Trade between symmetric countries, heterogeneous firms and the skill wage premium

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Abstract

This paper examines the effects of trade liberalisation between symmetric countries on the skill wage premium. I use a model of monopolistic competition with heterogeneous firms and two factors of production: skilled and unskilled labour. I introduce a correlation between productivity and skill intensity in the production process, which generates the empirically observed link between firm size, export status, wages and skill intensity. The entry and exit of firms following trade liberalisation has non-trivial effects on the demand for both types of labour, and therefore on their wages. I show that the impact of trade liberalisation on the skill premium depends on the type of trade costs considered, and on their initial size. While a decrease in the fixed costs of trade has a potentially non-monotonic effect, a drop in the variable trade costs yields an unambiguous and substantial increase in the skill premium. The calibration of the model to the U.S. economy shows that a reduction of the iceberg costs of trade from 1.5 to 1.1 can account for an increase in the skill premium of more than 10 percentage points, which is about a fourth of the observed rise in the 1980s and 1990s.

Keywords: Intra-industry trade, Firm heterogeneity, Trade liberalisation, Skill Premium

J.E.L. classification: F12, F16, J31

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1 Introduction

The strong rise in wage inequalities has, over the last decades, been one of the most debated political issues in industrial countries. The large increase in the returns to education (college wage premium[1]) from the early 1980s to the mid 1990s is well documented, and has been of dramatic magnitude from a historical perspective: Acemoglu (2002) shows that the U.S. college premium increased from 1.4 to 1.8 between 1979 and 1996. This trend has given rise to a large literature, which has spanned different fields of economics. Though prominent in the popular debate, international trade has never been a major explanation among economists, partly due to the weakness of its quantitative effect. A reason for this is that most trade studies have addressed the question using models based on North South heterogeneity[2], thereby leaving aside the bulk part of international trade, made of exchanges between industrial countries. It is only recently that trade economists have turned to intra-industry trade models as a potential determinant of the evolution of the skill premium.

In the last years, models of monopolistic competition with heterogeneous firms have been at the core of most developments in the international trade literature. Their popularity is based on their ability to match a number of well-established stylized facts linking firm characteristics to their export behaviour. Empirical studies such as Bernard and Jensen (1995, 1997) show that exporting firms are relatively skill intensive, while firms shutting down are less skill intensive than average (Bernard and Jensen (2007)). These facts suggest that trade liberalisation, due to its heterogeneous effects on different firms, can impact the skill wage premium through a reallocation of productive resources between firms. The aim of the present model is to explore this channel.

I develop a one-sector monopolistic competition framework with heterogeneous firms, in which firms produce with two factors of production: skilled and unskilled labour. I assume that firms are heterogeneous in the relative productivity of skilled labour, in the sense that some use skills more effectively than others. This establishes a correlation between productivity, skill intensity and exports: more productive firms are relatively skill intensive and export more than other firms, as confirmed by empirical studies. Following Melitz (2003), the present model is built on two types of trade costs: vari-

[1] The college premium is defined as the ratio of the wage of college graduates to the wage of non-graduates. I will refer to 'college premium' or 'skill premium' indifferently.

[2] Such as Heckscher-Ohlin types of model - see for example Krugman (2000) and Leamer (2000) on their relevance, or outsourcing models a la Feenstra and Hanson (2001)

able costs, which can be interpreted as transport costs or as tariffs, and fixed export costs, usually seen as administrative costs to export in a foreign country. As is standard in this type of models, these costs generate a partitioning between exporting and non-exporting firms.

The main contribution of this approach is to show that the evolution of the skill premium following trade liberalisation depends on two factors: the kind of trade costs considered and the magnitude of trade costs before liberalisation takes place (the 'initial' trade costs). On the one hand, a reduction in the variable costs of trade unambiguously raises the skill wage premium. On the other hand, a drop in the fixed costs of trade has an ambiguous, potentially non-monotonic effect on the skill premium. Indeed, for sufficiently low initial costs of trade, a further reduction in the fixed costs of exporting decreases the skill premium. I calibrate the model to match key variables of the U.S. economy and show that it has a substantial quantitative effect. A plausible reduction in the variable costs of trade between three identical countries (a reduction of the iceberg costs of trade from 1.5 to 1.1) can account for an increase in the skill premium of more than 10 percentage points, which is roughly a fourth of the observed rise in the 1980s and 1990s.

The core mechanism driving the results is the reallocation of productive resources between firms following trade liberalisation. A reduction in the variable costs of trade makes the export activity cheaper, so that exporting firms, which are more skill intensive than average, scale up their demand for labour (the first effect). Relatively unproductive firms are, on the other hand, driven out of the market, releasing much unskilled labour (the second effect). Both effects tend to raise the skill premium. The skill intensity of firms newly entering the export market (the third effect) is however undetermined and depends on the initial costs of trade. If initial trade costs are high, only productive firms, with a higher than average skill intensity enter the export market following liberalisation. This drives the skill premium further upwards. If the costs of trade are initially low, however, the firms entering the export market are relatively unskilled intensive, and their entry provides a countervailing force to the increase in the skill premium. I show that this third effect cannot overturn the first two, so that the skill wage premium unambiguously increases. On the other hand, if the fixed costs of exporting decrease, the first effect disappears, and the skill wage premium can decrease if trade is initially cheap. The third effect, which counteracts the rise in the skill premium as trade costs decrease, provides a rationale for the observed slowdown in the growth of the skill premium from the mid 1990s onwards[3].

[3] This fact has given rise to a literature contesting the standard skill-biased technological

This paper is related to the nascent literature discussing the link between the skill premium and intra-industry trade. Epifani and Gancia (2008) assume a correlation between the scale and the skill intensity of a sector, and follow Dinopoulos et al. (2002) who assume such a correlation at the firm level. With appropriate assumptions on preferences, the increase in scale inherent to trade liberalisation therefore exerts a bias towards skill demand and raises the skill wage premium. Both models use a representative firm's framework, so that the mechanisms driving the results are different from the present paper, which concentrates on factor reallocation between heterogeneous firms. Yeaple (2005) uses a monopolistic competition model in which ex-ante homogeneous firms choose between two production technologies with different complementarities to skills. High technology firms self-select into exporting while low technology firms remain purely domestic. There is no difference between firms of the same technology type. Labour consists of a continuum of skills, with high (low) skilled labour matched to the high (low) technology. Following trade liberalisation, some firms enter the export market and switch to the high technology, driving the skill premium up no matter which type of liberalisation occurs. The present model largely differs in its construction and conclusions. The source of heterogeneity is different, since I use two types of labour and a continuum of technologies. As in the standard models in the literature, firms receive an exogenous productivity and do not switch technology while entering the export market. This feature has an important impact on the results, since it drives the countervailing effect (the third effect) of liberalisation on the skill premium. The evidence on firms switching technologies upon entering the export market is at best mixed, while there is massive evidence that new exporters differ from continuing non-exporters long before their entry in the export market[4]. These facts suggest that the present model provides a useful complementary analysis to that of Yeaple (2005), with a stronger focus on firm heterogeneity. Finally, Yavas (2006) builds on Melitz (2003) to analyse the evolution of the skill premium, but does not allow for variable costs of trade, which is a major restriction considering both their empirical importance and their effect on the analysis.

The second strand of literature to which this paper relates is the rapidly

change hypothesis as an explanation for the rise in the skill premium, see Card and diNardo (2002).

[4]See Clerides et al. (1998), Bernard and Jensen (1999), Delgado et al. (2002) or Pavcnik (2002) among others. Some of these studies suggest that firms entering the export market have a high productivity growth before entry. Since this is contemporaneous with an increase in size, it cannot be however concluded that they switch technology but could be a pure size effect. Direct evidence on technology upgrading is still lacking.

expanding field of heterogeneous firm in trade[5]. The present model matches some important empirical features about exporting firms emphasised in this literature. Not surprisingly since I largely build on Melitz (2003), the fact that exporting firms are bigger and more productive is preserved, and conform to the conclusions of Bernard and Jensen (1995), Bernard et al. (2003) and many others. Additionally, the model typically generates higher average wages for exporting firms[6] due to their employing relatively more skilled labour, a feature which conforms to the results of Bernard and Jensen (1995, 1997). They provide a strong empirical case for my approach, arguing that 'the between plant movement of workers and wages, which are especially important in the increases in the aggregate wage gap, are largely determined by demand shifts across plants, and in particular by export related demand movements' (Bernard and Jensen, 1997, p.25).

