Hot and Spicy: Ups and Downs on the Price Floor and Ceiling at Japanese Supermarkets *

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Abstract

This paper develops a model of dynamic pricing with menu cost for stores with some local market power in retail markets. Our model derives frequent price changes among a few focal prices as the optimal price policy. The key assumption of the model is that customers differ in their willingness to pay, depending upon they purchase the product for immediate consumption, or for the inventory to avoid stock-out. The latter probability is important because the customers visit these stores occasionally and their shopping decisions are based upon overall consumption and shopping plan, not whether or not they need to purchase the particular item of our interest.

The model is then applied for the estimation of the demand fluctuations and pricing policy for two brands of curry pastes sold at 18 different supermarket stores in Japan. The empirical results strongly support the following key predictions of the model: (1) stores tend to lower the price when (a) the share of customers with inventory is lower, and when (b) the expected shopping intensity is higher; (2) the demand exhibits negative dependence on price duration at lower sales price, whereas the demand is positively dependent upon the price duration at high, regular price. This pattern is consistent with accumulation (when the price is low) and decumulation (when the price is high) of the inventory at home as the driving factor of short-run fluctuations in the demand.

JEL Classification Numbers L11, L81, D43, L16

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1 Introduction

Figure 1 speaks better than any other form of introduction in clarifying the objective of the paper. The figure plots daily movements of House Vermont Curry, a national brand of curry paste\(^1\) sold at a store belonging to one of the nationwide supermarket chains in Japan. The figure traces the price for roughly 5 years during which the price changed more than 300 times. This store was open for 1707 days during the observation period stretching 1889 days\(^2\). Remarkably, about 77% of the recorded prices are concentrated in only two specific levels in this brand, 253 and 198 yen (Figure 2). There are 64 other prices observed for this brand but these observations constitute only 378 days, or 22%, of the price observations. There are 364 price changes in the data. Among 363 completed spells, more than 70% of them are less than 6 days, with one day spell comprising 40% of these spells. Not surprisingly, frequent price changes among few focal prices induce customers to wait for the price markdown to concentrate their purchases: among the 7 stores which belong to one national chain of supermarket, the price was below 198 yen for 1,862 store-days, or 15% of 12,032 store days of the total observation, during which the stores sold 130,394 units, or, 45.6% of the total sales, 286,812.

What exactly are the underlying logic behind this type of pricing? Although it is clear that the customers do recognize such a pricing pattern and adjust their purchasing cycle to that of pricing, it is not at all obvious why these stores employ such a pricing policy.

This paper develops a model of sales by extending the Varian’s pioneering work [Varian(1980)]. We then empirically estimates the optimal pricing model with special emphasis on inventory accumulation by consumers.

A small but rapidly growing number of empirical researches use scanner data at retail stores to investigate the underlying mechanism of price adjustments at individual store/brand level. The intertemporal pricing pattern like the one we saw in Figure 1, is commonly observed among retailers\(^3\), although ours seem to exhibit exceptionally high frequency of price changes\(^4\). The most obvious

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1 Curry paste is sold generally in a package containing half-solid bars which can be cooked with vegetables and meats. Curry served with rice is extremely popular in Japan. The annual sales of the curry paste (or bar) is roughly 80 billion yen in recent years, which translates into 1.5 billion curry rice dishes cooked and consumed in Japanese household in a typical year. Two brands in our sample are chosen because these are by far the most well known and longtime best sellers among the curry pastes. The market shares of these two brands are relatively small; although no official statistics exist, the share is likely to be less than 10% for both brands. The House with commanding 40-45% market shares of the curry pastes have 5 major brands and each brand comes in 3 to 4 different versions (typically distinguished by the spicyness). Our House sample is one version of the five major brands offered by House. Curry paste in Japan is similar to Cambell’s soup cans in the United States: you are bound to find at least a few packages of curry pastes in any randomly picked Japanese house.

2 Missing observations due to no registered sales or store closing constitute 182 days.


4 For example, in Slades(1998), the average duration of price is about 5 weeks, compared to 5 to 25 days in our data (see Table 3). In Aguirregabiria, the distribution of average price duration (for 534 brands) ranges between 1 to 2.3 months (in his Table 2).
and natural starting point of the analysis is to consider the observed pattern as reflection of the underlying fluctuations in the demand schedule. For example, Slades (1998) estimates a menu cost model combined with a model of ‘customer capital’, the driving factor behind the price change in her model.

Varian’s influential paper [Varian (1980)] gave rise to a variety of random pricing models as equilibria supported by mixed strategies in pricing games. Among the models in this strand, one common implication is that the current demand is increasing in its past price because maintaining high price for extended periods is necessary to accumulate large size of shoppers, or, bargain hunters. Pesendorfer (1998) exploits this idea and develop a simple model of sales which is applied to explain pricing patterns of ketchup sold at a large supermarket. In this type of model, each price cycle ends with ‘sales’, which is short lived and generates large amount of sales. Another common implication of these models is that the length of the cycle depends upon the speed of accumulation of bargain hunters, in relation to the cost of delaying the sales. This suggests that exogenous changes in demand size predicts the occurrence of sales. Warren and Barsky (1995) actually finds supportive evidence showing that a large predictable increase in shoppers significantly raises the probability of sales. One attractive feature of models of sales is endogenous price cycles even in the absence of exogenous shocks. Large fluctuations in sales and occasional price mark-downs are outcome of intertemporal price discrimination.

This paper builds upon this intuitively appealing idea that the observed pricing patterns are used to dynamically price discriminate consumers. The driving force of the model is the inventory accumulation by customers. Customers differ in their reservation prices depending upon whether they purchase the product for immediate consumption or for the inventory at home. As a result, the reservation prices of two types of consumers differ, and, this is the source of price discrimination that retail stores exploit. Consumers try to concentrate their purchases when stores occasionally mark down the price. As a result, the sales pattern of such a store is characterized by extended periods of relatively stable and low level of sales, punctuated by a burst of sales for a short period of lower price. As a matter of fact, the correlation in our data between the current sales and the past price is positive and significant, suggesting the plausibility of the basic idea. As we will see later, the evidence also suggests that while the store charges high price, the number of customers waiting for the sale steadily increases, whereas the stock of these customers is rapidly depleted when the store

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5 In Varian’s original paper, and also in Sobel (1984), the game is played among oligopolistic firms. The mixed strategy in price setting game arises because stochastic sales, if successful, generates supra normal profits. To maintain the equilibria, it is necessary that success in sales occur randomly so that the expected profit during stochastic sales period is compatible to the profit during the no sale period. The driving factor behind the stochastic sale is accumulation of ‘shoppers’ who are willing to postpone consumption and wait for the sale. In Conlisk, Gerstener, and Sobel (1984), the game is played between a monopolist seller and its customer. The pricing pattern in his paper is such that the price is stable at high level for some time before it begins to decline gradually until all the shoppers hit the store at the lowest price period. The cycle then repeat itself. Gradual price decline is necessary to retain and accumulate large number of shoppers.
sets lower price for the product\textsuperscript{6}.

We develop a model that incorporate these features of the pricing policy in a unified framework\textsuperscript{7}. One of the key assumptions of the model is that the flow of customers are exogenously given not only for the stores but also to the customers. We justify this assumption on the ground that the stores in our sample sell a large variety of grocery items and it is unlikely that customers time their shopping solely on the basis of the price of any particular product. This implies that the pent-up demand for inventory cannot be disposed with within a single sales period. This crucial property gives rise to shorter but still non-negligible price durations even at prices below regular price.

The second salient feature of the model is the source of heterogeneity of customers. As we indicated above, we allow each customer to store the product at home. The heterogeneity arises because of the difference between the demand for immediate consumption and for the inventory. The cost associated with holding inventory comes from two distinct sources. First, grocery items purchased occupy limited storage space, an acute concern for the Japanese households. Another cost stems from the strong preference for freshness, and concerns over the deterioration of quality for foods stored for long time\textsuperscript{8}. We believe these costs are substantial, and arguably more important than the role played by time discount in the context of the pricing patterns in our sample\textsuperscript{9}.

\textsuperscript{6} This is in sharp contrast to the negative correlation between the current sales and the past prices found in the data that Slades (1998) uses. Moreover, high frequency of price changes, especially those between two focal prices suggest that the underlying factor responsible for the price change also has relatively high frequency. The fluctuation of customer capital seems somewhat implausible on this ground.

\textsuperscript{7} High frequency of price changes is consistent also with the fluctuations of the store inventory as the underlying factor, the avenue taken by Aguirregabiria (1999). Circumstantial evidence suggests strongly against the (S-s) inventory dynamics as the important factor responsible for the price changes in our data. The retail stores in our sample have virtually free and unlimited access to the inventory of their merchandise through the network of warehouses and delivery system. Most of the items in each store can be ordered overnight in very small units; very often an order of even one unit is assembled without any significant additional costs. The logistics involved in this network can be summarized as follows. Both major wholesalers and national retail chain stores have regional warehouses and highly sophisticated transportation network connecting these warehouses. Individual retail stores prepares orders for the next day (or the day after). The regional warehouse, upon receiving the orders, assembles the orders and stock them into tracks headed for the delivery. In so doing, hundreds of different kinds of merchandises are delivered to an individual store by a single trip. In a nutshell, the cost associated with replenishing inventory seems quite small and seems highly unlikely to be the major cause of price changes.

\textsuperscript{8} According to a PR officer of a major manufacturer of the curry paste, most of the curry pastes purchased will be consumed within two months from the purchase and those leftovers beyond two months are more likely to be disposed of, rather than consumed. He also noted, however, that the product sorted in room temperature will last at least one year without deterioration in quality.

\textsuperscript{9} Alternatively, the occasional price mark-down can be modeled as an dynamic price discrimination in the presence of heterogenous consumers (shoppers and non-shoppers). Although popular, we do not employ this assumption for the following reasons. First of all, bargain hunters do buy a large amounts to store up at home, the very behavior that we focus upon in our specification. Our model incorporates this crucial feature without the heterogeneity assumption. The distinction between bargain hunters and non-hunters are also
The sequel of the paper is organized as follows. In the next section, we develop a model of dynamic price discrimination in the spirit of Varian (1980) and Sobel (1984). We develop the model in several steps. First we make an intuitive explanation on why retail stores employ pricing policy such as the one we saw in Figure 1. We then start out with static price policy. The result of the static policy serves as the benchmark for the subsequent analysis. The substance of the analysis is in 2.4 in which we first characterize the optimal price policy without menu cost. The role of menu cost is analyzed next. In 2.5, we consider the impact of periodic increase in the size of shoppers. The analysis of the model with menu cost is supplemented by numerical examples in 2.6. In section 3, we offer a preliminary analysis of the data and motivates our empirical model specification. We demonstrate that the data strongly rejects models which predict negative effect of the past price on the current demand. The econometric model of pricing decision is developed and estimated in section 4. We provide three regression results. First, we estimate non-linear regressions and obtain the two key parameters of the model. The result is then used to construct the key unobservable variable, the ratio of customers without inventory. The second set of regressions show that this key variable is significant in the ordered probit model for the direction of price change. Finally, we estimate cross-section regressions on a few of key endogenous variables on three key parameters of the model. Section 5 concludes.

2 A Model of Dynamic Price Discrimination

This section develops a model of dynamic price discrimination for a retail store. In 2.1, we explain the basic set up. In 2.2, we start the analysis with simple static pricing and show that the retail store chooses from the two alternatives: static high price at which customers purchase only for the immediate consumption; and static low price at which the customers are marginally induced to buy the product also to store. In 2.3, we explain the intuition behind the dynamic price discrimination. In 2.5 we formally develop and analyze the model. In 2.6, we consider the impact of temporal variations in the shopping intensity and find that the store prefer to time low price period to heavy shopping days. The formal analysis is supplemented by numerical examples in 2.7.

2.1 Basic Setup

We consider a local monopolist retail store catering to a stable population of regular customers. We abstract from rivalry and price competitions among problematic when we apply the model to our data: the crucial difference between the two is that the bargain hunters are more patient (low time discount) than non-hunters. Highly frequent price mark-downs that we explained above and the fact that typical grocery items purchased are bought regularly by households (e.g., at least once every month) makes this types of specification unsuitable.
the retailers in such a market. The total size of customers is constant and normalized to unity. In anticipation of the sample stores used in the empirical analysis, we assume that the store sells a large variety of grocery items and its customers typically visit store on regular basis and purchase a variety of goods. Our analysis in this section focuses on the pricing of a particular brand of a good, holding as given all the other aspects of the store activities. We assume that each customer visits the store with probability \( s \) for any given day. In other words, shopping cost is very large for the rest of customers on that particular day. Thus \( s \) is also the number of shoppers every day. As we explained in Introduction, the assumption on shopping behavior is motivated by the fact the customers visit the store to purchase a large variety of grocery items and the shopping behavior is not dictated by the price of any particular item. Assuming exogenous probability of shopping is a convenient shortcut to characterize the environment in which the firm sets the price of an individual merchandise.

The customers are far-sighted and may purchase the good today for future consumption if the price is right. We assume a constant cost \( \varepsilon \) of holding one unit of inventory per unit of time. For simplicity, we assume that the storage cost for more than one unit is prohibitively large so that each customer has at most one unit of the good stored.

