Split-Ticket Voting: An Implicit Incentive Approach*

Galina Zudenkova†

November 19, 2008

Abstract

This paper applies an implicit incentive approach in the principal-agent framework to explain split-ticket voting, when citizens vote for candidates of different parties in the local municipal simultaneous elections (i.e. the elections of a mayor for the city hall and of a governor for the province). The principals (voters), in each period of an infinite horizon, reward the agents (mayor and governor) with reelection based on their observed performance but through an implicit reward rule. So the voters can influence the politicians’ performance only through the choice of an evaluation rule. We first show that, if the voters do vote split-tickets and if the politicians do care about overall representation of their party in governing bodies rather than their own reelection prospects, then the voters prefer a relative performance evaluation rule in the next period. Otherwise, they use an absolute performance evaluation rule to reappoint the politicians in the next period. Second, we show that the stationary probability that in the long-run the principals vote split-tickets is independent on the initial state, and is lower than the stationary probability that they do not vote split-tickets.

*JEL classification: D72, D82.

Key words: Split-ticket voting; Simultaneous elections; Implicit incentive contracts.

*The author thanks Luis C. Corchón, M.A. de Frutos, Ignacio Ortúño Ortín and Ronny Razin for helpful comments, suggestions and encouragement, and Iwan Baranlauy, Georges Casamatta, Micael Castanheira, Humberto Llavador, António M. Osório-Costa, Joe Ostroy, Katharina Wick and participants at 2008 ENTER Jamboree Meeting (Madrid), the Conference on Tournaments, Contests and Relative Performance Evaluation (Raleigh), IEA 2008 (Istanbul) and EEA-ESEM 2008 (Milan) for useful comments. The FPI Grant BES-2005-10017 of the Spanish Ministry of Education and Science is gratefully acknowledged. The usual disclaimer applies.

†Department of Economics, Universidad Carlos III de Madrid, C/Madrid, 126 28903-Getafe, Madrid, Spain. E-mail address: galina.zudenkova@uc3m.es.
1. Introduction

Split-ticket voting is a common feature of modern political systems, when citizens vote for candidates of different parties in two simultaneous elections (for example, the presidential and congressional elections in US, the mayor and governor elections in Spain). The literature has addressed the problem of split-ticket voting in the US elections in the context of strategic voting models analyzed by Alesina and Rosenthal (1995, 1996) and Chari, Jones and Marimon (1997). They show that citizens have incentives to vote split-tickets strategically since the policy outcome is a function of the composition of the executive and the legislature. However, Degan and Merlo (2007) have shown empirically that "by and large split-ticket voting is also consistent with sincere voting."\(^1\) So a full microfoundation of split-ticket voting in the US elections is still ahead of researchers. In its turn, the problem of split-ticket voting in the local municipal simultaneous elections (when citizens elect a mayor for the city hall and a governor for the province this city belongs to), has not been studied yet to our knowledge. This work aims to fill the gap.

In this paper, we apply implicit incentive contracting theory to shed light on split-ticket voting in the mayor and governor local elections that occur simultaneously. We study a principal-agent model of policy implementation, where the voters are principals and the politicians are office-motivated agents. The society, with an infinite horizon, is described as a discrete-time stochastic process that takes on two states: either a mayor and a governor are the members of the same party (in our framework this is interpreted as no split-ticket voting), or a mayor and a governor belong to different parties (that is interpreted as split-ticket voting). The process transition probabilities are determined endogenously by the interactions between the rational voters and the rational politicians where the voters reward (i.e. reelect) the politicians depending on their observed performance but through an implicit reward rule. We show that the stationary probability that in the long-run the society is in split-ticket voting state is independent on the initial state, and is lower than the stationary probability of no split-ticket voting state. In other words, in the long-run the society is found in split-ticket voting state less frequently than in no split-ticket voting state.

In our model the politicians want to be reelected for a next term so they are held accountable by the voters at the election moment and have incentives to satisfy the voters' wishes. Moreover, we assume that the politicians care about overall representation of their party in governing bodies, i.e. a mayor prefers her party comember to a politician of the rival party for the governor office (and vice versa for a governor). Each politician performs a single task pol-

\(^1\)Degan and Merlo (2007), p. 16.
icy, and elections are held every period. In their turn, the voters care about policies outcomes that are observable but not contractible. They evaluate the incumbents’ performance, and vote accordingly. Every period the voters decide what evaluation rule to use to reward the incumbents in the coming elections: absolute, relative or mixed performance evaluation rule (with the latter combining both absolute and relative performance evaluation). Obviously, the voters can influence the politicians’ performance through the choice of an evaluation rule. This rule is further required to be sequentially rational.

We show that the voters prefer to evaluate the mayor and governor from the same party with an absolute performance evaluation rule, under which the politician’s effort just increases her own reelection prospects and does not decrease her comember’s chances for reappointment. As for the mayor and governor from different parties, the voters’ choice of an evaluation rule depends on the degree of the politicians’ faithfulness to their political party. If the politicians are concerned about their own reelection prospects rather than their party overall representation in governing bodies, then the voters use an absolute performance evaluation rule, under which the reelection chances are more sensitive to effort that increases the politicians’ incentives. If the mayor (resp. governor) shows faithfulness to her political party, so she wants her comember to hold the governor (resp. mayor) office, then the voters prefer a relative performance evaluation rule to create competitive environment between the politicians from different parties. As for the mixed performance evaluation, the voters never use it since under this rule there is not as much competition as it is needed for the politicians from rival parties, and there is so much competition as it is not needed for the members of the same party.

Our results rest on the fundamental assumption about the politicians’ faithfulness to their political party, when a mayor (resp. governor) not only cares about her own reelection prospects but also about her party comember’s chances to be elected for the governor (resp. mayor) office. The principals choose a relative performance evaluation rule to create competitive environment that gives extra implicit incentives to the agents from different parties to perform better. If we relax the assumption about the politicians’ faithfulness to their party, then the competition effect weakens and the voters prefer an absolute performance evaluation rule, under which the reelection chances are more sensitive to effort that increases the politicians’ incentives.

This paper is related to a principal-agent literature on career concerns models pioneered by Holmström (1982) and then applied to the accountability of government agencies by Dewatripont, Jewitt and Tirole (1999a, b) and to the advocacy in the executive and legislative branches of government by Dewatripont and Tirole (1999). Several authors contrast elected
officials ("politicians", "elected regulators") with non-elected ones ("judges", "bureaucrats", "appointed regulators") to study allocation of decision-making powers in the society (see Alesina and Tabellini (2007, 2008), Besley and Coate (2003), Maskin and Tirole (2004)). In their turn, Gersbach and Liessem (2008) and Gersbach (2004) analyze elections as incentive element in politics to solve the problems of effort allocation among several tasks and of politicians’ competence, respectively.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 studies the politicians’ efforts under different performance evaluation rules. Section 4 analyzes the voters’ choice of an evaluation rule. Section 5 deals with the process dynamics. Finally, Section 6 concludes.

