Consideration Sets and Competitive Marketing

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Abstract

We study a market model in which competing firms use costly marketing devices to influence the set of alternatives which consumers perceive as relevant. The consumer in our model applies well-defined preferences to a "consideration set", which is a function of the marketing devices employed by the firms (irrelevant products that serve as “door openers”, persuasion by salespeople, advertising, etc.). We examine the implications of various assumptions regarding consumers’ formation of the consideration set on the distribution of products that firms offer in equilibrium, as well as on industry profits and consumer welfare. In particular, we study the way marketing devices are used in equilibrium and how effective they are in attracting consumers.

1 Introduction

We present a model of competitive marketing based on the notion that consumers are boundedly rational and that marketing interferes with their decision process. The standard model of consumer behavior assumes that the consumer has a clear perception of what is desirable (captured by the preference relation) and a clear perception of what is feasible, taking into account informational constraints (captured by the choice set). Our model retains the assumption that consumers have stable, unmanipulable preferences, while relaxing the assumption that they perfectly perceive what is feasible, thus allowing firms to manipulate that perception. Our objective is to examine the implication of this departure from the standard model on the nature of competition between firms, and on their use of marketing devices such as advertising or the use of “door opening” products.

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The cornerstone of our model is the idea that before consumers apply their preferences to a set of alternatives, they first construct what a “consideration set” - namely, the set of alternatives which they perceive as relevant for their decision. For a rational consumer, there is no distinction between the consideration set and the feasible set. However, for a boundedly rational consumer, the consideration set may be a strict subset of the feasible set, either because his attention was not drawn to certain products, or because he was not convinced that these products are relevant for his decision problem. More importantly, in our model firms can use marketing devices to manipulate the consideration set.

We borrow the concept of consideration sets from the marketing literature (see Roberts and Lattin (1997) for a review). The basic idea behind this concept is that consumers face an overwhelmingly large feasible set, and use screening criteria to reduce the number of “relevant” alternatives in order to simplify their decision problem. For instance, consumers may refuse to consider alternatives which are hard to compare with what they currently consume (see Medin, Goldstone and Markman (1995)). A recent study by Chakravarti and Janiszewski (2003) presents experimental evidence suggesting that when people are asked to select an alternative from a large set of heterogeneous alternatives, they tend to simplify their decision problem by focusing on a small subset of “easy-to-compare” options having alignable or overlapping attributes. A pair of alternatives is more likely to enter the consideration set as their alignability or number of overlapping features increase. One potential implication of this finding is that consumers who become familiar with a particular product are less likely to consider alternatives which are hard to compare with it.

To see the relevance of the concept in economic situations, imagine a store manager who wishes to get potential customers into his store. Once a customer is in the store, he can survey the products on display. If he finds something better than his outside option (e.g., a product he regularly buys elsewhere), he will purchase it at the store. However, first the manager has to get the customer inside the store - or, using the marketing literature’s terminology, to introduce the set of products sold at the store into the customer’s consideration set. For instance, the customer may have to be persuaded that one of the products at the store is superior to his outside option, according to some criterion (e.g., price or some aspect of quality), in order to agree to enter the store. After giving serious consideration to the “door opening” product, as well as the other products at your store, he may realize that by his preferences the outside option is better after all. However, the initial criterion persuades the customer that the array of products at the store merits serious consideration.
Alternatively, the potential customer may be unaware that the store sells potential substitutes to his outside option. To draw his attention to the store, the manager may need to put on display at the shop front a product which is similar to the customer’s outside option. By similarity we mean that the product and the outside option share a brand name, certain product features, a price range, etc. Often we find it easier to identify an alternative as relevant to our decision problem when it resembles what we are already familiar with. Think about the way you browse through the long list of presentations at a large conference - the title of a talk is more likely to catch your eye if it resonates with your research interests. When thinking more deeply about it, you may decide not to attend the talk after all, but the proximity makes the initial, instinctive attention grabbing easier.

Keyword-based Google search provides a modern-day analogue of the shop front story. Suppose that you are used to buying electric gadgets at a particular webstore. When you decide that it time to look for substitutes, you click a few keywords in Google. The keywords are based on your familiarity with a benchmark product, which is what you regularly buy or bought in the past. The Google search will elicit a competing store if and only if that store posts products that fit the keywords on its website. When you browse through the competing website, you may end up buying something altogether different from what you were originally familiar with. However, the similarity between the attention grabbing product and your benchmark, reflected by the keywords that fit both, is instrumental in getting the competing webstore into your consideration set.

To capture such examples, we present in Section 2 a model of consumer behavior, in which the consumer’s choice problem is given by a pair of menus of products, \((M_1, M_2)\), where \(M_1 (M_2)\) is the set of products offered by outlets the consumer is already (yet to be) familiar with. The consumer’s choice procedure is based on two primitives: a standard preference relation \(\succ\) and another binary relation \(R\), which we call the “consideration relation”. The consumer first identifies the set of most preferred products in \(M_1\). If one of these products relates to one of the products in \(M_2\) via \(R\), the consumer’s consideration set is \(M_1 \cup M_2\); otherwise, it is \(M_1\). The consumer chooses one of the most preferred products in the consideration set. This procedure contains rational choice as a special case, when \(x R y\) for every two products \(x, y\). In general, the procedure induces choice behavior which departs from rationality.

The heart of this paper is a pair of applications which embed the consideration-sets procedure in market environments where two identical firms compete over a homogeneous population of consumers who follow the procedure. The applications have the
property that if consumers were rational, the equilibrium outcome would be manifestly competitive and no marketing devices would be employed. Our objective is to explore the implications of the consideration-sets model on the competitiveness of the market outcome and the equilibrium use of marketing devices.

In Section 3, we analyze a model in which firms simultaneously choose menus of products and aim to maximize market share minus the fixed cost of adding products to the menu (where the cost of offering the grand set is below the value of getting half the market). Each firm is equally likely to play the role of $M_1$ or $M_2$ in the consumer’s procedure. When the consumer is rational, firms offer the consumers’ most preferred product as a singleton in Nash equilibrium. In contrast, as long as some product fails to relate via $R$ to the most preferred product, symmetric (mixed) Nash equilibrium induces a non-competitive outcome, in which firms offer the best product with probability below one. Under the same condition on $R$, the support of the equilibrium strategy includes non-singleton menus - i.e., menus containing “irrelevant alternatives” which the consumer never chooses. Irrelevant alternatives function as “loss leaders” that draw the consumers’ attention to better products in the menu. The expected cost of irrelevant alternatives may be viewed as a “deadweight loss” resulting from consumer bounded rationality.

Our main finding in this section is that for a large class of consideration relations, firms earn the same equilibrium payoffs as in the rational-consumer case. Thus, although the market outcome is not competitive, industry profits are. From this perspective, the function of irrelevant alternatives is to restore an aspect of market competition in the face of consumers’ imperfect ability to perceive the entire feasible set. The competitive-payoff result has an immediate welfare implication: the equilibrium outcome under consumer bounded rationality is Pareto inferior to the outcome under rational consumers. A more subtle corollary concerns the effectiveness of using irrelevant alternatives as a marketing device. Recall that the function of irrelevant alternatives is to “get the customer into the store”. It turns out that in any symmetric equilibrium that induces competitive industry profits, whenever the consumer considers a firm thanks to an irrelevant alternative in its menu, he necessarily ends up buying from that firm (unless the other firm offers the most preferred product). We refer to this result as the “effective marketing property”. Essentially, it means that whenever the marketing device succeeds in attracting initial attention it culminates in a sale.