The remainder of the paper is structured as follows. Section 2 develops the model. Section 3 derives the comparative statics of the skill premium following a marginal trade liberalisation. Section 4 provides a numerical solution to the model for illustrative and quantitative purposes. Section 5 concludes.

2 The model

2.1 Demand

The world consists of two identical countries. All consumers in each country are identical except for their income and share the same constant elasticity of substitution (C.E.S.) utility function over a continuum of varieties. Each consumer has utility:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where the set Ω represents all available varieties, $q(\omega)$ stands for the consumption of variety ω by the consumer, and ϵ is the elasticity of substitution between varieties, assumed to be strictly greater than one. Consumers preferences therefore exhibit the usual love of variety property following Dixit

[5]Melitz (2003), Bernard et al. (2003), Helpman et al. (2004), Bernard et al. (2004), Chaney (2006) among others.

[6]Under the sufficient and plausible condition that there is weakly less skilled than unskilled labour in the economy and that skilled labour is at least as productive as unskilled labour.

and Stiglitz (1977). Define:

$$C \equiv \left[\int_{\omega\in\Omega} q^1(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

as the optimal consumption bundle costing one unit of income. This optimal bundle is the result of the maximisation of utility with respect to the consumption of each variety and subject to the budget constraint: $PC = 1$, where P denotes the price index of the optimal bundle. This yields:

$$q^1(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\epsilon} C = p(\omega)^{-\epsilon} P^{\epsilon-1} \tag{3}$$

where P is:

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\epsilon} d\omega \right]^{\frac{1}{1-\epsilon}} \tag{4}$$

Since preferences are homothetic, the aggregate demand for a variety in a country is given by:

$$q(\omega) = p(\omega)^{-\epsilon} P^{\epsilon-1} I \tag{5}$$

where I denotes the aggregate income of all consumers in a country. This income consists of the proceeds of capital, labour and of profits. From (5), the demand for a variety decreases in its relative price and increases in national income.

2.2 Production

In each country, there is a continuum of firms, each producing a different variety. Production uses two factors, skilled (s) and unskilled (u) labour, internationally immobile and in fixed aggregate supply. They are combined in a C.E.S. production function:

$$y = \left[u^{\frac{\sigma-1}{\sigma}} + z^{\frac{1}{\sigma}} s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where $\sigma > 1$ is the elasticity of substitution between the two factors of production. This is an empirically founded assumption, as most studies estimate a parameter σ between 1 and 2 for industrial countries.[7] Firms are heterogeneous as to the productivity of skilled labour, indexed by z, which is the realisation of a random variable, drawn from an exogenously given continuous

[7] See Acemoglu (2002) p.20

distribution with support $[\underline{z}, \infty]$, where $\underline{z} \geq 1$ in order to ensure that skilled labour is more productive than unskilled labour. Acemoglu (2002) among others uses the same form of production function to study skill-biased technological change, where $z^{\frac{1}{\sigma}}$ increases exogenously over time[8]. In order to produce, a firm needs to pay a fixed cost f, in terms of capital K, at a unit price r. This assumption largely simplifies the labour market equilibrium conditions and is neutral for the study of the skill premium[9]. I assume that both countries are perfectly symmetric in every respect, so that factor prices are equal across countries.

The marginal costs of production (m) are the lowest possible costs for a firm to produce a unit of its variety. This is the solution to the minimisation problem:

$$\min_{u,s}(w_u u + w_s s) \quad \text{s.t. } y \geq 1 \tag{7}$$

where w_u and w_s are respectively the wage of unskilled and skilled labour. The first order conditions of the minimisation problem yield:

$$s = zuw^{-\sigma} \tag{8}$$

where $w = \frac{w_s}{w_u}$ is the skill premium. From (6) and (8), the unit unskilled and skill requirements are given by:

$$u = y\left(1 + zw^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \tag{9}$$

$$s = yzw^{-\sigma}\left(1 + zw^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}} \tag{10}$$

The marginal cost of production of a firm having drawn z are therefore given by:

$$m(z) = w_u(1 + zw^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{11}$$

The first equation states that the skill intensity of a firm increases in z and decreases in the skill premium. The second shows that marginal costs decrease with z. The following lemma follows directly:

[8]Note that z enters to the power $\frac{1}{\sigma}$ only for simplicity.

[9]Assuming that fixed costs are paid as a fraction of output as in Yeaple (2005) would here not be neutral for the skill premium since the composition of fixed costs would depend on the skill intensity of the firm. Assuming that these are paid in terms of skilled labour as in Ekholm and Midelfahrt (2005) would complicate the labour market conditions and generate an inverse relationship between size and skill intensity which runs counter to empirical evidence.

Lemma 1 *Skill intensive firms have lower marginal costs of production.*

This correlation between productivity and skill intensity is well established empirically, as shown by Idson and Oi (1999), Haltinwanger et al. (1999) and Bernard and Jensen (1995).

The domestic profits of a firm z ($\pi_d(z)$) are given by:

$$\pi_d(z) = (p - m(z))q_d - fr \tag{12}$$

Due to the monopolistic competition structure of the model, there is no strategic interaction between firms, which maximise their profits taking the average price P as given. The optimal decision of a firm is to set the price for its variety on its domestic market equal to a constant markup over marginal costs:

$$p_d(z) = \frac{\epsilon}{\epsilon - 1} m(z) \tag{13}$$

Using the demand equation (5) and the optimal price (13), the quantity sold by a firm on its domestic market is:

$$q_d(z) = \left(\frac{\epsilon}{\epsilon - 1}\right)^{-\epsilon} (m(z))^{-\epsilon} P^{\epsilon-1} I \tag{14}$$

A firm sells more the lower its marginal cost and the higher the average price of its competitors P. More productive firms are therefore larger and, from Lemma 1, more skill intensive. The domestic profits realised by a firm z are therefore:

$$\pi_d(z) = A(m(z))^{1-\epsilon} P^{\epsilon-1} I - fr \tag{15}$$

where for simplicity: $A \equiv \frac{1}{\epsilon}\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon}$. The profits of a firm are increasing in I, the level of income of the country, P and z, which indexes its productivity.

Due to the existence of fixed costs of production, not all firms find it profitable to produce. The least productive producing firm is the one having drawn a productivity $z = z^*$, at which it makes zero profits. All entrepreneurs having drawn a $z < z^*$ do not find it profitable to produce and stay out of the market. Setting $\pi(z^*) = 0$ in (15) and solving for the price index:

$$P^{\epsilon-1} = \frac{fr(m(z^*))^{\epsilon-1}}{AI} \tag{16}$$

This establishes a negative relationship between the price index P and the cutoff level z^*. If the price index is low, a firm faces very productive competitors on average, and the demand for its variety is low. It should therefore be relatively productive to be able to cover the fixed costs of production and make non-negative profits.

Using (9), (10), (11) and (14), the amount of unskilled and skilled labour employed by a firm $z \geq z^*$ for domestic production is:

$$u_d(z) = A(\epsilon - 1)\left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} P^{\epsilon-1} I w_u^{-\epsilon} \quad (17)$$

$$s_d(z) = zw^{-\sigma} A(\epsilon - 1)\left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} P^{\epsilon-1} I w_u^{-\epsilon} \quad (18)$$

Each firm also has the possibility to export to the other country if it finds it profitable. Exporting requires the payment of two additional types of costs. Iceberg costs $\tau \geq 1$ are the fraction of goods that must be produced in order for one unit of the good to arrive at destination. Shipping costs or tariffs are typical interpretations for such iceberg costs. Additionally, an exporting firm has to incur a fixed cost of exporting f_x, which reflects the additional costs of doing business abroad, of establishing a distribution network, etc. There is much empirical evidence about the importance of these fixed costs of exporting, which generate a partition of firms between non-exporting and exporting firms as long as: $\frac{f_x}{f}\tau^{\epsilon-1} > 1$. I assume that these fixed costs of exporting are paid in terms of capital, and that both types of trade costs are symmetric between the two countries.