2. The Model

Consider a representative city that has to elect mayor $M$ for local municipal government and governor $G$ for the province this city belongs to. The city is inhabited by a large number (formally a continuum) of individuals. The individuals live forever. At the beginning of each period $t$ the mayor and governor elections take place simultaneously and a winner is determined by the majority rule (mayor at the city level and governor at the province level). Politicians running for the elections belong to one of the two political parties, $L$ or $R$. In particular, we assume that there are exactly two candidates from the opposite parties—the incumbent and an opponent—at each election at period $t$.

We denote by $S$ the state where mayor $M$ and governor $G$ are members of the same party (either $L$ or $R$), and by $D$ the state where mayor $M$ and governor $G$ belong to different parties. Thus, we face a two-state discrete-time stochastic process $\{X_t, t = 0, 1, \ldots\}$. If $X_t = S$ (resp. $X_t = D$) we say that the process is in state $S$ (resp. $D$) at period $t$.

There is no cost of voting, and we assume that there are no abstainers. The voters are rational, and the politicians are office-motivated in a way we describe below. We assume that the participation constraints of the politicians are always satisfied, and there is no term limit. First, we describe the politicians’ preferences, and then we consider the voters’ preferences.

2.1. Politicians

While in office, at each period $t$ politician $i$ ($i = M, G$) is to implement a corresponding policy $p^i_t$. We assume that the policy outcomes are determined by the politician’s ability $\theta^i_t$ and the unobservable effort $a^i_t$ she exerts in the additive way:

$$p^i_t = \theta^i_t + a^i_t.$$
The politicians’ abilities are independent normally distributed random variables \( \theta_i^t \sim N(\bar{\theta}, \sigma^2) \). We assume that a random element of ability is present every period so that it can never be learned in advance, and that ability is evaluated equivalently by society and the politicians.\(^2\)

The mayor and governor from different parties incur higher costs of policy implementation than the ones from the same party, so at each period \( t \) the politicians carry a cost

\[
C\left(a_i^t\right) = \begin{cases} \frac{(a_i^t)^2}{2(1+c)} & \text{if } X_t = S \\ \frac{(a_i^t)^2}{2} & \text{if } X_t = D, \end{cases}
\]

where \( c > 0 \). The logic under this assumption is straightforward. While from the same party the politicians tend to give assistance to each other, so the mayor and governor administrations may conduct some joint projects that lead to a decrease in effort costs. As for the politicians from different parties, it is logical to assume more rival behavior and, therefore, more costly efforts.\(^3\)

Let us denote the reward for politician \( i \) at period \( t \) as \( U(a_i^t, \cdot) \).\(^4\) Both the mayor and governor maximize their net utility at each period \( t \) given by

\[
U(a_i^t, \cdot) - C(a_i^t).
\]

Without loss of generality we specify the mayor’s preferences throughout the paper. Note that, due to the symmetry, the governor’s problem is identical. First, once elected at \( t \) mayor \( M \) wants to be reelected herself at \( t + 1 \). Moreover, she cares about the chances of her party comembers to be elected for a governor office at \( t + 1 \). In case at \( t \) governor \( G \) is a member of the same party, mayor \( M \) prefers her to be reelected. Otherwise, mayor \( M \) wants her party comember to be elected for the governor office for a next term. Let us denote by \( \mu_t \) the value of holding office, by \( \lambda_t \) the value mayor \( M \) associates to her party comember’s holding governor office and by \( P_{t+1}^i (\cdot, \cdot) \) the probability of being reelected for a corresponding office \( i = M, G \) at \( t + 1 \). Therefore, mayor \( M \) holding office at \( t \) has the following reward function:

\[
U\left(a_i^M, a_i^G\right) = \begin{cases} \mu_t P_{t+1}^M (a_i^M, a_i^G) + \lambda_t P_{t+1}^G (a_i^G, a_i^M) & \text{if } X_t = S \\ \mu_t P_{t+1}^M (a_i^M, a_i^G) + \lambda_t (1 - P_{t+1}^G (a_i^G, a_i^M)) & \text{if } X_t = D. \end{cases}
\]

\(^2\)Alternatively, one could interpret \( \theta_i^t \) as a state of the world. In general, the mayor and governor policies pursue different goals, so we can assume that the mayor and governor administrations face independent states of the world.

\(^3\)Alternatively, one can consider the situation where the mayor and governor are to take some joint decision. Then it is easier to come to the agreement for the politicians from the same party than for the politicians from rival parties.

\(^4\)We do not define the politicians’ rate of time discount since there is no intertemporal trade-off here.
With the preferences of this type we try to reflect the politicians’ allegiance to their party with politicians caring about overall representation of their party in governing bodies.\(^5\) Still, the reasonable assumption here is that a mayor values her own office holding more than her party representation in the governor office, so \(0 \leq \frac{\mu_i}{\mu_t} \leq 1\). We call \(\rho_t \equiv \frac{\mu_i}{\mu_t}\) the degree of the politician’s faithfulness to her party at period \(t\), and assume that \(\rho_t\) is independently distributed according to a smooth, at least once differentiable, cumulative distribution function \(\Lambda(\cdot)\) with support \([0, 1]\).

### 2.2. Voters

Individuals differ in their preferences over political parties. To be more specific, we assume that some individuals always prefer party \(L\) to party \(R\), therefore they vote for candidates from party \(L\) at both elections, while other individuals are faithful to party \(R\), so they always ballot their votes for party \(R\) at both elections. Moreover, there is a group of individuals that share the same preferences, and which votes are believed to be decisive for the outcome of both elections at each \(t\). They are indifferent between political parties and care about the policy outcomes in each period according to linear utility function\(^6\)

\[ p_t^M + p_t^G. \]

We normalize the size (mass) of the decisive voters group to unity. The decisive voters face no coordination problem since they all participate in elections and share the same preferences. So our problem is analogous to the one with one decisive voter who determines the outcome of the elections. In what follows we call this decisive voters group just the voters.

Let us specify the first-best levels of effort, denoted by \(F_t\):

\[
F_t = \begin{cases} 
1 + c & \text{if } X_t = S \\
1 & \text{if } X_t = D.
\end{cases}
\] (2.1)

Therefore, in case mayor \(M\) and governor \(G\) belong to the same party the first-best level of effort is higher than the one in case of the mayor and governor belonging to different parties.\(^7\)

The intuition under this result is straightforward: if the politicians are from the same party it is cheaper for them to exert effort, so they perform better than the politicians from different parties. If performance \(p_t^i\) is verifiable and contractible, then the first-best is achieved by

\(^5\)Alternatively, one can think of the mayor and governor that are to interact with each other while in office. Then each prefers her own party comember to work with rather than a member of the rival party.

