Nominal Debt as a Burden on Monetary Policy*

Javier Díaz-Giménez  Giorgia Giovannetti  Ramon Marimon  Pedro Teles†

April 29, 2007

Abstract

We study the effects of nominal debt on the optimal sequential choice of monetary policy. We first analyze a model where nominal debt is the only source of time inconsistency of monetary policy. Without full commitment, the sequential optimal policy implies the depletion of the outstanding stock of debt progressively until the time inconsistency disappears. There is a resulting welfare loss due to the presence of nominal debt. We also discuss the more general case where monetary policy is time inconsistent even when debt is indexed. In that case with nominal debt the optimal policy converges to a time-consistent steady state with positive—or negative—debt, depending on the elasticity of substitution. Welfare can be improved if debt is nominal rather than indexed. The introduction of alternative forms of taxation may lessen the distortions due to the time inconsistency of monetary policy. Full commitment for the fiscal authority can override any commitment problem of the monetary authority.

JEL Classification Numbers: E40, E50, E58, and E60

*We would like to thank José-Victor Ríos-Rull, Jaume Ventura, Juan Pablo Nicolini and Isabel Correia for their comments, as well as the participants in seminars and conferences where this work has been presented. Díaz-Giménez, Giovannetti and Marimon gratefully acknowledges the financial support of the Spanish Ministerio de Ciencia y Tecnología (Grants: SEC2002-004318, ). Corresponding author: Ramon Marimon; Universitat Pompeu Fabra; Ramon Trias Fargas, 25-27; 08005 Barcelona (Spain); <ramon.marimon@upf.edu>.
†J. Díaz-Giménez: Universidad Carlos III and CAERP; G. Giovannetti: Università di Firenze; R. Marimon: Universitat Pompeu Fabra, CREi, CREA, CEPR and NBER, and P. Teles: Banco de Portugal, U. Católica Portuguesa and CEPR.
1 Introduction

Fiscal discipline has often been seen as a precondition to sustain price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. The policy discussion shows the concern with a time inconsistency problem associated with high levels of nominal debt that could be monetized. In this paper we analyze the implications for the optimal sequential design of monetary policy of public debt being nominal versus indexed. We take a simple model where a benevolent government has an incentive to monetize nominal debt, even if it is costly. We characterize the dynamic time consistent optimal policy when policy choices are made sequentially and we compare it to the optimal policy in an economy where debt is indexed.

We use a cash-in-advance model where agents start the period with predetermined money balances which they use for consumption during the period, as in Svensson (1985). The problem of the government is to finance exogenous government expenditures in the least distortionary manner. In this economy, an increase in the price level will deplete the real value of outstanding money and nominal debt, therefore reducing the need for distortionary taxation. However, this will also induce a decrease in present consumption. As shown by Nicolini (1998), the incentives to inflate, or deflate, depend on preferences and on whether debt is nominal or real.

We first consider a version of the model in Nicolini (1998) where, if debt is indexed, policy is time consistent, while if it is nominal there is always an incentive to inflate. As we show in this paper, this introduces dynamic distortions when optimal policy is chosen sequentially. Therefore, if the government could issue both real and nominal debt, only real debt would be issued. Instead, if there is only nominal debt, these distortions can only be mitigated by reducing the debt. We show that reducing asymptotically the debt is part of an optimal sequential policy.

We analyze the more general case where the Ramsey policy is time inconsistent even when there is only indexed debt. We show that, for that case where debt is indexed, depending on the value of the elasticity of substitution, a Markov perfect equilibrium will have debt be depleted to the point where it becomes negative and large enough in absolute value so that it is enough to finance all expenditures, or else that will increase asymptotically. If instead of indexed debt, debt is nominal, then there is another steady state where the value of debt (or assets) is enough to compensate for the incentives to inflate (or deflate) arising from the differing elasticities. In the case of log preferences, the steady state level of debt is zero. When instead the elasticity is higher than one, it is necessary to have positive assets for the two effects to be compensated, and the reverse is true when the elasticity is lower than one.

and an adhoc cost of repudiation that depends on the volume of debt. Because of the assumptions on the cost of repudiation, there are multiple equilibria. This multiplicity of equilibria in not there in the model we consider. We model the policy problem in a very different way and make it fully dynamic. In doing this we are able to understand how debt, either nominal or indexed, can be used as a state variable in affecting future policy.

Obstfeld (1997) and Ellison and Rankin (2005) assume that debt is real, and they focus only on monetary policy. They analyze Markov perfect equilibria when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. Obstfeld (1997) uses a model where money balances are not predetermined and therefore must consider an ad-hoc cost of a surprise inflation. Ellison and Rankin (2005) use the model in Nicolini (1998) with a class of preferences for which the level of real debt matters for the direction of the time inconsistency problem.

Nicolini (1998) shows that for preferences that are separable in leisure and isoelastic in consumption, monetary policy with indexed debt can either be time consistent or not, depending on the elasticity of substitution. For logarithmic utility, the Ramsey monetary policy with indexed debt is time consistent. In a previous version of this paper, Diaz et al (2004), we concentrated on that case and showed that with nominal debt, the Markov perfect equilibrium implied the asymptotic depletion of debt to the point where the time inconsistency was eliminated. Nominal debt creates an intertemporal distortion associated with the time inconsistency and unambiguously reduces welfare relatively to the case with indexed debt. Martin (2006) extended the positive results in that previous version of the paper by characterizing the Markov perfect equilibria with nominal debt for the general case, where the elasticity is different from one. We now extend the analysis to the general case and show that the welfare assessment is no longer unambiguous.\footnote{We thank the editor and two anonymous referees for this suggestion.}

There is a related literature on how optimal policies under commitment can be made time consistent by properly managing the portfolio of government assets and liabilities. The closest paper to ours is Persson, Persson and Svensson (2006)\footnote{See also Alvarez, Kehoe and Neumeyer (2004) and Lucas and Stokey (1983).}. They use a similar structure to the one in Nicolini (1998) and assume that the government can use both nominal and real debt and that there are no restrictions on debt being positive or negative.

We focus on Markov perfect equilibria. There is a recent literature on the characterization of the best sustainable equilibrium in similar optimal taxation problems. Reis (2006) studies the problem of the optimal taxation of capital and labor and shows that the path will have public debt be accumulated but not all the way to the first best.

In this paper, we start by analyzing the case with logarithmic utility where policy with indexed debt is time consistent. We first solve for the optimal policy when debt is nominal and there is full commitment. In this case, it is optimal to monetize part
of the initial nominal debt. This incentive to deplete the real value of nominal debt is present every period if there is no commitment. We then study optimal policy without commitment, so that policies are decided sequentially. We confine attention to Markov perfect equilibria. We call this equilibrium recursive as in Cole and Kehoe (1996), and in Obstfeld (1997). We characterize and compute numerically the equilibrium. The optimal inflation tax is non-stationary, and converges to the inflation tax that obtains when there is no government debt. We show that nominal debt is indeed a burden for monetary policy not only because it has to be serviced, but also because of the dynamic distortion associated with the time inconsistency. The full characterization and computation of the optimal policy in a recursive equilibrium with a state variable is an additional contribution of this paper.

Having characterized the recursive monetary equilibrium with nominal debt, then, as a benchmark, we characterize the optimal policy that obtains when debt is indexed. The solution, being stationary, is the same with and without commitment. Next, we compare the welfare properties of the equilibria in the different economies. We discuss how to make these comparisons meaningful, taking into account that in the nominal economies the initial conditions are in nominal terms and that in the indexed economy the initial condition for debt is in real terms. When the initial real liabilities are the same the economy with indexed debt has the highest welfare.