By a similar argument to the one presented for the domestic case, the optimal price charged by a firm on the export market is:

$$p_x(z) = \tau p_d(z) \quad (19)$$

An exporting firm charges the same mark-up on both markets due to the C.E.S. preferences, but faces higher costs of selling on the export market due to the iceberg costs. Using the demand equation (5), this translates into the following profits on the export market:

$$\pi_x(z) = \tau^{1-\epsilon} A w_u^{1-\epsilon}\left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} P^{\epsilon-1} I - f_x r \quad (20)$$

Setting these profits equal to zero defines the level z_x^* of the productivity parameter z that makes a firm indifferent between exporting or not. Plugging (16) for the price index in $\pi_x(z_x^*) = 0$ yields:

$$\left(1 + z_x^* w^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} = \frac{f_x}{f}\tau^{\epsilon-1}\left(1 + z^* w^{1-\sigma}\right)^{\frac{\epsilon-1}{\sigma-1}} \quad (21)$$

From the assumptions that $\frac{f_x}{f}\tau^{\epsilon-1} > 1$, it is immediate that the cutoff export level z_x^* is higher than the domestic cutoff level z^*. This generates the well-known partitioning between exporting and non-exporting firms, and by Lemma 1, the empirically established facts that exporting firms are more productive and more skill intensive than non-exporting firms. Furthermore, if $w > 1$[10], this ensures that exporters pay on average higher wages than non exporters, as many empirical studies confirm.

In the same way as for the domestic decision, the number of workers used for export production by firms $z \geq z_x^*$ are given by:

$$\begin{aligned} u_x(z) &= \tau^{1-\epsilon} u_d(z) & (22) \\ s_x(z) &= \tau^{1-\epsilon} s_d(z) & (23) \end{aligned}$$

Due to the iceberg costs, firms must employ more labour to be able to sell the same amount on the export market than on the domestic market. But the higher price they charge decreases the demand for their variety on the export market, and therefore the labour they employ. This second effect dominates, so that firms employ weakly less labour for their exports than for their domestic production.

2.3 Equilibrium

There is an exogenous constant mass of entrepreneurs M in each country. This conforms to Chaney (2006) and differs from the original Melitz (2003) framework in that there is no free entry condition, no dynamics of entry and exit, and that there are positive aggregate profits in the economy. The amount of factors available in a country is exogenous, and denoted as K for the stock of capital, U for the mass of unskilled labour and S for the mass of skilled labour. Using (17), (18), (22) and (23), the factor market equilibrium in each country is given by:

$$\begin{aligned} U &= M\left[\int_{z^*}^{\infty} u_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*}^{\infty} u_d(z)dG(z)\right] & (24) \\ S &= M\left[\int_{z^*}^{\infty} s_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*}^{\infty} s_d(z)dG(z)\right] & (25) \\ K &= M\left[(1-G(z^*))f + (1-G(z_x^*))f_x\right] & (26) \end{aligned}$$

These market clearing conditions, as well as (21), define the equilibrium.

[10]Sufficient for this is that $\underline{z} > 1$, i.e. that skilled labour is always more productive than unskilled labour, and that there are weakly more unskilled than skilled workers in the economy.

Since the primary interest of this paper is the evolution of the skill wage premium, it is worth noting that the equilibrium tuple (z^*, w) solves the following system of equations:

$$\frac{U}{S} = \frac{\int_{z^*}^{\infty} u_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*(z^*)}^{\infty} u_d(z)dG(z)}{\int_{z^*}^{\infty} s_d(z)dG(z) + \tau^{1-\epsilon}\int_{z_x^*(z^*)}^{\infty} s_d(z)dG(z)} \tag{27}$$

$$\frac{f_x}{f} = \left(\frac{1 + z_x^*(z^*)w^{1-\sigma}}{1 + z^* w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}} \tau^{1-\epsilon} \tag{28}$$

The first equation is the ratio of the two labour market equilibrium conditions (24) and (25), where the market equilibrium for capital (26) defines the implicit function $z_x^*(z^*)$. The second equation is the indifference condition of the cutoff exporting firm (21). Equations (27) and (28) allow to solve for the skill wage premium independently of the interest rate r.

A number of assumptions on the parameters is needed at this stage to ensure the existence of an equilibrium:

Assumption 1 $\frac{K}{M} \leq f$

Assumption 1, which is sufficient but not necessary for the results, means that capital is scarce in the sense that the stock of capital is not sufficient for all potential entrepreneurs in a country to pay the fixed costs of production. This is due to the construction of the model, which features a fixed mass of potential entrepreneurs and in which capital is only required for the payment of fixed costs, meaning that there is a maximum amount of capital which firms can demand. Imposing Assumption 1 ensures both that the capital market equilibrium can hold with equality, and that the ratio $\frac{z_x^*}{z^*}$ can become arbitrarily large[11].

Assumption 2 $\epsilon \geq \sigma$

This assumption requires that the varieties are better substitutes in the utility functions than skilled and unskilled labour in the production function. This is a rather innocuous assumption considering the numerous empirical estimations for these parameters. Estimates of σ are usually[12] comprised between 1 and 2 while ϵ tends to be higher, between 3 and 6[13].

[11] Under this assumption, if z^* is small enough, z_x^* has to go to infinity for (26) to hold. Even if there are only domestic firms on the market, capital is used up and has a positive price.

[12] Acemoglu (2002) p.20

[13] Bernard et al. (2003) and many others use 3.8, while a reasonable markup of 20 percent would require $\epsilon = 6$. Both σ and ϵ depend on the sector considered.

Proposition 1 *Under Assumptions 1 and 2, there exists a unique equilibrium.*

Proof. *See Appendix* ■

Under Assumptions 1 and 2, (27) and (28) respectively establish a positive and a negative relationship between w and z^*, as illustrated in figure 2. I show in the Appendix that the two curves defined in the (z^*, w) space intersect and that an equilibrium exists and is unique.

3 Trade liberalisation

The central aim of this model is to study the impact of a marginal perturbation in the costs of exporting on the wage gap between unskilled and skilled labour. I use two definitions of trade liberalisation: (i) a bilateral decrease in the iceberg costs, which can be interpreted as a reduction in tariffs or freight costs, (ii) a bilateral reduction in the fixed costs of exporting, which may occur due to the dismantling of certain types of non tariff barriers[14] or of any other measure hampering entry on the export market. Distinguishing between these two types of trade liberalisation is of importance for policy analysis. In order to derive the results, I conduct a comparative static analysis by totally differentiating (27) and (28) with respect to both trade costs measures f_x and τ.

3.1 Trade liberalisation as a marginal decrease in variable costs of exporting τ

I first consider the effect of a reduction in the iceberg costs τ on the skill premium w, and refer to the 'initial' situation for the equilibrium prevailing before any perturbation in the costs of trade.

Lemma 2 $\frac{dw}{d\tau}$ *has the same sign as* Δ*, defined as follows:*

$$\Delta \equiv \eta \int_{z_x^*}^{\infty} (1 + zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}} (Bzw^{-\sigma} - C) dG(z) + \xi \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}} - \xi \frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} \quad (29)$$

[14] Adapting a product to foreign regulation is a common example for fixed costs of exporting.

where $\eta, \xi < 0$ *and:*

$$Bz'w^{-\sigma} - C = w^{-\sigma}\left[\int_{z^*}^{\infty}(w^{1-\sigma}+z)^{\frac{\epsilon-\sigma}{1-\sigma}}(z'-z)dF(z) + \tau^{\epsilon-1}\int_{z_x^*}^{\infty}(w^{1-\sigma}+z)^{\frac{\epsilon-\sigma}{1-\sigma}}(z'-z)dF(z)\right] \tag{30}$$

for all $z' \geq z^*$

Proof. *See Appendix* ∎

First, note that $Bz'w^{-\sigma} - C$ represents the relative skill intensity of a given firm z'. It is positive (negative) if firm z' is more (less) skill intensive than the average. The above lemma can be interpreted as follows.

For a constant w, trade liberalisation has three effects on firms, as shown by the three components of (29). These are essentially identical to those highlighted in Melitz (2003), and are represented in figure 1.