\(^6\)Since the voters face no intertemporal trade-off, we do not define their rate of time discount.

\(^7\)This simple result comes from maximizing \(p_t^M + p_t^G - C(a_t^M) - C(a_t^G)\) with respect to \(a_t^i\), \(i = M, G\).
an optimal explicit contract by making the agents the residual claimants.\footnote{Consider the contract based on performance $U (p^i_t) = p^i_t - w$, where $w$ is defined by the agents’ ex ante binding participation constraint $E (U (p^i_t)) - C (a^i_t) = 0$. Then $w = \overline{\mu} + F_t - C (F_t)$, where $F_t$ is the first-best level of effort.} However, it is reasonable to assume that the final outcomes $p^M_t$ and $p^G_t$ (but not their composition between ability and effort) are observed at the end of period $t$ but not contractible. We believe that it is quite realistic assumption taking into account that public policies are difficult to reward with explicit contracts. It is more natural to use implicit incentive contracting in this situation.

At the end of each period $t$ the voters observe the policy outcomes, and at $t + 1$ elections they reward the incumbents according to their performance at period $t$, i.e. they reappoint the incumbent who has shown "good" results at period $t$. If the voters are not satisfied with the incumbent’s performance at $t$, they will vote for an opponent at $t + 1$.

There are different evaluation rules the voters can use to reward the politicians at $t + 1$ elections. To be more specific, we analyze three scenarios, namely, absolute performance evaluation, relative performance evaluation, and mixed performance evaluation with the latter combining both absolute and relative performance evaluation. Obviously the voters can influence the politicians’ behavior through the choice of an evaluation rule. This rule is further required to be sequentially rational, that is, no precommitment is allowed. We assume that all the model parameters are common knowledge, so the politicians have rational expectations about the evaluation rule that will be used by the voters for particular parameters values. Let us stress here that the game is played sequentially, so the politicians observe the elections outcome before exerting efforts. Thus, the politicians know whether the voters use the evaluation rule that has been rational for them or deviate. In the latter case the politicians conclude that, from this period on, the voters reappoint the incumbents randomly or use some unknown reelection rule that is not based on the politicians’ performance evaluation. As a result, from this period on, the politicians will exert zero effort to minimize their costs. The voters know this, so they have no incentives to deviate and always reward the incumbents according to a rule chosen in the previous period.

Thus the timing of events is as follows. At each period $t$: first, the elections take place, where the voters use the evaluation rule chosen at period $t - 1$; second, nature chooses the value of holding office, $\mu_t$, and the value of comember’s holding office, $\lambda_t$; then, the voters make the choice of an evaluation rule to be used at $t + 1$; then, the elected politicians exert efforts $a^i_t$; finally, nature chooses the politicians’ ability levels $\theta^i_t$, and policy outcomes are
observed.$^9$

We analyze the game backwards: first, we solve for the politicians’ efforts at period $t$ for the three evaluation rules; second, we concentrate on the voters’ choice of an evaluation rule to be used at period $t + 1$.

3. Politicians’ Efforts under Different Performance Evaluation Rules

3.1. Absolute Performance Evaluation

The simplest evaluation rule is based on rewarding the incumbents according to their absolute performance at period $t$. In particular, at period $t + 1$ the voters reappoint the incumbent for office $i$ if the policy outcome $p_{i}^{t}$ exceeds a threshold level $\bar{p}^{i}$ that we specify below. Therefore, the mayor’s reward becomes

$$U (a_{i}^{M}, a_{i}^{G}) = \begin{cases} 
\mu_{t} P \left( \{ p_{i}^{M} (a_{i}^{M}) > \bar{p}^{M} \} \right) + \lambda_{t} P \left( \{ p_{i}^{G} (a_{i}^{G}) > \bar{p}^{G} \} \right) & \text{if } X_{t} = S \\
\mu_{t} P \left( \{ p_{i}^{M} (a_{i}^{M}) > \bar{p}^{M} \} \right) + \lambda_{t} \left( 1 - P \left( \{ p_{i}^{G} (a_{i}^{G}) > \bar{p}^{G} \} \right) \right) & \text{if } X_{t} = D.
\end{cases}$$

The voters perfectly realize that if the politician is not elected at period $t$, another candidate of average ability $\bar{\theta}$ will take her place and will exert the equilibrium level of effort $\bar{a}^{i}$, so $\bar{p}^{i} = \bar{\theta} + \bar{a}^{i}$.

Let us compute the equilibrium level of effort at period $t$ under absolute performance evaluation. Using a normal distribution for the ability $\theta^{i}_{t}$, taking the first-order condition with respect to real efforts $a^{i}_{t}$ and imposing the equilibrium condition $\bar{a}^{i} = a^{i}_{t}$, we get the following result.

**Lemma 1.** Under the absolute performance evaluation rule the politicians’ equilibrium effort at period $t$, denoted by $A_{t}$, reads

$$A_{t} = \begin{cases} 
\frac{\mu_{t}(1+c)}{\sqrt{2\pi}\sigma^{2}} & \text{if } X_{t} = S \\
\frac{\mu_{t}}{\sqrt{2\pi}\sigma^{2}} & \text{if } X_{t} = D.
\end{cases} \tag{3.1}$$

Note that the equilibrium effort of the politicians from the same party is higher than the one of the politicians from different parties. The intuition under this result is straightforward:

$^9$Note that this timing makes sense only under the assumptions that ability can never be learned in advance, and that it is evaluated equivalently by the principals and the agents. If the former is relaxed, at period $t$ elections the principals will know the incumbents’ ability. If we relax the latter, different timing is to be considered, namely, the politicians learn their ability levels before exerting effort. In both cases, different models are to be analyzed that we leave for future research.
if the politicians are from the same party it is cheaper for them to carry out effort, so they perform better than the politicians from different parties.

How does the effort of the politicians under the absolute performance evaluation rule differ from the first-best level of effort at period $t$? Comparing (2.1) and (3.1), we conclude that the answer is not clear and depends on the parameters of the model. On the one hand, the higher the value of holding office $\mu_t$ is, the higher effort the politicians exert. On the other hand, a larger variance of ability $\sigma^2$ decreases the politicians’ efforts, that is, the more uncertain the ability is, the less effort the politicians exert. Intuitively, the higher the variance of ability $\sigma^2$ is, the more random the policy outcome $p_t^i$ is, that cuts down the politician’s incentives. At period $t$, the politicians’ behavior is socially optimal only for some particular parameters values (namely, $\mu_t = \sqrt{2\pi\sigma^2}$).