In Section 4 we give a further demonstration of the scope of our framework and study a model of competitive advertising. The consideration-sets procedure allows two products to be equivalent in terms of the preference relation yet distinct in terms of
the consideration relation. This means that the two products are inherently the same as far as the consumption experience is concerned, but they are framed differently and therefore differ in the set of products from which they attract attention. In our model of competitive advertising, each firm sells a single product and decides which features to include in it as well as which of these features to advertise. We assume that in terms of preferences, consumers care only about the number of product features, whereas in terms of the consideration relation, \( x \sim y \) if and only if the set of advertised features in \( y \) weakly contains the set of actual features in \( x \). We characterize symmetric Nash equilibria in this model. We reproduce the “competitive-payoff” result of Section 3. We also characterize the type of products that are offered in equilibrium, as well as the firms’ advertising decisions. A comparative statics result is of special interest: the equilibrium amount of advertising increases with the number of potential product features. The interpretation is that advertising becomes more intensive as the market environment becomes more complex.

To our knowledge, this paper is the first to incorporate the notion of (manipulable) consideration sets into economic modeling. More broadly, it contributes to a growing literature on market interaction between profit-maximizing firms and boundedly rational consumers. Rubinstein (1993) analyzes monopolistic behavior when consumers differ in their ability to understand complex pricing schedules. Piccione and Rubinstein (2003) study intertemporal pricing when consumers have diverse ability to perceive temporal patterns. Spiegler (2006a,b) analyzes markets in which profit-maximizing firms compete over consumers who employ a naive sampling procedure to evaluate each firm. Shapiro (2006) studies a model in which consumers have limited memory and firms use advertising to affect the likelihood that consumers remember past consumption experiences. DellaVigna and Malmendier (2004), Eliaz and Spiegler (2005,2006), and Gabaix and Laibson (2006) study interaction with consumers having limited ability to predict their future tastes. Mullainathan, Schwartzstein and Shleifer (2007) study the role of uninformative advertising when consumers apply “coarse reasoning”.

2 The Consumer’s Choice Procedure

Let \( X \) be a finite set of products. A menu is a non-empty subset of \( X \). A choice problem is a pair of menus \((M^1, M^2)\), where \( M^1 \) (\( M^2 \)) represents the set of products offered by outlets the consumer is already (yet to become) familiar with. Note that \( M^1 \cap M^2 \) may be non-empty. A choice correspondence assigns to each choice problem
\((M^1, M^2)\) a set of alternatives from \(M^1 \cup M^2\). The consumer’s choice procedure has two primitives: a standard preference relation \(\succeq\) over \(X\) and a binary relation \(R\) over \(X\), which we call the “consideration relation”. Slightly abusing notation, for any \(x \in X\), let \(R(x) = \{y \in X \mid xRy\}\). The procedure has several stages. First, the consumer identifies the set of \(\succeq\)-maximal products in \(M^1\), denoted \(\max_{\succeq}(M^1)\). The products in this set serve as benchmarks for the potential exploration of the new outlet. The consumer then constructs a “consideration set”, which is \(M^1 \setminus M^2\) if \(\exists x \in \max_{\succeq}(M^1) (R(x) \cap M^2) \neq \emptyset\), and \(M^1\) otherwise. The consumer then chooses a \(\succeq\)-maximal product in the consideration set. The procedure induces the following choice correspondence: 

\[
c(M^1, M^2) = \max_{\succeq}(M^1 \cup M^2) \quad \text{if} \quad \exists x \in \max_{\succeq}(M^1) (R(x) \cap M^2) \neq \emptyset,
\]

and

\[
c(M^1, M^2) = \max_{\succeq}(M^1) \quad \text{otherwise}.
\]

Two classes of consideration relations are of particular interest, as they fit the motivating examples given in the Introduction. The case of a complete and transitive \(R\) fits situations in which a salesperson tries to arouse the consumer’s interest in a set of products offered by a new outlet. The salesperson needs to provide an argument that will overcome the consumer’s initial resistance to explore new alternatives. An example of such an argument is that one of the products in the new outlet is superior to one of the benchmark products according to some preference criterion (price, an aspect of quality, etc.). This criterion need not overlap with the consumer’s preferences, because it merely convinces the consumer to give the new outlet a closer look, while the consumer’s preferences are based on that closer look. The case of a reflexive \(R\) fits situations in which the consumer is initially unaware of the new outlet, yet his attention can be drawn to the new outlet if it offers something similar to one of the consumer’s benchmark products. Reflexivity is a minimal property that should hold under any reasonable similarity relation. We shall return to these two classes of consideration relations in Section 3.

The consumer’s choice procedure departs from standard rational choice. First, the very notion of a choice problem distinguishes between pairs of menus \((M^1, M^2)\) and \((N^1, N^2)\) for which \(M^1 \cup M^2 = N^1 \cup N^2\). For instance, let \(X = \{a, b\}\), \(b \succ a\), and assume that \(R\) is the identity relation. Then, \(c(\{a\}, \{b\}) = \{a\}\) whereas \(c(\{a, b\}, \{b\}) = \{b\}\). This example exhibits a non-standard choice effect: adding a product \(x \in M^1\) to \(M^2\) may cause the consumer to switch from a product in \(M^1\) to some other product in \(M^2\).

Another non-standard choice effect induced by the procedure is the following: enlarging \(M^1\) may cause the consumer to switch from a product in \(M^1\) to some other product in \(M^2\). For instance, let \(X = \{a, b, d\}\), \(d \succ b \succ a\), and assume \(bRd\) and \(aRd\). Then, \(c(\{a\}, \{d\}) = \{a\}\) and \(c(\{a, b\}, \{d\}) = \{d\}\). This example also violates
the property known as Independence of Irrelevant Alternatives (IIA): adding \( b \) to the set of feasible alternatives reversed the choice from \( a \) to \( d \). (This is not a precise statement, since the formal definition of IIA is based on choice functions defined over menus rather than pairs of menus.) Note, however, that the procedure retains some aspects of rational choice. In particular, if \( c(M_1, M_2) \in M_2 \setminus M_1 \) and \( M_2 \subset M'_2 \), then \( c(M_1, M'_2) \in M'_2 \setminus M_1 \). Complete choice-theoretic characterization of the consideration-sets procedure is left for future work.

Our non-standard definition of a choice problem implies that the procedure differs from alternative decision models proposed in the literature, which share our attempt to capture decision makers with limited ability to rank the entire feasible set. For instance, the model of incomplete preferences (proposed by Aumann (1962) and developed by many others) assumes that the decision maker grasps the entire feasible set but is unable to rank some pairs of alternatives. Another related model is the Rational shortlist Method due to Mariotti and Manzini (2007), which like ours is based on a pair of binary relations. The first binary relation is used to select a “shortlist” and the second binary relation is used to select a unique product from the shortlist. In both these alternative models, the notion of a choice problem is standard. It follows that in order to draw a deep comparison between these non-standard decision models and ours, one needs to modify the former so that they are based on the same notion of a choice problem as the latter. Such a comparison lies outside the scope of this paper.

3 Competitive Use of Irrelevant Alternatives

In this section we present a market model that incorporates the choice procedure presented in Section 2. Two identical firms compete for a continuum of measure one of identical consumers. Each firm \( i \) simultaneously chooses a menu \( M_i \), which can be any non-empty subset of a finite set of products \( X = \{1, 2, ..., n\} \), \( n > 1 \). For each product, let \( c_x \) be a strictly positive fixed cost associated with adding \( x \) to the menu. The fixed cost of a menu \( M \) is \( c(M) = \sum_{x \in M} c_x \). Let \( \rho_i(M_1, M_2) \) denote the proportion of consumers who choose firm \( i \) under the strategy profile \((M_1, M_2)\). Firm \( i \)'s objective is to maximize \( \rho_i(M_1, M_2) - c(M_i) \). We assume that \( c(X) < \frac{1}{2} \) and \( c_1 > c_x \) for all \( x \neq 1 \).