First, following a decrease in the marginal costs of exporting, the most productive firms, which are initially exporting, find it profitable to scale up their production aimed at the export market (arrow 1 in figure 1). Second, the most productive among the initially non-exporting firms can now make weakly positive profits on the export market, in which they enter (arrow 2). This decreases the export cutoff level z_x^*. As this last channel tends to increase the demand for capital, the cutoff level of domestic production z^* needs to rise for the capital market equilibrium to hold. The least productive producing firms in the economy therefore drop out of the market (arrow 3), which is the third effect of trade liberalisation on firms. I will denote these three effects respectively as Effect 1, 2 and 3 in accordance with the arrows of figure 1 and the order in which they appear in (29).

These three effects have different consequences for the unskilled and skilled labour markets. Two of them have a clear positive effect on the relative demand for skilled labour. First, initially exporting firms are productive and relatively skill intensive. An expansion of their production (Effect 1) thus increases the demand for skilled labour more than that of unskilled labour, as shown by the negative sign of the first term in (29). This tends to raise w. Second, firms dropping out of the domestic market (Effect 3) are relatively unproductive and unskilled intensive. They therefore release much unskilled labour, and their effect on (29), represented by the second term, is also negative. These two effects are exactly conform to the empirical evidence of Bernard and Jensen (1995, 1997, 2007).

The impact of firms newly entering the export market (Effect 2) is however undetermined. It depends on the relative unskilled intensity of the cutoff export firms, represented by the third term in (29). A positive (negative) sign means that these firms are relatively skilled (unskilled) intensive. From (30), it is immediate that this sign depends on the initial z_x^*, which is a function of the initial size of the trade costs τ and f_x by (28). If trade is initially expensive, only very productive firms are able to export, and the cutoff export firm is relatively skill intensive. In this case, firms newly entering the export market following a marginal trade liberalisation are also skilled intensive, and increase the relative demand for skilled labour. On the other hand, if trade is initially cheap, unskilled intensive firms can benefit from the trade liberalisation by becoming able to export. If this is the case, the entry of firms with a lower than average skill intensity on the export market provides a countervailing force to the rise in the skill wage premium.

Central for the analysis is to determine whether the third effect described above can overturn the other two and cause a decrease in the skill premium. It appears that this cannot be the case.

Proposition 2 *A decrease in the variable costs of trade unambiguously increases the skill wage premium.*

Proof. *See Appendix* ■

The proposition states that the effect of new firms entering the export market cannot be strong enough to reduce the skill premium. The reason is that effect 2 cannot overturn effect 3, as can be seen from (29). The additional effect of initially exporting firms therefore guarantees that the skill wage premium strictly increases when τ decreases.

In the description of the three effects above, it was implicitly assumed that a rise in the relative demand for skilled labour raises the skill premium. As shown in the Appendix[15], this is indeed the case. The increase in the skill premium however does not have the same effect on all firms: larger firms, which are more skill intensive, see their cost rise by a higher proportion. Smaller firms therefore benefit from this countervailing force to trade liberalisation, which may even overcompensate the direct effect of a decrease in trade costs and yield a decrease in the cutoff level of firm z^*. Though relevant for quantitative purposes or for the welfare analysis, this indirect effect does not affect the qualitative result that a decrease in the variable trade costs of trade raises the skill premium.

A potential concern about the specification of the model is that the assumption of iceberg transportation costs may drive the results. The assumption on trade costs is indeed not neutral for the skill premium, as it requires

[15] Appendix A.2., equation (48) and its interpretation

that they are paid in terms of exported goods. Since exporting firms are more skill intensive than average, it implies that variable trade costs are relatively skill intensive. As long as a decrease in variable trade costs raises the amount of resources used for transportation[16], this effect may tend to increase the skill premium. A simple way of showing that it does not drive the result is to assume that trade costs are paid in terms of an asset, which is in infinitely elastic supply, has a price t and is held by a third country. I moreover assume that in order to export one unit of its output, a firm z has to pay a cost $tm(z)$. This assumption has the similarity with iceberg transport costs that more productive firms pay lower transportation costs per unit shipped[17] and allows me to concentrate on the relevant aspect for the skill premium. The marginal costs of selling to a foreign consumer are therefore given by:

$$m_x(z,t) = (1+t)w_u(1+zw^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{31}$$

Using the demand equation (5), the price index (16) and the results of the cost minimisation problem (8) gives the amount of labour used by a firm z for its exports:

$$\begin{aligned} u_{xt}(z) &= (1+t)^{-\epsilon}\frac{r}{w_u}(\epsilon-1)(1+zw^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}(1+z_x^*w^{1-\sigma})^{\frac{1-\epsilon}{\sigma-1}} && (32) \\ s_{xt}(z) &= u_{xt}(z)zw^{-\sigma} && (33) \end{aligned}$$

The only change to the equilibrium conditions is therefore that $\tau^{1-\epsilon}$ in (27) should be replaced by $(1+t)^{-\epsilon}$. It can be readily seen from (49) in Appendix that the comparative statics of the model with respect to t are qualitatively similar to those derived with respect to τ. This small extension therefore confirms that the result of Proposition 2 does not rely on the skill intensity of iceberg trade costs.

3.2 Trade liberalisation as a marginal decrease in the fixed costs of exporting f_x

Trade liberalisation can also be defined as a drop in the fixed costs of exporting f_x.

[16]It will be the case as long as the rise in trade implied by a decrease in costs is stronger than the gain due to the lower unit cost of trade.

[17]The assumption is plausible for insurance costs of trade or for ad valorem tariffs (more productive firms sell in this model goods of lower price), less so for freight costs.

Lemma 3 $\frac{dw}{df_x}$ *has the same sign as* Δ'*, defined as follows:*

$$\Delta' \equiv \theta(Bz_x^* w^{-\sigma} - C) + \kappa(Bz^* w^{-\sigma} - C) \tag{34}$$

where θ *and* κ *are defined in the Appendix*

Proof. *See Appendix* ■

The mechanism at stake is very similar to that highlighted in the case of variable costs of trade but the first effect in (29) (Effect 1) disappears, because a decrease in the fixed costs of exporting, though it raises the profits of all exporting firms, does not change their level of production and employment, which only depends on the marginal costs of exporting. Arrow 1 in figure 1 is therefore not relevant in the present case. Initially exporting firms therefore do not scale up their demand for labour. The only two effects remaining are the marginal effects of firms dropping out of the domestic market (Effect 3) and firms entering the export market (Effect 2), as can be seen from the two terms in (34).

Proposition 3 *A decrease in the fixed costs of exporting has an ambiguous effect on the skill wage premium. For* τ *small enough, there is a level of* f_x *below which a marginal reduction in the fixed costs of exporting decreases the skill wage premium.*

Proof. *See Appendix* ■

Comparing Propositions 2 and 3 highlights the qualitative difference between the two types of trade liberalisation for their impact on the skill premium. The proof of Proposition 3 requires to show that, contrary to the variable cost case, the effect on the skill premium of firms exiting the market (Effect 3) can here be overturned by the effect of new firms entering the export market (Effect 2) if these are sufficiently unskilled intensive.

As shown by (57) in Appendix, Δ' can be rewritten as:

$$\begin{aligned} \Delta' &= \gamma \left[\frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} - \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}} \right] \\ &- \xi \left[Bz_x^* w^{-\sigma} - C + \frac{g(z^*) f}{g(z_x^*) f_x} (Bz^* w^{-\sigma} - C) \right] \end{aligned} \tag{35}$$

where $\gamma, \xi > 0$.

A reduction in f_x has an impact on the cutoff levels z^* and z_x^* (effects 3 and 2) through two channels, which determine the relative sizes of both effects. First, the increase in the profits of exporting decreases z_x^* since some firms find it profitable to enter the export market. By the capital market equilibrium (26), z^* therefore increases. This first channel, given by the first line in (35), is exactly the one that prevailed in the case of a decrease in iceberg costs. From this channel, effect 2 cannot overturn effect 3, so that the skill premium tends to rise. As $z_x^* \to z^*$, the effect goes to zero. Second, the reduction in f_x has a direct effect on the capital market equilibrium (26). For a constant z_x^*, a marginal decrease in f_x releases capital, since exporting firms need to pay lower costs. The additional capital mitigates the increase in z^* that is needed for the capital market equilibrium to hold. The exit of unskilled intensive firms (effect 3) is therefore attenuated in comparison to the effect of firms entering the export market (effect 2). If these new exporting firms are sufficiently unskilled intensive, this second channel, which corresponds to the second line in (35), allows effect 2 to overturn effect 3, and therefore the skill premium to rise.

