The Structural Transformation Between Manufacturing and Services and the Decline in U.S. GDP Volatility*

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Abstract

For a single firm with a given volatility of total factor productivity at the gross output level (GTFP), the volatility of total factor productivity at the value added level (YTFP) increases with the share of intermediate goods in gross output. For a Cobb-Douglas production function in capital, labor and intermediate goods, YTFP volatility is equal to GTFP volatility divided by one minus the share of intermediate goods in gross output. In the U.S., this share is steadily around 0.6 for manufacturing and 0.38 for services during the 1960-2005 period. Thus, the same level of GTFP volatility in the two sectors implies a 55% larger YTFP volatility in manufacturing. This fact contributes to the higher measured YTFP volatility in manufacturing with respect to services. It follows that, as the services share in GDP increases from 0.53 in 1960 to 0.71 in 2005 in the U.S., GDP volatility is reduced. I construct a two-sector dynamic general equilibrium input-output model to investigate the role of the sectorial reallocation between manufacturing and services in reducing U.S. GDP volatility. Numerical results for the calibrated model economy suggest that the sectorial reallocation can account for almost a half of the 56% GDP volatility difference between the 1960-1983 and the 1984-2005 periods. When the highly volatile period 1973-1983 is excluded, the sectorial reallocation alone is able to account for the entire difference in GDP volatility between the 1960-1973 and the 1984-2005 periods.

JEL Classification: C67, C68, E25, E32.

Keywords: Volatility Decline, Business Cycle, Structural Change, Total Factor Productivity.

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1 Introduction

There is a large literature documenting the decline in output volatility in the U.S. over time.\(^1\) Several explanations have been advocated to explain this process: improved inventory management techniques, better monetary policy - Leduc and Sill (2007), better financial instruments - Jermann and Quadrini (2006), decline in aggregate TFP volatility - Arias et al. (2007), and structural change between manufacturing and services. In particular, Alcala and Sancho (2003) show that when appropriate chain-weighted index numbers are considered, the increase of the share of services in GDP is responsible for 30% of the reduction in output volatility from 1950 to 2002. However, there has not been any attempt to build a model of the sectorial reallocation between manufacturing and services that can account for the reduction in U.S. output volatility.

In this paper I study the cyclical behavior of the two broad sectors, manufacturing and services, and investigate how the increase in the services share of GDP reduces GDP volatility.\(^2\) Using data for the U.S from Jorgenson Dataset, 2007, I document two facts. First, manufacturing total factor productivity (TFP) volatility at the gross output level is higher than that of services during the period considered, 1960-2005. Second, manufacturing displays a higher share of intermediate goods in gross output with respect to services, 0.6 versus 0.38. As first shown in Bruno (1984) and Baily (1986), for a given volatility of TFP at the gross output level, value added TFP volatility is an increasing function of the share of intermediate goods in gross output.\(^3\) Given the observed difference in the share of intermediate goods, this implies that the same level of TFP volatility in the two sectors at the gross output level results in a TFP volatility at the value added level 55% larger in manufacturing than in services.

\(^1\)See McConnell and Perez-Quiros (2000) and Blanchard and Simon (2001) for instance.
\(^2\)To avoid confusion with sectorial output, in the paper I always refer to aggregate output, also defined as aggregate value added, as GDP.
\(^3\)See also EU KLEMS Growth and Productivity Accounts (2007).
Aggregate TFP depends on sectorial value added TFPs and on the share of each sector in GDP. The share of services in GDP in the U.S. is 0.53 in 1960 and 0.71 in 2005 according to Jorgenson Dataset. A larger share of services reduces aggregate TFP for two reasons. First, services display a smaller TFP volatility at the gross output level with respect to manufacturing so that an increase in the services share of GDP contributes to reduce GDP volatility. Second, the multiplier effect due to the share of intermediate goods in gross output, that is created when gross output TFP volatility is converted into value added TFP volatility, is smaller for services than for manufacturing, so that aggregate TFP is further reduced when the share of services in GDP increases.

I construct a two-sector, input-output, dynamic general equilibrium model to quantify the effect that the structural reallocation between manufacturing and services has on GDP volatility in the U.S. The two sectors, manufacturing and services, produce output using a Cobb-Douglas production function in capital, labor and intermediate goods purchased from the sector itself and from the other sector. Household’s preferences are non-homotetic so that the elasticity of services with respect to income is greater than one and that of manufacturing is smaller than one.\footnote{For a theory of the rise in the service sector see Buera and Kaboski (2008).} I perform linear quadratic (LQ) approximations around three steady states (SS). The first SS is found by calibrating the model so that the share of services in GDP matches that observed in the data in 1960, 0.53. The second steady state is found by increasing gross output TFP in the 1960 SS, by the growth factor observed in the data from 1960 to 2005, for both manufacturing and services. I call this 2005a SS. As income increases because of higher TFP levels in the two sectors, non-homotetic preferences imply that the services share of output enlarges while that of manufacturing shrinks. In the 2005a SS, the share of intermediate goods implied by the model is 0.64, lower than that observed in the data in 2005, 0.71. Finally, I construct a third steady state, 2005b. This is constructed by
increasing gross output TFP in the 1960 SS in the two sectors by a common growth factor, such that the share of services in GDP is equal to 0.71.

I then compute gross output TFP volatility for manufacturing and services during two sub-periods 1960-1983 and 1984-2005. I perform a LQ approximation around the 1960 SS, where the standard deviations of TFP shocks in the two sectors are taken from the data for the 1960-1983 period. Next, I perform the same approximation around the 2005a and 2005b SS, where the standard deviations of the shocks are the ones observed in the data between 1984 and 2005. I find that GDP volatility is 71% larger in the 1960 SS with respect to the 2005a SS and 84% larger with respect to the 2005b SS. In the data, GDP volatility is 56% larger in the 1960-1983 period with respect to the 1984-2005 period. Thus, the model is able to explain the entire decline in GDP volatility observed in the data.

I then perform a decomposition experiment to quantify how much the sectorial reallocation alone is responsible for the GDP volatility decline. To do this, I first compute gross output TFP volatility in manufacturing and services during the whole period considered, 1960-2005. I use these volatilities to perform LQ approximations around the three steady states. Thus, in the three approximations the standard deviation of the sectorial TFP shocks is the same and equal to that observed in the data during the whole period 1960-2005. It follows that any decline in GDP volatility across steady states is due to the change in the weights of the two sectors. GDP volatility is 14% larger in the 1960 SS with respect to the 2005a SS and 24% larger with respect to the 2005b SS. The latter represents almost half of the volatility decline observed in the data between the 1960-1983 and the 1984-2005 periods.

