -p\frac{\partial Q}{\partial t_c} + \sum (c_i + t_c + t_i)\frac{\partial q_i}{\partial t_c} + \sum q_i \\ &= \underbrace{\sum (p - c_i - t_c - t_i)(-\frac{\partial q_i}{\partial t_c})}_{\textit{Indirect } (=I_{dt_c})} + \underbrace{\sum q_i}_{\textit{Direct}} \end{aligned} \tag{15}$$

[27] The change in welfare is multiplied by -1 to account for a *reduction* in transport costs. Additionally, since we are considering the inverse of capacity by using z (de Palma and Leruth (1989)), then $K_z < 0$, explaining the plus in from of it.

[28] At this point we ignore revenue for infrastructure charging and expenses from captial investment as congestion is not present. This will be incorporated in the measure of social welfare in the next section.

and

$$\begin{aligned} EWB_{dt_k} &= -p\frac{\partial Q}{\partial t_k} + \sum (c_i + t_c + t_i)\frac{\partial q_i}{\partial t_k} + q_k \\ &= \underbrace{\sum (p - c_i - t_c - t_i)(-\frac{\partial q_i}{\partial t_k})}_{Indirect(=I_{dt_k})} + \underbrace{q_k}_{Direct} \end{aligned} \tag{16}$$

From the first line in both (15) and (16) it is apparent that the economy-wide benefits can be decomposed in three terms: *extra-revenue effect, industry-costs effect* and *direct benefits.*[29] The revenue effect comprises the marginal effect on industry revenue while the industry-costs effect reflects the impact on costs due to the recomposition of market shares. Together these two effects compose our measure of indirect effects. The second line in both (15) and (16) shows the economy-wide benefits as composed of *indirect* and *direct* benefits. The sign of this sum will depend on the reallocation of market shares generated with the infrastructure improvement, which in turn depends on the value of Θ.

Proposition 3 *The indirect benefits for both an idiosyncratic and common transport link marginal change reflect changes in efficiency associated with redistribution of ouput across firms, as given by,*

$$I_{dt_c} = \underbrace{p\sum \frac{s_i}{\eta}(-\frac{\partial q_i}{\partial t_c})}_{\leqslant 0} \tag{17}$$

$$I_{dt_k} = \underbrace{p\frac{s_k}{\eta}(-\frac{\partial q_k}{\partial t_k})}_{>0} + \underbrace{p\sum_{i\neq k}^{n} \frac{s_i}{\eta}(-\frac{\partial q_i}{\partial t_k})}_{<0} \tag{18}$$

Proof. See Appendix. ■

Both (17) and (18) show that the indirect effects result from the weighted sum across firms of marginal changes in quantities where the weights are the product of the (initial) price and corresponding firm mark-up. As discussed before it is possible that this reallocation generates negative indirect benefits which can compensate in magnitude the (positive) direct benefits.

Corollary 1 *In an oligopoly defined by (1) under symmetry, so that $c_i = c_j$ and $t_i = t_j$ for all $i \neq j$, indirect benefits for a common and idiosyncratic resource projects are given, respectively, by:*

$$I_{dt_c} = \frac{Q}{n\gamma}\left[n\left(\Theta + 1\right) - \Theta\right]$$

[29] Notice that for a reduction in the transport costs the welfare change has to be multiplied by -1 to account for the benefits.

and

$$I_{dt_k} = \frac{Q}{n\gamma}$$

Corollary 2 *In an oligopoly defined by (1) under linear demand, so that* $\Theta = 0$*, indirect benefits for a common and idiosyncratic resource projects are given, respectively, by:*

$$I_{dt_c} = \frac{Q}{\gamma}$$

and

$$I_{dt_k} = \frac{Q}{\gamma}\left[s_k\left(n+1\right)-1\right]$$

Corollary 1 points to a case where the reallocations (if any) are linked only with demand conditions. In particular, there are no reallocation effects for an idiosyncratic project while there is still possible to see reallocations for a common resource project. Corollary 2 refers to a situation where reallocations are possible only through technology conditions. In particular, there is no possibility for reallocation effects through a common resource project but the opposite occur for an idiosyncratic link project. For the latter case, the sign of the indirect effects depends on the sign of the term in square parenthesis on the RHS, which in turn define a threshold for the market share of the directly affected firm.[30]

The indirect effect in (17) and (18) can be also decomposed into one *general* effect and a *industry concentration* effect, given respectively by (*see appendi*x):

$$I_{dt_c} = \underbrace{p'Q}_{<0}\left[\underbrace{H\frac{\partial Q}{\partial t_c}}_{<0} + Q\underbrace{\sum s_i\frac{\partial s_i}{\partial t_c}}_{\lesseqgtr 0}\right] \tag{19}$$

and

$$I_{dt_k} = \underbrace{p'Q}_{<0}\left[\underbrace{H\frac{\partial Q}{\partial t_k}}_{<0} + Q\left[\underbrace{s_k\frac{\partial s_k}{\partial t_k}}_{<0} + \underbrace{\sum_{i\neq k} s_i\frac{\partial s_i}{\partial t_k}}_{\lesseqgtr 0}\right]\right] \tag{20}$$

with $H = \sum s_i^2$, as the *Hirschman-Herfindhal* index of industry concentration. The first term on the RHS in both (19) and (20) is, under our assumptions, negative and given respectively by propositions 1 and 2. This term can be termed as an *increased*

[30] This threshold is $s_k = \frac{1}{n+1}$ and was already derived in Newbery(1988).

competition effect which increase welfare through greater profits and consumer surplus. The second term is related to the recomposition of market shares and it could be negative if the reallocation of market share goes from the most efficient (*bigger*) to the less efficient (*smaller*) firms, as discussed before. This second term points to possible positive effects on welfare through higher market concentration under Cournot competition. (Farrell and Shapiro, 1990). The conditions under which the indirect benefits are negative, implying that the usual CBA benefit measure overstate the economy-wide benefits, can be summarized computing a thershold for r in (14).

Proposition 4 *For a non-linear demand ($\Theta \neq 0$), the sign of the indirect to direct benefits ratio after a common resource (marginal) project, r_{dt_c}, depends on the initial level of the Hirschman-Herfindhal index, H, according to:*

$$r_{dt_c} < 0 \iff H > \frac{\Theta + 1}{\Theta n} \tag{21}$$

while for an idiosyncratic resource project with linear or non-linear demand, the sign of the indirect to direct benefits, r_{dt_k}, depends on the (initial) level of the market share of the directly affected firm, s_k, according to:

$$r_{dt_k} < 0 \iff s_k < s_k^T = \frac{\sum_{i \neq k}^{n} s_i(1 + \frac{q_i p''}{p'})}{1 + \sum_{i \neq k}^{n}(1 + \frac{q_i p''}{p'})} \tag{22}$$

Proof. See Appendix.[31] ■

Then negative *economy-wide* benefits could even be negative under both type of projects, but under our assumptions will require extreme convexity of the demand function ($r < 1$).[32] Before finishing this sub-section it should be noticed that, with the appropiate equivalent interpretation of variables, parallel comparative statics for mill pricing can be derived. More especifically, the indirect effects under mill pricing will refer only to technological efficiency changes since the transport sector is separated from the firm (and there are no congestion costs). In this case, direct benefits will include not only the initial level of quantities transported but also the savings in transport costs associated with the reallocation of market shares across firms.[33] In the next section we investigate the effects of infrastructure improvement projects in a more general version of the model just presented, dealing with both delivered and mill pricing.

Corollary 3 *In an oligopoly defined by (1) under symmetry, so that $c_i = c_j$ and $t_i = t_j$ for all $i \neq j$, the ratio of indirect to direct benefits for a common and idiosyncratic resource project are given, respectively, by:*

$$I_{dt_c} = \frac{1}{\gamma}\left[(\Theta + 1) - \frac{\Theta}{n}\right]$$

[31] This result is reminiscent of a Braess paradox where, if properly identified, the elimination of the link for the firm with a small enough market share can actually increase welfare, as discussed in Lahiri and Ohno (1988).

[32] This possibility is similar to the Down/Thompson paradox identified in transport economics.

[33] See appendix $A.7$ for details.

and

$$I_{dt_k} = \frac{1}{\gamma}$$

Corollary 4 *In an oligopoly defined by (1) under symmetry, so that* $\Theta = 0$*, the ratio of indirect to direct benefits for a common and idiosyncratic resource project are given, respectively, by:*

$$I_{dt_c} = \frac{1}{\gamma}$$

and

$$I_{dt_k} = \frac{1}{\gamma}\left[(n+1) - \frac{1}{s_k}\right]$$

2.5 Infrastructure improventments: nonlinear costs

2.5.1 Preliminaries

Infrastructure improvement projects can take several forms in our setting. It could be, for example, that a common resource (e.g. road) experiences an upgrade affecting all the firms using it in a uniform manner, which is a relevant scenario for cities when different sets of firms reach the central market using alternative common resources (as well as their idiosyncratic links). Alternatively, one can consider an improvement in a single (or a set of) idiosyncratic link in the network. We confine ourselves to the case of a single idiosyncratic link improvement in the first part of this section. For this case two scenarios are considered: firstly with congestion effects in all idiosyncratic links but no congestion in the common resource; secondly we consider the opposite situation, no congestion in the idiosyncratic links but with congestion effects in the common resource. Later in the section we will study the effects of an improvement in the common transport link under congested use with fixed transport cost in the idiosyncratic links. Before undertaking the analysis a discussion on the differences in costs functions and welfare measures between delived and mill pricing settings, and some definitions needed for the rest of the paper, is provided.