Consumers choose a product according to the consideration-sets procedure. Given a strategy profile \((M_1, M_2)\), the choice problem is \((M_1, M_2)\) for one half of the population of consumers and \((M_2, M_1)\) for the other half. The interpretation is that for half the consumers, firm 1 is the incumbent - i.e., the firm they are familiar with whereas firm 2
is a new competitor, and vice versa for the other half. Consumer preferences are strict: $1 > 2 > \cdots > n$. We assume that $R(x) \neq \emptyset$ for every $x \in X$. When the consumer is rational - i.e., if $xRy$ for all $x, y \in X$, then firms play $\{1\}$ in Nash equilibrium and earn a payoff of $\frac{1}{2} - c_1$. We will refer to this outcome as the competitive outcome and to the equilibrium payoff as the \textit{competitive payoff}. Note that $\frac{1}{2} - c_1$ is also the max-min payoff for any admissible $R$.

To illustrate the model, let $n = 2$ and assume that $xRy$ if and only if $x = y$. Thus, when the strategy profile is $\{(1), (2)\}$, firms 1 and 2’s payoffs are $\frac{1}{2} - c_1$ and $\frac{1}{2} - c_2$, and when the strategy profile is $\{(1, 2), (2)\}$, the payoffs are $1 - c_1 - c_2$ and $-c_2$. The consideration relation in this example can be interpreted as follows. Each firm $i$ is a store and $M_i$ is the set of products it puts on display. When a consumer walks near a store he does not regularly shop at, his attention is drawn to that store only if it puts on display the same product he is used to buying at the other store.

In this example, there is a unique symmetric Nash equilibrium. The equilibrium strategy $\sigma$ is mixed: $\sigma\{2\} = 2c_2, \sigma\{1, 2\} = 2(c_1 - c_2), \sigma\{1\} = 1 - 2c_1$. The equilibrium has a few noteworthy features. First, the outcome is non-competitive: consumers end up buying the inferior product 2 with positive probability. Second, firms offer the non-singleton menu $\{1, 2\}$ with positive probability. In this menu, the product 2 is an “irrelevant alternative”, because the consumer never chooses it. Its function is to attract consumers’ attention to the better product in the menu. Third, firms earn a payoff of $\frac{1}{2} - c_1$ in equilibrium. This is the same payoff firms would earn in Nash equilibrium if consumers were rational (i.e., when $xRy$ for all $x, y \in \{1, 2\}$). Thus, although the equilibrium outcome is not competitive, industry profits are. The rest of this section is devoted to an exploration of the generality of these observations.

Let us begin with a few general results that characterize symmetric Nash equilibrium in this model for arbitrary consideration relations (which satisfy the above condition that $R(x) \neq \emptyset$ for all $x \in X$). The following is necessary and sufficient condition for a competitive equilibrium outcome.

\textbf{Proposition 1} \textit{Firms play $\{1\}$ with probability one in Nash equilibrium if and only if $xR1$ for every $x \neq 1$.}

The next result establishes that the use of irrelevant alternatives is a general property of non-competitive symmetric equilibria.
Proposition 2 If $x \not\in R_1$ for some $x \neq 1$, then in any symmetric Nash equilibrium $\sigma$, $S(\sigma)$ contains a non-singleton menu.

An irrelevant alternative is an extreme form of a “loss leader”: it is costly to offer and generates no direct benefit. Therefore, the equilibrium use of irrelevant alternatives is socially wasteful.

We now revisit two classes of consideration relations highlighted in Section 2: reflexive relations and complete and transitive relations. The identity relation ($x R y$ if and only if $x = y$) is an example of a reflexive consideration relation. It turns out that symmetric Nash equilibria have a particularly clean characterization in this case. Given a symmetric equilibrium strategy $\sigma$, let $\beta_\sigma(x)$ and $\alpha_\sigma(x)$ denote the probabilities that $x$ is offered as a $\succ$-maximal and as a $\succ$-inferior, “irrelevant” product. That is, $\beta_\sigma(x) = \sum_{M \in S(\sigma), b(M) = x} \sigma(M)$ and $\alpha_\sigma(x) = \sum_{M \in S(\sigma), x \in M \setminus \{b(M)\}} \sigma(M)$.

Proposition 3 If $R$ is the identity relation, then in any symmetric Nash equilibrium $\sigma$, $\beta_\sigma(x) = 2c_x$ and $\alpha_\sigma(x) = 2c_1 - 2c_x$ for all $x \neq 1$.

Thus, all products $x \neq 1$ are offered with the same probability $2c_1$ in symmetric equilibrium, where the better the product, the higher the probability it is offered as a $\succ$-maximal product and the lower the probability it is offered as an irrelevant alternative. Proposition 3 implies that firms earn a payoff of $\frac{1}{2} - c_1$ in any symmetric equilibrium. To see why, note that $\beta_\sigma(1) = 1 - \sum_{x \neq 1} \beta_\sigma(x) = 1 - 2 \sum_{x \neq 1} c_x > 0$. If $\{1\} \in S(\sigma)$, the payoff from this menu is clearly $\frac{1}{2} - c_1$. If all menus $M \in S(\sigma)$ with $1 \in M$ contain irrelevant alternatives $x \neq 1$, the cost of these products is exactly offset by the added market share they generate, by the characterization of $\beta_\sigma(x)$ for all $x \neq 1$.

Given the equilibrium characterization of $\beta_\sigma(\cdot)$ and $\alpha_\sigma(\cdot)$, we can calculate the fraction of consumers who switch a supplier in symmetric equilibrium. In order for a consumer to switch from one firm to the other, the best product in the former’s menu must be offered by the other firm as an irrelevant alternative. This leads to the following expression:

$$\sum_{x \neq 1} \beta_\sigma(x)\alpha_\sigma(x) = \sum_{x \neq 1} 4c_x(c_1 - c_x)$$

Our assumptions on menu costs ensure that the switching fraction is well-defined. Unlike the rational-consumer benchmark, it is strictly positive. Moreover, it behaves non-monotonically in menu costs. The reason is that as a product becomes more costly,
it is offered more frequently as a non-maximal product but less frequently as a maximal product. The switching fraction approaches a maximum of \((n - 1) \cdot c_1^2\) as the costs of all products \(x \neq 1\) cluster near \(c_1/2\).

The switching fraction is exactly equal to the expected cost of irrelevant alternatives, because for each \(x \neq 1\), the probability it is offered as an irrelevant alternative by each firm is by definition \(\alpha_\sigma(x)\), while by Proposition 3, its cost is equal to \(\beta_\sigma(x)/2\). We may view this expected cost as a “deadweight loss” of consumers’ bounded rationality, because it is a cost firms invest in products that consumers never choose. However, this deadweight loss is equivalent to consumers’ switching frequency.

We say that a consideration relation \(R'\) is richer than \(R\) if \(R(x) \subseteq R'(x)\) for all \(x \in X\). Among all reflexive consideration relations, the identity relation analyzed above is the least rich while the consideration relation that fits a rational consumer \((xRy\) for all \(x, y \in X\)) is the richest. In both of these extreme cases, we saw that firms earn competitive payoffs in symmetric Nash equilibrium. Intuitively, as the consideration relation becomes richer, the market friction due to consumers’ bounded rationality gets weaker, because the consideration set coincides with the feasible set for more strategy profiles. Therefore, one might expect that the competitive payoff result would hold for all reflexive consideration relations.

