On the Impact of Forward Contract Obligations in Multi-Unit Auctions

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Abstract

Several regulatory authorities worldwide have recently imposed forward contract obligations on electricity producers as a way to mitigate their market power. In this paper we investigate how such contractual obligations affect equilibrium bidding in electricity markets, or in any other auction-based market. For this purpose, we introduce forward contracts in a uniform-price multi-unit auction model with complete information. We find that forward contracts are pro-competitive when allocated to relatively large or efficient firms; however, they might be anti-competitive otherwise. We also show that an increase in contract volume need not always be welfare improving. From a methodological point of view, we aim at contributing to the literature on multi-unit auctions with discrete bids.

Keywords: Forward contracts, multi-unit auctions, discrete bids, market power, electricity, antitrust remedies, simulations.

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1 Introduction

Concerns over the exercise of market power in electricity markets have led several competition and regulatory authorities to impose forward contract obligations on the dominant producers. Such obligations have taken various forms, but they all share one important feature: they commit producers to receiving a fixed price for some fraction of their output before wholesale market competition takes place. The 'vesting contracts' introduced at privatization in the British electricity market (Wolfram (1999)) or the Competition Transition Costs for stranded costs recovery in place in Spain from 1998 to 2006 (Fabra and Toro (2005)), provide two well-known examples of such contract obligations. More recently, several European regulators are forcing large electricity producers to auction-off 'virtual power plants' (VPPs), which essentially work as forward sales. VPPs have been used as antitrust remedies in merger cases (e.g. EDF/EnBW, DONG/Elsam/Energi 2 or E.ON/MOL), following antitrust proceedings (as in the Italian AGCM case against ENEL), or in an attempt to dilute excessive market concentration (as in Spain).[1]

In this paper we investigate how such contractual obligations - which we encompass under the name of *forward contracts* - affect equilibrium bidding behaviour in electricity markets, or in any other auction-based market. Our purpose is two-fold. We seek to understand whether forward contract obligations contribute to reducing prices and removing the inefficiencies due to market power. Related to this, we also seek to identify the most effective way to allocate such obligations among (possibly) asymmetric firms.

There is already a large body of theoretical work on the impact of forward trading on the performance of oligopolistic markets.[2] In broad terms, the predictions of these papers are based on a two-stage game in which (symmetric) firms choose their level of contract coverage prior to competing in the spot market. In the second stage, forward sales induce firms to compete more fiercely given that spot market prices only affect their uncovered sales. This occurs regardless of whether firms compete by choosing quantities (as in Allaz and Vila (1993) or Bushnell (2006)), prices (as in Mahenc and Salanié (2004)), or continuous supply functions (as in Green (1992) or Newbery (1998)). However, once contracts are endogenized, the predictions of these models tend to differ. For instance, whereas under Cournot competition the subgame perfect equilibrium involves all firms selling forward contracts, under Bertrand competition all firms buy their own production forward. Hence, prices decrease

[1] Market power concerns in electricity markets have also fostered the establishment and promotion of forwards markets, as in the Pennsylvania-New Jersey-Maryland market (PJM) or in the Australian National Electricity Market (Bushnell *et al.* (2007); Wolak (2000, 2007)). Furthermore, the default service auctions used in several states of the US have implied an additional source of forward contracting for generators (Loxley and Salant (2004)). However, these cannot be considered to be exogenous contract obligations, which is the focus of this paper.

[2] There is also an extensive empirical literature that confirms that contracts affect the performance of spot markets. See Bushnell, Mansur and Saravia (2007), Fabra and Toro (2005), Hortacsu and Puller (2007), Kühn and Machado (2006), Mansur (2007) or Wolak (2000, 2007).

under Cournot competition but increase under Bertrand competition with respect to the no-contracts case. Furthermore, even when firms compete *à la* Cournot, forward contracts may turn out to be anti-competitive if there is an infinite number of contract rounds before the spot market opens. In particular, if firms are allowed to buy and sell contracts in all such rounds, there arise multiple equilibrium outcomes among which only the monopoly outcome is renegotiation-proof (Ferreira (2003)). Last, if firms interact for infinitely many periods in both the forward and spot markets, forward trading renders collusion more easily sustainable (Liski and Montero (2006); Green and Le Coq (2006)).

Unfortunately, the above papers are not directly applicable to assessing the effectiveness of forward contract obligations in electricity markets. Firstly, forward contract obligations are not endogenously chosen by firms but rather imposed by regulators. One could argue that it would then suffice to focus attention on the second stage of the games described above. However, we believe that a more detailed modelling of the institutional and structural features of electricity markets is necessary to fine-tune our predictions. Regarding institutional features, it is unnecessary to resort to either the Cournot of the Bertrand assumptions when studying competition among electricity producers. Instead, it is possible to model the actual market institution as electricity producers compete by submitting a *finite* number of price-quantity pairs to an auctioneer, who then dispatches generators in increasing bid order and pays them a price equal to the highest accepted bid. Regarding structural features, it is inadequate to assume perfect symmetry as it rarely holds in real markets. Since firms' capacity and cost asymmetries have profound effects on equilibrium market outcomes, ignoring them would miss an important determinant of the link between forward contract obligations and spot markets.

For these reasons, we have adopted the 'multi-unit auction approach' (von der Fehr and Harbord (1993) and García-Díaz and Marín (2003)).[3] Consistently with actual electricity market rules, it assumes that firms submit discrete supply functions to the pool;[4] furthermore, it allows to obtaining equilibrium predictions with no need to assume specific functional forms, thus imposing no constraints on the degree of capacity and/or cost asymmetry among firms. In sum, our approach captures essential institutional features of electricity markets while providing enough flexibility to reflect complex market structures.

In line with the existing literature, we find that forward contracts play a key role in shaping equilibrium market outcomes. Furthermore, relaxing the symmetry assumption also allows to uncovering new effects. We show that forward contracts do not alter one intrinsic

[3]Formally, we analyze a uniform-price multi-unit auction model in which quantity-bids are discrete, which is a common feature of virtually all auctions in practice. In this respect, our paper belongs to a small set of papers that analyze equilibrium behavior when bidders submit step bid functions (Kastl (2006); Kremer and Nyborg (2004)).

[4]Actual market rules limit the number of bids that may be submitted by an individual firm to a finite number. For example, in the original market design in England and Wales, generators were allowed to submit up to three incremental prices per unit, while in Spain, firms are allowed to submit up to 25 price-quantity pairs per generating unit.

prediction of the 'multi-unit auction approach': the fact that firms bid asymmetrically. More specifically, all firms but one (referred to as *non-price-setters*) behave as price-takers, while the remaining firm (referred to as the *price-setter)* sets the price at the level that maximizes its profits over the residual demand. However, forward contracts have a distinct impact on firms' strategies depending on their identity as either price-setters or non-price-setters. On the one hand, forward contracts reduce the price-setter's profit-maximizing price because they have the same effect as an inward shift in its residual demand. On the other hand, forward contracts have no impact on the non-price setters' bidding behaviour, as they continue to behave as price-takers irrespective of whether they hold contracts or not. If taken in isolation, these results would unambiguously indicate that forward contracts are pro-competitive.

However, assessing the impact of forward contracts on firms' incentives alone is not enough, as forward contracts also affect equilibrium existence. If firms are fully symmetric, equilibrium existence is not an issue. Hence, irrespective of who sets the price, an even increase in all firms' contract positions leads to a monotonic reduction in spot market prices, thus supporting the pro-competitive view of forward trading. However, when firms are asymmetric, the existence of an equilibrium in which a highly contracted firm sets a low price is not guaranteed, as one of the remaining firms may be better off deviating to set a higher price even at the expense of losing output. Hence, if only the high-price equilibria survive the introduction of contracts, these may turn out to be anti-competitive despite reducing some firms' incentives to raise prices.[5]

The extent to which contracts reduce (or increase) prices and improve (or worsen) welfare depends on several factors. We find that welfare depends non-monotonically on total contract volume: whereas a small amount of contracts may improve market performance with respect to the no-contracts case, a large amount of forward contracts may worsen it by destroying the low-price equilibria or by leading to inefficiently low prices. The distribution of forward contracts across firms is also critical: forward contracts have the potential to improve market performance only if they are allocated to firms with strong incentives to distort prices (typically, the large and efficient firms).

From a policy perspective, our analysis thus implies that forward contracts should be allocated in ways that align all firms' interests by (virtually) reducing their asymmetries. Paradoxical though it may seem, it is as important to mitigate the large firms' incentives to increase prices as it is to enhance those of the smaller competitors. This could be achieved by encouraging the intermediate to small firms in the industry to act as counterparts of the contractual obligations imposed on the dominant producers. Thus, restricting certain firms

[5]Let us stress that the reasons underlying this result are unrelated to the potential anti-competitive effects of contracts when firms hold long-positions (as in Mahenc and Salanié (2004)). In our model, contracts may have anti-competitive effects even when firms' positions are short. We conjecture that this result is not unique to our modelling approach, and that it would also arise in models that deliver multiple equilibria (such as the Supply Function approach developed by Klemperer and Meyer (1989) and first applied to electricity markets by Green and Newbery (1992)) whenever firms' asymmetries are explicitly modelled.

to enter into these contracts, as in the Spanish VPPs, is misplaced according to our model. Regarding contract volume, forcing firms to hold too few or too many forward contracts might be at best ineffective. Since the optimal contract volume ultimately depends on firms' cost structures and demand, it should be determined on a case-by-case basis.

Beyond the regulatory debate in electricity markets, this paper also aims at contributing to the analysis of forward contracts and spot markets within more general settings. Aside from the empirical and simulation studies, there are limited theoretical results on the impact of forward contracts on heterogenous firms, a feature that is pervasive to most real-world examples. Furthermore, besides electricity wholesale markets, there are several others in which forward contracts and auctions coexist, or markets which are organized in ways that make auction theory useful for understanding firms' strategic behaviour (Klemperer (2003)).

The paper is structured as follows. In the next section we describe the ingredients of the general model. In Section 3 we solve an illustrative example with the aim of presenting the main results of the paper in a simple way. Section 4 is devoted to the analysis of the general model. Section 5 investigates the impact of forward contracts on equilibrium outcomes, which are further illustrated by means of a simulation exercise in Section 6. Last, Section 7 concludes with a summary of the main results and policy implications that can be derived from the analysis. All proofs are contained in the Appendix.

2 Description of the Model

We consider a multi-unit uniform-price auction model in which $N \geq 2$ firms compete for the right to supply the market. Firm i's productive capacity K_i, $i = 1, ..., N$, is made of several units. Each unit has constant marginal costs of production up to the its capacity limit. By stacking firm i's units in increasing cost order, we construct its marginal cost curve, $c_i(q)$.[6] We use $C_i(q)$ to denote firm i's cost function, i.e., $C_i(q) = \int_0^q c_i(x)dx$.

Firms compete by submitting a finite number of price-quantity pairs to an auctioneer. In particular, each firm submits one price per production unit specifying the minimum price at which it is willing to supply the whole of the unit's capacity. Prices cannot exceed the 'market reserve price' P, which is at least equal to the marginal cost of the most expensive unit, i.e., $P \geq \max_i c_i(K_i)$. By stacking firms' price-quantity pairs in increasing order, we construct their bid functions, i.e., for firm i, $b_i(q) : [0, K_i] \rightarrow [0, P]$. Accordingly, bid functions are left continuous increasing step functions.

The auctioneer calls units to produce in increasing bid order, until total demand $D(p)$, with $D'(p) \leq 0$, is fully satisfied. The market clearing price or equilibrium price, denoted p^*, is set equal to the bid of the last accepted unit(s). Therefore, all units with bids strictly below p^* produce at capacity, whereas the unit(s) whose bid equals p^* will serve the residual

[6] We impose no constraints on the number of units firms have (other that it must be finite), and allow for all types of asymmetries (both in size and cost) among the units owned by a firm, as well as across firms.

demand. If more than one unit has been bid in at p^*, we assume that the units with lower marginal costs are called to produce first; if their marginal costs are equal, we assume that they split the residual demand equally.[7] The quantity allocated to firm i is denoted by q_i. The production by all dispatched units is paid at the equilibrium price.

