Market Share Discounts and Investment Incentives*

Igor Sloev†
Universidad Carlos III de Madrid,
Department of Economics
September, 2007

Abstract

The paper investigates pro- and anticompetitive effects the use of market share discounts (MSDs). While MSDs can be used for exploiting a dominant position and that may lead to a welfare reduction, MSDs also can serve as an efficient device of incentives creation. Particularly, if a final demand for an upstream manufacturer’s good depends on a promotional effort of a retailer, the manufacturer can effectively use MSDs to induce an optimal level of the retailer’s effort. Moreover, it is possible that MSDs have a positive impact both on the consumers’ surplus and the total industry profits. Thus the use of MSDs should not be treated as an anticompetitive practices a piori, but rather it has to be judged on a case-by-case basis.

*JEL classification: L22, L25, L42

Keywords: market-share discounts, vertical restraints, incentives.

---

*I am grateful to my supervisors Emmanuel Petrakis and Chrysovalantou Milliou for their helpful guidance.
†Economics Department, Universidad Carlos III de Madrid, c./Madrid 126, Getafe (Madrid), Spain; tel.:+34625418598; e-mail:isloev@eco.uc3m.es
1 Introduction

Vertical restraints, such as loyalty rebates, resale price maintenance, exclusive dealing and exclusive territories are often used in the dealing among manufacturers and retailers. In some cases vertical restraints serve anti-competitive purposes by leading, for instance, to the market exclusion of competitors or to the creation of entry barriers. In other cases these restraints are used to increase efficiency, for example, by eliminating double price marginalization, reaching an optimal level of production or by creating "right" incentive for vertically related firms. Still in all cases, vertical restraints are of considerable interest to antitrust practitioners.

In this paper I analyze a special type of vertical restraints, market share discounts (MSDs). In particular, I examine the incentives of manufacturers to apply MSDs as well as the impact of MSDs on retailer's investments in the promotion of the manufacturer's product and on welfare. MSDs are discounts that a manufacturer offers to its distributors or retailers if their sales of the manufacturer's brand comprise a sufficiently high percentage of their total sales of a given class of goods. Thus MSDs is a special type of discounts which are based on the quantities of goods that the retailer buys from both the manufacturer and its competitors.

The increasing number of cases related to such restraints confirms that manufacturers have begun to use more intensively this type of arrangements in recent years1. The case of Concord Boat Corporation versus Brunswick Corporation is one of well-known examples of the use of MSDs2. Brunswick manufactured and sold stern drive engines for recreational boats; it had a large share of the market (e.g., 75% in 1983). Beginning in the early 1980s, Brunswick (like its competitors) offered market share discounts. Boat builder customers who agreed to purchase a certain percentage of their engine requirements from Brunswick for a period of time (often a year, sometimes longer) received a discount off the list price for all engines purchased3. Some of the boat builders sued Brunswick, alleging among other claims, that these discount programs excluded competing stern drive engine

2See Concord BoatCorp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000)
3Particularly, an agreement to buy 70% of engine requirements from Brunswick might result in a 3% discount, agreement for 65% a 2% discount, and an agreement for 60% a 1% discount.
manufacturers from the market and amounted to monopolization. A trial court ruled that Brunswick’s pricing amounted to de facto exclusive dealing, and foreclosed rival suppliers of marine engines from the market. On appeal, that ruling was reversed on grounds that market conditions were not conducive to foreclosure.

An additional example is the case of Virgin Atlantic Airways Ltd. versus British Airways. British Airways (BA) used incentive programs that provided travel agencies with commissions, and corporate customers with discounts, for meeting specified thresholds for sales of BA tickets (sometimes expressed in market share terms). Virgin Atlantic claimed that the result was below cost pricing on certain transatlantic routes where Virgin and BA competed, with BA’s attendant losses subsidized by monopoly pricing on other BA routes. Virgin alleged that the below cost pricing slowed its expansion on the competitive routes. Both a district court and a court of appeals concluded that Virgin had failed to show below cost pricing.

In this paper I consider a vertically related two-level industry. At an upstream level a manufacturer and a competitive sector’s firms produce imperfect substitutes. At a downstream level there is only one retailer which trades both goods to final consumers. I consider two types of contract that manufacturer may offer to the retailer: a wholesale price contract and a market share discount contract.

It is supposed that the retailer can make a costly effort investment which results in an increase in the demand for the manufacturer’s good. By assumption this effort has no effects on the demand for the competitive sector’s firms good. The effort level is non-contractible hence either the wholesale price or MSDs may not be contingent on the retailer’s effort level. However the manufacturer may use either the wholesale price or MSDs in order to motivate the retailer accept the desired level of the effort. That allows analyzing a role of MSDs as a tool for the incentives creation as well as welfare effects of MSDs.

To highlight this role I begin with a consideration of a benchmark case - a particular case of the model when the retailer’s effort has no impact on the demand. For both the benchmark and the general case of the model and for both types of contract market outcomes and welfare effects are analyzed.

For the benchmark case the following result are shown. Comparing to the wholesale price settings, if MSDs applied, both the quantities of manufacturer’s

\footnote{Virgin Atlantic Airways Ltd. v. British Airways F.3d 256 (2d Cir. 2001).}
good and the manufacturer’s profit increase while the quantity of the competitive sector’s firm good decreases. The total profit of the industry decreases as well as consumers surplus decreases. The only agent that gains from MSDs is the manufacturer. That allows me conclude about an anticompetitive character of MSDs in these settings.

The main results obtained for the general case are the following. First, if the wholesale price contract applied the manufacturer may be not able to motivate the retailer to undertake the desired level of the effort. In this case the market outcome is the same as in the benchmark case of the wholesale price use. If MSDs applied then the manufacturer can design it such that the retailer makes the desired level of the effort and this level is the efficient one from the social point of view. Moreover both the industry profit and the consumers’ surplus are higher in the case of MSDs comparing with the wholesale price. Another important result is that while MSDs increase market share of the manufacturer it does not drive competing firms out of the market completely. Thus in terms of social welfare the outcome when MSDs applied dominates one when the wholesale price applied.

Finally, combining the results obtained for the benchmark and the general case, I conclude that judgment on either MSDs has the anti- or procompetitive effect crucially depends on features of market environment. While MSDs may serve for a profit redistribution between the manufacturer and the retailer and may lead to a decrease in social welfare it may also serve as an efficient instrument for an investment incentives creation and may result in an increase in a social welfare. Thus MSDs should not be treated as the anticompetitive practices a piori, but rather the treatment of MSDs should be based on the principle of a case-by-case basis.

Although there is growing number of paper examining different aspects of MSDs, pro- and anti-competitive effects of the market share discounts have not got enough attention in the economic literature yet.

The rent-shifting effects of MSDs is analyzed by Marx and Shaffer [2004] and Greenlee and Reitman [2004]. Marx and Shaffer [2004] examine a use of market share discounts, slotting allowances and predatory pricing in a three-party sequential contracting environment. In their model two sellers negotiate sequentially with one buyer. Market share discounts and slotting allowances are used to shift
rents between the contracting parties, with no short run consequences for social welfare. One result is that these rent shifting equilibrium generally result in both sellers remaining in the market. In the long run, they suggest that preventing the use of such devices will result in the adoption of strategies that are more likely to result in one of the sellers being excluded. However, the model does not explicitly analyze the welfare effect of such long term effects.

Greenlee and Reitman [2004] analyze the case when two firms compete selling their goods to the final consumers using loyalty rebates or wholesale price contract. They found that only one firm proposes market share discount in equilibrium and a welfare effects are the ambiguous. The welfare analysis in my paper shows also that the welfare effects crucially depend on a model specification.

Majumdar and Shaffer [2007] analyze a case when one manufacturer and a competitive fringe supply goods to a retailer who has private information about the state of demand. They examine conditions under which market-share contracts are profitable, and show that the full-information outcome can be obtained. They show as well that market-share discounts contracts are more profitable than all-units discounts contract.

Chioveanu and Akgun [2006] compare a manufacturer’s incentives to apply market share discounts, all-unit discounts and incremental-unit discounts. They show that under complete information all discounts are equivalent from both manufacturer’s and social viewpoints. Under uncertainty, risk attitude of the retailer can play a crucial role in the form of loyalty discount applied by the manufacturer.