3.2. Relative Performance Evaluation

In this subsection we consider a reelection rule that is based on rewarding the incumbents according to their relative performance. To be more specific, we assume that the voters compare their utilities from the policies implemented by mayor $M$ and by governor $G$ at period $t$. In reality public policies often pursue many different goals that could be hard to measure and compare. Still, we assume that the voters can compare their utility levels from implemented policies if not policies themselves. To be more specific, the voters use the following relative performance evaluation rule: to reappoint the incumbent for the mayor office if the voters’ utility from policy $p_t^M$ exceeds their utility from policy $p_t^G$ implemented by governor $G$ (and vice versa for governor reelection). Under this rule the mayor’s reward reads

$$U(a_t^M, a_t^G) = \begin{cases} 
\mu_t P \left( \{ p_t^M (a_t^M) \geq p_t^G (a_t^G) \} \right) + \lambda_t P \left( \{ p_t^G (a_t^G) \geq p_t^M (a_t^M) \} \right) & \text{if } X_t = S \\
\mu_t P \left( \{ p_t^M (a_t^M) \geq p_t^G (a_t^G) \} \right) + \lambda_t \left( 1 - P \left( \{ p_t^G (a_t^G) \geq p_t^M (a_t^M) \} \right) \right) & \text{if } X_t = D.
\end{cases}$$

So the voters compare their utilities from policies $p_t^M$ and $p_t^G$ between each other and reappoint the incumbent who has performed better, while vote for an opponent in place of the incumbent who has performed worse.

**Lemma 2.** Under the relative performance evaluation rule the politicians’ equilibrium effort at period $t$, denoted by $R_t$, is

$$R_t = \begin{cases} 
\frac{(\mu_t - \lambda_t)(1+c)}{2\sqrt{\pi}\sigma^2} & \text{if } X_t = S \\
\frac{(\mu_t + \lambda_t)}{2\sqrt{\pi}\sigma^2} & \text{if } X_t = D.
\end{cases}$$
The proof of this and all the following results are given in the Appendix. Let us compare
the effort of the politicians from different parties with the one of the politicians from the same
party presented in (3.2). The answer is ambiguous and depends on the problem parameters.
A higher value the mayor associates to her party comember’s holding the governor office, \( \lambda_t \),
increases the effort of the politicians from different parties, while it decreases the effort of the
politicians from the same party. The result comes from the nature of rewarding according
to the politicians relative performance. If the governor office is held by the mayor’s party
comember, the mayor wants her to be reelected for a next term. So she wants the governor’s
policy outcome not to be lower than her own that decreases the incentives to exert higher
effort. On the other hand, if the governor office is held by a member of the rival party, the
mayor does not want her to be reappointed, that gives her incentives to exert higher effort
for her policy outcome to exceed the one of the governor. Still, the effort of the politicians from
the same party can be higher than the one of the politicians from different parties in case
it is much cheaper for them to implement the policies, that happens if the cost decreasing
parameter \( c \) is high enough. Intuitively, if the politicians are from the same party and it is
much cheaper for them to carry out effort, then they have extra incentives to perform better
than the politicians from different parties. To resume,

\[
\begin{align*}
\text{if } 0 \leq \rho_t \leq \frac{1}{3} \text{ and } 0 < c < \frac{2\rho_t}{1-\rho_t} & \text{ then } R_t|_{X_t=S} < R_t|_{X_t=D} \\
\text{if } 0 \leq \rho_t < \frac{1}{3} \text{ and } c \geq \frac{2\rho_t}{1-\rho_t} & \text{ then } R_t|_{X_t=S} \geq R_t|_{X_t=D} \\
\text{if } \frac{1}{3} < \rho_t \leq 1 \text{ and } \forall c > 0 & \text{ then } R_t|_{X_t=S} < R_t|_{X_t=D},
\end{align*}
\]

where \( R_t|_{X_t=S} \) (resp. \( R_t|_{X_t=D} \)) denotes the equilibrium effort levels under the relative performance evaluation rule if the politicians are members of the same party at period \( t \) (resp.
of different parties). Thus, if the politicians are faithful enough to their party (so \( \rho_t \) is high
enough), then the mayor and governor from different parties exert higher effort than the ones
from the same party. Otherwise, the outcome depends on the cost decreasing parameter \( c \).
If effort is considerably cheaper for the politicians from the same party holding offices (so \( c \) is high enough), then this effect dominates, so the politicians from the same party carry
out higher effort than the ones from different parties. If the cost decreasing parameter is not
very high, then the politicians from different parties exert higher effort. In any case under
the relative performance evaluation the equilibrium level of effort exerted by the politicians
differs from the socially optimal one given by (2.1) except for some particular values of the
parameters, namely, \( \mu_t - \lambda_t = 2\sqrt{\pi \sigma^2} \) if \( X_t = S \) and \( \mu_t + \lambda_t = 2\sqrt{\pi \sigma^2} \) if \( X_t = D \).
3.3. Mixed Performance Evaluation

In this subsection we consider a reelection rule that is based on rewarding the politicians according to both their absolute and relative performance, where the latter consists of the comparison between the voters’ utilities from the policy outcomes achieved by mayor \( M \) and by governor \( G \) at the end of period \( t \). To be more specific, the voters use the following mixed performance evaluation rule: to reappoint the incumbent for the mayor office if the policy outcome \( p^M_t \) exceeds a threshold level \( \bar{p}^M \) (defined in the same way as in the absolute performance evaluation rule), and if the voters’ utility from policy \( p^M_t \) exceeds their utility from the policy \( p^G_t \) implemented by governor \( G \) (as in the relative performance evaluation rule). Under mixed performance evaluation the mayor’s reward at period \( t \) reads

\[
U(a^M_t, a^G_t) = \begin{cases} 
\mu_t P \{ \{ p^M_t (a^M_t) \geq \bar{p}^M \} \cap \{ p^M_t (a^M_t) \geq p^G_t (a^G_t) \} \} + \\
\lambda_t P \{ \{ p^G_t (a^G_t) \geq \bar{p}^G \} \cap \{ p^G_t (a^G_t) \geq p^M_t (a^M_t) \} \} & \text{if } X_t = S, \\
\mu_t (1 - P) \{ \{ p^G_t (a^G_t) \geq \bar{p}^G \} \cap \{ p^G_t (a^G_t) \geq p^M_t (a^M_t) \} \} + \\
\lambda_t (1 - P) \{ \{ p^M_t (a^M_t) \geq \bar{p}^M \} \cap \{ p^M_t (a^M_t) \geq p^G_t (a^G_t) \} \} & \text{if } X_t = D,
\end{cases}
\]

and vice versa for the governor’s reward. First, the voters care about the absolute performance of the politicians. Second, they pay attention to the relative performance as well, comparing their utilities from the policies implemented by the mayor and governor. In case both the mayor and governor have performed badly in absolute terms (policy outcomes do not exceed the corresponding thresholds), the voters do not reelect any of them. If in absolute terms the incumbents have shown satisfactory performance (policy outcomes exceed the corresponding thresholds), the voters compare the utilities from policies between each other and reappoints the incumbent who has performed better while votes for an opponent in place of the incumbent who has performed worst.