We proceed to analyzing the case of more general preferences where the elasticity of substitution is different from one. The optimal policy with indexed debt will no longer be stationary. Debt will either increase asymptotically or decrease, converging to the first best where there are enough assets to finance expenditures, and there is no longer a distortionary taxation problem. Instead, with nominal debt the policy converges to a distorted steady state that is no longer with zero debt. Rather, depending on a demand elasticity debt can be either positive or negative.

The assessment between the two regimes for the general case trades-off (i) a slow convergence to the first best, when debt is indexed, with a faster convergence to a distorted steady state still with negative debt, when debt is nominal, or (ii) a path with slowly increasing levels of debt (indexed debt) with a faster convergence to a stationary positive level of debt (nominal debt). In either case, when debt is relatively low, the equilibrium paths with nominal debt, where there is convergence to the distorted steady state, with either positive or negative debt, dominate in terms of welfare.

Finally, we test for the robustness of the results to the introduction of additional taxes. This is important since, in advanced economies, seigniorage is a minor source of tax revenues, and we want to know if our results still hold when government outlays are financed with other taxes. Specifically, we study the case of consumption taxes. First, we impose the natural assumption that taxes are chosen before the monetary policy decisions are made. In particular they are assumed to be chosen one period in advance. We find that the same equilibria result when there are both seigniorage and consumption

---

3In this respect, our work is closely related to the recent work of Krusell, Martín and Ríos-Rull (2003) who characterize the recursive equilibria that obtain in an optimal labor taxation problem.
taxes than when there is only seigniorage, provided that the optimal monetary policy distortions can be supported with strictly positive nominal interest rates.

When, instead, there is enough fiscal commitment, the fiscal authority can constrain the monetary authority to follow the Friedman rule, of zero nominal rates, from the outset. In this case, since negative interest rates cannot be sustained in equilibrium, the monetary authority has no incentive to monetize the debt and, as a result, it implements the optimal equilibrium with commitment.4

2 The model economy

In this section we describe the model economy with nominal debt. We follow very closely the structure in Nicolini (1998). The economy is a production economy with linear technology,

\[ c_t + g \leq n_t \] (1)

for every \( t \geq 0 \), where \( c_t \) and \( g \) are private and public consumption, respectively and \( n_t \) is labor. There is a representative household and a government. The preferences of the household are assumed to be linear in leisure and isoelastic in consumption,

\[ \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t], \] (2)

where \( u(c) = \frac{(c)^{1-\sigma} - 1}{1-\sigma} \). 0 < \( \beta < 1 \) is the time discount factor.

We assume that consumption in period \( t \) must be purchased using currency carried over from period \( t-1 \) as in Svensson (1985). This timing of transactions implies that the representative household takes both \( M_0 \) and \( B_0(1+i_0) \) as given and makes it costly an unexpected price increase. The specific form of the cash-in-advance constraint faced by the representative household is:

\[ P_t c_t \leq M_t \] (3)

for every \( t \geq 0 \), where \( P_t \) is the price of one unit of the date \( t \) consumption good in units of money and \( M_t \) are money balances acquired in period \( t-1 \) and used for consumption in period \( t \).

Each period the representative household faces the following budget constraint:

\[ M_{t+1} + B_{t+1} \leq M_t - P_t c_t + B_t(1+i_t) + P_t n_t \] (4)

See also Marimon, Nicolini and Teles, 2003.
where $M_{t+1}$ and $B_{t+1}$ denote, respectively, the stock of money and the stock of nominal government debt that the household carries over from period $t$ to period $t + 1$. The representative household faces a no-Ponzi games condition:

$$\lim_{T \to \infty} \beta^T \frac{B_{T+1}}{P_T} \geq 0$$

(5)

In each period $t \geq 0$, the government issues currency $M^g_{t+1}$ and nominal debt $B^g_{t+1}$, to finance an exogenous and constant level of public consumption $g$.\(^5\) Initially, we abstract from all other sources of public revenues. The sequence of government budget constraints is the following:

$$M^g_{t+1} + B^g_{t+1} \geq M^g_t + B^g_t (1 + i_t) + P_t g, \ t \geq 0$$

(6)

where $i_t$ is the nominal interest rate paid on debt issued by the government at time $t - 1$.\(^6\) The initial stock of currency, $M^g_0$, and initial debt liabilities, $B^g_0 (1 + i_0)$, are given. A government policy is, therefore, a specification of $\{M^g_{t+1}, B^g_{t+1}, g\}$ for $t \geq 0$.

2.1 A competitive equilibrium with nominal debt

**Definition 1** A competitive equilibrium for an economy with nominal debt is a government policy, $\{M^g_{t+1}, B^g_{t+1}\}^\infty_{t=0}$, an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}^\infty_{t=0}$, and a price vector, $\{P_t, i_{t+1}\}^\infty_{t=0}$, such that:

(i) given $M^g_0$ and $B^g_0 (1 + i_0)$, and $g$, the government policy and the price vector satisfy the government budget constraints described in expression (6);

(ii) when households take $M_0$, $B_0 (1 + i_0)$ and the price vector as given, the allocation maximizes utility (2), subject to the cash-in-advance constraints (3), the household budget constraints (4), and the no-Ponzi games condition (5); and

(iii) all markets clear, that is: $M^g_{t+1} = M_{t+1}$, $B^g_{t+1} = B_{t+1}$, and $g$ and $\{c_t, n_t\}^\infty_{t=0}$ satisfy the economy’s resource constraint (1), for every $t \geq 0$.

Given our assumptions on the utility of consumption $u$, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the economy’s resource constraint (1) and the household’s budget constraint (4) with equality, and that the first order conditions of the Lagrangian of the household’s problem are both

\(^5\)We assume that government expenditures, $g$, are given, although our analysis can easily be extended to the case of endogenous government expenditures.

\(^6\)Considering positive transfers that are decided endogenously by the government will not in general affect the analysis since they will be optimally set equal to zero. TECHNICAL ISSUE AT THE FIRST BEST.
necessary and sufficient to characterize the solution to the household’s problem. The cash in advance constraint, (3), will be binding in each period \( t \geq 0 \) if \( \frac{u'(c_t)}{\alpha} > 1 \). This will be a feature of the equilibria that we characterize. Since \( \frac{u'(c_t)}{\alpha} = 1 + i_t, t \geq 1 \), this will be the case whenever \( i_t > 0, t \geq 1 \). In period zero the cash in advance constraint will be binding whenever \( u'(c_0) > \alpha \).

The competitive equilibrium allocation of an economy with nominal debt can be characterized by the following conditions that must hold for every \( t \geq 0 \):

\[
\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, \ t \geq 0, \tag{7}
\]

\[
1 + i_{t+1} = \beta^{-1} \frac{P_{t+1}}{P_t}, \ t \geq 0, \tag{8}
\]

\[
c_t = \frac{M_t}{P_t}, \ t \geq 0, \tag{9}
\]

the government budget constraints (6) and the resource constraints (1) with equality, and the transversality condition (5)

\[
\lim_{T \to \infty} \beta^T \left( \frac{M_{T+1} + B_{T+1}}{P_T} \right) = 0 \tag{10}
\]

implied by optimality given the no-Ponzi games condition (5).

### 2.2 Indexed debt

An economy with indexed debt is an economy in all identical to the economy with nominal debt except in what concerns the government assets. The nominal interest rate adjusts with the price level so that \( \frac{B_t(1+i_t)}{P_t} \equiv b_t \) is now predetermined for every period \( t \geq 0 \). The budget constraints of the households can than be written as

\[
M_{t+1} + \frac{b_{t+1}}{1 + i_{t+1}} P_{t+1} \leq M_t - P_t c_t + b_t P_t + P_t n_t \tag{11}
\]

and the first order conditions (7)-(9) will be the same as before.