10 Our modeling choice is dictated by the data limitation in that the data does not cover any pair of retailers in close proximity to warrant direct empirical tests of the role of interactions among the competitors. Although the role of rivalry is no doubt an important empirical issue, we expect that the substance of the model developed here will be extended naturally to incorporate strategic interactions among retailers. Moreover, in view of the comparison of the monopoly model in Conlisk, Gertner and Sobel (1984) and oligopoly model in Sobel (1984), we also expect that the crucial property, i.e., price cycle punctuated by periodic price mark downs, remains valid in such an extended model, although Sobel's results \[\text{Sobel (1991)}\] suggests possible mixed strategy equilibria under such setting.

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2.2 Static Pricing

We start the analysis with a simple benchmark case wherein the store is exogenously constrained to offer time invariant price. Since the store cannot recognize the type of each customer, the only choice is the constant level of the price for the good\textsuperscript{14}. The pricing problem is further simplified by noting the fact that in effect only two types of customers exist in this case: those who are potentially willing to buy to store and those buying (only) for immediate consumption. Hence if the store decides not to cater to the former type of customers, the optimal price is obviously the maximum that it can charge without losing the first type customers. Hence we have

\[ p_H = u \]

In this case the demand for the good is given by

\[ D_H = sc \]

because only those who visit the store \textit{and} plan to consume the good immediately will buy the product. Therefore the profit per day under this policy is given by

\[ \Pi_H = (u - \omega)sc \] (1)

On the other hand, suppose that the store lowers the price enough to induce the second type of customers to buy. Denote by \( \pi \) the share of customers without one unit of inventory. On each day, \( s \) customers visit the shop, among which \( sc\pi \) of them purchase two units for immediate consumption and also to store, \( sc(1 - \pi) \) purchase one unit for consumption, \( s(1 - c)\pi \) also purchase one unit for inventory, and \( s(1 - c)(1 - \pi) \) do not purchase any. Summing up these, we get

\[ D_L = s(c + \pi) \]

Denote by \( p_L \) the optimal price in this case (yet to be determined). We have

\[ \Pi_L = (p_L - \omega)D_L = (p_L - \omega)s(c + \pi) \] (2)

By comparing (1) and (2), we obtain

\[ \Pi_H \leq \Pi_L \iff \frac{p_L - \omega}{u - \omega} \geq \frac{c}{c + \pi} \] (3)

\textsuperscript{14}The store can in principle use non-linear price (volume discount) to discriminate the customers. We assume away this alternative for the following reasons. First of all, through interview, we learned that the volume discount is rarely used. Although scanner technology substantially reduce the cost of price changes, the volume discount is singularly unsuitable and costly under the scanner technology. Price data is matched to individual item through bar code printed on the merchandise. To use volume discount, the store must prepare different packaging for multiple units purchase and register distinct code numbers.

Another reason to assume away volume discount is that it is an imperfect measure: although consumers purchasing multiple (2) units are necessarily those without inventory, the reverse is not true. Some customers buy only one unit to store, not for immediate consumption.
If \( p = p_L \) forever, far-sighted customer without inventory always purchase the good to store whenever she visits the store, whereas the inventory will be consumed with probability \( c \) which will not be replenished if she does not shop for the day, which occurs with probability \((1-s)\). Thus we get

\[
\Delta \pi \equiv \pi_t - \pi_{t-1} = c(1-s)(1-\pi_{t-1}) - s\pi_{t-1}
\]

Hence the steady state level of \( \pi \) is given by

\[
\pi^* = \frac{c(1-s)}{c(1-s) + s} \tag{4}
\]

Therefore the condition (3) is rewritten as

\[
\Pi_H \leq \Pi_L \iff \frac{p_L - \omega}{u - \omega} \geq \frac{c(1-s) + s}{1 + c(1-s)} \tag{3'}
\]

Let us now consider the determination of \( p_L \). Denote by \( V_i (i = 0, 1) \) the expected present value of net utility stream for a customer with \( i (=0,1) \) units of inventory of the good at home. We have

\[
\begin{align*}
 rV_0 &= sc(\tilde{u} + \tilde{v}) + s(1-c)\tilde{v} \\
 rV_1 &= -\varepsilon + s\tilde{u}c + c(1-s)(\tilde{u} - \tilde{v}) \\
 \tilde{u} &\equiv u - p_L \\
 \tilde{v} &\equiv W_L - p_L, \ W_L \equiv V_1 - V_0
\end{align*}
\]

Solving for \( \tilde{v} \), we get

\[
\tilde{v} = \frac{c(1-s)\tilde{u} - \varepsilon - rp_L}{r + s + c(1-s)}
\]

Setting the time discount rate \( r = 0 \), we get

\[
\tilde{v} = \frac{c(1-s)\tilde{u} - \varepsilon}{s + c(1-s)}
\]

Since the customer without inventory is willing to purchase to store the good if and only if doing so yields net gain in the expected utility. Hence the maximum that the store can charge is given by

\[
c(1-s)\tilde{u} - \varepsilon = 0
\]

or,

\[
p_L = W_L \equiv u - \frac{\varepsilon}{c(1-s)}
\]

Therefore, the previous condition (3’) can be written as

\[
\Pi_H \leq \Pi_L \iff R \leq R_S, \ R_S \equiv \frac{(1-s)}{1 + c(1-s)}, \ R \equiv \frac{\varepsilon}{(u - \omega)c(1-s)} \tag{5}
\]
Notice that $R$ is the ratio of effective cost of inventory $\left[\frac{1}{c(1-s)}\right]$ relative to the gains from trade, $(u - \omega)$, because $\frac{1}{c(1-s)}$ is the average duration of inventory. The equation above thus simply states that the static high (low) price is chosen if the ratio is larger (smaller) than the threshold, $\frac{(1-s)}{1+(1-s)}$. As we will see later, we get a similar condition for the interior solution for Hi-Lo pricing in the next section.

2.3 Optimality of Hi-Lo pricing policy: Intuition

In a static setting, the store cannot effectively price discriminate customers unless the store knows the type of each customer. The only choice left for the store is whether or not to price the good low enough to induce customers to buy for the inventory at home. Inspection of (3) suggests that this strategy is chosen if the share of customers without inventory is large. We also know that the customer will not purchase the good to store if the price is above the reservation level, $\hat{v}$.

Once we allow the firm to price discriminate by changing the price from time to time, the firm can exploit the above property that the lower (high) price increases profit when the average inventory at customers’ homes is low (high). In what follows, we demonstrate through several steps that a type of Hi-Lo pricing policy emerges as the optimal intertemporal price discrimination with menu cost. To proceed, we first describe intuitively the idea before we formally prove the optimality of such a policy.

Suppose the store employs a Hi-Lo pricing policy in the following manner: high price is posted for $[1 \tau_1] \equiv T_H$, and low price is posted for $[\tau_1+1, \tau_1+\tau_2] \equiv T_L$. Then the process is repeated. First of all, we notice that during the high price period, the customer will not buy the good to store up since the customer gets zero net utility even if she immediately consumes the good upon purchase. This implies that during the high price period, the average level of inventory at home will be depleted gradually as they are consumed so that the share of customers without inventory increases. The low price must be such that the customers are marginally induced to buy the good to store. The crucial question here is whether or not such a reservation price is constant. Intuition suggests that such a price will not be constant if the store employs Hi-Lo policy stated above. The point here is the fact that towards the end of the low price period, customers are willing to pay more to store up than they are at the beginning of the low price period. This is the case because customers get strictly positive net utility from buying and consuming the good at the low price level. The fact that such a valuable opportunity will soon end acts as an inducement to buy during the low price period. These arguments in turn suggest that the customer’s willingness to pay for the inventory should be declining during the high price period in anticipation of the low price periods ahead, although such a shadow price is immaterial during the high price period.

The pricing pattern that emerges from the argument above is the following. A typical cycle starts (arbitrarily) with high price period during which no in-
ventory accumulation occur and the inventory will be gradually depleted as the customers consume them. The actual demand during the high price period is confined to those without inventory who happens to shop on the day and also happens to get $u$ from consumption. At $\tau_1$, the price is reduced to the lowest level. The price then gradually rises during the low price period, mirroring the rising reservation price to store up the inventory. The cycle ends with the sudden jump at $\tau_1 + \tau_2$ back to the high price. Then the cycle is repeated.

Now consider the role of menu cost. It is clear that at high enough menu cost, the cyclical pricing policy described above will not be optimal. Instead either permanently high or low price policy will be taken. As we gradually lower the menu cost, the first pattern that will emerge is the one with minimum number of price change: i.e., a simple Hi-Lo pricing policy in which the ‘low’ price is kept constant until the price is returned to the high level. As we further lower the menu cost, the number of price adjustments during the low price period increases, more and more closely mirroring the price path without menu cost. As we will see shortly, however, without menu cost, the optimal policy calls for infinitely frequent alternations between high and low prices because doing so maximizes the sustainable size of $\pi$ consistent with low pricing.

### 2.4 Optimal Price Policy: Formal Analysis

In this sub-section, we present and analyze the dynamic pricing model. We first characterize the optimal purchase policy of the customers, assuming that the store employs stable Hi-Lo pricing policy described above. We then demonstrate indeed that the store optimally choose such a High-Lo pricing policy given the optimal response of the customers. In Appendix, we prove that the optimal policy starting from arbitrary level of $\pi$ indeed converges to the optimal stationary policy analyzed in this section.

#### 2.4.1 Customer’s Optimal Policy

Let us start with customers. As we specified above, the relevant choice for each of the customers is either to buy for the good as inventory or not. Since price never exceeds the reservation utility level, $u$, customers always purchase the good for immediate consumption.

We denote, as above, by $V_i^t$ ($i = 0, 1 : t = 1, \ldots, t_1 + t_2$) the asset value of consumption stream net of purchasing and inventory holding costs for customers with $i$ units of inventory in period $t$ during the cycle. We have

\begin{align*}
V_i^1 &= -c + c(1 - s)Max[0, u - W_{t+1}] \\
&\quad + csMax[u - p_{t+1}, u - W_{t+1}, 0] + V_{t+1}^1 \quad (6) \\
V_i^0 &= s(1 - c)Max[0, W_{t+1} - p_{t+1}] \\
&\quad + cs\{Max[u - p_{t+1}, 0] + Max[W_{t+1} - p_{t+1}, 0]\} + V_{t+1}^0 \quad (7) \\
W_t &= V_i^1 - V_i^0
\end{align*}
where it is understood $t+1 = 1$ if $t = t_1 + t_2$. These equations can be understood as follows. First, for those with one unit of inventory already at hand, she has to pay $\varepsilon$ per period for inventory holding. She will get to consume one unit of inventory with probability $c$. If she does not shop that day, she depletes the inventory by doing so and suffer capital loss given by $W_t$. By construction, postponing consumption is never optimal and $u - W_t$ is non-negative. If she happens to shop that day, she can replenish the inventory (in which case she gets $u - p_t$) or simply depletes the inventory (in which case she gets $u - W_t$). Those without inventory consumes the good only if she also shops on the day, upon which she gets $u - p_t$. Whether or not she purchases one more unit for inventory depends upon the sign of $W_t - p_t$. Denote by $T_H$ ($T_L$) the high (low) price period. Recall that by assumption,

$$
p_t = u \text{ for } t \in T_H,
\quad p_t \leq W_t \text{ for } t \in T_L
$$

The latter property is obtained because $W_t$ is the maximum price that consumers are willing to pay for the good to store. We can use these pricing policy to rewrite (6) and (7). The results are

$$
V_t^1 = -\varepsilon + c(u - W_{t+1}) + V_{t+1}^1 \text{ for } t \in T_H
V_t^1 = -\varepsilon + c(1 - s)(u - W_{t+1}) + cs(u - p_{t+1}) + V_{t+1}^1 \text{ for } t \in T_L \quad (6')
$$

$$
V_t^0 = V_{t+1}^0 \quad \text{ for } t \in T_H
V_t^0 = s(1 - c)(W_{t+1} - p_{t+1}) + cs(u - p_{t+1} + W_{t+1} - p_{t+1}) + V_{t+1}^0 \quad \text{ for } t \in T_L \quad (7')
$$

These two pairs of equations summarize the intertemporal pricing constraint for the store, assuming that the store follows the Hi-Lo pricing policy.

### 2.4.2 Optimal Pricing Policy without Menu Cost

We now move on to consider the store’s optimal policy. As we said above, we limit our attention to the choice among the set of stationary policy rules. Hence the available choice for the firm is simply to choose either constant price or Hi-Lo pricing. The question on the choice over the two will be discussed later. For the time being, we assume that the firm pursue a variant of Hi-Lo pricing policy. Without loss of generality, we assume that the cycle begins with the high price period, followed by the low price period.