To avoid the additional volatility due to the oil shocks, I repeat the quantitative exercise excluding the 1973-1983 period. I compute the volatility of manufacturing, services and GDP for the period 1960-1973 and compare it with the corresponding volatilities in the 1984-2005 periods.

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5The pre-1983 period and the post-1983 period are often considered in the literature to compare output volatility over time. See Arias et al. (2007), for instance.
period. GDP volatility is 22% higher in the first sub-period compared to the second. Instead GDP volatility in the period 1960-1983 is 56% higher than in the 1984-2005 period. In this way, the model is able to display a GDP volatility difference of 21% between the 1960 and the 2005a steady states and of 30% between the 1960 and the 2005b steady states. Thus, in this case also, the model accounts for the entire volatility decline in GDP.

I repeat the decomposition experiment in which the standard deviation of manufacturing and services does not change across steady states. This is computed from the data using the log-deviations from trend for the period 1960-1973 together with those of the 1984-2005 period. Thus, the reduction in GDP volatility only comes from the sectorial reallocation of manufacturing and services. In this case, the model accounts for a 14% difference between the 1960 and the 2005a steady state and of 22% between the 1960 and the 2005b. Thus, the model is able to account for the entire decline in GDP volatility between the 1960-1973 and the 1984-2005 periods.

The shift away from the manufacturing sector towards the services sector has been suggested as an explanation for lower output volatility in Dalsgaard et al. (2002). They suggest that the smaller share of inventories that services display over output with respect to manufacturing can be a source of smaller GDP volatility. In the model I present there is no inventory, and the reduction in GDP volatility is due to the change in the transmission mechanism of sectorial productivity shocks to the aggregate economy when the share of the services sector share in GDP increases. The sectorial reallocation explanation is not at work in reducing output volatility according to Blanchard and Simon (2001), Stock and Watson (2002) and McConnell and Perez-Quiros (2000). These papers perform simple counterfactual experiments to assess the contribution of structural change on GDP volatility. They construct counterfactual GDP series using the weights of the services sector in GDP in an early period and use the subsequent growth rates of the services sector. They do not find
evidence of an increase in output volatility in this way. Alcala and Sancho (2003) show the limitations of this approach. They show that using appropriate indices with time varying weights the result in McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002) does not hold and find that structural change accounts for 30% of the reduction in output volatility. For this reason, in the quantitative exercises in this paper I use a time-varying Tornqvist index to compute changes in the model’s GDP.

Finally, it is worth noting here that many papers in the literature on volatility decline take a structural break view. These papers consider the reduction in volatility between the pre-1983/84 and the post-1983/84 periods. However, the 1973-1984 period has been characterized by the oil shocks and high volatility of output and prices. When the pre-1973 and the post-1984 periods are considered, the difference in GDP volatility is around 22% instead of the 56% between the 1960-1983 and 1984-2005 periods. As shown in Blanchard and Simon (2001), the reduction in GDP volatility does not occur suddenly between the pre-1983/84 and the post-1983/84 periods, but it is a process that started at least in 1950 and was interrupted in the seventies and mid-eighties. Furthermore, output volatility has decreased across G7 countries.\(^6\) These facts together suggest that changes in policies of a single country, such as changes in monetary policy, cannot represent a satisfactory explanation of the volatility decline. Instead, the sectorial reallocation between manufacturing and services is a feature shared by all industrialized countries.

2 Services and Manufacturing Production Functions

Figure 1 reports the nominal share of intermediate goods in the manufacturing and in the services sector from 1960 to 2005. Given the small long-run variation of nominal input shares, assume that in each sector the representative firm produces output using a Cobb-Douglas

\(^6\)See table 1 in Stock and Watson (2002) for instance.
production function in intermediate goods $M$ and a function of capital and labor $f(K, N)$.\footnote{Assume $f(K, N)$ to be homogeneous of degree one in capital and labor.}

Markets are competitive so the firm takes the price of capital $r$, that of labor $w$, and that of intermediate goods $p$ as given. The maximization problem of the firm is

$$
\max_{K,N,M} \left\{ Af(K, N)^\alpha M^{1-\alpha} - rK - wN - pM \right\}.
$$

(1)

The first order condition of (1) with respect to intermediate goods implies that, at the optimal solution,

$$
M = (1 - \alpha)^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}} A^{\frac{1}{\alpha}} f(K, N).
$$

(2)

Using (2) in (1) I obtain

$$
\max_{K,N} \left\{ \alpha(1 - \alpha)^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}} A^{\frac{1}{\alpha}} f(K, N) - rK - wN \right\}.
$$

(3)

Note that $\alpha(1 - \alpha)^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}} A^{\frac{1}{\alpha}} f(K, N)$ represents the value added production function for the sector considered. Suppose that the relative price of intermediate goods $p$ is fixed.\footnote{On this point, note that in the data most of the intermediate goods used by each sector come from the sector itself. Thus, a large part of the price of intermediate goods for a sector is given by the same sector’s gross output price, implying a stable relative price $p$ over time.}
Changes in productivity at the value added level are driven by $A_t^{\frac{1}{\alpha}}$. In the real business-cycle literature, the volatility of a variable is measured by the standard deviation of the log-deviations of that variable from its Hodrick-Prescott (HP) filter. For a variable $A_t$ and its HP filter $\tilde{A}_t$, the log-deviation $\hat{a}_t$ at time $t$ is given by $\hat{a}_t = \log(A_t) - \log(\tilde{A}_t)$. Instead, for the variable $A_t^{\frac{1}{\alpha}}$ the log-deviation $\hat{a}_t$ at $t$ is given by $\hat{a}_t = (1/\alpha)[\log(A_t) - \log(\tilde{A}_t)]$. It follows that the value of $\alpha$ affects value added volatility through its effect on the sector’s gross output TFP, $A$. Consider the difference between the services sector, where $\alpha = 0.62$ and manufacturing, where $\alpha = 0.40$. Even when $A$ have the same volatility in the two sectors, that is, the same $\hat{a}$, the difference in $\alpha$ across sectors implies a value added TFP volatility in manufacturing 55% larger than in services. Thus, an increasing share of services in GDP represents a source of smaller volatility in the same GDP.\(^9\)

Finally, note that when TFP is measured on a value added basis, the entire term $\alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} p^{\frac{1-\alpha}{\alpha}} A_t^{\frac{1}{\alpha}}$ is captured. If $p$ changes over time, the term $p^{-\frac{1-\alpha}{\alpha}}$ causes a measurement bias when TFP changes are measured directly on a value added basis.\(^{10}\) This fact does not reflect any change in the production technology $Af(K,N)^\alpha M^{1-\alpha}$ so the relevant measure of TFP changes at the value added level is $A_t^{\frac{1}{\alpha}}$. However, $p$ affects value added volatility of the sector considered. In the general equilibrium model presented in section 4, $p$ is endogenously determined in equilibrium and its contribution to volatility is explicitly considered.