Delivered Pricing To be able to analyze more general cases (e.g. scale economies) we consider non-linear production and transport cost functions in terms of output and shipments, respectively.[34] More precisely, under delivered pricing, firm i's production and transport cost functions are given by,

$$\begin{aligned} C_i(q_i) &= c_i(q_i; a_i, d_i)q_i = \left(a_i + d_i\frac{q_i}{2}\right)q_i, & c_i > 0, d_i > (<)0 \\ T_i(q_i, Q; e_i, f_i, j) &= T_i^I(q_i; e_i, f_i) + T_i^C(q_i, Q; j) & e_i, f_i, j > 0 \end{aligned}$$

[34] Output and shipments in a Cournot model with only one demand node are the same thing. This is not the case for the model considered in the second part of the paper where more than one demand node is possible and each firm is located in each demand node.

where $T_i^I(q_i; e_i, f_i) = t_i(q_i)q_i = e_iq_i + f_i\frac{q_i^2}{2}$ as the total idiosyncratic cost function and $T_i^C(q_i, Q; j) = t_c(Q)q_i = j\frac{Q^2}{2}q_i$, the common resource total cost function. While for the production function we allow for both increasing ($d < 0$) or decreasing ($d > 0$) returns to scale, for the transport cost function only congestion effects are possible. Concerning transport costs, average costs functions are respectively given by,

$$
\begin{aligned}
t_i(q_i; e_i, f_i) &= e_i + f_i\frac{q_i}{2} && (23)\\
t_c(Q; j) &= j\frac{Q^2}{2} && (24)
\end{aligned}
$$

that is, total idiosyncratic costs can are *quadratic* in q_i. when $f_i \neq 0$. For the i-th firm the profit function becomes:

$$\pi_i(q_i, Q) = p(Q)q_i - C_i(q_i) - T_i(q_i) - (\tau^C + \tau_i^I)q_i \tag{25}$$

with τ^C and τ_i^I as (possible) infrastructure use charges. Due to delivered pricing it is optimal for the firm to incorporate in the price the congestion generated in its idiosyncratic link, then we assume the government does not issue a charge in these links ($\tau^I = 0$).[35] Associated first-order condition from (25) are:

$$
\begin{aligned}
p(Q) + p'(Q)q_i &= C_i'(q_i) + T_i^{I\prime}(q_i) + T_i^{C\prime}(q_i) + \tau^C && (26)\\
&= C_i'(q_i) + t_i(q_i) + t_i'(q_i)q_i + t_c(Q) + t_c'(Q)q_i + \tau^C\\
&= C_i'(q_i) + t_i(q_i) + \underbrace{t_i'(q_i)q_i}_{Idios.Cong.} + t_c(Q) + \underbrace{t_c'(Q)Q}_{Com.Cong.}\, s_i + \tau^C
\end{aligned}
$$

Then in equilibrium a firm totally internalize the congestion generated in the idiosyncratic link and only partially internalize the congestion it generates in the common link (Brueckner 2004), shown in the last term of the second line in (26), which can be re-written as, $s_i(Q\frac{\partial t_c}{\partial Q})$, after multiplying it by $\frac{Q}{Q}$. The firm equilibrium conditions under the assumed production and transport cost functions become,

$$p(Q) + p'(Q)q_i = a_i + d_iq_i + e_i + f_i\frac{q_i}{2} + f_i\frac{q_i}{2} + j\frac{Q^2}{2} + jQ^2s_i + \tau^C$$

Mill Pricing under Congested Transport Under mill pricing the transport cost levels arise as an equilibrium in the transport market as we assume that carriers are not controlled by the shippers (i.e. producers). In order to simplify the analysis we take the transport industry as competitive and user charges are optimal.[36] We assume that all suppliers of transport services in a particular link are homogeneous, sufficiently competitive with constant returns to scale and with unit supply cost equal to u, which without loss of generality we set to zero.[37] Other unit generalized costs of the trips

[35] The same applies to mill pricing as will be discussed later in this section.
[36] We discuss in detail the optimality of charges later in the section.
[37] See Kidokoro (2004) for a similar analysis.

-including monetized time costs-, incurred by the transport industry in the idiosyncratic and common links are given, respectively, by (23) and (24). The profits of the set of transport services suppliers aggregate to,

$$\begin{aligned}\pi_i^I &= (p_i^I - \tau_i^I)q_i - t_i q_i \\ \pi^C &= (p^C - \tau^C)Q - t_c Q\end{aligned}$$

where $(p_i^I - \tau_i^I)$ and $(p^C - \tau^C)$ correspond to net prices (after tolls) received from consumers. As in the delivered pricing case, we assume that $\tau_i^I = 0$ since final producers optimally incorporate in the price the congestion effects in their idiosyncratic link.[38] Perfect competition induce zero profits for the entire industry (in every link) implying a (marginal) transport rate for a trip in an idiosyncratic and common links, respectively, given by,

$$p_i^I = t_i \quad (27)$$
$$p^C = t_c + \tau^C \quad (28)$$

The generalized price for consumers in every link is given by an expression similar to (8), modified to account for the fact of a perfectly competitive industry in transportation, that is,

$$g = p_i^I + p^C + P_i, \qquad i = 1, ..., n \quad (29)$$

The pricing rule per-firm with congestion then will be given by, (*see appendix*)

$$P_k = c_k + (t_k' + t_c')q_k - q_k G', \qquad k = 1, ..., n \quad (30)$$

This is in paralell to van Dender (2005) and implies that the producer totally internalize in the price the congestion in the idiosyncratic link and only partially internalize the congestion created on the common link, which can be seen after rewritting (30) as,

$$P_k = c_k + t_k' q_k + t_c' Q s_k - q_k G', \qquad k = 1, ..., n$$

The pricing rule in (30) can again be rewritten in a form equivalent to (26), using (29).

Government, Consumer and Producer Surpluses A final word, before turning to the evaluation of indirect benefits, has to be said in relation to the different actors' surpluses. On the network operator (*government*) side the surplus is composed by the difference between user's charge revenue (*tolling*) minus the operation and capacity costs incurred for the level of service provided.[39]The government can charge a toll per unit of the good using the common resource of τ^C, and a toll per unit of the good using a particular idiosyncratic link of τ_i^I. Total revenues then correspond to the sum of revenue

[38] See the appendix for details on this.
[39] We assume, without loss of generality, that operation costs are zero.

across links, as in (31). Since we assume nill tolls in idiosyncratic links, then total revenue reduces to $R(q_i, Q; e_i, f_i, j) = \tau_i^I \sum q_i$. Following de Palma and Lerouth (1989) we assume an inverse relation between the parameters in the transport cost functions (e_i,f_i and g) and the capacity level in the respective link. Then it follows that the total capacity costs associated with a certain capacity level in the network corresponds to the multiplication of the inverse of the parameter times the cost of capital applicable to the respective link, as in (32). The government surplus under these conditions is then given by $GS = R - K$. User's charges revenue, R, and total capacity costs, K, are given respectively by:

$$R(q_i, Q; e_i, f_i, j) = \tau^C Q + \tau_i^I \sum q_i \tag{31}$$

$$K(q_i, Q; e_i, f_i, j) = \frac{\kappa^C}{j} + \sum \frac{\kappa_i^I}{z_i} \tag{32}$$

The other two components of social welfare are the consumer surplus and the sum of firms profits including possible link use charges. Consumer surplus for the delivered pricing case is given by $CS^D = \int_0^Q p(s)ds - Qp(Q)$, while for the mill pricing arrangement is given by $CS^M = \int_0^Q p(s)ds - Qg(Q)$, with g as in (29). Total profits are also depending on the pricing arrangement, with total profits under delivered pricing given by (25) while for mill pricing final i-th producer' profits ($\pi_i = \sum(P_i - c_i)q_i$) are summed up with the transport industry profits, with the latter being zero in equilibrium. Total social welfare under delivered and mill pricing arrangements is given by[40]:

$$W = \int_0^Q p(s)ds - \sum_{i=1}^n c_i q_i - \sum_{i=1}^n t_i q_i - \sum_{i=1}^n (t_c(Q) + \tau^C) q_i + \left[\sum \tau^C q_i - K(q_i, Q)\right] \tag{33}$$

[40] Under delivered pricing,

$$\begin{aligned} W^D &= CS^D + \sum_{i=1}^n \pi_i + GS \\ &= \int_0^Q p(s)ds - Qp(Q) + Qp(Q) - \sum_{i=1}^n \left[C_i(q_i) + T_i(q_i) + \tau^C q_i\right] \\ &\quad + R(q_i, Q) - K(q_i, Q) \end{aligned}$$

yielding (33). Under mill pricing,

$$\begin{aligned} W^M &= CS^M + \sum_{i=1}^n \pi_i + GS \\ &= \int_0^Q G(s)ds - Qg(Q) + \sum_{i=1}^n (P_i - c_i) q_i + \left[(p_i^I q_i - t_i q_i\right] \\ &\quad + \left[(p^C - \tau^C)Q - t_c Q\right] + R(q_i, Q) - K(q_i, Q) \\ &= \int_0^Q G(s)ds - \sum_{i=1}^n t_i q_i - \sum_{i=1}^n (t_c(Q) + \tau^C) q_i - \sum_{i=1}^n c_i q_i + \left[\sum \tau^C q_i - K(q_i, Q)\right] \end{aligned}$$

which after noticing the equivalence between $G(s)$ and $p(s)$ reduces also to (33).