The fact that effect 2 can overturn effect 3 if f_x decreases, but cannot if τ decreases, may at first seem driven by the construction of the model. Indeed, fixed costs of exports have an additional direct effect on the demand for capital that variable costs do not have. However, allowing for such direct effects of variable trade costs on the capital market would strengthen the results. Indeed, for a constant z_x^*, a lower τ would raise the demand for productive factors[18], thereby strengthening effect 2 compared to effect 3. This difference in the relative strength of effects 2 and 3 is therefore due to a property of heterogeneous firms models, in which, for a constant z_x^*, a decrease in τ raises the demand for production factors, while a decrease in f_x reduces it. This effect has, to my knowledge, never been explicitly pointed out.

4 Numerical simulations

In the present section, I calibrate the model to match key variables of the U.S. economy and study the quantitative impact of trade liberalisation on the skill premium. This serves the purpose of illustrating the results and testing their quantitative importance as well as to ensure that the restrictions on the parameters imposed in the theoretical part are quantitatively sensible. I find that a plausible multilateral reduction in variable trade costs has a substantial effect on the skill premium.

[18]The reason is that exporting firms would scale up their production for the export market.

4.1 Calibration

For all exogenous parameters of interest, I use values which are now well established in the literature. Following Bernard et al. (2003) and subsequent studies, I assume that the consumers' elasticity of substitution (ϵ) is approximately equal to 4. I set the elasticity of substitution between factors in the production function (σ) to 1.5, which is in the middle of the range suggested by Acemoglu (2002)[19]. I set $\frac{U}{S} = 1.4$, which was the value prevailing in the U.S. in 1995 according to Acemoglu (2002) and $f = 1$ without loss of generality.

The literature on heterogeneous firms suggests that the right tail of the distribution of firms domestic sales can be well approximated by a Pareto distribution. Axtell (2001), in a seminal contribution, estimates the parameter of this distribution using U.S. census data for 1997 to be 1.06 for large firms. In line with the literature, I assume that the productivity parameter z is Pareto distributed, i.e. that:

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^a \tag{36}$$

I further assume that $\underline{z} = 1$, which requires that skilled labour be always at least as productive as unskilled labour. The original model of Melitz (2003) has the nice property that a Pareto distribution of productivity yields a Pareto distribution for sales, a property that is not preserved in the present model. However, as argued in the appendix, for firms with large z, the distribution of sales converges to a Pareto distribution with parameter $\frac{a(\sigma-1)}{\epsilon-1}$. In order to be in line with Axtell (2001), I impose that $a = 1.06\frac{\epsilon-1}{\sigma-1} = 6.36$.

For the calibration of the model, I assume that $\tau = 1.3$ as in Ghironi and Melitz (2005), which is close to Obstfeld and Rogoff (2000). I calibrate $\frac{K}{M}$ and f_x such that: (i) $w = 1.8$, which corresponds to the estimation of the skill premium presented in Acemoglu (2002) for the U.S. in 1996. (ii) The percentage of exporting firms: $\frac{1-G(z_x^*)}{1-G(z^*)}$ is 21%, which is a common estimate for the U.S. and is close to the estimate for other industrial countries (Bernard et al. (2003), Ghironi and Melitz (2005)). In order to match these values, the fixed costs of export should be set at: $f_x = 0.916$ and the stock of capital $\frac{K}{M} = 0.765$. It is worth noting that Assumption 1 is then fulfilled.

This provides the benchmark case for the simulation. In the next step, I examine the impact of both types of trade costs on the skill premium.

[19] I checked the results for different values of the elasticities in the range usually estimated. It has only little influence on the quantitative results.

4.2 Results

Figure 3 isolates the effect of a change in τ for a given fixed cost of exports which remain at 0.916. This confirms the result of Proposition 2 and shows that a marginal decrease in the *variable* costs of trade from the reference situation would increase the skill premium. Indeed, a reduction of the iceberg trade costs from 1.3 to 1.1 increases the skill premium by four percentage points, which is not negligible.

Figure 4 on the other hand shows the effect of a decrease in the *fixed* costs of exports on the skill wage premium for a constant $\tau = 1.3$. The fixed costs of exports are allowed to decrease down to a lower limit of $f_x = 0.46$, below which there would be no partitioning between exporting and non-exporting firms (the condition $\tau^{\epsilon-1}\frac{f_x}{f} > 1$ would not hold anymore). As shown in Proposition 3, for sufficiently low trade costs a further decrease in f_x reduces the skill premium. This effect is relatively small, which is not surprising considering that it is driven solely by the cutoff firms. At the benchmark case of $f_x \approx 0.9$, a decrease in the fixed costs of export however still raises the skill premium, albeit by a very small amount.

The quantitative effects derived here are limited magnitude partly because of the two countries assumption. A multilateral liberalisation between many symmetric countries reinforces the effects at stake, since exporting firms scale up their exports to many countries (and their demand for labour) following a decrease in variable trade costs. I conduct a straightforward extension of the model to study an economy with three symmetric countries. I recalibrate the model in the same manner as above, and obtain $K = 1.075$ and $f_x = 0.905$. As can be seen from figures 5 and 6, the effects of trade liberalisation are larger in the three country case: a decrease in τ from 1.5 to 1.1 triggers an increase in the skill premium from 1.74 to 1.85, i.e. 11 percentage points. This constitutes a substantial quantitative effect considering that the total increase in the skill premium from 1979 to 1995 in the U.S. was of around 35 percentage points [20]. For the present range of parameter values, a further increase in the number of countries only marginally strengthen the quantitative results.

[20] from 1.45 to 1.8, see Acemoglu (2002) p.15.

5 Conclusion

This paper shows how the reallocation of productive resources between heterogeneous firms following trade liberalisation influences the skill premium. For this I use a model of monopolistic competition with heterogeneous firms and two factors of production: skilled and unskilled labour. I introduce a correlation between productivity and skill intensity in the production process, which generates the empirically observed link between firm size, export status, wages and skill intensity. Within this consistent framework, I analyse the different effects of two types of trade liberalisation, defined as a reduction in variable and fixed costs of trade, and stress that they lead to different effects for the skill premium. A decrease in variable costs of trade has an unambiguously positive effect on the skill premium, thereby widening inequalities, while a decrease in the fixed costs of trade mitigates inequalities if trade costs are initially low.

The core mechanism at stake in both types of liberalisation is the reallocation of labour from low productive, unskilled intensive firms to more productive firms, which are more skill intensive. The difference between the two scenarii is twofold. First, exporting firms, which are highly skill intensive, only scale up their demand for labour in the variable costs trade liberalisation. Second, more unproductive, unskilled intensive firms drop out of the market following a reduction in variable trade costs than following a decrease in fixed export costs. Both these channels account for the larger effect of a decrease in variable costs on the skill premium.

Though the model does not provide a full-flegded welfare analysis[21] this differentiation between two types of trade liberalisation may be of particular interest for policy analysis. This suggests that bilateral trade liberalisation, when concentrating on the dismantling on Non Tariff Barriers of the fixed cost nature or any measure hampering the access of exporters to a foreign market, may come at no costs in terms of inequalities between skilled and unskilled labour. This is however not the case for a decrease in tariffs.

[21]Numerical simulations suggest that, if profits and capital income accrue a third group in the population, unskilled as well as skilled workers see their welfare increase following a trade liberalisation in the range considered, albeit in different proportions.

Appendix

A1. Existence and Uniqueness

Proof of Proposition 1

The structure of the proof is as follows. Lemmas 4 and 5 show that the two equilibrium conditions (27) and (28) respectively establish a positive and a negative relationship between w and z^*, as illustrated in figure 2. This ensures that if an equilibrium exists, it is unique. Lemma 6 completes the proof by showing that the two curves defined by the equilibrium conditions in the (z^*, w) space do cross, and therefore that an equilibrium exists.