One last feature of the model is that firms are subject to *forward contracts.* We use τ_i to denote firm i's contract price, and $x_i \geq 0$ to denote firm i's contract quantity;[8] both τ_i and x_i are set before competition to supply the market takes place. Consequently, when the equilibrium price is p^*, firm i's profits are given by

$$\pi_i (p^*) = p^* q_i - C_i(q_i) + [\tau_i - p^*] x_i, \tag{1}$$

where the first two terms give the firm's spot market profits, and the last term gives the firm's contract profits. This expression allows to interpret contracts as being purely financial, i.e., firm i continues to supply all its quantity q_i to the market at p^* and the contract's counterpart, e.g. a big customer, continues to buy all its demand from the market at p^*. The contract requires firm i to pay (receive) the difference between the contract price and the market price times the contract quantity, $[\tau_i - p^*] x_i$, whenever positive (negative).

Alternatively, the above expression can be re-written as follows,

$$\pi_i (p^*) = p^* [q_i - x_i] - C_i(q_i) + \tau_i x_i. \tag{2}$$

This formulation allows to interpret contracts as physical contracts in the sense that firm i only supplies $[q_i - x_i]$ through the market, i.e., the difference between its production and its contracted quantity. If $q_i > x_i$ firm i is a net-seller, so it supplies a positive quantity through the market. Conversely, if $q_i < x_i$ firm i is a net-buyer, so it has to buy the difference from the market. Whereas firm i's net position is settled at the market price, $p^* [q_i - x_i]$, its contract quantity is settled at the agreed contract price, $\tau_i x_i$. Since both (1) and (2) are analytical equivalent, our analysis allows for both financial as well as physical contracts.

Firm i's problem is to choose a bid function that maximizes π_i given the bid functions submitted by its rivals. All aspects of the model are common knowledge among bidders.

In the remainder of the paper we will label equilibrium outcomes as *competitive* or *non-competitive.* An equilibrium outcome is said to be *competitive* if, irrespective of how firms bid, the outcome (prices and quantities) is the same *as if* firms bid at marginal costs. The *competitive price* and the *competitive quantities* are respectively denoted p^c and $\{q_1^c, ..., q_N^c\}$. All other equilibrium outcomes are referred to as *non-competitive.*

[7]This tie breaking rule is common in the literature, e.g. Fabra *et al.* (2006), among others.

[8]We adopt the convention that $x_i > 0$ corresponds to firm i selling contracts (i.e., taking a short-position). We do not allow firms to buy forwards since in real markets regulators impose sale obligations. However, in our set-up forward sales and forward purchases simply have reverse effects.

3 Illustrative Example

We start by analyzing a simple example in order to convey the intuitions of the main results of the paper. In particular, we fix $N = 2$ and assume that firm 1 and firm 2 own three production units, each with a capacity normalized to one, and constant marginal costs respectively given by $\{0, 1, 2\}$ and $\{0, 1, 3\}$. Demand is assumed to be perfectly inelastic at $D = 3$. Under these assumptions, the competitive outcome is characterized by $q_i^c = 3/2$, $p^c = 1$, and $\pi_i(p^c) = [1 - x_i] + \tau_i x_i$, for $i = 1, 2$.

In the absence of contracts, the competitive outcome cannot be sustained in equilibrium since one of the two firms would rather set the price at the marginal cost of its rival's third unit. For instance, if firm 2 bids at marginal costs, firm 1 responds optimally by bidding all its units at 3: it produces 1 instead of $q_1^c = 3/2$, but it raises the market price from $p^c = 1$ to 3, thus increasing its profits by 2. Similarly, if firm 1 bids its units at marginal costs, firm 2 can increase its profits by 1 by bidding its first two units at 2. In contrast to firm 1, firm 2 cannot profitably raise the price to 3 since firm 1 would then satisfy all demand. Therefore, there exist two equilibrium outcomes: a high-price equilibrium outcome in which firm 1 sets the price at 3, and a low-price equilibrium outcome in which firm 2 sets the price at 2. These outcomes can be sustained by several bid profiles, as the low bids are price-irrelevant. The only constraint put on such bids is that they must be low enough to discourage the firm that sets the price from undercutting them.

Let's now introduce contracts. Consider first the case in which only firm 1 holds contracts, i.e., $x_1 > x_2 = 0$. If $x_1 \in (0, 1]$, the same two equilibrium outcomes that arise in the no-contracts case can be sustained in equilibrium. Recall that under both equilibrium outcomes, each firm's production is at least equal to 1; given their contracts do not exceed 1, both firms are net-sellers. As such, they do not want to reduce the price, and increasing it would imply losing all production.

Assume now that $x_1 \in (1, 2]$. As opposed to the no-contracts case, the equilibrium in which firm 1 sets the price at 3 and sells one unit is no longer an equilibrium (as a net-buyer, firm 1 would rather bid at marginal costs in order to reduce the price from 3 to 1). However, if firm 1 bids at marginal costs, firm 2 is better-off bidding all its units at 2, which implies that only the low-price equilibrium outcome survives the introduction of contracts. Hence, contracts can result in lower prices.

Last, suppose that $x_1 \in (2, 3]$. Given that firm 1's incentives to depress the price are now stronger, if firm 2 bids at marginal costs, firm 1 responds optimally by bidding all its units at zero, thus driving the price down to zero. Even though this requires firm 1 to sell its second unit below marginal costs, its productive loss, -1, is more than compensated by its savings on its net-buying position, $-[1 - x_1] > 0$. Firm 2 cannot profitably deviate, given that it would face zero demand if it tried to increase the price. Hence, there are now two equilibrium outcomes: one in which the price is zero and another one in which the price equals 2. Therefore, contracts may lead to inefficiently low prices, i.e., below the competitive

	Equilibrium Prices	
	Firm 1 is contracted	Firm 2 is contracted
$x_i \in [0,1]$	$\{3,2\}$	$\{3,2\}$
$x_i \in (1,2]$	2	3
$x_i \in (2,3]$	$\{0,2\}$	$\{3,0\}$

Table 1: Equilibrium prices as a function of firms' forward contract positions

price.

To complete the example, let's now assume that only firm 2 holds contracts, i.e., $x_2 > x_1 = 0$. For brevity, we focus on the case $x_2 \in (1,2]$ in which contracts have a qualitatively different effect on equilibrium outcomes than the one described above. In particular, the equilibrium in which firm 2 sets the price at 2 and sells one unit does not exist since, as a net-buyer, firm 2 would rather bid at marginal costs in order to reduce the price from 2 to 1 (and thus save $-[1-x_2] > 0$). However, firm 1 would then respond by bidding all its units at 3, which implies that only the high-price equilibrium outcome survives the introduction of contracts. Hence, even though contracts reduce the contracted firm's incentives to increase the price, contracts may result in (weakly) higher prices whenever the equilibrium in which the contracted firm sets the price fails to exist. Table 1 summarizes these results.

To sum-up, this example illustrates that the effect of contracts on bidding incentives and equilibrium outcomes crucially depends on both their total volume and their distribution across firms. It shows that contracts reduce firms' incentives to increase prices. However, this does not necessarily lead to lower equilibrium prices as contracts might also jeopardize the existence of the low-price equilibria. Indeed, contracts might lead to (weakly) higher prices whenever they are allocated in sufficiently large quantity to the firm that already has the greatest incentives to set low prices. For similar reasons, an increase in contract volume might not necessarily be pro-competitive.

4 Analysis of the Model

In this section, we characterize equilibrium outcomes (equilibrium prices and quantities) in the general model. For this purpose, we first note that at any equilibrium resulting in a non-competitive outcome, firms' bidding behavior must be asymmetric.[9] More specifically, to support any equilibrium outcome with a price $p^* \neq p^c$, one firm must bid at p^* some unit with marginal costs other than p^*, and this unit must be at least partially dispatched. We refer to such firm as the *price-setter*. Furthermore, at a non-competitive equilibrium there

[9] Previous papers have already predicted asymmetric bidding behaviour (García-Díaz and Marín (2003), Fabra *et al.* (2006), and Crawford *et al.* (2007) among others). We note here that contracts do not alter this intrinsic feature of the model. Further note that if the equilibrium outcome is competitive, all firms trivially act as price-takers. Hence, such asymmetries do not arise.

exists *at least* and *at most* one price-setter, so that all other firms must behave differently. We refer to such firms as the *non-price-setters*.

The intuition behind this distinct bidding behavior relies on the fact that firms submit a finite number of price-quantity pairs, which implies that there is a positive output mass at the margin.[10] Price-competition to supply this positive mass ensures that in any non-competitive equilibrium there is a single price-setter. To see this, argue by contradiction and suppose that there is more than one price-setter. Assume first that $p^* > p^c$, so that at least one of the price-setters produces *less* than its competitive quantity. This could never be part of an equilibrium given that the price-setter that is selling less than its competitive quantity could expand its production by undercutting the other price-setter(s). Since there is a positive mass at the margin, the increase in the deviant's output outweighs the infinitesimal (if any) price reduction caused by its deviation. A similar reasoning applies to the case in which $p^* < p^c$. In particular, if there is more than one price-setter, one of them must be producing *more* than its competitive quantity. However, given that it could avoid production losses by bidding above p^*, this can never be part of an equilibrium. Last, at any equilibrium resulting in a non-competitive outcome there must be *at least* one price-setter. Otherwise, if all marginal units were bid in at marginal costs, the equilibrium price p^* would trivially be equal to the competitive price p^c.

Accordingly, in what follows we first fix the identity of the price-setter and non-price setters in order to characterize firms' optimal bidding behaviour conditional on their identities. We then characterize equilibrium outcomes by stating the conditions under which no firm prefers to reverse its identity given its rivals' bidding behavior. We end this section by discussing existence and multiplicity of equilibrium outcomes.

4.1 Equilibrium behavior by the non-price setters

Let firm i be the price-setter at p^*. We first show that all other firms $j \neq i$ behave as price-takers. Namely, they find it optimal to bid all their units at marginal costs, or at bids that yield the same outcome as marginal cost bidding. This result holds true regardless of whether firms hold contracts or not.

Proposition 1 *At any Nash equilibrium in which firm i is the price-setter, all other firms produce the same output as if they bid at marginal costs.*

The intuition underlying the above result is simple. Suppose that one of firm j's units with marginal costs strictly below the equilibrium price, p^*, has not been called to produce. That could never be part of an equilibrium since firm j could always achieve a positive

[10] Competition for the margin is also the driving force for the no-underpricing equilibrium in the multi-unit Treasury auctions analyzed by Kremer and Nyborg (2004). If one assumes smooth supply functions, as in Newbery (1998), there is no mass at the margin and such asymmetries in firms' bidding behavior do not arise.

increment in output by bidding such a unit slightly below p^*. Such a deviation may or may not affect the equilibrium price, but even if it does, the quantity effect always outweighs the price effect as the price reduction can be made arbitrarily small. Alternatively, suppose that one of firm j's units with marginal costs strictly above p^* has been called to produce. Again, that could never be part of an equilibrium since firm j could avoid losses by bidding such a unit slightly above p^*, with only (if any) an infinitesimal increase in the price. These results are valid independently of whether the firm is a net-seller or a net-buyer precisely because the price effect on the firm's net position is almost nil.

Proposition 1 implies that, in equilibrium, all units belonging to firms j with marginal costs *below* the equilibrium price p^* are dispatched at capacity, whereas all units belonging to firms j with marginal costs *above* p^* are not dispatched at all. Given the efficient tie-breaking rule, the issue of whether the non-price setters dispatch units with marginal costs *equal to* p^* depends on whether p^* is above or below the competitive price, p^c. If p^* is above (below) p^c, the price-setter is producing less (more) than its competitive quantity; as this implies that price-setter's marginal unit has marginal costs below (above) p^*, the marginal output will be allocated to the price-setter (non-price setters). This allows to specify the output produced by the non-price-setters in equilibrium as a function of the equilibrium price p^*.

Corollary 1 *At any equilibrium in which firm i is the price-setter and p^* is the equilibrium price, the quantity produced by firm $j \neq i$ is non-decreasing in p^* and it satisfies*

$$q_j^{NPS}(p^*) = \begin{cases} \max\{q : c_j(q) \leq p^*\} & \text{if } \; p^* < p^c, \\ q_j^c & \text{if } \; p^* = p^c, \\ \max\{q : c_j(q) < p^*\} & \text{if } \; p^* > p^c. \end{cases}$$

Note that to guarantee that non-price-setters produce $q_j^{NPS}(p^*)$, they do not need to bid at marginal costs as there are many other outcome equivalent strategies. However, their choice of bidding strategies is not inconsequential, since these will determine the shape of the residual demand faced by the price-setter and hence its optimal bidding behavior.