A competitive equilibrium for an economy with indexed debt is defined as a government policy, \( \{M_{t+1}', b_{t+1}', g_t\}_{t=0}^\infty \), an allocation \( \{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^\infty \), and a price vector, \( \{P_t, i_{t+1}\}_{t=0}^\infty \), such that the conditions (i), (ii) and (iii) of Definition 1 are satisfied when nominal liabilities are replaced by real liabilities, according to \( \frac{B_t(1+i_t)}{P_t} = b_t \), where \( b_t \) is predetermined.
2.3 Implementability

When choosing its policy the government takes into account the above equilibrium conditions. These conditions can be summarized with implementability conditions in terms of the allocations. In particular, the government budget constraint (6) with equality can be written as the implementability condition

\[ c_{t+1} U'(c_{t+1}) \frac{\beta}{\alpha} \beta z_{t+1} c_{t+1} = c_t + z_t c_t + g, \quad t \geq 0 \]  

(12)

where

\[ z_t \equiv \frac{B_t^g (1 + i_t)}{M^g_t} \]

To see this, notice that the constraint (6) with equality can be written in real terms as

\[ \frac{M^g_t}{P_t} + \frac{B^g_t}{P_t} = \frac{M^g_{t+1}}{P_t} + \frac{B^g_{t+1}(1 + i_t)}{P_t} + g \]

and, using the first order conditions of the households problem, (7), (8) and (9), \( \frac{M^g_t}{P_t} = c_t; \)

\[ \frac{M^g_{t+1}}{P_t} = \frac{M^g_{t+1} P_{t+1}}{P_t} = c_{t+1} \beta (1 + i_{t+1}) = c_{t+1} U'(c_{t+1}) \frac{\beta}{\alpha}; \]

\[ \frac{B^g_{t+1}(1 + i_{t+1})}{P_t} = \frac{B^g_{t+1}(1 + i_{t+1}) M^g_{t+1}}{M^g_t P_t} = z_t c_t, \] and

\[ \frac{B^g_{t+1}}{P_t} = \frac{B^g_{t+1}(1 + i_{t+1}) M^g_{t+1}}{M^g_t P_t} = \beta z_{t+1} c_{t+1}. \]

From the transversality condition (10) we have that \( \lim_{T \to \infty} \beta^T (c_{T+1} U'(c_{T+1}) \frac{\beta}{\alpha} + \beta z_{T+1} c_{T+1}) = 0, \) which implies that the present value government budget constraint takes the form

\[ \sum_{t=0}^{\infty} \beta^t \left( c_{t+1} U'(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = z_0 c_0 \]

(13)

This condition summarizes the competitive equilibrium restrictions on the sequence of consumption \( \{c_t\}_{t=0}^{\infty}. \)

2.4 Implementability with indexed debt

With indexed debt the government budget constraint (6) with equality can be written as the implementability condition

\[ c_{t+1} U'(c_{t+1}) \frac{\beta}{\alpha} + \beta b_{t+1} = c_t + b_t + g, \quad t \geq 0 \]  

(14)
The transversality condition (10) is written as
\[ \lim_{T \to \infty} \beta^T (c_{T+1} u'(c_{T+1}) \frac{\beta}{\alpha} + \beta b_{T+1}) = 0, \]
which implies that the present value government budget constraint takes the form
\[ \sum_{t=0}^{\infty} \beta^t \left( c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = b_0 \] (15)

This condition summarizes, when debt is indexed, the competitive equilibrium restrictions on the sequence of consumption \( \{c_t\}_{t=0}^{\infty} \). The comparison between the implementability conditions with nominal and indexed debt –(13) and (15), respectively – is revealing. The only difference is in the right hand side: \( z_0 c_0 \) with nominal debt, where \( z_0 \) is exogenous, and exogenous \( b_0 \) with indexed debt. In the case with nominal debt, the government can use policy to affect the real value of outstanding debt, but the means to do that is by affecting consumption.

Notice also that if an economy with nominal debt, and initial nominal liabilities \( z_0 \), the government policy results in a choice of \( c_0 \), then such economy has the same period zero, ex-post, real liabilities than an indexed economy with initial –but, predetermined– real liabilities \( b_0 = z_0 c_0 \). We will use such correspondence in comparing economies with nominal debt with economies with real debt.

3 Optimal policy with full commitment

In this section we compare optimal policies under full commitment when debt is indexed and when it is nominal. This is useful because, by observing how the optimal allocations differ in the initial period from the subsequent ones, we are able to understand whether policy is time consistent, and if not, why that is the case.

**Definition 2** A full commitment Ramsey equilibrium with indexed debt is a competitive equilibrium such that \( \{c_t\} \) solves the following problem:

\[ \max \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha(c_t + g)] \] (16)

subject to the implementability condition (15):
\[ \sum_{t=0}^{\infty} \beta^t \left( c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = b_0 \]

The other competitive equilibrium variables which are the government policy \( \{M_{t+1}^g, B_{t+1}^g\}_{t=0}^{\infty} \), the allocation \( \{M_t, B_t, n_t\}_{t=0}^{\infty} \), and the price vector, \( \{P_t, i_{t+1}\}_{t=0}^{\infty} \), are obtained using the competitive equilibrium conditions.
The optimal solution with commitment results in a constant consumption path from period one on, \(c_{t+1} = c_1, \ t \geq 0\). The intertemporal condition relating the optimal consumption in the initial period with the optimal stationary consumption in the subsequent periods is

\[
u'(c_0) - \alpha = \frac{u'(c_1) - \alpha}{1 - \frac{u'(c_1)}{\alpha} (1 - \sigma)}.
\] (17)

Clearly, when \(\sigma = 1\), the two consumptions will be equal and the solution will be time consistent. Instead when \(\sigma < 1\), (i.e., the intertemporal elasticity of substitution is \(1/\sigma > 1\)) the government prefers to tax more current consumption than future consumption, and, therefore, \(c_0 < c_1\). When \(\sigma > 1\), the government prefers to delay taxation and \(c_0 > c_1\).

**Definition 3** A full commitment Ramsey equilibrium with nominal debt is a competitive equilibrium such that \(\{c_t\}\) solves the following problem:

\[
\text{Max} \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - \alpha (c_t + g) \right]
\] (18)

subject to the implementability condition (13):

\[
\sum_{t=0}^{\infty} \beta^t \left( c_{t+1}u'(c_{t+1}) \frac{\beta}{\alpha} - (c_t + g) \right) = z_0 c_0
\]

As in the economies with indexed debt, it is optimal for the government to commit to a constant path of consumption (and nominal interest rates) from period one on, but consumption in period zero may differ. With nominal debt, the intertemporal condition relating consumption in period zero and period one is given by

\[
u'(c_0) - \alpha = \frac{u'(c_1) - \alpha}{1 - \frac{u'(c_1)}{\alpha} (1 - \sigma)}.
\] (19)

This condition makes explicit the additional motive for consumption in the two periods to diverge, when debt is nominal. In particular, comparing (19) with the intertemporal condition with indexed debt (17) it can be seen that as long as \(z_0 \equiv \frac{B_0(1+i_0)}{M_0} > 0\), \(c_0\) is relatively smaller— with respect to \(c_1\)— than the corresponding \(c_0\) of the economy with indexed debt. In other words, the incentive to monetize debt always results in relatively lower period zero consumption; whether this results in lower consumption in the initial period with respect to the future consumption will depend on how this effect interacts with the 'intertemporal elasticity' effect already present in the indexed economy. As in the economies with indexed debt, the government commits to a constant
path of consumption (i.e., of nominal interest rates) from period one on, but consumption in period zero may differ. It may differ—as in the indexed debt case—due to the ‘intertemporal elasticity effect’ or due to the ‘nominal effect’, and the later is always to monetize nominal debts and revalue nominal assets.

