Long-Term Asset Price Volatility and Macroeconomic Fluctuations*

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Price markups appear to be highly correlated with stock market values, whereas other financial measures of profitability exhibit much less variability and are weakly correlated with stock values. We study a variant of the neoclassical growth model that highlights the role of innovation, price markups, and leverage as main determinants of stock market volatility. The model confers a rather limited role to other macroeconomic forces such as TFP shocks, adjustment costs, interest rate policies, input costs, taxes, and labor and financial frictions. We develop some numerical methods to provide a variance decomposition of stock market values.

Keywords: Technological innovations, stock market, price markups, leverage, technology shocks, taxes, labor and financial frictions.

JEL Classification Numbers: E44, E32, G12.

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1 Introduction

In this paper we are concerned with the main macroeconomic determinants of long-term asset price volatility. Although in recent times a burgeoning literature has emerged in the frontier between macroeconomics and finance, there is no general consensus on the main driving forces of stock market prices – which remain a puzzle to economists. Furthermore, although there is an extensive empirical literature that uses isolated data to decompose the variance of stock values into the fundamental factors and expectations of future prices, there seems to be a shortage of numerical methods to study these sources of volatility. The numerical approximation can exploit all the information provided by a full-fledged model. This avoids small sample problems as well as the evaluation of the asset price over a single sample path. We study a general equilibrium model in which asset prices may be driven by technological innovations, product price markups, and TFP shocks. This formulation is quite flexible. Indeed, to check for the robustness of our results, we will allow for more general preferences, and other factors affecting the economy. Since our asset pricing equation does not admit a closed-form solution, we propose some numerical methods to perform a variance decomposition of the stock market value.

Three main issues have mostly been considered in the study of long-term asset market price volatility. First, as discussed below,[1] fundamental variables such as dividends and broader financial measures of profitability exhibit relatively low variability; further, these measures are weakly correlated with changes in stock market prices over the various time windows. We find that various measures of product price markups are better predictors of stock market price movements. These price markups may originate as a result of product innovations, patents, or monopoly power from market frictions and scarcity.

Second, asset market volatility is also disconnected from macroeconomic uncertainty of the real economy. Again, real economic aggregates exhibit relatively low variability, and are

[1] See LeRoy and Porter (1981), Shiller (1981), Campbell and Shiller (1988, 1989), and Cochrane (1992). Some authors appear to be slightly positive on the correlation of stock prices with fundamental values; see Barsky and De Long (1993), Donaldson and Kamstra (1996), Hall (2001), Larrain and Yogo (2003), and Boldrin and Peralta-Alva (2009).

weakly correlated with stock market values. Dynamic equilibrium models have been fairly successful in accounting for comovements of real economic aggregates but have failed to offer plausible explanations for the volatility of stock values based on the variability of economic fundamentals. In the neoclassical growth model, changes in total factor productivity (TFP), the relative price of capital, taxes, and frictions in labor and capital markets hardly generate any volatility of stock values [see Rouwenhorst (1995) for some numerical exercises]. Indeed, all these variables do not affect significantly the volatility and persistence of dividends and earnings under observable variations in consumption. Viewed in another way, capital is the only asset in the economy, and investment must fluctuate enormously to get desirable levels of volatility in stock values. The model's performance can be improved with adjustment costs [Christiano and Fisher (2003), and Jermann (1989)], but these costs need to be implausibly high to attain reasonable levels of volatility in capital stock values.

And third, asset market price volatility could be disconnected from other financial variables. Risk-free or uncontingent interest rates do not fluctuate as much as as stock market returns. Hence, we can observe long-time episodes such as decades with pronounced positive or negative equity premia. A great deal of research has focused on the equity premium [e.g., Bansal and Yaron (2004), Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), Guvenen (2009) and Jinnai (2009)], but further progress on the equity premium may require a better understanding of the volatility of stock market prices.

Our empirical analysis considers publicly and privately held corporations in an effort to capture potential impacts of recently founded corporations.[2] We shall focus on the market value of corporations (MVC): The sum of the market value of corporate equity and the book value of net debt. The joint consideration of equity and debt is convenient in our model and avoids the introduction of arbitrary corporate debt policies and dividends. As documented in Hall (2001), pay-outs to debt holders have been fairly erratic in recent decades.

Our model is a simplified variant of those in Romer (1990) and Comin and Gertler (2006), but our objectives are quite different. Romer (1990) is concerned with innovations and eco-

[2]In other words, we include companies since its foundation date; e.g., Google was founded in 1998 and went public in August 2004. Hence, it is included in our data since 1998.

nomic growth, and Comin and Gertler (2006) with a quantitative analysis of real economic fluctuations.[3] Technological innovations arrive exogenously to the economy. These innovations, however, cannot be readily put into use and undergo a process of adoption embedded in the production of new varieties of intermediate goods. Asset prices incorporate the option value of technological innovations that remain to be adopted. We decompose the value of the stock market into the value of installed capital, the value of technology goods, and the option value of adopting present and future innovations. Then, episodes of technology innovation, expected shocks to price markups and to real and financial variables may generate sudden fluctuations in the aggregate value of stocks. This propagation mechanism is somewhat present in the partial equilibrium setting of Abel and Eberly (2005), in the tree economy of Gârleanu, Panageas and Yu (2009), and in the learning model of Pástor and Veronesi (2009). In all these papers the value of the firm may differ from the replacement value of the stock of capital. In contrast to these authors, we carry out a quantitative general equilibrium analysis of the volatility of asset prices along with other macroeconomic fluctuations. Hence, the challenge for our model is to generate observed levels of volatility in stock markets while preserving the less pronounced volatility of real macroeconomic aggregates.

Our model can account for a sizable part of the volatility of stock market values. We also attest that this asset price volatility in the model stems from expectations of future asset prices rather than dividend growth. As in Greenwood and Jovanovic (1999) not all technological innovations will increase stock market prices, since the arrival of new technologies will depreciate the value of existing ones. To affect positively the stock market, technologies must command higher price markups. Apart from price markups, we also perform a quantitative study of the effects of various macroeconomic variables. A notable feature of these numerical exercises is that adjustment costs for capital investment, interest rate policies, taxes, changes in input prices, and labor and financial frictions may only have a significative impact on the volatility of asset values at the expense of implausible fluctuations in some

[3]In a later paper, Comin, Gertler and Santacreu (2009) build their analysis from our asset pricing equation of Proposition 3.1 below. Their empirical implementation, however, is fairly independent from ours. They do not include price markups and leverage.

other variables. The introduction of some of these frictions, however, may help explain the evolution of price-earning ratios and the correlation of stock values with real macroeconomic aggregates.

Various papers have analyzed the evolution of stock market values, but fail to study the variability and comovements of both the financial and real sectors. Geanakoplos, Magill and Quinzii (2004) contend that changes in stock values may be driven by demographic trends, whilst Lustig and Nieuwerburgh (2006) cite credit access from home equity collateral that may affect attitudes toward risk. Gomme, Ravikumar and Rupert (2011) document the inability of real business-cycle models to replicate the volatility of financial returns.

The paper will proceed as follows. In Section 2 we report some evidence on the correlation of stock market values with markups and other financial measures. Section 3 lays out our model of technology adoption and derives some qualitative properties of the solution with emphasis on a fundamental asset pricing equation in which the asset price is decomposed into the value of physical capital and the value of adopted and unadopted technologies for the production of intermediate products. This section also shows existence of an invariant distribution as a preliminary requirement for the applicability of our numerical methods. Section 4 is concerned with the calibration of the model. Section 5 reports various numerical experiments, and Section 6 provides numerical methods to analyze the volatility of asset prices. We conclude in Section 7 with a further evaluation of our findings. The Appendix contains the proofs of our main theoretical results.