### 3 Value Added and Gross Output TFP

In this section I analyze the behavior of TFP in the manufacturing and in the services sectors. Figure 2 reports TFP in the two sectors both at the gross output level - first panel - and at the value added level - second panel. Details of the calculations are given in the Data

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\(^9\)GDP can be computed as a weighted average of the value added of the two sectors in the economy.

\(^{10}\)The distortion generated by $p^{-\frac{1-\alpha}{\alpha}}$ favors the measurement of TFP at the gross output level. See Bruno (1984) for a theoretical and empirical study of effect of the term $p^{-\frac{1-\alpha}{\alpha}}$ on productivity changes.
Appendix. The first panel shows that TFP growth is similar in the two sectors at gross output level, with a growth factor between 1960 and 2005 of 1.33 for manufacturing and 1.31 for services. On the other hand, at the value added level the difference in TFP growth between the two sectors is more evident, with a 2.05 growth factor for manufacturing and a 1.54 for services. As showed in the previous section, when TFP at the gross output level is $A$, TFP at the value added level is $A^{\hat{\alpha}}$, where $\alpha$ is the capital and labor share in gross output, equal to 0.4 for manufacturing and to 0.62 for services. According to this analysis, the higher value added TFP growth commonly measured in manufacturing with respect to services is due to different production technologies at the gross output level - different $\alpha$ -, rather than to different growth rates of $A$.

Thus, economies that are more intensive in manufacturing than in services are likely to display a higher aggregate TFP growth rate with respect to economies that are more intensive in services, even when manufacturing and services TFP at the gross output level display the same growth rate. Echevarria (1997) shows that the transition from manufacturing to services implies a decline in aggregate TFP growth due to the smaller growth rate of services value added TFP with respect to manufacturing. In fact, gross output TFP growth is
similar in the two sectors, but the different intensity of intermediate goods in the production of manufacturing and services determines different growth rates of TFP at the value added level and consequently on aggregate TFP.\footnote{On this point, it is possible to show that the TFP of two countries that have the same supply side model economy - that is, including the same growth rate of gross output TFP growth in manufacturing and services - as the one I present in this paper, can grow at different rates depending on the share of services and manufacturing in GDP. I study the effects of sectorial composition on GDP growth in this sort of models in parallel research.}

The different intensity of intermediate goods in manufacturing and services is also able to rationalize the observation that growth and volatility are usually correlated. Economies whose production is more intensive in manufacturing tend to grow faster and have higher volatility with respect to economies more intensive in services. For a given growth rate and volatility component for TFP at the gross output level, $A$, the corresponding growth rate and volatility component at the value added level are determined by the value of $1/\alpha$. A small $\alpha$ implies both a high growth rate and a high volatility.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Standard deviations of Gross Output TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subperiod</td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.44%</td>
</tr>
<tr>
<td>Services</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

Table 1 reports the cyclical behavior of gross output TFP for manufacturing and services for different sub-periods. Manufacturing is 44% more volatile than services over the whole 1960-2005 period. During the 1960-1973 period TFP volatility is similar in the two sectors, 1.06% and 1.05%, increases for both during the oil-shocks era 1974-1985, 1.92% and 1.35%, and declines afterwards, 1.18% and 0.69%. However, in manufacturing TFP volatility increases more during the oil shocks and does not decline as much as the services sector in the subsequent period. Interestingly, manufacturing TFP volatility is larger in the 1986-2005 period than in the 1960-1973 period, 1.18% versus 1.06%. Finally, I report measures for two sub-periods which are usually considered in the literature to show the reduction in
GDP volatility, before and after 1983. For both sectors TFP volatility is lower in the second period, although the services sector displays a larger decline, 40% instead of 27%.

4 The Model

In this section I specify a two-sectors, input-output, general equilibrium model to quantify the effect of sectorial reallocation on output volatility.

4.1 Firms

There are two sectors in the economy, manufacturing and services. The representative firm in each sector, \(j = m, s\), produces output using a Cobb-Douglas production function in capital, labor, manufactured intermediate goods and intermediate services

\[
G_j = B_j \left( K_j^\alpha N_j^{1-\alpha} \right)^{\nu_j} \left( M_j^{\varepsilon_j} S_j^{1-\varepsilon_j} \right)^{1-\nu_j},
\]

(4)

where \(0 < \alpha < 1\), \(0 < \nu_j < 1\), \(0 < \varepsilon_j < 1\), \(K_j\) and \(N_j\) are the amounts of capital and labor, \(M_j\) the manufactured intermediate good, \(S_j\) is the amount of intermediate services and \(B_j\) is TFP.

To simplify the computation of the solution of the model, I exploit the Cobb-Douglas form of the production function to derive the net production function of each sector, defined as the total output of a sector minus the intermediate good produced and used in the same sector. The net production function for manufacturing is obtained by solving

\[
Y_m = \max_{M_m} \left\{ B_m \left( K_m^\alpha N_m^{1-\alpha} \right)^{\nu_m} \left( M_m^{\varepsilon_m} S_m^{1-\varepsilon_m} \right)^{1-\nu_m} - M_m \right\},
\]

and it is equal to

\[
Y_m = \Omega_m B_m^{\frac{1}{\varepsilon_m (1-\nu_m)}} K_m^{\frac{\alpha}{1-\varepsilon_m (1-\nu_m)}} N_m^{\frac{1}{1-\varepsilon_m (1-\nu_m)}} S_m^{\frac{(1-\varepsilon_m)\nu_m}{1-\varepsilon_m (1-\nu_m)}}
\]

(5)

where \(\Omega_m = [1 - \varepsilon_m (1 - \nu_m)] [\varepsilon_m (1 - \nu_m)]^{\frac{\varepsilon_m (1-\nu_m)}{1-\varepsilon_m (1-\nu_m)}}\). Note that TFP in the net production function \(Y_m\) is a function of gross output TFP, \(B_m\), and it depends on the share of man-
ufactured intermediate goods in total manufacturing gross output, $\varepsilon_m\left(1-\nu_m\right)$. The net production function in the manufacturing sector becomes

$$ Y_{m,t} = A_{m,t} \left(K_{m,t}^{\alpha} N_{m,t}^{1-\alpha}\right)^{\theta} S_t^{1-\theta}, \quad (6) $$

where

$$ A_m = \Omega_m B_m^{1-\varepsilon_m(1-\nu_m)}, \quad (7) $$

$0 < \theta < 1$ is equal to $\frac{\nu_m}{1-\varepsilon_m(1-\nu_m)}$ and $S = S$.\(^{12}\)

The net production function in the services sector is found accordingly and it is given by

$$ Y_{s,t} = A_{s,t} \left[\left(K_t - K_{m,t}\right)^{\alpha}(N_t - N_{m,t})^{1-\alpha}\right]^{\gamma} M_t^{1-\gamma}, \quad (8) $$

where $0 < \gamma < 1$, $K_t$ and $N_t$ are the amount of capital and labor in the economy at time $t$, and $M_t$ is the amount of manufacturing good used as intermediate good in the services sector.