This intuition turns out to be false, as the following counter-example demonstrates. Let \(X\) be the set of all three-digit binary numbers, excluding 000. Assume that \(111 \succ x\) for every \(x \neq 111\). Finally, assume that \(xRy\) if \(x\) and \(y\) have at least two identical digits. This \(R\) is richer than the identity relation. Assume that \(c_{111} = \frac{1}{3}\), whereas \(c_x = c < \frac{1}{30}\) for all \(x \neq 111\). It can be shown that there exists a continuum of symmetric equilibria with the following properties: (i) the support of the equilibrium strategy consists of \{111, 110\}, \{111, 101\}, \{111, 001\}, \{100\}, \{010\} and \{001\}; (ii) the equilibrium payoff is strictly above the competitive level of \(\frac{1}{2} - c_{111}\).

Nevertheless, we conjecture that for generic cost structures, firms indeed earn the competitive payoff in symmetric Nash equilibrium under any reflexive consideration relation. We now prove this conjecture for two restricted domains of reflexive relations, which capture different aspects of similarity, inspired by Tversky (1977) and Rubinstein (1988).

1. Equivalence relations. A consideration relation \(R\) is an equivalence relation if it is reflexive, symmetric and transitive. This case fits situations in which products are divided into mutually exclusive categories, such that two products are deemed similar if they belong to the same category.
2. **Linear similarity.** Let $\psi : X \to \mathbb{R}$ be a one-to-one function that assigns to each product a distinct location along the real line. Assume that for every $x \in X$, $y \in R(x)$ if $\psi(y)$ belongs to some arbitrary neighborhood of $\psi(x)$. We refer to such $R$ as a linear consideration relation. It fits situations in which products are represented by points on a efficiency frontier in $\mathbb{R}^2_{++}$ (e.g., soft drinks are characterized by tastiness and healthiness, such that the tastier the beverage, the less healthy it is), where two products are similar if they lie close to each other along the frontier. In general, a linear similarity relation need not be symmetric. More importantly, it is typically intransitive. However, it is obviously reflexive, since any neighborhood of $\psi(x)$ necessarily contains itself.

**Proposition 4** Suppose that $R$ is an equivalence relation or a linear consideration relation. Then, in any symmetric Nash equilibrium firms earn a payoff of $\frac{1}{2} - c_1$.

For an arbitrary reflexive consideration relation, we are able to obtain a competitive-payoff result for sufficiently low menu costs. This result is based on the following lemma.

**Lemma 1** Suppose that $R$ is reflexive. Let $\sigma$ be a symmetric Nash equilibrium strategy. Then, $\beta_\sigma(x) \leq 2c_x$ for every $x \neq 1$.

This lemma is interesting in its own right, as it establishes a relation between the probability that a product is offered as a $\succ$-maximal product and its menu cost. The lemma also implies that as menu costs tend to zero, $\beta_\sigma(1)$ converges to one. Thus, as far as the consumer is concerned, when menu costs are small the equilibrium outcome is close to the rational-choice benchmark.

**Proposition 5** If $c(X) < 2^{-n}$, then firms earn a payoff of $\frac{1}{2} - c_1$ in symmetric Nash equilibrium under any reflexive consideration relation.

Let us now turn to the other class of consideration relations highlighted in Section 2, namely the class of complete and transitive relations.
Proposition 6 If \( R \) is complete and transitive, relation, then in any symmetric Nash equilibrium firms earn a payoff of \( \frac{1}{2} - c_1 \).

The implications of competitive equilibrium payoffs

Competitive-payoff results have an immediate welfare implication. Suppose that \( x \not\in R 1 \) for some \( x \neq 1 \). Let \( \sigma \) be a symmetric Nash equilibrium strategy that induces competitive payoffs. Then, the equilibrium outcome is Pareto inferior to the equilibrium outcome in the rational-consumer case. The reason is simple. On one hand, Proposition 1 implies that firms offer products \( x \neq 1 \) with positive probability, hence the consumer is worse off than in the rational-consumer benchmark. On the other hand, by assumption firms earn the same payoffs than in the rational-consumer case. Thus, in our model consumer bounded rationality leads to a clear welfare deterioration relative to the full-rationality benchmark.

Competitive-payoff results imply another, subtler corollary regarding the equilibrium effectiveness of irrelevant alternatives as marketing devices.

Definition 1 (Effective Marketing) A mixed strategy \( \sigma \) satisfies the effective marketing property if for every \( M, M' \in S(\sigma) \) with \( b(M') \neq 1 \), \( M \) beats \( M' \) whenever \( b(M') \not\in Rb(M) \) and \( R(b(M')) \cap M \neq \emptyset \).

The interpretation is as follows. When \( b(M') \not\in Rb(M) \) yet \( R(b(M')) \cap M \neq \emptyset \), a consumer for whom \( M' \) is the “incumbent’s menu” and adds \( M \) to his consideration set only because of an irrelevant alternative this menu contains. That is, if the firm that plays \( M \) deviated to \( \{b(M)\} \), this consumer would fail to consider it. The effective marketing property means that whenever the consumer considers a firm only because of a marketing device it utilizes, he ends up buying from that firm. To use a vivid image, if a customer enters the fishmonger’s store only because of his “fresh fish” calls, then ultimately he buys from him.

This property is not obvious. For instance, let \( X = \{1, 2, 3, 4\} \), \( 1 \succ 2 \succ 3 \succ 4 \), \( 2R4 \) and \( 2 \not\in R3 \). Suppose that \( \{2\}, \{3, 4\} \in S(\sigma) \). Then, when \( \{2\} \) plays the role of \( M^1 \) in the consumer’s choice problem, the consumer adds \( \{3, 4\} \) to his consideration set only because this menu includes 4, yet he ends up buying from the firm that offers \( \{2\} \). Such a state of affairs is impossible in symmetric equilibrium in which firms earn competitive payoffs.
Proposition 7: Let \( \sigma \) be a symmetric Nash equilibrium strategy that induces competitive payoffs. Then, \( \sigma \) satisfies the effective marketing property.

To get an intuition for this result, observe that competitive payoffs imply that the menu cost of an irrelevant alternative must offset exactly the increased market share it generates. Thus, if \( \sigma \) be a symmetric Nash equilibrium strategy that induces competitive payoffs and \( \{2\}, \{3, 4\} \in S(\sigma) \), then firms are indifferent between offering \( \{3, 4\} \) and \( \{3\} \). But this means that if a firm played \( \{1, 4\} \), the irrelevant alternative 4 would generate an even greater market share than when it is offered in conjunction with 3, because \( \{1, 4\} \) beats \( \{2\} \) whereas \( \{3, 4\} \) does not. Thus, the net gain from adding 4 to 1 is strictly positive, which means that a firm can earn a payoff above the competitive level, a contradiction.

Let us summarize the insights our model of competitive marketing has generated. First, the socially wasteful use of irrelevant alternatives is an integral part of equilibrium behavior. Second, for large classes of consideration relations, industry profits are competitive in equilibrium, even though the market outcome is non-competitive. This means that the outcome is Pareto-inferior to the rational-consumer benchmark. Finally, competitive-payoff results imply that irrelevant alternatives are effective as marketing devices in equilibrium, in the sense that whenever they are responsible for drawing the consumer’s attention to the menu, he ends up making a choice from this menu.

4 Competitive Advertising

The model in Section 3 assumes that consumer preferences are strict. In particular, this rules out the existence of two products \( x, y \in X \) such that \( x \sim y \) and \( R(x) \neq R(y) \).