In particular, for every \( \sigma \) there is a \( z_0 \) such that the optimal consumption path is constant from period zero on and, therefore, policy is time consistent. Such \( z_0 \) is obtained by solving, for \( c \), the following steady state budget equation

\[
\beta \left[ \frac{u_c(c)}{\alpha} - 1 \right] - (1 - \beta) c - g = (1 - \beta) \left[ -\frac{u_c(c)}{\alpha} (1 - \sigma) \right] c
\]

and then setting \( z_0 = -\frac{u_c(c)}{\alpha} (1 - \sigma) \). Notice that \( z_0 \) is negative, zero or positive, depending on whether \( \sigma < 1 \), \( \sigma = 1 \) or \( \sigma > 1 \).

In summary, as shown in Nicolini (1998), with a unitary elasticity of substitution, the optimal policy with indexed debt is time consistent. The time consistency is lost, for those same preferences, if, instead, debt is nominal. Under more general preferences, there is a time inconsistency even when debt is indexed. Nominal debt in that case can either worsen or alleviate the time inconsistency.

The intuition for these results is the following: Under indexed debt, the optimal policy problem is the following: The Ramsey planner can use the nominal interest rate to tax consumption from period one on and the price level, through the cash in advance constraint, to tax consumption in period zero. The utility function is separable in leisure and homothetic in consumption goods so that it is clearly optimal to set the same inflation tax rate from period one on. Consumption will be stationary from that period on. In deciding whether to tax more or less in period zero, relatively to the subsequent periods, the planner compares elasticities. When the elasticity of substitution (of future consumption) is equal to one, i.e. \( \sigma = 1 \), it will tax the two goods, today and tomorrow, equally, and the policy is time consistent. For lower \( \sigma \), the elasticity is higher and it is optimal to tax more today. The reverse is true for higher \( \sigma \).

With nominal debt there is an additional reason to inflate in the initial period, to reduce the real value of outstanding nominal liabilities. When \( \sigma = 1 \), that creates a time inconsistency where there wasn’t one. When \( \sigma < 1 \), that adds to the incentives already present, for higher taxes in the initial period, while when \( \sigma > 1 \), the incentives can be offset.

It should also be noticed that if public debt was negative the incentive for the government is to revalue these assets and, therefore, the effect of nominal debt/assets works in the opposite direction. Therefore, in the presence of assets, with \( \sigma < 1 \), the time inconsistency problem is mitigated—and, possibly, offset—, while with \( \sigma > 1 \) is aggravated.

\(^7\)Ellison and Rankin (2005) show that this possibility of having time consistent optimal policies, for specific initial real liabilities, can also occur with indexed debt for some forms of non CRRA preferences.
We can compare the welfare of an economy with nominal debt with an otherwise identical economy with indexed debt, provided that both have the same initial real liabilities. More formally, if we consider an economy with initial nominal liabilities $z_0$ with an associated optimal initial consumption $c_0(z_0)$, we then compare such economy with an economy with initial indexed debt $b_0 = b_0(z_0) \equiv z_0 c_0(z_0)$. Conversely, if we consider an economy with indexed debt and initial real liabilities $b_0$, we then compare such economy with an economy with initial nominal liabilities $z_0 = b_0^{-1}(b_0)$.

**Proposition 1** Consider two economies with initial money stock $M_0$. One of them has initial nominal debt $B_0(1+i_0) > 0$, and the other has initial indexed debt $b_0$. Suppose, under full commitment, $b_0 = \frac{B_0(1+i_0)}{P_0}$, where $P_0$ is the period zero price in the economy with nominal debt. Then the economy with nominal debt always gives lower welfare independently of the value of $\sigma$.

**Proof:** In order for the two problems to be comparable we restrict $z_0 c_0 = b_0$, so that the budget constraints of the two problems are the same,

$$\beta c_1 \left[ \frac{u'(c_1)}{\alpha} - 1 \right] - (1 - \beta) c_0 = g + (1 - \beta) b_0.$$ 

Then the solution of the problem with indexed debt gives the highest welfare.

The intuition for this result is that once we take away the gains from unexpectedly inflating away nominal debt there only remain the distorting costs associated with the ‘nominal debt effect’. More precisely, the indexed economy is characterized by having nominal interest rates adjusting to price changes as to guarantee the predetermined value of real liabilities (i.e., $B_t(1+i_t) \equiv b_t$). With nominal debt, real liabilities are not predetermined, but with rational expectations (and no uncertainty), ex-post real returns correspond to agents’ ex-ante expected values. However, for the initial period there are no prior consumption plans and, therefore, there is the possibility of a ‘free lunch’ for the government. We exclude such possibility by imposing a form of ‘rational expectations for the initial period’: $\frac{B_0(1+i_0)}{M_0} \equiv b_0$. Such condition not only makes indexed and nominal debt economies with full commitment comparable, but also makes optimal policies with full commitment comparable with optimal policies without full commitment. Without full commitment, policies are decided sequentially and the special features of ‘initial period’ in the full commitment economy become the recurrent features in the economy without commitment. However, being recurrent features, agents have rational expectations and in equilibrium there are no ‘free lunches;’ therefore, the appropriate reference for the no commitment case is a full commitment economy without an initial ‘free lunch’.

We now turn to the central part of the paper: the analysis of economies without commitment. The welfare comparison is more interesting in that context, when we

---

8 Notice that the so-called, ‘timeless perspective’ simply disregards this initial period problem (see, for example, Woodford XXX).
compare the recursive equilibria with nominal and indexed real debt. When there is no commitment, in the Markov perfect equilibria, we can show that nominal debt can either increase or lower welfare.

In the case of log utility where monetary policy is time consistent when debt is indexed, if debt was nominal, welfare would be lower provided, again, that the two problems are made comparable by eliminating the gains from the initial depletion of nominal debt. This is clear both from the result above and the result established later that a recursive equilibrium does not increase welfare relatively to the commitment solution. In that case of log utility, nominal debt generates intertemporal distortions that can only lower welfare. If, instead, monetary policy is time inconsistent when debt is indexed, there is already a source of intertemporal distortions. Adding another distortion, by introducing the incentives to deplete nominal debt may improve welfare.

4 Recursive (Markov Perfect) monetary equilibria

In this section we consider that the government cannot commit to a future path of policy actions and, therefore, chooses its monetary policy sequentially. We restrict the analysis to the case where such sequential choices do not depend on the whole history up to period $t$ but can depend on the pay-off relevant state variables —as in Markov Perfect equilibria— and, therefore, sequential optimal choices are recursively given by stationary optimal policies. In particular, in the case with nominal debt, government policy in a primal form is recursively defined by $c_t = C(z_t)$ and the corresponding state transition $z_{t+1} = z'(z_t)$. Similarly, with indexed debt, policy is recursively defined by $c_t = C(b_t)$ and the corresponding state transition $b_{t+1} = b'(b_t)$. Agents have rational expectations and, therefore, their consumption plans are consistent with the government policy choices and the corresponding state transitions. Our definitions of recursive monetary equilibria take these elements into account.