Given the structure of the supply side of this economy, the relative price of services with respect to manufacturing, $p_s/p_m$, is independent of the quantities produced of the two goods. To see this, note that in perfect competition each firm sets the price equal to the marginal cost, that is

$$ p_m = \frac{(r^{\alpha}w^{1-\alpha})^{\theta} p_s^{1-\theta}}{\Phi_1 A_m}, \quad (9) $$

and

$$ p_s = \frac{(r^{\alpha}w^{1-\alpha})^{\gamma} p_m^{1-\gamma}}{\Phi_2 A_s}, \quad (10) $$

where $\Phi_1$ is a function of $\alpha$ and $\theta$ and $\Phi_2$ a function of $\alpha$ and $\gamma$.\(^{13}\) By solving (9) and (10) for $p_m$ and $p_s$ I can write

$$ \frac{p_s}{p_m} = \frac{\left(\Phi_1 A_m\right)^{\gamma+\theta-\theta\gamma}}{\left(\Phi_2 A_s\right)^{\gamma+\theta-\theta\gamma}}, \quad (11) $$

\(^{12}\)The derivation of (6) is important at the moment of calibrating the model.

\(^{13}\) $\Phi_1 = (\alpha\theta)^{\alpha\theta} \left[(1-\alpha)\theta\right]^{\left(1-\alpha\right)\theta} \left(1-\theta\right)^{1-\theta}$ and $\Phi_2 = (\alpha\gamma)^{\alpha\gamma} \left[(1-\alpha)\gamma\right]^{\left(1-\alpha\right)\gamma} \left(1-\gamma\right)^{1-\gamma}$. 

12
Thus, the relative price of the two goods is technologically determined, i.e., (11) represents the technological rate of transformation between the two goods. It depends only on the parameters of the production functions and not on the quantity produced in the two sectors. This result depends on the input-output structure of the model together with the same capital and labor aggregator for the two firms, $K_j^\alpha N_j^{1-\alpha}$, with $j = m, s$.

In general equilibrium, the representative household needs to know aggregate resources to take consumption and investment decisions. The aggregate resources available for consumption and investment purposes at time $t$ can be found by defining an aggregate production function. An aggregate production function for this economy can be obtained by solving the following maximization problem

$$\max_{K_{m,t},N_{m,t},M_t,S_t} \left[ A_{m,t} \left( K_{m,t}^{\alpha} N_{m,t}^{1-\alpha} \right)^\theta S_t^{1-\theta} - M_t \right]$$

subject to

$$A_{s,t} \left[ (K_t - K_{m,t})^\alpha (N_t - N_{m,t})^{1-\alpha} \right]^\gamma M_t^{1-\gamma} = S_t.$$

The solution to problem (12) gives the maximum amount of manufacturing that can be produced in the economy when $C_s = 0$, that is, when the services sector serves only as an intermediate sector. The solution to problem (12) is

$$V_{m,t} = \Phi_3 A_{m,t}^{\frac{1}{\gamma + \theta - \gamma \theta}} A_{s,t}^{\frac{1-\gamma}{\gamma + \theta - \gamma \theta}} K_t^\alpha N_t^{1-\alpha},$$

where $\Phi_3$ is a function of $\gamma$ and $\theta$.\textsuperscript{14} By dividing (13) by (11) it is possible to obtain the maximum amount of services that can be produced when the manufacturing sector produces only intermediate goods

$$V_{s,t} = \Phi_4 A_{m,t}^{\frac{1}{\gamma + \theta - \gamma \theta}} A_{s,t}^{\frac{1-\gamma}{\gamma + \theta - \gamma \theta}} K_t^\alpha N_t^{1-\alpha},$$

where $\Phi_4$ is a function of $\gamma$ and $\theta$.\textsuperscript{15}

\textsuperscript{14}$\Phi_3 = \frac{1}{\gamma + \theta - \gamma \theta} \left( \frac{1}{\gamma + \theta - \gamma \theta} \right)^{\frac{\theta}{\gamma + \theta - \gamma \theta}} \left( \frac{\gamma + \theta - \gamma \theta}{\gamma + \theta - \gamma \theta} \right)^{\frac{1-\gamma}{\gamma + \theta - \gamma \theta}}$

\textsuperscript{15}$\Phi_4 = \Phi_3 / \left( \frac{\gamma + \theta - \gamma \theta}{\gamma + \theta - \gamma \theta} \right)$. 

13
Using the definition of $A_m$ given by (7) and the corresponding definition for $A_s$, (13) becomes

$$V_{m,t} = \Phi_3 \Omega_1 \frac{1}{\gamma + \theta - \varphi} \Omega_3 \frac{1}{\gamma + \theta - \varphi} B_{m,t}^{1-\epsilon_m(1-\nu_m)} \frac{1}{\gamma + \theta - \varphi} B_{s,t}^{1-\epsilon_s(1-\nu_s)} \frac{1}{\gamma + \theta - \varphi} K_t^\alpha N_t^{1-\alpha},$$

and (14) becomes

$$V_{s,t} = \Phi_4 \Omega_1 \frac{1}{\gamma + \theta - \varphi} \Omega_3 \frac{1}{\gamma + \theta - \varphi} B_{m,t}^{1-\epsilon_m(1-\nu_m)} \frac{1}{\gamma + \theta - \varphi} B_{s,t}^{1-\epsilon_s(1-\nu_s)} \frac{1}{\gamma + \theta - \varphi} K_t^\alpha N_t^{1-\alpha}.$$  

Equations (15) and (16) represent the economy’s value added in two extreme cases, one in which only manufacturing is consumed - or invested - and services is an intermediate sector, and another in which the opposite situation holds. Note that TFP in the two cases is different and TFP volatility also. To see this, I calculate the exponent of $B_m$ and $B_s$ in (15) and (16) using Jorgenson 2007 data. The values are reported in Table 2.