Greenlee et al [2004] analyze a use of bundled market share discounts by multi-product monopolist. They show that it may exclude an equally efficient competitor that produces single-product, and welfare effect is ambiguous.

Ordover and Shaffer [2007] show that when market share discounts implemented by a dominant firm, who may have easier access to financing then a rival, it can sometimes exclude equally-efficient rival and lower overall welfare.

The theoretical literature on discounts has not generally considered efficiency based reasons using market share discounts. One exception is Mills’s [2004] paper. Mills has examined competitive effects of a vertically differentiated product manufacturer implementing market share discounts in sales to its distributors. His

---

5The term "loyalty rebates" usually used as a synonym for market share discounts.
central idea is that market share discounts are not mainly an exclusionary device, but rather a device for inducing merchandising services that help consumers make well-informed decisions and augment market performance. In order to show this Mills investigates a case when an upstream firm negotiates a separate contract with every retailer to determine the size and the division of the firms’ joint profits. In this case every downstream firm has an incentive to make an effort to promote the upstream firm’s good whenever it is optimal from a social welfare point of view. Then the author shows that the same result can be implemented by the upstream firm when it uses MSDs. To get this result Mills (implicitly) assumes that a level of promotion effort of the downstream firm is contractible. This assumption is crucial for getting Mill’s result. In contrast I assume that the effort level is not contractible that allows to reveal a role of MSDs as an efficient mechanism for incentives creation.

The rest of paper proceeds as follows. Section 2 describes the model. Section 3 considers the benchmark case. Section 4 analyses the general case. Section 5 provides the welfare analysis and a numerical example and section 6 concludes.

2 The Model

There is one retailer, $R$, which which sells two substitute goods to final consumers. The first good is produced by a brand-name upstream manufacturer, $M$. It is supposed that the brand manufacturer produces with a constant marginal cost, $c \geq 0$. The second good is produced by a competitive industry. The marginal cost of producing of the second good is zero.

It is supposed that the retailer can make a costly effort investment which will increase the demand for the manufacturer’s good. For example, it can be a case that consumers are not perfectly informed about the manufacturer’s good quality and the retailer can provide consumers with information. The level of the effort is discrete, $e = \{0, 1\}$. It is supposed that the effort can not be made by the brand manufacturer and, moreover, the effort level is not contractible. A cost of effort is denoted by $E > 0$. 
A representative consumer has utility function of the form:

\[ U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2), \]

where \( q_1, q_2 \) are quantities purchased by the consumer, \( b \in (0, 1) \) is the degree of goods differentiation and the parameter \( A \) depends on the retailer’s effort level: \( A(1) = A_1 \geq A(0) = 1 \). It is supposed that \( 1 - b - c > 0 \). The consumer surplus is:

\[ CS(q_1, q_2, p_1, p_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - q_1p_1 - q_2p_2. \]

The utility function (1) generates the demand for the manufacturer’s good, \( p_1 = A(e) - q_1 - bq_2 \), which depends on the retailer’s effort level, and the demand for the competitive sector’s firms good \( p_2 = 1 - q_2 - bq_1 \), which does not depend on the effort level.

I consider two types of contract that the manufacturer uses in dealing with the retailer: a wholesale price contract, which specifies a constant per-unit price, \( \omega \), and market-share discounts. In the latter case the manufacturer’s contract specifies parameters \( \{t_L, t_H, \bar{s}\} \) that form a menu of prices:

\[ t_{MSD} = \begin{cases} 
  t_L & \text{if } s \geq \bar{s} \\
  t_H & \text{if } s < \bar{s} 
\end{cases}, \]

where \( s = q_1/(q_1 + q_2) \) denote the share of the manufacturer’s good in a total sales of the retailer, \( \bar{s} \) is the market share threshold that the retailer must meet in order to buy at the reduced price \( t_L \), and the price \( t_H: t_H > t_L \) is the manufacturer’s price for the case if the retailer does not meet the market share requirement.

Let’s \( t \) denote either the single single price \( \omega \) or the menu of prices \( t_{MSD} \) depending on the type of contract applied.

All producers compete in prices and as a result competitive sector’s firms set prices equal to the marginal cost and obtain zero profit because of competition \( a la \) Bertrand among them.

The retailer’s profit is:

\[ \pi^R = (A(e) - q_1 - bq_2 - t)q_1 + (1 - q_2 - bq_1)q_2 - eE. \]
The profit of the brand manufacturer is:

\[ \pi^M = q_1(t - c) \]

where \( t \) is either a wholesale price or a menu of prices.

The timing in the model is the following.

At the first stage the manufacturer and competitive sector’s firms simultaneously set their prices. The manufacturer sets the menu of prices, \( t_{MSD} \) of the form (2) or the wholesale price \( \omega \).

At the second stage the retailer takes a decision on the effort level \( e = \{0, 1\} \) and levels of quantities \( q_1 \) and \( q_2 \).

The model analysis is presented in the following way. I begin with investigation of a special case of the model when \( A_1 = A_0 = 1 \). This case is considered as a benchmark for a comparison with a general case \( A_1 > A_0 = 1 \). The condition \( A_1 = A_0 \) implies that the retailer’s effort has no effects on the consumer’s demand and the manufacturer has no reason to motivate the retailer to undertake the costly effort. For both the wholesale price and MSDs contracts I analyze market outcomes and examine an effect of the contract type on a welfare. That allows to reveal if MSDs have the procompetitive or the anticompetitive effect in the benchmark settings.

Then I examine the role of the contract type (wholesale price vs. MSDs) in the case when the consumer’s demand depends on the retailer’s effort level, \( A_1 > A_0 = 1 \). Again I consider market outcomes for both types of contract.

Profit functions are subscribed by indexes MSDs and WP for cases when MSDs and the wholesale price is applied respectively.

3 Benchmark case: No effort investment

3.1 Wholesale price contract

First let’s consider the retailer’s problem:

\[
\max_{q_1, q_2} \pi^R_{WP}(q_1, q_2; \omega) = (1 - q_1 - bq_2 - \omega)q_1 + (1 - q_2 - bq_1)q_2.
\]
The first order conditions are:

\[
\begin{align*}
1 - \omega - 2q_1 - 2bq_2 &= 0 \\
1 - 2bq_1 - 2q_2 &= 0
\end{align*}
\]

and the solution of the system is: \(q_1(\omega) = \frac{1-b-\omega}{2(1-b^2)}, q_2(\omega) = \frac{1-b+b\omega}{2(1-b^2)}\).

The profit of the retailer as a function of the price \(\omega\) is:

\[
\pi^R_{WP}(\omega) = \frac{2 - 2b(1 - \omega) - 2\omega + \omega^2}{4(1 - b^2)}.
\]

Now the problem of the manufacturer can be written as:

\[
\max_{\omega} \pi^R_{WP}(\omega) = q_1(\omega)(\omega - c) = \frac{1 - b - \omega}{2(1 - b^2)}(\omega - c)
\]

and it has the solution: \(\omega = \frac{1}{2}(1 - b + c)\).

The market outcome is characterized by quantities produced by the manufacture and competitive sector’s firms, \(\left\{ q_1^{WP} = \frac{1-b-c}{4(1-b^2)}, q_2^{WP} = \frac{2-b^2+bc}{4(1-b^2)} \right\}\), the price of the manufacturer, \(\omega = \frac{1}{2}(1 - b + c)\), the final markets prices \(\left\{ P_1^{WP} = \frac{1}{4}(3 - b + c), P_2^{WP} = \frac{1}{2} \right\}\) and the profits of the retailer and the manufacturer, \(\left\{ \pi^R_{WP} = \frac{5+3b^2-2b(1-c)-(2-c)c}{16(1-b^2)}, \pi^M_{WP} = \frac{(1-b-c)^2}{8(1-b^2)} \right\}\).

### 3.2 MSDs contract

The retailer’s profit maximization problem is:

\[
\max_{q_1,q_2} \pi^R_{MSD}(q_1, q_2; t_{MSD}) = (1 - q_1 - bq_2 - t_{MSD})q_1 + (1 - q_2 - bq_1)q_2
\]

s.t. \(t_{MSD} = \begin{cases} t_L & \text{if } s \geq \bar{s} \\ t_H & \text{if } s < \bar{s} \end{cases}\)

Note that the retailer always has an option to trade the competitive sector’s firm good only. In this case the retailer’s profit is a solution of the problem:

\[
\max_{q_2} \pi^R = (1 - q_2)q_2
\]
and it is equal to 1/4. This value plays a role of the retailer’s "reservation profit" in a sense that the retailer for sure gets at least this in equilibrium.

**Lemma 1** The equilibrium values of \( \{t^L, t^H, \bar{s}\} \) are such that the retailer meets the market share threshold, \( s \geq \bar{s} \).

**Proof.** See the Appendix.

The Lemma 1 says that in an equilibrium the manufacturer’s price \( t_{MSD} \) is such that the retailer always meet the market share threshold and buys at the price \( t_L \). The intuition is the following. If the retailer does not meet the threshold, that is \( s < \bar{s} \), and it buys at the price \( t_H \) then the outcome does not change if the manufacturer sets prices \( \{t'_L, t'_H, \bar{s}'\} \) such that \( t'_L = t_H, \bar{s}' = s \) and \( t'_H \) is the prohibitively high. Now, if the manufacturer increases slightly the market share threshold \( \bar{s}' > s \) then the retailer buys more for the same price and the manufacturer’s profit is higher. Thus \( s < \bar{s} \) can not hold in equilibrium.

Hence the exact value of the manufacturer’s price \( t_H \) does not play a role provided it is high enough. Without loss of generality we can put \( t_H = +\infty \).

**Corollary 1** In equilibrium it must be that \( s = \bar{s} \).

**Proof.** See the Appendix.

Note that if in the equilibrium it could be that \( s \neq \bar{s} \) it would implies that the market outcome is the same as in the case of wholesale price and manufacturer has no possibility to increase its profit by setting an appropriate levels of \( \bar{s} \) and \( t_L \). Which is contra-intuitive.

As a result of the Corollary 1, the profit of the retailer can be written as:

\[
\max_{q_1, q_2} \pi^R_{MSD}(q_1, q_2; t_L) = (1 - q_1 - bq_2 - t_L)q_1 + (1 - q_2 - bq_1)q_2 \\
\text{s.t. } \frac{q_1}{q_1 + q_2} = \bar{s}
\]

The first order condition gives the solution:

\[
q_1(t_L, \bar{s}) = \frac{\bar{s}(1 - \bar{s}t_L)}{2(1 - \bar{s}(1 - b)(1 - \bar{s}))}.
\]
Thus the retailer’s profit as a function of \( t_L \) and \( \bar{s} \) is

\[
\pi_{MSD}^R(t_L, \bar{s}) = \frac{(1 - \bar{s}t_L)^2}{4(1 - 2\bar{s})(1 - b)(1 - \bar{s})}.
\]

The manufacturer’s profit maximization problem now can be written as:

\[
\max_{t_L, \bar{s}} \pi_{MSD}^M = q_1(t_L, \bar{s})(t_L - c)
\]

s.t. \( q_1(t_L, \bar{s}) = \begin{cases} \frac{\pi(t_L)}{2(1-2\bar{s}(1-b)(1-\bar{s}))} & \text{if } \pi_{MSD}^R(t_L, \bar{s}) \geq \frac{1}{4} \\ 0 & \text{if } \pi_{MSD}^R(t_L, \bar{s}) < \frac{1}{4} \end{cases} \]

**Lemma 2** In equilibrium the manufacturer extracts all retailer’s profit above the reservation profit level.

**Proof.** See the Appendix. \( \blacksquare \)

The last proposition means that in the equilibria the equality \( \pi_{MSD}^R(t_L, \bar{s}) = \frac{1}{4} \) holds. This gives a correspondence between a price \( t_L \) and market share threshold \( \bar{s} \) which must hold in the equilibria:

\[
\bar{s}(t_L) = \frac{2(1 - b - t_L)}{2(1 - b) - t_L^2}.
\]

Now the profit maximization problem of the manufacturer becomes:

\[
\max_{t_L} \pi_{MSD}^M(t_L) = q_1(\bar{s}(t_L), t_L)(t_L - c) = \frac{1 - b - t_L}{2(1 - t_L)(1 - b) + t_L^2}(t_L - c)
\]

and it has the unique solution:

\[
t_L^* = \frac{(2 - c)(1 - b) - D_1}{1 - b - c},
\]

where \( D_1 = const = \sqrt{(1 - b^2)(2(1 - c)(1 - b) + c^2)} \).

Plugging (4) into (3), we obtain the equilibrium values of \( \bar{s}^* \) which together with \( t_L^* \) determines other equilibrium values.

The market outcome is characterized by quantities produced by the manufacturer and competitive sector’s firms, \( \{ q_1^* = \frac{1-b-c}{2D_1}, q_2^* = \frac{1-b+c}{2D_1} \} \), the market share threshold, \( \bar{s}^* = \frac{1-b-c}{(1-b)(2-c)} \), the manufacturer’s price, \( t_L^* = \frac{(2-c)(1-b)-D_1}{1-b-c} \), the final
market prices $\{p_1^* = \frac{2D_1-(1-b^2)(1-c)}{2D_1}, p_2^* = \frac{2D_1+b^2-1}{2D_1}\}$ and the profits of the retailer and the manufacturer, $\{\pi_{MSD}^R = 1/4, \pi_{MSD}^M = \frac{D_1+b^2-1}{2(1-b^2)}\}$.

Let’s note that $b(1-c) < 1$ implies $s^* = \frac{1-b-c}{(1-b)(2-c)} < 1$. Hence we can formulate the following proposition.

**Proposition 1** *Although the share of the manufacturer is higher in the case of use of MSDs, the manufacturer never sets the market threshold equals to 1.*

Thus the competitive sector never moved from the market completely.

### 3.3 MSDs vs. wholesale price contract

In what follows I compare the outcomes in the case of MSDs with those in the case of the wholesale price contract.

**Proposition 2** *Comparing with the wholesale price contract the use of MSDs leads to*

1) an increase in the manufacturer’s market share $s$,
2) the retailer buys at higher price, that is $t_L > \omega$,
3) an increase in the manufacturer’s output, $q_1$,
4) an increase in the manufacturer’s profit,
5) a decrease in the final market price for the manufacturer’s good $p_1$,
6) an increase in the final market price for the good $p_2$,
7) a decrease in the competitive sector’s firms output $q_2$,
8) a decrease in the retailer’s profit,
9) a decrease in the consumer surplus.

**Proof.** See the Appendix.

Thus the manufacturer, which has some degree of market power, uses MSDs to increase its both output and price and to extract all the retailer’s profit above its reservation level. In the case of MSDs all agents but the manufacturer lose. Hence, in the presented environment, MSDs can be treated as an anticompetitive tool.
4 Effort Investment

4.1 Wholesale price contract

The profit maximization problem for the retailer is:

$$\max_{q_1, q_2, e} \pi_{WP}^R (\omega) = (A(\omega) - q_1 - bq_2 - \omega)q_1 + (1 - q_2 - bq_1)q_2 - eE$$

where $e \in \{0, 1\}$, $A(0) = 1$, $A(1) = A_1$.

The first order conditions in respect to $q_2$ and $q_1$ are:

$$\begin{cases} A(e) - 2q_1 - 2bq_2 - \omega = 0 \\ 1 - 2bq_1 - 2q_2 = 0 \end{cases}$$

It gives a solution: $q_1(\omega, e) = \frac{A(e) - b - \omega}{2(1 - b^2)}$, $q_2(\omega, e) = \frac{1 - A(e)b + b\omega}{2(1 - b^2)}$.

The retailer’s profit as a function of the effort level $e$ and the price $\omega$ is:

$$\pi_{WE}^R (\omega, e) = \left( \frac{A(e) - b - \omega}{4(1 - b^2)} \right) \left( \frac{1 - A(e)b + b\omega}{4(1 - b^2)} \right) - eE.$$  

Retailer’s profit functions $\pi_{WP}^R (\omega, e)|_{e=0}$ and $\pi_{WP}^R (\omega, e)|_{e=1}$ are decreasing in $\omega$ functions with the following relation of slopes:

$$\left. \frac{\partial \pi_{WP}^R (\omega, e)}{\partial \omega} \right|_{e=1} = \frac{-2A_1 + 2b + 2t}{4(1 - b^2)} < \frac{-2b + 2t}{4(1 - b^2)} = \left. \frac{\partial \pi_{WP}^R (\omega, e)}{\partial \omega} \right|_{e=0}. \quad (5)$$

Thus if two profits functions have an intersection that intersection is unique.

Suppose the functions intersect and let’s $\widehat{\omega}$ denote the intersection point. Then the following conditions holds:

$$\begin{cases} \pi_{WP}^R (\omega, e)|_{e=1} > \pi_{WP}^R (\omega, e)|_{e=0} & \text{if } \omega < \widehat{\omega} \\ \pi_{WP}^R (\omega, e)|_{e=1} < \pi_{WP}^R (\omega, e)|_{e=0} & \text{if } \omega > \widehat{\omega} \end{cases}$$

which means that if $\omega < \widehat{\omega}$ then the retailer’s profit is higher if it makes the effort and if $\omega > \widehat{\omega}$ then the retailer’s profit is higher if its level of the effort is $e = 0$. Together with (5) it implies that the lower is the price $\omega$, the higher is the retailer’s gain from the effort investment, $\pi_{WP}^R (\omega, 1) - \pi_{WP}^R (\omega, 0)$.

Now let’s restrict parameters of the model to rule out trivial cases. In order to insure that $\widehat{\omega} > 0$ and hence the effort $e = 1$ may be implemented I make the
following assumption.

**Assumption 1.** Given the manufacturer’s price $\omega = 0$ the retailer’s profit higher if it makes the effort investment, that is

$$\pi_{WE}(\omega, e)\big|_{\omega=0,e=1} > \pi_{WE}(\omega, e)\big|_{\omega=0,e=0}. \quad (6)$$

In other words (6) implies that, if the manufacturer’s price is low enough, the retailers will make the effort investment. It can be rewritten as: $\frac{(A_1-1)(A_1+1+2b)}{4(1-b^2)} > E$. In the fact, the assumption 1 rule out cases when the cost of effort is "too high" ($E \rightarrow +\infty$) or the result of the effort investment is "too small" ($A_1 \approx 1$). The latter case was considered as the benchmark case. If assumption 1 does not hold there is no possibility to implement the effort level $e = 1$.

The price $\widehat{\omega}$ is determined by the equation:

$$\pi_{WP}(\widehat{\omega}, e)\big|_{e=1} = \pi_{WP}(\widehat{\omega}, e)\big|_{e=0},$$

which has a solution:

$$\widehat{\omega} = \frac{1}{2} \left( A_1 + 1 - 2b \right) - \frac{2E(1-b^2)}{A_1 - 1}.$$

Now, the profit maximization problem of the manufacturer may be written in the form:

$$\max_{\omega} \pi_{WP} = q_1(\omega, e)(\omega - c) = \begin{cases} \frac{A_1-b-\omega}{2(1-b^2)}(\omega - c) & \text{if } \omega \leq \widehat{\omega} \\ \frac{1-b-\omega}{2(1-b^2)}(\omega - c) & \text{if } \omega > \widehat{\omega}. \end{cases}$$

The function $\pi_{WP}(\omega, e)\big|_{e=1} = \frac{A_1-b-\omega}{2(1-b^2)}(\omega - c)$ gets its maximum at the point $\omega = \frac{1}{2}(A_1 - b + c)$ thus in an equilibria the manufacturer’s price such that $\omega \leq \frac{1}{2}(A_1 - b + c)$. The manufacturer’s profit function is kinked in the point $\widehat{\omega}$ and it increases in both intervals $\omega \in [0, \widehat{\omega}]$ and $\omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)]$.

**Assumption 2.** The parameters $A_1, E, b$ are such that: $\widehat{\omega} < \frac{1}{2}(1 - b + c)$.

The inequality in the assumption 2 may be rewritten as $\frac{(A_1-1)(A_1-1-b-c)}{4(1-b^2)} < E$ and it implies that the effort cost is not "too small" or the effect of the effort investment is not "too high". It is out of the interest because in these the retailer
makes the effort investment regardless the type of manufacturer’s contract.

Let’s note that the point \( \omega = \frac{1}{2}(1 - b + c) \) is the point of the maximum of the \( \pi_{WP}^m(\omega, e) \) as \( \pi_{WP}^m(\omega, e) \) for any \( \omega \leq \frac{1}{2}(A_1 - b + c) \), the assumption 2 implies that

\[
\max_{\omega \in [0, \widehat{\omega}]} \pi_{WP}^m = \pi_{WP}^m|_{\omega = \widehat{\omega}} = \frac{A_1 - b - \widehat{\omega}}{2(1 - b^2)}(\widehat{\omega} - c)
\]

(7)

Hence if the manufacturers wants to implement the level of the effort \( e = 1 \) it sets the price \( \omega = \widehat{\omega} \).

Now let’s consider the manufacturer’s profit at the interval \( \omega \in (\widehat{\omega}, \frac{1}{2}(A_1 - b + c)] \). While \( \omega > \widehat{\omega} \) the retailer does not undertake the effort investment and the manufacturer’s profit \( \pi_{WP}^m = \frac{1-b-c}{2(1-b^2)}(\omega - c) \) reaches the maximum at the point

\[
\max_{\omega > \widehat{\omega}} \pi_{WP}^m(\omega, e)|_{e=0} = \frac{(1 - b - c)^2}{8(1 - b^2)}.
\]

(8)

Combining (7) and (8) we conclude that:

\[
\max_{\omega} \pi_{WP}^m = \max \left\{ \frac{(1 - b - c)^2}{8(1 - b^2)}, \frac{A_1 - b - \widehat{\omega}}{2(1 - b^2)}(\widehat{\omega} - c) \right\}
\]

with

\[
\omega^* = \arg \max_{\omega} \pi_{WP}^m = \begin{cases} 
\widehat{\omega} & \text{if } \frac{A_1 - b - \widehat{\omega}}{2(1 - b^2)}(\widehat{\omega} - c) \geq \frac{(1-b-c)^2}{8(1-b^2)} \\
\frac{1}{2}(1 - b - c) & \text{otherwise}
\end{cases}
\]

In order to concentrate on the role of MSDs as a tool of investment incentives creation I accept one more assumption.

**Assumption 3** \( \frac{A_1 - b - \widehat{\omega}}{2(1 - b^2)}(\widehat{\omega} - c) < \frac{(1-b-c)^2}{8(1-b^2)} \).

The assumption 3 may be written in the form:

\[
E > \frac{(A_1 - 1)(1 - b - c) + \sqrt{(A_1 - 1)(A + 1 - 2b - 2c)}}{4(1 - b^2)}
\]

and it implies that neither the effect or the effort should be "too high" or the cost
of effort "too small" as well as the rate of goods substitution should be close to 1.

While the assumption 1 implies the possibility of implementation of the effort level $e = 1$ and the assumptions 2 implies that the effort $e = 1$ is not implemented with necessity in the equilibrium, the assumption 3 restricts the model parameters such that if the wholesale price contract applied the equilibrium level of the effort is $e = 0$.

Thus given assumptions 1-3 if the wholesale price contract applied the manufacturer's profit maximization implies the zero-level of the retailer's effort. This immediately results in that the equilibrium outcome coincides with the benchmark wholesale price outcome.

4.2 MSDs contract

The profit maximization problem of the retailer is:

$$\max_{q_1, q_2, e} \pi^{R}_{MSD} = (A(e) - q_1 - b q_2 - t_{MSD}) q_1 + (1 - q_2 - b q_1) q_2 - e E$$

(9)

where $t_{MSD} = \begin{cases} t_L & \text{if } s \geq \bar{s}, A(0) = 1; A(1) = A_1, e \in \{0, 1\}. \\ t_H & \text{if } s < \bar{s} \end{cases}$

Lemma 3 In the equilibrium the condition $s = \bar{s}$ holds and the manufacturer's price $t_H$ is prohibitively high.

Proof. See the Appendix.

Hence in the equilibrium the retailer chooses $q_1, q_2$ such that $q_1/(q_1 + q_2) = \bar{s}$ and buys at the price $t_L$. Plugging $q_2 = q_1 \frac{1 - \bar{s}}{\bar{s}}$ into (9) and solving the first order conditions we get the optimal level of $q_1$:

$$q_1(\bar{s}, t_L, e) = \frac{\bar{s}(1 + \bar{s}(A(e) - 1 - t_L))}{2(1 - 2\bar{s}(1 - b)(1 - \bar{s}))}. $$

The profit of the retailer as the function of the decision variable $e$ given $\{t_L, \bar{s}\}$ is:

$$\pi^{R}_{MSD}(e; t_L, \bar{s}) = \frac{(1 - \bar{s} + \bar{s}(A(e) - t_L)^2}{4(1 - 2\bar{s}(1 - b)(1 - \bar{s}))} - e E. $$

16
Given \( \{t_L, \bar{s}\} \) the retailer makes the effort if and only if

\[
\begin{align*}
\pi^R_{MSD}(e; t_L, \bar{s})|_{e=1} & \geq \pi^R_{MSD}(e; t_L, \bar{s})|_{e=0} \\
\pi^R_{MSD}(e; t_L, \bar{s})|_{e=1} & \geq \frac{1}{4}
\end{align*}
\] (10)

The first inequality in (10) is an incentives constraint and it implies that for the retailer it is profitable to make the effort \( e = 1 \). The second inequality in (10) is a participation constraint and it implies that the profit of the retailer is greater or equal to its reservation profit.

If the values of \( \{t_L, \bar{s}\} \) are such that

\[
\begin{align*}
\pi^R_{MSD}(e; t_L, \bar{s})|_{e=1} & \geq \pi^R_{MSD}(e; t_L, \bar{s})|_{e=0} \\
\pi^R_{MSD}(e; t_L, \bar{s})|_{e=0} & \geq \frac{1}{4}
\end{align*}
\] (11)

then the retailer chooses the effort \( e = 0 \).

Now the manufacturer’s profit maximization problem is:

\[
\begin{align*}
\max_{t_L, \bar{s}} \pi^M_{MSD} = \quad q_1(t_L, \bar{s})(t_L - c), \quad \text{(12)}
\end{align*}
\]

s.t. \( q_1(t_L, \bar{s}) = \begin{cases} \pi(1 + \pi(A(1-t_L)) - \frac{1}{2}(1-\bar{s})(1-b)(1-\bar{s})) \text{ if the condition (10) holds} \\ \pi(1-\pi-t_L) \text{ if the condition (11) holds} \\ 0 \text{ otherwise} \end{cases} \)

Let’s first consider the manufacturer’s profit in the case if the price \( t_{MSD} \) such that (10) holds which implies the retailer makes the effort \( e = 1 \).

**Lemma 4** In the equilibria the manufacturer extracts all the retailer’s profit above the reservation level.

**Proof.** See the Appendix. □

Thus the condition \( \pi^R_{MSD}(e; t_L, \bar{s})|_{e=1} \geq \frac{1}{2} \) binds and this determine the equilibrium correspondence on \( t_L \) and \( \bar{s} \) of the form:

\[
t_L(\bar{s}) = A_1 - 1 + \frac{1 - \sqrt{1 + 4E\sqrt{1 - 2\bar{s}(1-b)(1-\bar{s})}}}{\bar{s}}
\] (13)
Plugging (13) into the manufacturer’s profit function (12) and solving the first order conditions we get the optimal value of $s$:

$$s^* = \frac{A_1 - b - c}{(1-b)(1+A_1-c)}.$$

Now, the optimal value of $t_L$ is:

$$t_L^* = A_1 - 1 + \frac{(1-b)(A_1 + 1 - c) - D_2}{A_1 - b - c},$$

where $D_2 = const = \sqrt{(1-b^2)(1+(A_1-c)(A_1-c-2b))\sqrt{(1+4E)}}$.

The profit of the manufacturer is

$$\pi_{MSD}^R(t_L^*, s^*, e)|_{e=1} = \frac{(1 + 4E)(1 + (A - c)(A - c - 2b) - D_2)}{2D_2}.$$

Thus by setting $\{t_L^* = A_1 - 1 + \frac{(1-b)(A_1 + 1 - c) - D_2}{A_1 - b - c}, \ t_H = \infty, \ s^* = \frac{A_1 - b - c}{(1-b)(1+A_1-c)}\}$ the manufacturer motivates the retailer to make the effort investment $e = 1$.

The profits $\{\pi_{MSD}^M|_{e=1} = \frac{(1+4E)(1+(A-c)(A-c-2b)-D_2)}{2D_2}, \ \pi_{ME}^R|_{e=1} = 1/4\}$, the outputs $\{q_1 = \frac{(A_1-b-c)(1+4E)}{2D_2}, \ q_2 = \frac{(1-Ab+bc)(1+4E)}{2D_2}\}$ and prices $\{p_1 = A_1 - \frac{(4E+1)(A_1-c)(1-b^2)}{2D_2}, \ p_2 = 1 - \frac{(4E+1)(1-b^2)}{2D_2}\}$ are realized in this case.

Now let’s consider the profit of the manufacturer in the case if it motivates the retailer to choose the zero-level of the effort. Given the price $t_L$ and the threshold $\overline{s}$ the retailer’s profit is $\pi_{MSD}^R(e; t_L, \overline{s})|_{e=0} = \frac{(1-\overline{s})^2}{4(1-2\overline{s}(1-b)(1-\overline{s}))}$. The manufacturer’s maximization problem is:

$$\max_{t_L, \overline{s}} \pi_{MSD}^M = \frac{\overline{s}(1 - \overline{s}t_L)}{2(1 - 2\overline{s}(1-b)(1-\overline{s}))}(t_L - c),$$

s.t. (11) holds

The participation constraint $\pi_{MSD}^R(e; t_L, \overline{s})|_{e=0} = \frac{1}{4}$ gives the correspondence $t_L(\overline{s})$ that guarantees the reservation level of the profit to the retailer:

$$t_L(\overline{s}) = \frac{1 - \sqrt{1 - 2\overline{s}(1-b)(1-\overline{s})}}{\overline{s}}.$$  (14)
Plugging (14) into (12) and solving the first order conditions we get the optimal value of $\bar{s} = \frac{1-b-c}{(1-b)(2-c)}$.

The optimal value for $t_L$ is:

$$t_L = \frac{(1-b)(2-c-D_3)}{1-b-c},$$

where $D_3 = const = \sqrt{(1-b^2)(1+1-c)(1-c-2b)}$.

The profit of the manufacturer in this case is

$$\pi^M_{MSD}(t_L, \bar{s}, e)|_{e=0} = \frac{(1 + (1-b)(1-c-2b)-D_3)}{2D_3}.$$

Thus if

$$\pi^M_{MSD}(t^*_L(e), \bar{s}^*(e), e)|_{e=1} \geq \pi^M_{MSD}(t^*_L(e), \bar{s}^*(e), e)|_{e=0}$$

the manufacturer sets the price $t^*_L = A_1 - 1 + \frac{(1-b)(A_1+1-c)-D_2}{A_1-b-c}$, market share threshold $\bar{s}^* = \frac{A_1-b-c}{(1-b)(1+A_1-c)}$ and the equilibrium retailer's level of the effort is $e = 1$, otherwise the manufacturer sets the price $t^*_L = \frac{(1-b)(2-c-D_3)}{1-b-c}$, the market share threshold $\bar{s}^* = \frac{1-b-c}{(1-b)(2-c)}$ and the equilibrium retailer's level of the effort is $e = 0$. For the purpose of the paper I am interested in the former case. Let's $\Omega$ denote the set of parameters $(A_1, b, c, E)$ for which the conditions (15) holds as well as assumptions 1-3 do. The following technical lemma states that the set $\Omega$ is the non-degenerated set.

**Lemma 5** There is a compact set of the parameters of the model $(A_1, b, c, E) \in \Omega$ such that the inequality (15) and assumptions 1, 2, 3 are compatible.

**Proof.** The numerical example in the part 5.1 proves that $\Omega$ contains at least one point. Moreover because all functions used in (15) and assumptions 1-3 are continuous the required conditions hold in some neighborhood of the provided point. $lacksquare$

Thus if the wholesale price contract applied the manufacturer may be not able to motivate the retailer to undertake the desired level of the effort, while market share discounts allows to the manufacturer to design the menu of prices such that the retailer makes the desired level of the effort.
Hence we can conclude that MSDs can be used by the manufacturer as an
efficient device for the investment incentives creation. Certainly, the manufacturer
gains from MSDs use. To analyze MSDs impacts from the social point of view I
conduct a welfare analysis.

5 Welfare analysis

The representative consumer’s surplus is:

\[
U(q_1, q_2) = A(e)q_1 + q_2 - \frac{1}{2}(q_1^2 + 2bq_1q_2 + q_2^2) - q_1p_1 - q_2p_2.
\]

Proposition 3 For the set of the parameters $\Omega$ the following statements are cor-
rect together:

1. When MSDs applied the manufacturer may design the menu of prices such
that the retailer’s level of the effort is $e = 1$ When the wholesale price applied the
level of the effort $e = 0$ is implemented in the equilibria.
2. Comparing with the wholesale price case the total profit of the industry is
higher when MSDs applied.
3. Comparing with the wholesale price case the total output is higher when
MSDs applied.
4. The consumer surplus as well as the total welfare is higher when MSDs
applied.

Proof. The proposition immediately follows the numerical example in the part
5.1 and the Lemma 5

The intuition here is the following. The retailer is motivated to make the costly
effort only if the quantity of the manufacturer’s good that it resells is high enough.
That means that the manufacturer’s wholesale price should be small enough. Thus
the manufacturer faces a trade-off: either to set a lower wholesale price to shift
the demand upward or to set a higher price and to remain with the same demand
curve. The gain of the manufacturer from an increase in the demand can be smaller
then its losses from the price reduction. Thus the wholesale price contract may be
not enough to implement the desired level of retailer’s effort. If MSDs applied than
the manufacturer may use the market threshold to enforce the retailer to buy more manufacturer’s good, up to the level when the costly effort becomes profitable for the retailer. The effort investment shifts the demand for the manufacturer’s good and increases both the manufacturer’s profit and the consumer surplus.

5.1 A numerical example

Let’s consider a numerical example with the following values of the parameters: \( A_1 = 1.5, b = 0.7, c = 0.15, E = 0.2 \).

First I consider the case of the wholesale price \( \omega \). Given the parameters of the model the retailer is indifferent either to make the effort investment or not if and only if

\[
\pi_{WP}^R(\omega, e)|_{e=0} = \pi_{WP}^R(\omega, e)|_{e=1},
\]

that is

\[
\frac{(1 - \omega)(1 - b - t)}{4(1 - b^2)} + \frac{1 - b + b\omega}{4(1 - b^2)} = \frac{(A_1 - \omega)(A_1 - b - \omega)}{4(1 - b^2)} + \frac{1 - A_1b + b\omega}{4(1 - b^2)} - E,
\]

with the solution

\[
\hat{\omega} = \frac{(A_1 - 1)[A_1 + 1 - 2b] - 4E(1 - b^2)}{2(A_1 - 1)} = 0.142.
\]

The manufacturer’s profit in this case is negative:

\[
\pi_{WP}^M(\hat{\omega}) = q_1(\hat{\omega})(\hat{\omega} - c) = \frac{A_1 - b - \hat{\omega}}{2(1 - b^2)} (\hat{\omega} - c) = -0.051.
\]

For any price above the \( \hat{\omega} \) the retailer prefers to choose the zero-level of the effort. Thus the effort investment \( e = 1 \) may not be implemented.

The wholesale price \( \omega \) that maximizes the manufacturer’s profit is \( \omega = \frac{1}{2}(1 - b + c) = 0.225 \) and the profit is

\[
\pi_{WP}^M = \frac{1 - b - \omega}{2(1 - b^2)} (\omega - c) = 0.73 \cdot 0.075 = 0.0055.
\]
The equilibrium prices and quantities are \((p_{1WP}, p_{2WP}) = (0.6125, 0.5)\) and \((q_{1WP}, q_{2WP}) = (0.0735, 0.4485)\) respectively; the profit of the retailer is \(\pi_{WP}^R = 0.2528\); the consumer surplus is \(CS_{WP} = 0.1264\). Thus the total surplus is \(TS_{WP} = 0.3847\).

If MSDs applied then the manufacturer sets the price \(t_L = 0.1611\) and the market share threshold \(\bar{s} = 0.922\). The retailer is indifferent between two scenarios. The first is to make the effort \((e = 1)\) and to set the optimal prices \((p_{1MSD}, p_{2MSD}) = (0.8303, 0.504)\). The quantities in this case are \((q_{1MSD}, q_{2MSD}) = (0.6323, 0.0535)\). The second scenario is not to trade the manufacturers good at all and to set \(p_2 = 1/2\) and \(q_2 = 1/2\). The retailer’s profit in both cases is \(\pi_{MSD}^R = 1/4\). It is assumed that in this case the retailer makes the effort investment. Then the profit of the manufacturer is \(\pi_{MSD}^M = 0.007\), the consumer surplus is \(CS_{MSD} = 0.225\). Thus the total surplus is \(TS_{MSD} = 0.482\).

The results are confirmed in the example are the following: comparing with the wholesale price MSDs result in:

1) an increase in the manufacturer’s output, \(q_1\), and a decrease in the competitive sector’s firms output \(q_2\),

2) an increase in the manufacturer’s profit and a decrease in the retailer’s profit,

3) the retailer buys at the lower price, that is \(t_L < \omega\),

4) an increase in both the final market prices \(p_1\) and \(p_2\),

5) an increase in the total industry’s profit, an increase in the consumer surplus and, as a result, an increase in the total welfare

### 6 Conclusion

The paper investigates effects of the use of MSDs. In the first part of the model the case without an effort possibility is considered. It is shown that the manufacturer, which has some degree of market power, can use MSDs to extract additional profit through an increase in its market share and a decrease in the market share of competitors. This way of the MSDs use can be treated as the anticompetitive because it leads to decrease in both the total industry profit and the consumers’ surplus.
The second part of analysis considers the case when the retailer is able to make a costly effort investment that shift upward the demand for the manufacturer’s good. In this case MSDs can be used to motivate the retailer to make the efficient level of the effort investment. It happens because the MSDs use guarantees that quantity of the manufacturer’s good sold by the retailer is high enough and this provides the incentives for the retailer to make the effort investment. It is shown that this outcome not always may be reached through the use of the wholesale price contract. The main result is that MSDs can lead to increase in both the total industry profit and the social surplus. Hence the total welfare in the case of MSDs may be higher comparing with the case of the wholesale price.

One possible extension of the model may be in a consideration of the case of many heterogenous retailers. It can be that in this case the optimal menu of prices should include as many non-degenerated price as well as market thresholds as many retailers are at the downstream level. That may allow to satisfy incentives compatibility constraints for each of them separately. At the same time when the manufacturer designs the optimal price menu it must take into account that the required market share threshold may be reached by a retailer just through a reduction of the share of competitors without making a costly effort investment. Thus, to expand results presented for the case of many retailers, a deep formal analysis of extended model is required.

Another possible extension of the model is in comparison of the result of the MSDs use with the results of other non-linear price schemes. Particular interest is in comparison of MSDs and quantities discounts. The quantities discounts usually is not considered as anticompetitive discounts and their use is not restricted by law. If it is shown that MSDs lead to more preferable from the social point of view outcome then the quantities discount does, it will give more reasons to treat MSDs as a efficiency increasing, procompetitive tool. One possibility of getting this result may be in consideration of a case of stochastic demand when the use of the quantities discounts can involve difficulties related to an absolute value of a discount threshold. MSDs may be free of these difficulties in the case when both demands, for the manufacturer’s good and for the competitive sector’s firms good, have the same shock. I leave these extensions for the future investigation.
7 References


"Roundtable on loyalty or fidelity discounts and rebates", DAFFE meeting, May, 2002.

A Appendix

Proof of the Lemma 1. I proof the statement by contradiction.

Suppose, in the equilibrium the manufacturer sets \( \{ t^e_L, t^e_H, s^e \} \) and the retailer does not meet the market share threshold. That is \( s = \frac{q^e_1}{q^e_1 + q^e_2} < s^e \), where \( \{ q^e_1, q^e_2 \} \) and \( s^e \) are equilibrium quantities and the equilibrium market share of the manufacturer respectively.

Because in the equilibrium the market threshold restriction is not met, the level of the market threshold \( s^e \) has no effect on market outcome. In this case
the equilibrium price $t_H'$ coincides with one in the case of wholesale price $t_H = \frac{1}{2}(1 - b + c)$. As a result, the equilibrium retailer’s profits equals one in the case of the wholesale price, $\pi_{RMSD}^H = \frac{5 - 3b^2 - 2b(1-c) - (2-c)c}{16(1-b^2)}$.

Note that the retailer’s profit is higher that its reservation profit. It is because of:

$$5 - 3b^2 - 2b(1-c) - (2-c)c > \frac{1}{4} \iff 5 - 3b^2 - 2b(1-c) - (2-c)c > 4(1-b^2) \iff (b+c)^2 - 2(b+c) + 1 > 0 \iff (1-b-c)^2 > 0,$$

were the last inequality is obviously true.

Let’s construct new menu of prices $t' = \{t'_L, t'_H, s'\}$ in the form:

$$t'_L = t''_H$$
$$t''_H = +\infty$$
$$s' = s^e + \delta$$
where $\delta > 0$.

Now let’s show that the new price $t'$ gives the higher profit to the manufacturer.

Because $t''_H = +\infty$, the retailer has either to meet the market share threshold or to trade the competitive sector’s good only. In the latter case its profit equal to the reservation profit. In the former case, the retailer faces the same manufacturer’s price $t'_L = t''_H$ but it has to adjust quantities $q'_1, q'_2$ to meet the market share threshold. The optimal adjustment implies decreasing in the competitive sector’s good quantity $q_2$ and increasing in the manufacturer’s quantity $q_1$. Because of continuity of the retailer’s profit function in $q_1$ and $q_2$, for $\delta$ small enough we have that the new retailer’s profit is still higher than its reservation profit. Thus, if the new price $t'$ is offered then the retailer chooses new quantity $q'_1 > q^e_1$. Given the manufacturer’s price remains the same, $t'_L = t''_H$, the profit of the manufacturer is higher. Thus $\{t'_L, t'_H, s'\}$ were not the equilibrium values which contradicts to the assumption. ■

**Proof of the Corollary 1.** I proof the statement by contradiction. Let’s $\{t_L, t_H, \overline{s}\}$ and $s$ be the equilibrium manufacturer’s menu of prices and the equilibrium manufactures market share respectively. By Lemma 1 $s \geq \overline{s}$ and the retailer buys at the price $t_L$.

Suppose $s > \overline{s}$. Note that small changes in $t_L$ leads to small changes in the equilibrium quantities of $q_1, q_2$ and the condition $s > \overline{s}$ still holds.

If $t_L$ is higher (lower) than the equilibrium manufacturer’s wholesale price
Proof of the Proposition 2.  

1). The manufacturer’s market share is \( s = \bar{s} \) still holding. Thus in equilibrium \( t_L = \frac{1}{2}(1-b+c) \) and the condition \( \pi_{MSD}^{R} > \frac{1}{4} \) holds. Now, if the manufacturer sets \( s' = s + \delta \) then the retailer has either to to trade the competitive sector’s firms good only or adjust quantities \( q_1, q_2 \) to meet new threshold requirement. In the former case the retailer obtains its reservation profit only while in the latter case its profit changes only slightly and it still remains higher than its reservation profit. Thus the retailers chooses to by more manufacturer’s good at the same price. The profit of the manufacturer is higher that contradict to assumption that \( \{t_L, t_H, \bar{s}\} \) is equilibrium menu of prices. 

Proof of the Lemma 2.  

By Lemma 1 and Corollary 1 \( s = \bar{s} \) and hence

\[
\pi_{MSD}^{M} = q_1(t_L', \bar{s})(t_L' - c) = \frac{\pi(1-\bar{s}t_L')}{2(1-2\bar{s}(1-b)(1-\bar{s}))}(t_L' - c).
\]

Let’s show that \( \frac{\partial \pi_{MSD}^{M}}{\partial t_L} = \frac{s^2(c-2t_L)+s}{2(1-2s(1-b)(1-s))} \geq 0. \)

First, because of \( 2s(1-b)(1-s) \leq \max_2 s(1-s) (1-b) = \frac{1-b}{2} < 1 \implies 2(1-2s(1-b)(1-s)) > 0. \) Hence the denominator is positive.

Second, the nominator is positive because

\[
s^2(c-2t_L) + s > \min_{s}s^2(c-2t_L) + s = [s^2(c-2t_L) + s]_{s=\pi_{MSD}^{M}-c} = 0
\]

for any \( t_L \geq c \)

Thus, for any given level of \( \bar{s} \), \( \pi_{MSD}^{M} \) is a non-decreasing in \( t_L' \) function. Thus the manufacturer sets \( t_L' \) to be as high as possible until \( \pi_{MSD}^{R} \geq \frac{1}{4}. \)

The retailer’s profit \( \pi_{MSD}^{R} \) is the deceasing in \( t_L' \) function for any \( 0 < t_L < 1. \) Thus the manufacturer set price such that \( \pi_{MSD}^{R} = \frac{1}{4}. \) 

Proof of the Proposition 2.  

1). The manufacturer’s market share is \( s = \bar{s} = \frac{1-b-c}{(1-b)(2-c)} \) in the case of MSDs and it is \( s^{WP} = \frac{q_1}{q_1+q_2} = \frac{1-b-c}{(1-b)(3-c+2b)} \) in the case of the WP. Because \( (3-c+b) > 2 > (2-c) \) we have that \( \bar{s} > s^{WP} \)

2). Now I show that \( t_L^{*} = \frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) = \omega, \)

where \( D_1 = \sqrt{(1-b^2)(2(1-c)(1-b) + c)^2}. \)

\[
\frac{(2-c)(1-b)-D_1}{1-b-c} > \frac{1}{2}(1-b+c) \iff \frac{2(2-c)(1-b) - (1-b)^2 + c^2 > 2D_1}{1-b-c} \iff \frac{(1-b^2) + (2(1-b)(1-c) + c^2) > 2D_1}{1-b-c} \iff \sqrt{(1-b^2)^2} + \sqrt{(2(1-c)(1-b) + c^2)^2} > 2D_1 \iff
\]

26
(\sqrt{(1-b^2)} - \sqrt{[2(1-c)(1-b) + c^2]})^2 > 0. 
Moreover 1 - b^2 = 2(1-c)(1-b) + c^2 \leftrightarrow 1 - b - c = 0 which contradicts to the assumption. Thus \( t_L^* > \omega. \)

3). \( q_{1MSD} = \frac{1-b-c}{2D_1} > \frac{1-b-c}{4(1-b^2)} = q_{1WP} \leftrightarrow D_1 < 2(1-b^2) \leftrightarrow 2(1-c)(1-b) + c^2 < 4(1-b^2). \)
By the assumption \( c < 1 - b \)
\( 2(1-c)(1-b) + c^2 < 2(1-c)(1-b) + (1-b)^2 = \)
\( = (1-b)[2(1-c) + (1-b)] = (1-b)[3 - 2c - b] < \)
\( < 3(1-b) < 4(1-b^2). \)

4). \( q_{1MSD} > q_{1WP} \) and \( t_L^* > \omega \) gives that \( \pi_{MSD}^M > \pi_{WP}^M. \)

5). Competitive sector’s firms outputs in cases of WP and MSDs contracts are \( q_{2WP}^2 = \frac{2-b-b^2+bc}{4(1-b^2)} \) and \( q_{2MSD}^2 = \frac{1-b+bc}{2D_1} \) respectively.

First, let’s note that \( D_1 > 1 - b^2 \) because of
\( \sqrt{(1-b^2)}[2(1-c)(1-b) + c^2] \geq \min_c \sqrt{(1-b^2)}[2(1-c)(1-b) + c^2] = \)
\( = \sqrt{(1-b^2)}[2(1-c)(1-b) + c^2] \bigg|_{c=1-b} = (1-b^2) \)
Thus \( q_{2MSD}^2 = \frac{1-b+bc}{2D_1} < \frac{1-b+bc}{2(1-b^2)} = \frac{2-b+b^2}{2(1-b^2)}. \)
Now let’s see that \( 2 - 2b + 2bc < 2 - b - b^2 + bc \leftrightarrow \)
\( b(b+c-1) < 0 \) which holds by assumptions. Thus \( q_{2MSD}^2 < q_{2WP}^2. \)

6) and (7). The changes in prices are the immediate result of changes in quantities. Thus both the increase in \( q_1 \) and the decrease in \( q_2 \) results in the decrease in \( p_1 \) and the increase in \( p_2. \)

8). \( \pi_{WP}^M = \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > 1/4 = \pi_{MSD}^R \)
because of \( \frac{5-3b^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4} \leftrightarrow \)
\( 5 - 3b^2 - 2b(1-c) - (2-c)c > 4(1-b^2) \leftrightarrow \)
\( (b+c)^2 - 2(b+c) + 1 > 0 \leftrightarrow (1-b-c)^2 > 0. \)

9). Substituting equilibrium values of prices and quantities for both cases of the wholesale price and MSDs we get that the consumers’ surpluses are:
\( CS_{WP}^W = \frac{5-4b^2+(b+c)^2}{32(1-b^2)} \) and \( CS_{MSD}^W = \frac{1}{8}. \) Thus \( CS_{WP}^W > CS_{MSD}^W \leftrightarrow \frac{5-4b^2+(b+c)^2}{32(1-b^2)} > \frac{1}{8} \leftrightarrow 1 + (b+c)^2 - 2(b+c) > 0 \leftrightarrow (1-b-c)^2 > 0 \) where the last inequality is obviously true. ■

**Proof of the Lemma 3.** Suppose, in the equilibrium the manufacturer sets \( \{t_L^*, t_H^*, s^*\} \) and the retailer does not meet the market share threshold, \( s \neq s^*. \)
Because in the equilibrium \( s \neq \bar{s} \), the level of the market threshold \( \bar{s} \) has no effects on quantities \( q_1^e, q_2^e \).

Suppose that \( e^* = 1 \). In this case the equilibrium price (either \( t_H^e \) if \( s < \bar{s} \) or \( t_L^e \) if \( s > \bar{s} \)) coincides with the price \( \hat{\omega} \). But this contradicts to the assumption 3 which says that the manufacturers profit is higher if its price is \( \omega^* = \frac{1}{2}(1 - b + c) \) and \( e = 0 \). Thus if \( e^* = 1 \) it must be that \( s = \bar{s} \).

Suppose that \( e^* = 0 \). Then because of \( s \neq \bar{s} \) the manufacturers price (either \( t_H^e \) if \( s < \bar{s} \) or \( t_L^e \) if \( s > \bar{s} \)) equals to its wholesale price \( \omega^* = \frac{1}{2}(1 - b + c) \). As a result, the equilibrium retailer’s profits is \( \pi^R_{MSD} = \frac{5-3c^2-2b(1-c)-(2-c)c}{16(1-b^2)} > \frac{1}{4} \).

Let consider new menu of prices \( t' = \{t'_L, t'_H, \bar{s}'\} \) in the form:
\[
\begin{align*}
  t'_L &= t'_H \\
  t'_H &= +\infty \\
  \bar{s}' &= s^e + \delta
\end{align*}
\]
where \( \delta > 0 \).

Now let’s show that the new price \( t' \) gives the higher profit to the manufacturer.

Because \( t'_H = +\infty \), the retailer have either to meet the market share threshold \( \bar{s}' \) or to trade the competitive sector’s good only. To exclude the manufacturer’s good from the trade is not profitable because in this case the retailer obtains its reservation profit only. In the former case, the retailer has to adjust quantities \( q_1^e, q_2^e \) to meet the market share threshold. As well retailer may change its effort level to \( e = 1 \). Regardless changes in the effort level, the optimal adjustment implies an increase in the manufacturer’s quantity \( q_1 \). Because of continuity of the retailer’s profit function in \( q_1 \) and \( q_2 \), for \( \delta \) small enough the new retailer’s profit is still higher than its reservation profit. Thus, if the new menu of prices \( t' \) is offered then the retailer chooses new quantity \( q_1' > q_1^e \). Given the manufacturer’s price remains the same, \( t'_L = t_H^e \), the profit of the manufacturer is higher. Thus \( \{t'_L, t'_H, s^e\} \) were not the equilibrium values which contradicts to assumption.

**Proof of the Lemma 4.** Suppose, in the equilibrium the manufacturer sets \( \{t'_L, t'_H, s^e\} \). By Lemma 3 \( s = \bar{s} \).

Suppose the equilibrium level of the retailers effort is \( e^* = 0 \). Then all arguments of the Lemma 2 applied with small difference in the following way.

First, \( \pi^M_{MSD}(t'_L, \bar{s}^e) = q_1(t'_L, \bar{s}^e)(t'_L - c) = \frac{\bar{s}^e(1-\bar{s}^e)\Gamma(t'_L - c)}{2(1-2\bar{s}^e(1-b)(1-s)^2)(t'_L - c)}, \)
with \( \frac{\partial \pi^M_{MSD}}{\partial t'_L} = \frac{s^e(2c-2t_L+s)}{2(1-2s(1-b)(1-s))} \geq 0 \) for any \( t_L < \frac{(1+c)s}{2s} \).
Thus, for any given level of $\bar{e}$, $\pi^M_{MSD}$ is non-decreasing in $t^*_L$ function. for $t_L < \frac{(1+cs)}{2s}$. If $\pi^R_{MSD}(t^*_L, \bar{e})|_{e^*=0} > 1/4$ then the manufacturer may increase its profit by an increase in $t^*_L$. The retailer’s response on an increase in $t^*_L$ may imply changes in both the effort level and in quantities $q_1, q_2$. Regardless changes in the retailer’s effort level, the profit of the manufacturers increases for any $t_L < \frac{(1+cs)}{2s}$. The profit of the retailer decreases in $t_L$ and it is less then $\frac{1}{4}$ at $t_L = \frac{(1+cs)}{2s}$. Thus, if $e^* = 0$, the optimal manufacturer’s price is such that $\pi^R_{MSD}|_{e=0} = \frac{1}{4}$.

Now suppose the equilibrium level of the retailers effort is $e^* = 1$ and the retailer’s profit is $\pi^R_{MSD}(t^*_L, \bar{e})|_{e=1} > 1/4$. If the manufacturer increase its price to $t^*_L = t^*_L + \delta$ then the retailers response may imply changes in both the effort level and in quantities $q_1, q_2$. If retailer changes the effort level, then $\pi^R_{MSD}|_{e=0} = \frac{1}{4}$ as it was shown above. Given that $\delta > 0$ is small enough, an adjustment in quantities still provide the retailer with $\pi^R_{MSD}(t^*_L, \bar{e})|_{e=1} > 1/4$. The profit of the manufacturer $\pi^M_{MSD}(t_L, \bar{e})|_{e=1}$ increases while the retailer’s profit $\pi^R_{MSD}|_{e=1}$ decreases for $t < \frac{1+(A_1-1)s+cs}{2s}$ with $\pi^R_{MSD}|_{e=1} < 1/4$ if $t = \frac{1+(A_1-1)s+cs}{2s}$. Thus, if $e^* = 1$, the manufacturer optimal price is such that $\pi^R_{MSD}(t_L, \bar{e})|_{e=1} = 1/4$. ■