As it turns out, the analysis without menu cost is not only simpler but also the nature of optimal pricing policy can be seen far more clearly in this benchmark case. Without menu cost, the firm can costlessly change prices provided that during the ‘low’ price periods, $p_t \leq W_t$ is satisfied. Obviously the store sets price such that:

$$p_t = W_t \text{ for } t \in T_L$$
Substituting the equality for $p_t$ in (6') and (7'), and then subtracting (7’) from (6’) for both sides, we obtain:

\[
W_{t+1} = (1 - c)^{-1} \{ W_t + \varepsilon - cu \} \quad \text{for } t \in T_H
\]

\[
W_{t+1} = \{1 - c(1 - s)\}^{-1} \{ W_t + \varepsilon - c(1 - s)u \} \quad \text{for } t \in T_L
\]

Corresponding changes in $\pi_t$ for each period is given by

\[
\pi_t = c + (1 - c)\pi_{t-1} \quad \text{for } t \in T_H
\]

\[
\pi_t = c(1 - s) + (1 - c(1 - s) - s)\pi_{t-1} \quad \text{for } t \in T_L
\]

Denote by $W$ and $\pi$, respectively the initial level of each variable at $t = 0$. We can solve respective difference equations to obtain:

\[
W_t = W_H - \left(1 - \frac{1}{1 - c}\right)^t (W_H - W) \quad \text{for } t \in T_H
\]

\[
W_t = W_L + \eta^{t - t_1} \{ W_H - W_L - \left(1 - \frac{1}{1 - c}\right)^t_1 (W_H - W) \} \quad \text{for } t \in T_L
\]

\[
W_H = \frac{cu - \varepsilon}{c}, \quad W_L = \frac{c(1 - s)u - \varepsilon}{c(1 - s)}, \quad \eta = \frac{1}{1 - c(1 - s)}
\]

\[
\pi_t = 1 - (1 - c)^t(1 - \pi) \quad \text{for } t \in T_H
\]

\[
\pi_t = \pi^* + \{1 - \pi^* - (1 - c)^{t_1}(1 - \pi)\} \{1 - c(1 - s) - s\}^{t - t_1} \quad \text{for } t \in T_L
\]

As we limit our analysis to the stationary price policy, the price cycle must repeats itself after $(t_1 + t_2)$ periods. Thus we require:

\[
\pi_{t_1 + t_2} = \pi
\]

\[
W_{t_1 + t_2} = \overline{W}
\]

Using these terminal conditions, we obtain:

\[
\overline{W} = (DE - 1)^{-1} \{ E(D - 1)W_H + (E - 1)W_L \}
\]

\[
D \equiv \left(1 - \frac{1}{1 - c}\right)^{t_1} = A^{-1}, \quad E \equiv \eta^{t_2} = \left\{1 - \frac{1}{1 - c(1 - s)}\right\}^{t_2}
\]

Using these expressions, equations in (10) are re-written as follows:

\[
W_t = W_H - W_\Delta (DE - 1)^{-1} (E - 1)(1 + c)^t \quad \text{for } t \in T_H
\]

\[
W_t = W_L + \eta^{t - t_1} W_\Delta (DE - 1)^{-1} (D - 1) \quad \text{for } t \in T_L
\]

\[
W_\Delta \equiv \frac{W_H - W_L}{c(1 - s)} > 0
\]
Notice that $W_t$ decreases during the high price period in reflection of the expected price decline in the low price period. By the same token, $W_t$ increases during the low price period anticipating the price increase back to the high price.

Next, the time path of $\bar{w}_t$ is given by

$$\bar{w}_t = 1 - (1-c)^t(1-AB)^{t-1}(1-B)(1-A)(1-\pi^*)$$ for $t \in T_H$

$$\bar{w}_t = \pi^* + \{1-c(1-s)-s\}^{t-t_1}(1-AB)^{-1}(1-A)(1-\pi^*)$$ for $t \in T_L$

(15)

Let us now compute the store’s net profit for a representative cycle. The amount of sales is given by

$$Q_t = sc\bar{w}_{t-1}$$ for $t \in T_H$

$$Q_t = s(c + \bar{w}_{t-1})$$ for $t \in T_L$

(16)

We can use these to obtain the net profit for a representative cycle as follows.

$$M = (u-\omega)sc \sum_{t=1}^{t_1} \bar{w}_{t-1} + s \sum_{j=1}^{t_2} (W_{t+j} - \omega)(c + \bar{w}_{t+j-1})$$

(17)

We can substitute (14) and (15) for $W_t$ and $\bar{w}_t$ in (17). After tedious but straight-forward computations and re-arranging terms, we obtain:

$$M = (u-\omega)sc t_1 + (W_L - \omega)(c + \pi^*)t_2$$

$$+\left\{ W_L - \omega - \{c(1-s) + s\}(u-\omega)\right\} \times \frac{(1-\pi^*)(1-A)(1-B)}{(c(1-s) + s)(1-AB)}$$

$$+ \frac{W_A(D-1)(E-1)(c + \pi^*)}{(DE - 1)c(1-s)} + \frac{W_A(D-1)(1-\pi^*)(1-A)(1-F)}{(DE - 1)(1-AB)s}$$

$$F = BE = \left\{1 - c(1-s) - s\right\}^{t_2} < 1$$

(17')

The optimal policy maximizes the net profit per period:

$$\{t_1^*, t_2^*\} \equiv \text{argmax}_{\{t_1, t_2\}} \frac{M}{t_1 + t_2}$$

(18)

As it turns out, without menu cost, the optimal policy calls for infinitely frequent price changes in order to minimize the deviation of $\bar{w}$ around the steady state. On the other hand, the ratio of $t_1^*$ to $t_2^*$ converges to a positive finite value. To establish the claim, we employ the following Lemma.

**Lemma 1** Consider a function:

$$\psi(x; a, b, \alpha, \beta) \equiv \frac{(1-a^{ax})(1-b^{bx})}{x(1-a^{ax}b^{bx})}, \quad 0 < a, b < 1, \quad \alpha, \beta > 0$$

$\psi(x)$ is monotonically decreasing in $x$ in $(0, \infty]$ and

$$\psi_0(a, b, \alpha, \beta) = \lim_{x \to 0} \psi(x) = \frac{\alpha \beta \log(a) \log(b)}{-(\alpha \log(a) + \beta \log(b))}$$
Proof. The result is immediate by using the L’Hopital’s rule twice: i.e., differentiate the denominator and the numerator twice and evaluate them at $x = 0$.

**Lemma 2** Consider a function:

$$\phi(x; a, b, \alpha, \beta) \equiv \frac{(a^{\alpha x} - 1)(b^{\beta x} - 1)}{x(a^{\alpha x} b^{\beta x} - 1)}, \ a, b > 1, \ \alpha, \beta > 0$$

$\psi(x)$ is monotonically decreasing in $x$ in $(0, \infty]$ and

$$\phi_0(a, b, \alpha, \beta) = \lim_{x \to +0} \phi(x) = \frac{\alpha \beta \log(a) \log(b)}{(a \log(a) + b \log(b))}$$

**Proof.** Apply the same argument as in Lemma 1.

**Lemma 3** Consider a function:

$$\xi(x; a, b, d, \epsilon, \alpha, \beta) \equiv \frac{(1 - a^{\alpha x})(d^{\epsilon x} - 1)\{1 - (b\epsilon)^{\beta x}\}}{x(1 - a^{\alpha x} d^{\epsilon x})\{a^{\alpha x} e^{\beta x} - 1\}}$$

$$0 < a, b, d, \epsilon, \alpha, \beta > 0, \ b\epsilon < 1$$

$\psi(x)$ is monotonically decreasing in $x$ in $(0, \infty]$ and

$$\xi_0(a, b, d, \epsilon, \alpha, \beta) = \lim_{x \to +0} \xi(x) = \frac{\alpha^2 \beta\{\log(a) \times \log(b)\}\{\log(b) + \log(\epsilon)\}}{-\{a \log(a) + b \log(b)\}\{a \log(d) + b \log(\epsilon)\}}$$

**Proof.** Apply the L’Hopital’s rule thrice.

We now rewrite the maximand (17').

$$\widetilde{M} \equiv \frac{M}{sT} = (u - \omega)e\theta + (W_L - \omega)(c + \pi^*)(1 - \theta)$$

$$+ \frac{[(W_L - \omega) - \{c(1 - s) + s\}(u - \omega)](1 - \pi^*)}{c(1 - s) + s} \times$$

$$\psi[T; 1 - c, 1 - c(1 - s) - s, \theta, 1 - \theta]$$

$$+ \frac{W\Delta(c + \pi^*)}{s} \phi[T; \{\frac{1}{1 - c}\}, \frac{1}{1 - c(1 - s)}, \theta, 1 - \theta]$$

$$+ \frac{W\Delta(1 - \pi^*)}{s} \times$$

$$\xi[T; 1 - c, 1 - c(1 - s) - s, (\frac{1}{1 - c}), \frac{1}{1 - c(1 - s)} \theta, 1 - \theta]$$

where the following new variables are introduced:

$$T \equiv t_1 + t_2$$

$$\theta \equiv \frac{t_1}{T}$$

Using lemma 1 through lemma 3, we obtain the following result.
Lemma 4  The optimal policy is obtained by setting $T \to 0$, and choosing $\theta$ that maximizes

$$
\tilde{M}(\theta) = (u - \omega) \theta + (W_L - \omega)(c + \pi^*)(1 - \theta) \\
+ \left[ (W_L - \omega) - \{c(1 - s) + s\}(u - \omega)\right](1 - \pi^*) \times \\
\psi_0[1 - c, 1 - c(1 - s) - s, \theta, 1 - \theta] \\
+ \frac{W_\Delta(c + \pi^*)}{c(1 - s)} \phi_0 \left[ \frac{1}{1 - c}, \frac{1}{1 - c(1 - s)} \right] \theta, 1 - \theta] \\
+ \frac{W_\Delta(1 - \pi^*)}{s} \times \\
\xi_0[1 - c, 1 - c(1 - s) - s, \frac{1}{1 - c}, \frac{1}{1 - c(1 - s)}] \theta, 1 - \theta]
$$

(19)

Proof. Immediate from Lemma 1 through 3 since the first two terms of the maximand is independent from $T$, whereas the remaining three terms are maximized by setting $T \to 0$.

Having converted the maximization problem into the choice of $\theta$, we next establish the following:

Lemma 5  The optimal policy entails the following choices:

- **Static High Price Policy** $\theta^* = 1$
- **Dynamic Hi-Lo Policy** $0 < \theta^* < 1$
- **Static Low Price Policy** $\theta^* = 0$

The value function $\tilde{M}$ is applicable to all of these cases. In particular, we have

$$
\tilde{M}(1) = (u - \omega)c \\
\tilde{M}(0) = (W_L - \omega)(c + \pi^*)
$$

Moreover, if the parameters $(c, s)$ are sufficiently small positive numbers\(^\text{15}\), the following conditions characterize the partitions of the parameter space into each of the three types of the optimal policy.

$$
\theta^* = 0 \text{ if } R \equiv \frac{c}{(u - \omega)c(1 - s)} \leq R_L \equiv \frac{c(1 - s)^2}{c(1 - s) + s} \\
0 < \theta^* < 1 \text{ if } R_L < R < R_U \equiv \frac{1}{c(1 - s) + s} \\
\theta^* = 1 \text{ if } R \geq R_U
$$

(20)

Proof. See Appendix.

The conditions (20) for an interior solution can be interpreted as follows. As we saw in 2.2, the ratio $R$ signifies the relative importance of the normalized

---

\(^{15}\)Since the choice of the time unit is arbitrary, we can choose the unit small (but still finite) enough so that parameters $c, s, \text{and } \varepsilon$ are all small positive numbers.
inventory cost to the size of the surplus. The lemma above states that this ratio must lie within the interval given above in order for the store to choose Hi-Lo pricing policy. If the ratio is too small, the store optimally set the price low permanently, whereas the ratio exceeds the upper limit, the store always choose Hi price. The upper and lower thresholds in (20) can be compared to the threshold for static low and high price policies in 2.2. We have

$$R_L < R_S < R_U$$

Thus the static threshold lies in between the lower and the upper threshold for the interior solution of the Hi-Lo policy.

Another property of Hi-Lo price policy is that the price level during the low price period is higher than the static low price, $W_L$. This can be confirmed from (10).

$$W_i \geq \frac{D(E - 1)W_L + (D - 1)W_H}{DE - 1} > W_L$$

The store can induce customers to purchase the good for inventory at a price higher than the reservation level under the static low price policy precisely because the price discount is known to be temporary (Hi-Lo pricing policy). As a result, the level of inventory is always lower than $\pi^* = \frac{c(1-s)}{n(1-s)}$, the level of inventory at the steady state under static low price policy$^{16}$.

### 2.4.3 Optimal Pricing Policy with Menu Cost

We now introduce the cost of changing prices. Then, each of Hi-Lo pricing policy is characterized by $(k + 1)$ - tuple of positive integers denoted by

$$T \equiv \{t_1, t_2^1, t_2^2, \ldots, t_k^j\}, k \geq 1$$

where the firm adopts the following price policy:

$$p_t = u \text{ for } t \in T_H$$

$$p_t = p_L^j \text{ for } t \in T_L^j$$

$$T_H = [1, t_1]$$

$$T_L^j = [t_1 + \sum_{n=1}^{j-1} t_2^n + 1, t_1 + \sum_{n=1}^{j} t_2^n], j = 1, 2, \ldots, k$$

Viz., the firm changes the price $(k+1)$ times during the cycle and incurs $(k+1)\sigma$ of menu costs. Since the cycle repeats itself, $k + 1 \geq 2$. For the first $t_1$ periods, the price is set at the reservation utility level, $u$. At $t = t_1 + 1$, price is reduced and the low price periods begins. For the first $t_2^1$ periods, the price is set at $p_L^1$. Then the price is changed to $p_L^j$ which lasts $t_2^j$ periods, and so on, until the price is finally increased back to $u$ after $t_1 + \sum_{n=1}^{k} t_2^n$ periods.