4.1 Indexed debt

**Definition 4** A recursive monetary equilibrium with indexed debt is a value function $V(b)$ and policy functions $C(b)$ and $b'(b)$ such that $c = C(b)$ and $b' = b'(b)$ solve

$$V(b) = \max_{c,b'} \{ u(c) - \alpha(c + g) + \beta V(b') \}$$

subject to the implementability constraint

$$C(b')u_c(C(b')) \frac{\beta}{\alpha} + \beta b' = c + g + b, \quad t \geq 0$$

In order to characterize the recursive equilibrium, notice that the first order condition for $c$ is:

$$u_c(c) - \alpha = \lambda, \quad (20)$$
and for \( b' \),
\[
V_b(b') + \lambda \left( \frac{C_b(b')u_c(C(b'))}{\alpha} (1 - \sigma) + 1 \right) = 0.
\]
while the envelope condition is given by
\[
V_b(b) = -\lambda
\]
These equations imply the following intertemporal condition along an equilibrium path,
\[
\frac{u_c(c')}{\alpha} - 1 = 1 + \frac{u_c(C(b'))}{\alpha}C_b(b') (1 - \sigma)
\]  
(21)

It follows that there is a steady state if and only if \( C_b(b') = 0 \) or \( \sigma = 1 \). In the log case (\( \sigma = 1 \)), the steady state is independent of \( b \) and, therefore, given any sustainable level of initial debt, \( b_0 \), such level is maintained and, correspondingly, the consumption path is constant.

The first best is also a steady state, independently of the value of \( \sigma \). Indeed at the first best, where the stationary level of debt is negative and large enough in absolute value to cover expenditures, an increase in the level of assets does not affect consumption \( C_b(b_0) = 0 \)\(^9\).

Comparing the equilibrium intertemporal condition of the economy without commitment (21) with the corresponding condition of the economy with full commitment, (17), we see that the ‘intertemporal elasticity effect,’ when \( \sigma \neq 1 \), is weighted by \(-C_b(b')\), which is the marginal decrease of consumption in response to an increase in debt, as a function of the level of debt. In the numerical computations of the recursive equilibria, \( C_b(b') \) is always negative, except at the first best where it is zero. Indeed, if there were lump sum taxes an increase in debt would not lower consumption, so that at the first best, \( C_b(b_0) = 0 \).

With \( C_b(b') \) negative, the intertemporal condition (21) implies that when \( \sigma < 1 \) the consumption path is increasing, while when \( \sigma > 1 \) the consumption path is decreasing.

[INSERT Figure 1. \( b'(b) \) policies for \( \sigma = 0.6, 1 \) and 1.4]

We turn now to our computed results\(^{10}\). Figure 1 shows the optimal debt policies, \( b'(b) \); in particular it shows \( b'(b) \) for the cases \( \sigma = 1, \sigma = 0.6, \) and \( \sigma = 1.4, \) the last two as examples of the cases \( \sigma < 1 \) and \( \sigma > 1 \), respectively. As we already said, in the

\(^{9}\)Technically there is a need to consider positive transfers, meaning that there is free disposal of extra revenues by the government.

\(^{10}\)See the Appendix for a discussion of the choice of parameters and a description of the algorithm. Although here we only report computations for few specific values, a large number of robustness tests have been performed.
log case, the policy function \( b'(b) \) coincides with the 45 degree line; i.e, the initial level of debt is maintained forever. With \( \sigma < 1 \) the policy function \( b'(b) \) lies below the 45 degree line, except at the level of debt \( b^* < 0 \), that allows to achieve the first best level of consumption where there is no distortionary taxation. The inflation tax is higher initially so that debt may be depleted, and then assets accumulated, to the point where there are enough assets to finance all expenditures, and the first best may be attained. At the first best, the ‘intertemporal elasticities effect’ disappears.

When \( \sigma > 1 \), the policy function \( b'(b) \) lies above the 45 degree line, except at the first best level of debt \( b^* < 0 \). Furthermore, \( \frac{b'(b)}{b} < \beta^{-1} \), so that debt is accumulated, at a rate lower than \( \beta^{-1} - 1 \). The first best is also a steady state when \( \sigma > 1 \), but it is not the asymptotic state and, therefore, the ‘intertemporal elasticity effect’ never disappears. In general, for \( \sigma \) close to one, \( C_0(b') \) is very close to zero, meaning that the convergence to the first best when \( \sigma < 1 \), or the accumulation of debt when \( \sigma > 1 \), is very slow.

Figure 2a shows the equilibrium consumption paths from a common initial value \( b_0 \) (for the cases \( \sigma = 1 \), \( \sigma = 0.6 \), and \( \sigma = 1.4 \)) and Figure 2b the corresponding debt paths. These figures illustrate the remarks previously made.

4.2 Nominal debt

As we have seen, in an economy with full commitment with nominal debt, there is a ‘nominal effect’ since the government has an incentive to partially monetize the debt in the initial period. In an economy without commitment, such distorting effect is present every period and, therefore, will also be anticipated by households. As in the economy with full commitment, such ‘nominal effect’ interacts with the possible ‘intertemporal elasticity effect’.

**Definition 5** A recursive monetary equilibrium with nominal debt is a value function \( V(z) \) and policy functions \( C(z) \) and \( z'(z) \) such that \( c = C(z) \) and \( z' = z'(z) \) solve

\[
V(z) = \max_{\{c,z'\}} \{ u(c) - \alpha (c + g) + \beta V(z') \} \tag{22}
\]

subject to the implementability constraint

\[
C(z') u_c(C(z')) \frac{\beta}{\alpha} + \beta z' C(z') = z c + c + g \tag{23}
\]
To characterize the recursive monetary equilibrium, notice that the first order conditions of the problem described above are

\[ u_c(c) - \alpha = \lambda (1 + z) \]  \hfill (24)

and

\[ V_z(z') + \lambda \left( \frac{C_z(z') u_c(C(z'))}{\alpha} \right) (1 - \sigma) + C'(z') \left[ 1 + \varepsilon_c(z') \right] ] = 0 \]  \hfill (25)

where \( \varepsilon_c(z) = \frac{z C_z(z)}{C(z)} \) is the elasticity of \( C(z) \), and the envelope condition is

\[ V_z(z) = -\lambda c \]  \hfill (26)

This implies

\[ \frac{u_c(c')}{u_c(c)} - 1 - \frac{1 + \varepsilon_c(z')}{1 + z} \left( 1 + \varepsilon_c(z') \right) \left[ 1 + \frac{u_c(C(z'))C_z(z')}{z' \alpha} (1 - \sigma) \right] \]  \hfill (27)

For \( z' \neq 0 \), this can be written as

\[ \frac{u_c(c')}{u_c(c)} - 1 = \frac{1 + z'}{1 + z} \left( 1 + \varepsilon_c(z') \left[ 1 + \frac{u_c(C(z'))}{z' \alpha} (1 - \sigma) \right] \right) \]  \hfill (28)

As in the case with indexed debt, the first best (where government assets are enough to finance expenditures) is a steady state, since at the first best, \( C_z(z) = 0 \), and therefore \( \varepsilon_c(z) = 0 \).

However, with nominal debt there is another steady state, where nominal debt, \( \pi \), is such that

\[ 1 + \frac{u_c(C(\pi))}{\pi \alpha} (1 - \sigma) = 0 \]

provided the corresponding steady state implementability condition is satisfied, i.e., \( \pi u_c(\pi) \frac{\beta}{\alpha} + \beta \pi c = (1 + \pi) \pi + g \). That is, substituting \( \pi \) this last equation reduces to \( \frac{\pi u_c(\pi)}{\alpha} [1 - (1 - \beta) \sigma] = c + g \). Therefore, as long as \( (1 - \beta) \sigma < 1 \), there is a solution for \( \pi \) and, correspondingly, for \( \pi \).