2.5.2 An infrastructure project on a single idiosyncratic link with fixed common link cost

We consider in this sub-section that $e_i = 0$.and without loss of generality $g = 0$. Under these conditions, and based on the optimal pricing rules, optimal tolls are set to zero ($\tau^C = \tau_i^I = 0$). Define for each firm i the following two terms:[41]

$$\delta_i(q_{i,}Q) = p'\left(1 + s_i\Theta\right), \quad \text{and} \quad \varphi_i(q_{i,}Q) = p' - C_i'' - T_i^{I\prime\prime} = p' - d_i - f_i$$

with $\Theta(Q) = \frac{p''Q}{p'}$ as before.[42] The following assumptions are made:

$$\begin{aligned}
\mathbf{A1} &: \ p' < 0, \ p'' \textit{is continuous}, \ \textit{all } i \\
\mathbf{A2} &: \ \delta_i(q_i, Q) < 0, \ \textit{and } \varphi_i(q_i, Q) < 0, \ \textit{for all } q_i \leq Q, \textit{all } i \\
\mathbf{A3} &: \ s_i \ \textit{for all firms is} \ \text{positive} \ \textit{at the Cournot equilibrium}
\end{aligned}$$

Our definition of a transport infrastructure improvement project at this stage comprise a change in capacity for a particular idiosyncractic link k, that in turns affects the level of f_k. The first step to take in order to distinguish *direct* from *induced* effects will be to differentiate total social welfare to obtain a measure of the economy-wide benefits. Differentiating (33) with respect to f_k, we obtain:

$$\begin{aligned}
\frac{\partial W}{\partial f_k} &= p\sum\frac{\partial q_i}{\partial f_k} - \frac{\partial\sum C_i(q_i)}{\partial f_k} - \frac{\partial T_k^I(q_k)}{\partial f_k} - \frac{\partial\sum_{i\neq k}^n T_i^I(q_i)}{\partial f_k} \\
&\quad - \frac{\partial\sum T_i^C}{\partial f_k} - \frac{\partial K(q_i,Q)}{\partial f_k} \\
&= -\frac{q_k^2}{2} + \sum(p - c_i - d_i q_i - f_i q_i)\frac{\partial q_i}{\partial f_k} - \kappa_k^I
\end{aligned} \tag{34}$$

The third line (34) is already giving us information concerning the optimal level of link k capacity. The last term correspond to the marginal cost of increasing capacity (capital cost per-unit κ_k) which for an optimum should equalize the (*economy-wide*) benefits, corresponding to the first two terms. We will focus now on those two terms but later we will interpret our results on induced benefits in terms of the optimal rule for capacity.

Taking (34) as economy-wide benefits (EWB) minus capital costs changes, as in (13), we will focus now on the composition of the former. It is apparent from the

[41] These terms are needed in order to get meaninful expressions for the change in welfare in the presence of an infrastructure improvement. They are discussed here since our assumptions to obtain unique and stable equilibriums rely heavily on them. Their economic meaning will be clearer later in this section.

[42] Note that φ_i include cases of both decreasing ($d < 0$) and increasing ($d > 0$) returns to scale in production.

third line in (34) that the first term correspond to direct effects, composed of the initial level of transport services times the change in the transport charge ($q_k \frac{\partial t_k(q_k)}{\partial f_k} = q_k \frac{q_k}{2}$) after the change in capacity ($\frac{1}{f_k}$). The second term, comprise the change in productive efficiency (*extra-revenue effect plus industry-costs effect).* In order to get a more detailed expression for EWB, a closed-form expression for $\frac{\partial q_i}{\partial f_k}$ is needed. This is provided in the following proposition (*see the appendix for details and proofs*).

Proposition 5 *Let δ_i and φ_i be defined as above and assume $A1 - A3$ to hold. Under these conditions the following claim holds:*

$$\frac{\partial Q}{\partial f_k} = \frac{q_k}{\varphi_k \Delta} < 0; \frac{\partial q_k}{\partial f_k} = \frac{q_k(1 + \sum_{i \neq k} \theta_i)}{\varphi_k \Delta} < 0; \frac{\partial q_i}{\partial f_k} = \frac{-q_k \theta_i}{\varphi_k \Delta} > 0, \quad for\ i \neq k \tag{35}$$

where θ_i,χ_i and Δ are given by

$$\theta_i = \frac{\delta_i}{\varphi_i} = \frac{p'(1 + s_i \Theta)}{p' - d_i - f_i} > 0,\ and\ \ \Delta = 1 + \sum \theta_i = 1 + p' \sum \frac{(1 + s_i \Theta)}{p' - d_i - f_i} > 0 \tag{36}$$

Proof. See Appendix ∎

Proposition 5 implies that an idiosyncratic infrastructure project always increases firm's k output by more than it decreases other firms' outputs, so the industry output always increases.The magnitude of the effects is affected by the directly affect firm scale parameter, d_k, through φ_k.and the rest of firms scale economies through Δ. The higher the d_i's the stronger the reallocation of market shares.

Using the first order conditions in (26) together with *Proposition* 5, the EWB can be rewritten as:

$$\begin{aligned} EWB &= \frac{q_k^2}{2} + p' \sum q_i \frac{\partial q_i}{\partial f_k} = \frac{q_i^2}{2} + p' q_k \frac{\partial q_k}{\partial f_k} + p' \textstyle\sum_{i \neq k} q_i \frac{\partial q_i}{\partial f_k} \\ &= \frac{q_k^2}{2} + \underbrace{p' q_k \frac{q_k \left(1 + \sum_{i \neq k} \theta_i\right)}{\varphi_k \Delta}}_{>0} + \underbrace{p' \textstyle\sum_{i \neq k} q_i \frac{q_k(-\theta_i)}{\varphi_k \Delta}}_{<0} \end{aligned} \tag{37}$$

which shows a further decomposition of the indirect benefits (in the last two terms on the RHS). The second term is positive while the third one negative, that is the contribution to indirect benefits from the directly affected firm are positive while negative for the rest.[43] Let us define now the term

$$\nu_k = \frac{\varphi_k \Delta}{p'} > 0,$$

[43] This result from strategic substitutability in Cournot competition.

which is the inverse of the *passthrough* effect of a change in f_k on the equilibrium price p $(\frac{df_k}{dp})$, that is, the inverse of the product of p' and $\frac{\partial Q}{\partial f_k}$.We can rewrite (37) in more compact form as:

$$EWB = Q\underbrace{\frac{q_k p'}{\varphi_k \Delta}}_{\frac{\partial p}{\partial f_k}>0}\left[\underbrace{s_k\left(1+\frac{\nu_k}{2}+\sum_{i\neq k}\theta_i\right)-\sum_{i\neq k}\theta_i s_i}_{\lesseqgtr 0}\right]$$

Notice that $\frac{Qq_kp'}{\varphi_k\Delta}>0$, so the economy-wide effects are greater or smaller than zero depending on the sign of the term in square parenthesis, which lead us to the following result:

Proposition 6 *Let s_i be i's market share and θ_i ,ν_k and Δ as defined above. The following claim holds:*

$$EWB < 0 \Leftrightarrow s_k < \bar{s}_k = \frac{\sum_{i\neq k}\theta_i s_i}{1+\frac{\nu_k}{2}+\sum_{i\neq k}\theta_i}$$

for the quadratic transport cost function.

Proof. See Appendix. ■

This implies that an infrastructure improvement affecting firm k, generates posite benefits at an economy-wide level if and only if firm-k market share is initially above the critical level defined by $\bar{s}_k$. From this point on we would like to get an explicit expression for the multiplier, m, in (14). For the linear case, the direct effects are, as discussed before, simply the initial level of shipments. On the other hand, for the quadratic case we would have to account not only for the benefits arising to the existing freight transported but also for the change in the transport rate as the new equilibrium implies an increase in shipments from production site k and in the presence of congestion externalities in the idiosyncratic links this effect might be accounted for in a more general transport CBA. In order to take part of this effects we will assume a naive CBA in which only the congestion effects on the link subject to improvement are accounted for.

Applying this to (37) we get:

$$EWB - D = p'q_k\frac{q_k\left(1+\sum_{i\neq k}\theta_i\right)}{\varphi_k\Delta} + p'\sum_{i\neq k} q_i\frac{q_k\left(-\theta_i\right)}{\varphi_k\Delta}$$

which represents the benefits arising from the infrastructure improvement at the economy-wide level. This have to be compared with those measured at the transport market itself.

Following Newbery(1998) we can compute a rate of adjustment that will have to be applied to the user's benefits arising in a CBA in order to take account of the induced effects.

$$\begin{aligned} r &= \frac{EWB - \frac{q_k^2}{2}}{\frac{q_k^2}{2}} \\ &= \frac{p' q_k \frac{q_k(1+\sum_{i\neq k}\theta_i)}{\varphi_k \Delta} + p' \sum_{i\neq k} q_i \frac{q_k(-\theta_i)}{\varphi_k \Delta}}{\frac{q_k^2}{2}} \\ &= \frac{2}{v_k}\left[s_k(1+\textstyle\sum_{i\neq k}\theta_i) - \sum_{i\neq k}\theta_i s_i\right] \end{aligned}$$

where $\frac{1}{v_k} = \frac{\partial p}{\partial f_k} > 0$ under our assumptions, which in turn leads us to,

Proposition 7 *Let s_i be i's market share and θ_i and Δ as defined above. The following claim holds:*

$$r < 0 \Leftrightarrow s_k < \overline{s}_k^T = \frac{\sum_{i\neq k}\theta_i s_i}{1+\sum_{i\neq k}\theta_i}$$

and $\overline{s}_k < \overline{s}_k^T$ by a factor that varies with v_k.which is a parameter related to the respond of the equilibrium price to a change in transport costs.