<table>
<thead>
<tr>
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<th>Table 2</th>
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<tbody>
<tr>
<td>$B_{m,t}$ in $V_{m,t}$</td>
<td>$\frac{1}{\gamma + \theta - \varphi}$ $\frac{1}{\gamma + \theta - \varphi}$ $\epsilon_m(1-\nu_m)$ $1.82$</td>
</tr>
<tr>
<td>$B_{s,t}$ in $V_{m,t}$</td>
<td>$\frac{1}{\gamma + \theta - \varphi}$ $\frac{1}{\gamma + \theta - \varphi}$ $\epsilon_s(1-\nu_s)$ $0.44$</td>
</tr>
<tr>
<td>$B_{m,t}$ in $V_{s,t}$</td>
<td>$\frac{1}{\gamma + \theta - \varphi}$ $\frac{1}{\gamma + \theta - \varphi}$ $\epsilon_m(1-\nu_m)$ $0.28$</td>
</tr>
<tr>
<td>$B_{s,t}$ in $V_{s,t}$</td>
<td>$\frac{1}{\gamma + \theta - \varphi}$ $\frac{1}{\gamma + \theta - \varphi}$ $\epsilon_s(1-\nu_s)$ $1.44$</td>
</tr>
</tbody>
</table>

Assume that the productivity shock at the firm level is the same for both firms, that is, $B_m = B_s = B$. The exponent of $B$ becomes 2.26 in (15) and 1.72 in (16), with a difference between the two cases of 31%. That is, TFP volatility depends on the units output is measured in. Output composition changes over time and becomes more intensive in services with respect to manufacturing. The calculation above suggests that the structural change that occurs between manufacturing and services can account for a part of the reduced output volatility in the U.S.

### 4.2 Households

The model economy is inhabited by a large number of identical households. Households in this economy have preferences over manufacturing and services. The consumption index at
date $t$ is given by

$$c_t = \left[ b c_{m,t}^\rho + (1 - b) (c_{s,t} + \bar{s})^\rho \right]^\frac{1}{\rho}, \quad (17)$$

with $\bar{s} > 0$, $\rho < 1$ and $0 < b < 1$, where $c_{m,t}$ and $c_{s,t}$ are the per capita consumption levels in manufacturing and services. The parameter $\bar{s}$ is interpreted as home production of services.\footnote{See, for instance, Kongsamut, Rebelo and Xie (2001) for this interpretation.} The utility function in (17) displays an income elasticity of demand smaller than one for manufacturing and larger than one for services. As income increases the expenditure share on services increases with respect to that of manufacturing.

### 4.3 The Planner’s Problem

Given the demand and the supply side of the economy I can state the problem of a benevolent social planner that maximizes the discounted lifetime utility of the representative household. A benevolent social planner in this economy solves the following stochastic recursive problem

$$V(k, z_m, z_s) = \max_{c_{m,t}, c_{s,t}, k} \{ \log(c) + \beta E[V(k', z_{m}', z_{s}')|z_m, z_s] \} \quad (18)$$

subject to

$$c = \left[ b c_{m}^\rho + (1 - b) (c_{s} + \bar{s})^\rho \right]^\frac{1}{\rho}, \quad (19)$$

$$\phi c_s + c_m + k' - (1 - \delta)k = y_m(B_m, B_s, k), \quad (20)$$

$$\phi = \left( \Phi_1 \Omega_m B_m^{\frac{1}{1-\epsilon_m}(1-\sigma_m)} \right)^{\frac{\gamma}{\gamma + \beta - \sigma_m}}, \quad (21)$$

$$B_m = \bar{B}_m e^{z_m} \quad \text{and} \quad B_s = \bar{B}_s e^{z_s}, \quad (22)$$

$$z_{m}' = \rho_z z_m + \epsilon_{m}', \quad \text{with} \ \epsilon_m \sim N(0, \sigma_m^2), \quad (23)$$

$$z_{s}' = \rho_z z_s + \epsilon_{s}', \quad \text{with} \ \epsilon_s \sim N(0, \sigma_s^2), \quad (24)$$
where the prime indicates the value of a variable in the next period and $E$ is the expectations operator. Manufacturing is the numeraire of the economy, $c_s$, $c_m$, and $k' - (1 - \delta)k$ are consumption in the services sector, in the manufacturing sector and investment - which is a manufacturing good - all in per capita terms. The aggregate production function, expressed in units of manufacturing and in per capita terms, $y_m(B_m, B_s, k)$, is given by (15) divided by $N$. The rate of technological transformation between manufacturing and services, $\phi$, is given by (11).\footnote{This is an economy without distortions so the first and the second welfare theorems hold. Thus, the relative price of services with respect to manufacturing in the competitive equilibrium, $p_s$, is equal to the rate of technological transformation between services and manufacturing in the centralized economy, $\phi$.} Total factor productivity at the gross output level in each sector, $B_m$ and $B_s$, is the product of a level component, $\bar{B}_m$ and $\bar{B}_s$ respectively, and a cycle component, $e^{z_m}$ and $e^{z_s}$, respectively. Each period, a shock affects the cycle component of each sector’s total factor productivity. The process for this shock is $z'_m = \rho_z z_m + \epsilon'_m$ for manufacturing and $z'_s = \rho_z z_s + \epsilon'_s$ for services. The shocks $\epsilon_m$ and $\epsilon_s$ are i.i.d. over time with zero mean.

### 5 Strategy and Results

The purpose of the paper is to quantify the role of structural change in reducing GDP volatility in the U.S. In this country, the share of services in consumption is around 0.53 in 1960 and 0.71 in 2005. Figure 3 reports the pattern of this share during the period 1960-2005. As services is the less volatile component of GDP, as showed in the previous sections, an increase in the services share of GDP tends to reduce GDP volatility.

A steady state for the economy presented in the previous section exists.\footnote{See the Appendix for the steady state derivation.} To assess the contribution of the structural change to the GDP volatility decline I compare GDP volatility in different steady states.\footnote{The two-sector model presented in this paper does not display a balanced growth path (BGP). In general, multi-sector growth models do not display a BGP, unless under restrictive assumptions on the utility or the production functions, as in Kongsamut, Rebelo and Xie (2001) and Ngai and Pissarides (2007). The latter authors provide conditions for BGP existence in a model with structural change and intermediate goods. In the class of models they consider, all sectors have the same Cobb-Douglas technology in capital,} The data I use are from Jorgenson dataset, 2007. The
parameters defining the elasticity of output with respect to inputs in the production functions (4), (6) and (8) are easily derived from the data given the Cobb-Douglas assumption. The depreciation rate \( \delta = 0.08 \), and the subjective discount factor \( \beta = 0.96 \), are common values in the literature. The autoregressive parameter \( \rho_z = 0.95 \) is set as in Cooley and Prescott (1995). The parameter governing the elasticity of substitution between manufacturing and services, \( \rho = -2 \), is consistent with the value used in Rogerson (2008) and Ngai and Pissarides (2007). The value of the level of gross output productivity in the services sector in 1960, \( \bar{B}_s \), is normalized to one.