An interesting interpretation of such a situation is that \( x \) and \( y \) are inherently the same product as far as the consumption experience is concerned, but they differ in the way they are framed. Thus, allowing weak preferences enables us to capture situations in which firms decide not only which product to sell but also how to present it to consumers. In a classical model of price competition with advertising, Butters (1977) assumes that firms attract consumers’ attention by posting ads. This basic “advertising technology” can be incorporated into our framework: let every product \( x \) come in two variants, advertised and unadvertised, denoted \( x^a \) and \( x^u \), such that (i) \( x^a \sim x^u \), and (ii) for any \( x, y \in X \) and any \( i \in \{a, u\} \), \( y^i R x^a \) and \( y^i R x^u \). Our model goes beyond
Butters (1977) and other existing models in providing language for formulating a rich variety of “advertising technologies”, as we now demonstrate.

Let \( F = \{1, ..., K\} \) be a finite set of product features. Let \( X \) be the set of all pairs \((P, A)\), where \( A \subseteq P \subseteq F \) and \( P \neq \emptyset \). Firms choose simultaneously single products from \( X \). A mixed strategy is a probability distribution over the set of admissible pairs \((F, A)\). A firm’s cost of offering \((P, A)\) is \( c(P, A) = c_p \cdot |P| + c_a \cdot |A| \), where \( c_p, c_a > 0 \). Assume that \( K(c_p + c_a) < \frac{1}{2} \).

Given a strategy profile \((x_1, x_2)\), the consumer’s choice problem is equally likely to be \((\{x_1\}, \{x_2\})\) or \((\{x_2\}, \{x_1\})\). Assume that \((P, A) \succ (P, A')\) if \(|P| > |P'|\) and assume that \((P, A) R (P', A')\) if \( A' \supseteq P \). We interpret \( P \) as the set of actual features of the product the firm chooses to sell, while \( A \) is the set of product features which the firm chooses to advertise. In order to persuade a consumer to consider a substitute to the product he is already familiar with, the advertisement must state that the new product has all the features the old product has. However, only closer scrutiny will reveal whether the new product is strictly better than the old one and thus merits switching a supplier. We discuss alternative specifications of \( R \) below.

Recall that in the previous section, a firm could ensure being considered by the consumer by offering the grand set. Similarly, in the current example, a firm can ensure being considered by offering \((F, F)\). Note that when the consumer is rational, there is no need to advertise because he will always consider both alternatives. Consequently, In Nash equilibrium both firms will offer \((F, \emptyset)\) and earn a payoff of \( \frac{1}{2} - Kc_p \). These are also the max-min strategy and max-min payoff under the current specification of \( R \).

**Proposition 8** Let \( \sigma \) be a symmetric Nash equilibrium strategy \( \sigma \). Then:

(i) \( \sigma(\{k\}, \emptyset) = 2c_a \) for all \( k = 1, ..., K \).
(ii) \( \sum_{A \subseteq F} \sigma(F, A) = 1 - 2Kc_a \).
(iii) \( \sum_{k \in A} \sigma(F, A) = 2(K - 1)c_p \) for all \( k = 1, ..., K \).

Thus, in symmetric equilibrium firms offer either the lowest-quality products (containing a single feature) or the highest-quality product (containing all features). When they offer the latter, they advertise each feature with probability \( 2(K - 1)c_p \). It is easy to verify that firms earn the “competitive payoff” \( \frac{1}{2} - Kc_p \) in equilibrium. An “effective marketing property” holds trivially, since the only advertised product is the highest-quality one. Thus, when a consumer’s benchmark product is not the most
preferred one and he considers the other product only because of its advertising, he ends up buying the latter.

The equilibrium amount of advertising can be defined as $4K(K - 1)c_p$, each firm advertises each feature with probability $2(K - 1)c_p$. Suppose that we switch from an environment $(K, c_p, c_a)$ to the environment $(K', c'_p, c'_a)$ which satisfies $K' > K$, $K'c'_p = Kc_p$ and $K'c'_a = Kc_a$. Products in the latter environment are more complex because they have a greater number of potential features, yet overall production and advertising costs are the same in both environments. The equilibrium amount of advertising in the environment $(K', c'_p, c'_a)$ is thus $(K' - 1)/(K - 1)$ times higher than in the environment $(k, c_p, c_a)$. In other words, a more complex product structure gives rise to a greater amount of equilibrium advertising, even when we control for costs.

In this example, we defined $R$ in terms of the relation between one firm’s actual product features and the other firm’s advertised product features. In some situations it is more natural to define $R$ in terms of the relation between the two firms’ advertised product features - for instance, $(P, A)R(P, A')$ if $A' \supset A$, or $(P, A)R(P, A')$ if $|A| > |A'|$. The story is that the consumer does not know the actual terms of the product he currently consumes (think of a subscription to a mobile phone service). Only when he looks deeply into the matter he can tell whether his current deal is better than competitors’ offers. However, his decision to initiate this meticulous comparison depends on advertising - specifically, when by the firms’ own admission the competing offer is better in some sense (having additional features relative to the status quo, or simply a greater number of features) than the benchmark. It can be shown that under these two alternative specifications of $R$, firms earn the competitive payoff $\frac{1}{2} - Kc_a$ in symmetric Nash equilibrium. We have been unable to formulating an interesting general property of $R$ which unifies these competitive-payoff results with Proposition 8.

5 Concluding Remarks

Our contributions in this paper are threefold. First, we provide a framework which captures the notion of (manipulate) consideration sets and can be applied to a rich variety of marketing situations. Second, we characterize the use of marketing devices such as advertising or “loss leading” products in competitive environments, as well as the effectiveness of these devices, both in terms of the amount of consumer attention they attract and in terms of the market share they generate. Finally, we demonstrate that firms may fail to exploit consumers’ bounded rationality to earn collusive payoffs,
because the use of marketing devices induces competitive industry profits. We conclude by discussing several features which are absent from the models of competitive marketing presented in Sections 3 and 4.

5.1 Incorporating prices

Throughout this paper, we abstracted from price setting and assumed that firms try to maximize market share minus fixed costs. Our motivation was analytic simplicity: just as in older marketing models - a prime example of which is the Hotelling strategic location model - it is easier to start with a model in which firms care only about market share and only then incorporate price setting. At any rate, extending our models in this direction is clearly important.

The following is an example of such an extension of the competition-in-menus model of Section 3. Let \( X = A \times P \), where \( A \) is a finite set of product types and \( P \) is a finite set of (non-negative) prices. Consumer preferences satisfy the following condition: for any \( a \in A \), \( (a, p') \succ (a, p) \) if \( p' < p \). Define the consideration relation \( R \) as follows: \( (a, p)R(a', p') \) if and only if \( a = a' \) and \( p \leq p' \). This means that consumers consider a new outlet if and only if it offers their benchmark product at a lower price. Adding items to the menu carries a fixed cost, as in the model of Section 3.

Aside from checking the robustness of the main results of Section 3 to this new extension, a new challenge that arises is to identify the equilibrium structure of price dispersion. One type of price dispersion occurs when a firm offers the same product at two different prices in the same menu. Let \( \sigma \) be a symmetric Nash equilibrium strategy, and assume there exists a menu \( M \in S(\sigma) \) that contains both \( (a, p) \) and \( (a, p') \), where \( a \in A \) and \( p' > p \). Because \( R(a, p') \subset R(a, p) \) and \( b(M) \succeq (a, p) \succ (a, p') \), each firm can profitably deviate by removing \( (a, p') \) from \( M \), a contradiction. Therefore, this type of price dispersion cannot exist in equilibrium. Another type of price dispersion exists when \( S(\sigma) \) contains two menus \( M, M' \) such that \( b(M) = (a, p) \) and \( b(M') = (a, p') \), \( p' \neq p \). This means that in equilibrium, the consumer buys the same product at different possible prices. Whether this type of price dispersion is possible in equilibrium is a problem we leave for future research.