Proposition 1 also implies that, conditionally on the identity of the price-setter, we can write the non-price-setters' profits as a function of the equilibrium price p^*. Consider a candidate equilibrium at which firm i is the price-setter at p^*. Equilibrium profits for the remaining firms j, $j \neq i$, are given by,

$$\pi_j^{NPS}(p^*) = \max_{q \in [0, D(p))} \{p^*[q - x_j] - C_j(q)\} + \tau_j x_j.$$

The following Lemma derives properties of the non-price setters' profit function.

Lemma 1 *Let p^* be the equilibrium price.*

(i) If $p^* \geq p^c$, then $\pi_j^{NPS}(p^*)$ is an increasing (non-decreasing) function of p^* for all j such that $x_j < q_j^c$ ($x_j = q_j^c$).

(ii) If $p^* \leq p^c$, then $\pi_j^{NPS}(p^*)$ is a decreasing (non-increasing) function of p^* for all j such that $x_j > q_j^c$ ($x_j = q_j^c$).

Trivially, the profits made by a firm are increasing (decreasing) in p^* if such a firm is a net-seller (net-buyer). However, since a firm's net position is in itself an endogenous outcome, we can not state in general whether a non-price-setter's profits are increasing or decreasing in the equilibrium price. Still, we can be sure that if $x_j < q_j^c$ firm j will be better off when p^* is raised above p^c since $p^* \geq p^c$ implies $q_j^c \leq q_j^{NPS}(p^*)$, which in turn guarantees that firm j is a net-seller. The reverse is true if $x_j > q_j^c$.

4.2 Equilibrium behavior by the price-setter

The price-setter's problem is simply to choose the price that maximizes its profits over the residual demand induced by its rivals' bidding behavior, i.e., total demand minus the quantity that the other firms are willing to supply at lower prices. Formally, the price-setter must choose

$$p_i^*(b_{-i}) \in \arg\max_p \pi_i^{PS}(p; b_{-i}),$$

where

$$\pi_i^{PS}(p; b_{-i}) = p\left[q_i(p; b_{-i}) - x_i\right] - C\left(q_i(p; b_{-i})\right) + \tau_i x_i,$$

and,

$$q_i(p; b_{-i}) = \max\left\{0, D(p) - \sum_{j \neq i} q_j(p; b_{-j})\right\}.$$

Furthermore, at the equilibrium price, optimal behavior by the non-price setters (i.e., consistent with Proposition 1) implies that the price-setter sells

$$q_i^{PS}(p^*) = D(p^*) - \sum_{j \neq i} q_j^{NPS}(p^*),^{11}$$

and makes profits

$$\pi_i^{PS}(p^*) = \max_p \left\{p\left[q_i^{PS}(p) - x_i\right] - C_i\left(q_i^{PS}(p)\right)\right\} + \tau_i x_i$$

Since both the cost function and the residual demand are step-functions, the price-setter's profit function may fail to be differentiable, so that the price-setter's profit-maximizing price might not be obtained as the solution to a first order condition.

Thus, in order to understand the price-setter's bidding incentives, let us work directly with changes in firm i's profits when it raises the price from p to some $p' > p$,

$$\pi_i(p'; b_{-i}) - \pi_i(p; b_{-i}) = [p' - p]\left[q_i(p'; b_{-i}) - x_i\right] - \int_{q_i(p'; b_{-i})}^{q_i(p; b_{-i})} \left[p - c_i(q)\right] dq. \qquad (3)$$

As in any standard monopoly problem, the price increase implies greater revenues through the firm's net position - the first term in (3), - but it also implies a profit loss due to the

[11] For prices other than p^* this condition may not hold since the non-price-setters need not be bidding at marginal costs.

output reduction - the second term in (3). Accordingly, the price-setter's incentives to raise the price are stronger the bigger its net position is, the less elastic its residual demand is and the smaller the price-cost margin on its lost production is. It then follows that firm i's profit-maximizing price given its rivals' strategies, $p_i^*(b_{-i})$, is non-increasing in its contract cover x_i. This mimics the standard result that smaller firms (here, firms with smaller net positions) have weaker incentives to raise the price.[12]

Inspection of (3) further shows that it is never optimal for a price-setter that holds fewer contracts than its competitive quantity, $x_i < q_i^c$, to set a price below the competitive one, $p^* < p^c$. To see this, note that firm i could marginally increase the price above p^* and still remain a net-seller. This would allow the firm to earn more through its positive net-position - the first term in (3), - while saving the margin between the price and its marginal costs, which is negative for prices below the competitive one - the second term in (3). However, despite the fact that $p^* \geq p^c$, the price-setter remains a net-seller in equilibrium. Intuitively, if firm i were a net-buyer in equilibrium, it could always increase its profits by marginally reducing the price. The reversed arguments apply for the case in which the price-setter holds more contracts than its competitive quantity, $x_i > q_i^c$.

Proposition 2 *In an equilibrium in which firm i is the price-setter, (i) if $x_i < q_i^c$, then $p^* \geq p^c$ and $x_i \leq q_i$; (ii) if $x_i = q_i^c$, then $p^* = p^c$ and $x_i = q_i$; and (iii) if $x_i > q_i^c$, then $p^* \leq p^c$ and $x_i \geq q_i$.*

Proposition 2 thus relates the primitives of the model (contract quantities and firms' costs, which determine firms' competitive quantities) to the equilibrium outcome. In particular, it allows to predict whether the price-setter will raise or reduce the equilibrium price above or below the competitive one, and whether it will be a net-seller or a net-buyer in equilibrium.

To conclude, let us compare the profits made by the price-setter and the non-price-setters in equilibrium. Given that the price-setter behaves as a monopolist over the residual demand, one may be tempted to (wrongly) believe that the price-setter's role is an appealing one, when just the opposite is true.

Lemma 2 $\pi_i^{PS}(p^*) \leq \pi_i^{NPS}(p^*)$ *for any equilibrium price p^*.*

As stated in the previous Lemma, for any equilibrium price p^*, the price-setter's profits are bounded above by the profits it could obtain as a non-price-setter. Both the price-setter and the non-price setters are paid the same price. However, unlike the non-price setters, the price-setter either sells less (when $p^* > p^c$) than if it behaved as a price-taker, in which case it gives up a positive profit margin on its reduced production, or it produces more (when $p^* < p^c$), in which case it must be incurring in productive losses.

[12] Discreteness in the bidding functions is the reason why the profit-maximizing price is not strictly decreasing in the contract quantity. Indeed, Newbery (1998) finds that prices are *strictly* decreasing in contract cover when firms' bidding functions are continuous.

Since we have already characterized firms' optimal behaviour conditional on their identities, we are now ready to characterize the equilibria of the game. This requires to assess whether the price-setter prefers to become a non-price-setter and *vice-versa*, an issue which we analyze next.

4.3 Equilibrium characterization

An equilibrium outcome will be a collection of quantities produced by the non-price setters and a price chosen by the price-setter such that no firm wants to deviate neither by changing its quantity or price choice, nor by changing its identity. The following Theorem provides necessary and sufficient conditions for equilibrium bidding.

Theorem 1

(i) A strategy profile constitutes a Nash equilibrium in which firm i sets the price at $p^* \geq p^c$ if and only if the following three conditions hold:

1). $p^* = p_i^*(b_{-i})$ and $q_j = q_j^{NPS}(p^*)$ for all $j \neq i$;

2). $\pi_i^{PS}(p^*) \geq \pi_i^{NPS}(p)$ for all $p < p^*$ such that $q_i^{NPS}(p) + \sum_{j \neq i} q_j(p; b) = D(p)$; and

3). $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ for all $j \neq i$ such that either $p_j^*(b_{-j}) > p^*$ and $x_j < q_j\left(p_j^*; b_{-j}\right)$ hold or $p_j^*(b_{-j}) < p^*$ and $x_j > q_j\left(p_j^*; b_{-j}\right)$ hold.

(ii) A strategy profile constitutes a Nash equilibrium in which firm i sets the price at $p^* \leq p^c$ if and only if the following three conditions hold:

1). $p^* = p_i^*(b_{-i})$ and $q_j = q_j^{NPS}(p^*)$ for all $j \neq i$;

2). $\pi_i^{PS}(p^*) \geq \pi_i^{NPS}(p)$ for all $p > p^*$ such that $q_i^{NPS}(p) + \sum_{j \neq i} q_j(p; b) = D(p)$; and

3). $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ for all $j \neq i$ such that either $p_j^*(b_{-j}) < p^*$ and $x_j > q_j\left(p_j^*; b_{-j}\right)$ hold or $p_j^*(b_{-j}) > p^*$ and $x_j < q_j\left(p_j^*; b_{-j}\right)$ hold.

Consider part (i) of Theorem 1 - part (ii) follows the reversed arguments. In equilibrium, one firm sets the price that maximizes its profits over the residual demand, $p^* = p_i^*(b_{-i})$, and all other firms behave as price-takers given p^* (Proposition 1). Since all firms are already optimizing conditionally on their identities, the only relevant deviations are those by which the price-setter becomes a non-price-setter and *viceversa*. If the price-setter became a non-price setter, it would do so in order to increase its production, implying that the only relevant deviations by the price-setter are those that involve a price reduction. Furthermore, for the non-price-setters, it is never profitable to deviate by setting the price at its profit maximizing price if this results in a reversal of its net-position.

To see this, consider a non-price-setter that is a net-seller at the candidate equilibrium. Suppose first that its profit-maximizing price as a price-setter is below the equilibrium price. If such a firm deviated to become the price-setter, it would receive less for its net-sales and it would have to incur in productive losses on its increased production, rendering its deviation unprofitable. Suppose instead that its profit-maximizing price as a price-setter is above the equilibrium price. Inspection of equation (3) again shows that it cannot be profitable for such a firm to deviate if it became a net-buyer after the price increase, as both terms in the equation would be negative. Hence, we need not impose any additional condition on this subset of firms - other than behaving optimally as non-price setters - given that they cannot profitably become the price-setter. The reversed arguments show that the only relevant deviations by the net-buyers involve a price reduction, which is unprofitable whenever it forces the deviant to become a net-seller.

4.4 Equilibrium existence and multiplicity

We now analyze whether equilibrium existence is guaranteed, and whether there is multiplicity of bid function equilibria and/or multiplicity of equilibrium outcomes.[13] For this purpose, it is useful to distinguish between two cases: (i) the polar cases, in which all firms' are either net-sellers or net-buyers at the competitive outcome (i.e., either $x_i \leq q_i^c$ or $x_i \geq q_i^c$ for all firms), and (ii) the mixed cases in which at least two firms' net positions have opposite signs. The polar cases are the empirically relevant ones (in practice, firms rarely hold that many contracts so as to exceed their competitive quantities), but we will also cover the mixed cases for completeness.

4.4.1 Polar cases

Under the polar cases, all firms agree as to whether the equilibrium price should be set above or below the competitive one (Proposition 2). This has important implications for equilibrium behavior. In particular, there is always an equilibrium in which the non-price-setters bid at marginal cost. Furthermore, conditionally on the identity of the price-setter, any other equilibrium is outcome equivalent to the equilibrium at which the non-price setters bid at marginal costs.

Proposition 3 *Index firms by their profit-maximizing prices when rivals bid at marginal costs, i.e.,* $p_i^*(c_{-i}) \geq p_{i+1}^*(c_{-i+1})$.

(i) If $x_i \leq q_i^c$ *for all firms, there exists a pure-strategy equilibrium in which firm* i *sets the price at* $p^* = p_i^*(c_{-i}) \geq p^c$ *while firms* $j \neq i$ *sell* $q_j = q_j^{NPS}(p^*)$ *if and only if* $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ *for all firms* j *such that* $p_j^*(c_{-j}) > p^*$.

(ii) If $x_i \geq q_i^c$ *for all firms, there exists a pure-strategy equilibrium in which firm* i *sets the price at* $p^* = p_i^*(c_{-i}) \leq p^c$ *while firms* $j \neq i$ *sell* $q_j = q_j^{NPS}(p^*)$ *if and only if* $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ *for all firms* j *such that* $p_j^*(c_{-j}) < p^*$.