To better understand the distortions present in the economy with nominal debt without commitment, it is useful to consider the log case, where intertemporal condition (27) can be rewritten as

\[ \left[ 1 + \frac{1 + (1+i)B}{M} \right] = \left[ 1 + \frac{1 + (1+i)^B}{M} \right] \left[ 1 + \varepsilon_c \left( \frac{(1+i)^B}{M^\pi} \right) \right]^{-1} \]  \hfill (29)
where the elasticity $\varepsilon_c(z')$ is negative and less than one in absolute value (as our computations show).

This intertemporal equation reflects the different distortions present as a result of debt being nominal and policy decisions being sequential. The term $\left[1 + \frac{(1+\gamma)B}{M}\right]$ results from the discretionary incentive to reduce the real value of nominal debt. It is present in the problem with commitment only in the initial period (equation (22)). Higher consumption today means that the price level will have to be lower, which in turn means that the real value of outstanding nominal debt will be higher, and therefore future distortionary taxation will also be higher. Hence, the benefits for the benevolent government of higher consumption today are lower. The term $\left[1 + \varepsilon_c \left(\frac{(1+\gamma)B}{M}\right)^{\beta}\right]$ results from the dynamic nature of this problem. Higher consumption today, and a lower price today, also means that future debt will be higher, which has the cost of exacerbating the time inconsistency problem in future decisions. For the case with $\sigma \neq 1$, these two effects are compounded with the ‘intertemporal elasticity effects’ and, as we have seen, the interaction of all these effects may result in stationary solutions not present in the economy with indexed debt.

[INSERT Figure 3. $z'(z)$ policies for $\sigma = 0.6, 1$ and $1.4$]

We turn now to the computed results. Figure 3 shows the optimal debt policies. In particular, in parallel with Figure 1, it shows $z'(z) - z$ for the cases $\sigma = 1, \sigma = 0.6$, and $\sigma = 1.4$. In all three cases, $z'(z) - z$ is decreasing and, as we have shown before, it intersects zero at $\overline{z} = 0$, when $\sigma = 1$, $\overline{z} < 0$, when $\sigma < 1$ and $\overline{z} > 0$, when $\sigma > 1$. Since $z'(z) - z$ is decreasing these steady states also define the asymptotic behavior of nominal debt paths.

In contrast with the indexed debt case of Figure 1, when $\sigma = 1$ debt is no longer constant, but it is progressively depleted until the ‘nominal effect’ disappears at $\overline{z} = 0$. When $\sigma < 1$ debt is also progressively depleted and, then, assets are accumulated but, in contrast with the indexed case, this process does not lead to the first best steady state, but to a lower level of assets $\overline{z}$, $z^* < \overline{z} < 0$. The first best remains as an isolated, unstable, steady state. When $\sigma > 1$, debt is not accumulated without bound –as with indexed debt– but it is accumulated or progressively depleted until it reaches the steady state $\overline{z} > 0$ in which the ‘nominal’ and the ‘intertemporal elasticity’ effects cancel out. Figure 4 shows consumption and debt paths starting at a positive level of nominal liabilities $z_0 > 0$ below the steady state $\overline{z}$ for $\sigma = 1.4$.

[INSERT Figure 4. Consumption and debt paths for a common $z_0$ for $\sigma = 0.6, 1$ and 1.4]

---

$^{11}$ Myopic governments that would not take into account the effect of their choices on the state variables of future government decisions would be solving a problem where only the first term would be present.
4.3 Welfare comparisons

In comparing economies with and without commitment, the result that the recursive equilibrium is in general less efficient than the full commitment Ramsey equilibrium is straightforward. The full commitment Ramsey solution is the choice of a sequence of consumption \( \{c_t\}_{t=0}^{\infty} \) that maximizes utility (16) in the set of competitive equilibrium sequences defined either by (15) with indexed debt or (13) with nominal debt. If the maximum is unique, it follows that, since the recursive equilibrium is a competitive equilibrium, if different, it must give strictly lower welfare. More formally,

**Proposition 2** Consider two identical economies with indexed debt (nominal debt) with the same initial liabilities \( b_0 (z_0) \). The full commitment Ramsey equilibrium, if unique and different from the no-commitment Recursive equilibrium, must provide strictly higher welfare.

The result is clear when we think of the recursive equilibrium as the equilibrium of a game between successive governments. The recursive equilibrium, imposes as additional restrictions the optimality of decisions by future governments.

As we have seen (Proposition 1), with full commitment indexed debt is unambiguously more efficient than nominal debt, when the pure monetization effect is not considered; i.e., when there is no ‘free lunch’ for the government in period zero. Without commitment, along a recursive equilibrium path, there are no ‘free lunches’; does this mean that indexed debt is better than nominal debt? The following proposition states the result.

**Proposition 3** Consider two economies without commitment and initial money stock \( M_0 \). One of them has initial nominal debt \( B_0(1 + i_0) > 0 \), and the other has initial indexed debt \( b_0 \). If \( b_0 = \frac{B_0(1 + i_0)}{\pi_0} \), then if \( \sigma = 1 \) the welfare in the economy with indexed debt is higher than in the economy with nominal debt. However, for \( \sigma \neq 1 \) the welfare in the economy with nominal debt can be higher (or lower) than in the economy with indexed debt, depending on \( b_0 \).

The first result that for the log case nominal debt provides lower welfare than indexed debt is a corollary of propositions 1 and 2. In the log case, with indexed debt, policy is time consistent, so that the full commitment and the recursive equilibria coincide. From Proposition 1, the full commitment equilibrium with nominal debt provides lower welfare than with indexed debt, and the recursive equilibrium provides even lower welfare.

Figure 5 shows the last part of the proposition. It compares the value functions with indexed debt, \( V_i \), with the value functions with nominal debt \( V_n \) as functions of real liabilities \( b \); that is, \( V_n(b^{-1}(b)) \), where \( b(z) \equiv zC(z) \).

[INSERT Figure 5. Value functions \( V_n \) and \( V_i \) for \( \sigma = 1 \) (5a), 0.6 (5b) and 1.4 (5c)]
Figure 5a illustrates the unambiguous result that with unitary elasticity indexed debt dominates nominal debt in terms of welfare. In Figure 5b it can be seen that when $\sigma < 1$ there is a range of assets and debts for which nominal debt Pareto dominates indexed debt; in particular, even if the distorted steady state requires negative debt, $\overline{z} < 0$ for low enough values of positive debt, nominal debt gives higher welfare than indexed debt (when $\sigma = 0.6$). This is due to the fact that with nominal debt the progressive depletion of debt (and accumulation of assets) only has to reach the value of assets $\overline{z}$, $z^* < \overline{z} < 0$, while with indexed debt asset accumulation must lead to the first best.

For larger values of debt, welfare with indexed debt is higher; that is, if the initial value of debt is much higher than $\overline{z}$, it is better to have a slow convergence to the first best, rather than a faster convergence to the distorted steady state.

Figure 5c shows that, when $\sigma > 1$ there can be a relatively large range of initial values of debt for which nominal debt results in higher welfare than indexed debt. In particular, only for relatively high values of debt, the path with indexed debt, where debt is accumulated asymptotically gives higher welfare than the optimal path with nominal debt, where debt converges to the distorted steady state $\overline{z} > 0$.

There are two main points to take from this analysis. The first one is the results above are an application of the principle that adding a distortion to a second best problem can actually improve welfare. Nominal debt adds a dynamic distortion to the recursive equilibrium. In the case where policy is time consistent with indexed debt, adding that distortion reduces welfare. When policy is time inconsistent with indexed debt, there are two dynamic distortions, due to the differing elasticities and to nominal debt. In that case, adding the distortion from nominal debt can actually raise welfare.