5.2 Consumer Heterogeneity

Throughout this paper, we restricted attention to populations of homogeneous consumers. Since consumers in our model are characterized by two primitives, \( \succeq \) and \( R \), heterogeneity may exist in both dimensions. Consider heterogeneity in \( R \) first. Suppose
that the firms face a continuum of consumers, all of whom have the same preferences \(\succ\). However, consumers differ in their consideration relation. It can be shown that in the model of Section 3, extended to allow for heterogeneous consideration relations, if we hold the distribution of consumer types fixed and let menu costs tend to zero, then firms earn the competitive payoff \(\frac{1}{2} - c_1\) in any symmetric Nash equilibrium.

However, it is not true that for a fixed cost structure, if the competitive-payoff result holds for any consideration relation in the collection \(\mathcal{R} = \{R^1, \ldots, R^K\}\), then it must hold for a heterogeneous consumer population with some distribution over \(\mathcal{R}\). To see why, let \(X = \{1, 2\}\) and suppose that \(R^1\) is the identity relation whereas \(xR^2y\) for all \(x, y \in X\). It is easy to show that for a certain range of distributions over \(\{R^1, R^2\}\), the symmetric Nash equilibrium strategy has a support \(\{1\}, \{2\}, \{1, 2\}\), which means that firms earn an expected payoff above the competitive level (because the menu \{1\} attracts consumers with \(R^2\) away from the menu \{2\}).

We avoided heterogeneous consumer preferences because the rational-consumer benchmark would be sensitive to the exact distribution over consumer types, and we wanted a clear benchmark. Some of our results are probably extreme as a result of this modeling strategy. First, when the set of preferences in the population is sufficiently rich, every product will be found to be optimal by some consumers, and therefore no menu can contain “irrelevant alternatives”. Second, the effective marketing property is unlikely to hold under heterogeneous preferences. One of the challenges of extending our models in this direction is to formulate the analogues of these two effects when consumer preferences are diverse.

6 Proofs

Throughout this section, \(\sigma\) denotes a symmetric Nash equilibrium strategy and \(\mathcal{M}_\sigma^x\) denotes the set of menus \(M \in S(\sigma)\) for which \(b(M) = x\). We adopt the definitions of \(\beta_\sigma(x)\) and \(\alpha_\sigma(x)\) given in Section 3.

Proof of Proposition 1

Assume \(xR^11\) for every \(x \neq 1\). Clearly, if both firms play \{1\}, no firm has an incentive to deviate to another menu because \{1\} beats any menu \(M \notin \mathcal{M}_\sigma^1\), while any other menu \(M \in \mathcal{M}_\sigma^1\) attains the same market share and costs more. Now suppose that there exists a non-competitive Nash equilibrium \((\sigma_1, \sigma_2)\). Let \(M \in S(\sigma_1) \cup S(\sigma_2)\) such that \(b(M') \succeq b(M)\) for all \(M' \in S(\sigma_1) \cup S(\sigma_2)\). Without loss of generality, let \(M \in S(\sigma_1)\). Note that \(b(M) \neq 1\) and that \(M\) does not beat any menu in \(S(\sigma_2)\). Suppose that
firm 1 deviates from $M$ to $X$. Then, the firm increases its market share by at least \( \frac{1}{2}[\beta_{\sigma_2}(1) + (1 - \beta_{\sigma_2}(1)) = \frac{1}{2} \), because the deviation prevents being beaten by menus $M' \in \mathcal{M}_\sigma^1$, and it allows beating any menu $M' \notin \mathcal{M}_\sigma^1$ in $S(\sigma_2)$. The cost of this deviation is below $\frac{1}{2}$, hence the deviation is profitable.

Now assume $x \not\in \mathcal{R}_1$ for some $x \neq 1$. If both firms play $\{1\}$, then it is profitable for any firm to deviate from $\{1\}$ to $\{x\}$, thereby increasing its payoff from $\frac{1}{2} - c_1$ to $\frac{1}{2} - c_x$.

\[ \text{Lemma 2} \quad \beta_{\sigma}(1) > 0. \]

\textbf{Proof.} Assume the contrary. Given symmetry of equilibrium, a firm’s equilibrium payoff is below $\frac{1}{2}$. If the firm deviates to the pure strategy $X$, it ensures that all consumers choose it, yielding a payoff of $1 - c(X) > \frac{1}{2}$, hence the deviation is profitable.

\[ \text{Proof of Proposition 2} \]

Assume the contrary that $S(\sigma)$ consists of singletons only. By Lemma 2, $\{1\} \in S(\sigma)$. At the same time, by Proposition 1, there exists $x \neq 1$ such that $\sigma\{x\} > 0$. Suppose that a firm deviates from $\{1\}$ to $X$. The cost of this deviation is $c(X) - c_1$. The benefit from this deviation is \( \frac{1}{2} \sum_{x \neq 1, x \in \mathcal{R}_1} \sigma\{x\} \). From the firms’ decision not to carry out this deviation, we conclude that \( \frac{1}{2} \sum_{x \neq 1, x \in \mathcal{R}_1} \sigma\{x\} \leq c(X) - c_1 \). Suppose that $\sigma\{\{x\}\} > 0$ for some $x \neq 1$, $x \in \mathcal{R}_1$. Let $x^*$ denote the $\succ$-minimal such product. Suppose that a firm deviates from $\{x^*\}$ to $\{1, x^*\}$. The cost of this deviation is $c_1$. The benefit from this deviation is at least \( \frac{1}{2} \sigma\{1\} + \frac{1}{2} \sum_{x \neq 1, x \in \mathcal{R}_1} \sigma\{x\} \), because the deviation prevents being beaten by $\{1\}$ and allows beating any $\{x\}$ with $x \neq 1$, $x \in \mathcal{R}_1$. From the firms’ decision not to carry out this deviation, we conclude that \( \frac{1}{2} \sigma\{1\} + \frac{1}{2} \sum_{x \neq 1, x \in \mathcal{R}_1} \sigma\{x\} \leq c_1 \). Combining the two inequalities, we obtain $\frac{1}{2} \leq c(X)$, a contradiction. It follows that $\sigma\{\{x\}\} = 0$ for every $x \neq 1$, $x \in \mathcal{R}_1$. Therefore, $\{1\}$ generates a payoff of $\frac{1}{2} - c_1$. Now consider the $\succ$-maximal product $y \neq 1$ for which $\sigma\{\{y\}\} > 0$. By our previous step, $y \not\in \mathcal{R}_1$. Therefore, $\{y\}$ generates a payoff of at least $\frac{1}{2} - c_y > \frac{1}{2} - c_1$, a contradiction.}

\[ \text{Lemma 3} \quad \text{For every} \ M' \in S(\sigma) \ \text{there exists} \ M \in \mathcal{M}_\sigma^1 \ \text{that does not beat} \ M'. \]

\textbf{Proof.} Let $M' \in S(\sigma)$ be a menu which is beaten by all $M \in \mathcal{M}_\sigma^1$. Suppose that a firm deviates from $M'$ to $X$. Then, it increases its payoff by \( \frac{1}{2} [\beta_{\sigma}(1) + (1 - \beta_{\sigma}(1))] - [c(X) - c(M')] > 0 \), hence this deviation is profitable.
Lemma 4  Suppose that firms earn a payoff above \( \frac{1}{2} - c_1 \) under \( \sigma \). Then, every menu \( M \in \mathcal{M}_\sigma^1 \) contains a product \( y \neq 1 \) such that \( zRy \) for some \( z \neq 1 \) with \( \beta_\sigma(z) > 0 \).