Consider part (i) of the Proposition above. By Proposition 2, $x_i \leq q_i^c$ implies $p^* \geq p^c$. Furthermore, since all firms $j \neq i$ are price-takers, it follows that all firms in equilibrium

[13] We will refine the equilibrium set by restricting attention to strategies that are not weakly-dominated. Weak-dominance arguments eliminate below marginal cost bidding for quantities above the firm's contract cover, above marginal cost bidding for quantities below the firm's contract cover, and bidding units out of marginal cost order (see Lemma A.1 in the Appendix).

are net-sellers. Accordingly, the only relevant deviations are those of the non-price setters whose profit-maximizing price is above the equilibrium price: becoming a price-setter at a lower price cannot be profitable as the deviant would produce less, and it would sell its positive net position at a lower price. The price-setter cannot profitably deviate: on the one hand, it is already optimizing given its identity; on the other hand, if it became a non-price setter, the resulting price would be the competitive price, which cannot be profitable by revealed preference. Similarly, when all firms are net-buyers in equilibrium, as in part (ii) of Proposition 3, the only relevant deviations are those of the non-price setters with a profit-maximizing price below the equilibrium price, as raising the price is unprofitable.

The above implies that equilibrium existence is guaranteed under both cases.[14] If $x_i \leq q_i^c$ for all firms, no firm would like to deviate from the highest price equilibrium; if $x_i \geq q_i^c$ for all firms, no firm would like to deviate from the lowest price equilibrium.

Corollary 2 *Index firms by their profit-maximizing prices when rivals bid at marginal costs, i.e.,* $p_i^*(c_{-i}) \geq p_{i+1}^*(c_{-i+1})$.

(i) If $x_i \leq q_i^c$ *for all firms, the equilibrium in which firm* 1 *sets the price always exists.*

(ii) If $x_i \geq q_i^c$ *for all firms, the equilibrium in which firm* N *sets the price always exists.*

For a given price-setter, there exist multiple bid function profiles that constitute an equilibrium (all those satisfying Theorem 1). This derives from the fact that firms only care about one point in their bid functions, the one corresponding to the market clearing price. Furthermore, the price that maximizes the price-setter's profits depends on the bid functions submitted by the non-price setters, thus paving the way for this multiplicity of equilibrium bid functions to also generate multiplicity in equilibrium outcomes. Fortunately, this is not the case: for a given price-setter, all the equilibria result in the same market price and output allocation across firms.[15]

Proposition 4 *Suppose that either* $x_i \leq q_i^c$ *or* $x_i \geq q_i^c$ *for all firms. Conditionally on the identity of the price-setter, all the equilibria are outcome equivalent.*

Consider the case in which all firms are net-sellers at the competitive outcome, and assume that there exists an equilibrium in which all the non-price-setters bid at marginal costs and the price-setter, firm i, bids all its units at its profit-maximizing price.[16] Proposition 4 above guarantees that there cannot exist any other equilibrium in which firm i is the price-setter that results in a different equilibrium outcome. If in equilibrium one of the non-price

[14] In contrast, in Kastl (2006)'s model, incomplete information implies that "existence of an equilibrium in a model of uniform-price auction with restricted space is an open question."

[15] In contrast, multiplicity of equilibrium outcomes is pervasive in auctions with continuous bid functions (see Back and Zender (1993), Klemperer and Meyer (1989) and Wilson (1979)).

[16] More specifically, we assume that the price-setter bids all units at p_i^* except for those units for which bidding at p_i^* is weakly dominated, in which case they are bid in at marginal costs.

setters bids *above* marginal costs and this results in a different equilibrium price, such a non-price-setter would be producing less than under marginal cost bidding, thereby contradicting Proposition 1. Instead, if one of the non-price setters bids some units *below* marginal costs, the resulting equilibrium outcome must remain unchanged by revealed preference arguments. Last, if the price-setter uses a different bid function, it should also result in the same market price and lead (by Proposition 1) to the same output allocation as the original equilibrium. When all firms are net-buyers, the reverse arguments apply.

Finally, a different source of multiplicity of equilibrium outcomes may arise due to the coexistence of several equilibria which differ in the identity of the price-setter. This (potential) multiplicity was highlighted in the illustrative example provided in Section 3 but it holds more generally, as shown next.

Corollary 3 *Index firms by their profit-maximizing prices as price-setters when their rivals bid at marginal costs, i.e.,* $p_i^*(c_{-i}) \geq p_{i+1}^*(c_{-i+1})$.

(i) Suppose that $x_i \leq q_i^c$ *for all firms* $i = 1, ..., N$. *If the equilibrium in which firm* k *is the price-setter exists, the equilibria in which firms* $i < k$ *are the price-setters also exist. Alternatively, if it does not exist, the equilibria in which firms* $i > k$ *are the price-setters do not exist either,* $k = 2, ..., N$.

(ii) Suppose that $x_i \geq q_i^c$ *for all firms* $i = 1, ..., N$. *If the equilibrium in which firm* k *is the price-setter exists, the equilibria in which firms* $i > k$ *are the price-setters also exist. Alternatively, if it does not exist, the equilibria in which firms* $i < k$ *are the price-setters do not exist either,* $k = 1, ..., N-1$.

In the case in which all firms are net-sellers (net-buyers) at the competitive outcome, existence of a candidate equilibrium implies that all other candidate equilibria with higher (lower) equilibrium prices also exist. To understand this result, note that the profits that a firm achieves as a price-setter are given, but the profits it makes as a non-price setter are increasing (decreasing) in the equilibrium price (Lemma 1). Hence, if none of the firms has incentives to deviate from a candidate equilibrium, it must also be the case that none of them wants to deviate from a candidate equilibrium with a higher (lower) price. For similar reasons, if a candidate equilibrium does not exist, any other candidate equilibrium with lower (higher) prices does not exist either.

4.4.2 Mixed cases

Consider now the mixed cases, in which some firms hold fewer contracts than their competitive quantities while others hold more. Similar results regarding equilibrium existence and multiplicity as those found under the polar cases also arise under the mixed cases as long as *in equilibrium* all firms are either net-sellers or net-buyers. The intuition is simple: if all firms' net position at the candidate equilibrium have the same sign, none of the non-price setters would like to deviate in order to increase or reduce the price.

Under the remaining cases, net-sellers and net-buyers have conflicting interests concerning the level of the equilibrium price. An over-contracted firm would set a price below marginal costs, whereas an under-contracted firm would set a price above. This conflict of interests has profound effects on equilibrium behavior. In particular, and in contrast with the polar cases, the non-price setters' choice of bidding strategies is not inconsequential as it affects both equilibrium existence as well as the multiplicity of equilibrium outcomes.

Furthermore, equilibria in which the non-price setters bid at marginal cost may fail to exist as marginal cost bidding exacerbates the aforementioned conflict of interests. To see this, note that marginal cost bidding makes it easier for an over-contracted firm to set a low price, as it reduces the number of units that it has to bid below marginal costs in order to drive the market price below the competitive one. Since firms with fewer contracts may find it profitable to set a much higher price, an equilibrium may fail to exist. However, if one allows the non-price setters to follow any (undominated) strategy, equilibrium existence is guaranteed, as shown next.

Proposition 5 *Under the mixed cases, an equilibrium always exists. However, in this equilibrium the non-price setters may not be bidding at marginal costs.*

The effect of enlarging the set of the non-price setters' bidding strategies is not only to restore equilibrium existence. In contrast to the polar cases (Proposition 4), it also implies that there may arise multiple equilibrium outcomes even when the identity of the price-setter is fixed. To see this, consider a duopoly game in which demand is inelastic. Firm 1 has no contracts whereas firm 2's contracts do not exceed total demand,[17] and $p_1^*(c_2) > p^c > p_2^*(c_1)$. To make the analysis interesting, suppose that at $p_1^*(c_2)$ firm 2 is a net-buyer. By Theorem 1, it is then an equilibrium for firm 1 to bid all its units at $p_1^*(c_2)$ and for firm 2 to bid them at marginal costs given that: 1) both firms are optimizing conditional on their identities; 2) if firm 1 became the non-price setter the price would be driven down to p^c, which by revealed preference is unprofitable; and 3) if firm 2 became the price-setter and reduced the price, it would face total demand and hence become a net-seller, which would render the deviation unprofitable.

However, it is also an equilibrium for firm 2 to bid at zero up to its contract quantity and to bid its remaining units at marginal costs. The best response by firm 1 is then to set a price above $c_2(x_2)$, given that at any lower price firm 1's residual demand is constant. Note that at the new candidate equilibrium firm 2 would be a net-seller, and that the new price would exceed $p_1^*(c_2)$ given that at $p_1^*(c_2)$ firm 2 was a net-buyer. No firm has incentives to deviate from this equilibrium given that: 1) both firms are optimizing conditional on their identities; 2) if firm 1 became the non-price setter the price would be driven down to zero; and 3) firm 2 is a net-seller and hence does not find it profitable to reduce the price.

[17] More precisely let us assume that $D > \overline{x}_2 \geq x_2$, where $\overline{x}_2$ (formally defined in the Appendix).is the maximum quantity whose marginal costs do not exceed $c_2(x_2)$.

The two equilibria described above are not outcome equivalent despite the fact that firm 1 is the price-setter under both of them. Furthermore, one cannot derive a general Pareto ranking among them: by revealed preference, the price-setter is strictly better-off at the equilibrium in which the non-price setter bids at marginal costs; however, the non-price setter might be worse-off or better-off depending on the value of x_2.

To conclude, under the polar cases (i) equilibrium existence is always guaranteed and, (ii) conditionally on the identity of the price-setter, the equilibrium outcome is unique. Hence, any potential multiplicity of equilibrium outcomes must derive from the coexistence of equilibria in which different firms act as price-setters. These results may not hold in some mixed cases.

5 The Impact of Forward Contracts

We now analyze the impact of forward contracts on equilibrium outcomes under two alternative specifications: a symmetric oligopoly and an asymmetric duopoly. This allows to assessing the relationship between firms' asymmetries and the effects of forward contracts, an issue which, as far as we are aware of, has not been studied elsewhere.

5.1 Symmetric firms

First, assume that firms are symmetric in all respects (including their contract quantities), and consider the effects of increasing their contract coverage on prices and productive efficiency.[18]

Lemma 3 *Assume there exist N symmetric firms. If contracts are symmetrically distributed among them, i.e., $x_1 = ... = x_N = x$,*

(i) equilibrium prices are non-increasing in x; and

(ii) productive efficiency is non-decreasing in x for $x \leq q^c$, and non-increasing in x for $x \geq q^c$.

Since firms are fully symmetric, there exist N equilibrium outcomes that only differ in the identity of the price-setter. As firms' contract coverage is increased, the equilibrium price is reduced from the level that results at the no-contracts case to the competitive price, when all firms are fully contracted. Hence, up to the competitive quantity, contracts unambiguously contribute to reduce prices and improve productive efficiency. However, as total contract coverage is further increased, firms start exercising monopsony power, leading to prices below the competitive price. Furthermore, since the price-setter produces more than at the competitive outcome, productive inefficiencies re-emerge. Hence, contracts have

[18] The symmetry assumption guarantees that we are always under the polar cases, implying that both equilibrium existence and uniqueness of equilibrium outcomes are guaranteed.

the potential of being welfare improving, but they may also reduce it if held in sufficiently large quantities.

For given contracts, similar results also arise in Allaz and Vila (1993)'s and Bushnell (2007)'s Cournot models, as well as in Newbery (1998)'s Supply Function Equilibrium model. Our contribution here is to show that these results are robust to the introduction of a different mode of competition *provided that* firms are fully symmetric.

5.2 Asymmetric firms

We now focus on the duopoly case while allowing for all types of asymmetries among them. In order to guarantee equilibrium existence and uniqueness of equilibrium outcomes, we shall assume that firms hold contracts that never exceed their competitive quantities. Also, this assumption implies that any potential inefficiency cannot be explained by firms holding an excessive amount of contracts, as in point (ii) of Lemma 3 above. Similarly, in order to isolate the distinct effects of contracting by each firm, we shall assume that only one firm is contracted at a time. These assumptions make the analysis clearer without biasing its main conclusions.

The following Proposition summarizes the effects of contracts on equilibrium prices.

Proposition 6 *Assume that there exist $N = 2$ (possibly) asymmetric firms, indexed by their profit-maximizing prices when rivals bid at marginal costs at the no-contracts case, $p_1^*(c_2) \geq p_2^*(c_1) > p^c$. There exists $0 \leq x_i' < x_i'' \leq q_i^c$, $i = 1, 2$, such that,*

(i) as compared to the no-contracts case, contracts by firm 1 lead to (weakly) lower prices. In contrast, whereas contracts by firm 2 below x_2' also lead to (weakly) lower prices, contracts above x_2' lead to (weakly) higher prices. Furthermore,

(ii) there is a non-monotonic relationship between contract volume and equilibrium prices. Specifically, increasing firm i's contracts up to x_i'' leads to (weakly) lower prices, but increasing contracts above x_i'' has the opposite effect, $i = 1, 2$.