Proof. See Appendix ■

To summarize, a cost reduction in a idiosyncratic transport link for firm k increases welfare if only if its market share is bigger that $\overline{s}_k$ and generates a positive extra benefits not captured by a traditional $CBA\ iff\ s_k > \overline{s}_k^T$. Both critical levels for the market share of the affected firm are determined by four market forces: the first and second order derivatives of inverse demand, p' and p''; other firms' market share, $s_i; i \neq k$; and the second order derivatives of the others' s production and transportation costs functions, $C_i''(q_i)$ and $T_i''(q_i)$.which entail scale effects in production.

Corollary 5 *For an oligopoly as in (1) with $d_i = \widetilde{d}$, $\forall\ i$. and $A.1 - A.3$ holding, the threshold in Proposition 7 is given by,*

$$\overline{s}_k^T = \frac{\Theta + H}{[1+n+\Theta]}$$

Corollary 5 points to the absence of assymetric scale effects in production, then the threshold will depend only on general demand conditions and concentration driven by differences in c_i's.

2.5.3 An infrastructure project on a single idiosyncratic link with a congestible common link

The previous infrastructure project analyzed corresponded to an expansion in capacity in a particular link connecting just one firm with an arterial link that brings all firms to market. The arterial link was also considered big enough to carry all the traffic without experimenting any kind of congestion effects. In this section we drop this last assumption and investigate the consequences of expanding capacity in an idiosyncratic link (without congestion) when a common link ahead in the network is subject to congestion. In terms of average transport cost functions in (23) and (24) we assume $f_i = 0$, while $e_i > 0$ and $j > 0$. Define, as before, for each firm i the following two terms:

$$
\begin{aligned}
\delta_i(q_{i,}Q) &= p'\left(1 + s_i\Theta\right), \\
\varphi_i(q_{i,}Q) &= p' - C_i'' - T_i'' = p' - d_i - t_c'(Q), \text{and} \\
\zeta_i(q_i, Q) &= t_c'(Q) + q_i t_c''(Q) = t_c'(1 + s_i\Phi) > 0
\end{aligned}
$$

where Φ $(= \frac{t_c''(Q)Q}{t_c'(Q)})$ is defined as the elasticity of the slope of the common resource cost function.

Taking the differential of (33) with respect to e_k, we get:

$$
\begin{aligned}
\frac{\partial W}{\partial e_k} &= p\sum \frac{\partial q_i}{\partial e_k} - \frac{\partial \sum C_i(q_i)}{\partial e_k} - \frac{\partial T_k^I(q_k)}{\partial e_k} - \frac{\partial \sum_{i\neq k} T_i^I(q_i)}{\partial e_k} \\
&\quad - \frac{\partial \sum_{i=k} T_i^C}{\partial e_k} + \sum \tau^C \frac{\partial q_i}{\partial e_k} - \frac{\partial K(q_i, Q)}{\partial e_k} \\
&= -q_k + \textstyle\sum_{i=1}(p - c_i - d_i q_i - e_i - t_c(Q) - t_c'(Q)q_i)\displaystyle\frac{\partial q_i}{\partial e_k} \\
&\quad - \textstyle\sum_{i=1}\sum_{h\neq i} t_c'(Q) q_i \displaystyle\frac{\partial q_h}{\partial e_k} + \sum \tau^C \frac{\partial q_i}{\partial e_k} - \kappa_k^I \\
&= -q_k + \textstyle\sum_{i=1}(p - c_i - d_i q_i - e_i - t_c(Q) - Q t_c'(Q) s_i)\displaystyle\frac{\partial q_i}{\partial e_k} \\
&\quad - \textstyle\sum_{i=1}(1 - s_i) Q t_c'(Q)\displaystyle\frac{\partial q_i}{\partial e_k} + \sum \tau^C \frac{\partial q_i}{\partial e_k} - \kappa_k^I
\end{aligned}
\tag{38}
$$

From the last two lines in (38) and concerning the common resource toll we consider two extreme cases.[44] First, we assume the toll different from zero and optimal in the transport sector in the sense that it generates complete internalization of the externalitiy but has no impact on the mark-up exercised by the firm at the plant level, then:

[44] Note that the first term in the last line in (38) comes from decomposing -$Qt_c'(Q)\frac{\partial Q}{\partial e_k} = -\sum_{i=1}\sum_{h=1} t_c'(Q) q_i \frac{\partial q_h}{\partial e_k}$ into $-\sum_{i=1} t_c'(Q) q_i \frac{\partial q_i}{\partial e_k}$ $-\sum_{i=1}\sum_{h\neq i1} t_c'(Q) q_i \frac{\partial q_h}{\partial e_k}$ and can be rewritten as $-\sum_{i=1}\sum_{h\neq i} t_c'(Q) q_h \frac{\partial q_i}{\partial e_k}$ and further as $-\sum_{i=1} t_c'(Q)(Q - q_i)\frac{\partial q_i}{\partial e_k}$, which after multiplying by $\frac{Q}{Q}$ leads to the first term in the last line in (38).

$$\frac{\partial W}{\partial e_k} = -q_k + \sum_{i=1}(p - c_i - d_i q_i - e_i - t_c - Qt_c' s_i - \sum_{i=1}(1-s_i)Qt_c')\frac{\partial q_i}{\partial e_k} + \sum \tau^C \frac{\partial q_i}{\partial e_k} - \kappa_k^I$$

This implies that the optimal toll is firm specific, τ_i^C, and equal to $(1 - s_i)Qt_c'(Q)$. The second case occurs when there is not tolling policy, $\tau_i^C = 0$, so there is an externality that each producer impose on one another when shipping to the central market. The change in social welfare is then given by,

$$\frac{\partial W}{\partial e_k} = -q_k + \sum_{i=1}(p - c_i - d_i q_i - e_i - t_c - Qt_c' s_i)\frac{\partial q_i}{\partial e_k} - \sum_{i=1}(1-s_i)Qt_c' \frac{\partial q_i}{\partial e_k} - \kappa_k^I$$

The *total* externality term $(= -\sum_{i=1}(1 - s_i)Qt_c'(Q)\frac{\partial q_i}{\partial e_k})$ can be decomposed into a positive and negative terms, according to the type of firm imposing the externality. In order to get a more detailed expression for (38) we differentiate the appropiate first order conditions in (26) with respect to e_k getting the following result (*see appendix for details*):

Proposition 8 *Let δ_i, ζ_i and φ_i be defined as above and assume $A1-A3$ to hold. Under these conditions the following claim holds:*

$$\frac{\partial Q}{\partial e_k} = \frac{1}{\varphi_k \Delta} < 0;\ \frac{\partial q_k}{\partial e_k} = \frac{1 + \sum_{i \neq k} \theta_i}{\varphi_k \Delta} < 0;\ \frac{\partial q_i}{\partial e_k} = \frac{-\theta_i}{\varphi_k \Delta} > 0,\ for\ i \neq k \tag{39}$$

where θ_i and Δ are given by:

$$\theta_i = \frac{\delta_i - \zeta_i}{\varphi_i} = \frac{p'(1 + s_i\Theta) - t_c'(1 + s_i\Phi)}{p' - d_i - t_c'(Q)} > 0,\ and\ \ \Delta = 1 + \sum \theta_i > 0 \tag{40}$$

Proof. The proof is in the appendix. ■

Proposition 8 implies that in the presence of congestion in an arterial transport link, an infrastructure project in a link preceding it and serving only firm k,[45] always increases firm's k output by more than it decreases other firms' outputs, so the output of the whole industry always increases. It is interesting to see that appart from the economies of scale effect already discussed in the previous sub-section, the magnitude of the effects decreases with the first and second derivatives of the congestion function, as shown by θ_i.Venables (1999) discusses a similar effect for the case of commuting on the same network shape.and characterizes as *Braess* paradox type of result.[46]

[45] The following result can be generalized to the case that the improvement affect a subset of firms.

[46] Note that this effects is also a consequence of assuming strategic substitutability in outputs.