Proof. By Lemma 2, \( \beta_\sigma(1) > 0 \). By Lemma 3, \( \beta_\sigma(x) = 0 \) for all \( x \not\sim_1 \), \( x \neq 1 \). On one hand, if a menu \( M \in \mathcal{M}_\sigma^1 \) contains a product \( y \neq 1 \) such that \( \beta_\sigma(z) = 0 \) for all \( zRy \), then it is profitable to remove \( y \) from \( M \). On the other hand, if \( \{1\} \in S(\sigma) \), then the payoff this menu generates is above \( \frac{1}{2} - c_1 \) only if \( \beta_\sigma(z) > 0 \) for some \( z \neq 1 \), \( z \not\sim_1 \). But this means that any menu \( M \in \mathcal{M}_\sigma^1 \) beats a menu \( M \in S(\sigma) \) with \( b(M) = z \), contradicting Lemma 3. ■

Proof of Proposition 6
Assume the contrary. Let \( Y \) denote the set of products \( y \) that belong to some \( M \in \mathcal{M}_\sigma^1 \) and for which there exists a product \( z \neq 1 \) such that \( zRy \) and \( \beta_\sigma(z) > 0 \). By Lemma 4, \( Y \not= \emptyset \). Because \( R \) is complete and transitive, there exists an \( R \)-maximal product \( y^* \) in \( Y \), such that any \( M \in \mathcal{M}_\sigma^1 \) containing \( y^* \) beats any \( M' \in S(\sigma) \) which is beaten by some other \( M'' \in \mathcal{M}_\sigma^1 \), thus contradicting Lemma 3. ■

Proof of Proposition 4
Assume the contrary. For every \( M \in \mathcal{M}_\sigma^1 \), let \( \Delta(M) \equiv \sum_{z \neq 1, zRy, y \in M} \beta_\sigma(z) - 2 \sum_{y \in M} c_y \). By Lemma 4, \( \Delta(M) > 0 \). For every \( M \in \mathcal{M}_\sigma^1 \), define \( B(M) \) as the set of products \( z \neq 1 \) for which \( \beta_\sigma(z) > 0 \) and \( zRy \) for some \( y \in M \) (recall that \( y \neq 1 \)). Suppose that \( B(M) \cap B(M') = \emptyset \) for some pair of menus \( M, M' \in \mathcal{M}_\sigma^1 \). If a firm deviates from \( M' \) to \( M' \cup M \), it increases its payoff by \( \Delta(M) > 0 \), hence the deviation is profitable. It follows that \( B(M) \cap B(M') \neq \emptyset \) for any pair of menus \( M, M' \in \mathcal{M}_\sigma^1 \). When \( R \) is a linear similarity relation, these pairwise intersections imply that \( \cap_{M \in \mathcal{M}_\sigma} B(M) \neq \emptyset \), in contradiction to Lemma 3. When \( R \) is an equivalence relation, these pairwise intersections imply that \( B(M) \) is identical for all \( M \in \mathcal{M}_\sigma^1 \), again contradicting Lemma 3. ■

Proof of Lemma 1
Assume the contrary. Let \( x \) be the \( \succ \)-minimal product for which \( \frac{1}{2} \beta_\sigma(x) > c_x \). Suppose that there exists a menu \( M \in S(\sigma) \) such that \( b(M) \succ x \) and \( x \not\sim_1 \) for all \( y \in M \). Then, \( M \) does not beat any menu \( M' \) with \( b(M') = x \). If a firm deviates from \( M \) to \( M \cup \{x\} \), then since \( b(M) \succ x \), the probability that some menu \( M'' \) with \( b(M'') \succ b(M) \) beats \( M \) does not change. Therefore, by reflexivity of \( R \), the deviation increases the firm’s payoff by at least \( \frac{1}{2} \beta_\sigma(x) - c_x > 0 \), hence it is profitable. It follows that for every \( M \in S(\sigma) \) for which \( b(M) \succ x \), there exists some \( y \in M \) such that \( x \not\sim_1 \), so that \( M \) beats any \( M' \) with \( b(M') = x \).
Now consider a menu \( M \in S(\sigma) \) with \( b(M) = x \) (there must be such a menu, since by assumption, \( \frac{1}{2} \beta_\sigma(x) > c_x > 0 \)), and suppose that a firm deviates to \( M \cup \{1\} \). The cost of this deviation is \( c_1 \), whereas the gained market share is at least \( \frac{1}{2} \sum_{y \geq x} \beta(y) \). The reason is that first, \( M \cup \{1\} \) beats any menu \( M' \) with \( b(M') = x \); and second, whereas prior to the deviation every menu \( M' \in S(\sigma) \) with \( b(M') > x \) beat \( M \) (as we showed in the previous paragraph), after the deviation no menu beats \( M \cup \{1\} \). In order for this deviation to be unprofitable, we must have \( \frac{1}{2} \sum_{y \geq x} \beta(y) \leq c_1 \). By the definition of \( x \), \( \frac{1}{2} \beta_\sigma(z) \leq c_z \) for all \( z < x \). Adding up these inequalities, we obtain
\[
\frac{1}{2} \sum_{y \in X} \beta(y) \leq c_1 + c\{x + 1, \ldots, n\} < c(X)
\]
But since the L.H.S of this inequality is by definition \( \frac{1}{2} \), we obtain \( \frac{1}{2} - c(X) \leq 0 \), contradicting condition (ii).

**Proof of Proposition 5**

Let \( c(X) < 2^{-n} \) and assume there exists a symmetric Nash equilibrium in which firms’ payoff exceeds \( \frac{1}{2} - c_1 \). By Lemma 4, every menu in \( \mathcal{M}_\sigma^1 \) contains some other product apart from 1. For each \( M \in \mathcal{M}_\sigma^1 \), there exists another menu \( M' \in S(\sigma) \), such that \( b(M') \neq 1 \) and \( b(M')\{y \in M, y \neq 1\} \). If a firm deviates from \( M' \) to \( M' \cup \{1\} \), it increases its payoff by at least \( \frac{1}{2} \sigma(M) - c_1 \). In order for this deviation to be unprofitable, we must have \( \frac{1}{2} \sigma(M) \leq c_1 \). Summing over all menus in \( \mathcal{M}_\sigma^1 \), we obtain
\[
\frac{1}{2} \beta_\sigma(1) \leq c_1 \cdot |\mathcal{M}_\sigma^1|.
\]
Clearly, \( |\mathcal{M}_\sigma^1| < 2^{n-1} \). By Lemma 1, \( \beta_\sigma(x) = 2c_x \) for every \( x \neq 1 \). Adding up these inequalities, we obtain
\[
\frac{1}{2} \leq c(X) + (2^{n-1} - 1) \cdot c_1 < 2^{n-1} \cdot c(X),
\]
which contradicts our initial assumption on \( c(X) \).

**Proof of Proposition 7**

Assume firms earn competitive payoffs under \( \sigma \). Assume there are at least two distinct products, \( x, x' \in M \), such that \( b(M')Rx \) and \( b(M')Rx' \). This implies \( 2c_x < \sum_{yRx, b(M)\supseteq y} \beta_\sigma(y) \leq \sum_{y \neq 1} \beta_\sigma(y) \). The reason is that since firms find it optimal to include \( x \) in \( M \), the market share generated by \( x \) alone must be strictly higher than its cost. Otherwise, given the overlap between the menus from which \( x \) and \( x' \) draw attention, it would be strictly profitable to remove \( x \) from \( M \). By Lemma 3, \( y \in M \) for every \( y \neq 1 \) for which \( \beta_\sigma(y) > 0 \). Therefore, if a firm deviates from its equilibrium strategy to \( \{1, x\} \), it earns a payoff of \( \frac{1}{2} - c_1 + \frac{1}{2} \sum_{yRx, y \neq 1} \beta_\sigma(y) - c_x > \frac{1}{2} - c_1 \), a contradiction.