The other point is a quantitative point. In the calibrated economy, debt will have to be substantially higher than output for it to be better to have indexed debt, rather than nominal debt. It is preferable to have a relatively fast convergence to either positive or negative debt associated with the distorted steady state, with nominal debt, rather than have a slow accumulation of debt or decumulation all the way to the first best.

5 Additional taxes

In most advanced economies, seigniorage is a minor source of revenue, and government liabilities are financed mostly through consumption and income taxes. In this section we show two basic results, regarding the introduction of taxes in our economy. First, we show that the introduction of taxes, while reducing the need to raise revenues through seigniorage, may not change the characterization of equilibria with respect to the economies with monetary policy only, analyzed in the previous sections. This is the case if the fiscal authority simply sets taxes one period in advance and, subsequently, the monetary authority sets its policy. This adds realism to the analysis of the interaction of fiscal and monetary policies. Second, we show that, in contrast, if there is full commitment on the part of the fiscal authority that makes its policy choices in the initial
period, before the monetary authority does, then the full commitment outcome can be achieved even if there is no commitment on the part of the monetary authority. We show that it is part of such policy to finance all the outstanding government liabilities with the consumption tax, and to constrain the monetary authority to implement a zero nominal interest rate.

We show these results introducing consumption taxes, \( \{\tau_t\}_{t=0}^{\infty} \). The analysis easily generalizes to the introduction of other taxes. However, it is not the purpose of the paper to provide a complete characterization of all possible fiscal instruments, neither to consider different games between fiscal and monetary authorities.

### 5.1 The model economy with consumption taxes

When the government levies consumption taxes, the household problem becomes:

\[
\max \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t]
\]  

subject to:

\[
P_t (1 + \tau_t) c_t \leq M_t \tag{31}
\]

\[
M_{t+1} + B_{t+1} \leq M_t - P_t (1 + \tau_t) c_t + B_t (1 + i_t) + P_t n_t \tag{32}
\]

and to:

\[
\lim_{T \to \infty} \beta^T \frac{B_{T+1}}{P_T} \geq 0 \tag{33}
\]

Now, the marginal conditions (7), (8) and (9) characterizing the households’s optimal choice become:

\[
\frac{u'(c_{t+1})}{\alpha} = (1 + i_{t+1})(1 + \tau_{t+1}) \tag{34}
\]

\[
1 + i_{t+1} = \beta^{-1} \frac{P_{t+1}}{P_t} \tag{35}
\]

and

\[
c_t = \frac{M_t}{P_t (1 + \tau_t)} \tag{36}
\]
These conditions must hold for every \( t \geq 0 \). Notice that (34) reflects the fact that the household makes plans based on expectations about both interest rates and taxes. The intertemporal condition (35) is exactly the same as expression (8) while the cash-in-advance constraint (36) now includes consumption taxes.

The sequence of government budget constraints in this economy is now given by:

\[
P_t g + M^g_t + B^g_t(1 + i_t) \leq P_t \tau_t c_t + M^g_{t+1} + B^g_{t+1}
\]

while the feasibility conditions (1) do not change. The implementability conditions (14) can be written as

\[
c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} + \beta z_{t+1} c_{t+1} (1 + \tau_{t+1}) = c_t + z_t c_t (1 + \tau_t) + g
\]

which is formally identical to (14).

5.2 Optimal monetary policy when the fiscal authority moves one period in advance

We now consider the case where tax decisions for some period \( t \) must be made one period in advance, and may depend only on the state at \( t - 1 \). In this case we can define the new state variable \( \tilde{z}_t \equiv z_t (1 + \tau_t) \), and the problems are isomorphic to the problems in the previous sections, since the implementability condition (38) reduces to

\[
c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} + \beta \tilde{z}_{t+1} c_{t+1} = c_t + \tilde{z}_t c_t + g
\]

which is formally identical to (14).

There is an additional restriction that the nominal interest rate must be nonnegative. This constraint was satisfied when seigniorage was the only source of revenue, but it is not necessarily satisfied in this case.

We can model the interaction between the monetary and the fiscal authority in two ways. One way is to assume that they jointly decide as a single authority. In this case the analysis goes through as before and we obtain the same results in terms of allocations. The alternative is to assume some given strategy for taxes as a function of the state. As long as the nominal interest rates are away from the lower bound of zero nominal interest rates, the problem for the monetary authority has the same structure as before, and therefore the same results go through, even if part of the government liabilities are financed with taxes.
In summary, the monetary authority faces the same problem with consumption taxes than with only seigniorage, for any degree of monetary commitment. Therefore, the allocations for the various types of debt and monetary policy commitment technologies are exactly the same as those that obtained before. This result is established in the following subsections:

**Consumption taxes and indexed debt.** In the economy with indexed debt indexed debt is

\[ b_t = z_t c_t (1 + \tau_t) = \tilde{z}_t c_t, \]

so that the implementability condition with indexed debt can be written as

\[ c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} + \beta b_{t+1} = c_t + b_t c_t + g. \]  

This is the implementability condition in the economy without taxes. Whether taxes are decided one period in advance or not, the problem with indexed debt is exactly the same with or without taxes.

The analysis above for the case without taxes, goes though when there are taxes, as long as taxes are decided one period in advance when debt is nominal.

### 5.3 Optimal fiscal policy with commitment

In the three regimes discussed above we do not determine exactly how the equilibrium allocations are supported since the household only cares about the effective nominal rate of return, \((1 + i)(1 + \tau)\).

It turns out that whenever the discretionary incentives are for a higher current tax, then it is always possible to set taxes in a way that the resulting monetary policy follows the Friedman rule of zero nominal interest rates, even though in our economy there is no efficiency gain from following such a rule\(^{12}\).

To see this, suppose that the stock of debt is nominal and that there is full commitment to fiscal policy. Let the fiscal authority set, for \( t \geq 0, \tau_{t+1} = \tau(\tilde{z}_t) = \tau(\tilde{z}_0) \), where \( \tau(\tilde{z}_0) \) corresponds to the tax rate that fully finances the government liabilities in the allocation that obtains with full commitment, from period one on. That is,

\[ (1 + \tau(\tilde{z}_0)) = \frac{u'(c^F)}{\alpha} \]

\(^{12}\)This may not be true in a more general model economy. For instance, this is not true if we introduce a distinction between cash and credit goods. In this case, the Friedman rule would eliminate the distortion between cash and credit goods created by the cash-in-advance constraint. This notwithstanding, the distortions introduced by the presence of a positive stock of nominal debt would still be there, just as in the economy with only cash goods.
If, at any $t > 0$, the monetary authority tries to monetize part of the existing stock of nominal debt and to use the resulting revenues to increase future consumption —say, maintaining a constant $\bar{c}$ from then on— then, it must be the case that $c_t < c^F < \bar{c}$. Given that

$$(1 + \tau(z_0))(1 + \bar{r}) = \frac{u'(\bar{r})}{\alpha} \tag{43}$$

and that (42) must be satisfied, the interest rate would have to be negative, $\bar{r} < 0$. Negative interest rates can not be an equilibrium in this economy since then the household would like to borrow unboundly. Therefore, given that it is not possible to raise future consumption with negative taxes, there is no gain in partially monetizing the stock of nominal debt in period zero. In this case monetary policy is time consistent. Therefore if there is no commitment to monetary policy, a fully committed fiscal authority who wants to maximize utility (2), will set $\tau_{t+1} = \tau(z_0), t \geq 0$.\(^{13}\) The following proposition summarizes this result:

**Proposition 4** Assume that fiscal authorities maximize the welfare of the representative household and can fully commit to their policies. Then the equilibrium allocation is the optimal equilibrium allocation that obtains when there is a single Ramsey planner, regardless of the degree of commitment of the monetary authority.