**Lemma 3.** Under the mixed performance evaluation rule the politicians’ equilibrium effort at period \( t \), denoted by \( Y_t \), is given by

\[
Y_t = \begin{cases} 
\frac{(\sqrt{2}+1)\mu_t - \lambda_t}{4\sqrt{\pi}\sigma^2} & \text{if } X_t = S, \\
\frac{(\sqrt{2}+1)\mu_t + \lambda_t}{4\sqrt{\pi}\sigma^2} & \text{if } X_t = D.
\end{cases}
\]  

(3.3)

Let us compare the effort of the politicians from different parties with the one of the politicians from the same party presented in (3.3). The intuition is analogous to the case of
\[
\begin{array}{c|c|c}
X_t = S & X_t = D \\
\hline
F_t & 1 + c & 1 \\
A_t & \frac{\mu_t(1+c)}{\sqrt{2\pi\sigma^2}} & \frac{\mu_t}{\sqrt{2\pi\sigma^2}} \\
R_t & \frac{(\mu_t-\lambda_t)(1+c)}{2\sqrt{2\pi\sigma^2}} & \frac{(\mu_t+\lambda_t)}{2\sqrt{2\pi\sigma^2}} \\
Y_t & \frac{(\sqrt{2+1}\mu_t-\lambda_t)(1+c)}{4\sqrt{\pi\sigma^2}} & \frac{(\sqrt{2+1}\mu_t+\lambda_t)}{4\sqrt{\pi\sigma^2}} \\
\end{array}
\]

Table 3.1: First best effort \((F_t)\) and equilibrium effort levels under Absolute \((A_t)\), Relative \((R_t)\) and Mixed \((Y_t)\) performance evaluation rules in cases of politicians from the same party \((X_t = S)\) and politicians from different parties \((X_t = D)\) at period \(t\)

relative performance evaluation. To be more specific,

\[
\begin{align*}
\text{if } 0 & \leq \rho_t \leq \frac{\sqrt{2}+1}{3} \text{ and } 0 < c < \frac{2\rho_t}{\sqrt{2+1} - \rho_t} \text{ then } Y_t|_{X_t=S} & < Y_t|_{X_t=D} \\
\text{if } 0 & \leq \rho_t \leq \frac{\sqrt{2}+1}{3} \text{ and } c \geq \frac{2\rho_t}{\sqrt{2+1} - \rho_t} \text{ then } Y_t|_{X_t=S} & \geq Y_t|_{X_t=D} \\
\text{if } \frac{\sqrt{2}+1}{3} & < \rho_t \leq 1 \text{ and } \forall \ c > 0 \text{ then } Y_t|_{X_t=S} & < Y_t|_{X_t=D},
\end{align*}
\]

where \(Y_t|_{X_t=S}\) (resp. \(Y_t|_{X_t=D}\)) denotes the equilibrium effort levels under the mixed performance evaluation rule if the politicians are members of the same party at period \(t\) (resp. of different parties). Therefore, if the politicians are faithful enough to their party (so the mayor (resp. governor) cares both about her own reelection prospects and her party comember’s chances to hold the governor (resp. mayor) office, i.e. \(\rho_t\) is high), then the mayor and governor from different parties exert higher effort than the ones from the same party. In case the politicians care about their own reelection prospects rather than their party overall representation in governing bodies (i.e. \(\rho_t\) is low), the outcome depends on the cost decreasing parameter \(c\). If it is considerably cheaper for the politicians from the same party to exert effort (i.e. \(c\) is high enough), then this effect dominates, and the politicians from the same party exert higher effort. Otherwise, the politicians from different parties carry out higher effort. Note that the equilibrium level of effort exerted by the politicians at period \(t\) differs from the socially optimal one in (2.1) except for some particular parameter values, namely, \((\sqrt{2} + 1) \mu_t - \lambda_t = 4\sqrt{\pi\sigma^2}\) if \(X_t = S\) and \((\sqrt{2} + 1) \mu_t + \lambda_t = 4\sqrt{\pi\sigma^2}\) if \(X_t = D\).

Table 3.1 summarizes the first best effort level and equilibrium effort levels at period \(t\) under different evaluation rules. In what follows we solve for the voters’ choice of an evaluation rule to reward the incumbents at period \(t+1\).
4. Voters’ Choice of Evaluation Rule

What evaluation rule do the voters prefer to reappoint the incumbents at period \( t + 1 \)? The voters have rational expectations about the politicians’ effort for any realization of \( X_t \) and \( \rho_t \). Thus, to decide on an evaluation rule they just maximize their expected utility given the realizations of \( X_t \) and \( \rho_t \). The politician’s expected ability is equal to \( \bar{\theta} \), so the voters prefer an evaluation rule under which the politicians are expected to exert higher effort.

First, we analyze the case where mayor \( M \) and governor \( G \) are members of the same party at period \( t \), i.e. \( X_t = S \). It is straightforward to see that

\[
A_t|X_t=S > Y_t|X_t=S > R_t|X_t=S
\]

for each \( c > 0 \) and each \( \rho_t \in [0,1] \), so if the offices are held by the members of the same party they exert higher effort under the absolute performance evaluation rule. The result is due to the fact that the mayor wants to be reelected herself and she wants the governor to be reappointed as well. She knows that under the absolute performance evaluation, her effort just affects her own reelection prospects: the higher the effort is, the greater the chances to be reappointed are. Under the relative (as well as mixed) performance evaluation her effort affects both her own and the governor reelection prospects: the higher the effort is, the greater the mayor’s chances and the lower the governor’s chances to be reappointed are. The same intuition works in the case of a governor. This trade-off explains why the politicians from the same party exert higher effort under the absolute performance evaluation rule. Therefore, if \( X_t = S \) for any realization of \( \rho_t \) the voters will use the absolute performance evaluation rule to reelect the incumbents at period \( t + 1 \).

Second, we consider the case where mayor \( M \) and governor \( G \) belong to different parties at period \( t \), i.e. \( X_t = D \). It is easy to check that

\[
\begin{align*}
\text{if } 0 \leq \rho_t \leq \sqrt{2} - 1 & \text{ then } A_t|X_t=D \geq Y_t|X_t=D \geq R_t|X_t=D \\
\text{if } \sqrt{2} - 1 < \rho_t \leq 1 & \text{ then } A_t|X_t=D < Y_t|X_t=D < R_t|X_t=D.
\end{align*}
\]

If the degree of the politician’s faithfulness to her party, \( \rho_t \), is low, so the politicians care mainly about their own reelection and not about their party overall representation in governing bodies, then the politicians exert higher effort under the absolute performance evaluation rule. The result is due the fact that in this case the mayor mainly wants to be reappointed herself, so under the absolute performance evaluation rule her reelection chances depend on her effort and her random ability, while under the relative (as well as mixed) performance evaluation rule the governor performance affects the mayor’s reelection prospects as well.
Therefore, under the latter more of the policy outcome is due to the randomness, so the reappointment chances are less sensitive to effort that decreases the mayor's incentives (and vice versa for the governor). In other words, if the politicians are not too faithful to their party—so they care much more about their own reelection prospects than about their party comembers' chances to hold office—then the voters prefer to use the absolute performance evaluation rule to reward the incumbents at period $t + 1$.