2 Stock Market Volatility and Markups

Figure 1 plots the evolution of the S&P index and the MVC. Both series have been filtered by taking out our best fit for a deterministic exponential trend. It can be observed that both aggregates display similar long-term cyclical behavior. Note that peak values occur in 1900, 1929, 1965 and 2000. Therefore, the amplitude of these long-term cycles can be up to 35 years. Jovanovic and Rousseau (2001) associate these long fluctuations in the stock market with three technological revolutions: Electricity, World War II, and IT. These authors document long lags in the operation and diffusion of new technologies. Nicholas (2008) claims that innovation was a main driver of the stock market run-up of the late 1920s.

Figure 2 decomposes the evolution of the MVC for different company cohorts in the recent IT revolution. Market capitalization relative to corporate value added is decomposed into the values of four different groups of companies: (*i*) The incumbents, (*ii*) Companies originating in 1970-1980, (*iii*) Companies originating in 1980-1990, and (*iv*) Companies originating after 1990. As one can see, most added stock value belongs to new corporations. These newcomers may reflect the value added of local technology adopters. Hence, the S&P index and the MVC are likely to differ in periods of sharp changes in leverage ratios and pronounced technological shocks associated with significant waves of potential entrants. But our story is not only a story of technology adoption, since markup changes may originate from many other sources besides technology.

In order to have an empirical counterpart of the aggregate markup in our model, we obtain markup estimates for high-tech corporations listed in the Compustat data base. Our estimates are based on US non-financial companies in Compustat North America data base reporting positive R&D expenditures over 1950–2012.[4] We use various related criteria to define these high-tech companies. First, companies are ranked by R&D intensity. Within the subsample of companies with positive R&D expenditures, we generate three subsamples comprising the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. The price markup of a company is defined from the total revenue over the cost ratio[5] and the aggregate markup is then obtained as a weighted average of company markups, using the share of company revenues.

Figure 3 confirms that the evolution of company markups for different vintages resembles the share of these companies over the aggregate stock market value shown in Figure 2. This evidence reveals that the markup for the top 50% and 75% companies with the highest ratio

[4]We consider the US set of companies (FIC=USA) in Compustat reporting positive R&D (positive values on XRD), non-financial (SIC codes out of the interval 6000–6999 and INDFMT=INDL), non-ADR, and with non-missing values in the variables REVT, COGS, XSGA, PRCC_C, and CSHO.

[5]Variables REVT and COGS in Compustat.

of R&D expenditure over total revenue, which is closely associated with the evolution of the aggregate stock market value, is mainly driven by newcomers' markups.

These same patterns are replicated in Table 1, where we compare the variability of markups with the shares of stock market values and some other cash flow measures over different vintages. Hence, this table groups different subperiods to show the evolution of the share of the stock market value of each company vintage against the evolution of the share of every other cash-flow measure for that same vintage. We include various cash flow measures; these measures are spelled out in the Appendix. Markups are reported as fractions of the average markup in the corresponding sample. Hence, a value above 1 means that this vintage commands higher markups than the average markup in the economy. Note that the share of the stock market value of the companies originating in 1980-2000 increases over time; further, these newcomers command higher markup values in all the markup measures *MU100*, *MU75*, and *MU50*. The financial literature has considered different measures of clash flow to analyze the volatility of the stock market. The most common cash flow measures are dividends and earnings. As discussed by Barsky and DeLong (1993), both series present low frequency fluctuations. Both measures have some shortcomings discussed in the literature. Dividends may be subject to optimal payout policies [cf. Marsh and Merton (1987)], and hence they may not be contemporaneously correlated with stock market values. Moreover, accounting earnings could be an inappropriate measure to capture profitability [cf. Campbell and Shiller (1988) and Novy-Marx (2013)]. Although Fama and French (2006) show that accounting earnings has some power to predict the cross section of returns, Novy-Marx (2013) argue that gross profitability becomes more relevant. Hence, following Novy-Marx (2013), we consider gross profits and earnings before extraordinary items. These are cleaner accounting measures which should be more closely associated with economic profitability. Additionally, we also consider some popular measures like *EBITDA*, *EBIT*, and operating income (before and after depreciation). The main problem is that these cash flow measures do not exhibit clear patterns. In fact, *EBE* and *NI* do not follow the evolution of *MVC*, and dividends are less volatile. The cash flow measures closer to economic profitability, *GP*, *OIADP*, *OIBDP*, *EBIT*, and *EBITDA*, resemble the dynamic behavior of the stock market. Hence, the evolution of the stock market value of different vintages is tightly linked to the evolution of markups, and to a lesser degree to the set of cash flow measures potentially capturing economic profitability (i.e., *GP*, *OIADP*, *OIBDP*, *EBIT*, and *EBITDA*). Indeed, these other financial measures present very low variability, and do not follow very clear patterns.

To confirm the lack of correlation of all these other financial measures, Table 2 displays changes in MVC and changes in these other variables over 10-year intervals. More specifically, the table reports growth rates of markups and financial variables over 10-year time periods. We include an additional measure of cash flow, *DIV2*, which is computed by taking out investment and wages from corporate value added. Similar profitability measures have been reported by other authors [cf. McGrattan and Prescott (2005), Larrain and Yogo (2007), and Peralta-Alva and Boldrin (2009)]. Markups and all other variables do very poorly in the 1950s. The run-up of the 1950s may have to do with external political forces, and our limited sample in COMPUSTAT. For all the other time intervals, traditional profitability measures seem poorly correlated with MVC, but our markup measures appear highly correlated. Again, while our measures of markup for high-tech companies are closely associated with the evolution of the market value over corporate value added, some of the considered cash flow measures (*DIV*, *DIV2*, *EBE*, *NI*) exhibit very low variability. The lack of volatility in the considered cash flow measures implies that the evolution of the stock market value is mainly driven by fluctuations in future expected returns rather than future expected cash flows [cf. Campbell and Shiller (1988)].

To delve into this evidence, Tables 3-4 compute correlations of MVC with every other financial variable over various time intervals. That is, these are correlations of growth rates over various time intervals. Note from Table 3 that markups are very highly correlated with MVC when we consider 5- to 30-year intervals. Our markup measures may display correlation coefficients of the order of 0.80, whereas the correlation of MVC with dividends is usually about 0.10. In general, growth rates of MVC are mildly correlated with growth rates of our cash flow measures. Table 4 computes these correlations after filtering out frequencies below five years. Hence, Table 4 is concerned with secular trends, and hence it is expected that markups variations will be less correlated with MVC variations when we average over 10-year time periods.

In line with this evidence, Figure 4 suggests that corporate markups may account for long-term movements of MVC. Hence, supporting our model predictions, the markup of high-tech companies – identified by their R&D intensity – is highly associated with the evolution of the market value of US corporations. This result is also consistent with alternative definitions of high-tech companies. In Figure 5 we consider two additional identification procedures of high-tech companies: (1) Companies with SIC codes 281, 283, 284, 289, 357, 367, 381, 384, and (2) Companies whose stocks are currently traded in the NASDAQ stock exchange. Figure 5 shows markup estimates and the price-revenue ratio (PR) for these samples of high-tech companies. Here again our estimated markups exhibit a clear association with the stock market which is even stronger than in Figure 4. This graphical evidence is confirmed by the contemporaneous correlation estimates shown in Table 5.

Figure 6 shows the evolution of two alternative definitions of aggregate markup. *MU* is a weighted average of all the company markups in Compustat without considering R&D intensity, and *MU2* is the revenue over cost ratio for all the companies in Compustat. This figure shows that these two alternative definitions of markup do not reproduce the evolution of the stock market. Moreover, as seen in Table 1, *MU* is not helpful to understand the stock market value for different vintages. As in our above cross-section analysis, some companies with high markups seem to be the main drivers of stock market fluctuations. Hence, we are not suggesting that a comprehensive measure of the average markup would be highly correlated with stock market values.