Point (i) of Proposition 6 compares equilibrium prices with and without contracts, conditioning on the identity of the contracted firm. If all contracts are allocated to the firm with the high profit-maximizing price, contracts (weakly) reduce prices with respect to the no-contracts case. This occurs regardless of whether the equilibrium in which the contracted firm sets the price exists or not, as when it does not, the price will be set by the uncontracted firm, which has the low profit-maximizing price.

This conclusion might be reversed when all contracts are allocated to the firm with the low profit-maximizing price. Whereas it is still true that contracts (weakly) reduce prices when *both* equilibria exist (for $x_2 \leq x_2'$), this is no longer the case when the equilibrium in which the contracted firm sets the price does not exist (for $x_2 > x_2'$). Given that the equilibrium price will then be set by the uncontracted firm, which has the high profit-maximizing price, contracts in this case may result in (weakly) higher prices as compared to the no-contracts case.

Point (ii) of Proposition 6 assesses the desirability of increasing contract volume. It shows that, irrespective of whether contracts are allocated to one firm or another, an increase in contract volume does not always lead to price reductions. Indeed, an increase in contract volume may lead to (weakly) higher prices whenever such an increase in contracts destroys the equilibrium in which the contracted firm sets the price (for instance, for $x_1 > x_1''$ the equilibrium in which firm 1 sets a price below $p_2^*(c_1)$ no longer exists).

The above results are illustrated in the numerical example provided in Section 3.

To sum up, our analysis highlights two key factors as determinants of whether contracts are pro-competitive or anti-competitive: the allocation of contracts across firms as well as their total volume. The analysis shows that contracts have anti-competitive effects when allocated to firms which, even in the absence of contracts, have weak incentives to raise prices (i.e., the firms with low profit-maximizing prices, which are typically the small and/or the inefficient ones). However, contracts may be pro-competitive and lead to lower prices when allocated to firms which would otherwise have strong incentives to raise prices (i.e., typically the large and/or the efficient ones). Furthermore, we have found that *more is not always better*. That is, even when contracts are allocated to the firm with greatest incentives to distort prices, an increase in its contract cover might lead to higher prices. As a corollary, contracts should be allocated to firms with strong incentives to raise prices, and contract quantities should not be neither 'too small' nor 'too large'- the optimal contract volume ultimately depends on firms' cost structures and demand.

6 Simulating the Impact of Forward Contracts

We next apply the theoretical model to simulate equilibrium bidding behaviour and market outcomes in the Spanish electricity market during 2005. The aim is to illustrate with real data the strategic effects of contracts that we have described in the previous section. Appendix B contains details on the Spanish electricity market as well as on the procedures we have followed to compute firms' marginal costs.

We have considered alternative scenarios regarding total contract volume and its distribution across firms. In particular, under the assumption that only the two main firms (Endesa and Iberdrola) behave strategically, we have computed both the competitive as well as the equilibrium market outcomes under the no-contracts case and the cases in which either Endesa (END) or Iberdrola (IB) hold contracts, ranging from 1,000 to 8,000 MWs.[19]

[19] Since the simulations are conducted on an hourly basis over a year, there are at least 8,760 and at most (if both firms act as price-setters) 17,520 equilibrium market outcomes under each of the 17 cases considered, plus the 8,760 competitive outcomes (these are the same regardless of whether firms hold contracts or not)- summing over 300,000 simulated market outcomes in total. Simulations have been produced by ENERGEIA, a simulation programme developed by the authors.

	No Contracts		END 2,000		END 5,000		IB 6,000		IB 8,000	
Price-setter	IB	END	IB	END	IB	END	IB	END	IB	END
Peak load	50.0	50.0	50.0	50.0	50.0	*–	50.0	50.0	*–	50.0
75%	11.2	15.0	11.2	*11.6	11.2	*–	*–	15.0	*–	15.0
50%	–	15.9	*5.2	*10.7	*5.2	*–	–	15.9	–	15.9
25%	23.4	23.6	23.4	23.6	23.4	*–	23.4	23.6	23.4	23.6

Table 2: The impact of forward contracts (MWs) on markups $\frac{p^{*}-p^{c}}{p^{*}}$ in the Spanish electricity market during 2005

Note on Table 2: The table reports the simulated mark-ups $\frac{p^{*}-p^{c}}{p^{*}}$ for four demand levels (the year's peak load, and the 75%, 50% and 25% demand percentiles). The results are divided in columns, depending on the identity of the price-setter. A table entry is left empty if, for the associated demand level and contract volumes, there is not an equilibrium in which such a firm behaves as price-setter. An asterisk denotes that the equilibrium has changed with respect to the no-contracts case.

Table 2 reports the markups that result from comparing the simulated equilibrium price to the price that would arise in a competitive market (Borenstein *et al.* (2002)).[20] Markups are computed at four demand levels (expressed in percentiles), under the no-contracts case and under the cases in which Endesa has contracted either 2,000 or 5,000 MWs, and Iberdrola has contracted either 6,000 or 8,000 MWs (results for all other cases are qualitatively similar).[21] By comparing the markups across firms at the no-contracts case, we can readily verify that Endesa's profit-maximizing price exceeds that of Iberdrola's for all demand levels considered.

Let us first consider the effects of contracts when allocated to the firm with the high profit-maximizing price, Endesa. First, contracts may reduce Endesa's profit-maximizing price as a price-setter; this is for instance the case when Endesa contracts 2,000 MWs and demand is at its 50% or 75% percentiles. Second, contracts may give rise to a new equilibrium in which Iberdrola sets a lower price; this is for instance the case when Endesa contracts either 2,000 or 5,000 MWs and demand is at its 50% percentile. Last, contracts may eliminate certain equilibria at which Endesa sets the price; this is for instance the case when Endesa contracts 5,000 MWs for all demand levels. Therefore, contracts by Endesa have (weakly) pro-competitive effects.

[20] We have chosen to report these markups rather than prices for clarity. Nevertheless, note that both markups and prices illustrate identical results to the extent that the competitive price is the same regardless of which firm sets the price.

[21] The markups are based on the equilibria at which the non-price-setter bids at marginal costs. For the vast majority of cases, this equilibrium generates the unique equilibrium outcome. Whenever there are multiple equilibrium outcomes, the unreported outcomes result in higher markups.

	Min	Max
END Contracts	Δ Payments	
1,000 MWs	-84	-107
2,000 MWs	-143	-194
3,000 MWs	-377	-410
4,000 MWs	-457	-577
5,000 MWs	-439	-608
6,000 MWs	-456	-632
7,000 MWs	-548	-639
8,000 MWs	-709	-654

	Min	Max
IB Contracts	Δ Payments	
1,000 MWs	-24	-78
2,000 MWs	-54	-161
3,000 MWs	-88	-222
4,000 MWs	-117	-280
5,000 MWs	-181	-379
6,000 MWs	-200	-434
7,000 MWs	-169	-437
8,000 MWs	-171	-437

Table 3: The impact of forward contracts on total payments to producers (Million €) for the Spanish electricity market during 2005

Note on Table 3: Total payments to producers under the competitive outcome are 9,599 M€; the minimum value under the no-contracts case is 11,422 M€, while the maximum is 11,728 M€. The table reports how these figures change when forward contracts are introduced. Given that there might be multiplicity of equilibrium outcomes, the Min and the Max columns report the minimum and maximum change in total payments.

However, such conclusion is reversed when contracts are allocated to the firm with the low profit-maximizing price, Iberdrola. More specifically, contracts by Iberdrola have (weakly) anti-competitive effects when they destroy the low-price equilibrium outcomes. This is the case when Iberdrola contracts either 6,000 or 8,000 MWs and demand is at its 75% percentile.

The effects of contracts reported so far vary with the demand level, e.g. whereas at very high or very low demand levels contracts barely have no effect on equilibrium outcomes, their effect for intermediate demand levels can go in either direction depending on contract volume and contract allocation. In real markets, since demand changes over time while contract volumes remain fixed, the overall effect of contracts will depend on the relative occurrence of periods in which contracts are either pro-competitive or anti-competitive. Therefore, with illustrative purposes, we have assessed the effect that contracts would have had on the Spanish electricity prices during 2005 by computing total payments to producers over the year.

Table 3 reports the change in total payments when contracts are introduced. Given that there may be multiplicity of equilibrium outcomes depending on which firm sets the price, we have reported the minimum and the maximum change in payments. Under all contract cases, total payments to generators go down, thereby indicating that the pro-competitive effects of contracts seems to dominate over the anti-competitive ones. However, the anti-competitive effects can also be inferred from these figures as they account for the non-monotonic relationship between payments to producers and total contract volume. For instance, such non-monotonicity arises when Iberdrola's contracts are increased above 6,000 MWs, when savings are reduced from 200 M€ to either 169 M€ or 171 M€.

7 Conclusions

In this paper we have analyzed the impact of forward contracts on the performance of spot markets in a model that tries to capture the essential institutional and structural features of electricity markets. Instead of assuming either Cournot or Bertrand competition, we have tried to model the actual market rules that govern most electricity markets in practice. In particular, we have assumed that firms compete by submitting step functions to the auctioneer, who then sets prices and dispatches production in increasing bid order. Furthermore, we have put no restrictions on either the market demand function - which could be either downward-sloping or price-inelastic,- or the firms' cost functions - which could result in either constant or step-wise increasing marginal costs, and could be symmetric or asymmetric across firms. Thus, the model is flexible enough so as to make it comparable with other more stylized models at the same time as it allows for all degrees of complexity. Indeed, we have used it to simulate real electricity market outcomes in order to illustrate the model predictions.

We have found that the scope of contracts to improve market performance crucially depends on both its volume and its distribution across firms. If contracts are symmetrically distributed across symmetric firms, an increase in contracts up to firms' competitive quantities is welfare improving. This supports the pro-competitive view of forward trading found in previous papers. However, this prediction might be reversed when firms are asymmetric. Indeed, contracts might lead to higher prices and reduced welfare if they jeopardize the existence of the low price equilibria. This may be the case when contracts are allocated to those firms with weak incentives to distort prices, e.g. typically the small and/or the inefficient ones. On the contrary, contracts are welfare improving whenever they are allocated to large and efficient firms with strong incentives to raise prices. These results suggest that the policy debate should focus on how allocate forward contract obligations rather than on their desirability *per se*.

We have focused on exogenously given contracts since we believe, in line with Bushnell *et al.* (2007), that many "vertical arrangements [in electricity markets] are better understood and can reasonably be considered to be exogenous." Furthermore, without knowledge of how exogenously given contracts affect market performance, it would be difficult to inform policy-makers on how to design forward contract obligations, e.g. how much virtual power to auction-off and to whom. Still, a further step of the analysis would be to allow for more general types of contracts by investigating the incentives to sign new contracts and hence their endogenous distribution across firms.[22] The current paper provides the needed first

[22] Our paper also abstracts from other types of dynamic effects. For instance Newbery (1998) shows that if

step to perform such an analysis.

To conclude, even though our analysis has been inspired by the workings of electricity markets, we believe that its implications have broader applicability. Since the most relevant features of our model are not unique to electricity markets, its conclusions could be applied to other contexts. Indeed, many modern markets are organized around a centralized auction site where all transactions are executed at market prices, e.g. order driven periodic auctions in financial markets, auctions for on-line services, or markets for inputs such as gas.

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potential entrants can sign contracts, the market becomes contestable. On the contrary, conventional wisdom suggests that contract positions may also protect an incumbent from the adverse effect of a reduction in the spot price and could therefore be used to deter new entry. Bushnell and Ishii (2007) show that just as contracts change firms' incentives in the spot market, so do they influence investment decisions and thus, long run market performance. Last, Schultz (2005) shows that if contracts are negotiated in a sequential manner, firms may have an incentives to raise spot market prices in order to influence the buyers's willingness to pay for those contracts.