In case the politicians are faithful to their political party—$\rho_t$ is quite high—then the voters will use the relative performance evaluation rule. As we mentioned above, under the relative performance evaluation rule the policy outcome is more random, that weakens the politicians' incentives. However, rewarding based on relative performance gives additional incentives to the politicians from different parties to carry out higher effort (when the politician has to perform better for the incumbent from the rival party not to be reappointed).

We sum up the results in the following proposition.

**Proposition 1.** If the mayor and governor are members of the same party at period $t$ ($X_t = S$), then the voters use the absolute performance evaluation rule to reward the incumbents at period $t + 1$.

If the mayor and governor belong to different parties at period $t$ ($X_t = D$), then

i) if $\rho_t \in [0, \sqrt{2} - 1]$ the voters prefer the absolute performance evaluation rule,

ii) if $\rho_t \in (\sqrt{2} - 1, 1]$ they use the relative performance evaluation rule,

where the degree of the politician’s faithfulness to her party, $\rho_t$, is distributed according to a distribution function $\Lambda(\cdot)$ with support $[0, 1]$.

Note that the mixed performance evaluation rule is never used by the voters. This result is due to the fact that this evaluation rule is a kind of mixture of the other two rules, so in case of the politicians from different parties, higher effort is due to poor competition effect (so relative performance evaluation is chosen), while in case of the politicians from the same party, higher effort is achieved with no competition at all (so absolute performance evaluation is used). In its turn, the mixed performance evaluation cuts down both on the incentives for the politicians from different parties in comparison with the relative performance evaluation, and on the incentives for the politicians from the same party in comparison with the absolute performance evaluation.
5. Stochastic Process Dynamics

Note that whenever the process is in state $k$, there is a fixed probability that it will next be in state $l$. We denote this probability by $P_{kl} \equiv P(X_{t+1} = l \mid X_t = k)$. Thus, we establish the following lemma.

**Lemma 4.** Stochastic process $\{X_t, t = 0, 1, \ldots\}$ is a Markov chain with the following matrix of one-step transition probabilities, $\mathbf{P}$:

$$
\mathbf{P} = \begin{bmatrix}
P_{SS} & P_{SD} \\
P_{DS} & P_{DD}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} \\
1 - \frac{1}{2} \Lambda(\sqrt{2} - 1) & \frac{1}{2} \Lambda(\sqrt{2} - 1)
\end{bmatrix}
$$

where $\Lambda(\cdot)$ is a distribution function of the degree of the politician’s faithfulness to her party, $\rho_t$.

The $n$-step transition probability $P_{kl}^n \equiv P(X_{t+n} = l \mid X_t = k)$ is the probability that the process in state $k$ will be in state $l$ after $n$ additional periods. The $n$-step transition matrix $\mathbf{P}^n$ may be obtained by multiplying the matrix $\mathbf{P}$ by itself $n$ times:

$$
\mathbf{P}^n = \begin{bmatrix}
P_{SS}^n & P_{SD}^n \\
P_{DS}^n & P_{DD}^n
\end{bmatrix} = \mathbf{P}^n = \begin{bmatrix}
\frac{2-\Lambda(\sqrt{2}-1)+2^{-n}(\Lambda(\sqrt{2}-1)-1)^n}{3-\Lambda(\sqrt{2}-1)} & \frac{1-2^{-n}(\Lambda(\sqrt{2}-1)-1)^n}{3-\Lambda(\sqrt{2}-1)} \\
\frac{2^n(2^{\Lambda(\sqrt{2}-1)-1}-1)(2^{-\Lambda(\sqrt{2}-1)})}{3-\Lambda(\sqrt{2}-1)} & \frac{1+2^{-n}(\Lambda(\sqrt{2}-1)-1)^n(2-\Lambda(\sqrt{2}-1))}{3-\Lambda(\sqrt{2}-1)}
\end{bmatrix}
$$

Note that

$$
\lim_{n \to \infty} \mathbf{P}^n = \begin{bmatrix}
\frac{2-\Lambda(\sqrt{2}-1)}{3-\Lambda(\sqrt{2}-1)} & \frac{1}{3-\Lambda(\sqrt{2}-1)} \\
\frac{2^{-\Lambda(\sqrt{2}-1)}}{3-\Lambda(\sqrt{2}-1)} & \frac{1}{3-\Lambda(\sqrt{2}-1)}
\end{bmatrix}
$$

Thus, there is a limiting (or stationary) probability that the process will be in state $S$ (resp. $D$) after a large number of periods, and this value is independent on the initial state and is equal to $\frac{2-\Lambda(\sqrt{2}-1)}{3-\Lambda(\sqrt{2}-1)}$ (resp. $\frac{1}{3-\Lambda(\sqrt{2}-1)}$). Note that this probability can be interpreted as the long-run proportion of time that the Markov chain is in the corresponding state. Thus, we conclude that in the long-run the mayor and governor offices are held more often by the members of the same party than by the politicians from rival parties.

In our framework, state $S$ (resp. state $D$) happens when the voters do not vote (resp. do vote) split-tickets to reward the incumbents for their performance in the previous period. Note that the voters’ decision whether to vote split-tickets or not is modeled as the outcome of an evaluation rule chosen in the previous period. In other words, the voters do choose on an evaluation rule to reward the incumbents, and then just vote (or do not vote) split-tickets to follow the chosen rule. The following proposition summarizes the discussion.
Proposition 2. The stationary probability that in the long-run the voters do vote split-tickets is independent on the initial state, and is equal to \( \frac{1}{3 - \Lambda(\sqrt{2} - 1)} \).

6. Conclusion

This paper applies a principal-agent approach to explain the split-ticket voting phenomenon in the local simultaneous elections. In our analysis, the principals (voters) reward the agents (politicians) through different implicit reward rules. The voters choose an evaluation rule based on either absolute, or relative, or mixed (i.e. both absolute and relative) performance of the politicians. In their turn, the politicians can be faithful to their political party caring about overall representation of their party in governing bodies.

We show that the stationary probability that in the long-run the principals vote split-tickets is independent on the initial state, and is lower than the stationary probability that they do not vote split-tickets. So, in the long-run the society is found in split-ticket voting state less frequently than in no split-ticket voting state.

We have focused on single task policies with outcomes determined by the politician’s ability and effort in the additive way. However, in reality public policies can pursue many goals, so it is of interest to study split-ticket voting problem under more realistic assumption of multiple tasks policies when the problem of effort allocation among tasks can create policy trade-offs. Alternatively, policy outcomes could be determined by ability and effort in the multiplicative way rather than in the additive one. We leave these extensions of the model for future research.