Finally, to provide additional evidence on the robustness of the correlation between markups and the stock market, we compute markup estimates on disaggregated data. From the Fama-French 5-sector industrial decomposition, we decompose our initial set of companies into three sectors.[6] Figure 7 shows our industrial sector markups together with the *PR*

[6]Fama and French 5-industry classification specifies **Cnsmr:** Consumer Durables, Non Durables, Wholesale, Retail, and Some Services (Laundries, Repair Shops); **Manuf:** Manufacturing, Energy, and Utili-

ratio. Although the comovement between markups and the stock market is still present, it is attenuated. Table 5 shows some contemporaneous correlations for these series.

3 The Model

The economy is populated by a continuum of identical households. At every time $t = 0, 1, ...$, each agent demands quantities of the aggregate consumption good, supplies labor inelastically, and trades in the equity and bond markets. The aggregate consumption good is produced by a single firm with a constant returns to scale technology. Three inputs are involved in the production of this final commodity: Capital accumulated by the firm, labor, and a composite intermediate good. Both the firm and the consumer act competitively in all markets, but the sector of intermediate goods is composed of a continuum of monopolistic competitors. The range of available intermediate goods can be expanded by a fixed set of local adopters upon the arrival of new technologies. As in Romer (1990), an increase in the varieties of intermediate goods allows for a more efficient use of resources and augments capital and labor productivity. The remaining source of change in productivity is an exogenous shock to the TFP of the final good production function. Proposition 3.1 below puts forward an asset pricing equation which will be a main building block in our empirical investigation.

3.1 The household

The representative household supplies one unit of labor inelastically, and has preferences over infinite streams of consumption. Preferences are represented by the expected discounted objective:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} \right] \tag{1}$$

ties; **HiTec:** Business Equipment, and Telephone and Television Transmission; **Hlth:** Healthcare, Medical Equipment, and Drugs; **Other:** Mines, Construction, Building Materials, Transportation, Hotels, Business Services, and Entertainment. Based on this classification we define our sectors as **Sector 1:** Cnsmr and Manuf, **Sector 2:** HiTec, and **Sector 3:** Hlth and Other.

where $c_t \geq 0$ denotes the quantity of consumption at t, with $0 < \beta < 1$, $0 \leq h < 1$, and $\sigma \geq 0$. Observe that this utility function includes habit persistence for positive h [Boldrin, Christiano and Fisher (2001)].

The agent may participate in financial markets by trading shares of an aggregate stock a_t and quantities of a risk-free bond b_t. The aggregate stock yields a stochastic dividend d_t, and the bond sells at a predetermined gross interest rate R_t. For initial asset holdings a_0, b_0, the optimization problem of the agent is to choose a stochastic sequence of consumption, shares of the aggregate stock, and units of the risk-free bond $\{c_t, a_{t+1}, b_{t+1}\}_{t\geq 0}$ to attain the maximum utility in (1) subject to the sequence of budget constraints

$$c_t + q_t a_{t+1} + b_{t+1} = \omega_t + (q_t + d_t)a_t + R_t b_t + T_t \tag{2}$$

$$q_t a_{t+1} + b_{t+1} \geq 0, \ t = 0, 1, 2, ..., \tag{3}$$

for given stock prices q_t, rates of interest R_t, exogenous wages ω_t, and lump-sum transfers T_t. Note that (3) is a simple borrowing limit which in this representative agent economy entails no loss of generality.

3.2 The production sector

The firm producing the final good accumulates capital and buys labor and intermediate goods. The firm's TFP is stochastic, and represented by a random variable θ_t. At every date t there is a mass A_t of intermediate goods that enter into the production of the final good. These intermediate goods are bundled together in a composite good M_t defined by a CES technology $M_t = [\int_0^{A_t} m_{s,t}^{\frac{1}{\vartheta_t}} ds]^{\vartheta_t}$ where $m_{s,t}$ denotes the amount of intermediate good s bought by the firm at time t and $\vartheta_t > 1$ follows an exogenous stochastic process to be specified below.

Given initial conditions k_0, B_0, the firm chooses stochastic sequences of investment, labor, debt, and intermediate goods $\left\{i_t, l_t, B_{t+1}, (m_{s,t})_{s\in[0,A_t]}\right\}_{t\geq 0}$ so as to maximize the present value of dividends:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \eta_t d_t^f \right] \tag{4}$$

subject to

$$d_t^f \equiv (1-\tau)\left[Y_t - \Omega_t\left(i_t + \omega_t l_t + \int_0^{A_t} p_{s,t} m_{s,t} ds\right) - R_t B_t + B_{t+1}\right] \tag{5}$$

$$Y_t \equiv \theta_t\left[\gamma\left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\rho} + (1-\gamma)M_t^{\rho}\right]^{\frac{1}{\rho}},\ 0<\gamma<1,\ -\infty<\rho<1 \tag{6}$$

$$k_{t+1} = (1-\delta)\,k_t + g(i_t/k_t)k_t \tag{7}$$

$$l_t \geq 0, B_t \leq \bar{B}_t. \tag{8}$$

Here, η_t is a state price[7] converting income of period t to period 0, and $p_{s,t}$ denotes the price of intermediate good s at time t. Our definition of dividends in (5) includes financial leverage. Observe that the amount of debt is bounded by the exogenous process $\bar{B}_t$. We include a tax on dividends $0 \leq \tau < 1$ and a friction Ω_t which may reflect trends in factor prices or financial costs. This friction can be made extensive to the sector of intermediate goods below, or it may only apply to some inputs so that the quantitative impact of these distortions may be linked to the functional form of the aggregate production function (6). The physical capital stock depreciates at a constant rate $0 \leq \delta < 1$. Capital accumulation is also subject to adjustment costs which are represented by function g. This latter function is positive and concave with $g(\delta) = \delta$ and $g'(\delta) = 1$.

Besides state prices, interest rates, wages, and the above distortions, the firm considers that TFP and price markups evolve exogenously. Stochastic variables θ_t and ϑ_t are governed by the following stationary first-order autoregressive processes

$$\begin{aligned} ln(\theta_t) &= \psi^{\theta} ln(\theta_{t-1}) + \sigma_{\theta}\varepsilon_t^{\theta} & (9)\\ ln(\vartheta_t) &= \psi_0^{\vartheta} + \psi_1^{\vartheta} ln(\vartheta_{t-1}) + \psi_2^{\vartheta}\varepsilon_t^{\vartheta} & (10)\end{aligned}$$

where $\psi^{\theta}, \psi_1^{\vartheta} \in (0,1)$, ψ_2^{ϑ}, $\sigma_{\theta} > 0$, $\varepsilon_t^{\theta} \overset{iid}{\sim} N(0,1)$, and $ln(\varepsilon_t^{\vartheta}) \overset{iid}{\sim} N(0,\sigma_{\vartheta})$.

Monopolistic competition prevails in the market for intermediate goods. Each variety s is supplied by an independent producer. For simplicity, the production process adopts the following form: One unit of good s requires only one unit of the final good. Then, producer

[7]For our economy with a representative household, state price η_t will correspond to the shadow value of income at time t over the same quantity at time 0.

of variety s picks an optimal pricing strategy $p_{s,t}$ and quantity $m_{s,t}$ from inspection of the downward-sloping demand for the product by the firm producing the aggregate commodity – after assuming a fixed set of prices and quantities for all other varieties. More precisely, for each time period t producer of variety s maximizes the amount of profits:

$$\pi_{s,t} \equiv \max_{m_{s,t} \geq 0} \{p_{s,t} m_{s,t} - m_{s,t}\} \tag{11}$$

where $p_{s,t}$ should be viewed as a function of $m_{s,t}$ from the inverse demand

$$p_{s,t} = \left(\frac{m_{s,t}}{M_t}\right)^{\frac{1-\vartheta_t}{\vartheta_t}} p_t \tag{12}$$

with $p_t = \left(\int\limits_0^{A_t} p_{s,t}^{\frac{1}{1-\vartheta_t}} ds\right)^{1-\vartheta_t}$.