For convenience, I denote the right hand side of (27) as $H(z^*, w)$ and its numerator and denominator respectively as $B(z^*, w)$ and $C(z^*, w)$:

$$\begin{aligned} B(z^*, w) &\equiv \int_{z^*}^{\infty} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} g(z)dz + \tau^{1-\epsilon} \int_{z_x^*(z^*)}^{\infty} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} g(z)dz \\ C(z^*, w) &\equiv \int_{z^*}^{\infty} zw^{-\sigma} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} g(z)dz + \tau^{1-\epsilon} \int_{z_x^*(z^*)}^{\infty} zw^{-\sigma} \left(1 + zw^{1-\sigma}\right)^{\frac{\epsilon-\sigma}{\sigma-1}} g(z)dz \\ H(z^*, w) &\equiv \frac{B(z^*, w)}{C(z^*, w)} \end{aligned}$$

Lemma 4 *Equation (27) establishes a continuous monotonic positive relationship between z^* and w.*

Proof. The proof uses the implicit function theorem on (27):

- Step 1: $\frac{\partial H(z^*,w)}{\partial z^*} < 0$

From (26):

$$\frac{dz_x^*}{dz^*} = -\frac{g(z^*)f}{g(z_x^*)^* f_x} \tag{37}$$

Using this relationship in the derivative of $H(z^*, w)$ with respect to z^* and rearranging:

$$\frac{\partial H(z^*, w)}{\partial z^*} = g(z^*)(1 + z^* w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}} \left[\frac{Bz^* w^{-\sigma} - C}{1 + z^* w^{1-\sigma}} - \frac{Bz_x^* w^{-\sigma} - C}{1 + z_x^* w^{1-\sigma}}\right] < 0 \tag{38}$$

where the inequality holds when there is partitioning between exporting and non-exporting firms because $\frac{Bz^*w^{-\sigma}-C}{1+z^*w^{1-\sigma}} < \frac{Bz_x^*w^{-\sigma}-C}{1+z_x^*w^{1-\sigma}}$. Indeed, $Bz^*w^{-\sigma} - C$ is negative and smaller than $Bz_x^*w^{-\sigma} - C$, and divided by $1 + z^*w^{1-\sigma} < 1 + z_x^*w^{1-\sigma}$, which yields the above inequality.

- Step 2: $\frac{\partial H(z^*,w)}{\partial w} > 0$

$$
\begin{aligned}
\frac{\partial H}{\partial w}C^2 &= \frac{\sigma}{w}BC \\
&+ (\epsilon-\sigma)w^{-\sigma}\left[\int_{z^*}^{\infty}\kappa(z)z^2w^{-\sigma}dzB - \int_{z^*}^{\infty}\kappa(z)zdzC + \tau^{1-\epsilon}\int_{z_x^*}^{\infty}\kappa(z)z^2dzB - \tau^{1-\epsilon}\int_{z_x^*}^{\infty}\kappa(z)zdzC\right]
\end{aligned}
$$

where $\kappa(z) \equiv (1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}-1}g(z)$

$\frac{\sigma}{w}BC$ is strictly positive. Since $\epsilon > \sigma$ by Assumption 2, it is sufficient for the above term to be positive that the square bracket be weakly positive. Using $\kappa(z)$ to rewrite B and C and simplifying, a sufficient condition for $\frac{\partial H(z^*,w)}{\partial w}$ to be positive is:

$$
\int_{z^*}^{\infty}\kappa(z)z^2dz\int_{z^*}^{\infty}\kappa(z)dz - \left(\int_{z^*}^{\infty}\kappa(z)zdz\right)^2 + \tau^{2-2\epsilon}\left(\int_{z_x^*}^{\infty}\kappa(z)z^2dz\int_{z_x^*}^{\infty}\kappa(z)dz - \left(\int_{z_x^*}^{\infty}\kappa(z)zdz\right)^2\right)
$$
$$
+\tau^{1-\epsilon}\left(\int_{z^*}^{\infty}\kappa(z)z^2dz\int_{z_x^*}^{\infty}\kappa(z)dz + \int_{z_x^*}^{\infty}\kappa(z)z^2dz\int_{z^*}^{\infty}\kappa(z)dz - 2\left(\int_{z^*}^{\infty}\kappa(z)zdz\int_{z_x^*}^{\infty}\kappa(z)zdz\right)\right) \geq 0 \quad (39)
$$

The first line above is positive by a direct application of the Cauchy-Schwarz inequality:

$$
\int\kappa(z)zdz = \int\kappa(z)^{\frac{1}{2}}\kappa(z)^{\frac{1}{2}}zdz \leq \left(\int\kappa(z)dz\right)^{\frac{1}{2}}\left(\int\kappa(z)z^2dz\right)^{\frac{1}{2}} \quad (40)
$$

By the same reasoning, the second line would also be positive if $z^* = z_x^*$. On the other hand, as $z_x^* \to Z$, the second line would become zero. To ensure that the second line is positive for any z_x^*, I differentiate it with respect to z_x^*, which I denote ζ:

$$
\zeta \equiv 2\left[\int_{z^*}^{\infty}\kappa(z)zdz\right]\kappa(z_x^*)z_x^* - \left[\int_{z^*}^{\infty}\kappa(z)dz\right]\kappa(z_x^*)z_x^{*2} - \left[\int_{z^*}^{\infty}\kappa(z)z^2dz\right]\kappa(z_x^*) \quad (41)
$$

Completing the square and using the Cauchy-Schwarz inequality shows that ζ is negative for all z_x^*. The second line in (39) is therefore weakly positive, and $\frac{\partial H(z^*,w)}{\partial w} > 0$.

- Step 3: By the implicit function theorem, steps 1 and 2 show Lemma 4.

Lemma 5 *Equation (28) establishes a continuous monotonic negative relationship between z^* and w.*

Proof. The proof uses the implicit function theorem on (28):

Define :

$$
J(z^*,w) = \left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}} - \frac{f_x}{f}\tau^{\epsilon-1} \quad (42)
$$

Using (37):

$$
\frac{\partial J}{\partial z^*} = \frac{1-\epsilon}{\sigma-1}w^{1-\sigma}\frac{(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{(1+z^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}+1}}\left[1+\frac{(1+z^*w^{1-\sigma})}{(1+z_x^*w^{1-\sigma})}\frac{g(z^*)f}{g(z_x^*)f_x}\right] < 0 \quad (43)
$$

$$
\frac{\partial J}{\partial w} = \frac{(\epsilon-1)w^{-\sigma}}{(1+z^*w^{1-\sigma})(1+z_x^*w^{1-\sigma})}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}(z^*-z_x^*) < 0 \quad (44)
$$

where the last inequality is strict whenever there is partitioning between exporting and non-exporting firms.

The implicit function theorem immediately implies Lemma 5.

Lemma 6 *Under Assumption 1, the two curves defined by Lemmas 4 and 5 intersect.*

Proof.

- Limits of the curve defined by Lemma 4.

For $w \to 0$, $H(z^*, w) \to 0$. In order for $\frac{U}{S} = H(z^*, w)$ to hold, z^* has to decrease as much as possible. From Assumption 1, this is attained at a level $\tilde{z} > \underline{z}$ defined by:

$$K = (1 - G(\tilde{z}))f \tag{45}$$

I denote the wage attained at $\tilde{z}$ by $\tilde{w}$, below which the equality cannot hold. Similarly, if $w \to \infty$, $H(z^*, w) \to \infty$, which requires that z^* increases as much as possible. z^* is required to be smaller than the upper bound $\overline{z}$, defined as the solution to:

$$\frac{K}{M} = (f + f_x)(1 - G(\overline{z})) \tag{46}$$

The wage defined by (27) at $\overline{z}$ is denoted as $\overline{w}$.

- Limits of the curve defined by Lemma 5:

$$\lim_{w \to 0} J(z^*, w) = \left(\frac{z_x^*}{z^*}\right)^{\frac{\epsilon-1}{\sigma-1}} - \tau^{\epsilon-1}\frac{f_x}{f}$$

Under Assumption 1 and since $\tau^{\epsilon-1}\frac{f_x}{f} > 1$, the limit of $J(z^*, w)$ will be equal to zero for some $z^* \in [\tilde{z}, \overline{z}]$.

$$\lim_{w \to \infty} J(z^*, w) = 1 - \frac{f_x}{f}\tau^{\epsilon-1}$$

In order for $J(z^*, w)$ to remain equal to zero, $\frac{z_x^*}{z^*} \to \infty$. Under Assumption 1, this is the case for $z^* \to \tilde{z}$.