Using the appropiate first order conditions as in (26), the EWB arising from a marginal change in e_k for the case of positive and optimal τ_i^C become:

$$\begin{aligned}
EWB_{\tau_i^C>0} &= q_k + p' \textstyle\sum_{i=1} q_i \dfrac{\partial q_i}{\partial e_k} - \textstyle\sum_{i=1} \tau_i^C \dfrac{\partial q_i}{\partial e_k} \\
&= q_k + \underbrace{p' q_k \frac{\partial q_k}{\partial e_k}}_{>0} + \underbrace{p' \textstyle\sum_{i\neq k} q_i \frac{\partial q_i}{\partial e_k}}_{<0} - \underbrace{\tau_k^C \frac{\partial q_k}{\partial e_k}}_{<0} - \underbrace{\textstyle\sum_{i\neq k} \tau_i^C \frac{\partial q_i}{\partial e_k}}_{>0}
\end{aligned}$$

with, as before, $\nu_k = \frac{\varphi_k \Delta}{p'} > 0$ and $\frac{Qp'}{\varphi_k \Delta} > 0$. For the case of zero τ^C we have,

$$\begin{aligned}
EWB_{\tau_i^C=0} &= q_k + p' \textstyle\sum_{i=1} q_i \dfrac{\partial q_i}{\partial e_k} + \textstyle\sum_{i=1}(1-s_i) Q t_c'(Q) \dfrac{\partial q_i}{\partial e_k} \\
&= q_k + \underbrace{p' q_k \frac{\partial q_k}{\partial e_k}}_{>0} + \underbrace{Q t_c'(Q)(1-s_k) \frac{\partial q_k}{\partial e_k}}_{<0} \\
&\quad \underbrace{+p' \textstyle\sum_{i\neq k} q_i \frac{\partial q_i}{\partial t_k}}_{<0} + \underbrace{Q t_c'(Q) \textstyle\sum_{i\neq k}(1-s_i) \frac{\partial q_i}{\partial e_k}}_{>0}
\end{aligned}$$

which after replacing terms from Proposition 8 become, respectively:

$$EWB_{\tau_i^C>0} = q_k + \underbrace{p' q_k \frac{1+\sum_{i\neq k}\theta_i}{\varphi_k \Delta}}_{>0} + \underbrace{p' \textstyle\sum_{i\neq k} q_i \frac{-\theta_i}{\varphi_k \Delta}}_{<0} - \underbrace{\tau_k^C \frac{1+\sum_{i\neq k}\theta_i}{\varphi_k \Delta}}_{<0} - \underbrace{\textstyle\sum_{i\neq k} \tau_i^C \frac{-\theta_i}{\varphi_k \Delta}}_{>0}$$

and

$$\begin{aligned}
EWB_{\tau_i^C=0} &= q_k + \underbrace{p' q_k \frac{1+\sum_{i\neq k}\theta_i}{\varphi_k \Delta}}_{>0} + \underbrace{Q t_c'(Q)(1-s_k) \frac{1+\sum_{i\neq k}\theta_i}{\varphi_k \Delta}}_{<0} \\
&\quad \underbrace{+p' \textstyle\sum_{i\neq k} q_i \frac{-\theta_i}{\varphi_k \Delta}}_{<0} + \underbrace{Q t_c'(Q) \textstyle\sum_{i\neq k}(1-s_i) \frac{-\theta_i}{\varphi_k \Delta}}_{>0}
\end{aligned}$$

When assuming a CBA that incorporates congestion effects, we have as *indirect* benefits:

$$EWB_{\tau_i^C>0} - q_k + \textstyle\sum_{i=1} \tau_i^C \dfrac{\partial q_i}{\partial e_k} = p' q_k \dfrac{1+\sum_{i\neq k}\theta_i}{\varphi_k \Delta} + p' \textstyle\sum_{i\neq k} q_i \dfrac{-\theta_i}{\varphi_k \Delta}$$

which represent the benefits arising from the infrastructure improvement appart from the ones that would be reported in a CBA. This have to be compared with those measured at the transport maket itself.

$$\begin{aligned} r &= \frac{EWB_{\tau_i^C>0} - q_k + \sum_{i=1}\tau_i^C\frac{\partial q_i}{\partial e_k}}{q_k - \sum_{i=1}\tau_i^C\frac{\partial q_i}{\partial e_k}} \\ &= \frac{p'q_k\frac{1+\sum_{i\neq k}\theta_i}{\varphi_k\Delta} + p'\sum_{i\neq k}q_i\frac{-\theta_i}{\varphi_k\Delta}}{q_k - \tau_k^C\frac{1+\sum_{i\neq k}\theta_i}{\varphi_k\Delta} - \tau_i^C\sum_{i\neq k}q_i\frac{-\theta_i}{\varphi_k\Delta}} \end{aligned}$$

where $\frac{p'}{\varphi_k\Delta} = \frac{1}{v_k} > 0$, which (for a positive denominator) leads us to,

Proposition 9 *Let s_i be i's market share and θ_i and Δ as defined above. The following claim holds:*

$$r < 0 \Leftrightarrow s_k < \overline{s}_k^T = \frac{\sum_{i\neq k}\theta_i s_i}{1+\sum_{i\neq k}\theta_i}$$

and $\overline{s}_k < \overline{s}_k^T$ by a factor that varies with v_k.which relates to the respond of the equilibrium price to a change in transport costs.

2.5.4 An infrastructure project on a common congestible link with fixed idiosyncratic link costs

In this section we investigate the case of an infrastructure improvement in a congestion-prone common resources with fixed idiosyncratic costs. In terms of average transport cost function in (23) and (24) we assume as before $f_i = 0$, $e_i > 0$ and $j > 0$. Keeping the definition of δ_i, φ_i and ζ_i from the previous section, we take the differential of (33) with respect to j,yielding:

$$\begin{aligned} \frac{\partial W}{\partial j} &= p\frac{\partial Q}{\partial j} - \frac{\partial\sum C_i(q_i)+T_i(q_i)}{\partial j} + \sum\tau^C\frac{\partial q_i}{\partial j} - \frac{\partial K(q_i,Q)}{\partial j} \\ &= -\frac{\partial t_c}{\partial j}Q + \sum_{i=1}(p - c_i - d_iq_i - e_i - t_c(Q) - t_c'(Q)q_i)\frac{\partial q_i}{\partial j} \\ &\quad -\sum_{i=1}\sum_{h\neq i}t_c'(Q)q_i\frac{\partial q_h}{\partial j} + \sum\tau^C\frac{\partial q_i}{\partial j} - \kappa^C \\ &= -\frac{Q^2}{2}Q + \sum_{i=1}(p - c_i - d_iq_i - e_i - t_c(Q) - Qt_c'(Q)s_i)\frac{\partial q_i}{\partial j} \\ &\quad -\sum_{i=1}(1-s_i)Qt_c'(Q)\frac{\partial q_i}{\partial j} + \sum\tau^C\frac{\partial q_i}{\partial j} - \kappa^C \end{aligned}$$

Proposition 10 *Let δ_i, ζ_i and φ_i be defined as above and assume $A1 - A3$ to hold. Under these conditions the following claim holds:*

$$\frac{\partial Q}{\partial j} = \frac{1}{\Delta}\sum_{i=1}\frac{Q^2}{2\varphi_i} < 0;\ \frac{\partial q_i}{\partial j} = \frac{Q^2}{2\varphi_i} - \theta_i\frac{\partial Q}{\partial j} < 0$$

where θ_i *and* Δ *are given by:*

$$\theta_i = \frac{\delta_i - \zeta_i}{\varphi_i} = \frac{p'\left(1 + s_i\Theta\right) - t'_c(1 + s_i\Phi)}{p' - d_i - t'_c(Q)} > 0, \text{ and } \Delta = 1 + \sum \theta_i > 0$$

Proof. The proof is in the appendix. ∎

Assume there is no toll implemented, then the EWB are given by:

$$\begin{aligned} EWB_{\tau_i^C=0} &= \frac{Q^2}{2}Q + \underbrace{p'\sum\nolimits_{i=1} q_i \frac{\partial q_i}{\partial j}}_{>0} + \underbrace{Qt'_c(Q)\sum\nolimits_{i=1}(1-s_i)\frac{\partial q_i}{\partial j}}_{<0} \\ &= \frac{Q^2}{2}Q + p'\sum\nolimits_{i=1} q_i\left(\frac{Q^2}{2\varphi_i} - \frac{\theta_i}{\Delta}\sum\nolimits_{i=1}\frac{Q^2}{2\varphi_i}\right) \\ &\quad + Qt'_c(Q)\sum\nolimits_{i=1}(1-s_i)\left(\frac{Q^2}{2\varphi_i} - \frac{\theta_i}{\Delta}\sum\nolimits_{i=1}\frac{Q^2}{2\varphi_i}\right) \end{aligned}$$

When assuming a CBA that incorporates congestion effects, we have as *indirect* benefits:

$$EWB_{\tau_i^C>0} - \frac{Q^2}{2}Q - \sum\nolimits_{i=1}(1-s_i)Qt'_c(Q)\frac{\partial q_i}{\partial j} = p'\sum\nolimits_{i=1} q_i \frac{-\theta_i}{\varphi_k \Delta}$$

which represent the benefits arising from the infrastructure improvement appart from the ones that would be reported in a CBA. This have to be compared with those measured at the transport maket itself.

$$\begin{aligned} r &= \frac{EWB_{\tau_i^C>0} - \frac{Q^2}{2}Q - \sum_{i=1}(1-s_i)Qt'_c(Q)\frac{\partial q_i}{\partial j}}{\frac{Q^2}{2}Q + \sum_{i=1}(1-s_i)Qt'_c(Q)\frac{\partial q_i}{\partial j}} \\ &= \frac{p'\sum_{i=1} q_i\left(\frac{Q^2}{2\varphi_i} - \frac{\theta_i}{\Delta}\sum_{i=1}\frac{Q^2}{2\varphi_i}\right)}{\frac{Q^2}{2}Q + \sum_{i=1}(1-s_i)Qt'_c(Q)\frac{\partial q_i}{\partial j}} \end{aligned}$$

where $\frac{p'}{\varphi_k \Delta} = \frac{1}{v_k} > 0$, which (for a positive denominator) leads us to,