$^{16}$ From (12), it is immediate that $\bar{\pi} > \pi^*$. Moreover, using (11), we know $\pi_t \geq \bar{\pi}$ for any $t$.  

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The maximand with menu cost is given by

$$M = -(k + 1)\delta + (u - \omega)sc \sum_{t=1}^{t_1} \pi_{t-1} + s \sum_{j=1}^{k} \sum_{m=t_j}^{t_j+1} (p^j_L - \omega)(c + \pi_{m-1})$$

We relegate lengthy computations to Appendix. The derivation process is analogous to those without menu cost. The maximand is given by

$$M = -(k + 1)\sigma + (u - \omega)sc \frac{(1 - \pi^*)(1 - A)(1 - B)}{(1 - AB)}$$

$$+ s \sum_{j=1}^{k} (p^j_L - \omega)(c + \pi^*)t^j_2$$

$$+ s \sum_{j=1}^{k} (p^j_L - \omega) \frac{B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j)(1 - A)(1 - \pi^*)}{(1 - AB)\{c(1 - s) + s\}}$$

$$= -(k + 1)\sigma + (u - \omega)sc t_1 + s \sum_{j=1}^{k} (p^j_L - \omega)(c + \pi^*)t^j_2$$

$$+ \frac{(1 - \pi^*)(1 - A)(1 - B)}{(1 - AB)\{c(1 - s) + s\}} \times$$

$$[s \sum_{j=1}^{k} (p^j_L - \omega)(1 - B)^{-1} B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j)$$

$$- s\{c(1 - s) + s\}(u - \omega)]$$

where $B_j$ and $B$ are given by

$$B_j \equiv \{1 - c(1 - s) - s\}t^j_1,$$

$$B \equiv \{1 - c(1 - s) - s\}t^2_2 \equiv B_1 B_2 B_3 \ldots B_h,$$

and $p^j_L$ is given by

$$p^j_L = [c(1 - s) + s\lambda]^{-1}(\lambda - 1)\mu + [c(1 - s) + s\lambda]^{-1}\lambda[c(1 - s) + s]\{(\Theta_{1h}D - 1)^{-1} \times$$

$$[\Theta_{1j-1}(D - 1)W + \Theta_{0j-1}(\Theta_{1h}D - 1) - \Theta_{1j-1}D\Theta_{0k}]\mu\}$$

Then, the optimal solution is given by

$$\{T^*, k\} \equiv \{t^1_1, t^2_1, \ldots, t^k_1, k^*\} = argmax \left[ \frac{M}{t_1 + \sum_{j=1}^{k} t^j_2} \right]$$

Unfortunately, it is practically impossible to characterize the optimal solution. In the case of interior solution (i.e., Hi-Lo policy, rather than either static high or low price policy), we have, however,
because the optimal solution strikes the balance between the menu cost and the loss from setting the length of the price cycle too long (the optimal cycle length is zero). This implies that for a Hi-Lo policy to be optimal, it is necessary that

$$M^*|_{\sigma=0} > M^*|_{\sigma>0} + \frac{(k+1)\sigma}{T^*}$$

Therefore, for an interior solution with menu cost, the necessary condition is more stringent than the one without menu cost because by construction the necessary (and sufficient condition without menu cost is given by setting $\sigma = 0$ above, in which case we recover (21).

$$R_L|_{\sigma>0} < R < R_U|_{\sigma>0}$$

$$R_L \equiv \frac{c(1-s)^2}{c(1-s)^2 + s(1-s)^2} < R_L|_{\sigma>0},$$

$$R_U|_{\sigma>0} < R_U \equiv \frac{1}{(1+c)}$$

### 2.5 Christmas Bargain Effects, Sales Promotion, and Hi-Lo Price Cycle

In the base model, we assumed that the daily size of customers visiting the store is constant. As we show in the next section, however, the data shows a large variation in the number of visitors within a week, a month, or a year. In this section we analyze the effect of deterministic variations of $s$.

The concentrations of price mark-down and sales during the heavy sales periods is noted by Warner and Barsky (1995) who collects a variety of retail sales and pricing data in the United States and find that 'frequent markdowns in the intensive shopping period prior to Christmas, and a tendency for such sales to occur in weekends' (p.322). Our model can be modified to incorporate deterministic fluctuations in the shopping probability. Such seasonality is common in retail stores: weekends are heavy shopping periods, so are the Christmas-New Year periods. The business is generally slow in mid-Winter and Summer days, etc. Faced with such patterns, a store can time its price cycle to increase profit. Intuitions suggest that the cycle should be timed in such a way that the price should be low in shopping days.

The underlying mechanism is again that of price discrimination. We noted in the beginning that the Hi-Lo pricing in our model is a compromised form of price discrimination: instead of perfectly price discriminate between bargain hunters from the other shoppers, retail stores lower the price when the share of the bargain hunters is large. In our model, bargain hunters are those who buy the good to store rather than for immediate consumption. Timing the low
price period to heavy shopping days help contribute to increase the share of purchase by the bargain hunters during the period because as the low price period continues the share of bargain hunters decrease gradually (decrease in \( \pi \)). By timing the low price period to heavy shopping days, the store can accelerate the sales to the targeted bargain hunters within a shorter period\(^{17}\).

As a result, the sales at low price to non-bargain hunters (those buying for immediate consumption) is reduced, thus enhancing the effectiveness of the dynamic price discrimination. Compared to this strategy, setting price high in heavy shopping days is less profitable because the size of potential customers is reduced.

The observation that the store benefits from timing the low price period to heavy shopping days suggest that the store has an incentive to artificially generate fluctuations in the shopping intensity\(^{18}\). If the number of shoppers during the low price periods can be increased by advertisements and other sales promotion activities, the store can reduce the length of low price period and hence reduce the sales at low price to those customers purchasing for immediate consumption.

To demonstrate this point, suppose that a retail store can pinpoint the sales promotion and advertising activities to increase the shopping intensity of a particular day during the price cycle from \( s \) to \((1 + \gamma)s\) (\( \gamma > 0 \)). Denote by (*) the optimal policy corresponding to a particular configuration of parameters:

\[
P^* = \arg\max_{P^*} \left[ \frac{M(P^* : \Theta_0)}{t_1 + t_2} \right]
\]

\[\Theta_0 \equiv \{\omega_0, \varepsilon_0, u_0, c_0, s_0, \sigma_0\}\]

Consider:

\[
\Delta = \frac{M(P^* : \Theta_1)}{t_1 + t_2} - \frac{M(P^* : \Theta_0)}{t_1 + t_2}
\]

\[\Theta_1 \equiv \{\omega_0, \varepsilon_0, u_0, c_0, \sigma_0, \{s_t\}\}\]

\[
\{s_t\} = \begin{cases} s_0 & \text{if } t \notin t_S \\ (1 + \gamma)s_0 & \text{if } t \in t_S, \gamma > 0 \end{cases}
\]

The increase in profit measured by \( \Delta \) depends upon the timing of the shock within the cycle. Hence we denote:

\[
\Delta = \Delta(t_S)
\]

\(^{17}\)It is also likely that the share of bargain hunters actually increases, either because bargain hunters are better informed and/or the retailers invests in advertising to entice bargain hunters to visit during the period. These factors (absent in our model) reinforce our argument that the stores benefit from setting the low price sales during the heavy shopping periods.

\(^{18}\)This finding also justifies the comment in interview by a director in charge of pricing at a national chain of supermarket: manufacturer-sponsored special sales are always accompanied by corresponding sales promotions including newspaper advertisement, in-house demonstrations, special display, etc. Chevalier, Kashyap, and Rossi (2000) also finds similar patterns of counter-cyclical markups for a wide variety of items sold at a large supermarket chains in the Mid West of the United States.
Lemma 6 Suppose $t_1^*, t_2^* > 0$. Viz, the optimal policy calls for Hi-Lo pricing. Then, $t_1^* + 1 = \arg\max_i (\Delta(t_S))$.

Proof. See Appendix. □

In 3.2, we show that one of the supermarket chains does adopt such a policy. At one of the supermarket chains, the day 20 of each month is designated as a store wide sales day, and this chain regularly runs TV commercials and newspaper ads about this special sales day. We find that, on the day 20, the price of curry paste is almost always marked down from the previous day, only to be increased on the next day, 21st.

2.6 Numerical Examples of Optimal Pricing Policy

The optimal pricing policy with menu cost is highly non-linear and conventional battery of comparative analysis are powerless to examine the characteristics of the optimal pricing policy. We therefore provide a variety of numerical examples of these policies in this subsection to illustrate the impact of changes in parameters on the pricing policy.\(^{19}\)

We start with the description of the benchmark equilibrium as shown in Table 2. In this benchmark case, we set: $u = 10, \omega = 5.11, s = .35, c = .19, \varepsilon = .35,$ and $\sigma = .05$. The optimal policy\(^{20}\) is a price cycle of 6.76 days, roughly one week, in which the high price is set for the first .67 of the cycle, or 4.53 days, then the price is cut to 7.3 from the high price, 10. This first low price period lasts 1.15 days, then the price is increased slightly to 7.47, which last for 1.08 days, and the cycle ends by returning to the high price level. I.e., $k^* = 2$

The average discount during the low price period is 26% (i.e., the average price during the low price period is about 7.4). The maximand (per unit of time) in this case is 1.03977. Compared to this value of the maximand at the optimal policy, it is, respectively .33%, .52%, 1.23%, 2.07%, and 10.64% lower if $k = 1, 3, 4, 5$ and 0 (static pricing policy). On the other hand, this maximand is 8.52% smaller than the maximum value which we would obtain if the menu cost, $\sigma$, is zero.

Table 1 shows the comparative statics results which is obtained by varying one of the parameters and keeping the rest of the parameters at the benchmark case.

The impacts of changes in $s$ are shown in the second row of the Table 1 and they are quite intuitive. Both cycle length and average price duration are increasing in $s$. As the customer shops more frequently, the benefit of holding inventory declines. Hence to induce customers to store inventory, the shops need to discount more during the low price period and also shorten the low price period relative to the length of the cycle. Eventually, beyond the threshold value of $s$, it becomes optimal to sell always at the high price. Similarly, with $s$

\(^{19}\)The computer program for numerical examples shown in this subsection is written by MATLAB (v.5.3) and available upon request from the authors.

\(^{20}\)We ignore the integer constraints except for the choice of $k$. 

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below the threshold, it is optimal to set the price always at the low threshold level so that customers without inventory always purchase to store.

Similar logic can be applied for the impact of a change in $c$. An increase in $c$ increases the probability of stock-out and hence raises the reservation price of the inventory. As a result, low price period increases but the average discount during the low price period becomes smaller. Eventually, beyond the threshold value, the optimal policy calls for static low price policy.

An increase in the marginal cost ($\omega$) obviously raises the average price not only by shortening the relative share of the low price period but also by raising the average price during the period.

The impact of $\varepsilon$ is quantitatively large: the impact is through its direct effect on the cost of holding inventory at household. As a result, the store must set lower price during the low price period, whereas the consequent decline in the profit from low price sales reduces the share of the low price period. The impacts on price level and discounts are relatively small and it is unclear if the results shown in the Table 1 are robust.

In Table 2 we show the impact of a change in the menu cost on the optimal choice of $k$. To begin with, the optimal policy calls for longer cycle length and larger value of $k$ as we decrease $\sigma$ (bottom row). Starting with the benchmark value of .05, the optimal policy changes from $k = 2$ to $1$ at $\sigma = .091$ or larger. Reducing $\sigma$ further, optimal policy changes to $3$ at or around $\sigma = .0074$, and then to $4$ at $\sigma = .00059$. Correspondingly, the cycle length (not shown in Table 3) gets shorter and shorter as we decrease $\sigma$. For example, at $\sigma = .24$ (the maximum value in our numerical example), the cycle length is 16.6 days, whereas it is less than 2.5 days when optimal $k$ first becomes 3. Notice $\sigma = .05$ corresponds to .5% of the high price. The simulations results suggest that thrice or more frequent price changes during the low price period can be found only in extreme configuration of parameter values and optimal policy, e.g., extremely small menu costs (less than .1% of the price tag) and extremely short cycle length (less than 3 days). The results are hence consistent with our data in that most of price changes are between regular high price and regular low price, i.e., HiLo pricing policy. The rest of the simulation results shown in Table 2 adds more evidence pointing the optimality of HiLo type pricing policy. Aside from the changes in menu cost, we did not find any parameter configurations in which optimal policy calls for more than three ($k > 3$) price changes. Frequent price changes during a cycle is typically dominated by either a static low or static high price policy.

3 Data Explorations and Preliminary Empirical Analysis

In this section we introduce our data taken from Nikkei Data Base. This section also explores the data, especially to ascertain the plausibility of alternative theories advanced in the existing literature in this field.
3.1 Data

The data used in this paper consists of daily observations on actual price and quantities sold for the two competing brands of curry paste at 18 different stores which belong to one of the six national and regional supermarket chains in Japan. Unfortunately, none of these stores are in close proximity against the other stores in the data to explicitly analyze the impact of strategic interactions. The sample period is July 1st 1991 to August 31, 1996. During the entire period, retail prices in Japan remained stable and there was virtually no discernible effect of ongoing or future inflation. (CPI rose only by 3.6% between 1991 and 1996).