Combining Lemmas 4, 5 and 6 completes the proof of Proposition 1. A graphical representation summarising the proof is given in figure 2.

A2. Comparative statics

In order to see how the skill wage premium w responds to a change in the costs of trade $T \in \{f_x, \tau\}$, I use Cramer's rule on the following system:

$$\begin{bmatrix} \frac{\partial J(z^*,w)}{\partial w} & \frac{\partial J(z^*,w)}{\partial z^*} \\ \frac{\partial H(z^*,w)}{\partial w} & \frac{\partial H(z^*,w)}{\partial z^*} \end{bmatrix} \begin{bmatrix} dw \\ dz^* \end{bmatrix} = \begin{bmatrix} -\frac{\partial J(z^*,w)}{\partial T} \\ -\frac{\partial H(z^*,w)}{\partial T} \end{bmatrix} d\tau \tag{47}$$

This yields:

$$\frac{dw}{dT} = \frac{\frac{\partial J(z^*,w)}{\partial z^*}\frac{\partial H(z^*,w)}{\partial T} - \frac{\partial J(z^*,w)}{\partial T}\frac{\partial H(z^*,w)}{\partial z^*}}{\frac{\partial J(z^*,w)}{\partial w}\frac{\partial H(z^*,w)}{\partial z^*} - \frac{\partial J(z^*,w)}{\partial z^*}\frac{\partial H(z^*,w)}{\partial w}} \tag{48}$$

From the proof of Proposition 1, it is immediate that the denominator of the right hand side above is positive. The sign of the effect of trade liberalisation on the skill wage premium is therefore the sign of the numerator, which depends on the trade costs considered.

Proof of Lemma 2

$$\frac{\partial H(z^*,w)}{\partial \tau} = \frac{\epsilon-1}{C^2}\tau^{-\epsilon}\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bw^{-\sigma}z-C)dG(z) \tag{49}$$

$$\frac{\partial J(z^*,w)}{\partial \tau} = (1-\epsilon)\frac{f_x}{f}\tau^{\epsilon-2} \tag{50}$$

Using (48) and the partial derivatives of $H(z^*,w)$ and $J(z^*,w)$ immediately yields Lemma 2 where:

$$\eta = -\frac{(\epsilon-1)^2}{\sigma-1}w^{1-\sigma}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\left[(1+z^*w^{1-\sigma})^{-1}+(1+z_x^*w^{1-\sigma})^{-1}\frac{g(z^*)f}{g(z_x^*)f_x}\right]\frac{\tau^{-\epsilon}}{C^2}$$

$$\xi = (1-\epsilon)\frac{f_x}{f}\tau^{\epsilon-2}g(z^*)(1+z^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}$$

Proof of Proposition 2

From the first step of the proof of Lemma 4, it holds that: $\frac{Bz^*w^{-\sigma}-C}{1+z^*w^{1-\sigma}} < \frac{Bz_x^*w^{-\sigma}-C}{1+z_x^*w^{1-\sigma}}$. This, in combination with the definition of $\xi < 0$, shows that the second and third terms in (29) are, taken together, negative. Since $\eta < 0$, it remains to show that:

$$\phi \equiv \int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bzw^{-\sigma}-C)dG(z) > 0 \tag{51}$$

From the definitions of B and C, it can be derived that:

$$\phi \propto \frac{\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}zdG(z)}{\int_{z_x^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}dG(z)} - \frac{\int_{z^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}zdG(z)}{\int_{z^*}^{\infty}(1+zw^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}dG(z)} > 0 \tag{52}$$

The above term is the difference between two weighted sums of z, the first for $z \geq z_x^*$, the second for $z \geq z^*$. Since $z_x^* > z^* > 1$, the above term is positive. Differentiating the weighted sum with respect to the lower bound shows this fact formally.

Proof of Lemma 3

$$\frac{\partial H(z^*,w)}{\partial f_x} = \frac{1-G(z_x^*)}{f_x}\frac{\tau^{1-\epsilon}}{C^2}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}(Bz_x^*w^{-\sigma}-C) \tag{53}$$

$$\frac{\partial J(z^*,w)}{\partial f_x} = -\frac{\tau^{\epsilon-1}}{f}+\frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x} \tag{54}$$

Using (48) and the partial derivatives of $H(z^*,w)$ and $J(z^*,w)$ immediately yields Δ' in Lemma 2 where:

$$\kappa = -g(z^*)(1+z^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}\left[-\frac{\tau^{\epsilon-1}}{f}+\frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x}\right] \tag{55}$$

$$\begin{aligned}\theta = &-\frac{\epsilon-1}{\sigma-1}\frac{1-G(z_x^*)}{fC^2}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}w^{1-\sigma}\left[(1+z^*w^{1-\sigma})^{-1}+(1+z_x^*w^{1-\sigma})^{-1}\frac{fg(z^*)}{f_xg(z_x^*)}\right]\\ &+g(z^*)\frac{(1+z^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{1+z_x^*w^{1-\sigma}}\left[-\frac{\tau^{\epsilon-1}}{f}+\frac{\epsilon-1}{\sigma-1}\frac{w^{1-\sigma}}{1+z_x^*w^{1-\sigma}}\left(\frac{1+z_x^*w^{1-\sigma}}{1+z^*w^{1-\sigma}}\right)^{\frac{\epsilon-1}{\sigma-1}}\frac{1-G(z_x^*)}{g(z_x^*)f_x}\right]\end{aligned} \tag{56}$$

Proof of Proposition 3

Rearranging Δ' gives:

$$\begin{aligned}\Delta' = &\frac{1-\epsilon}{\sigma-1}w^{1-\sigma}(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-\sigma}{\sigma-1}}\frac{1-G(z_x^*)}{f(1+z^*w^{1-\sigma})}\left[Bz_x^*w^{-\sigma}-C+\frac{g(z^*)f}{g(z_x^*)f_x}(Bz^*w^{-\sigma}-C)\right]\\ &+\frac{(1+z_x^*w^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}}}{f_x}g(z^*)\left[\frac{Bz^*w^{-\sigma}-C}{1+z^*w^{1-\sigma}}-\frac{Bz_x^*w^{-\sigma}-C}{1+z_x^*w^{1-\sigma}}\right]\end{aligned} \tag{57}$$

For $\tau=1$, as $f_x\to f$, $z_x^*\to z^*$ and the first line in the above equation approaches zero, while the second line is strictly positive. For higher f_x and (or) τ, however, the first line is negative, while the second is ambiguous, so that it is not possible to draw more elaborate conclusions without further assumptions.

A3. Calibration

As assumed for the calibration, z is drawn from a Pareto distribution with lower bound equal to one and parameter a:

$$G(z) = 1-z^{-a} \tag{58}$$

I now define the cumulative distribution function of z conditional on firm z producing:

$$\Gamma(z) = 1-z^{*a}z^{-a} \tag{59}$$

Domestic sales ($n(z)$) of a given z firm are given from (13) and (5) by:

$$n(z) = \lambda(1+zw^{1-\sigma})^{\frac{\epsilon-1}{\sigma-1}} \tag{60}$$

where $\lambda \equiv \epsilon A w_u^{1-\epsilon} P^{\epsilon-1} I$. Solving for z in (60) gives the productivity of a firm as a function of its domestic sales.

$$z(n) = w^{\sigma-1}\left[\left(\frac{n}{\lambda}\right)^{\frac{\sigma-1}{\epsilon-1}} - 1\right] \tag{61}$$

A simple transformation of variable gives the c.d.f. of firms domestic sales conditional on their producing as:

$$F(n) = \Gamma(z(n)) = 1 - z^{*a}\left(w^{\sigma-1}\left[\left(\frac{n}{\lambda}\right)^{\frac{\sigma-1}{\epsilon-1}} - 1\right]\right)^{-a} \tag{62}$$