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Appendix

Notation

The following pieces of notation are used throughout the Appendix. We will denote by $\underline{x}_i$ the maximum quantity that firm i can produce at marginal costs strictly below the marginal costs of producing its contract quantity, $c_i(x_i)$, and by $\overline{x}_i$ the maximum quantity that firm i can produce with marginal costs not exceeding $c_i(x_i)$. Note that $x_i \in [\underline{x}_i, \overline{x}_i]$. Formally,

$$\begin{aligned} \overline{x}_i &\equiv \max\{q : q \in [0, K_i] \ \ and \ c_i(q) \leq c_i(x_i)\}, \text{ and} \\ \underline{x}_i &\equiv \max\{q : q \in [0, K_i] \ \ and \ c_i(q) < c_i(x_i)\}, \end{aligned}$$

Similarly, we will denote by $\underline{s}_i(p)$ the maximum quantity that firm i can produce at marginal costs strictly below p, and by $\overline{s}_i(p)$ the maximum quantity that firm i can produce with marginal costs not exceeding p. Formally,

$$\begin{aligned} \underline{s}_i(p) &= \max\{q : q \in [0, K_i] \ \ and \ c_i(q) < p\}, \text{ and} \\ \overline{s}_i(p) &= \max\{q : q \in [0, K_i] \ \ and \ c_i(q) \leq p\}. \end{aligned}$$

Last, note that the market clearing price p^* depends on the bid profiles, and that the quantity allocated to each firm will depend on both p^* and b. Nevertheless to simplify notation, in the proofs that follow we will suppress these arguments whenever clear from the context.

A Proofs

Lemma A.1. *For firm i, it is weakly dominated (i) to bid below marginal cost for quantities such that $c_i(q) > c_i(\overline{x}_i)$, as well as (ii) to bid above marginal cost for quantities such that $c_i(q) < c_i(\underline{x}_i)$.*

Proof of Lemma A.1. (i) Let us fix the bid functions submitted by firms other than i at $b_{-i}(q)$, and compare the profits made by firm i with the strategies $b'_i(q)$ and $\widehat{b}_i(q)$ which only differ in the bids for units above $\overline{x}_i$. Whereas under $b'_i(q)$ those units are bid in at marginal costs, under $\widehat{b}_i$ they are bid in at prices below marginal cost. More precisely,

$$b'_i(q) = \begin{cases} b_i(q) & \text{if } \ q \leq \overline{x}_i \\ c_i(q) & \text{if } \ q > \overline{x}_i \end{cases} \qquad \text{and} \qquad \widehat{b}_i(q) = \begin{cases} b_i(q) & \text{if } \ q \leq \overline{x}_i \\ \widetilde{b}_i(q) \in [b_i(\overline{x}_i), c_i(q)) & \text{if } \ q > \overline{x}_i. \end{cases}$$

Let $\widehat{b}$ (respectively, b') denote the bidding profile $(\widehat{b}_i(q), b_{-i}(q))$ (respectively, $(b'_i(q), b_{-i}(q))$). We first note that the equilibrium price under $\widehat{b}$ is no larger than under b' as $\widehat{b}_i(q) \leq b'_i(q)$ while $b_{-i}(q)$ coincides under both profiles. Consequently $\widehat{p} \leq p'$. Furthermore, if $\widehat{p} < p'$ then $\widehat{q}_i \geq q'_i$ and $q'_i \geq \overline{x}_i \geq x_i$ as $\widehat{b}_i(q) = b'_i(q)$ for all $q \leq \overline{x}_i$. Profits at the two profiles are given by

$$\begin{aligned}\pi_i(b') &= [q_i' - x_i]\, p' - C_i(q_i') + \tau_i x_i \text{ and} \\ \pi_i\left(\widehat{b}\right) &= [\widehat{q}_i - x_i]\,\widehat{p} - C_i(\widehat{q}_i) + \tau_i x_i,\end{aligned}$$

with $\widehat{q}_i \geq q_i' \geq \overline{x}_i \geq x_i$. Consequently,

$$\pi_i(b') = \pi_i\left(\widehat{b}\right) + \underbrace{[q_i' - x_i]\,[p' - \widehat{p}]}_{\geq 0} - \underbrace{\int_{q_i'}^{\widehat{q}_i} [\widehat{p} - c_i(q)]\, dq}_{\leq 0} \geq \pi_i\left(\widehat{b}\right)$$

where $\int_{q_i'}^{\widehat{q}_i} [\widehat{p} - c_i(q)]\, dq \leq 0$ as either $\widehat{q}_i = q_i'$ so that the integral is zero or $q_i' < \widehat{q}_i$ and then $\widehat{p} \leq c_i(q)$ for all $q > \overline{x}_i$ and hence for $q \in [q_i', \widehat{q}_i]$. Since profits under b' are no smaller than under $\widehat{b}$, the statement follows.

(ii) Let us fix the bid functions submitted by firms other than i at $b_{-i}(q)$, and compare the profits made by firm i with the strategies $b_i'(q)$ and $\widehat{b}_i(q)$ that only differ in the bids for units below $\underline{x}$, with

$$b_i'(q) = \begin{cases} c_i(q) & \text{if } \; q \leq \underline{x}_i \\ b_i(q) & \text{if } \; q > \underline{x}_i \end{cases} \qquad \text{and} \qquad \widehat{b}_i(q) = \begin{cases} \widetilde{b}_i(q) \in (c_i(q), c_i(\underline{x}_i)] & \text{if } \; q \leq \underline{x}_i \\ b_i(q) & \text{if } \; q > \underline{x}_i. \end{cases}$$

We first note that the equilibrium price under $\widehat{b}$ is no smaller than under b' as $\widehat{b}_i(q) \geq b_i'(q)$ while $b_{-i}(q)$ coincide under both profiles. Consequently $\widehat{p} \geq p'$. Furthermore, if $\widehat{p} > p'$ then $\widehat{q}_i \leq q_i'$ and $q_i' \leq \underline{x}_i \leq x_i$ as $\widehat{b}_i(q) = b_i'(q)$ for all $q > \underline{x}_i$. The difference in profits at the two profiles is hence

$$\pi_i(b') = \pi_i\left(\widehat{b}\right) - \underbrace{[\hat{q}_i - x_i]\,[\widehat{p} - p']}_{\leq 0} + \underbrace{\int_{\hat{q}_i}^{q_i'} [p' - c_i(q)]\, dq}_{\geq 0} \geq \pi_i\left(\widehat{b}\right).$$

Since profits under b' are no smaller than under $\widehat{b}$, the statement follows. ■

Proof of Proposition 1. Since we have to show that $q_j \in \left[\underline{s}_j(p^*), \overline{s}_j(p^*)\right]$, we first prove that $q_j \geq \underline{s}_j(p^*)$. Assume for the sake of contradiction that there is some firm j, $j \neq i$, for which $q_j < \underline{s}_j(p^*)$ with $q_j \geq \underline{x}_j$ as weakly dominated strategies are not allowed. As there is a finite number of units, there exists $\hat{\epsilon} > 0$ such that no unit has been bid in at prices in the interval $(p^* - \hat{\epsilon}, p^*)$. Pick an $\epsilon < \hat{\epsilon}$ and consider the following deviation by firm j :

$$b_j'(q) = \begin{cases} b_j(q) & \text{for} \quad 0 \leq q < q_j \\ p^* - \epsilon & \text{for} \quad q \in \left[q_j, \underline{s}_j(p^*)\right] \\ b_j(q) & \text{for} \quad \underline{s}_j(p^*) < q \leq K_j \end{cases}.$$

Let the resulting bidding profile be denoted by b', where $b' = (b_1(q), .., b_j'(q), .., b_N(q))$, with associated equilibrium price p' and with $q_j' = q_j(b') \geq q_j$. The difference between firm j's profits under b and b' is given by:

$$\pi_j(b') - \pi_j(b) = [p' - p^*]\left[q_j' - x_j\right] + \int_{q_j}^{q_j'} [p^* - c_j(q)]\, dq.$$

There are two cases to consider: (a) If $p' = p^* - \epsilon$, then the difference in profits is positive: the second term of the above equation is positive by the definition of $\underline{s}_j(p^*)$ as $p^* > c_j(q)$ for any positive $q < q_j' \leq \underline{s}_j(p^*)$, whereas the first term can be made arbitrarily small by taking ϵ small enough. (b) If $p' = p^*$, the deviant now sells the additional output $\left(q_j - \underline{s}_j(p^*)\right)$ at a price above its marginal costs and thus gets more profits. The derived contradictions prove the first result.

We now show that $q_j \leq \overline{s}_j(p^*)$.By contradiction, assume there is some firm j, $j \neq i$, for which $q_j > \overline{s}_j(p^*)$. By elimination of weakly dominated strategies, this requires $q_j \leq \overline{x}_j$. Consider the following deviation by firm j :

$$b_j'(q) = \begin{cases} b_j(q) & \text{for} \quad 0 \leq q < \overline{s}_j(p^*) \\ p^* + \epsilon & \text{for} \quad q \in [\overline{s}_j(p^*), q_j] \\ b_j(q) & \text{for} \quad q_j < q \leq K_j \end{cases}.$$

Let the resulting profile be denoted by b', with equilibrium price p' and with $q_j' = q_j(b') \leq q_j$. The difference between firm j's profits under b and b' is given by:

$$\pi_j(b') - \pi_j(b) = [p' - p^*]\left[q_j' - x_j\right] + \int_{q_j'}^{q_j} [c_j(q) - p^*]\, dq.$$

There are two cases to consider: (a) If $p' = p^* + \epsilon$, then the difference in profits is positive: the second term of the above equation is positive given that $c_j(q) > p^*$ for any positive $q > q_j' \geq \overline{s}_j(p^*)$, whereas the first term can be made arbitrarily small by taking ϵ small enough. (b) If $p' = p^*$, the deviant now reduces its output by $(q_j - \overline{s}_j(p^*))$ and therefore reduces its losses. As in either case, the deviation is profitable we ran into a contradiction that proves the second result. ■

Proof of Lemma 1. (i) We first note that $q_j \geq \underline{s}_j(p^*)$ holds for all $j \neq i$ because of Proposition 1. Thus if $p^* \geq p^c$, then $\underline{s}_j(p^*) \geq \underline{s}_j(p^c)$ given that $\underline{s}_j$ in a non-decreasing function of p. Consequently, $\pi_j^{NPS}(p^*) = \max_q \{p^*[q - x_j] - C_j(q)\} + \tau_j x_j$ is an increasing function of p^* as $q_j^{NPS}(p^*) \geq q_j^c > x_j$. Finally, if $q_j^c = x_j$, firms i's profits are increasing in p^* if $q_j > q_j^c$, and they are independent of p^* if $q_j = q_j^c$.

(ii) Since $q_j^{NPS}(p^*) \leq q_j^c$ if $p^* \leq p^c$, then, using similar arguments to those above, $q_j^{NPS}(p^*) \leq q_j^c \leq x_j$ implies that $\pi_j^{NPS}(p^*)$ is a decreasing function of p^* if $q_j^c < x_j$ and it is non-increasing if $q_j^c = x_j$. ■

Proof of Lemma 2. If $p^* \geq p^c$ then $q_i^{PS}(p^*) = D(p^*) - \sum_j q_j^{NPS}(p^*) = D(p^*) - \sum_{j \neq i} \underline{s}_j(p^*)$. Since $D(p^*) \leq \sum_i \underline{s}_i(p^*)$ then $q_i^{PS}(p^*) \leq \underline{s}_i(p^*)$, so that the result follows as

$$\begin{aligned} \pi_i^{PS}(p^*) &= p^*\left[q_i^{PS}(p^*) - x_i\right] - C\left(q_i^{PS}(p^*)\right) + \tau_i x_i \\ &\leq p^*\left[\underline{s}_i(p^*) - x_i\right] - C\left(\underline{s}_i(p^*)\right) + \tau_i x_i = \pi_i^{NPS}(p^*). \end{aligned}$$

If $p^* < p^c$ then $q_i^{PS}(p^*) > \bar{s}_i(p^*)$ and

$$\pi_i^{PS}(p^*) = \pi_i^{NPS}(p^*) + \int_{\bar{s}_i(p^*)}^{q_i^{PS}(p^*)} [p^* - c_i(q)]\, dq < \pi_i^{NPS}(p^*),$$

as $p^* < c_i(q)$ for all $q > \bar{s}_i(p^*)$. ■

Proof of Proposition 2. (i) We first show that $x_i < q_i^c$ implies $p^* \geq p^c$. Argue by contradiction and assume $x_i < q_i^c$ and $p^* < p^c$ where $p^* = p_i^*(b_{-i})$. Since $p^* < p^c$, then by appealing to Proposition 1 it must hold that $q_j^{NPS}(p^*) \leq q_j^c$, so that $q_i^{PS}(p^*) \geq q_i^c$ and hence $q_i^{PS}(p^*) \geq x_i$. As there is a finite number of units, there exists $\hat{\epsilon} > 0$ such that no unit has been bid in at prices in the interval $(p^*, p^* + \hat{\epsilon})$. Pick an $\epsilon < \hat{\epsilon}$ such that $p^* + \epsilon < p^c$ and suppose that firm i deviates from setting the price at p^* to $p^* + \epsilon$. We argue that this deviation increases firm i's profits, contradicting equilibrium behaviour, i.e., contradicting $p^* = p_i^*(b_{-i})$. To see this, note that the difference in profits is given by

$$\pi_i^{PS}(p^* + \epsilon) - \pi_i^{PS}(p^*) = \epsilon [q_i' - x_i] + \int_{q_i'}^{q_i} [c_i(q) - p^*] \, dq,$$

where $q_i' = q_i(p^* + \epsilon) \leq q_i = q_i(p^*)$.