Proposition 11 *Let* s_i *be i's market share and* θ_i *and* Δ *as defined above. The following claim holds:*

$$r < 0 \Leftrightarrow s_k < \overline{s}_k^T = \frac{\sum_{i \neq k}\theta_i s_i}{1 + \sum_{i \neq k}\theta_i}$$

and $\overline{s}_k < \overline{s}_k^T$ *by a factor that varies with* v_k*.which relates to the respond of the equilibrium price to a change in transport costs.*

2.6 Non-Marginal Infrastructure Projects: Linear and Non-linear demand

For a non-marginal change we first look at the linear demand case which is tractable analytically. Using the set-up in sub-Section 2.5 with $\Theta = 0$, the change in welfare after an infrastructure improvement is given by,

$$\begin{aligned}
\Delta W_{(\Delta e_k, f_k)} &= CS(p^1) - CS(p^0) + \textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1))(q_i^0 + \Delta q) \\
&\quad - \textstyle\sum_{i=1}(p^0 - C_i(q_i^0) - T_i(q_i^0))q_i^0 \\
&= -\Delta p\left(Q^0 + \frac{1}{2}\Delta Q\right) + \Delta p Q^0 - \textstyle\sum_{i=1}\left(\Delta t_i(q_i)q_i^0 + \Delta C_i(q_i)q_i^0\right) \\
&\quad + \textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1))\Delta q \\
&= \underbrace{\frac{1}{2}\Delta p\Delta Q - \textstyle\sum_{i=1}\left(\Delta t_i(q_i)q_i^0 + \Delta C_i(q_i)q_i^0\right)}_{Direct} + \underbrace{\textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1))\Delta q}_{Indirect}
\end{aligned}$$

for and change in j with fixed idiosyncratic links costs, we have:

$$\begin{aligned}
\Delta W_{(\Delta j)} &= CS(p^1) - CS(p^0) + \textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1, Q^1))(q_i^0 + \Delta q) \\
&\quad - \textstyle\sum_{i=1}(p^0 - C_i(q_i^0) - T_i(q_i^0, Q^0))q_i^0 \\
&= -\Delta p\left(Q^0 + \frac{1}{2}\Delta Q\right) + \Delta p Q^0 \\
&\quad - \textstyle\sum_{i=1}\left(\Delta t_i(q_i)q_i^0 + \Delta C_i(q_i)q_i^0\right) - \left(t_c(Q^1)q_i^1 - t_c(Q^0)q_i^0\right)q_i^0 \\
&\quad + \textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1) - t_c(Q^1)q_i^1)\Delta q \\
&= \underbrace{\frac{1}{2}\Delta p\Delta Q - \textstyle\sum_{i=1}\left(\Delta t_i(q_i)q_i^0 + \Delta C_i(q_i)q_i^0\right)}_{Direct} + \underbrace{\textstyle\sum_{i=1}(p^1 - C_i(q_i^1) - T_i(q_i^1))\Delta q}_{Indirect}
\end{aligned}$$

Results from simulations..

3 Multiple Demand Nodes: Spatial Price Equilibrium

3.1 Surplus Equivalence in Complex Networks with Congestion Effects

The focus of this section is on inter-regional (e.g. between cities) freight flows. There are two main classes of network models which have been used to analyze inter-region freight movements: spatial price equilibrium models (SPM) and freight network equilibrium models (Harker, 1986). The first class focuses on the producer-consumer-shippers interactions in the economy without explicitly determining the microeconomics of these activities. The transportation sector is represented by a directed graph with nodes and links. The transportation costs are not derived from a model of carrier behavior, but are stated as fixed values or as functions of the flows on a discrete network. Additionally producers' and consumers' behavior is incorporated by defining supply and demand functions for each region. In the general case -independently of the producer's industrial organization- the shippers are assumed to behave according to the following two equilibrium principles :

·If there is a flow of commodity i from any pair of regions (k,l), then the marginal cost of commodity i in k plus the transportation costs from k to l will equal the marginal revenue of the commodity in l.

·If the marginal costs of commodity i in k plus the transportation costs from k to l is greater than the marginal revenue of commodity i in l, then there will be no flow from k to l.

The SPM were originally qualitatively described in Enke (1951), and formalized by Samuelson (1952) and Takayama and Judge (1964, 1970). In our work we use formulations and extensions from Florian and Los (1982), Friesz et al (1983) and Dafermos and Nagurney (1985), among others.

The second major class of predictive intercity freight models is the freight network equilibrium models, in which the focus is on the shipper-carrier interaction. The generation of trips from each region is imposed and assumed to be known in this type of models, in contrast with the SPM that solves for the trip (e.g. trade flows) generation via the interaction between supply and demand functions embedded in a network. An attempt to integrate both types of models is presented in Harker and Friesz (1986a, 1986b) as generalized spatial price equilibrium, and it will be later on discussed as a possible extension of the model used here. In the rest of the paper, the focus will be on SPM.

Spatial price equilibrium is also one of the two most studied steady-state concepts of network equilibrium (Friesz, 1985). The other type is the user equilibrium which is mostly employed in urban passenger networks but as pointed out in Florian and Los (1981) and Harker (1985), can also be fully consistent with SPM featuring multi-paths for a given origin-destination (O-D) pair. This last point will be emphasized when considering transshipment nodes and average cost pricing in the transportation sector .

3.1.1 Network and Industry Configuration

The network topology for a homogeneous one-commodity economy with single paths between each OD pairs is introduced in this sub-section. The discrete shipper network is represented by a finite-directed graph, $G[L, W]$, with L and W denoting the full set of nodes and arcs, respectively. The indices i, j, k, and l refer to nodes of the network. Define $W = \{w = (ij); i \in L, j \in L\}$, to be the set of all origin-destination pairs connecting pairs of regional centroids of trade represented by pairs of nodes in L . All interactions within agents in the same centroid are conducted through the price system so an implicit assumption of zero transport cost for trade within each centroid is at stake .

For each region $l \in L$, S_l is the supply quantity in this region and D_l is the demand quantity. The flow between O-D pair $(ij) \in W$ is denoted by $T_{(ij)}$. Conservation of flow in every region implies:

$$S_l - D_l + \sum_{i \in L} \sum_{(il) \in W} T_{(il)} - \sum_{j \in L} \sum_{(lj) \in W} T_{(lj)} = 0 \tag{41}$$

for all $l \in L$. The market clearing condition requires that total demand equal total supply, which is obtained summing up over all $l \in L$ in (41):

$$\sum_{l \in L} D_l - \sum_{l \in L} S_l = 0$$

For each O-D pair, a function $c_{(ij)}(T_{(ij)}, K_{(ij)})$ is defined as the average (marginal private) cost incurred in transporting an unit of good between i and j. This is the cost that has to be paid when shipping between OD pair $(ij) \in W$, and depends negatively in infrastructure capacity in the relevant link, $K_{(ij)}$, and positively in the transport flow, $T_{(ij)}$ in that link . Furthermore $c_{(ij)}(.,.)$ is continuous, monotone and strictly convex in $T_{(ij)}$, implying that we are focusing on situations where congestion effects occur in each link. There is a cost per-unit of infrastructure provided that will represent the cost side of a CBA in our context, but we assumed it null at this point in order to focus on user's benefits. Allowing this cost to be different from zero will made possible to analyze the interaction of first and second best policies in infrastructure provision within the surplus equivalence question (van den Bergh and Verhoef, 1996).

Let us define, for each region $l \in L$, $\Psi_l(S_l)$ as the inverse supply function with associated long-run supply function $S_l(\Psi_l)$, and $\phi_l(D_l)$ as the inverse demand function with associated long-run demand function $D_l(\phi_l)$. These functions are assumed, without loss of generality, separable in prices and quantities for simplicity in computation of equilibrium and simulations without loss of generality (Friesz et.al. 1983).

3.2 Cournot-Nash Competition

In order to simplify the discussion of the Cournot-Nash competition case we make the extreme assumption that there is only one firm controlling production in each region-market. As pointed out by Harker (1986) our assumption is not restrictive in the sense that any particular region, in which two or more firms operate, can always be decomposed into sub regions in which only one firm operates. Hashimoto (1984) has extended the model of Takayama and Judge to a spatial Nash non-cooperative equilibrium model. He considers one firm per-region which competes a la Cournot-Nash with firms in other regions. Harker (1983) and Nagurney (1993) also present a spatial oligopoly model where the profit function of each firm considers the transportation costs from the production plants to the demand markets. Harker (1986) compares the spatial oligopoly game with the spatial price equilibrium, totally competitive market, and with two possible monopoly situations. All the four approaches have a network structure.