There are three parameters left, \( \bar{s}, b, \) and \( B_m \). Two targets used to calibrate these parameter are the services share in GDP in the data in 1960, 0.53, and a share of market services in total consumption of services in 1960 of 15%. The third target is the share of services in GDP obtained when \( \bar{B}_m \) and \( \bar{B}_s \) are raised by the growth factor observed in the data between 1960 and 2005. This growth factor is 1.33 for manufacturing and 1.31 for labor and intermediate goods, and different TFP levels. The model presented here does not possess these characteristics, as the production functions of the two sectors differ both in the elasticity parameters and in the TFP levels. As a BGP for this economy does not exist I perform steady state comparisons.
services. In the data, the share of services in GDP in 2005 is 0.71. However, the maximum share of services in GDP that the model can generate is 0.64.

Thus, with the basic calibration I construct two steady states. I label the first 1960 steady state. This is obtained by solving the model with the parameters reported in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Share of $K$ in value added</td>
<td>0.34</td>
<td>Data</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>Share of $K_m$ and $N_m$ in $G_m$</td>
<td>0.40</td>
<td>Data</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Share of $M$ in manufac. interm.</td>
<td>0.70</td>
<td>Data</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Share of $K_m$ and $N_m$ in $Y_m$</td>
<td>0.70</td>
<td>Data</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>Share of $K_s$ and $N_s$ in $G_s$</td>
<td>0.62</td>
<td>Data</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Share of $S$ in services interm.</td>
<td>0.70</td>
<td>Data</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share of $K_s$ and $N_s$ in $Y_s$</td>
<td>0.84</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.08</td>
<td>Literature</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subject. discount rate</td>
<td>0.96</td>
<td>Literature</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elasticity between manuf. and serv.</td>
<td>-2</td>
<td>Literature</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autoregressive parameter</td>
<td>0.95</td>
<td>Literature</td>
</tr>
<tr>
<td>$\bar{B}_s$</td>
<td>Initial GTFP level in services</td>
<td>1</td>
<td>Normaliz.</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Home production of services</td>
<td>5.5</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$b$</td>
<td>Weight of manuf. in preferences</td>
<td>0.12</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\bar{B}_m$</td>
<td>Initial GTFP level in manufacturing</td>
<td>10.5</td>
<td>Calibrated</td>
</tr>
</tbody>
</table>

The second steady state is constructed by increasing $\bar{B}_m$ and $\bar{B}_s$ in the 1960 steady state, 10.5 and 1, by the growth factor observed in the data between 1960 and 2005 in the two sectors, 1.33 and 1.31 respectively. The implied share of services in GDP is 0.64 and I label this the 2005a steady state. Finally, I construct a third steady state, 2005b, by increasing $\bar{B}_m$ and $\bar{B}_s$ in the 1960 steady state, 10.5 and 1, by a common factor that makes the share of services in GDP be 0.71.\(^{20}\)

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviations of GDP</td>
</tr>
<tr>
<td>1960-2005</td>
</tr>
<tr>
<td>1960-1983</td>
</tr>
<tr>
<td>1984-2005</td>
</tr>
</tbody>
</table>

\(^{20}\)This common factor is 2.
In table 4 I report the standard deviation of GDP for the sub-periods 1960-1983 and 1984-2005 and for the whole period.\textsuperscript{21} The standard deviation in the first sub-period is 56\% higher than in the second sub-period. This confirms the general result encountered in the literature of a large decline in volatility between the two sub-periods.

In the model, GDP is computed as a Tornqvist index of the growth rates of manufacturing and services value added, and it is set to one in the first period of each simulation. For each steady state, I perform the LQ approximation and I simulate 1000 times the economy for 120 years, where the model period is one year. The Hodrick-Prescott parameter is $\lambda = 100$, consistent with yearly data.

In Table 5, I report the average results over the 1000 simulated economies for the 1960 steady state.\textsuperscript{22} The volatility of the error terms for the stochastic processes of the two sectors is given by $\sigma_m = 1.53\%$ and $\sigma_s = 1.12\%$, which are computed from the data for the 1960-1983 period. The first entry in table 5 is value added volatility when expressed in manufacturing terms, that is when output is expressed by (15).

The second entry is value added volatility when expressed in terms of services, as by (16). In the first case, output volatility is 65\% larger. The numerical analysis confirms that the measurement units of output are crucial in determining the volatility level. However, GDP is a composite of both manufacturing and services, so none of the standard deviations of $V_m$ and $V_s$ reported in table 5 corresponds to the standard deviation of aggregate value added

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{S.D.} & \textbf{V_m} & \textbf{V_s} & \textbf{Ratio} \\
\hline
& 2.74\% & 1.66\% & 1.65 \\
& (0.29\%) & (0.17\%) & (1.71) \\
\hline
\end{tabular}
\caption{Standard Deviations of the Simulated Economy}
\end{table}

\textsuperscript{21}Standard deviations are computed as in standard business cycle exercises. The Hodrick-Prescott parameter used to filter the series is $\lambda = 100$, consistent with annual data.

\textsuperscript{22}For the methodology to perform linear quadratic approximations I follow Diaz-Gimenez (1999).
Table 6 reports the comparison across the three steady state constructed, 1960, 2005a and 2005b. The first row reports the 1960 steady state. The standard deviations of the error terms $\epsilon_m$ and $\epsilon_s$ are calibrated using the data for the 1960-1983 period, $\sigma_m = 1.53\%$ and $\sigma_s = 1.12\%$. The share of services in GDP is 0.52 in the model versus 0.53 in the data. The standard deviation of GDP is 1.93\% while it is 2.23\% in the data. The second row reports the 2005a steady state, where TFP has increased by 1.33 for manufacturing and by 1.31 for services with respect to the 1960 steady state. Furthermore, the standard deviations of the errors $\epsilon_m$ and $\epsilon_s$ are computed using data from the 1984-2005 sector. The share of services in GDP increases to 0.64 from 0.52 of the 1960 steady state. However, it does not reach the 0.71 observed in the data in 2005. The standard deviation of GDP is 1.13\% compared to a 1.43\% in the data. Thus GDP volatility is 71\% higher in the first steady state in the model while in the data it is 56\% higher in the 1960-1983 period with respect to the 1984-2005 one. The third row of table 6 reports the 2005b steady state. This is constructed by increasing TFP in manufacturing and services to reach a 0.71 share of services in GDP. The TFP growth factor of the two sectors with respect to the 1960 steady state is 2. In this case GDP volatility is even smaller with respect to the 1960 steady state with a 84\% difference between the latter and the former. The model is then able to replicate the entire decline in GDP volatility in the data.