There are two cases to consider: (a) If $q_i' = q_i$, then the difference in profits, $\epsilon [q_i' - x_i]$, is positive given that $q_i' = q_i \geq q_i^c > x_i$. (b) If $q_i' < q_i$, then it must be still the case that $q_i' > q_i^c$ as $p^* + \epsilon < p^c$ and no other unit is bid in at $(p^*, p^* + \hat{\epsilon})$. Consequently $\epsilon [q_i' - x_i] > 0$ and $\int_{q_i'}^{q_i} [c_i(q) - p^*] \, dq > 0$ as $p^* < p^c \leq c_i(q)$ for any $q > q_i^c$. As the firm is better-off deviating,a contradiction is reached.

Second, we show that $x_i < q_i^c$ implies $x_i \leq q_i$. Argue by contradiction and assume $x_i \leq q_i^c$ and $x_i > q_i$. Since $x_i \leq q_i^c$ then $p^* \geq p^c$ by the proof above. Let's first assume $p^* > p^c$. As the rivals are dispatching all units with marginal costs below p^*, then $q_i \leq q_i^c$ and $p^* > c_i(q_i)$. Consider the following deviation by firm i: all units that were originally bid in at p^* plus the extra units needed to cover q_i^c are bid in at $p^* - \epsilon$, where again $\epsilon < \hat{\epsilon}$, and $p^* - \epsilon > p^c$; the remaining units are bid in as originally. Since the equilibrium price is now $p^* - \epsilon > p^c$ firm i produces $q_i^c \geq q_i' = q_i(p^* - \epsilon; b_{-j}) \geq q_i(p^*; b_{-j}) = q_i$. Firm i's deviation gains are given by

$$\pi_i^{PS}(p^* - \epsilon) - \pi_i^{PS}(p^*) = \epsilon [x_i - q_i] + \int_{q_i}^{q_i'} [p^* - \epsilon - c_i(q)] \, dq > 0,$$

where the last inequality follows from the fact that $x_i > q_i$ and $p^* - \epsilon > p^c \geq c_i(q)$ for all $q \leq q_i'$ as $q_i' \leq q_i^c$. The existence of a profitable deviation gives the desired contradiction.

Last, let's assume $p^* = p^c$. As this implies $q_i = q_i^c$, $x_i \leq q_i^c$ and $x_i > q_i$ would contradict each other.

(ii) If $x_i = q_i^c$ and $p^* = p^c$, which trivially imply $x_i = q_i$, firm i has no incentives to deviate. If firm i reduced the price to $p' < p^* = p^c$, its profits would decrease by $\int_{q_i}^{q_i'} [p' - c_i(q)] \, dq < 0$, given that $p' < p^* = p^c \leq c_i(q)$ for all $q > q_i = q_i^c$. If it increased the price to $p' > p^* = p^c$, its profits would decrease by $\int_{q_i'}^{q_i} [c_i(q) - p'] \, dq < 0$, given that $p' > p^* = p^c \geq c_i(q)$ for all $q < q_i = q_i^c$.

(iii) The proof follows the same steps and (reversed) arguments as the one in part (i); it is hence omitted. ■

Proof of Theorem 1. (i) *[Only if]* Suppose there exists a pure-strategy equilibrium $(b_1, .., b_n)$ in which firm i sets the price at $p^* \geq p^c$ and firms' payoffs are $\pi_i^{PS}(p^*)$ and $\pi_j^{NPS}(p^*)$, $i, j = 1, ...N$, $j \neq i$. If this is the case then Condition 1). follows from Propositions 1 and 2, and both Conditions 2). and 3). follow trivially from the definition of Nash equilibrium.

[If] We have to show that no firm profits by deviating from strategies that satisfy conditions 1). to 3). Consider first the non-price setters j, $j \neq i$. By 1). we only need to consider deviations that imply a change in identity. Let us distinguish between those firms with $x_j > q_j$ and those with $x_j \leq q_j$. For a firm j with $x_j > q_j$ the only relevant deviations are those that allow to become the price-setter at a price below p^*. To see this, compare for any $p' \geq p^*$,

$$\begin{aligned} \pi_j^{NPS}(p^*) &= p^*[q_j - x_j] - C(q_j) + \tau_j x_j \text{ with} \\ \pi_j^{PS}(p') &= p'\left[q_j' - x_j\right] - C(q_j') + \tau_j x_j, \end{aligned}$$

where $q_j = q_j^{NPS}(p^*) \geq q_j' = q_j(p')$. The difference in profits is given by

$$\pi_j^{PS}(p') - \pi_j^{NPS}(p^*) = [p' - p^*]\left[q_j' - x_j\right] + \int_{q_j'}^{q_j} [c_j(q) - p^*]\, dq. \tag{4}$$

The first term of the above expression is negative given that $p' > p^*$ and $q_j' \leq q_j < x_j$. The second term is also negative given that $p^* \geq c_j(q_j) \geq c_j\left(q_j'\right)$, where the first inequality follows from Proposition 1. Hence, those firms j with $x_j > q_j$ and $p_j^*(b_{-j}) \geq p^*$ will not deviate. To assess whether firms j with $x_j > q_j$ have incentives to deviate by reducing the price, note that for any $p' < p^*$ the difference in profits becomes,

$$\pi_j^{PS}(p') - \pi_j^{NPS}(p^*) = [p' - p^*]\left[q_j' - x_j\right] + \int_{q_j}^{q_j'} [p^* - c_j(q)]\, dq, \tag{5}$$

where now $q_j \leq q_j'$. The second term of the above expression is negative given that $p^* \geq p^c$ and Proposition 1 imply $p^* < c_j(q)$ for all $q > q_j$. The first term is also negative if $x_j < q_j'$. Since deviating to p_j^* is the most profitable deviation, it follows that firms with $p_j^*(b_{-j}) < p^*$ and $x_j \leq q_j\left(p_j^*\right)$ will not deviate. Finally, since 3). ensures that firms j with $p_j^*(b_{-j}) < p^*$ and $x_j > q_j\left(p_j^*\right)$ do not want to become the price-setter either, one can conclude that no deviation by firms j with $x_j > q_j$ is profitable.

Consider now firms j such that $x_j \leq q_j$. The only relevant deviations are those that allow to become the price-setter at a price $p_j^* > p^*$. Deviating to a price lower than p^* is not profitable given that $x_j \leq q_j \leq q_j'$ implies that the first term in (5) is negative. Hence, those firms j with $x_j \leq q_j$ and $p_j^*(b_{-j}) \leq p^*$ will not deviate. Those firms j with $x_j \leq q_j$ and $p_j^*(b_{-j}) > p^*$ will not deviate if $x_j \geq q_j\left(p_j^*\right)$ given that (4) would then be negative. Last, since by 3). firms j with $p_j^*(b_{-j}) > p^*$ and $x_j < q_j\left(p_j^*\right)$ do not want to become the price-setter either, no deviation by the non-price setters is profitable.

Consider now firm i. Since Condition 1 holds, the only deviations we have to consider are those that would allow firm i to become a non-price setter at a price below p_i^*. Since this deviation is ruled out by Condition 2, we can conclude that the strategy profile $(b_1, .., b_n)$ constitutes a Nash equilibrium.

(ii) The proof follows the same steps and (reversed) arguments as the one in part (i); it is hence omitted. ■

Proof of Proposition 3. (i) We first note that the set of sufficient conditions for a strategy profile to constitute a Nash equilibrium follow from Theorem 1 part (i). Condition 2). of the Theorem, i.e., $\pi_i^{PS}(p^*) \geq \pi_i^{NPS}(p)$ for all $p < p^*$ such that $q_i^{NPS}(p) + \sum_{j \neq i} q_j(p; b) = D(p)$, holds trivially as the only price at which firm i can be a non-price setter is p^c and this deviation is ruled out by the fact that $p^* = p_i^*$ is a maximizer, i.e., $\pi_i^{PS}(p^*) \geq \pi_i^{PS}(p^c) = \pi_i^{NPS}(p^c)$ as rivals are bidding at marginal costs. Condition 3). also holds: $x_i \leq q_i^c$ (which by Proposition 2 implies $p^* \geq p^c$) and $q_j = q_j^{NPS}(p^*) \geq q_j^c$, rule out deviations by the non-price-setters to lower prices as Lemmas 1 and 2 imply $\pi_j^{NPS}(p^*) \geq \pi_j^{NPS}(p) \geq \pi_j^{PS}(p)$ for any $p \leq p^*$ given that $x_j \leq q_j^c$ for all firms j. Hence, we only need to show that an equilibrium exists with $p^* = p_i^* \geq p^c$ at which firm i is the price-setter and profits are given by $\pi_i^{PS}(p^*)$ and $\pi_j^{NPS}(p^*)$ for all $j \neq i$. We prove this by constructing a strategy profile that results in such an equilibrium. Consider the strategy profile b according to which all firms but firm i bid at marginal costs and firm i submits the strategy $b_i(q; \bar{p}_i^*)$ where

$$b_i(q; \bar{p}_i^*) = \begin{cases} c_i(q) & \text{if} \quad q < \underline{x}_i, \\ \max\{\bar{p}_i^*, c_i(q)\} & \text{if} \quad q \geq \underline{x}_i. \end{cases}$$

We first note that the equilibrium price under the proposed profile, p^*, must be equal to p_i^*. The proposed profile cannot result in $p^* > p_i^*$ since at any such price p^* all firms are bidding at marginal costs, and $p^* > p_i^* \geq p^c$, a contradiction. The proposed profile cannot result in $p^* < p_i^*$ either as at any such price, $q_i = \underline{x}_i$, implying that firm i's profits would be negative, which contradicts that p_i^* maximizes firm i 's profits as $\pi_i^{PS}(p_i^*) \geq \pi_i^{PS}(p^c) \geq 0$ given that $x_i \leq q_i^c$.

Last, we need to show that firms are best responding to each other. Under profile b, firm i cannot profitably deviate since $p^* = p_i^*(c_{-i})$ implies that it is already maximizing over its residual demand. For firms j, $j \neq i$, any profitable deviation must result in the deviant firm becoming the price-setter. Such a deviation can only be profitable by firms j with $p_j^* > p_i^*$ as by appealing to lemmas 1 and 2 we have that

$$\pi_j^{NPS}(p_i^*) \geq \pi_j^{NPS}(p) \geq \pi_j^{PS}(p) \text{ for any } p \leq p_i^*.$$

For prices above p_i^* firm i is bidding at marginal costs, so that if firm j with $p_j^* > p_i^*$ deviates it will set the price at $p_j^*(c_{-j})$. Since $\pi_j^{NPS}(p^*) \geq \pi_j^{PS}(p_j^*)$ holds for all $j \neq i$ such that $p_j^*(c_{-j}) > p^*$, such deviations are not profitable either. As firms are best responding to each other, the proposed profile b constitutes an equilibrium as claimed.

(ii) The proof follows the same steps and (reversed) arguments as the one in part (i); hence, we only provide here the pieces of information which are specific to this case. In particular, a strategy profile that results in such an equilibrium is a strategy profile b according to which all firms but firm i bid at marginal costs and firm i submits the strategy $b_i(q; \underline{p}_i^*)$, where

$$b_i\left(q; \underline{p}_i^*\right) = \begin{cases} \min\left\{\underline{p}_i^*, c_i(q)\right\} & \text{if } q \leq \bar{x}_i, \\ c_i(q) & \text{if } q > \bar{x}_i. \end{cases}$$

■

Proof of Proposition 4.

Consider the case in which $x_i \leq q_i^c$ for all firms. First, suppose there exist at least two equilibria: one in which firm i sets the price at p^* and all other firms bid at marginal costs and another one in which firm i sets the price at p' and at least one firm j, $j \neq i$, bids some units *above* marginal costs, $b'_j(q) > c_j(q)$. Let the bid profiles submitted by the non-price setters under both equilibria be respectively denoted b_{-i} and b'_{-i}. Three cases may arise:

(a) Equilibrium prices under both profiles are equal, $p' = p^*$, so that both equilibria are outcome-equivalent.