The model we implement here follows closely Harker (1985). If Q ($= L$ initially) denotes the set of firms operating in the market and each firm $q \in Q$'s control only one production site. The total amount demanded (= the total amount supplied) in region l ∈ L is given by D_l. Defining D_{lq} as the amount shipped by firm q to region l (or the amount demanded by the consumers in region $l \in L$ from firm q), and as the amount supplied by all other firms to region $l \in L$:

$$\tilde{D}_{lq} = \sum_{j \in Q, j \neq q} D_{lj}$$

and

$$D_l = \sum_{j \in Q} D_{lj}$$

we can defined a spatial Cournot-Nash equilibrium model as the maximization to:

$$\sum_{l \in L} \theta_l (D_{lq} + \tilde{D}_{lq}) D_{lq} - \int_0^{S_q} \psi_q(s) ds - \sum_{(qj) \in W} \int_0^{T_{(qj)}} c_{(qj)}(s, K^0_{(qj)}) ds$$

subject to an optimal strategy vector xq, followed by all other firms (including supplies, demands and transport flows). The amount supplied by all other firms to region $l \in L$, , is taken as given in the perspective of a particular firm . At this point, we also assume $c_{(ij)}(T_{(ij)})$ as taken as given, even in the presence of congestion charging. This last assumption could be seen controversial when a more realistic view of the interaction shipper-carrier is considered in a Cournot-Nash environment since a partial market power should be recognized for either size in this context. If for example, transport services are produced under oligopoly conditions then in the presence of congestion a partial internalization of the externality created when transporting using a fixed capacity should be recognized. This is precisely the point raised recently for airline transportation in Brueckner (2004) and Verhoef and Pelps (2004). The assumption of no partial internalization will be modified later in the paper when considering more general networks and simulations.

For the mathematical program formulated before there are associated Kuhn-Tucker conditions. Important insight can be obtained from these conditions:

$$(\theta_l (\tilde{D}_{lq} + D_{lq}) + D_{lq} \partial \theta_l (\tilde{D}_{lq} + D_{lq}) / \partial D_{lq} - \pi_l) D_{lq} = 0$$

for all $l \in L$

$$(\theta_l (\tilde{D}_{lq} + D_{lq}) + D_{lq} \partial \theta_l (\tilde{D}_{lq} + D_{lq}) / \partial D_{lq} - \pi_l) \leq 0 \qquad D_{lq} \geq 0$$

$$(\theta_q(\tilde{D}_{lq} + D_{qq}) + D_{qq}\partial\theta_q(\tilde{D}_{lq} + D_{qq})/\partial D_{qq} - \pi_q)D_{qq} = 0$$

for q

$$(\theta_q(\tilde{D}_{lq} + D_{qq}) + D_{qq}\partial\theta_q(\tilde{D}_{lq} + D_{qq})/\partial D_{qq} - \pi_q) \leq 0 \qquad D_{qq} \geq 0$$

$$(-\psi_q + \pi_q)S_q = 0$$

$$-\psi_q + \pi_q \leq 0 \qquad S_q \geq 0$$

$$(-c_{(qj)} + \pi_j - \pi_q)T_{(qj)} = 0$$

$$-c_{(qj)} + \pi_j - \pi_q \leq 0 \qquad T_{(qj)} \geq 0$$

for all $(q, j) \in W$

$$-c_{(qj)} + \pi_j - \pi_q \leq 0 \qquad T_{(qj)} \geq 0$$

The first two conditions imply that if there is demand in a particular region for the product of firm q, π_l will equal the marginal revenue for firm q in that region. Similarly conditions three and four imply that if there is demand in the region that the firm is located, then π_q will equal the marginal revenue for firm q in its location. The last four conditions concern costs of production and transportation. Conditions six and seven imply that if there is production, π_q will equal the marginal cost of production whereas the last two conditions state that if there is flow between region j and the production site of firm q then the cost of transportation plus the marginal cost of production will equal the marginal revenue for firm q in region j. An important distinction with respect to the perfect collusion case is that the marginal revenue functions here are based on a residual demand, instead of the total demand in a particular market. This residual demand is the part of total demand that firm q considers it is facing, after subtracting the expectation of supply for that market from other firms.

3.3 A three regions economy

To be able to uncover important issue previously not discussed in the literature and to prepare the field for simulation results shown at the end of the paper we restrict the analytical discussion to a three region economy.

The network configuration is shown in Fig. 1. The pattern of trade is assumed as indicated by the arrows going from one region to the other. In particular, we have positive trade from region 1 to 2 and region 2 to 3. Trade flow from region 1 to 2 take two different paths, one using a direct link to this region and the other employing region 3 as a transshipment node. This latter result is a consequence of (1) and the Wadropian equilibrium assignment of trade flows. In particular, both for the case of user equilibrium and social optimal equilibrium the transport cost from region 1 to 2 will coincide with the sum of the transport costs from region 1 to 3 plus transport cost from region 3 to 2. That is the used paths between OD pair $\{12\}$, directly or indirectly through node 3, have the same cost and in terms of the supply of transportation services the supply of transport services for the latter should be added vertically to comprise the total multi-mode (e.g. multi-path) transport services supply as discussed in Jara-Diaz and Friesz (1986) and Glazer and Niskanen (1991).

The trade pattern assumed incorporates as special cases configurations of trade previously analyzed in the literature and other cases that involve only two regions or that do not incorporate trade through transshipments nodes. More specificity, it will incorporate as special cases three different arrangements: (a) only one region exporting and one region importing; (b) one region exporting and two importing; and (c) one region exporting and only one importing using a transshipment node. This will imply that our assumptions on the patterns of trade comprise the general case which includes as special cases the previous three patters. We begin with the perfect competition case.

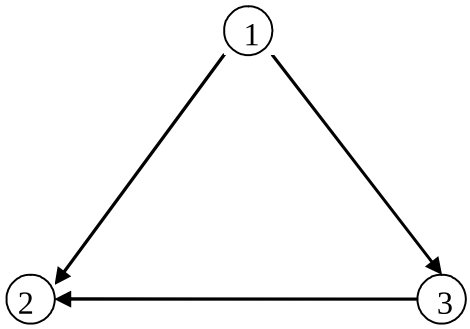

Fig.1 Three nodes Network

Results from simulations...

4 Conclusions

In this paper we study the conditions, magnitude and sign of indirect/induced benefits arising from an infrastructure improvement for a Cournot oligopoly embedded in a transport network. The applications of the findings of the paper are many, but the emphasis was on inland transportation (e.g. roads, trains). In terms of policy prescriptions we pointed out to the implications of non-zero induced effects on CBA practice, in line with previous related work of Friedlaender (1981) and Friedlaender and Mathur (1982). Possible relevant extensions of the present framework are the consideration of economies of scale in capacity expansion and multimodality as in Braeutigam (1979) and the analysis of the interaction of distortions in both the product and transport market as in Arnott and Yan (2000) and Krauss (2002) and Nilsson (1992).

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A Appendix

A.1 Lemma 3

Taking the sum over (3) we obtain (6),which defines an equilibrium relationship between total production and average costs. In parallel to (5), equation (6) shows that in equilibrium total output depends on the sum but not on the distribution of production and transport costs. It shows that given n, and with $p' < 0$, there is a negative monotone relationship between $\sum c + \sum t$ and the equilibrium industry output. In turn, given Q, each firm's output q_i is determined recursively by (6), using (3). **Q.E.D**

A.2 Propostion 1

Differentiating the first-order-conditions in (3) with respect to t_c, we have:

$$\left(p' + q_i p''\right) \frac{\partial Q}{\partial t_c} + p' \frac{\partial q_i}{\partial t_c} - 1 = 0, \qquad i = 1, ..., n$$

replacing Θ in this expression yields,

$$p'(1 + \Theta s_i) \frac{\partial Q}{\partial t_c} + p' \frac{\partial q_i}{\partial t_c} - 1 = 0, \qquad i = 1, ..., n \tag{42}$$

At this point it becomes clearer the choice of $\delta_i(q_i, Q)$ and $\varphi_i(q_i, Q)$. The latter correspond to the collection of terms multiplying $\frac{\partial q_i}{\partial t_c}$ in (51) while the former comes from the only term multiplying $\frac{\partial Q}{\partial t_c}$. More precisely,

$$\begin{aligned} p' + q_i p'' &= p' + \frac{q_i}{Q} \frac{p'' Q}{p'} p' \\ &= p'(1 + s_i \Theta) \end{aligned} \tag{43}$$

When (43) stricly negative and a range for Θ is assumed, an extra restriction for s_i will apply. For example, if $\Theta > 0$ (inverse demand is concave) then $s_i < \frac{1}{\Theta}$.The first term in Proposition 1 follows after adding up the n equations in (42). The rest of the propostion follows from substituting $\frac{\partial Q}{\partial t_c}$ in (42) and rearranging. **Q.E.D**

A.3 Proposition 2

Differentiating (3) with respect to t_k and using Θ as before, we get:

$$p'(\Theta s_i + 1)\frac{\partial Q}{\partial t_k} + p'\frac{\partial q_i}{\partial t_k} - 1 = 0, \qquad i = k \tag{44}$$

$$p'(\Theta s_i + 1)\frac{\partial Q}{\partial t_k} + p'\frac{\partial q_i}{\partial t_k} = 0, \qquad i \neq k \tag{45}$$

The system of equation can be represented in matrix form:

$$\begin{bmatrix} p' & .. & 0 & 0 & .. & 0 & p'(\Theta s_1 + 1) \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & p' & 0 & .. & 0 & p'(\Theta s_k + 1) \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & 0 & .. & p' & p'(\Theta s_n + 1) \end{bmatrix} \begin{bmatrix} -1 & .. & -1 & -1 & 1 \end{bmatrix} \begin{pmatrix} \frac{\partial q_1}{\partial t_k} \\ .. \\ \frac{\partial q_k}{\partial t_k} \\ .. \\ \frac{\partial q_n}{\partial t_k} \\ \frac{\partial Q}{\partial t_k} \end{pmatrix} = \begin{pmatrix} 0 \\ .. \\ 1 \\ .. \\ 0 \\ 0 \end{pmatrix}$$