It follows that for any \( M, M' \in S(\sigma) \) such that \( M \) beats \( M' \), there is exactly one product \( x \in M \) for which \( b(M')Rx \). Because \( M \) beats \( M' \), by definition \( M \) must contain at least one product \( x \in M \) for which \( b(M')Rx \). Assume that \( b(M') \neq 1 \) and \( b(M')\{y \in M, y \neq b(M)\} \), and yet \( M \) does not beat \( M' \) - i.e., \( b(M') \supseteq b(M) \). By the fact that \( M \in S(\sigma) \), and by the fact that in equilibrium firms choose to include
x \neq b(M) \text{ in } M, \text{ we can infer that the gain in market share from adding } x \text{ is at least as large as } c_x. \text{ Using the property we proved in the previous paragraph, we can write } 2c_x \leq \sum_{yRx, b(M) \succ y} \beta_\sigma(y). \text{ Since } 1 \succ b(M') \succeq b(M), \text{ this implies } 2c_x < \sum_{yRx, y \neq 1} \beta_\sigma(y). \text{ But as we saw, this inequality leads to a contradiction. }

**Proof of Proposition 3**

Because identity is an equivalence relation, Proposition 4 implies that firms earn competitive payoffs in symmetric Nash equilibrium. Observe that under the identity consideration relation, M beats M' if and only if b(M) \succ b(M') and b(M') \in M. Suppose that \alpha_\sigma(x) = 0 \text{ for some } x \neq 1. \text{ Then, if a firm plays } \{x\}, \text{ it earns } \frac{1}{2} - c_x, \text{ which is above the competitive level, a contradiction. Therefore, } \alpha_\sigma(x) > 0 \text{ for all } x \neq 1. \text{ Let } M \in S(\sigma) \text{ be a menu that includes some } x \neq 1 \text{ as a non-maximal product. The identity relation is reflexive. Therefore, by Lemma 1, } \beta_\sigma(x) \leq 2c_x. \text{ If the inequality is strict, it is profitable for a firm to deviate from } M \text{ to } M'\setminus\{x\}. \text{ It follows that } \beta_\sigma(x) = 2c_x. \text{ But this means that removing a non-maximal product } x \text{ from any menu } M \in S(\sigma) \text{ with } b(M) \succ x \text{ does not affect the payoff that the menu generates against } \sigma. \text{ In particular, for every } x \neq 1, \text{ the payoff generated by the menu } \{x\} \text{ is } \frac{1}{2}[1 - \alpha_\sigma(x)] - c_x = \frac{1}{2} - c_1. \text{ Thus, } \alpha_\sigma(x) = 2c_1 - 2c_x.

**Proof of Proposition 8**

The proof consists of two stages. First, we establish that firms earn competitive payoffs in symmetric Nash equilibrium. We then use this result to derive the conditions on symmetric equilibrium.

The competitive-payoff result is based on three lemmas which are analogous to Lemmas 2, 3 and 4 in the model of Section 3. Let us state them without proof, as the proof proceeds on essentially the same lines. Define \( \mathcal{A}_\sigma^F = \{A \subseteq P \mid (P, A) \in S(\sigma)\}. \)

**Lemma 5** \( \mathcal{A}_\sigma^F \neq \varnothing. \)

**Lemma 6** For every \((P, A) \in S(\sigma)\) with \(P \subseteq F\) there exists \(A' \in \mathcal{A}_\sigma^F\) such that \(A' \subseteq P\).

For any \(A \subseteq F\), let \(\Delta(A) = \frac{1}{2} \sum_{P \subseteq A, P \subseteq F} \sigma(P, A')\) be the gain in market share from advertising the subset of features \(A\) when the set of actual product features is \(F\) and the opponent plays the mixed strategy \(\sigma\). The definition of \(\Delta(\cdot)\) implies a superadditivity property: for all \(A, A' \subseteq F\), \(\Delta(A \cup A') \geq \Delta(A) + \Delta(A') - \Delta(A \cap A')\).
Lemma 7 Suppose that firms earn a payoff above $\frac{1}{2} - Kc_p$ under $\sigma$. Then, $\Delta(A) - c_a \cdot |A| = \delta > 0$ for every $A \in A^F_\sigma$.

Suppose that firms earn a payoff above $\frac{1}{2} - Kc_p$ under $\sigma$. Let us now show that $\Delta(\cap_{A \in A^F_\sigma} A) > 0$, violating Lemma 6. If $A^F_\sigma$ is a singleton, this follows immediately from Lemma 7. Suppose that $A^F_\sigma$ contains at least two subsets $A, A'$. By Lemma 7, $\Delta(A) - c_a \cdot |A| = \delta > 0$. By superadditivity of $\Delta(\cdot)$, if $\Delta(A \cap A') = 0$, then the payoff from the subset $A \cup A'$ is at least $2\delta + c_a \cdot |A \cap A'| > \delta$. Therefore, it is profitable for a firm to deviate from $(F, A)$, say, to $(F, A \cup A')$. It follows that $\Delta(A \cap A') > 0$ for every two $A, A' \in A^F_\sigma$. We can then prove inductively that for every $l = 3, ..., |A^F_\sigma|$, every collection of $l$ subsets $A^1, ..., A^l \in A^F_\sigma$ satisfies $\Delta(\cap_{i=1,...,l} A^i) > 0$. In particular, this means that $\Delta(\cap_{A \in A^F_\sigma} A) > 0$, contradicting Lemma 6. We have thus shown that firms earn $\frac{1}{2} - Kc_p$ under $\sigma$.

Our next step is to show that $A^F_\sigma = \emptyset$ for every $P$ satisfying $1 < |P| < K$. Assume the contrary - i.e., that there exists $(P, A) \in S(\sigma)$ with $1 < |P| < K$. Let us distinguish between two cases. First, suppose that $A^F_\sigma = \emptyset$. In this case, it is profitable to deviate from $(P, \emptyset)$ to some $(P', \emptyset)$ satisfying $|P'| = |P| - 1$. Second, suppose that $A^F_\sigma$ contains some non-empty subset $A$. In order for $(P, A)$ to be a best-reply, it must generate a payoff weakly above $(P, \emptyset)$. Therefore, $\frac{1}{2} \sum_{A' \subseteq A, P' \subseteq P} \sigma(P', A') - c_a \cdot |A| \geq 0$. But this means that $\Delta(A) - c_a \cdot |A| > 0$, because $\sum_{A' \subseteq A, P' \subseteq P} \sigma(P', A') \geq \sum_{A' \subseteq A, P' \subseteq P} \sigma(P', A') + \sigma(P, A) > \sum_{A' \subseteq A, P' \subseteq P} \sigma(P', A')$. It follows that the strategy $(F, A)$ generates a payoff strictly above $\frac{1}{2} - Kc_a$, contradicting our previous step.

Clearly, if $A^F_\sigma(k)$ is non-empty, then $A^F_\sigma(k) = \emptyset$, because there is no incentive to advertise a product with the minimal number of features. If $A^F_\sigma(k)$ is empty, then $k \notin A$ for every $A \in A^F_\sigma$; since advertising feature $k$ generates no added market share. But this means that if a firm plays $(\{k\}, \emptyset)$, it earns a payoff of $\frac{1}{2} - c_p$, a profitable deviation. It follows that for every $k = 1, ..., K$, $A^F_\sigma(k) = \emptyset$. The payoff from the strategy $(\{k\}, \emptyset)$ is $\frac{1}{2} - c_p - \frac{1}{2} \sum_{k \in A} \sigma(F, A) = \frac{1}{2} - Kc_p$, which gives the expression for $\sum_{k \in A} \sigma(F, A)$. This concludes the proof.

References