(b) Equilibrium prices differ, with $p' < p^*$.

Note that $b'_{-i}(q) \geq b_{-i}(q)$ implies that $q_i\left(p; b'_{-i}\right) \geq q_i(p; b_{-i})$ for any p. By revealed preference,

$$\pi_i^{PS}(p^*; b_{-i}) > \pi_i^{PS}(p'; b_{-i}) \text{ and } \pi_i^{PS}\left(p^*; b'_{-i}\right) < \pi_i^{PS}\left(p'; b'_{-i}\right) \text{ implying}$$

$$\begin{aligned} p^*\left[q_i(p^*; b_{-i}) - x_i\right] - C_i(q_i(p^*; b_{-i})) &> p'\left[q_i(p'; b_{-i}) - x_i\right] - C_i(q_i(p'; b_{-i})) \text{ and} \\ p^*\left[q_i\left(p^*; b'_{-i}\right) - x_i\right] - C_i\left(q_i\left(p^*; b'_{-i}\right)\right) &< p'\left[q_i\left(p'; b'_{-i}\right) - x_i\right] - C_i\left(q_i\left(p'; b'_{-i}\right)\right). \end{aligned}$$

Rearranging terms,

$$\int_{q_i(p^*; b_{-i})}^{q_i\left(p^*; b'_{-i}\right)} \left[p^* - c_i(q)\right] dq < \int_{q_i(p'; b_{-i})}^{q_i\left(p'; b'_{-i}\right)} \left[p' - c_i(q)\right] dq. \tag{6}$$

Since $p' < p^*$ and marginal cost functions are non-decreasing, we must have

$$q_i\left(p^*; b'_{-i}\right) - q_i(p^*; b_{-i}) < q_i\left(p'; b'_{-i}\right) - q_i(p'; b_{-i}).$$

Furthermore, since at least one non-price-setter is bidding above marginal costs under b', then $q_i\left(p^*; b'_{-i}\right) \geq q_i(p^*; b_{-i})$. Consequently, $q_i\left(p'; b'_{-i}\right) > q_i(p'; b_{-i})$. As this implies $\sum_{j \neq i} q_j\left(p'; b'_{-j}\right) < \sum_{j \neq i} q_j(p'; b_{-j})$, at least one non-price-setter must have unit(s) with marginal cost below p' which are not dispatched under b', so that Proposition 1 is violated, a contradiction with equilibrium behaviour.

(c) Equilibrium prices differ, with $p' > p^*$.

Again, $q_i\left(p'; b'_{-i}\right) \geq q_i(p'; b_{-i})$. By revealed preference, equation (6) applies. Thus, if $q_i(p'; b_{-i}) = q_i\left(p'; b'_{-i}\right)$ then the left hand side of (6) must be negative, which contradicts Proposition 2, i.e., that p^* is an optimal price for firm i when the rivals bid at b_{-i}, so that $q_i\left(p'; b'_{-i}\right) > q_i(p'; b_{-i})$ must hold. As this implies $\sum_{j \neq i} q_j\left(p'; b'_{-j}\right) < \sum_{j \neq i} q_j(p'; b_{-j})$, a non-price-setter must have unit(s) with marginal cost below p' which are not dispatched under b', violating Proposition 1 and hence contradicting equilibrium behaviour.

Last, suppose there exist at least two equilibria: one in which firm i sets the price at p^* and all other firms bid at marginal costs; and another one in which firm i sets the price at p' and at least some firm j, $j \neq i$, bids some units *below* marginal costs, $b'_j(q) < c_j(q)$. Again, let the bid profiles submitted by the non-price setters under both equilibria be respectively denoted b_{-i} and b'_{-i}. Since $x_i \leq q_i^c$ for all firms, then $p^* \geq p^c$ and $x_j \leq q_j$ so that, by revealed preference arguments, firm j may only bid below marginal costs those units that are already dispatched under the first equilibria. Hence, $q_i\left(p^*; b'_{-i}\right) = q_i\left(p^*; b_{-i}\right)$. As this implies that firm i must still find it optimal to set the price at p^* when all other firms bid at b'_{-i}, then $p^* = p'$ and both equilibria must be outcome equivalent. This shows our claim. ■

Proof of Corollary 3. (i) Suppose $x_i \leq q_i^c$ for all firms $i = 1, ..., N$. First, assume that the equilibrium in which firm k is the price-setter *exists*. From Proposition 3 part (i) $\pi_r^{NPS}(p_k^*) \geq \pi_r^{PS}(p_r^*)$ must hold for all firms $r < k$. Since by Lemma 1 the non-price-setters' profits are increasing in p, it follows that $\pi_r^{NPS}\left(p_{k-i}^*\right) \geq \pi_r^{NPS}(p_r^*)$ for any $i = 1, .., k-1$. Consequently, $\pi_r^{NPS}\left(p_{k-i}^*\right) \geq \pi_r^{NPS}(p_r^*) \geq \pi_r^{NPS}(p_k^*) \geq \pi_r^{PS}(p_r^*)$ implies that the equilibria in which firms $\{1, ..., k-1\}$ are the price-setters also exist.

Last, assume that the equilibrium in which firm k is the price-setter *does not exist.* Since firms $j > k$ do not have any profitable deviation, it must be the case that for some firm $r < k$, $\pi_r^{NPS}(p_k^*) < \pi_r^{PS}(p_r^*)$. Since the non-price-setters' profits are increasing in p, it follows that $\pi_r^{NPS}\left(p_{k+i}^*\right) \leq \pi_r^{NPS}(p_k^*)$ for any $i = 1, .., N-k$. Consequently, $\pi_r^{NPS}\left(p_{k+i}^*\right) \leq \pi_r^{NPS}(p_k^*) < \pi_r^{PS}(p_r^*)$ implies that the equilibria in which firms $\{k+1, ..., N\}$ are the price-setters do not exist either.

(ii) Suppose $x_i \geq q_i^c$ for all firms. It suffices to reverse the direction of the inequalities with respect to the ones above. ■

Proof of Proposition 5. Existence of an equilibrium trivially follows from Theorem 3 in Jackson and Swinkels (2005). Note further that existence can be guaranteed by appealing to Reny's better reply security (see corollary 5.2 to Theorem 3.1 in Reny (1999)): due to the efficient tie-breaking rule, bidders' payoffs are secure and their sum is upper semi-continuous, so that an equilibrium always exists.

To see why marginal cost bidding by the non-price setters may not generate an equilibrium, consider the following example. A duopoly composed of firms 1 and 2 face demand $D(p) = 20 - p$. Both firms' production units have capacity normalized to one, and their marginal costs are given by

$$c_1(q) = \begin{cases} 4 & \text{if} \quad 0 \leq q \leq 4 \\ \lceil q \rceil & \text{if} \quad 4 \leq q \leq 20 \end{cases} \qquad \text{and} \qquad c_2(q) = \begin{cases} \frac{\lceil q \rceil}{2} & \text{if} \quad 0 \leq q \leq 11 \\ \frac{11}{2} & \text{if} \quad 11 \leq q \leq 16 \\ \frac{\lceil q \rceil}{2} & \text{if} \quad 16 \leq q \leq 20 \end{cases}.$$

Finally, let their contract positions be $x_1 = 0$ and $x_2 = 15$. If firm 1 acts as the price setter, then $p_1^* = 5.5$, $D(p_1^*) = 14.5$, $q_2 = 10$ and $\pi_2^{NPS}(p_1^*) = -55$. However, firm 2 prefers to deviate and set the price at 5.25. It would then sell 14.75 units (since $x_1 = 0$, firm 1 is not offering any unit below 5.5) and would make profits equal to -54.938. Since the deviation is profitable, the candidate price $p_1^* = 5.5$ fails to be in equilibrium. If firm 2 acts as the price-setter, then $p_2^* = 4$, $D(p_2^*) = 16$, $q_1 = 4$ and $\pi_1^{NPS}(p_2^*) = 0$. However, firm 1 prefers to deviate and set the price at 4.5. It would then sell 0.5 units (firm 2 is bidding $x_2 = 15$ units at prices no larger than 4) and it would make profits equal to 0.25. Since the deviation is profitable, the candidate price $p_2^* = 4$ fails to be in equilibrium.[23] Nevertheless, for this market configuration it is an equilibrium for firm 2 to set the price by bidding $\min\{4.5, c_2(q)\}$ for $q \leq 15$ and marginal cost for the remaining units, while firm 1 follows a flat strategy at 4.5, i.e., $b_1(q) = \max(4.5, c_1(q))$. This equilibrium results in $p^* = 4.5$. ■

B Details on Simulations

The Spanish electricity market is organized similarly to many other wholesale electricity markets around the world. In particular, most transactions take place through an organized exchange, that operates on an hourly basis according to the rules described in Section 2 (see Crampes and Fabra (2005) for more details). The market structure is highly concentrated, with the two largest firms - Endesa and Iberdrola - controlling almost 60% of total thermal capacity, more than 80% of total hydro capacity, and approximately 40% of total renewables. Even though the shares of these technologies on total production vary across years, in 2005 hydro and renewables contributed to cover 8% and 11% of total demand, respectively. Table 4 summarizes the market structure of the main generators in the Spanish electricity market.

In order to conduct the simulations, we have first computed firms' marginal cost curves following similar techniques as in previous papers (Fabra and Toro (2005)).[24] In particular, we have estimated each thermal unit's marginal production costs on a daily basis, taking into account the type of fuel it burns, the cost of the fuel, the plant's heat rate (i.e., the efficiency rate at which each plant converts the heat content of the fuel into output), the short-run variable cost of operating and maintaining it, and the costs of its CO_2 emissions. In addition, for coal plants, we have added an estimate of the costs of transporting coal from the nearest harbor where it is delivered to the plant where it is consumed. Lastly, each unit's generation capacity has been reduced by its estimated outage rate. By aggregating the capacities of each firm's thermal units in increasing marginal cost order, we have obtained estimates of firms' thermal marginal cost curves for each day of the year.

Furthermore, we have assumed that the marginal costs of producing electricity with hydro and renewables equal zero. The production coming from such sources has therefore

[23] Note that non-existence of the equilibrium at which the non-price setter bids at marginal cost is the exception rather than the rule. In the example discussed above, if $x_2 \leq 14.5$, then $p_1^* = 5.5$ is an equilibrium. Similarly, if $x_2 > 15$, then $p_2^* = 4$ is an equilibrium.

[24] The data used in the simulations have been obtained from various sources, including The National Energy Commission (CNE), Red Eléctrica de España (REE), OMEL and UNESA.

Firm/ Technology	Nuclear	Coal	CCGT	Oil-Gas	Total	Shares
Endesa	3,511	5,511	1,170	1,779	10,918	33%
Iberdrola	3,222	1,225	3,704	3,050	8,456	26%
Unión Fenosa	702	1,946	1,559	747	4,954	15%
Gas Natural	–	–	2,729	–	2,729	8%
Hidrocantábrico	155	1,549	390	–	2,094	6%
Others	–	909	2,144	731	3,784	11%

Table 4: Installed Thermal Capacity (MW) by Firm and Technology in the Spanish Electricity Market, 2005 (Source: REE)

been added to the left of each firm's thermal marginal costs curve in order to construct their overall marginal costs curves. We have chosen not to use actual data on hydro production, as it is already the result of firms' strategic decisions. Instead, poundage hydro generation has been set to peak-shave demand on a monthly basis, taking into account maximum hydro flows.[25] Both run-of-river hydro as well as renewables' production have been uniformly spread across time. Hydro stocks, run-of-river hydro flows and renewable energy are monthly estimates of a representative year.

Demand has been assumed to be price-inelastic at the actual hourly demand levels that were observed in 2005. Furthermore, we have set the price cap at 120€/MWh, below its explicit 180€/MWh level, with the aim of reflecting issues such as the threat of entry or regulatory intervention. Nevertheless, setting the price at either 120 or 180€/MWh does not change the qualitative nature of the results.

[25] As it is by now well understood (Bushnell (2003)), firms could strategically shift hydro from peak to off-peak hours, thereby distorting the efficient use of hydro resources. A full analysis of this issue is out of the scope of this section. However, despite assuming competitive bidding for hydro units, hydro still affects firms' strategic decisions through its impact on their inframarginal output.