### 6 Concluding comments

This paper discusses the different ways in which nominal and indexed debt affect the sequential choice of optimal monetary and debt policies. To this purpose, we study a general equilibrium monetary model where the costs of an unanticipated inflation arise from a cash-in-advance constraint with the timing as in Svensson (1985), and where government expenditures are exogenous. In our environment, as in Nicolini (1998), when the utility function is logarithmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a Ramsey government.

In contrast, for the same specification of preferences, when the initial stock of government debt is nominally denominated, a time inconsistency problem arises. In this case, the government is tempted to inflate away its nominal debt liabilities. When the government cannot commit to its planned policies, progressively depleting the outstanding stock of debt is part of an optimal sequential policy consists, so that such policy converges asymptotically to zero debt liabilities. Optimal nominal interest rates in this case are also decreasing and converge asymptotically.

\(^{13}\)Marimon, Nicolini, and Teles (2003) make a similar argument.
Such equilibrium path is not chosen when the initial stock of government debt is nominally denominated and the government –as a Ramsey planner– can fully commit to its planned policies. In this case, it is optimal to increase the inflation tax in the first period, and to keep a lower and constant inflation tax for the rest of the future. This is also the case, even if there is no commitment, if the government can choose a portfolio of real and nominal debt, since then it is optimal to end period zero with only real debt liabilities and never issue nominal debt afterwards. That the Ramsey outcome can be made time consistent by having only real debt liabilities exemplifies, in our context, the existing results due to Persson, Persson and Svensson (2006) and others.

In the rational expectations equilibria of our economies there are no surprise inflations. Still, for a given initial real value of outstanding debt, the most efficient equilibrium is the one that obtains when debt is indexed, the equilibrium with nominal debt and full commitment comes second, and the equilibrium with nominal debt and no commitment is the least efficient. This result highlights the sense in which nominal debt can indeed be a burden on optimal monetary policy.

We extend the analysis to the general preference case where the elasticity of substitution is not one. In that case, policy is time inconsistent even when debt is indexed. The interaction of the two sources of dynamic distortions can overturn the result above and it may actually be the case that nominal debt provides higher welfare. This is an application of the principle that in a second best adding a distortion may actually increase welfare.

It should be noted that the source of the inefficiencies and of the monetary policy distortions discussed in this paper is not the desire to run a soft budgetary policy that increases the debt liabilities of the government. Every policy discussed in this article is an optimal policy, subject to the appropriate institutional and commitment constraints, and it is implemented by a benevolent and far-sighted government who does not face either uncertainty or the need for public investment, and who would, therefore, prefer to reduce debt liabilities. The source of the inefficiencies is the distortion created by the lack of commitment that results from the mere existence of an outstanding stock of nominal debt. Therefore, our results highlight the need to implement policy and institutional arrangements that either guarantee high commitment levels, or that reduce the allowed levels of nominal debt.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that fiscal policy choices are predetermined, we show that the optimal policy problem has the same characterization, provided that the revenues levied through seigniorage are enough to allow for an optimal monetary policy with non-negative interest rates. Instead, as in Marimon, Nicolini and Teles (2003), if there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, choose to fully finance government liabilities, and the resulting monetary policy is the Friedman rule of zero nominal interest rates. Moreover, this policy results in the equilibrium that obtains in the economy with full commitment.
References


1 Algorithm

Let \( u(c) = (c^{1-\sigma} - 1)/(1 - \sigma) \), then to compute the monetary equilibria numerically, we solve the following dynamic programs:

\[
V(x) = \max \left\{ u(c) - \alpha (c + g) + \beta V(x') \right\}
\]

subject to

\[
\frac{\beta}{\alpha} C(b')^{1-\sigma} + \beta b' = c + g + b
\]

when \( x = b \) and the debt is indexed, or subject to

\[
\frac{\beta}{\alpha} C(z')^{1-\sigma} + \beta z'C(z') = (1 + z)c + g + b
\]

when \( x = z \) and the debt is nominal.

The Bellman operators associated with these problems are:

\[
V_{n+1}(x) = T[V_n(x)] = \max \left\{ u(c) - \alpha (c + g) + \beta V_n(x') \right\}
\]

subject to expression (2) when \( x = b \) and the debt is indexed, or subject to expression (3) when \( x = z \) and the debt is nominal.

To solve these problems, we use the following algorithm:

- Step 1: Choose numerical values for parameters \( \alpha, \beta, \sigma, \) and \( g \).
- Step 2: Define a discrete grid on \( x \).
- Step 3: Define a decreasing discrete function \( C_n(x) \)
- Step 4: Define an initial discrete function \( V_n(x) \) and iterate on the Bellman operator defined above until we find the converged \( V^*(x), x^*(x), C^*(x) \)
- Step 5: If \( C^*(x) = C_n(x) \), we are done. Else, update \( C_n(x) \) and go to Step 3.

2 Calibration

We choose \( \beta = 0.98 \) and \( g = 0.01 \) and we carry out three numerical exercises with the following sets of parameter values.
(a) $\alpha = 1.5$ and $\sigma = 0.6$

(b) $\alpha = 2.5$ and $\sigma = 1.0$

(c) $\alpha = 4.0$ and $\sigma = 1.4$

for both a model economy with nominal debt and a model economy with indexed debt.

3 Numerical Solutions

We report solutions to the numerical exercises for the model economy with nominal debt in Figures 1, 2 and 3, and for the model economy with nominal debt in Figures 4, 5 and 6.

4 Welfare Comparisons

To compare the welfare gains of losses associated with each type of debt we represent the value functions of each pair of model economies in Panels A, B, and C of Figure 7. Notice that, to make the comparisons meaningful, first we compute $V^*_N(z)$, then we define $b = zC^*(z)$, then we compute $V^*_I(b)$ and our figure is in fact a double x-axis figure since we report $V^*_N(z)$ and the corresponding $V^*_I(b)$ in the same x-value.
Figure 1: Solutions for the model economy with nominal debt and $\sigma = 0.6 \ (nb = 2001)$

Panel A: $z'(z)$

Panel B: $z'(z) - z$

Panel C: $C(z)$

Panel D: $V(z)$
Figure 2: Solutions for the model economy with nominal debt and $\sigma = 1.0$ ($nb = 2001$)
Figure 3: Solutions for the model economy with nominal debt and $\sigma = 1.4$ ($nb = 2001$)
Figure 4: Solutions for the model economy with indexed debt and $\sigma = 0.6$ ($nb = 2001$)

Panel A: $b'(b)$
Panel B: $b'(b) - b$
Panel C: $c(b)$
Panel D: $v(b)$
Figure 5: Solutions for the model economy with indexed debt and $\sigma = 1.0$

Panel A: $b'(b)$  

Panel B: $b'(b) - b$

Panel C: $c(b)$  

Panel D: $v(b)$
Figure 6: Solutions for the model economy with indexed debt and $\sigma = 1.4$ ($nb = 2001$)
Figure 7: The Welfare Costs of Nominal Debt ($N$) and Indexed Debt ($I$)

Panel A: $V_I[zC(z)]$ and $V_N(z)$ for $\sigma = 0.6$

Panel B: $V_I[zC(z)]$ and $V_N(z)$ for $\sigma = 1.0$

Panel C: $V_I[zC(z)]$ and $V_N(z)$ for $\sigma = 1.4$