However, the results in table 6 derive from two effects. One is that TFP declines both
in manufacturing and services between the two periods 1960-1983 and 1984-2005. And the second is that the services share in GDP increases. As services are the less volatile component of GDP, an increase in the services share reduces GDP volatility. To quantify the effect of sectorial reallocation I perform a decomposition experiment. I run LQ approximations around the three steady states by leaving unchanged the standard deviation of the errors $\epsilon_m$ and $\epsilon_s$. In this way, the reduction in GDP volatility across steady states comes only from the sectorial reallocation between services and manufacturing. 

The first row of table 7 reports the 1960 steady state in the decomposition experiment. GDP volatility is 1.73% in the model compared to a 2.23% in the data. The second row reports the steady state 2005a. The volatility of GDP is 1.52%. The ratio of volatility in the 1960 over the 2005 steady states is 1.14. The third row of table 7 reports the 2005b steady state. GDP volatility is 1.39%. Thus, volatility in the 1960 steady state is 24% higher than in the 2005b steady state. The difference in volatility among the three steady states comes only from the change in the composition of output. In the 2005a steady state the share of services in GDP is 0.64. In the 2005b steady state the share is 0.71 as in the data, so the effect on volatility is higher than in the 2005a steady state.

The results encountered confirm that structural change contributes to the decline in output volatility in the U.S. Structural reallocation is able to explain a volatility 14% larger in the 1960 with respect to the 2005a steady state and 24% larger in the 1960 with respect to the 2005b steady state.
6 Oil Shocks and Volatility

The period going from 1973 to 1983 has been characterized by high macroeconomic instability. High rates of inflation and the slowdown in productivity growth created shocks which are absent in the rest of the 1960-2005 period. For this reason I perform the quantitative analysis of the previous section disregarding the period 1973-1983 when computing volatility.

Table 8 reports the comparison across the three steady state. The difference with table 6 lies in the first row. The standard deviation $\sigma_m$ and $\sigma_s$ are computed from the data for the period 1960-1973, whereas in table 6 the corresponding value was computed for the period 1960-1983. The standard deviation of GDP in the data is also computed for the period 1960-1983. The rest of the table is the same as in table 6 except for the last two columns. The standard deviation of GDP is only 22% higher in the 1960-1973 period with respect to the 1984-2005 one. Furthermore, the standard deviation of $\sigma_m$ is virtually the same in the two sub-periods. It follows that the decline in GDP volatility in the model is due to a reduction in $\sigma_s$, from 0.8 to 0.69, and to the sectorial reallocation between manufacturing and services. The model can explain a 21% difference between the 1960 and the 2005a steady state and and 30% difference between the 1960 and the 2005b steady state. As in table 6, the model is able to explain the entire decline in GDP volatility observed in the data.

Table 9 reports the decomposition experiment when the 1973-1983 period is excluded from the computation of standard deviations. In this case $\sigma_m$ and $\sigma_s$ are computed for the
period 1960-1972 together with the 1984-2005 period. As in table 7, the reduction in GDP volatility across steady states is due only to the sectorial reallocation between manufacturing and services, as $\sigma_m$ and $\sigma_s$ are constant. The model is able to explain a difference in GDP volatility of 14% between the 1960 and the 2005a steady state and a 22% difference between the 1960 and the 2005b steady state. This is exactly the GDP volatility difference observed in the data between the 1960-1973 and the 1984-2005 periods. Thus, when the highly volatile subperiod 1973-1983 is excluded from volatility computation the structural reallocation alone is able to explain the reduction in GDP volatility observed in the U.S.

## 7 Conclusions

The structural transformation between manufacturing and services in modern economies is a well established fact. At the same time, the reduction in GDP volatility has not occurred at once, but appears to be a continuous process. In this paper I show that the same TFP volatility at the gross output level for the manufacturing and the services sector delivers a difference in TFP volatility at the value added level of 55% between the two sectors. The reason lies in the manufacturing technology at the gross output level, more intensive in intermediate goods than services. It follows that a decline in gross output TFP volatility in the services sector contributes more to reduce GDP volatility than the same decline in manufacturing. The quantitative analysis suggests that the sectorial reallocation can

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23 This means that the log-deviations from the HP filter for the years 1973-1983 are excluded from the computation of $\sigma_m$ and $\sigma_s$. 

23
account for a difference up to 24% between volatility in the 1960-1983 and 1984-2005 periods, compared to a 56% difference in the data. When the highly volatile period 1973-1983 is excluded, the sectorial reallocation is able to explain the entire decline in GDP volatility observed between the 1960-1973 and the 1984-2005 periods.

The different intensity of intermediate goods in the production of manufacturing and services have also important implications for the growth rate of an economy. Echevarria (1997) shows that the sectorial reallocation between manufacturing and services implies a decline in aggregate TFP growth because services display a lower valued added TFP growth with respect to manufacturing. The analysis presented in this paper supports this finding, and I show that the difference in TFP growth rates at the value added level is due to the different intensity of intermediate goods in the technology of the two sectors, rather than to a different TFP growth rate at the gross output level. This fact is important because it implies that although gross output TFP grows at the same rate in the two sectors, aggregate TFP is not uniquely determined but depends on the weights of the two sectors in GDP. It follows that the demand side, which determines the size of the two sectors in the economy through preferences, affects aggregate TFP. I study the implications of sectorial reallocation on growth in parallel research.
8 Data Appendix

All data except the GDP series are from Jorgenson Dataset, 2007.\textsuperscript{24} The series for GDP is the annual Real GDP series from the Federal Reserve Bank of St. Louis.\textsuperscript{25}

Jorgenson dataset, 2007, provides data for 35 sectors from 1960 to 2005. It reports, for each sector, the value and the price of output and the value and the price of capital, labor and 35 intermediate goods coming from each of the 35 sectors. Values are in millions of current dollars and prices are normalized to 1 in 1996. Variables are defined as: $q_k = \text{quantity of capital services}$, $p_k = \text{price of capital services}$, $q_l = \text{quantity of labor inputs}$, $p_l = \text{price of labor inputs}$, $q_{m,j} = \text{quantity of intermediate goods inputs from sector j}$ and $p_{m,j} = \text{price of intermediate goods inputs from sector j}$. For gross output, $p^p$ is the price of output that producers receive, and $q$ the quantity of gross output. Thus, $q = (q_k p_k + q_l p_l + q_{m,j} p_{m,j})/p^p$, where $q_m$ is an index of individual $q_{m,j}$ and $p_m$ is an index of individual $p_{m,j}$.