3.2 Price Changes: Frequencies, Duration Dependence, and Store Characteristics

Table 3 provides several key indicators which we use to characterize the pricing policy of the stores. We notice immediately that the sample data exhibit extremely high frequency of price changes. Pesendorfer (1998) uses similar scanner sales data on two brands of ketchup sold at US supermarket stores. The durations of prices are shorter for lower prices, characteristics shared in our data. The durations in his data are much longer, however. In our sample, average price durations are less than 10 days in the first chains and none of the sample stores have durations longer than 60 days even at the regular high price level. In Slade (1998, 1999), she reports that 80% of weekly price observations are zero price changes, which implies unconditional mean duration of price is larger than one month.

Although both Pesendorfer and Slade indicate the existence of focal price levels, our data shows not only the existence but that they are actually dominant: typically, only two focal prices constitute a majority of daily price observations. In the subsequent empirical analysis, we exploit this finding and use the following representation of the price patterns:

\[
\text{prange}_t =
\begin{cases} 
1 & \text{if } p_t < \text{pregLow} \\
2 & \text{if } p_t = \text{pregLow} \\
3 & \text{if } \text{pregLow} < p_t < \text{pregHigh} \\
4 & \text{if } p_t = \text{pregHigh}
\end{cases}
\]

We investigated if there exists any simple time pattern of price changes in the data. We thoroughly checked the relative frequency of price changes in both directions by the day of a week, day of a month, by month, on holidays, all of them for each store. We found a pair of statistically significant regularities. Among the seven stores that belong to a same national chain of supermarket (chain 1), we found that the prices of both brands are reduced on they day 20th of a month, only to be increased again on 21st. As it turns out, the day 20th of each month is a regular monthly store sales day, common to all the stores of this chain. Except for this pattern, we failed to detect any simple regularity.
in price changes, although we do find mild seasonality in the overall frequency of price changes over a year\textsuperscript{21}. Somewhat unexpectedly, pricing policies of the two competing brands exhibit significant positive correlations. Table 4 shows relative frequencies of prices conditional upon the price of the competing brand. Table 4 shows the tendency that the probability of higher price is significantly larger when the price level of the competing brand is also higher, and vice versa.

Although we found no simple rule for price changes, they are far from random pricing either. Using conventional probit model and survival analysis (results not shown), we find robust and strong negative duration dependence of price changes, even after controlling for a variety of seasonabilities.

These findings are consistent with our modeling specifications in several dimensions. First of all, they suggest strongly that the retailers set these prices under their initiative and with information they have. Given the strong positive correlation of the prices of two competing brands within each store, it is extremely unlikely that upstream suppliers coincidentally adjust respective prices. The evidence indicates that prices of the two brands reflect common factors relevant to the optimal pricing policy. The time pattern we found on the day 20th is a prime example, as the store sales day of a month is likely to be accompanied by significant sales promotion and advertising activities, which in turn significantly increases the size of the customers visiting the store. Given the extreme high frequency of price changes, it is also unlikely that the changes in the wholesale price are the driving force behind these pricing policies. The evidence also rejects any simple random pricing as a description of the data. As we will see later, our results indicate that large day-to-day fluctuations in the number of customers is the most likely candidate.

3.3 'Stylized' Facts and Alternative Models

As we sketched in Introduction, frequent price changes without any apparent time trend or seasonality is observed regularly in many grocery items sold typically in supermarket stores. Recurrent price increases and decreases without accompanying changes in costs can be an optimal pricing policy only if some type of the inter-temporal linkage in demand is important. As pointed out correctly in Slade (1998), many candidate explanations imply gradually decaying negative impact of the past price on current demand. Think, for example, a customer capital model. Lower than a 'fair' price gradually accumulates goodwill or customer capital so that the (current) demand size also increases. Similar patterns can be predicted if consumers over time grow tastes or habits in consumption of a good: lower price attracts first time customers and the stores can build up the customer base. Information imperfection is yet another possibility.

\textsuperscript{21}For example, take weekly fluctuations in the relative frequency of price changes: for House brand, Tuesdays have the highest frequency in both directions (11.2 and 9.7%, respectively), and at 6.3%, Friday is the lowest for the price increase, whereas Thursday figure is the lowest at 6.8% for the decrease. S&B brand exhibits similar pattern: Tuesdays register the highest frequency in both directions (8.4 and 6.2%), with Fridays being lowest for the increase (4.4%) and Thursdays lowest (4.6%) for the decrease.
consistent with such a pattern. Suppose customers collect price information only occasionally so that, at any point of time, customers differ in the time elapsed since they collected price information for the last time. In this case, lower prices set in the past induce some of the customers to shop, resulting again in gradually decaying negative effect of the past price on current demand.

On the other hand, the current demand depends positively on the past prices in the case of models of sales. The longer the time elapsed since the last sales, the larger is the size of the pent-up demand waiting for the sales. Table 5 shows the results of the regressions on the current demand on past prices. There is no doubt that in both brands the current demand is positively correlated with the past prices (in one, two, and four weeks). It might also be noted here that regressions indicate significant and sizable effects of the current price of the competing brand.

3.4 Dependence of the current sales on the current Price and its Duration

Although the positive dependence of sales volume on the past prices is a characteristic shared by many (including our own) variants of the model on sales, our model differ sharply form the others in the prediction on the effect of the duration of the high price on the current demand during the high price period. The dependence of the current sales on the level of \( \pi \) implies that the sales monotonically decline during the low price period as the customers continue to build up the inventory at home, whereas the sales monotonically increases over time during the high price period as the customer gradually depletes the inventory. In a nutshell, the sales depends negatively on the price duration during the low price, whereas the sales depends positively on the duration during the high price period.

Table 6 shows the panel regression on the sales, incorporating differential impacts of the price durations on the sales. As the model in this paper predicts, the duration has generally significant negative impact on the sales, with only exception on price range 3, House brand case (the coefficient is insignificant), whereas at the regular high price (price range 4), the durations have positive impact on the sales. It might be noted again that the price of the competing brand has significant positive impact on sales, as we found in preliminary regressions in Table 5. Recall the finding in Table 4 that prices of the two competing brands are highly correlated. Taken together, these facts suggest that the prices of the two brands do respond to a common demand shock which is store specific and highly volatile.

In summary, we found that neither simple time-dependent rule or explanations based upon information imperfection, consumer habit formation, or customer capital model can account for the statistical regularities documented above. On the other hand, these facts are consistent with our model of sales. In particular, our model is the only one that predicts differential impacts of the past prices on the current sales.
4 Empirical Analysis

The base model developed in section 2 is deterministic, giving rise to infinite sequence of price cycles of identical length and pattern. As is clear from Figure 1, the observed pricing patterns are highly irregular, and in particular, we find large variations in the length and magnitude of price cycles.

As we will show below, the two most important temporal variations which we believe responsible for the fluctuations are (a) changes in the number of customers per period, (b) occasional but sharp changes (declines) in wholesale price of the curry paste.

In order to take account of these impacts, we use regression result for each store and brand, whose specification is identical to those panel regressions in Table 6. Since the regressions in Table 6 estimate the daily sales, not the size of the daily visitors to the store, we need to filter out the temporal variations of the demand, given the size of the visiting customers to a store. For this purpose, we use the fitted value of the sales and subtract the estimated effects of (1) those due to the duration as they reflect the gradual changes in $\pi$, and (2) the effect of price range dummy variables as they represent the average size of $\pi$, and (3) the effect of $c_{proxy}$, as this is the proxy for the temporal variations in $c$. Although we have no data on the wholesale price, we learned through interview that (1) at regular low price ($prange2$), stores intentionally avoid advertising the lower price level and these discounts are unaccompanied by corresponding wholesale price changes, (2) occasional sales lasting for a short periods (at most 3 days) at prices below regular low price ($prange1$) is always accompanied not only by corresponding reductions in wholesale price but also by promotion and advertising activities conducted jointly with the store and wholesalers. Based upon these findings we treat the impacts of these occasional sales as those reflecting the increase in the shopping intensity, $s$. Thus we include the estimated effect of the dummy variable for the lowest price range because we expect that the dummy primarily represents the increased customer awareness and increase in $s$.

To sum up, we used the following procedure. First, we ran the following regression for each brand-store observations (indexed by $i$).

$$
\log(sales_{it}) = \alpha_0 + \sum_k \alpha_{1k} \times dummy_{kt} + \alpha_2 c_{proxy t} + \\
\sum_{j=1}^4 \alpha_{3j} prange_{jt} + \alpha_{4j} prange_{jt} \times duration_{jt} + \\
(+\alpha_{5j} prange_{jt} \times \{duration_{jt}\}^2) + u_{it}
$$

Denoting by $\hat{\gamma}$ the estimated coefficients, we obtain the shopping intensity measure as follows:

$$
\hat{s}_{it} = \hat{\alpha}_0 + \sum_k \hat{\alpha}_{1k} \times dummy_{kt} + \hat{\alpha}_2 c_{proxy t} + \\
\sum_{j=1}^4 \hat{\alpha}_{3j} prange_{jt} + \hat{\alpha}_{4j} prange_{jt} \times duration_{jt} + \\
(+\hat{\alpha}_{5j} prange_{jt} \times \{duration_{jt}\}^2) + u_{it}
$$
We obtain two intensity measures for each case, depending upon whether or not the terms in the parenthesis in the equation above is included: henceforth we denote by type 1 [type 2] the measure without [with] the parenthesized term.

In view of the theoretical model in this paper, the amount of sales fluctuates over time as (1) $\pi$ changes over time, (2) $c$ fluctuates over time, (3) $s$ changes over time for exogenous reasons (such as heavy shopping in weekends, lower shopping rate in February), and (4) advertisement and sales promotion increases $s$. The variable $\tilde{s}_t$ is obtained by subtracting from the fitted value of sales, $\log(sales_t)$, the effects due to (1) and (2) in the sales regressions and incorporate the effects of (3) and (4) as measured in the sales regressions. Finally, we normalized the variable by taking the anti-log and then adjusting the average to unity.

4.1 Empirical Analysis of the Demand Shifts

One of the key characteristics of the model in our paper is that the composition and the size of the demand systematically evolves depending upon the price policy chosen by a store. Recall that the demand is given by

$$Q_t = s_t(c_t + \pi_{t-1}) \quad \text{if} \quad t \in t_L$$

$$Q_t = s_t c_t \pi_{t-1} \quad \text{if} \quad t \in t_H$$

Wherein we now allow $s$ and $c$ to vary over time. $(16')$ can be solved for $\pi$ to get

$$\pi_{t-1} = \frac{Q_t}{s_t - c_t} \quad \text{if} \quad t \in t_L$$

$$\pi_{t-1} = \frac{Q_t}{s_t c_t} \quad \text{if} \quad t \in t_H$$

Recall also that the following equations determine the time path of $\pi$.

$$\Delta \pi \equiv \pi_t - \pi_{t-1} = c_t(1 - \pi_{t-1}) \quad \text{if} \quad t \in t_H$$

$$\Delta \pi = c_t(1 - s_t)(1 - \pi_{t-1}) - s_t \pi_{t-1} \quad \text{if} \quad t \in t_L$$

We can eliminate $\pi$ from the system above to get

$$Q_{t+1} = \frac{s_t + 1}{s_t} (1 - c_t)(1 - s_t) - s_t Q_t + s_{t+1} c_t (1 - s_t) \quad \text{if} \quad t \in t_L$$

$$Q_{t+1} = \frac{s_t + 1}{s_t} c_{t+1} c_t (1 - c_t) Q_t + s_{t+1} c_{t+1} c_t \quad \text{if} \quad t \in t_H$$

These pair of equations in $(23)$ can be estimated using one of non-linear estimation methods once we have data on $Q_t, s_t, c_t$. In the subsection above we have obtained the estimate $\tilde{s}_t$ except that the average of the sales is unknown. Similarly, our data on consumption, $c_{proxy_t}$ needs the estimate of the average level. Finally, our data on gross sales needs to be re-adjusted to incorporate unknown size of the customer (which is normalized to unity in the theoretical model in section 2). Hence we have

$$s_{ik} = \lambda_{ik} s_t \tilde{s}_t \equiv \lambda_{ik} s_t$$

$$c_{ik} = \lambda_{ik} c_{proxy_t} \equiv \lambda_{ik} s_t$$

$$Q_{ik} = \frac{Q_t}{X_t}$$
In the last equation, $Q^i_{t}$ is the observed sales volume, and $X^{ik}$ is the unknown size of the regular customer for store $i$, brand $k$.