Production of intermediate goods may be discontinued because of exogenous factors. Let ϕ be the probability of survival of a technology at every date t. Let $V_{s,t}$ be the present value of operating technology s from the beginning of time t:

$$V_{s,t} = \mathbb{E}_t \left[\sum_{r=t}^{\infty} \frac{\eta_r}{\eta_t} \phi^{r-t} \pi_{s,r}\right]. \tag{13}$$

By the symmetry embedded in our model, $\pi_{s,t}$ and $V_{s,t}$ are the same for all s.

3.3 Technology adoption

Technological innovations arrive exogenously to the economy. The average stock of technological innovations Z_t evolves according to the law of motion

$$Z_t = \phi Z_{t-1} + \mu x_{t-1} \tag{14}$$

with normalizing constant $\mu > 0$ and

$$ln x_t = \psi^x ln x_{t-1} + \sigma_x \varepsilon_t^x \tag{15}$$

where $\psi_x \in (0,1), \sigma_x > 0,$ and $\varepsilon_t^x \overset{iid}{\sim} N(0,1)$.

Technologies are put into use by local adopters. The adoption sector is composed of a continuum of agents $i \in [0,1]$ that behave competitively. Each adopted technology sells at price V_t to a producer of intermediate goods. Let A_t^i be the stock of already adopted technologies by agent i, and $\lambda(H_t^i)$ the probability of adopting a new technology after investing the amount of resources H_t^i. An adopter can undertake a diversified menu of projects, and hence we assume that her aggregate productivity is not subject to uncertainty. The stock A_{t+1}^i follows the law of motion

$$A_{t+1}^i = \lambda(H_t^i)\phi\left[Z_t^i - A_t^i\right] + \phi A_t^i. \tag{16}$$

The optimal amount of expenditure H_t^i is derived from the following Bellman equation in which the value function is the option value J_t^i of a new technology:

$$J_t^i = \max_{H_t^i}\left\{-H_t^i + \phi\mathbb{E}_t\left[\frac{\eta_{t+1}}{\eta_t}\left(\lambda(H_t^i)V_{t+1} + \left(1 - \lambda(H_t^i)\right)J_{t+1}^i\right)\right]\right\}. \tag{17}$$

As is well known, this equation can be computed recursively by the method of successive approximations. It follows that the optimal amount of expenditure H_t^i is the same for all i. We then let the aggregate stock of adopted technologies $A_{t+1} = \int A_{t+1}^i di$.

3.4 Equilibrium and asset prices

In the present model, given the tax τ and the sequence of frictions $\{\Omega_t\}_{t\geq 0}$, the exogenous state variables are the stock of available technologies Z_t, the addition of new varieties x_t, the TFP index θ_t, and the price markup ϑ_t. The endogenous state variables are the capital stock k_t, the stock of adopted technologies A_t, and consumption c_{t-1} (if $h > 0$). The remaining variables are determined as solutions of the model from the above optimization problems, market clearing and feasibility conditions, and laws of motion of the exogenous state variables.

As suggested before, we adopt the convention that the stock market value includes all the above three production sectors. That is, $q_t a_{t+1}$ comprises the value of the objective (4) for the firm producing the final good, plus the discounted net value of profits over the set of intermediate goods and technology adoption. Hence, the aggregate dividend $d_t \equiv$

$d_t^f + \pi_t A_t - H_t(Z_t - A_t)$. In what follows we assume that the aggregate net supply of the asset equals one (i.e. $a_{t+1} = 1$) so that q_t corresponds to the value of the stock market. Thus, market clearing in the stock and bond markets requires $a_{t+1} = 1$, and $b_{t+1} = B_{t+1}$. For the aggregate commodity, market clearing holds if

$$YN_t \equiv Y_t - A_t m_t = c_t + i_t + H_t(Z_t - A_t) \tag{18}$$

where Y_t denotes gross production of the final good and m_t is the quantity of each variety of intermediate good produced. Hence, $A_t m_t$ is the cost of producing the composite intermediate good, and YN_t is the value added in this economy. Therefore, output can be broken down into consumption, investment in physical capital, and investment in adopting new technologies.

The first-order conditions for the representative household correspond to the usual no-arbitrage conditions for the aggregate stock and the risk-free bond:

$$1 = \mathbb{E}_t \left[\frac{\eta_{t+1}}{\eta_t} \left(\frac{d_{t+1} + q_{t+1}}{q_t} \right) \right] \tag{19}$$

$$1 = \mathbb{E}_t \left[\frac{\eta_{t+1}}{\eta_t} R_{t+1} \right]. \tag{20}$$

The firm producing the final good will always demand positive amounts of each factor. Hence, the first-order conditions for the maximization of the objective in (4) will always hold with equality, and the imposed restriction on the amount of leverage should be taken into consideration when such constraint is binding. In the adoption sector, optimal positive expenditure in new varieties requires:

$$1 = \lambda'(H_t) \phi \mathbb{E}_t \left[\frac{\eta_{t+1}}{\eta_t} (V_{t+1} - J_{t+1}) \right]. \tag{21}$$

It follows that for a concave function $\lambda(H)$ the optimal expenditure H is positively correlated with the expected difference between the value of adopted and non-adopted varieties.

The next proposition is central to our study. Iterating forward over our above asset pricing equations we show that the value of the stock market comprises the value of adopted technologies and the option value to adopt new technologies.[8]

[8] Following our approach, this asset pricing equation has been used in the empirical work of Comin, Gertler and Santacreu (2009), and Kung and Schmid (2011).

Proposition 3.1 *The stock market value*

$$q_t = (1-\tau)\left(p_t^k k_{t+1} - B_{t+1}\right) + V_t^+ A_t + J_t^+ \left(Z_t - A_t\right) + \xi_t \tag{22}$$

where $V_t^+ \equiv V_t - \pi_t, J_t^+ \equiv J_t + H_t, \xi_t \equiv \mathbb{E}_t\left[\sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} J_r\left(Z_r - \phi Z_{r-1}\right)\right]$, *and* $p_t^k \equiv \frac{\Omega_t}{g_t'}$.

Therefore, the value of the stock market incorporates five components: The value of installed capital, the amount of debt, the value of adopted technologies, the option value of inventions currently available but not yet adopted, and the present value of future inventions expected to happen. These latter components are further sources of volatility in the stock market over the value of capital and adopted technologies. We will analyze the dynamic evolution of these components – as well as their correlation with real macro aggregates – under perturbations of the exogenous state variables. For the purposes of our quantitative analysis, we consider corporate equity and debt $q_t + B_{t+1}$, which is referred as the market value of corporations (MVC). This avoids modeling of corporate debt policies and pay-outs to debt holders. As already discussed, pay-outs to debt holders have been quite erratic in recent decades [Hall (2001)].

3.5 Existence of and ergodic invariant distribution

As we are dealing with the volatility of stock values it is worth pointing out that our model displays long-term cycles. Hence, our simulated moments are drawn from stationary equilibria rather than from long transitional dynamics. Following Peralta-Alva and Santos (2011) we now show the existence of an ergodic invariant distribution. This result provides the theoretical foundations for our numerical approach. Note that our model contains several state variables and the equilibrium cannot be recast as the solution of a social planing problem. Indeed, this existence result holds under some forms of habit formation in preferences, monopolistic power, and pecuniary and non-pecuniary distortions.