Appendix

Throughout the Appendix we skip the subscript \( t \)–that denotes the period–for the sake of notation and use capital letters for the variables in place of lower-case ones. In their turn, lower-case letters will denote the realizations of these variables. Let us use \( \Phi \) to denote the normal distribution function and \( \varphi \) for the corresponding density.

A. Proof of Lemma 2

Under the relative performance evaluation rule the probability of being reelected for office \( M \) reads

\[
P^M (a^M, a^G) = P \left( \{ p^M (a^M) \geq p^G (a^G) \} \right) = P \left( \{ \theta^M - \theta^G \geq a^G - a^M \} \right).
\]
The politicians’ abilities $\Theta^M$ and $\Theta^G$ are independent normally distributed random variables, so by the convolution formula $\Theta^M - \Theta^G \sim N(0, 2\sigma^2)$. So the previous formula yields

$$P^M (a^M, a^G) = 1 - \Phi_{\Theta^M - \Theta^G} (a^G - a^M). \quad (A.1)$$

The politician holding office $M$ maximizes

$$\begin{cases}
\mu P^M (a^M, a^G) + \lambda P^G (a^G, a^M) - \frac{(a^M)^2}{2(1+c)} & \text{if } X = S \\
\mu P^M (a^M, a^G) + \lambda (1 - P^G (a^G, a^M)) - \frac{(a^M)^2}{2} & \text{if } X = D
\end{cases}$$

with respect to $a^M$, where $P^G (a^G, a^M)$ is symmetric to (A.1). The first-order condition with respect to $a^M$ yields

$$\begin{cases}
\mu \varphi_{\Theta^M - \Theta^G} (a^G - a^M) - \lambda \varphi_{\Theta^G - \Theta^M} (a^M - a^G) = \frac{a^M}{1+c} & \text{if } X = S \\
\mu \varphi_{\Theta^M - \Theta^G} (a^G - a^M) + \lambda \varphi_{\Theta^G - \Theta^M} (a^M - a^G) = a^M & \text{if } X = D.
\end{cases}$$

In the equilibrium the mayor and governor exert the same level of effort $a^M = a^G$, so

$$\varphi_{\Theta^M - \Theta^G} (a^G - a^M) = \varphi_{\Theta^M - \Theta^G} (a^M - a^G) = \varphi_{\Theta^M - \Theta^G} (0) = \frac{1}{2\sqrt{\pi} \sigma^2}$$

Therefore, the equilibrium level of effort under the relative performance evaluation rule—denoted by $R$—reads

$$R = \begin{cases}
\frac{(\mu - \lambda)(1+c)}{2\sqrt{\pi} \sigma^2} & \text{if } X_t = S \\
\frac{(\mu + \lambda)}{2\sqrt{\pi} \sigma^2} & \text{if } X_t = D.
\end{cases}$$

**B. Proof of Lemma 3**

Under the mixed performance evaluation rule the probability of being reelected for office $M$ reads

$$P_M (a^M, a^G) = P \left( \{p^M (a^M) \geq p^G (a^G) \} \cap \{p^M (a^M) \geq p^G (a^G) \} \right) = P \left( \{\theta^M \geq \theta^G \} \cap \{\theta^M - \theta^G \geq a^G - a^M \} \right).$$

We know that by the convolution formula $\Theta^M - \Theta^G \sim N(0, 2\sigma^2)$.

Applying the usual theory of changes of variables within an integral, we can find the joint distribution of the variables $(U, V) \equiv (\Theta^M, \Theta^M - \Theta^G)$. By the independence of $\Theta^M$ and $\Theta^G$, $f_{\Theta^M, \Theta^G} (\theta^M, \theta^G) = \varphi_{\Theta^M} (\theta^M) \varphi_{\Theta^G} (\theta^G)$. Let us denote the mapping of this problem by $\Delta (\theta^M, \theta^G) = (u, v)$ where

$$\begin{align*}
u &= \theta^M - \theta^G \\
v &= \theta^M - \theta^G
\end{align*}$$

$$u = \theta^M$$

$$v = \theta^M - \theta^G$$
with inverse \( \Delta^{-1}(u,v) = (\theta^M, \theta^G) \) where
\[
\begin{align*}
\theta^M &= u \\
\theta^G &= u - v.
\end{align*}
\]
The Jacobian of \( \Delta^{-1} \) is given by
\[
J(u,v) = \begin{vmatrix}
\frac{\partial \theta^M}{\partial u} & \frac{\partial \theta^M}{\partial v} \\
\frac{\partial \theta^G}{\partial u} & \frac{\partial \theta^G}{\partial v}
\end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1.
\]
Therefore, random variables \((U,V) \equiv (\Theta^M, \Theta^M - \Theta^G)\) have the joint density function
\[
f_{\Theta^M, \Theta^M - \Theta^G}(u,v) = f_{\Theta^M, \Theta^G}(\theta^M(u,v), \theta^G(u,v)) |J(u,v)| = \varphi_{\Theta^M}(\theta^M(u,v)) \varphi_{\Theta^G}(\theta^G(u,v)) = \frac{1}{2\pi\sigma^2} \exp \left\{ -\frac{(u-\bar{\theta})^2 + (u-v-\bar{\vartheta})^2}{2\sigma^2} \right\},
\]
that does not factorize as the product of a function of the first variable and a function of the second, so random variables \(\Theta^M\) and \(\Theta^M - \Theta^G\) are not independent.

We can write now the probability of being reappointed for office \(M\) as
\[
P_M(a^M, a^G) = 1 + \mathcal{F}_{\Theta^M, \Theta^M - \Theta^G}(p^M - a^M, a^G - a^M) - \Phi_{\Theta^M}(p^M - a^M) - \Phi_{\Theta^M - \Theta^G}(a^G - a^M).
\]
(B.1)

Therefore, under mixed performance evaluation the problem of the politician holding office \(M\) is to maximize
\[
\left\{ \begin{array}{ll}
\mu P_M(a^M, a^G) + \lambda P^G(a^G, a^M) - \frac{(a^M)^2}{2(1+c)} & \text{if } X = S \\
\mu P_M(a^M, a^G) + \lambda (1 - P^G(a^G, a^M)) - \frac{(a^M)^2}{2} & \text{if } X = D
\end{array} \right.
\]
with respect to \(a^M\), where the probability of being reelected for office \(G\), \(P_G(a^G, a^M)\), is symmetric to (B.1). The first-order condition with respect to \(a^M\) reads
\[
\left\{ \begin{array}{ll}
\mu \left( \frac{\partial}{\partial a^M} \mathcal{F}_{\Theta^M, \Theta^M - \Theta^G}(p^M - a^M, a^G - a^M) + \varphi_{\Theta^M}(p^M - a^M) + \varphi_{\Theta^M - \Theta^G}(a^G - a^M) + \frac{\lambda}{1+c} \right) & \text{if } X = S \\
\mu \left( \frac{\partial}{\partial a^M} \mathcal{F}_{\Theta^G, \Theta^G - \Theta^M}(p^G - a^G, a^M - a^G) + \varphi_{\Theta^M}(p^M - a^M) + \varphi_{\Theta^M - \Theta^G}(a^G - a^M) - \frac{\lambda}{1+c} \right) & \text{if } X = D.
\end{array} \right.
\]
(B.2)