Combining everything together we have:

$$
\overline{Q}_{t+1} = \nabla_t [ a_{11} \tilde{s}_{t+1} \overline{Q}_t + a_{12} \tilde{s}_{t+1} \tilde{e}_t \overline{Q}_t + a_{13} \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t + a_{14} \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t + a_{15} X \tilde{s}_{t+1} \{ \tilde{c}_t \}^2 \tilde{e}_t ]
$$

$$
+ (1 - \nabla_t) [ a_{21} \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t - a_{22} \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t + a_{23} X \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t + a_{24} \tilde{s}_{t+1} \tilde{c}_t \overline{Q}_t ]
$$

(23’)

where $\nabla$ is unity during the low price period and is zero otherwise. The set of restrictions are:

$$
a_{11} = a_{21} = 1
$$
$$
a_{12} = a_{22} = \lambda_c
$$
$$
a_{13} = \lambda_s \lambda_c
$$
$$
a_{14} = \lambda_s
$$
$$
a_{15} = a_{23} = \lambda_s \lambda_c^2
$$
$$
a_{16} = \lambda_s^2 \lambda_c^2
$$

(24)

The entire system given by (23’) and (24) can be estimated by maximum likelihood estimation of non-linear regressions. Specifically, we employed the following procedure. We had obtained an estimate of the average size of shoppers to each store in the sample. We used this as our proxy for $X^{ik}$. This leaves us with the two unknowns, $\lambda_c$ and $\lambda_s$ which we estimated as using maximum likelihood non-linear estimation. The results are shown in Table 7. Except for stores 8, 9, 13 and 16 for House and store 16 for S&B brands, estimated coefficients are positive and less than unity. Most of them are also highly significant. Since estimation requires positive sales figures for the current and the previous period, we lose many observations in some store/brand combinations and the regressions performed relatively poorly in those cases. Overall, these estimates provide us with intuitively reasonable magnitudes for the two key parameters, $\lambda_c$ and $\lambda_s$ : the average consumption probability ranges .2 to 1% per day, whereas the average shopping probability ranges between .1 to .3.

### 4.2 Ordered Probit Model of Price Changes

The second important prediction of the model is that the pricing policy depends crucially upon the level of $\pi$. The probability of price decline should be monotonically increasing in $\pi$, whereas the price increase probability decreases
monotonically with $\pi$. This prediction can be posited as an ordered probit model:

$$
\Delta p \equiv p_t - p_{t-1}
$$

$$
\Delta p > 0 \text{ if } \Psi > P^U
$$

$$
\Delta p = 0 \text{ if } P^D < \Psi < P^U
$$

$$
\Delta p < 0 \text{ if } \Psi < P^D
$$

$$
\Psi \equiv \alpha_1 \pi + \alpha_2 \pi \text{Low} + \alpha_3 \text{Low} + \alpha_4 s^e + \text{dummies}
$$

$$
\alpha_1 < 0, \alpha_2 > 0, \alpha_3 \leq 0, \alpha_4 < 0
$$

(25)

wherein the $\Psi$ equation above, $\text{Low}$ is a dummy variable which is unity whenever the price is below regular high price ($p < \text{prange}4$), and $s^e$ is the expected shopping intensity for the next two weeks. One of the two unknown threshold values, $P^U, P^D$, can be set arbitrarily at, say, zero and the remaining unknowns together with other parameters should be estimated. We expect $\alpha_1$ to be negative as the store is more likely to lower the price when the share of customers without inventory is higher. This impact is smaller, however, if the price is already below the regular price ($\alpha_2 > 0$). We cannot sign $\alpha_3$ because the lower price today reduces the probability of further price decline whereas the current price may predict further decline in price to the extent that the current price predicts future increase in shoppers if the retailers are better informed than econometrician (us) about the number of shoppers in the future (see below). We expect that the expected increase in shoppers should induce the store to reduce price, hence we expect $\alpha_4 < 0$. We use the fitted value of the regression on shopping intensity used to construct $\pi$ series$^{22}$.

To test these predictions of the model in the data, we first construct the time series for the unobservable variable, $\pi_t$. We use the estimates of the two key parameters $s$ and $c$ obtained above in order to dynamically construct $\pi$ series using:

$$
\bar{\pi}_t = \hat{s}_t
$$

$$
\bar{\pi}_t = \hat{c}_t \text{proxy}_t
$$

$$
\hat{\pi}_t = \pi_t + (1 - \bar{\pi}_t) \hat{\pi}_{t-1} \text{ for } t \in T_H
$$

$$
\bar{\pi}_t = \pi_t(1 - \bar{\pi}_t) + (\pi_t(1 - \bar{\pi}_t) + \bar{\pi}_t) \bar{\pi}_{t-1} \text{ for } t \in T_L
$$

The equations above generate the time series for a given initial condition, i.e., the value of $\pi$ in the initial period. In order to eliminate the effect of initial condition on the generated series, we used two alternatives. First, we arbitrarily pick an initial value and we computed the empirical density of $\pi$. Then we choose the next initial condition by drawing from this probability distribution. The process is continued until we obtain the convergence of the probability distribution. Alternatively, we can simply eliminate some portion of the early observations of $\pi$. The results of the probit analysis hardly differed irrespective of the methods used.

$^{22}$ In view of the RHS variables used to forecast shopping intensity, it is reasonable to assume that the store can perfectly predict those RHS variables in the near future.
Table 8 reports the ordered probit model given by (25). We report only the estimations based upon the first method\(^{23}\). The results shown are obtained by pooling all the observations for the stores for which we obtained theoretically correct estimates of \( s \) and \( c \), i.e., they are all positive and less than unity. The results are highly robust and the estimated coefficients are of correct signs and significant at 1% level\(^ {24}\). In particular, the constructed series of \( \pi \) is highly significant and an increase in \( \pi \) increases (lowers) the probability of price decline (increase). We also find that the expected increase in the size of shoppers has also significant positive (negative) impact on the probability of price decline (increase).

4.3 Cross Section Variations in the Estimated Coefficients

The model presented in this paper has five free parameters \((\omega, \varepsilon, \sigma, s, \text{ and } c)\) and the sixth parameter, \( u \), can be used as a numeraire. We obtained estimates of \( s \) and \( c \) in the nonlinear estimation model reported above. This leaves us with three remaining parameters. Since we have no strong reasons to believe that customers across stores and /or brands differ in their costs of holding inventory at home, we treat \( \varepsilon \) as an unknown but common constant across brands and stores.

Next, given the scanner technology and inventory and logistic system common to supermarket stores in our sample, we assume that the cost of changing price tags, \( \sigma \) does not vary across brands or stores within a chain. Moreover, the menu cost should be independent from the sales size\(^ {25}\). Thus we postulate the menu cost per unit of customer is given by:

\[
\sigma_{jk} = \bar{\gamma}_k / \bar{Q}_{jk}
\]

wherein \( \bar{\gamma}_k \) is menu cost specific to chain \( k \) and \( \bar{Q}_{jk} \) is the average customer size of brand \( i \) at store \( j \), which we normalized to unity in the theoretical model.

In section 2.6, we obtained the following comparative statics results using

\(^{23}\)The results are virtually identical under alternative specifications of the initial value of \( \pi \) or with alternative estimators for \( \bar{v} \).

\(^ {24}\)Although we cannot sign \( \alpha_3 \), a comparison of the regression results without \( s^e \) (not shown in Table 8) against those with \( s^e \) shows that \( \alpha_3 \) is always smaller in the absolute value for the latter cases. This is consistent with our expectation because the part of the predictive power of the current price on future sales is taken away by inclusion of \( s^e \).

\(^ {25}\)There is no requirement that the price tag be attached to the individual merchandise. All of the sample stores in our data use scanners and the system is linked to the price tags on the shelves of each store. As shown in whether or not the stores are required to attach the price tag on individual merchandise makes significant difference in the cost of changing prices.
numerical simulations.

\[ \log \{ E[p_{rk}] \} = \alpha_0 + \alpha_1 \log(s_{rk}) + \alpha_2 \log(c_{rk}) + \alpha_3 \log(\omega_{rk}) - \alpha_4 \log(\varepsilon) \]

\[ \log \{ E \frac{T}{k+1} \} = \beta_0 + \beta_1 \log(s_{rk}) - \beta_2 \log(c_{rk}) + \beta_3 \log(\omega_{rk}) + \beta_4 \log(\varepsilon) \]

\[ \log \{ E \frac{T_H}{T} \} = \gamma_0 + \gamma_1 \log(s_{rk}) - \gamma_2 \log(c_{rk}) + \gamma_3 \log(\omega_{rk}) + \gamma_4 \log(\varepsilon) \]

wherein all the parameters (\( \alpha, \beta, \gamma \)) are positive. Denote by \( \Delta \) log-differencing across stores and brands within a chain. Using (26) above to substitute for \( \sigma_{jk} \), the equations above can be written to get 'within a chain' regressions.

\[ \Delta \log \{ E[p_{rk}] \} = \delta_0 + \delta_1 \Delta \log(s_{rk}) + \delta_2 \Delta \log(c_{rk}) + \delta_3 \Delta \log(\omega_{rk}) + \delta_4 \Delta \log(\varepsilon) + u \]

\[ \Delta \log \{ E \frac{T}{k+1} \} = \theta_0 + \theta_1 \Delta \log(s_{rk}) - \theta_2 \Delta \log(c_{rk}) + \theta_3 \Delta \log(\omega_{rk}) + \theta_4 \Delta \log(\varepsilon) + u \]

\[ \Delta \log \{ E \frac{T_H}{T} \} = \zeta_0 + \zeta_1 \Delta \log(s_{rk}) - \zeta_2 \Delta \log(c_{rk}) + \zeta_3 \Delta \log(\omega_{rk}) + \zeta_4 \Delta \log(\varepsilon) + u \]

wherein chain specific constants and error terms (\( u \)’s) represent the impact of the unobservable, \( \Delta \omega_{jk} \). Note also that \( \varepsilon \)’s are cancelled out in these 'within a chain' regressions.

Table 9 shows the 'within a chain' estimates of these parameters in (27a) through (27c). The first column is within chain panel OLS and the second set of regression results are SUR (seemingly unrelated regressions) run for joint estimations of \( \alpha, \beta, \) and \( \gamma \).

The results are broadly consistent with the results of numerical examples but most of the estimated coefficients for \( s \) and \( c \) are not statistically significant. Some of them are even wrong signs. It should be noted, however, that none of the coefficients are statistically significant and wrong sign. Moreover, the coefficients on \( \log(\omega_j) \) are statistically significant and signs are correct in all of the three regressions.

5 Conclusion

In this paper, we developed a dynamic model of sales and applied it to the pricing policy of Japanese supermarkets, with the results consistent and sup-
porttive of the main thrust of the theoretical model. In particular, we found that the customers respond to occasional price markdown by accumulating the low priced merchandise as inventory at home, which in turn are gradually consumed during the high price period. Such an behavior is confirmed by the regression results demonstrating negative (positive) price duration dependence of the sales during the low (high) price period. We also estimated the determinants of the price changes and found that the share of customers without inventory have statistically significant impact on price changes.

These facts are not easily reconcilable with other competing models of sales. For example, customer capital model is at variance with the data because the model predicts negative dependence of the sales on the past prices, which as we saw is strongly contradicted by the data. Variants of models of sales developed in the past also have difficulty in one way or the other once confronted with the data. First of all, although the duration of sales (low price) is typically shorter than the price durations at higher price level, it can be hardly approximated by a burst of sales completed within a single period, a prediction shared by these models. They are silent on exactly how long sales periods last, not to mention when they start. Random pricing models of sales are also at variance with the strong negative duration dependence of price changes.

Our model scored better than any of these candidate models but we still leave many important questions left unanswered.

One of the major unexplored issues is the interactions between brands within a store. As we noted in section 3, statistical evidence indicates significant correlations between the pricing of the two brands within a store26. It seems quite likely that customers can easily switch between brands in response to occasional sales of one of the brands. It is not clear at this moment how the model should be altered once we incorporate brand substitutions into the model.

Another issue of interest is the linkage between pricing and sales promotion activities. We found evidence in support of the close coordination between pricing and sales promotion but our results fall short of identifying the mechanism of the interactions.

In more general terms, this paper can be considered as a first step towards comprehensive analysis of the retail store pricing and sales promotion activities. Our results indicate that supermarket stores do induce fluctuations in shopping intensity and purchase patterns by employing sophisticated dynamic pricing and sales promotion.

References


26 Lach and Tsiddon (1996) also found significant within store price synchronization in wines but not in meat products. Their findings are consistent with our model of sales.