Theorem 3.2 *For the above economy there exists an ergodic invariant distribution* μ^*.

Under more limited assumptions we can actually prove global convergence to a unique ergodic invariant distribution. As a matter of fact, in all calibrations below the model seems to have a unique stable ergodic invariant distribution.
The existence of a globally convergent invariant distribution implies that all exogenous and endogenous variables will eventually reach the ergodic set with probability one. It seems then adequate to simulate the model using a high-order perturbation method [Schmitt-Grohé and Uribe (2004)] that takes into account the high volatility of stock market prices. To check for accuracy of the computed solution, we have combined this approximation method with a numerical dynamic programming algorithm [Santos (1999)] for the computation of Bellman's equation (17).

4 Calibration of the Model

The most salient features of the empirical implementation of our model are our definitions of dividends and of the market value of corporations that includes public and privately held companies, and our estimation of the exogenous markup process. There are many other issues regarding the elasticity of substitution ρ and parameters defining the process of technology adoption, but these latter parameter values do not seem to have a critical influence on the volatility of stock values.

4.1 Data sources

Our purpose is to match different statistics of medium-term fluctuations observed in the data. Following Comin and Gertler (2006) we define medium-term cycles as those within a frequency band of 2 to 50 years. We use annual data from 1960 to 2007. Each nominal variable is transformed in real terms through the GDP implicit price deflator taken from NIPA, Table 1.1.4. We also transform each aggregate variable in per-capita terms. Population is taken from NIPA, Table 7.1. We take the logarithm of each variable and use the band pass filter of Christiano and Fitzgerald (2003) over various frequencies.[9] Output (YN) is the corporate value added from NIPA, Table 1.14. Investment (I) is the sum of Investment in Private Non-residential and Residential Fixed Assets of US Corporations (Standard Fixed Asset Tables 4.7 and 5.7). The replacement value of corporate capital (K) is the sum of nonresidential and residential tangible corporate fixed assets (Standard Fixed Asset Tables 4.1 and 5.1). Consumption (C) is measured as the sum of non-durables and services (NIPA, Table 1.1.5). The Solow residual (SR) is taken from the Bureau of Labor Statistics (BLS) private business sector. Wages are measured as compensation of employees from the NIPA, Table 1.14. The number of patent applications (PT) will proxy adopted technologies. PT is obtained from the U.S. Patent and Trademark Office for 1970–2004, and from the Historical Statistics of the U.S. series W–99 for 1960 to 1970. The interest rate (R) is the short-term commercial paper rate from Robert Shiller's web page: *http : //www.econ.yale.edu/ shiller/data.htm*. The market value of corporations (MVC) has been computed following Peralta-Alva (2007). It is the sum of corporate's market value of equity and book value of net debt. As discussed in McGrattan and Prescott (2003), the difference between market and book values of net debt is very small. Dividends (D) are defined from corporate value added by taking out both wages and investment. We should emphasize that stocks and dividends in the model are meant to include joint ventures and startups. The impact of these new companies may squarely be missed by some other aggregate stock indices like the S&P.

4.2 Baseline calibrations

We start with a basic calibration of the model with no habit persistence (i.e., $h = 0$ in the above utility function), and without taxes and frictions (i.e., $\tau = 0$ and $\Omega = 1$). Our baseline calibration of parameter values is displayed in the first column of Table 6. There are several ingredients in this calibration exercise. First, various standard parameters are taken from the literature. We include here parameters defining the utility function and adjustment costs. Second, some parameters were selected in order to match some long-run ratios in

[9] This filtering method is appropriate for difference-stationary and trend-stationary processes.

the model's deterministic steady state. This obviously includes all parameters defining the risk-free rate, the investment ratio, the ratio of intermediate goods in total production and the income share of wages. Third, regarding the sector of technology adoption [equation (17)], where there could be a wide range of microeconomic empirical estimates, parameter values are selected to match some macroeconomic data. As a matter of fact, to avoid a very high sensitivity of optimal expenditure H, we postulate an expenditure function that becomes fairly parsimonious with R&D expenditures. And fourth, for the estimation of the elasticity of substitution parameter ρ, and the laws of motion for TFP, markups, and technology innovation, we use a simulation-based estimation procedure along the lines of Santos (2010). This exercise yields an optimal estimation of the covariance matrix for these three shocks. The estimation of the covariance matrix may be of independent interest as it suggests how unobservable shocks to TFP, markups and technological innovations may be correlated in the data. The evolution of the markup process is also estimated from a sample of Compustat companies.

We assume an inelastic labor supply. It is well known that standard RBC models do not generate enough volatility in worked hours [see Cooley and Prescott (1995) and Kydland (1995)]. We could improve the performance of the model in this dimension by incorporating labor indivisibilities, or variable effort. However, these labor market refinements do not change significantly asset pricing volatility. Parameter σ in the utility function is set to 5, which is within the range of empirical estimates for many studies. We choose values for the set of parameters $(\beta, \alpha, \gamma, \delta)$ in line with the aforementioned business cycle literature. Parameter β is fixed at 0.95, leading to an annual interest rate of 5.26%. We make $\alpha = 0.26$ based on evidence of the average share of labor costs over corporate value added. The share of materials in gross output is assumed to be 0.5; this is in accordance with estimates for the manufacturing sector [see Jaimovich and Floetotto (2008)]. In the corresponding model with no uncertainty, this income share implies a steady-state value for $\gamma = 0.7$.

We reproduce the average investment to capital ratio in the data by assuming an annual depreciation rate δ of 0.09 in (7). We specify the adjustment cost function as in Jermann

(1998):

$$g(i/k) = \frac{\delta^{\frac{1}{\varsigma}}}{1-\frac{1}{\varsigma}}(\frac{i}{k})^{1-\frac{1}{\varsigma}} + \frac{\delta}{1-\varsigma} \tag{23}$$

where the positive parameter ς is the elasticity of the investment to capital ratio with respect to Tobin's q. We let $\varsigma = 8$, in line with empirical evidence [see Jermann (1998), Jinnai (2009) and references therein]. As discussed below, our results present low sensitivity to this parameter.

We assume that the data counterpart for the number of adopted technologies (i.e., $A_{t+1} - \phi A_t$) is the number of patent applications PT. Following empirical estimates in Hall (2007), the survival rate of each intermediate product ϕ is set to 0.98. The probability of adoption is determined by an exponential function

$$\lambda(H_t) = \Lambda H_t^{\kappa} \tag{24}$$

with $\Lambda > 0$ and $\kappa \in (0, 1)$. We assign parameter values in conjunction with the laws of motion for A and Z below to replicate the volatility and persistence of patents and the ratio of R&D expenditures over output. The steady-state value for probability $\lambda(H)$ is 0.166, which yields an average adoption time of six years. Parameter κ then determines the volatility of expenditures in technology adoption. We come close to this volatility for κ equal to 0.80. These parameter values, imply that the mean value of the ratio of adoption expenditures over corporate value added is 2.33 percent. This figure is roughly the ratio found in the data for both corporate and non-corporate expenditures. In our simulated exercises the optimal law of motion for the ratio of adoption expenditures over output ($RH_t \equiv H_t(Z_t - A_t)/YN_t$) is approximated by the linear form

$$RH_t = R_0 + R_1(Z_t - A_t) \tag{25}$$

where R_0 and R_1 are constants with $R_1 > 0$. This approximate policy function has low computational cost, and tracks down the volatility of R&D data in a more parsimonious way.