\textsuperscript{24}Downloadable at http://www.economics.harvard.edu/faculty/jorgenson.

\textsuperscript{25}Downloadable at http://research.stlouisfed.org/fred2/categories/106
33) Finance, insurance and real estate, 34) Personal and business services, 35) Government enterprises.

I construct indices of gross output, capital, labor and intermediate goods for both manufacturing and services. Gross output for each sector is constructed using chain-weighted Fisher indices. The aggregate labor series in each broad sector, manufacturing and services, is computed as

$$\Delta \ln N_t = \sum_{j=1}^{I} \chi_{j}^{n} \Delta \ln N_{jt},$$  \hspace{1cm} (25)$$

where each $\Delta \ln N_{jt}$ is the growth rate of the labor index in sector $j$ at $t$. $I = 27$ for manufacturing and $I = 8$ for services. The weight $\chi_{j}^{n}$ represents the average of the previous and current period share of labor compensation of sector $j$ in total labor compensation of the broad sector - manufacturing or services.

The aggregate capital series in each broad sector, manufacturing and services, is computed as

$$\Delta \ln K_t = \sum_{j=1}^{I} \chi_{j}^{k} \Delta \ln K_{jt},$$  \hspace{1cm} (26)$$

where each $\Delta \ln K_{jt}$ is the growth rate of the capital index in sector $j$ at $t$. $I = 27$ for manufacturing and $I = 8$ for services. The weight $\chi_{j}^{k}$ represents the average of the previous and current period share of capital compensation of sector $j$ in total capital compensation of the broad sector - manufacturing or services.

The index of intermediate goods in manufacturing and services is constructed in the following way. In a first step, I construct for each of the 35 sectors, chain-weighted Fisher price and quantity indices of the intermediate goods used. In a second step, I construct chain-weighted Fisher quantity indices for manufacturing and services from individual sectors quantity and price indices constructed in the first step.

\hspace{1cm} \textsuperscript{26}This type of index is suggested by the U.S. National Product and Income Accounts (NIPA) to construct real value added. See Bureau of Economic Analysis (2006) for details.
Gross output TFP in manufacturing and services is constructed in each period as

$$TFP_{GO}^i = \frac{G_i}{(K_i^{\alpha_i} N_i^{1-\alpha_i})^{\nu_i} (R_i)^{1-\nu_i}},$$

where, $i = \text{manufacturing, services}$, $G_i$ is the gross output quantity index for sector $i$, $K_i$, $N_i$, and $R_i$ are the capital, labor and intermediate goods indices, $1 - \nu_i$ is the intermediate goods share in gross output and $\alpha_i$ is the capital share of value added for sector $i$. Value added TFP is constructed as

$$TFP_{VA}^i = (TFP_{GO}^i)^{\frac{1}{\nu_i}}.$$

### 9 Appendix: Non-Stochastic Steady State

To derive the non-stochastic steady state of the model, I write the deterministic version of the planner’s problem

$$\max_{c_{m,t},c_{s,t}} \sum_{t=0}^{\infty} \beta^t \log \left\{ [bc_{m,t}^\rho + (1-b)(c_{s,t} + \bar{s})^\rho]^\frac{1}{\rho} \right\}$$

subject to

$$\phi_t c_{s,t} + c_{m,t} + k_{t+1} - (1 - \delta)k_t = \frac{V_{m,t}}{N},$$

$$\phi_t = \left( \Phi_1 \Omega_m B_{m,t}^{1-\epsilon_m(1-\nu_m)} \right)^\gamma \frac{\gamma}{\gamma + \theta + \theta_{\gamma}}.$$  

Here, the aggregate production function in manufacturing terms, $V_{m,t} = \Theta K_i^{\alpha_i} N_i^{1-\alpha_i}$, is the same function given in (15) with

$$\Theta = \Phi_3 \Omega_m \frac{1}{\gamma + \theta + \theta_{\gamma}} \Omega_s \frac{1-\theta}{\gamma + \theta - \theta_{\gamma}} B_{m,t} \frac{1}{\gamma + \theta - \theta_{\gamma}} B_{s,t} \frac{1}{\gamma + \theta + \theta_{\gamma}}.$$  

The first order conditions of (27) with respect to $c_{s,t}$, $c_{m,t}$ and $k_{t+1}$ deliver the following two conditions

$$c_{s,t} = \frac{c_{m,t}}{\phi_t^{1/(1-\rho)}} \left( \frac{1-b}{b} \right)^{1/(1-\rho)} - \bar{s}$$

(28)
and 

\[
\frac{1}{\beta} \left[ \beta c_{m,t}^{\rho - 1} c_{m,t+1}^{\rho - 1} + (1 - b) (c_{s,t+1} + \bar{s})^{\rho} \right] = \alpha \Theta k_{t}^{\alpha - 1} + 1 - \delta
\]  

(29)

In steady state \( c_{m,t} = c_{m}, \ c_{s,t} = c_{s}, \ k_{t} = k \) and \( \phi_{t} = \phi \) so (29) becomes

\[
\frac{1}{\beta} = \alpha \Theta k^{\alpha - 1} + 1 - \delta,
\]

which can be solved for the steady state per capita capital level

\[
k = \left( \frac{\alpha \Theta}{1/\beta - 1 + \delta} \right)^{\frac{1}{1 - \alpha}}. \tag{30}\]

By using (30) in the production function \( V_{m,t}, \) I am able to obtain the per-capita steady state value added (GDP) in terms of manufacturing

\[
V_{m}/N = \Theta \left( \frac{\alpha \Theta}{1/\beta - 1 + \delta} \right)^{\frac{\alpha}{1 - \alpha}}.
\]

Finally, in steady state, \( k_{t+1} - (1 - \delta)k_{t} = k - (1 - \delta)k = \delta k. \) The budget constraint becomes

\[
\phi c_{s} + c_{m} + \delta k = V_{m}/N. \tag{31}\]

By using (28) in (31) I obtain

\[
c_{m} = \frac{V_{m}/N - \delta k + \phi \bar{s}}{\phi^{-\frac{1}{1 - \rho}} \left( \frac{1 - b}{b} \right)^{\frac{1}{1 - \rho}} + 1}, \tag{32}\]

and using (32) in (28) I obtain the steady state level of \( c_{s}. \)
References