6 Tables and Figures

Table 1 Effects of Changes in Parameters

| $T$  | $\frac{\Delta T}{T}$ | $\frac{\Delta H}{H}$ | $M$ | $E(p)$ | $E(p_t | t \in T_L)$ |
|------|----------------------|-----------------------|-----|--------|---------------------|
| cycle length | price duration | share of high price periods | maximand | mean price (time average) | mean price in $T_L$ |
| $s$ | +2.59 | +3.87 | +1.05 | +0.65 | +0.18 | +0.09 |
| $c$ | -0.60 | -0.80 | -0.36 | +1.29 | -0.22 | +0.08 |
| $\omega$ | +3.16 | +4.90 | +1.49 | -1.44 | +0.24 | +0.10 |
| $\delta$ | +4.59 | +6.31 | +1.32 | -0.45 | +0.18 | -0.16 |
| $\sigma$ | +1.13 | +1.39 | +0.12 | -0.05 | +0.04 | -0.04 |

Notes: the figure in each cell shows the average % change of an endogenous variable (column) when one of the parameters (row) is increased by 1% when the rest of the parameters are set at the benchmark values. In computing the figures, only the results from interior equilibria are used (we exclude equilibria wherein static pricing policies are optimal).
Table 2 Optimal number of price changes during a cycle

<table>
<thead>
<tr>
<th>Parameter range</th>
<th>static low</th>
<th>k = 2</th>
<th>k = 1</th>
<th>k &gt; 2</th>
<th>static high</th>
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</thead>
<tbody>
<tr>
<td>s</td>
<td>[.122, .999]</td>
<td>&lt;.296</td>
<td>[.296, .423]</td>
<td>.423, .521</td>
<td>n.a.</td>
</tr>
<tr>
<td>c</td>
<td>[.036, .999]</td>
<td>&lt;.141</td>
<td>[.141, .237]</td>
<td>.237, .374</td>
<td>n.a.</td>
</tr>
<tr>
<td>ω</td>
<td>2.26, 9.65</td>
<td>&lt;3.74</td>
<td>[3.74, 5.63]</td>
<td>5.63, 6.11</td>
<td>n.a.</td>
</tr>
<tr>
<td>s</td>
<td>[.122, .999]</td>
<td>&lt;.284</td>
<td>[.284, .396]</td>
<td>.396, .460</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter range</th>
<th>static policy</th>
<th>k = 1</th>
<th>k = 2</th>
<th>k = 3</th>
<th>k ≥ 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>[.00013]</td>
<td>high price if σ &gt; .24</td>
<td>[.088, .24]</td>
<td>[.0073, .088]</td>
<td>[.00063, .0073]</td>
</tr>
</tbody>
</table>

+we programmed simulation model up to k=5. Obviously, as we reduce σ further, the optimal number of price changes increases indefinitely. We find that k=5 overtakes k=4 at .00013.
Table 3  Summary Statistics of Price Data: House Brand

<table>
<thead>
<tr>
<th>store#</th>
<th>price (yen)</th>
<th>prange shares (%)</th>
<th>price duration (days) at prange=</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>chain #</td>
<td>mean</td>
<td>min.</td>
<td>max.</td>
</tr>
<tr>
<td>1 [1]</td>
<td>221</td>
<td>128</td>
<td>253</td>
<td>13.5</td>
</tr>
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<td>2 [1]</td>
<td>199</td>
<td>145</td>
<td>253</td>
<td>22.3</td>
</tr>
<tr>
<td>3 [1]</td>
<td>218</td>
<td>130</td>
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Table 5 Panel Regression of the Sales on the Current and Past Prices

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<th>S&amp;B Brand Price</th>
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<td>past</td>
<td>current</td>
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<td>0.974 x 10^{-4}</td>
<td>-0.022***</td>
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<tr>
<td>2 weeks</td>
<td>-0.030***</td>
<td>0.326 x 10^{-2}</td>
<td>-0.022***</td>
<td>0.186 x 10^{-2}</td>
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<tr>
<td>4 weeks</td>
<td>-0.031***</td>
<td>0.551 x 10^{-2}</td>
<td>-0.022***</td>
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<td>1 week</td>
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<td>0.321 x 10^{-2}</td>
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<td>2 weeks</td>
<td>-0.030***</td>
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<td>4 weeks</td>
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<td>0.691 x 10^{-2}</td>
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†All regressions are fixed effect panel regressions and aside from those shown in the table, they include the following variables; year, month, day of the week, holidays, the day before shops are closed, and the day after shops are closed (all of these dummy variables), and cproxy, representing seasonal fluctuations in the consumption. ‡*** indicates significant at 1%, ** and * correspond to 5% and 10%, respectively.
Table 6 Panel Regressions of the Sales on Price Range and Durations

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1 Both regressions include the following variables; year, month, day of the week, holidays, the day before shops are closed, and the day after shops are closed (all of these dummy variables), and cproxy, representing seasonal fluctuations in the consumption. The variable labeled 1-3 indicate price range of the competing brand, i.e., in the case of House (S&B) brand, rivalprange$x$ is a dummy variable which is unity if the price of S&B (House) curry is in the price range $x$ (=1,2,or 3)
## Table 7 Non-Linear Regressions on Sales

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<td>.128</td>
<td>.00875 ( ** )</td>
<td>.157</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td></td>
<td>.628</td>
<td>601 ( ** )</td>
<td>-.0244 ( ** )</td>
<td>.071 ( ** )</td>
</tr>
</tbody>
</table>
Table 7 Non-Linear Regressions on Sales (cont’d.)

<table>
<thead>
<tr>
<th>Chain #</th>
<th>Store #</th>
<th>S&amp;B Brand</th>
<th>type 1</th>
<th>type 2</th>
<th>N</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_c$</td>
<td>$\lambda_s$</td>
<td>$\lambda_c$</td>
<td>$\lambda_s$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>.00518</td>
<td>.4141</td>
<td>.00529</td>
<td>.1471</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td>.00271</td>
<td>.4142</td>
<td>.00281</td>
<td>.1378</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td></td>
<td>.00999</td>
<td>.1682</td>
<td>.00997</td>
<td>.1736</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
<td>.00501</td>
<td>.1562</td>
<td>.00506</td>
<td>.1548</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td></td>
<td>.00899</td>
<td>.0791</td>
<td>.00890</td>
<td>.0748</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td></td>
<td>.00632</td>
<td>.1803</td>
<td>.00660</td>
<td>.1466</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td></td>
<td>.00372</td>
<td>.0968</td>
<td>.00395</td>
<td>.0863</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
<td>.00818</td>
<td>.1139</td>
<td>.00814</td>
<td>.1079</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
<td>.00293</td>
<td>.2091</td>
<td>.00285</td>
<td>.1947</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
<td>.00429</td>
<td>.2981</td>
<td>.00428</td>
<td>.2976</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td></td>
<td>.00979</td>
<td>.3058</td>
<td>.00949</td>
<td>.3700</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
<td>.0204</td>
<td>.1188</td>
<td>.0185</td>
<td>.1497</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td></td>
<td>.0539</td>
<td>.0258</td>
<td>.0205</td>
<td>.0813</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td></td>
<td>.00463</td>
<td>.3217</td>
<td>.00549</td>
<td>.2277</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td></td>
<td>.00569</td>
<td>.1251</td>
<td>.00667</td>
<td>.1351</td>
</tr>
</tbody>
</table>

All the regressions are estimated by non-linear maximum likelihood estimation and they include first-order serial correlation corrections. Based upon standard error computed from heteroscedastic consistent matrix, all the coefficients are significant at 1% unless otherwise noted: * not significant at 1% but significant at 5%, ** not significant at 5%.
Table 8 Ordered Probit Model of Price Changes

<table>
<thead>
<tr>
<th></th>
<th>House Brand</th>
<th></th>
<th>S&amp;B Brand</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coef.</td>
<td>Pr(Δp&lt;0)</td>
<td>Pr(Δp=0)</td>
<td>Pr(Δp&gt;0)</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-2.05</td>
<td>.387^1</td>
<td>.0052^2</td>
<td>-.382^3</td>
</tr>
<tr>
<td>( \text{low} \times \pi )</td>
<td>.120</td>
<td>-.0216</td>
<td>.0003</td>
<td>.0214</td>
</tr>
<tr>
<td>( \text{low} )</td>
<td>-0.67</td>
<td>.0744</td>
<td>-.0010</td>
<td>-.0734</td>
</tr>
<tr>
<td>( \text{shopping intensity} )</td>
<td>-0.077</td>
<td>.0136</td>
<td>.00018</td>
<td>-.0134</td>
</tr>
<tr>
<td>( \text{storesale} )</td>
<td>-1.04</td>
<td>.184</td>
<td>-.0025</td>
<td>-.181</td>
</tr>
<tr>
<td>( \text{storesale} + 1 )</td>
<td>0.95</td>
<td>-.165</td>
<td>.0022</td>
<td>.163</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.086</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of samples</td>
<td>21872</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-14467</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (*) the RHS is an index variable; 0 for price decrease, 1 for no change, and 1 for price increase, respectively, from the previous day. All the estimated coefficients are correct signs and significant at 1% significance level. (1,2,3) Numbers shown are the estimated effect of an increase in each variable on the probability that \( Δp < 0, = 0, > 0 \), respectively. (†) We use the estimated values of \( \pi_1 \) shown in Table. See the explanations in the main text.
Table 9 Cross Section Regressions on Estimated Coefficients

<table>
<thead>
<tr>
<th></th>
<th>$s = \hat{s}_1$</th>
<th></th>
<th>$s = \hat{s}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s$</td>
<td>$c$</td>
<td>$Q$</td>
</tr>
<tr>
<td>$\log(E_{p_{1_t}})$</td>
<td>-2.951</td>
<td>2.357</td>
<td>-1.856***</td>
</tr>
<tr>
<td>SUR</td>
<td>-2.952</td>
<td>2.355</td>
<td>-1.856***</td>
</tr>
<tr>
<td>$\log(E_{p_{1_t}})$</td>
<td>.0230</td>
<td>.0480</td>
<td>.0416***</td>
</tr>
<tr>
<td>SUR</td>
<td>.0228</td>
<td>.0489</td>
<td>.0416***</td>
</tr>
<tr>
<td>$\log(E_{t+1})$</td>
<td>-2.793</td>
<td>.0738</td>
<td>-.107**</td>
</tr>
<tr>
<td>SUR</td>
<td>-2.792</td>
<td>.0738</td>
<td>-.107**</td>
</tr>
</tbody>
</table>

All the regressions above include dummy variables for each chain (1 through 7). [SUR] indicates the results for seeming unrelated regressions in which three equations are simultaneously estimated incorporating possible covariations in the error terms, $\gamma$, indicates correct signs and $**(*)$ indicates 1%(5%) significance level.
Figure 1 Daily Price of *House* Brand Curry Paste at Store 1 of Chain 1
Figure 2 Frequency Distribution of House Brand Curry Paste at Store 1 of Chain 1
Mathematical Appendix for Hot and Spicy: Ups and Downs on the Price Floor and Ceiling at Japanese Supermarkets

Kenn Ariga, Kenji Matsui, and Makoto Watanabe

1 Proof of Lemma 4

By differentiations of $\psi_0$, $\phi_0$, and $\xi_0$, we obtain:

$$
\frac{d\psi_0}{d\theta} = \frac{\log(a) \times \log(b) \times [\theta^2 \log(a) - (1 - \theta)^2 \log(b)]}{[\theta \log(a) + (1 - \theta) \log(b)]^2}
$$

$$
\frac{d\phi_0}{d\theta} = \frac{\log(d) \times \log(e) \times [\theta^2 \log(d) - (1 - \theta)^2 \log(e)]}{[\theta \log(d) + (1 - \theta) \log(e)]^2}
$$

$$
\frac{d\xi_0}{d\theta} = \frac{\{\log(a) \times \log(b)\} \{\log(b) + \log(e)\}}{\theta^2 \log(a) \times \log(d) - \theta(1 - \theta)^2 \log(a) \times \log(e)}
$$

$$
\begin{align*}
&= \frac{1}{\theta^2 \log(a) \times \log(d) - \theta(1 - \theta)^2 \log(a) \times \log(e)} \\
&\times \left\{ \log(b) \times \log(e) - \log(d) \times \log(b) - (2 - 3\theta)(1 - \theta)^2 \log(b) \times \log(e) \right\} (A1)
\end{align*}
$$

where, $a = 1 - c$, $b = 1 - c(1 - s) - s$, $d = (1 - c)^{-1}$, $e = \{1 - c(1 - s)\}^{-1}$.

Using these expressions, the total differentiation of the maximand is given by

$$
\frac{dM(\theta)}{d\theta} = \{\mu^2(1 - s)\}^{-1} [C_u(\theta) \tilde{u} + C_\xi(\theta) \tilde{\xi}]
$$

$$
C_u(\theta) \equiv (1 - s) \{\mu c(1 - s) - s(1 - \mu) \frac{d\psi_0}{d\theta}\}
$$

$$
C_\xi(\theta) \equiv \mu c(1 - s) \{1 + c(1 - s)\} - s(1 - s) \frac{d\psi_0}{d\theta}
$$

$$
+ \mu s \{1 + c(1 - s)\} \frac{d\phi_0}{d\theta} + \mu s(1 - s) \frac{d\xi_0}{d\theta}
$$

$$
\mu \equiv c(1 - s) + s,
$$

$$
\tilde{u} \equiv u - \omega,
$$

$$
\tilde{\xi} \equiv \frac{\varepsilon}{c(1 - s)}.
$$

Now the following properties are immediate from (A1):

1. $\frac{d\psi_0}{d\theta}$ and $\frac{d\phi_0}{d\theta}$ are both monotonically decreasing in $\theta$, strictly positive at $\theta = 0$, and negative at $\theta = 1$. 

\[ \frac{d\theta}{d\theta} \] is strictly convex in \( \theta \), is zero at \( \theta = 0 \), reaches a strictly positive value at \( \theta(0 < \theta < 1) \) and negative at \( \theta = 1 \).

Using these properties, we find:

\( 3 \) \( C_u(\theta) \) is monotonically increasing in \( \theta \) and strictly positive in \([0,1]\)

\( 4 \) \( C_\varepsilon(\theta) \) is monotonically decreasing in \( \theta \) and strictly positive in \([0,1]\).