Parameter values for the exogenous stochastic processes (9)-(10) and (15), and parameter ρ are selected from a loss function defined over weighted second-order moments of output YN, investment I, market value of corporations MVC, Solow residual SR, and patent applications PT. The estimated value of ρ in the production function is -0.6. Krusell *et al.* (2000) provide some estimates for the elasticity of substitution between capital and skilled labor which are consistent with our estimation of ρ. In the model with no uncertainty, we get that the steady-state value for the intermediate producers' gross markup ϑ is equal to 1.18. This value is rather low as compared with the range of available estimates [Rotemberg and Woodford (1995) provide an overview of microeconomic evidence]. But as the markup shock stems from a log-normal distribution, by Jensen's inequality the simulated mean for parameter ϑ is 1.38 which seems a more reasonable value.

We can see from Table 6 that the markup shock seems highly persistent with autoregressive coefficient $\psi_1^{\vartheta} = 0.968$. This key autoregressive parameter is in line with estimated values of New-Keynesian models. For instance, Smets and Wouters (2007, Table 4, p. 597) report a mean autocorrelation parameter equal to 0.90 in a model with price stickiness. However, this parameter jumps to 0.97 when the degree of price stickiness is moved to a minimal value. It is worth pointing out that our estimation of the price markup shock uncovers a link to technological innovations – which may be reflected in changes in the elasticity of substitution of intermediate goods. Thus, in Table 6 we get $Corr\left\{\varepsilon^{x}, ln(\varepsilon^{\vartheta})\right\} = 0.70$. Technological innovations are therefore associated with high price markups [Broda and Weinstein (2010)]. Moreover, $Corr\left\{\varepsilon^{\theta}, ln(\varepsilon^{\vartheta})\right\} = 0.81$, which suggests strong correlated effects between the Solow residual and the price markup.

4.3 Further markup estimates

Our simulations above are based on an exogenous markup process estimated through a simulation-based estimation procedure. We can obtain independent estimates of the markup process for high-tech corporations listed in the Compustat data base. These high-tech companies can be associated with the intermediate sector of adopters and producers in our model. We use various related criteria to define these high-tech companies. In all cases, the empirical estimates provide support for our previous calibration of the markup process.

Following the procedure described in Section 2, we create six subsamples comprising the top 50%, 60%, 70%, 80%, 90%, and 100% companies with the highest ratio of R&D expenditure over total revenue. For each subsample, the price markup of a company is defined from the total revenue over cost ratio. The aggregate markup is then obtained as a weighted average of company markups, using the share of company revenues. After taking logs and detrending, we present estimates of the AR(1) process for the aggregate markup in Table 7. Observe that these estimates are very close to the ones presented in Table 6. More precisely, the persistence parameter ψ_1^ϑ has been calibrated to 0.969 and it is in the range of estimated values $\widehat{\psi}_1^\vartheta \in [0.9434, 0.9827]$, and the volatility parameter σ_ϑ calibrated in the interval $[0.15, 0.2135]$ is in the range of estimated values $\widehat{\sigma}_\vartheta \in [0.1413, 0.3862]$.

Alternative definitions of high-tech companies provide similar results. For instance, the aggregate markup could be defined over all companies with SIC codes: 281, 283, 284, 289, 357, 367, 381, and 384. In this case, we obtain the following point estimates for the AR(1) process: $\widehat{\psi}_1^\vartheta = 0.9490$, and $\widehat{\sigma}_\vartheta = 0.1805$.

Moreover, estimations based on oil prices provide similar parameter values. Using data from Dvir and Rogoff (2009), our estimates for an univariate regression of oil prices are: $\psi_1^\vartheta = 0.93$ and $\sigma_\vartheta = 0.09$ for 1900–2008, $\psi_1^\vartheta = 0.96$ and $\sigma_\vartheta = 0.10$ for 1950–2008, and $\psi_1^\vartheta = 0.93$ and $\sigma_\vartheta = 0.12$ for 1970–2008.

5 Numerical Experiments

This section contains several numerical experiments to assess the model's predictions. To learn about the influence of some external forces in the dynamics of the model, Figures 8–10 exhibit impulse-response functions for shocks in the TFP index θ, the stock of available technologies Z, and the price markup ϑ respectively. For our benchmark calibration of the model, positive changes in θ and ϑ lead to extended increases in the market value of corporations $MVC_t \equiv q_t + B_{t+1}$, with more pronounced effects for changes in ϑ. Widening the range of available technologies Z may actually decrease MVC, as the arrival of new technologies depresses the price of existing ones. Therefore, this impulse-response analysis illustrates that changes in ϑ account for most of the impact on MVC.

5.1 Second-order moments

These moments are obtained from equilibrium paths with 3000 observations, where the first 1000 observations have been dropped to avoid influence of initial conditions. To evaluate the robustness of our results we consider the following three calibrations.

Model I (the basic model): Our benchmark calibration that excludes habit persistence (i.e., $h = 0$) and frictions impacting input costs (i.e., $\Omega = 1$).

Model II (habit persistence): Our benchmark calibration does not generate enough volatility of the risk-free interest rate. As is well known [see Boldrin, Christiano and Fisher (2001)], this volatility can be increased through habit persistence. The calibration procedure goes along the lines of our previous calibration exercise, and it illustrates that our results on the volatility of the stock market do not hinge upon the lack of variability of the risk-free rate. Parameter values for the exogenous stochastic processes (9)-(10) and (15), and parameters ρ, h are chosen from a loss function which now incorporates the volatility of the risk-free rate. The second column of Table 6 contains the estimated parameter values.

Model III (input costs frictions): We capture shocks to input costs through variable Ω. This variable follows a first-order autoregressive process

$$ln(\Omega_t) = \varphi^{\Omega} ln(\Omega_{t-1}) + \varepsilon_t^{\Omega} \text{ with } \varepsilon_t^{\Omega} \overset{iid}{\sim} N(0, \sigma^{\Omega}). \tag{26}$$

The proceeds of this distortion are rebated back to the consumer as lump-sum transfers. As in our previous calibrations, we now perform a simulation-based estimation exercise for the exogenous stochastic processes (9)-(10), (15) and (26), and parameters ρ, h. The third column of Table 6 contains the estimated parameter values.

Table 8 reports on the volatilities of Models I, II and III. In Model I the volatility of MVC is 16.61 as compared to 23.96 in the data. Therefore, the model can provide over two-thirds of the actual volatility in the data. The volatilities of consumption C, and investment I are in line with the business cycle literature. The volatilities of SR, PT, MVC, the return

RC of MVC and dividends D are pretty similar for the three models. Model II improves on the volatility of R and Model III *substantially* improves on the volatility of the price-dividend ratio PD. Should adjustment costs be taken out of Model I, then the volatility of investment goes from 11.27 percent to 11.99 percent, and the stock market volatility goes from 16.61 percent to 16.42 percent. Hence, the introduction of adjustment costs leads to a mild drop in the volatility of capital investment. This minor effect on the volatility of the stock market seems to occur because adjustment costs may crowd out expenditures in technology adoption.

Figure 11 decomposes the MVC in Model III as in our fundamental asset pricing equation (22). Roughly, the value of installed physical capital ranges between 15 and 50 percent of the corporate value, the value of the sector of intermediate goods ranges between 25 and 65 percent of the corporate value, whereas the option value of adopting future technologies lies between 10 and 30 percent of the corporate value. These figures seem quite plausible, and it should be understood that the relative weight of capital is simply affected by the volatility of all other components. That is, the replacement value of capital is fairly smooth. It is common in the macroeconomic literature [e.g., Hall (2001)] to see that the weight of capital in the total stock value may get down to one fourth of its peak value in periods of high activity or technological innovations.