For large n:

$$1 - F(n) \approx b^a w^{-a(\sigma-1)} \lambda^{\frac{a(\sigma-1)}{\epsilon-1}} n^{-a\frac{\sigma-1}{\epsilon-1}} \tag{63}$$

The log right tail probability of the distribution of domestic firm size (measured as sales or as employment) are therefore a straight line with coefficient: $-a\frac{\sigma-1}{\epsilon-1}$

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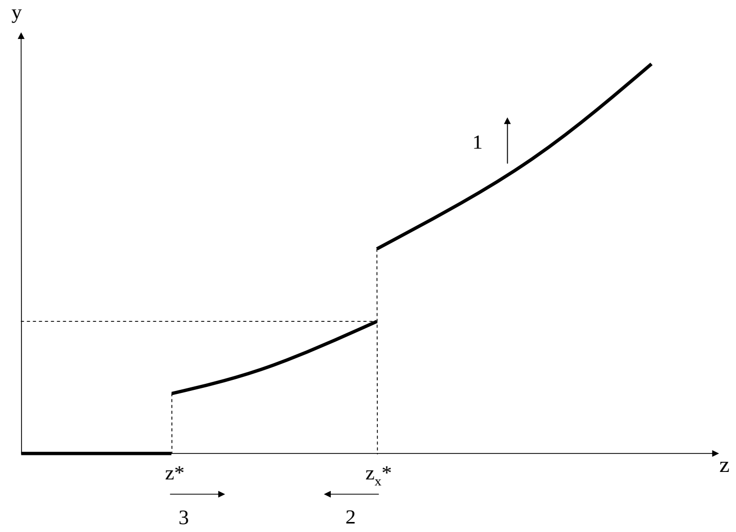

Figure 1: Effects of a reduction in τ on the production of different z firms for a fixed w.

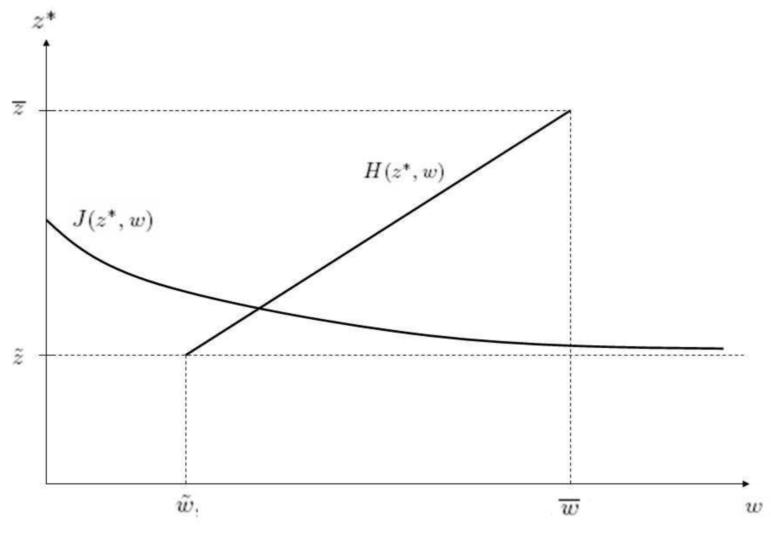

Figure 2: Sketch of the proof of existence and uniqueness of equilibrium. $H(z^*, w)$ stands for (27), $J(z^*, w)$ for (28).

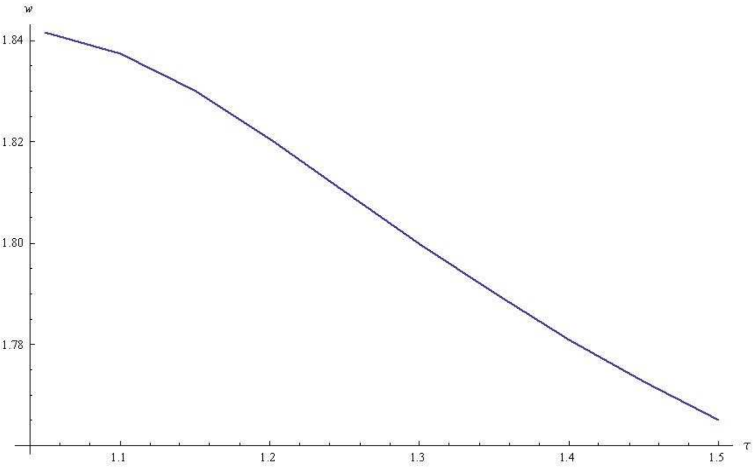

Figure 3: The skill premium as a function of τ for $f_x = 0.916$, 2 country case.

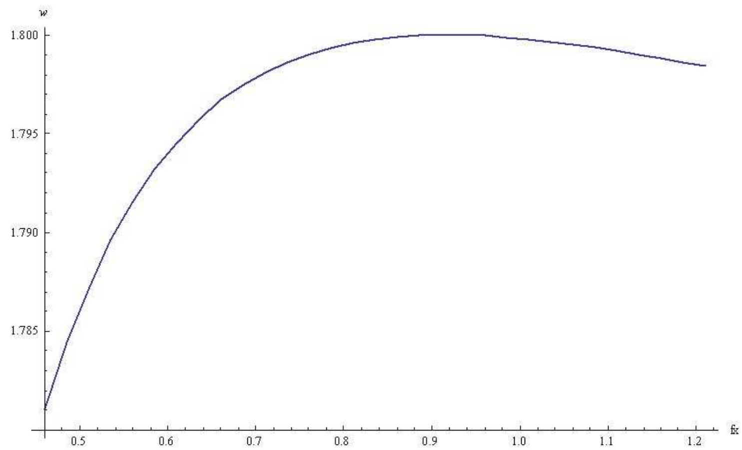

Figure 4: The skill premium as a function of f_x for $\tau = 1.3$, 2 country case.

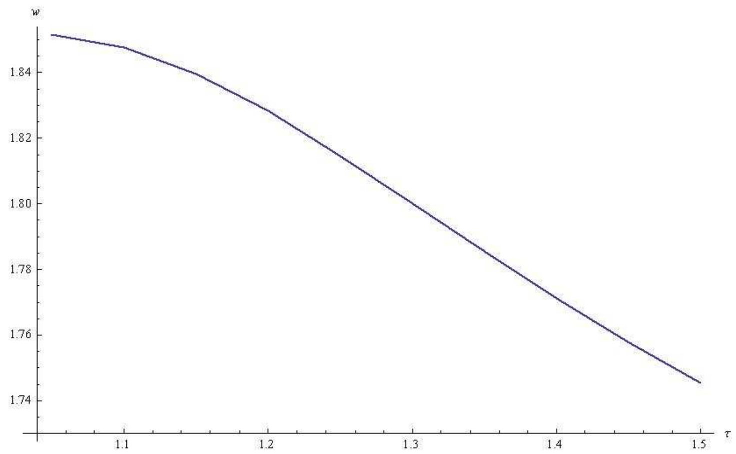

Figure 5: The skill premium as a function of τ for $f_x = 0.906$, 3 country case.

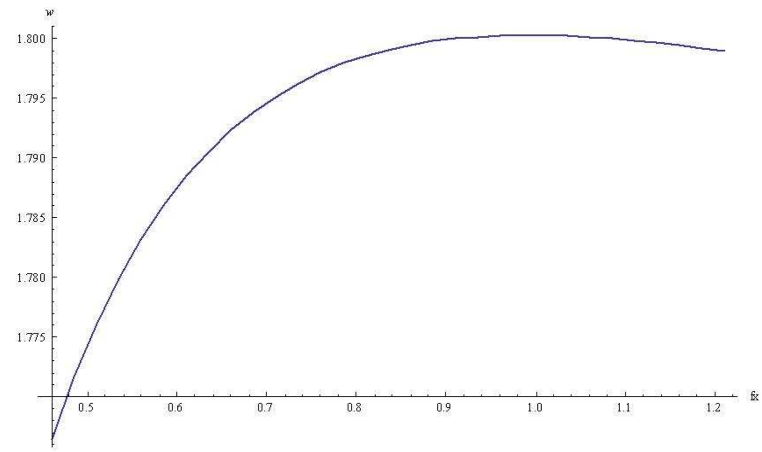

Figure 6: The skill premium as a function of f_x for $\tau = 1.3$, 3 country case.