As under absolute performance evaluation the voters expect \(\bar{p}^i = \bar{\theta} + \bar{v}^i\), where \(\bar{v}^i\) is the equilibrium level of effort. Imposing the equilibrium requirement \(\bar{a}^i = a^i\), \(i = M, G\), and taking into account that in the equilibrium the mayor and governor exert the same level of
effort $a^M = a^G$, we analyze the first-order condition (B.2) term by term. First, by Leibnitz's Rule for differentiating under an integral sign

$$
\frac{\partial}{\partial a^M} F_{\Theta M, \Theta M - \Theta G} (p^M - a^M, a^G - a^M) = \frac{\partial}{\partial a^M} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (x, y) \, dy \right) \, dx =
$$

$$
\int_{-\infty}^{\infty} \frac{\partial}{\partial a^M} \left( \int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (x, y) \, dy \right) \, dx - \int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (p^M - a^M, y) \, dy =
$$

$$
\int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (x, a^G - a^M) \, dx - \int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (p^M - a^M, y) \, dy =
$$

$$
- \int_{-\infty}^{\infty} f_{\Theta M, \Theta M - \Theta G} (x, 0) \, dx - \int_{-\infty}^{0} f_{\Theta M, \Theta M - \Theta G} (\bar{y}, y) \, dy =
$$

$$
- \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} \exp \left\{ -\frac{2(x - \bar{y})^2}{2\sigma^2} \right\} \, dx - \int_{-\infty}^{0} \frac{1}{2\pi\sigma^2} \exp \left\{ -\frac{(-y)^2}{2\sigma^2} \right\} \, dy =
$$

$$
- \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \left( \frac{a^2}{2} \right)}} \exp \left\{ -\frac{(x - \bar{y})^2}{2 \left( \frac{a^2}{2} \right)} \right\} \, dx - \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{y^2}{2\sigma^2} \right\} \, dy =
$$

$$
- \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{1}{\sqrt{2}} = - \frac{1}{\sqrt{2\pi\sigma^2}} \left( \frac{1}{\sqrt{2}} + 1 \right).
$$

Then,

$$
\varphi_{\Theta M} (p^M - a^M) = \varphi_{\Theta M} (\bar{y}) = \frac{1}{\sqrt{2\pi\sigma^2}}
$$

and

$$
\varphi_{\Theta M - \Theta G} (a^G - a^M) = \varphi_{\Theta G - \Theta M} (a^M - a^G) = \varphi_{\Theta M - \Theta G} (0) = \frac{1}{\sqrt{2\pi \left( 2\sigma^2 \right)}} = \frac{1}{2\sqrt{\pi\sigma^2}}.
$$

Finally,

$$
\frac{\partial}{\partial a^M} F_{\Theta G, \Theta G - \Theta M} (p^G - a^G, a^M - a^G) = \frac{\partial}{\partial a^M} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} f_{\Theta G, \Theta G - \Theta M} (x, y) \, dy \right) \, dx =
$$

$$
\int_{-\infty}^{\infty} \frac{\partial}{\partial a^M} \left( \int_{-\infty}^{\infty} f_{\Theta G, \Theta G - \Theta M} (x, y) \, dy \right) \, dx = \int_{-\infty}^{\infty} f_{\Theta G, \Theta G - \Theta M} (x, a^M - a^G) \, dx =
$$
\[
\int_{-\infty}^{\bar{\nu}} f_{G\epsilon G-M}(x, 0) \, dx = \frac{1}{\sqrt{2\pi}\sigma^2} \int_{-\infty}^{\bar{\nu}} \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{(x-\bar{\nu})^2}{2\left(\frac{\sigma^2}{2}\right)} \right\} \, dx = \frac{1}{4\sqrt{\pi}\sigma^2}.
\]

Therefore, the first-order condition (B.2) yields

\[
Y = \begin{cases} 
\frac{((\sqrt{2}+1)\mu-\lambda)(1+c)}{4\sqrt{\pi}\sigma^2} & \text{if } X = S \\
\frac{((\sqrt{2}+1)\mu+\lambda)}{4\sqrt{\pi}\sigma^2} & \text{if } X = D,
\end{cases}
\]

where \(Y\) stands for the equilibrium effort level under the mixed performance evaluation rule.

C. Proof of Lemma 4

One-step transition probabilities read

\[
P_{SS} = P \left( \{p^M \geq \bar{p}^M\} \cap \{p^G \geq \bar{p}^G\} \right) + P \left( \{p^M < \bar{p}^M\} \cap \{p^G < \bar{p}^G\} \right) = \frac{1}{2}
\]

\[
P_{SD} = P \left( \{p^M \geq \bar{p}^M\} \cap \{p^G < \bar{p}^G\} \right) + P \left( \{p^M < \bar{p}^M\} \cap \{p^G \geq \bar{p}^G\} \right) = \frac{1}{2}
\]

\[
P_{DD} = \Lambda (\sqrt{2} - 1) \left( P \left( \{p^M \geq \bar{p}^M\} \cap \{p^G \geq \bar{p}^G\} \right) + P \left( \{p^M < \bar{p}^M\} \cap \{p^G < \bar{p}^G\} \right) \right) + 
\]

\[
(1 - \Lambda (\sqrt{2} - 1)) \left( P \left( \{p^M \geq p^G\} \cap \{p^G \geq p^M\} \right) + P \left( \{p^M < p^G\} \cap \{p^G < p^M\} \right) \right) = 
\]

\[
\frac{1}{2} \Lambda (\sqrt{2} - 1)
\]

\[
P_{DS} = \Lambda (\sqrt{2} - 1) \left( P \left( \{p^M \geq \bar{p}^M\} \cap \{p^G < \bar{p}^G\} \right) + P \left( \{p^M < \bar{p}^M\} \cap \{p^G \geq \bar{p}^G\} \right) \right) + 
\]

\[
(1 - \Lambda (\sqrt{2} - 1)) \left( P \left( \{p^M \geq p^G\} \cap \{p^G < p^M\} \right) + P \left( \{p^M < p^G\} \cap \{p^G \geq p^M\} \right) \right) = 
\]

\[
1 - \frac{1}{2} \Lambda (\sqrt{2} - 1)
\]

where \(\Lambda(\cdot)\) is a distribution function of the degree of the politician’s faithfulness to her party, \(\rho\).

References