Table 9 reports on the persistence of the variables. All three models can fairly well reproduce all the autocorrelations observed in the data for variables YN, C, I, SR, PT, MVC, and R. Model III, however, improves on the persistence of D, PD, and R. This seems key for Model III to account for the volatility of PD in Table 8. It follows from Table 9 that our results on the volatility of MVC and PD do not rely on undesirable high persistence of macro aggregates.

Table 10 displays contemporaneous correlations of output YN with other macro aggregates. Note that these empirical estimates are associated with high standard errors, and hence the confidence intervals are usually fairly wide. In most of the business cycle literature, YN is highly correlated with SR over the shorter term cyclical component, and YN is also highly correlated with MVC. In our models, we still see a high correlation of YN with SR, but YN is more mildly correlated with MVC. Again, Model III *substantially* improves upon the correlations of YN with each of the following variables: C, MVC, D, PD and R.

Finally, Table 11 presents contemporaneous correlations between PD and remaining macro variables. By and large, we observe that the simulated moments are quite close to the empirical ones. Note that Model III exhibits a much better performance, since it *substantially* improves upon the correlations of PD with each of the following variables: YN, C, I, and SR. Still, Model III fails in regards to the correlation between PD and C, between PD and MVC, and between PD and R. Below, we interpret this lack of success of our model as absence of monetary propagation mechanisms. Further, Campbell (1999) reviews the international evidence for some major countries, and finds that the correlation estimates between the risk-free rate R and some other macroeconomic aggregates are not very robust.

In summary, all models generate a reasonable volatility of the market value of corporations MVC along with the volatilities of real macroeconomic variables. The volatility of MVC is not driven by excessive high persistence (i.e., close to unit roots) of real macro aggregates. Autocorrelations of the variables are roughly the same in both the models and the data. The introduction of habit persistence generates a more desirable volatility of the risk-free interest rate R, which is still compatible with a high volatility for MVC. Supply shocks given by pecuniary frictions in input costs lead to a better volatility of the price-dividend ratio PD and improved autocorrelations of dividends D and risk-free rate R. These latter two variables are also better correlated with output YN. None of the models yields good correlations between PD and C, between PD and MVC, and between PD and R. As discussed presently, additional financial frictions such as borrowing constraints may improve the performance of these latter statistics.

5.2 Extensions

Taxes: Taxes on corporate profits and dividends could greatly affect the stock market value as well as endogenous investment and dividends [cf. McGrattan and Prescott (2005) and

Poterba (2004)]. We have considered an exogenous process for taxes that is meant to fit the evolution of taxes on dividends in the US as reported in McGrattan and Prescott (2003). This tax policy had a very small effect on the stock market. We should also remark that some activist fiscal policies on taxes and allowances for depreciation [Auerbach (2009)] have a damping effect on stock market values. After analyzing various arbitrary tax policies, we have concluded that taxes may strongly affect the volatility of asset values as they can change optimal dividend policies, but these desirable changes in the volatility of stock values are only obtained at the expense of excessive volatility in some real variables such as capital investment and consumption.

Labor frictions: The model with variable labor does not significantly improve upon the volatility of the stock market [see Rouwenhorst (1995)]. The introduction of variable labor brings the same problems encountered in the business cycle literature [e.g., Kydland (1995)]. Moreover, sticky wages, labor market rigidities and additional shocks to labor markets seem to have a minor influence on the long-term volatility of the stock market. General and nested CES production functions for capital and labor and intermediate goods did not lead to much improvement of the current results. As a proxy for labor distortions, we have experimented with a persistent shock in the shares of labor and capital income. This distortion generated too much volatility in capital investment. Figure 12 plots the evolution of the income share of labor, capital and intermediate goods for Model III. We can see that the income share of labor hovers around 60 percent and it is also reasonably volatile. This share seems in accord with other studies [cf. Krusell *et al*. (2000)]. Finally, restricting the analysis to a Cobb-Douglas production function lowers slightly the volatility of MVC and generates much less fluctuations in the income share of labor and dividends.

Borrowing constraints, monetary policy, and leverage: As compared to the data, in our Model III the correlation between PD and C is too high, and the correlation between PD and MVC is too low. The correlation between PD and C may be improved by tighter borrowing limits. Kehoe and Levine (2001) illustrate how various types of borrowing arrangements may change the volatility of consumption and asset prices. Kehoe and Levine consider a model with two types of agents. The computation of asset pricing models with heterogenous agents is a complex topic that goes well beyond our original objectives. Similar considerations should apply to improve the correlation between PD and MVC. In the recent economic crisis we have seen low PD ratios associated with tighter borrowing limits, a lower quality of collateral, as well as higher credit risk. Hence, it seems that introduction of some monetary factors may lower the correlation between PD and MVC from our original computations.

Leverage – short-term and long-term debt – appears to be an important source of stock market volatility, and this is reflected in our model as we introduce arbitrarily debt policies for the aggregate firm that resemble actual data. In our complete markets setting these policies are easy to simulate since we just need to keep track of corporate debt and equity. We can also assess the relevance of corporate debt from the data: The volatility of corporate equity in our data set is 28.81 percent whereas the volatility of the market value of corporations is 23.96. The difference between these two figures seems a lower bound for the effects of debt on the volatility of stock prices. Indeed, by the Modigliani-Miller theorem in a frictionless economy this difference would be ascribed to leverage. With financial frictions it is reasonable to expect that debt may have much bigger effects.

6 Variance Decomposition of PD

To further our understanding of the sources of stock market volatility, we also provide a variance decomposition of the price-dividend ratio (PD). Numerical simulation of non-linear equilibrium models has certain advantages over traditional econometric estimation – concerned with sampling error. Hence, our techniques for numerical simulation should be of independent interest.

A lot of research has been devoted to explore sources of volatility of stock prices, e.g., see Gilles and LeRoy (1991) and references therein. Some authors [Campbell and Shiller (1988) and Cochrane (1992, 2008)] argue that PD can mainly be explained by expected future returns, whereas expected future dividend growth lacks explanatory power. We should nevertheless remark that these results are not generally accepted. Some authors have questioned their econometric significance [see Ang (2002) and Ang and Bekaert (2007)] and others have found strong inconsistencies over data samples [see Boudoukh *et al*. (2007) and Larrain and Yogo (2008)]. This ongoing debate provides further motivation for our numerical work. We focus on the computation of the population moments of the model's invariant distribution for asset prices and returns. Hence, the computed population moments are not subject to the sampling error found in data analysis.

Following Campbell and Shiller (1988), we now derive an approximate expression for PD by a log linearization over an observed sample path. Let $pd_t \equiv ln\left(\frac{MVC_t}{D_t}\right)$, $\Delta d_t \equiv ln(\frac{D_t}{D_{t-1}})$, $r_t \equiv ln\left(\frac{MVC_t+D_t}{MVC_{t-1}}\right)$, where D_t refers to our definition of dividends from Section 3, and MVC_t is the market value of corporations at the end of t. Then, from these definitions, we get the following equation

$$pd_t = -r_{t+1} + \Delta d_{t+1} + ln\left[1 + exp\left(pd_{t+1}\right)\right]. \tag{27}$$

By a first-order Taylor approximation for $ln\left[1 + exp\left(pd_{t+1}\right)\right]$ at the expected value $\mathbb{E}\left[pd_t\right]$, it must hold that

$$pd_t \approx \nu - r_{t+1} + \Delta d_{t+1} + \rho pd_{t+1}, \tag{28}$$

where $\nu = ln\left[1 + exp\left(\mathbb{E}[pd_t]\right)\right] - \rho exp\left(\mathbb{E}[pd_t]\right)$ and $\rho \equiv exp\left(\mathbb{E}[pd_t]\right) / \left(1 + exp\left(\mathbb{E}[pd_t]\right)\right)$. Iterating forward over this difference equation over pd_{t+1} for N periods, we must have

$$pd_t \approx \nu \frac{1-\rho^N}{1-\rho} - \sum_{s=1}^{N} \rho^{s-1} r_{t+s} + \sum_{s=1}^{N} \rho^{s-1} \Delta d_{t+s} + \rho^N pd_{t+N}. \tag{29}$$