Consequently, the interior solution to the maximization problem, if it exists, is given by

\[
\begin{align*}
\theta^* &= \arg\max_{\theta} \left\{ C_u(\theta) \right\}
\end{align*}
\]

Now evaluate \( C_u(\theta^*) \) and \( C_\varepsilon(\theta^*) \) at \( \theta = 0 \). We have

\[
\frac{C_u(0)}{C_\varepsilon(0)} = \frac{c(1-s)^2}{(e(1-s) + s)^2 + c(1-s)^2}
\]

where we have used the approximation for small positive \( x \):

\[-\log(1-x) \approx x\]

Similarly, at \( \theta = 1 \):

\[
\frac{C_u(1)}{C_\varepsilon(1)} = \frac{1}{1 + c}
\]

Then conditions given in (20) follow immediately.

2 Convergence to the Stationary Hi-Lo Pricing

Consider a sequence of Hi-Lo pricing cycles, each of which consists of high price periods follow by the low price periods. Index by \( n (=1,2,3,\ldots) \) each of these cycles. The optimal policy for each cycle is denoted by

\[
P^* = \arg\max_{P^*} \left\{ M(P : \pi, \Theta) \right\}
\]

\[
P = (k, t_1, \{t_j^w\}, j=1,2,\ldots,k)
\]

\[
\Theta = \{\omega, \varepsilon, u, c, s, \delta\}
\]

subject to \( \pi(0) = \pi \)

The optimal policy can be written as

\[
P^* = \Psi(\pi, \Theta)
\]

Using (9), we obtain:

\[
\pi t^*_1 t^*_2 = [1 - \pi^* - A(t_1^*(\pi))B(t_2^*(\pi))] + \pi^* + A(t_1^*(\pi))B(t_2^*(\pi))\pi
\]
Since \(\pi_{t_1^*+t_2^*}\) is the initial value of \(\pi\) for the next Hi-Lo price cycle, we obtain:

\[
\pi_{n+1} = [1 - \pi^* - A(t_1^*(\pi_n))B(t_2^*(\pi_n))] + \pi^* + A(t_1^*(\pi_n))B(t_2^*(\pi_n))
\]

Wherein \(\pi_n\) is the initial value of \(\pi\) for the \(n\)-th cycle. Rewriting the above we have

\[
\pi_{n+1} = \Phi(\pi_n)\pi_n,
\]

\[
0 < \Phi < 1
\]

\[
\pi_n \equiv \pi_n - \pi
\]

\[
\pi \equiv 1 - [1 - AB]^{-1}(1 - B)(1 - \pi^*)
\]

The mapping \(\Phi\) given above is contraction and we can apply conventional argument to confirm that the \(\{\pi_n\}\) converges monotonically towards the fixed point, \(\pi\).

3 Optimal Hi-Lo Policy with Menu Cost

In order to compute the willingness to pay for the good to store, we use the following notations:

\[
\begin{align*}
W_t &= W^H_t \text{ for } t \in T_H \\
W_t &= W^j_t \text{ for } t \in T^j_L
\end{align*}
\]

Since the difference equation (8) is still applicable to \(W^H_t\), we obtain:

\[
W_t = W_H - (\frac{1}{1-c})^t(W_H - W) \text{ for } t \in T_H
\]

Hence we have:

\[
W_{t+1} = W_H - D(W_H - W)
\]

where

\[
D \equiv (\frac{1}{1-c})^t_1,
\]

as before. The difference equation for \(W^j_t\) is given by

\[
W_t = -\varepsilon + c(1-s)(u - W_{t+1}) - s(W_{t+1} - p^j_L) + W_{t+1} \text{ for } t + 1 \in T^j_L
\]
Which can be solved to obtain:

\[ W_t = W_{t-1}^j + \lambda \tau_t^j (W_{t-1}^j - W_{t-1}^j) \]

\[ \lambda = \frac{1}{1 - c(1 - s) - s} > 1 \]  \hspace{1cm} (A2)

\[ \tau_t^j = t - \{ t_1 + \sum_{n=1}^{j-1} t_n^j \} \]

\[ \tau_0^j = t_1 + \sum_{n=1}^{j-1} t_n^j \]

\[ W_{t-1}^j = \frac{c(1 - s)u + sp_t^j - \varepsilon}{c(1 - s) + s} \]  \hspace{1cm} (A3)

Notice that, as in the case without menu cost, \( W_t \) increases during the low price period and decreases in the high price period. Therefore, in order to meet the constraint during each sub-period in which price is fixed, \( p_t \leq W_t \) must hold with equality at the beginning of each sub-period. Viz., we have

\[ p_t^j = W_{t+1}^j \text{ for } \forall j, j = 1, 2, \ldots k \]  \hspace{1cm} (A4)

In order to solve the system, we first solve for \( p_t^j \) and \( W_{t-1}^j \):

\[ W_{t-1}^j = \frac{\mu + s\lambda W_{t-1}^j}{c(1 - s) + s\lambda} \]  \hspace{1cm} (A5)

\[ p_t^j = \frac{(\lambda - 1)\mu + \lambda[c(1 - s) + s]W_{t-1}^j}{c(1 - s) + s\lambda} \]  \hspace{1cm} (A6)

Moreover we have:

\[ W_{t-1}^j \equiv W_{t-1}^j = W_{t-1}^j - D(W_{t-1}^j - W) \]

\[ W_{t-1}^j = W_{t-1}^j + E_{j-1}(W_{t-1}^j - W_{t-1}^j) \text{ for } j = 2, 3, \ldots k. \]

\[ E_j \equiv \lambda^{t_j^j} \]

Substituting the above expressions for \( W_{t-1}^j \) and re-arranging, we get:

\[ W_{t-1}^j = H_{0j-1} W_{t-1}^j + H_{1j-1} \mu \]

\[ H_{0j} = \frac{E_j - 1}{c(1 - s) + s\lambda} \]

\[ H_{1j} = \frac{c(1 - s)E_j + s\lambda}{c(1 - s) + s\lambda} \]
which has a general solution in the form given by

\[ W_{\tau_0} = H_{1j-1}H_{1j-2} \cdot H_{11} W_{\tau_1} + (H_{1j-1}H_{1j-2} \cdot H_{12} H_{01} + H_{1j-1} H_{11} \cdot H_{13} H_{02} + \cdots + H_{1j-1} H_{0j-2} + H_{0j-1}) \mu \]  

(A7)

The terminal condition is

\[ W_{\tau_{k+1}} = W \]

which can be solved to obtain:

\[ \bar{W} = (\Theta_{1k} D - 1)^{-1} H_{1i}, \quad j = 1, 2, \ldots, k \]

\[ \Theta_{0j} = \sum_{m=1}^{j} H_{0m} \prod_{i=m+1}^{j} H_{1i}, \quad j = 1, 2, \ldots, k \]

Then this solution can be substituted back into (A7) to obtain \( W_{\tau_0} \):

\[ W_{\tau_0} = (\Theta_{1k} D - 1)^{-1} \left[ \Theta_{1j-1} (D - 1) W_H + \{ \Theta_{0j-1} (\Theta_{1k} D - 1) - \Theta_{1j-1} D \Theta_{0k} \} \mu \right] \]

This solution in turn can be substituted back into (A5) and (A6) to obtain the solutions for \( p_L^j \) and \( W_L^j \). For the time path of \( \pi_t \), the equations derived for the case of no-menu cost are still applicable: they are reproduced here for convenience:

\[ \pi_t = 1 - (1 - c)^t (1 - AB)^{-1} (1 - B)(1 - \pi^*) \quad \text{for} \quad t \in T_H \]

\[ \pi_t = \pi^* + \{1 - c(1 - s) - s\}^{t-1} (1 - AB)^{-1} (1 - A)(1 - \pi^*) \quad \text{for} \quad t \in T_L \]

where it should be understood that \( B \) is defined as

\[ B = \{1 - c(1 - s) - s\}^{t-1} \equiv B_1 B_2 B_3 \ldots B_t \]

\[ B_j = \{1 - c(1 - s) - s\}^{t-1} \]

We utilize these equations to obtain the representation of the maximand in terms of the policy variables. To begin, the maximand is

\[ M = -(k + 1) \sigma + (u - \omega) sc \sum_{t=1}^{t_1} \pi_{t-1} + s \sum_{j=1}^{k} \sum_{m=\tau_0+1}^{j+1} (p_L^j - \omega)(c + \pi_{m-1}) \]

The first and second terms of the maximand are rewritten as

\[ -(k + 1) \sigma + \frac{(u - \omega) s(c t_1 - \frac{(1 - \pi^*)(1 - A)(1 - B)}{1 - AB})}{1 - AB} \]
We use
\[
\sum_{m=\tau_0}^{\tau_0^{i+1}} \pi_{m-1} = t_2 \rho^* + \frac{B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j)(1 - A)(1 - \pi^*)}{(1 - AB)\{c(1 - s) + s\}}
\]
to rewrite the last term as
\[
s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)(c + \pi_{m-1})
\]
\[
\sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)(c + \pi^*)t_2
\]
\[
+s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)\frac{B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j)(1 - A)(1 - \pi^*)}{(1 - AB)\{c(1 - s) + s\}}
\]
Hence the maximand is given by
\[
M = -(k + 1)\sigma + (u - \omega)s\{ct_1 - \frac{(1 - \pi^*)(1 - A)(1 - B)}{(1 - AB)}\}
\]
\[
+s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)(c + \pi^*)t_2
\]
\[
+s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)\frac{B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j)(1 - A)(1 - \pi^*)}{(1 - AB)\{c(1 - s) + s\}}
\]
\[
= -(k + 1)\sigma + (u - \omega)s\{ct_1 + s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)(c + \pi^*)t_2 \}
\]
\[
+ \frac{(1 - \pi^*)(1 - A)(1 - B)}{(1 - AB)\{c(1 - s) + s\}} \times
\]
\[
[s \sum_{j=1}^{\tau_0^{i+1}} (p_L^j - \omega)(1 - B)^{-1} B_1 B_2 B_3 \ldots B_{j-1} (1 - B_j) + s\{c(1 - s) + s\}(u - \omega)\]
\]
wherein \( p_L^j \) is given by
\[
p_L^j = [c(1 - s) + s\lambda]^{-1}(\lambda - 1)\mu
\]
\[
+ [c(1 - s) + s\lambda]^{-1}\lambda[c(1 - s) + s]\{\Theta_{ik} D - 1\}^{-1} \times
\]
\[
[\Theta_{ij-1}(D - 1)W_H + \{\Theta_{ij-1}\Theta_{ik} D - 1 - \Theta_{ij-1} D \Theta_{ik}\} \mu]
\]
}\( A8 \)
4 Proof of Lemma 6

We have

\[ \Delta(t_S) = \gamma(u - \omega)s\pi_{t_S - 1} \text{ if } t_S \in T_H \]
\[ \Delta(t_S) = \gamma(W_{t_S} - \omega)s(e + \pi_{t_S - 1}) \text{ if } t_S \in T_L \]

Note that \( \Delta(t_S) \) is increasing in \( t_S \) if \( t_S \in T_H \) because \( \pi_{t_S - 1} \) increases as the customers gradually deplete their inventory during the high price period. On the other hand, we can confirm easily that \( \Delta(t_S) \) is decreasing in \( t_S \) if \( t_S \in T_L \). Hence to prove the claim, it is sufficient to show: \( \Delta(t_1) < \Delta(t_1 + 1) \). We have

\[ \Delta(t_1) = \gamma(u - \omega)s\pi_{t_1 - 1} \]
\[ \Delta(t_1 + 1) = \gamma(W_{t_1 + 1} - \omega)s(e + \pi_{t_1}) \]

We rewrite (9) to get

\[ \pi_{t_1 - 1} = \frac{\pi_{t_1} - c}{1 - c} \]

and we know also

\[ W_{t_1 + 1} - \omega > W_L - \omega = (1 - R)(u - \omega) \]

Thus we have

\[ \Delta(t_1) = \gamma(u - \omega)s\frac{\pi_{t_1} - c}{1 - c} \]
\[ \Delta(t_1 + 1) > \Delta_1 = \gamma(1 - R)(u - \omega)s(e + \pi_{t_1}) \]

Then we have

\[ \Delta_1 > \Delta(t_1) \text{ iff } \]
\[ (1 - R)(e + \pi_{t_1}) > \frac{\pi_{t_1} - c}{1 - c} \]

Rewriting the above, the condition is

\[ R < \frac{\pi_{t_1}(1 - 2c) + c}{(e + \pi_{t_1})(1 - c)} \]

Since the RHS is decreasing in \( \pi_{t_1} \), we have

\[ \frac{\pi_{t_1}(1 - 2c) + c}{(e + \pi_{t_1})(1 - c)} \geq \frac{1}{(1 + c)} \equiv R_U \]

On the other hand, from Lemma 5, we know

\[ 0 < \theta^* < 1 \text{ if } \]
\[ R_L < R < R_U \]
if the menu cost is zero. With menu cost, *a fortiori* the interior solution \((t^*_1, t^*_2 > 0)\) implies

\[ R < R_U \]

Thus we obtain

\[ \Delta(t_1 + 1) > \Delta_1 > \Delta(t_1) \]