For $i = 1, ..., n$, divide row i by p' and add it to the last row, then an extra step leads to first part of *proposition 1.* Using this result in the original system of equations proofs the rest of the proposition. **Q.E.D**

A.4 Proposition 3

Using Lemma 1 in firts term in the second line of both (15) and (16) gives the result. **Q.E.D.**

A.5 Decomposition of Indirect Benefits for Common and Idiosyncratic Resource Projects

Using the first order condition (3) in the indirect benefits term in both (15) and (16) yields, respectively,

$$\sum(p - c_i - t_c - t_i)(-\frac{\partial q_i}{\partial t_c}) = \sum p' q_i \frac{\partial q_i}{\partial t_c} \tag{46}$$

and

$$\sum(p - c_i - t_c - t_i)(-\frac{\partial q_i}{\partial t_k}) = \sum p' q_i \frac{\partial q_i}{\partial t_k} \tag{47}$$

Note thta $\sum q_i dq_i = \frac{d[\sum q_i^2]}{2} = \frac{d[Q^2 H]}{2} = HQdQ + \frac{Q^2}{2}dH$, which after multiplying by $\frac{1}{dt_c}$ and $\frac{1}{dt_k}$ and replacing them in (46) and (47) respectively, yields:

$$\sum p' q_i \frac{\partial q_i}{\partial t_c} = p'Q\left[H\frac{\partial Q}{\partial t_c} + \frac{Q}{2}\frac{\partial H}{\partial t_c}\right] \tag{48}$$

and,

$$\sum p' q_i \frac{\partial q_i}{\partial t_k} = p'Q\left[H\frac{\partial Q}{\partial t_k} + \frac{Q}{2}\frac{\partial H}{\partial t_k}\right] \tag{49}$$

From the definition of H, we have $\frac{\partial H}{\partial t_c} = 2\sum s_i \frac{\partial s_i}{\partial t_c}$ and $\frac{\partial H}{\partial t_k} = 2\sum s_i \frac{\partial s_i}{\partial t_k}$. Replacing these expressions in (48) and (49) and using in the expressions firm output changes in propositions 1 and 2 yields (19) and (20) **Q.E.D**

A.6 Proposition 4

A.6.1 Common Resource

Replacing in the second line of (**??**) the marginal change in firm i-th production from (**??**), yields:

$$\begin{aligned} p\sum \frac{s_i}{\eta}\left(-\frac{\partial q_i}{\partial t_c}\right) &= p\sum \frac{s_i}{\eta}\left(-\frac{\Theta(1-s_i n)+1}{p'\gamma}\right) \\ &= \frac{p}{\eta p'\gamma}\sum s_i(\Theta(1-s_i n)+1) \\ &= \underbrace{\frac{nQ}{\gamma}}_{Q\frac{\partial p}{\partial t_c}}\left[\underbrace{\frac{\Theta+1}{n} - \Theta\sum s_i^2}_{\gtrless 0}\right] \end{aligned}$$

dividing this by the direct benefit, $\sum q_i$, and rearranging yields,

$$r_{dt_c} = \underbrace{\frac{n}{\gamma}}_{\frac{\partial p}{\partial t_c}}\left[\underbrace{\frac{\Theta+1}{n} - \Theta\sum s_i^2}_{\gtrless 0}\right]$$

with the value of the second (*multiplicative*) term in the RHS giving the result. **Q.E.D**

A.6.2 Idiosyncratic Resource

Replacing in the second line of (**??**) the marginal change in firm i-th production from (**??**) and (**??**), yields:

$$
\begin{aligned}
&= p\frac{s_k}{\eta}(-\frac{\partial q_k}{\partial t_k}) + p\sum_{i\neq k}^{n}\frac{s_i}{\eta}(-\frac{\partial q_i}{\partial t_k}) \\
&= \frac{p}{\eta p'\gamma}\left[-s_k(n+\sum_{i\neq k}\frac{q_i p''}{p'}) - \sum_{i\neq k}s_i(-(1+\frac{q_i p''}{p'}))\right] \\
&= \underbrace{\frac{Q}{\gamma}}_{Q\frac{\partial p}{\partial t_k}}\left[\underbrace{s_k(n+\sum_{i\neq k}\frac{q_i p''}{p'}) - \sum_{i\neq k}s_i(1+\frac{q_i p''}{p'})}_{\gtrless 0}\right]
\end{aligned}
$$

dividing this by the direct benefit, $\sum q_i$, and rearranging yields,

$$
r_{dt_k} = \underbrace{\frac{1}{\gamma}}_{\frac{\partial p}{\partial t_k}}\left[\underbrace{(n+\sum_{i\neq k}\frac{q_i p''}{p'}) - \frac{\sum_{i\neq k}s_i(1+\frac{q_i p''}{p'})}{s_k}}_{\gtrless 0}\right]
$$

with the value of the second (*multiplicative*) term in the RHS giving the result. **Q.E.D**

A.7 Indirect Benefits with Mill Pricing

Social welfare in this case will be given by,

$$
\int_0^Q G(s)ds - \sum_{i=1}^{n}(t_i - t_c)q_i - \sum_{i=1}^{n}c_i q_i
$$

which is the same as the one for the delivered case, with the only difference that the transport cost are directly faced by consumers. This last distinction implies that transport cost savings -after an infrastructure improvement- are counted as direct benefits and not indirect as before. Economy-wide benefits, for common and idiosyncratic links, will then be given respectively by,

$$
EWB = \underbrace{\sum(g_i - c_i)(-\frac{\partial q_i}{\partial t_c})}_{Indirect} + \underbrace{\sum q_i - \sum(t_i + t_c)\frac{\partial q_i}{\partial t_c}}_{Direct}
$$

$$
EWB = \underbrace{\sum(g_i - c_i)(-\frac{\partial q_i}{\partial t_k})}_{Indirect} + \underbrace{q_k - \sum(t_i + t_c)\frac{\partial q_i}{\partial t_k}}_{Direct}
$$

but, using (8) these two expressions become,

$$EWB = \underbrace{\sum (P_i - c_i)(-\frac{\partial q_i}{\partial t_k})}_{Indirect} + \underbrace{\sum q_i}_{Direct}$$

$$EWB = \underbrace{\sum (P_i - c_i)(-\frac{\partial q_i}{\partial t_k})}_{Indirect} + \underbrace{q_k}_{Direct}$$

from which can be obtained measures of indirect benefits as in (46) and (47) after replacing the first term in the RHS by (9).

A.8 Mill Pricing with Congestion

Following van Dender (2005) we can derive the pricing rule for firm k-th under congestion maximizing the following problem with respect to $P_k, q_k, P_j, j \neq k$,

$$\vartheta = (P_k - c_k)q_k + \sum_{i=1}^{n} \lambda_i (G - P_i - p_i^I - p^C)$$

which after replacing (27) and (28) can be re-written as,

$$\vartheta = (P_k - c_k)q_k + \sum_{i=1}^{n} \lambda_i (G - P_i - t_i(q_i) - t_c(Q) - \tau^C)$$

Kuhn-Tucker conditions are,

$$\begin{aligned} q_k &= \lambda_k \\ P_k &= c_k + \lambda_k (t_i' + t_c') - G' \sum_{i=1}^{n} \lambda_i \\ \lambda_j &= 0, \forall j \neq k \end{aligned}$$

solving the system gives the pricing rule per-firm:

$$P_k = c_k + q_k t_i' + q_k t_c' - q_k G', \qquad k = 1, ..., n \tag{50}$$

A.9 Proposition 5

Differentiating the first order conditions (26) with respect to f_k it follows that:

$$\begin{aligned} (p' + q_k p'')\frac{\partial Q}{\partial f_k} + p'\frac{\partial q_k}{\partial f_k} &= \frac{\partial T_k^{I\prime}}{\partial f_k} + (C_k'' + T_k'')\frac{\partial q_k}{\partial f_k}, \qquad i = k, \ and \\ (p' + s_i \Theta)\frac{\partial Q}{\partial f_k} + p'\frac{\partial q_i}{\partial f_k} &= (C_i'' + T_i'')\frac{\partial q_i}{\partial f_k} \qquad all \ \ i \neq k \end{aligned} \tag{51}$$

Solving the system of n equations in (51) as in $A.3$ we get the result. **Q.E.D**

A.10 Proposition 6

$$
\begin{aligned}
(p' + q_i p'')\frac{\partial Q}{\partial e_k} + p'\frac{\partial q_k}{\partial e_k} &= 1 + (C_k^{''} + T_k^{''})\frac{\partial q_k}{\partial e_k} + (T_c' + q_i T_c'')\frac{\partial Q}{\partial e_k} + T_c'\frac{\partial q_k}{\partial e_k}, \qquad (52)\\
i &= k,\ and\\
(p' + s_i\Theta)\frac{\partial Q}{\partial e_k} + p'\frac{\partial q_i}{\partial e_k} &= (C_k^{''} + T_k^{''})\frac{\partial q_i}{\partial e_k} + (T_c' + q_i T_c'')\frac{\partial Q}{\partial e_k} + T_c'\frac{\partial q_k}{\partial e_k}\\
all\ \ i &\neq k
\end{aligned}
$$

Solving the system of equations in (52) as in $A.3$ we get the result: