Eliciting Risk and Time Preferences

by

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Abstract. We design experiments to jointly elicit risk and time preferences for the adult Danish population. We find that joint elicitation results in estimates of discount rates that are dramatically lower than those found in previous studies. Estimation of latent time preferences requires that one specify a theoretical structure to understand risk and time choices, but we show that our main results, based on exponential discounting and expected utility theory, are robust to popular alternative specifications such as hyperbolic discounting and prospect theoretic formulations of choice under uncertainty. We also report evidence favoring standard, exponential discounting over hyperbolic. These results have direct implications for attempts to elicit time preferences, as well as debates over the appropriate domain of the utility function when characterizing risk aversion and time consistency.

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Utility functions are characterized in three dimensions, reflecting preferences over goods, time and uncertainty. The utility function conventionally characterizes preferences over goods defined by a time period and a state of nature, preferences over the temporal allocation of goods, and preferences over outcomes as realizations of uncertain states of nature. This broad characterization includes most alternatives to conventional expected utility theory.[1] We focus on the utility function for money, collapsing the choice over goods down to just one good so that there is no choice option with respect to goods. We jointly elicit risk and time preferences, using controlled experiments with field subjects in Denmark.

We show that *joint elicitation of risk and time preferences results in significantly lower discount rates than separate elicitation of discount rates.* The reason is that one can then infer discount rates defined in terms of temporally dated utility, rather than assuming that individuals are risk neutral and that discount rates are defined in terms of temporally dated money. Since subjects have concave utility of money functions, the implied discount rates are lower than when one incorrectly assumes a linear utility of money.

We use data from a field methodology developed by Harrison, Lau, Rutström and Sullivan [2005] (HLRS) to elicit both risk and time preferences from the same respondents. They used relatively simple experimental procedures that have evolved in the recent literature to study each. These experimental procedures are presented in section 2, and build on the risk aversion experiments of Holt and Laury [2002] (HL) and the discount rate experiments of Coller and Williams [1999] (CW) and Harrison, Lau and Williams [2002] (HLW). Data is collected in the field in Denmark, to obtain a sample that offers a wider range of individual socio-demographic characteristics than usually found in subject pools recruited in colleges, as well as a sample that can be used to make inferences about the preferences of the adult population of Denmark. These experiments are "artefactual field experiments" in the terminology of Harrison and List [2004], since

[1] The major exception for present purposes is the approach of Kreps and Porteus [1978], which allows for preferences over the timing of the resolution of uncertainty.

we essentially take lab experiments to field subjects.

We initially specify the relationship between risk and time preferences using standard assumptions: an exponential discount function, a constant relative risk aversion (CRRA) utility function, and expected utility theory (EUT). We evaluate the implied likelihood function of this specification for the data collected in our experiment. Maximum likelihood estimates, presented in section 3, imply moderate risk aversion and discount rates around 8% per annum. This is significantly lower than the discount rates around 25% per annum inferred from procedures that assume risk neutrality.

Our estimates rely on parametric functional forms, which is just to say that theory is needed to measure these latent preferences correctly. So it is important to consider alternative functional forms, even if the ones we initially chose are canonical. In section 3 we consider more flexible utility functions that allow for hyperbolic specifications of the discount rate function, and (separable and cumulative) prospect theory specifications. We find that our results are robust to the use of these alternative functional forms.

There are only a few studies that address the joint elicitation of risk and time preferences directly using monetary incentives and procedures familiar to experimental economists. None of these studies consider the formal theoretical link between elicited risk attitudes and individual discount rates that is our focus. We review this related literature in section 4. Finally, section 5 concludes with implications of our research.

1. Experimental Procedures for Eliciting Risk and Time Preferences

Our experimental procedures are documented in detail in HLRS, so we focus here just on the basics. In brief, each subject was asked to respond to four risk aversion tasks and six discount rate tasks. Each such task involved a series of binary choices, typically 10 per task. Thus each subject typically provides 100 binary choices that can be used to infer risk and time preferences.

A. Risk Preferences: Measuring Risk Aversion

Holt and Laury [2002] (HL) devise a simple experimental measure for risk aversion using a multiple price list (MPL) design.[2] Each subject is presented with a choice between two lotteries, which we can call A or B. Table 1 illustrates the basic payoff matrix presented to subjects in our experiments.[3] The first row shows that lottery A offered a 10% chance of receiving 2,000 DKK and a 90% chance of receiving 1,600 DKK. The expected value of this lottery, EV^A, is shown in the third-last column as 1,640 DKK, although the EV columns were not presented to subjects. Similarly, lottery B in the first row has chances of payoffs of 3,850 and 100 DKK, for an expected value of 475. Thus the two lotteries have a relatively large difference in expected values, in this case 1,165 DKK. As one proceeds down the matrix, the expected value of both lotteries increases, but the expected value of lottery B becomes greater relative to the expected value of lottery A.

The subject chooses A or B in each row, and one row is later selected at random for payout for that subject. The logic behind this test for risk aversion is that only risk-loving subjects would take lottery B in the first row, and only risk-averse subjects would take lottery A in the second last row. Arguably, the last row is simply a test that the subject understood the instructions, and has no relevance for risk aversion at all. A risk neutral subject should switch from choosing A to B when the EV of each is about the same, so a risk-neutral subject would choose A for the first four rows and B thereafter.

We take each of the binary choices of the subject as the data, and estimate the parameters of a latent utility function that explains those choices using an appropriate error structure to account for the panel nature of the data. Once the utility function is defined, for a candidate value of the parameters of that function, we can construct the expected utility of the two gambles, and then use a

[2] The MPL appears to have been first used in pricing experiments by Kahneman, Knetsch and Thaler [1990], and has been adopted in recent discount rate experiments by CW. It has a longer history in the elicitation of hypothetical valuation responses in "contingent valuation" survey settings, as discussed by Mitchell and Carson [1989; p. 100, fn. 14].

[3] As explained in HLRS the task also varied across subjects in terms of the width of the intervals and the number of rows.

linking function to infer the probability of the observed choice. We discuss statistical specifications in more detail in section 3.

We undertake four separate risk aversion tasks with each subject, each with different prizes designed so that all 16 prizes span the range of income over which we seek to estimate risk aversion. The four sets of prizes are as follows, with the two prizes for lottery A listed first and the two prizes for lottery B listed next: (A1: 2000 DKK, 1600 DKK; B1: 3850 DKK, 100 DKK), (A2: 2250 DKK, 1500 DKK; B2: 4000 DKK, 500 DKK), (A3: 2000 DKK, 1750 DKK; B3: 4000 DKK, 150 DKK), and (A4: 2500 DKK, 1000 DKK; B4: 4500 DKK, 50 DKK). At the time of the first phase of the experiments, the exchange rate was approximately 6.55 DKK per U.S. dollar, so these prizes range from approximately $7.65 to $687.

We ask the subject to respond to all four risk aversion tasks and then randomly decide which task and row to play out. In addition, the large incentives and budget constraints precluded paying all subjects, so each subject is given a 10% chance to actually receive the payment associated with his decision.

B. Time Preferences: Measuring Individual Discount Rates

The basic experimental design for eliciting individual discount rates (IDRs) was introduced in CW and expanded in HLW and CHR. Subjects were given payoff tables such as the one illustrated in Table 2, with 10 symmetric intervals. In this example, Option A offered 3000 DKK in one month and Option B paid 3000 DKK + x DKK in seven months, where x ranged from annual rates of return of 2.5% to 50% on the principal of 3000 DKK, compounded quarterly to be consistent with general Danish banking practices on overdraft accounts.[4] The payoff tables provided the annual and annual effective interest rates for each payment option, and the experimental instructions defined these terms by way of example. Subjects were asked to choose between Option

[4] Across all time horizons considered by HLW, payoffs to any one subject could range from 3,000 DKK up to 12,333 DKK. The exchange rate when the HLW experiments were conducted in mid-1997 was approximately 6.7 DKK per US dollar, so this range converts to $450 and $1,840.

A and B for each of the 20 payoff alternatives, and one decision row was selected at random to be paid out at the chosen date. If a risk-neutral subject prefers the 3000 DKK in one month then we can infer that the annual discount rate is higher than x%; otherwise, we can infer that it is x% per day or less.[5]

We use the multiple-horizon treatment from HLW. From the perspective of the task faced by the subjects, the only variations are that the instrument is now computerized, and subjects are presented with 6 discount rate tasks, corresponding to 6 different time horizons: 1 month, 4 months, 6 months, 12 months, 18 months, and 24 months.[6] In each task subjects are provided two future income options rather than one "instant income" option and one future income option. We follow HLW and use a FED of one month in all tasks. For example, they were offered 3000 DKK in one month and 3000 DKK + x DKK in 7 months, so that we interpret the revealed discount rate as applying to a time horizon of 6 months. This avoids the potential problem of the subject facing extra risk or transactions costs with the future income option, as compared to the "instant" income option. If the delayed option were to involve such additional transactions costs, then the revealed discount rate would include these subjective transactions costs. By having both options entail future income we hold these transactions costs constant.

Each subject responded to all six discount rates tasks and one task and row was chosen at

[5] We assume that the subject does not have access to perfect capital markets, as explained in CW (p.110) and HLW (p.1607ff.). This assumption is plausible, but also subject to checks from responses to the financial questionnaire that CW, HLW and we ask each subject to complete. We also assume that subjects consume the monetary amounts in options A and B at the time stated in the instrument, and do not smooth consumption as the result of the outcomes from these tasks. These are common assumptions in the discounting literature, as noted by Frederick, Loewenstein and O'Donoghue [2002; p.380]. The effects of allowing for field borrowing and lending opportunities on elicited discount rates for risk neutral subjects are discussed by CW and HLW; Harrison, Harstad and Rutström [2004] discuss the general implications of allowing for extra-experimental trading opportunities on inferences from experimental responses. The effects of allowing for consumption smoothing are harder to identify, since they require a more elaborate specification of the intertemporal choices of the individual. The upshot is that we are eliciting time preferences over money endowments, and these can only be interpreted as time preferences over money consumption when these assumptions are imposed.

[6] The design mimics the format used by HL in their risk aversion experiments: in that case the rows reflected different probabilities of each prize, and in this case the rows reflect different annual effective rates of return. We exploit this similarity of format in the design of our computerized interface to subjects, and in the use of trainers in the risk aversion task as a generic substitute for trainers in the discount rate task.

random to be played out. Future payments to subjects were guaranteed by the Danish Ministry of Economic and Business Affairs, and made by automatic transfer from the Ministry's bank account to the subject's bank account. This payment procedure is similar to a post-dated check, and automatic transfers between bank accounts are a common procedure in Denmark. Finally, each subject was given a 10% chance to receive actual payment. Thus, each subject faced a 10% chance of receiving payment in the risk preference task as well as a 10% chance in the time preference task.

C. Experimental Procedures and Data

The sample for the field experiments was designed to generate a representative sample of the adult Danish population. The experiments were conducted over 20 sessions, between June 2 and June 24, 2003, in 19 locations spread over Denmark, and a total of 253 subjects participated. Sample weights for the subjects of the experiment can be constructed using the sample design, stratifying by county, age group, and sex. These weights will be used to generate unbiased estimates for the adult Danish population.

2. Identifying Risk and Time Preferences

A. General Statement

Consider the identification of risk and time preferences in the canonical case of mainstream economic theory. Specifically, assume EUT holds for the choices over risky alternatives, that subjects employ a CRRA utility function defined over the prizes they made choices over, and that discounting is exponential. Then a plausible axiomatic structure on preferences is known from Fishburn and Rubinstein [1982; Theorem 2] to imply the stationary structure

$$U(M_t) = 1/(1+\delta)^{\tau}\, U(M_{t+\tau}) \qquad (1)$$

where $U(M_t)$ is the utility of monetary outcome M_t for delivery at time t, δ is the discount rate, τ is the horizon for later delivery of a monetary outcome, and U is a utility function for money that is stationary over time. Thus δ is the discount rate that makes the present value of the *utility* of the two monetary outcomes M_t and $M_{t+\tau}$ equal. Most analyses of discounting models implicitly assume that the individual is risk neutral,[7] so that (1) is instead written in the more familiar form

$$M_t = 1/(1+\delta)^{\tau} M_{t+\tau} \tag{2}$$

in which δ is the discount rate that makes the present value of the two *monetary outcomes* M_t and $M_{t+\tau}$ equal.

To state the obvious, (1) and (2) are not the same. This observation has considerable implications for the identification of discount rates from observed choices over M_t and $M_{t+\tau}$. These choices are, in fact, the observed data from our experiments. As one relaxes the assumption that the decision maker is risk neutral, it is apparent from Jensen's Inequality that the implied discount rate decreases since U(M) is concave in M. Thus one cannot infer the individual discount rate without knowing or assuming something about their risk attitudes.

B. Parametric Structure

We can quickly put some familiar parametric structure on this statement of the identification problem. Let the utility function be the CRRA specification

$$U(m) = m^{1-r}/(1-r) \tag{3}$$

where $r \neq 1$ is the CRRA coefficient, and $U(m) = \ln(m)$ for $r = 1$. With this parameterization, $r = 0$ denotes risk neutral behavior, $r > 0$ denotes risk aversion, and $r < 0$ denotes risk loving. To relate this specification to the risk aversion choices in our experiment, one can calculate the implied bounds on the CRRA coefficient for each row in Table 1, and these are in fact reported by HL [2002; Table 3]. These intervals are shown in the final column of Table 1. Thus, for example, a subject that made 5 safe choices and then switched to the risky alternatives would have revealed a CRRA interval between 0.14 and 0.41, and a subject that made 7 safe choices would have revealed a CRRA interval between 0.68 and 0.97, and so on. Thus the binary choices of the subject can be

[7] See Keller and Strazzera [2002; p. 148] and Frederick, Loewenstein and O'Donoghue [2002; p.381ff.] for an explicit statement of this assumption, which is often implicit in applied work.

explained by different values of the CRRA coefficient, and the coefficient estimated using standard maximum likelihood procedures (explained in detail below).

The experimental evidence from the field is that CRRA is 0.67 in Denmark (HLRS, p.148), close to the results obtained in comparable laboratory experiments in the United States by HL and Harrison, Johnson, McInnes and Rutström [2005].[8] The general conclusion from these studies is that *the utility of money function is concave* over the domain of prizes relevant for these experiments.

Of course, the other two parametric components of the specification include the assumption of EUT over risky lotteries, and the assumption of constant, exponential discounting.

C. Statistical Specification

It is easy to write out the likelihood function for all of the choices that our subjects made, and thereby jointly estimate the parameter *r* of the utility function and the discount rate δ.

Consider first the contribution to the overall likelihood from the risk aversion responses. Probabilities for each outcome k_n, $p(k_n)$, are those that are induced by the experimenter, so expected utility is simply the probability weighted utility of each outcome in each lottery. Since there were two outcomes in each lottery, the EU for lottery i is

$$EU_i = \sum_n [\, p(k_n) \times U(k_n) \,] \tag{4}$$

for n = 1, 2.

A simple stochastic specification from Holt and Laury [2002] is used to specify likelihoods conditional on the model. The EU for each lottery pair is calculated for a candidate estimate of r, and the ratio

$$\nabla EU = EU_R^{1/\mu} \,/\, (EU_R^{1/\mu} + EU_L^{1/\mu}) \tag{5}$$

calculated, where EU_L is the left lottery in the display and EU_R is the right lottery, and μ is a structural "noise parameter" used to allow some errors from the perspective of the deterministic

[8] Harrison, Lau and Rutström [2004] examine the Expo-Power specification for field subjects in Denmark and conclude that RRA is constant over the income considered in these lotteries. However, there is evidence of increasing RRA in laboratory experiments in the United States (Holt and Laury [2002][2005]).

EUT model. The index ∇EU is in the form of a cumulative probability distribution function defined over differences in the EU of the two lotteries and the noise parameter μ.[9] Thus, as $\mu \to 0$, this specification collapses to the deterministic choice EUT model, where the choice is strictly determined by the EU of the two lotteries; but as μ gets larger and larger the choice essentially becomes random. This is one of several different types of error story that could be used.[10] The index in (5) is linked to the observed choices by specifying that the right lottery is chosen when $\nabla EU > 0.5$.

Thus the likelihood of the risk aversion responses, conditional on the EUT and CRRA specifications being true, depend on the estimates of r and μ given the above statistical specification and the observed choices. The conditional log-likelihood is

$$\ln L^{RA}(r, \mu; y, X) = \textstyle\sum_i [(\ln (\nabla EU) \mid y_i=1) + (\ln (\nabla EU) \mid y_i=0)] \tag{6}$$

where $y_i = 1(0)$ denotes the choice of the right (left) lottery in risk aversion task i, and X is a vector of individual characteristics.

Turning to the discount rate choices, a similar specification is employed. Equation (4) is replaced by the present value of the utility of the two outcomes, conditional on some assumed discount rate, and equation (5) is defined in terms of those present values instead of the expected utilities. The present value of the utility of M_t at t is just

$$PV_L = U(M_t) \tag{7}$$

and the present value of the utility of $M_{t+\tau}$ at $t+\tau$ is

$$PV_R = 1/(1+\delta)^\tau \, U(M_{t+\tau}) \tag{8}$$

where the subscripts L and R refer to the left and right options in the choice tasks presented to

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[9] An alternative approach might be to define an index as the difference of the EU's of the two lotteries, and then specify some cumulative distribution function to link it to the observed choices. For example, the cumulative standard normal distribution leads to the probit specification.

[10] See Harless and Camerer [1994], Hey and Orme [1994] and Loomes and Sugden [1995] for the first wave of empirical studies including some formal stochastic specification in the version of EUT tested. There are several species of "errors" in use, reviewed by Hey [1995][2002], Loomes and Sugden [1995], Ballinger and Wilcox [1997], and Loomes, Moffatt and Sugden [2002]. Some place the error at the final choice between one lottery or the other after the subject has decided deterministically which one has the higher expected utility; some place the error earlier, on the comparison of preferences leading to the choice; and some place the error even earlier, on the determination of the expected utility of each lottery.

subjects, illustrated in Table 2. The parametric form for the utility function in (7) and (8) is the CRRA form given in (3), so we can rewrite these as

$$PV_L = M_t^{1-r}/(1-r) \tag{7'}$$

$$PV_R = [1/(1+\delta)^\tau] \times [M_{t+\tau}^{1-r}/(1-r)] \tag{8'}$$

An index of the difference between these present values, conditional on δ and *r*, can then be defined as

$$\nabla PV = PV_R^{1/\nu} \,/\, (PV_R^{1/\nu} + PV_L^{1/\nu}) \tag{9}$$

where ν is a noise parameter for the discount rate choices, just as μ was a noise parameter for the risk aversion choices. It is not obvious that $\mu = \nu$, since these are cognitively different tasks. Our own priors are that the risk aversion tasks are harder, since they involve four outcomes compared to two outcomes in the discount rate tasks, so we would expect $\mu > \nu$. Error structures are things one should always be agnostic about since they capture one's modeling ignorance, and we allow the error terms to differ between the risk and discount rate tasks.

Thus the likelihood of the discount rate responses, conditional on the EUT, CRRA and exponential discounting specifications being true, depend on the estimates of r, δ and ν given the above statistical specification and the observed choices. The conditional log-likelihood is

$$\ln L^{DR} (r, \delta, \nu; y, X) = \textstyle\sum_i [\, (\ln (\nabla PV) \mid y_i=1) + (\ln (\nabla PV) \mid y_i=0)\,] \tag{10}$$

where y_i =1(0) denotes the choice of the right (left) option in discount rate task i, and X is a vector of individual characteristics.

The joint likelihood of the risk aversion and discount rate responses can then be written as

$$\ln L\, (r, \delta, \mu, \nu; y, X) = \ln L^{RA} + \ln L^{DR} \tag{11}$$

and maximized using standard numerical methods. Our implementation uses version 9 of *Stata*.[11] The statistical specification allows for the deliberate survey design described earlier. In particular, we allow for the fact that subjects in one county were selected independently of subjects in other

[11] We document the *Stata* syntax for this estimation at http://exlab.bus.ucf.edu. We also provide all source code and data for the estimates reported here.

counties, as well as the possibility of correlation between responses by the same subject.[12]

D. Estimates

Table 3 presents maximum likelihood estimates from our experiments.[13] Panel A presents the estimates allowing for risk aversion, and panel B presents the effects of constraining the model to assume risk neutrality. From panel A we obtain estimates of the CRRA parameter of 0.65, close to the 0.67 reported in HLRS (p.148) using different statistical methods. The risk-neutral estimate is also close to the estimate of 24.2% reported in HLRS (p. 151) using the same data and different statistical methods, and the 28.1% reported in HLW using a prior series of comparable field experiments in Denmark. Critically, we obtain a point estimate of the discount rate of 8.2%, strikingly lower than the estimate in panel B of 24.9%. To evaluate the statistical significance of adjusting for a concave utility function we test the hypothesis that the estimated discount rate assuming risk aversion is the same as the discount rate estimated assuming risk neutrality. We easily reject this null hypothesis. Thus, *allowing for risk aversion makes a significant difference to the elicited discount rates.*

The estimates of the error terms in Table 3 also exhibit an interesting pattern. Recall that estimates of 0 indicate that no noise is present in the decision process, so we first observe that there is evidence of some noise since the *p*-values for each of ν and μ are statistically significant. However, there is a larger estimate of noise for the risk aversion tasks than the discount rate tasks in panel A,

[12] The use of clustering to allow for "panel effects" from unobserved individual effects is common in the statistical survey literature. Clustering commonly arises in national field surveys from the fact that physically proximate households are often sampled to save time and money, but it can also arise from more homely sampling procedures. For example, Williams [2000; p.645] notes that it could arise from dental studies that "collect data on each tooth surface for each of several teeth from a set of patients" or "repeated measurements or recurrent events observed on the same person." The procedures for allowing for clustering allow heteroskedasticity between and within clusters, as well as autocorrelation within clusters. They are closely related to the "generalized estimating equations" approach to panel estimation in epidemiology (see Liang and Zeger [1986]), and generalize the "robust standard errors" approach popular in econometrics (see Rogers [1993]). Wooldridge [2003] reviews some issues in the use of clustering for panel effects, in particular noting that significant inferential problems may arise with small numbers of panels.

[13] The data consists of observations from 253 subjects. Our data is a panel resulting in 7,928 observations for risk aversion choices and 15,180 for discount rate choices, 4.6% of which are expressions of indifference.

consistent with our prior that the risk aversion tasks were cognitively harder.

From the estimated covariance matrix, we observe a correlation of -0.92 between the estimate of r and the estimate of δ. This is exactly what the comparison of panels A and B in Table 3 imply. As r decreases from 0.65 to 0, which is the implicit value in panel B, the individual discount rate δ increases significantly.[14]

3. Alternative Specifications and Functional Forms

Although the basic insight that one should elicit risk and time preferences jointly seems simple enough, it is clear that identification of specific estimates does rely on assuming certain functional forms. The specifications considered in section 2 and Table 3 are canonical, and important in their own right given their place in the literature. Can we say that our main conclusion is robust to alternative specifications and functional forms? One concern is with the effect of allowing for heterogeneity, in the sense that we allow estimable parameters to be linear functions of observable individual characteristics (section A). Another concern is with alternative discounting functions, such as those assumed in hyperbolic discounting models (section B). There might also be a concern about the effects of using preference structures favored by critics of EUT, such as those based on prospect theory (section C).[15] Finally, we consider statistical specifications that allow us to directly estimate the weight of the evidence favoring the exponential discounting model when one allows concave utility functions (section D).

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[14] The discount rate is *not* the same thing as the intertemporal elasticity of substitution, which is the inverse of the risk aversion coefficient under the standard, temporally-separable model (Chavas [2004; pp. 145-146]). Using parlance from the general equilibrium calibration literature, the discount rate is the slope of the trade-off between temporally dated utility bundles, and not the elasticity of substitution between those bundles at some point. That elasticity is not constrained by the slope, *per se*.

[15] It would be possible to employ more flexible utility functions than CRRA that are consistent with EUT, but evaluation of such extensions are likely to be less informative to those that doubt the basic specification. As noted earlier, Harrison, Lau and Rutström [2004] have considered more flexible specifications of the utility function for these data, and conclude that the evidence is consistent with CRRA for these data and these income domains.

A. Heterogeneity of Preferences

It is an easy matter to allow each parameter in (11) to be linear function of observable characteristics of individuals and/or treatment effects. For example, we could allow for the CRRA coefficient r to depend on the sex of the subject, so that we would estimate

$$\hat{r} = \hat{r}_0 + (\hat{r}_{FEMALE} \times FEMALE) \tag{12}$$

where $\hat{r}_0$ is the estimate of the constant and $\hat{r}_{FEMALE}$ shows the difference in risk for females. In general we will report unconditional estimates of the parameters, since that is sufficient for our methodological purposes. But it is important to know that the specification extends easily, since demographic effects are of considerable policy significance.

Allowing for demographic effects for r and δ makes no difference to our conclusions. We include binary indicators for sex, aged less than 30, aged between 40 and 50, aged over 50, living alone, having children, owning one's own home or apartment, being retired, being a student, having some post-secondary education, having substantial higher education, have a lower income level in 2002 (below 300,000 kroner), having a higher income level in 2002 (500,000 kroner or more), living in the greater Copenhagen area, and living in a larger city of 20,000 or more. We also include variables measuring the number of people in the household. Each of the core parameters r and δ is specified as a linear function of these characteristics, and the model estimated using maximum-likelihood. We also estimated the model assuming that everyone was risk-neutral, but allowing for demographic heterogeneity for the estimates of δ.

Figure 1 displays kernel density estimates of the predicted discount rates from each estimation. The averages are virtually identical to those estimated without controls for demographics, as one would expect. The distribution of elicited discount rates in Figure 1 reflects variations across types of subjects in our sample. For example, older subjects that are not retired tend to have slightly higher discount rates, *ceteris paribus* other characteristics. Retirement tends to lower discount rates, as does the completion of substantial higher education. We test whether these demographic effects on discount rates are the same across the risk neutral and risk averse models, and confirm that they are not.[16] Thus, controlling for heterogeneity in risk attitudes is important to correctly identify heterogeneity in discount rates.

B. Hyperbolic Discounting

The earliest hyperbolic specifications assumed that individuals had discount rates that declined with the horizon they faced, in contrast to later quasi-hyperbolic specifications that posit an initial decline and then constant (per period) discount rates.[17] The most common functional form of the older literature is due to Herrnstein [1981], Ainslie [1992] and Mazur [1987], and would replace

$$PV_R = 1/(1+\delta)^\tau \, U(M_{t+\tau}) \tag{8}$$

with

$$PV_R = 1/(1+\alpha\tau) \, U(M_{t+\tau}) \tag{8*}$$

for $\alpha > 0$.

Maximum likelihood estimates using (8*) can be obtained using the same methods used for the exponential specification, and are reported in Panel A of Table 4. We estimate α to be 0.093 when one allows a concave utility function, and 0.26 when one imposes risk neutrality.

Figure 2 shows the implied hyperbolic estimates of annual discount rates against the time horizon in years. Clearly the hyperbolic model shows evidence of some slight decline in discount rates with horizon, but the quantitative magnitude of the decline is much smaller when one allows for concave utility functions. Furthermore, the level of discount rates is lower in the latter case, consistent with our inference assuming exponential discounting. Figure 3 shows how comparable the elicited discount rates are using exponential and hyperbolic discounting when one allows for concave utility functions.

[16] Further discussion of demographic effects on risk attitudes and discount rates in Denmark is provided by HLW, Harrison, Lau and Rutström [2004] and HLRS.

[17] The Quasi-Hyperbolic specification was first used by Phelps and Pollak [1968] to represent imperfect altruistic preferences across generations. Laibson [1997] then developed it as a model for individual discounting behavior. Our use of a "front end delay" on receipt of the earlier option implies that we cannot test the Quasi-Hyperbolic specification against the standard Exponential specification, unless one assumes that the "passion for the present" lasted longer than our front end delay.

We can also evaluate the effect of using a more general hyperbolic specification[18] proposed by Prelec [2004; p.526]. This specification replaces (8) with

$$PV_R = \exp\{-\beta\tau^{\alpha}\}\ U(M_{t+\tau}) \tag{8**}$$

The exponential discounting model emerges as a limiting case as α tends to 1. One can think of the parameter α as characterizing the "decreasing impatience" of the decision-maker, a smoother and inverse counterpart of the notion of a "passion for the present" in quasi-hyperbolic characterizations. As α takes values below 1, the discount function takes on the familiar shape of earlier hyperbolic specifications. One can also think of the parameter β as characterizing time preferences in the usual sense (Prelec [2004; p.524]). The instantaneous discount rate implied by this discount function is $\alpha\beta t^{\alpha-1}$, which collapses to β as $\alpha \rightarrow 1$. If we use this specification (8**) instead of the exponential discounting specification (8) or the hyperbolic specification (8*) we obtain essentially the same results: elicited discount rates are dramatically lower when one allows for concave utility functions.

C. Prospect Theory

Tversky and Kahneman [1992] propose a popular parametric specification of prospect theory. There are two components, the utility function[19] and the probability weighting function. Tversky and Kahneman assume a power utility function defined separately over gains and losses. Since we do not have any losses in our tasks, we only need consider the function defined over the gain domain. We employ the comparable CRRA function specified in (3), which is virtually the same as the simple power function.

There are two variants of prospect theory, depending on the manner in which the probability weighting function is combined with utilities. The original version proposed by Kahneman and Tversky [1979] posits some weighting function which is separable in outcomes, and has been

[18] Loewenstein and Prelec [1992] and Prelec [2004; p.515] proposed a similar generalized hyperbolic function which has fragile numerical properties when used for estimation.

[19] Often called a value function by some.

usefully termed Separable Prospect Theory (SPT) by Camerer and Ho [1994; p. 185]. The alternative version, proposed by Tversky and Kahneman [1992], posits a weighting function defined over the cumulative probability distributions. In either case, the weighting function proposed by Tversky and Kahneman [1992] has been widely used: it assumes weights w(p) given by

$$w(p) = p^{\gamma}/[\, p^{\gamma} + (1-p)^{\gamma}\,]^{1/\gamma} \tag{13}$$

where p is the probability of the outcome, w(p) is the implied "decision weight" used to weight the utility of final outcomes, and γ is a parameter to be estimated. When $\gamma=1$ this function collapses to the standard EUT specification that w(p) = p. Alternative probability weighting functions have been proposed, and could also be considered, but our objective is to illustrate robustness to the major alternatives in use.

Assuming that SPT is the true model, prospective utility PU is defined in much the same manner as when EUT is assumed to be the true model. Thus, since we have no losses and no loss aversion parameters, the SPT utility function is used instead of the EUT utility function, and w(p) is used instead of p, but the steps are otherwise identical. The same error process is assumed to apply when the subject forms the preference for one lottery over the other. Thus the difference in prospective utilities is defined similarly as

$$\nabla PU = PU_R^{1/\mu} \,/\, (PU_R^{1/\mu} + PU_L^{1/\mu}) \tag{5*}$$

where the PU for lottery i is

$$PU_i = \textstyle\sum_n [\, w[p(k_n)] \times U(k_n)\,] \tag{4*}$$

instead of (4).

Assuming that CPT is the true model, the transformation of lotteries into prospective utility is more involved. The weighting function is given by (13), but the prospective utility of lottery i is now defined as

$$PU_i = [\, w(p_2)U(k_2) + (1- w(p_2))U(k_1)] \tag{4**}$$

where $k_2 > k_1$. The difference in prospective utilities under CPT is then defined in the same way as (5*), using the same error process when subjects form their preferences.

Panels B and C of Table 4 contain the maximum likelihood estimates assuming separable and cumulative prospect theory. The use of probability weighting lowers the estimated degree of risk aversion slightly, from 0.65 in the baseline model to 0.55 and 0.44 respectively. This leads to an increase in the estimated discount rates, from 8.0% in the baseline model to 10.6% and 13.0%, respectively. Figure 4 extends Figure 1 to display the effect of allowing for SPT or CPT on the predicted distribution of discount rates, allowing for demographic effects on all core parameters (viz., r, γ and δ).

Although our objective is not to test the strength of PT *vis-à-vis* EUT, there is some evidence here in favor of probability weighting since EUT is a special case of the PT where γ=1.[20] Nevertheless, the effect that probability weighting has on discount rates is relatively minor compared to the effect of allowing for risk aversion, which is our main focus here. Again, this illustrates how theory is needed to infer discount rates, but indicates that the basic results appear to be robust to these alternative specifications.

We have also interacted the hyperbolic specification of discount functions in (8*) with the SPT and CPT specifications, and find no significant change in our main results.

D. Mixture Specifications

Finally, we consider the sensitivity of our conclusions to a statistical specification that allows each observation to potentially be generated by more than one latent data-generating process. Our motivation is to better identify the effect of jointly eliciting risk and time preferences on the weight of the evidence for exponential discounting. Consider Figure 2 again, which assumes that all of the data was generated by one hyperbolic data-generating process. The striking decline in discount rates under risk neutrality is significantly muted when one corrects for concave utility functions.

[20] Harrison and Rutström [2005] provide a more complete statistical characterization of the experimental evidence for EUT and PT, allowing for loss frames and *both* models to potentially generate each observed choice. They find that each latent choice process accounts for roughly half of the observed choices, and that one can identify demographic characteristics of subjects associated with each process.

Nonetheless, Figure 3 shows that there remains some difference in the effect of horizon on elicited discount rates when one compares exponential and hyperbolic specifications that allow for concave utility. How can we evaluate the significance of this difference, and use these data to inform us about the relative importance of each specification?

Finite mixture models provide an ideal statistical framework to address this question.[21] Consider the mixture of exponential discounting models and generalized hyperbolic models, defined by (8) and (8**). We assume that EUT characterizes behavior in all other respects.[22] The mixture likelihood function is then

$$\ln L\ (r, \delta, \alpha, \beta, \mu, \nu, \pi; y, X) = \ln L^{RA} + [\pi \times \ln L^{DR\text{-}E}] + [(1\text{-}\pi) \times \ln L^{DR\text{-}H}] \tag{14}$$

where π is a parameter, to be estimated and constrained such that $0 \leq \pi \leq 1$, giving the probability that a given observation[23] is generated by the exponential discounting model. In (14) the likelihood contributions $L^{DR\text{-}E}$ and $L^{DR\text{-}H}$ refer to the exponential and hyperbolic models, respectively.

Table 5 provides maximum likelihood estimates of this model. The first result is that the estimate of the mixture probability indicates that only 34.1% of the observations can be characterized as being generated by the exponential model. This would appear, then, to suggest that the weight of the evidence supports the (generalized) hyperbolic discounting model. However, the second result from Table 5 is that the generalized hyperbolic collapses to the exponential, since α is estimated to be 1.012, and is statistically indistinguishable from 1. Thus the mixture model suggests that there are really just two exponential discounting processes generating these data: one with a discount rate of 14.3% (the δ parameter estimate), and another with a discount rate of 5.1% (the β

[21] Mixture models have an astonishing pedigree in statistics: Pearson [1894] examined data on the ratio of forehead to body length of 1000 crabs to illustrate "the dissection of abnormal frequency curves into normal curves,...". In modern parlance he was allowing the observed data to be generated by two distinct Gaussian processes, and estimated the two means and two standard deviations. Modern surveys of the evolution of mixture models are provided by Everitt [1996] and McLachlan and Peel [2000].

[22] Harrison and Rutström [2005] and Harrison, Humphrey and Verschoor [2005] examine mixture models defined over lottery choices in which observations may be generated by EUT or SPT, and find evidence to support both latent processes.

[23] One could alternatively define a grand likelihood in which observations or subjects are completely classified as following one model or the other on the basis of the latent probability π. El-Gamal and Grether [1995] illustrate this approach in the context of identifying behavioral strategies in Bayesian updating experiments.

parameter, given that $\alpha \approx 1$),[24] The weighted average of these two estimates is 8.2% (= 0.341×14.3% + 0.659×5.1%), which is identical to the estimate for δ obtained when we simply assumed one latent exponential process generated the observed data (see the δ estimate in Panel A of Table 3). What we end up with is simply another reflection of the preference heterogeneity discussed earlier.

We therefore conclude that the evidence favors standard, exponential discounting over hyperbolic discounting.[25]

4. Related Literature

There are several studies that note the connection between concave utility functions and individual discount rates, but we are aware of only two studies address the joint elicitation of risk and time preferences directly using monetary incentives.[26]

A. Studies Using Hypothetical Tasks

Chapman [1996] draws the correct formal link between estimation of individual discount rates and concavity of the utility function, but does not elicit risk attitudes. Instead she uses hypothetical questions to elicit individual discount rates over money and health, and then estimates individual discount rates based on various assumptions about the risk attitudes of the subjects.

Kirby and Santiesteban [2003] argue correctly that the hyperbolic behavior found in many studies may be confounded by the concavity of the utility function and transactions costs with respect to the delayed payment option. However, they do not control for risk attitudes in their evaluation of different discounting functions.

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[24] In fact, we confirm this by specifying a mixture model of two exponential processes and generating the same results.

[25] We do not consider quasi-hyperbolic discounting here, since we employ a front end delay in our experimental design. Coller, Harrison and Rutström [2003] report lab experiments with no front end delay, and use a mixture model similar to the one employed here to evaluate the weight of the evidence in favor of exponential and quasi-hyperbolic models. They conclude that roughly 50% of the observations in that setting are consistent with each latent process.

[26] There are some studies that undertake joint statistical estimation of discount rates and risk attitudes from field data or survey data. For example, see Abdulkadri and Langemeier [2000], Issler and Piquera [2000] and van Praag and Booij [2003].

B. Studies Using Monetary Incentives

Anderhub, Güth, Gneezy and Sonsino [2001] (AGGS) use the Becker-DeGroot-Marschak (BDM) procedure to elicit certainty equivalents for lotteries with varied payoff dates.[27] They used undergraduate economic students in Israel as subjects. Each subject provided either a buying or a selling price for each of three lotteries that paid out the day of the experiment, two weeks from the day of the experiment, and four weeks from the day of the experiment. The lotteries differ only with respect to the timing of payments. One decision was chosen at random to be played out. AGGS find no statistical difference between certainty equivalents across different time horizons. They find a marginally significant positive relationship between the degree of risk aversion and the discount rates implied by the timing of payments.[28] The differences between the elicitation tasks in our design and that of AGGS reflect a tradeoff between compactness of experimental procedures and transparency of the task required of subjects. While our elicitation mechanism is logically equivalent to the BDM, we believe the binary decisions in the MPL are less of a cognitive burden for subjects. Moreover, the AGGS design elicits a single value from subjects that reflects both risk and time preferences, while we examine these preferences separately.

Eckel, Johnson and Montmarquette [2005] (EJM) conduct a field study of time and risk preferences. Their subjects are recruited from low income neighborhoods in Montreal. Subjects in these experiments are given 64 "compensated" questions, one of which is chosen at random for payment. Time preferences are elicited by presenting subjects with choices between payoffs that occur at different times. Time horizons for the later payments ranged from 2 days to 28 days, and most early payments had a front end delay of one day, one week, or two weeks. The value for most questions started at approximately $72 CAD, with a few questions presenting values around $26 CAD. The distribution of annual discount rates implied by the questions was lumpy, with values of

[27] Albrecht and Weber [1997] use a similar design, albeit with hypothetical rewards, and find no significant relationship between risk aversion and individual discount rates in the gain domain.

[28] AGGS are able to effect delayed payments by distributing post-dated checks the day of the experiment, thereby reducing any differences between immediate and delayed payments due to subject expectations regarding transactions costs of future payments.

10%, 50%, 200% and 380%. In fact, Eckel, Johnson and Montmarquette [2005; p.258] report short-term discount rates *averaging* 289% per annum, consistent with the earlier literature on discount rate elicitation.[29] Given the variance of responses, EJM construct a measure of time preferences that rely on four questions using the 14 days time horizon only. Risk preferences are elicited in a similar fashion by presenting subjects with choices between lotteries in random order, where most choices involved a "less risky" lottery that paid a single amount with certainty.[30] The expected value of the lotteries ranged from $40 CAD to $120 CAD. EJM find that subjects who choose the less risky lotteries have significant higher individual discount rates, but they do not estimate the relationship between risk and time preferences, or consider alternative functional forms.

5. Conclusions and Implications

We find that credible estimates of discount rates rely on the joint estimation of risk and time preferences. When one assumes that subjects are risk neutral when in fact they are risk averse, the estimated discount rates are significantly biased upwards. Most of the literature has been aware of this bias, but until now has not been able to formalize the estimation of time preferences to the point of decomposing it. We also show that this effect is independent of which of a wide range of alternative specifications is adopted, including hyperbolic discounting and prospect theory. This specification robustness is important given the role of parametric theory in allowing the identification of latent time preferences from observed choices.

Our results have direct implications for future efforts to elicit time preferences. The obvious one is to jointly elicit risk and time preferences, or at least to elicit risk preferences from a sample

[29] Rates as high as this are actually quite common in the extensive psychology literature where procedures are quite different from ours (e.g., Kirby and Maraković [1996]). The use of hypothetical scenarios, scrambling of the ordering of choices, and the absence of information on interest rates are common. Following CW, who also review earlier economics experiments that do not use hypothetical scenarios, it is now common to present subjects with an ordered series of choices to reduce simple confusion, and to present the interest rate information.

[30] Each of the first ten questions presented two lotteries with the same expected value, while the expected value of the less risky lottery was lower than the expected value of the more risky lottery in the last four questions. The predicted value of CRRA is 0.78, which is similar to other experimental evidence.

drawn from the same population, so that inferences about time preferences can be conditioned appropriately. There are also broader implications for the testing of theories of choice over time. Many "discounting anomalies" have been pointed to in the literature (see Frederick, Loewenstein and O'Donoghue [2002] for a review), and it is unclear *a priori* how the proper accounting for concave utility functions affects these anomalies. Similarly, if one adopts a characterization of EUT defined in terms of income, and thereby avoid the problems posed by Rabin [2000], one has to add assumptions about behavior over time. Rubinstein [2002] draws the important connection between adopting the assumption that utility is defined over income and the question of temporal consistency of preferences. When EUT is defined consistently over lifetime wealth, such consistency assumptions are implied; but when EUT is defined over income, consistency assumptions need to be added and evaluated, in the spirit of the older literature on the "asset integration hypothesis." Our results support the view that one can characterize EUT over income and still obtain time-consistent specifications, at least across temporal income choices made at the same point in time.

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Table 1: Typical Payoff Matrix in the Risk Aversion Experiments

Lottery A				Lottery B				EV^A	EV^B	Difference	Open CRRA Interval if Subject Switches to Lottery B
p	DKK	p	DKK	p	DKK	p	DKK	DKK	DKK	DKK	
0.1	2000	0.9	1600	0.1	3850	0.9	100	1640	475	1165	-∞, -1.71
0.2	2000	0.8	1600	0.2	3850	0.8	100	1680	850	830	-1.71, -0.95
0.3	2000	0.7	1600	0.3	3850	0.7	100	1720	1225	495	-0.95, -0.49
0.4	2000	0.6	1600	0.4	3850	0.6	100	1760	1600	160	-0.49, -0.15
0.5	2000	0.5	1600	0.5	3850	0.5	100	1800	1975	-175	-0.15, 0.14
0.6	2000	0.4	1600	0.6	3850	0.4	100	1840	2350	-510	0.14, 0.41
0.7	2000	0.3	1600	0.7	3850	0.3	100	1880	2725	-845	0.41, 0.68
0.8	2000	0.2	1600	0.8	3850	0.2	100	1920	3100	-1180	0.68, 0.97
0.9	2000	0.1	1600	0.9	3850	0.1	100	1960	3475	-1515	0.97, 1.37
1	2000	0	1600	1	3850	0	100	2000	3850	-1850	1.37, ∞

Note: The last four columns in this table, showing the expected values of the lotteries and the implied CRRA intervals, were not shown to subjects.

Table 2: Payoff Table for 6 Month Time Horizon in the Discount Rate Experiments

Payoff Alternative	**Payment Option A** (pays amount below in 1 month)	**Payment Option B** (pays amount below in 7 months)	**Annual Interest Rate** (AR, in percent)	**Annual Effective Interest Rate** (AER, in percent)	**Preferred Payment Option** (Circle A or B)	
1	3,000 DKK	3,075 DKK	5	5.09	A	B
2	3,000 DKK	3,152 DKK	10	10.38	A	B
3	3,000 DKK	3,229 DKK	15	15.87	A	B
4	3,000 DKK	3,308 DKK	20	21.55	A	B
5	3,000 DKK	3,387 DKK	25	27.44	A	B
6	3,000 DKK	3,467 DKK	30	33.55	A	B
7	3,000 DKK	3,548 DKK	35	39.87	A	B
8	3,000 DKK	3,630 DKK	40	46.41	A	B
9	3,000 DKK	3,713 DKK	45	53.18	A	B
10	3,000 DKK	3,797 DKK	50	60.18	A	B

Table 3: Baseline Estimates of Risk and Time Preferences

Parameter	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
			A. Allowing a Concave Utility Function (Risk Aversion)		
r	0.646	0.040	0.000	0.567	0.725
δ	0.082	0.011	0.000	0.060	0.103
μ (for RA)	0.129	0.014	0.000	0.101	0.158
ν (for DR)	0.047	0.006	0.000	0.035	0.058
			B. Assuming a Linear Utility Function (Risk Neutrality)		
δ	0.249	0.014	0.000	0.222	0.276
ν (for DR)	0.132	0.009	0.000	0.115	0.149

Figure 1: Estimated Discount Rates
Allowing for Heterogeneity of Preferences
Assuming exponential discounting and EUT

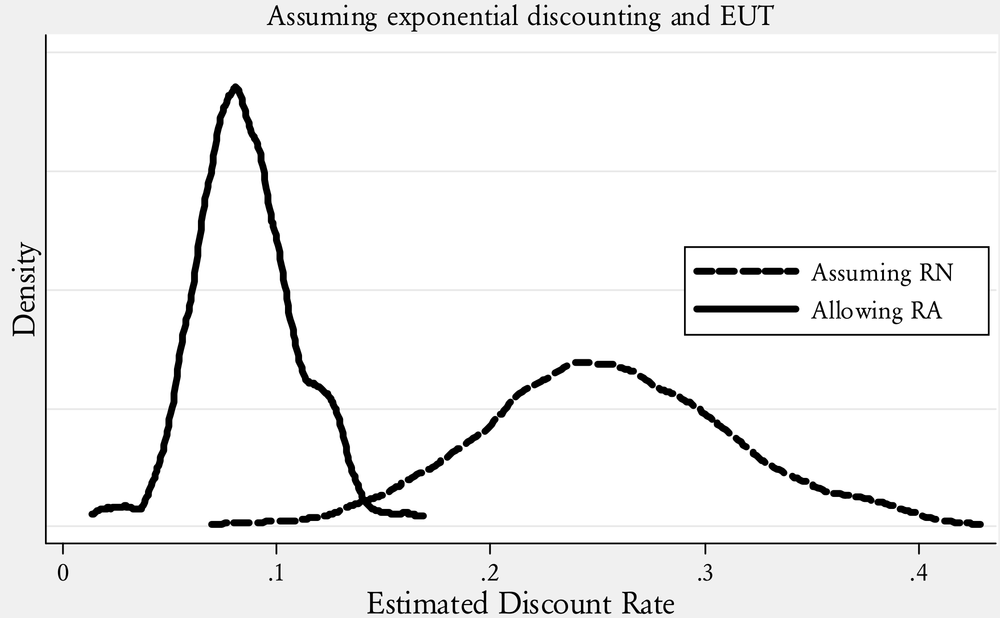

Table 4: Alternative Estimates of Risk and Time Preferences

Parameter	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
			A. Assuming Hyperbolic Discounting		
r	0.611	0.038	0.000	0.537	0.685
α	0.093	0.010	0.000	0.073	0.114
μ (for RA)	0.145	0.015	0.000	0.116	0.174
ν (for DR)	0.052	0.006	0.000	0.041	0.063
			B. Assuming Separable Prospect Theory		
r	0.546	0.029	0.000	0.488	0.603
γ	0.620	0.043	0.000	0.536	0.704
			0.000 (H_0: γ=1)		
δ	0.106	0.010	0.000	0.088	0.125
μ (for RA)	0.116	0.007	0.000	0.102	0.130
ν (for DR)	0.060	0.005	0.000	0.050	0.070
			C. Assuming Cumulative Prospect Theory		
r	0.442	0.040	0.000	0.363	0.522
γ	0.681	0.030	0.000	0.623	0.739
			0.000 (H_0: γ=1)		
δ	0.132	0.013	0.000	0.107	0.157
μ (for RA)	0.143	0.010	0.000	0.123	0.163
ν (for DR)	0.074	0.007	0.000	0.059	0.088

Figure 2: Assuming Hyperbolic Discounting

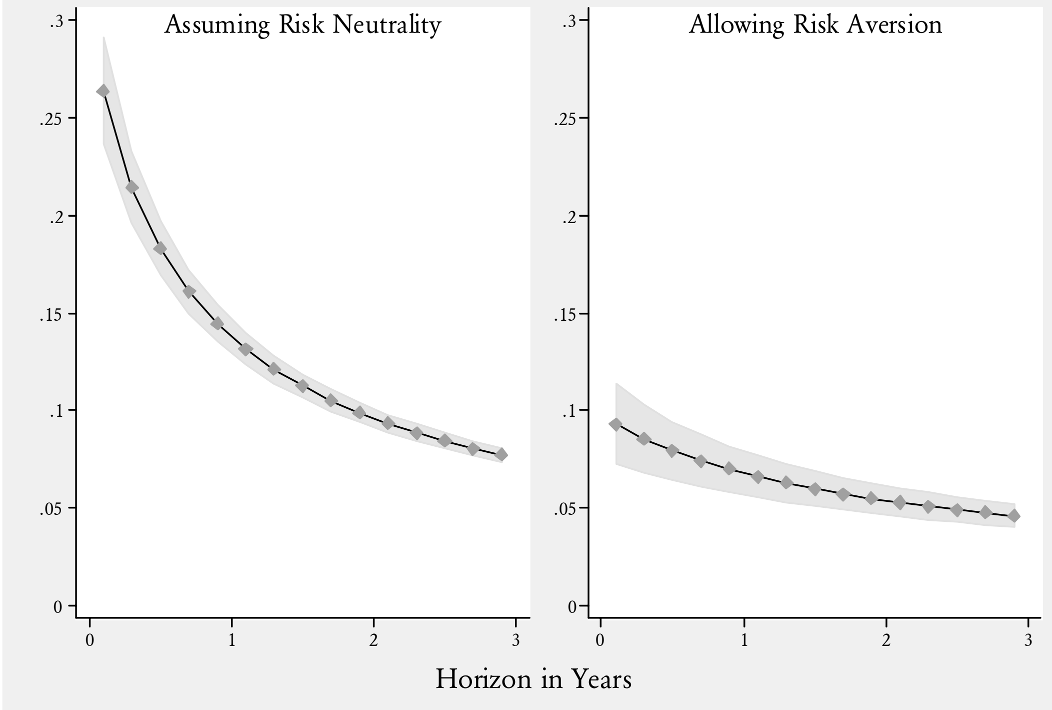

Figure 3: Comparing Exponential and Hyperbolic Discount Rates

Assuming concave utility functions and EUT

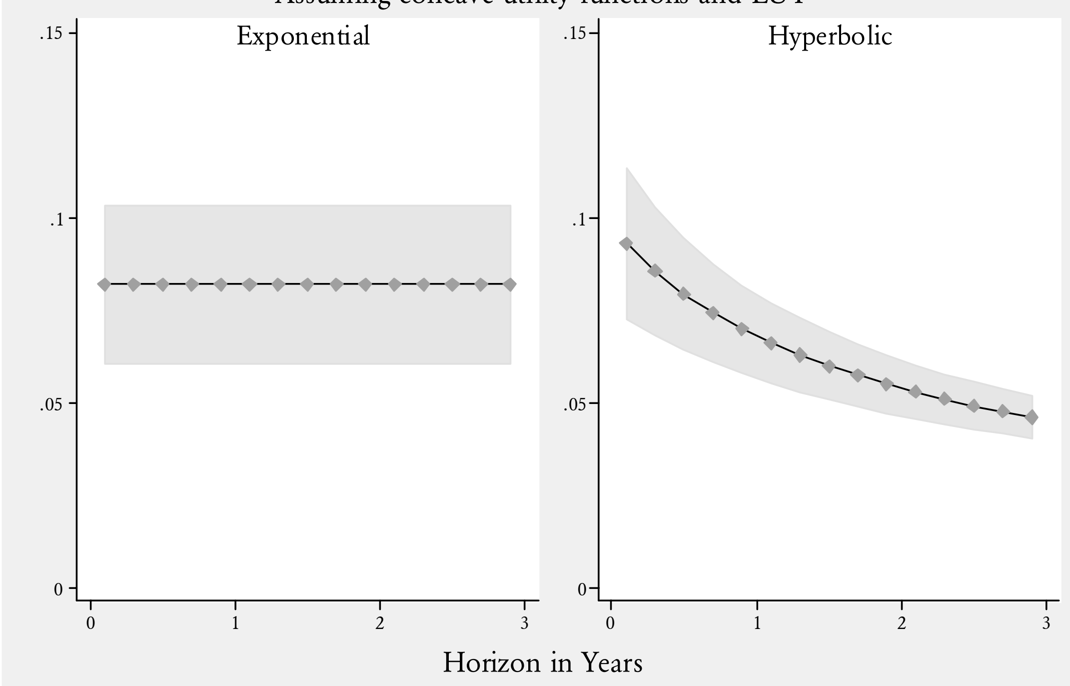

Figure 4: Estimated Discount Rates and Prospect Theory
Assuming exponential discounting and allowing for full demographics effects

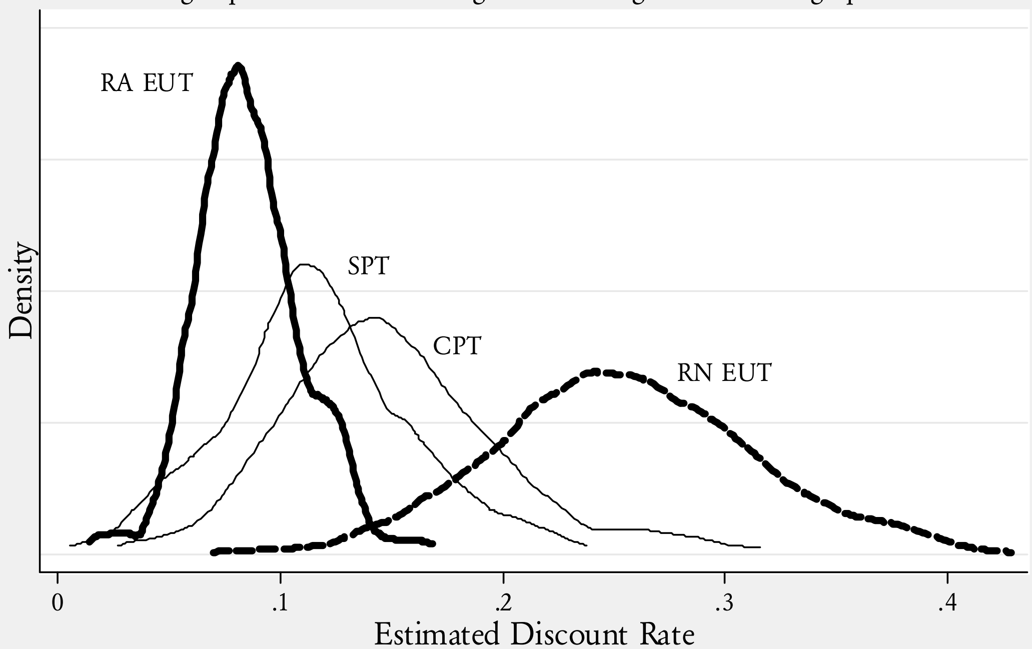

Table 5: Mixture Model Estimates of Risk and Time Preferences

Parameter	Estimate	Standard Error	*p*-value	Lower 95% Confidence Interval	Upper 95% Confidence Interval
		A. Risk Aversion			
r	0.645	0.040	0.000	0.566	0.724
		B. Exponential Discounting			
δ	0.143	0.019	0.000	0.107	0.180
		C. Generalized Hyperbolic Discounting			
β	0.051	0.008	0.000	0.035	0.067
α	1.012	0.089	0.000	0.837	1.187
			0.888 (H_0: α=1)		
		D. Stochastic Errors			
μ (for RA)	0.129	0.014	0.000	0.100	0.157
ν (for DR)	0.026	0.004	0.000	0.019	0.035
		E. Mixture Probability for Exponential Discounting			
π	0.341	0.045	0.000	0.252	0.430