Applying the conditional expectations operator $\mathbb{E}_t$, multiplying both sides of (29) by $pd_t - \mathbb{E}\left[pd_t\right]$, and dividing by $Var(pd_t)$, we obtain the variance decomposition [e.g., see Cochrane (1992)]:

$$CVAR_{N,r} \equiv \frac{-Cov\left\{\mathbb{E}_t\left[\sum_{s=1}^{N} \rho^{s-1} r_{t+s}\right], pd_t\right\}}{Var(pd_t)} \times 100, \tag{30}$$

$$CVAR_{N,d} \equiv \frac{Cov\left\{\mathbb{E}_t\left[\sum_{s=1}^{N} \rho^{s-1} \Delta d_{t+s}\right], pd_t\right\}}{Var(pd_t)} \times 100, \tag{31}$$

$$CVAR_{N,pd} \equiv \frac{Cov\left\{\mathbb{E}_t\left[\rho^N pd_{t+N}\right], pd_t\right\}}{Var(pd_t)} \times 100. \tag{32}$$

Observe that these ratios represent the fraction of the variance of pd that can be attributed to fluctuations of expected future returns, dividend growth, and the volatility of the terminal component pd_{t+N}, respectively. It is important to realize that the above variance decomposition is based upon the following assumptions: (*i*) Equation (27) considers the realized return r_{t+1} whereas our pricing equation (19) holds under the discounting operator $\mathbb{E}_t\left[\frac{\eta_s}{\eta_t}(\cdot)\right]$ for $s \geq t$, and (*ii*) Equation (28) comes from a first-order Taylor approximation. In order to circumvent these shortcomings we now propose new techniques of analysis based upon numerical simulation of our asset pricing equation (19). We provide variance decompositions of pd under the following two methods:

Method 1: This procedure builds upon a linear approximation of the summation terms (30)-(31); see the Appendix for further technical details. The evolution of our state variables and the pricing kernel $\mathbb{E}_t\left[\frac{\eta_s}{\eta_t}(\cdot)\right]$ are computed through a high-order approximation. Hence, the linear approximation only applies for the computation of the summation terms in the variance decomposition of (30)-(31) evaluated by the expectations operator over the non-linear law of motion of the state variables. The presumption is that the effect of higher order terms will be small for the variance decomposition. Then, $\mathbb{E}_t\left[\sum_{s=1}^{\infty}\rho^{s-1}\hat{r}_{t+s}\right]$ and $\mathbb{E}_t\left[\sum_{s=1}^{\infty}\rho^{s-1}\Delta d_{t+s}\right]$ are expressed as linear functions of the state variables, where $\hat{r}_{t+1} \equiv -ln\left(\frac{\eta_{t+1}}{\eta_t}\right)$. Using long simulations we compute the following ratios:

$$NCVAR_{1,r} \equiv \frac{-Cov\left\{\mathbb{E}_t\left[\sum_{s=1}^{\infty}\rho^{s-1}\hat{r}_{t+s}\right], pd_t\right\}}{Var(pd_t)} \times 100, \tag{33}$$

$$NCVAR_{1,d} \equiv \frac{Cov\left\{\mathbb{E}_t\left[\sum_{s=1}^{\infty}\rho^{s-1}\Delta d_{t+s}\right], pd_t\right\}}{Var(pd_t)} \times 100. \tag{34}$$

Method 2: This is a simple procedure to assess the size of the non-linear approximation errors neglected under Method 1. Again, the stock price is defined as the expected discounted value of dividends under operator $\mathbb{E}_t\left[\frac{\eta_s}{\eta_t}(\cdot)\right]$ rather than using the realized return r_{t+1}. Indeed, both $\frac{\eta_s}{\eta_t}$ and d_t are functions of the state variables, and both terms interact in a nonlinear way in the computation of pd. Hence, we propose the following numerical approximation of the variance decomposition based on the computation of two objects: A constant-dividend ratio pd^r and a constant-discounting ratio pd^d. More precisely, pd^r is computed from the exact pd ratio of the model by letting $\Delta d_t = 0$, for all $t \geq 0$, and pd^d is computed from the exact

pd ratio by letting $\frac{\eta_{t+1}}{\eta_t} = \beta$, so that expected future dividends are discounted by β from the model. We can then define the following ratios:

$$NCVAR_{2,r} \equiv \frac{-Cov\left\{pd_t^r, pd_t\right\}}{Var(pd_t)} \times 100, \tag{35}$$

$$NCVAR_{2,d} \equiv \frac{Cov\left\{pd_t^d, pd_t\right\}}{Var(pd_t)} \times 100. \tag{36}$$

As before, these second-order population moments can be calculated by model simulation. As already stressed, these statistics do not depend on the Campbell-Shiller approximation (29) and are computed using operator $\mathbb{E}_t\left[\frac{\eta_s}{\eta_t}(\cdot)\right]$.

For Model III, under (33)-(34) we get $NCVAR_{1,r} = 91.23$ and $NCVAR_{1,d} = 10.13$. Hence, for Method 1 about 91 percent of the variance decomposition corresponds to changes in the expected value of future state prices, whilst only 10 percent of the variance decomposition corresponds to changes in expected dividend growth. For Method 2, under (35)-(36) we get $NCVAR_{2,r} = 83.64$ and $NCVAR_{2,d} = 15.39$. Hence, almost 84 percent of the variance decomposition corresponds to changes in the expected value of future state prices, whilst only 15 percent of the variance decomposition corresponds to changes in expected dividend growth. Interestingly, for both Methods 1 and 2 the sum of the components $NCVAR_r$ and $NCVAR_d$ is very close to 100. Furthermore, under both methods the variance of pd is mainly explained by news associated with the discounting factor. Method 1 attributes more variability to asset returns, which may stem from computational errors of the linear approximation, but these errors are relatively small.

These numerical methods allow us to compute the sources of volatility of PD in our model. Now, let us compare our numerical procedures with commonly used econometric methods. Under (30)-(32) we calculate $CVAR_{N,r}$, $CVAR_{N,d}$ and $CVAR_{N,pd}$ for both the data and arbitrarily long simulated paths under Model III. Data statistics are calculated over our annual set of observations for the time period 1960-2007. Model statistics are calculated using an equilibrium path. The estimated values are presented in Table 12 for several terminal periods N. According to these estimates, for Model III the fraction of the variance of pd that can be associated with expected dividend growth is never greater than 15 percent.

Observe from this table that actual data attaches a negative weight to expected dividend growth, which suggests an over-reaction of stock prices to changes in expected future returns with values around 120 percent. Although these estimates may change over data samples, they underscore the role of fluctuations of expected asset returns in the volatility of the price-dividend ratio.

In conclusion, we get similar variance decompositions for the above methods and for the computation of $CVAR_{N,r}$ and $CVAR_{N,d}$ in Table 12. In both cases the variability attributed to expected changes in state prices in Model III is around 85 percent. As in Campbell and Shiller (1988), this confirms that the approximation errors for these empirical tests seem to be small. From a methodological point of view, it is therefore reassuring that we get similar variance decomposition results for both computational and commonly used econometric methods. As a matter of fact, our numerical procedures provide a further validation of empirical tests because our computations of the variance decomposition for Model III are not subject to sampling error.

We carried out the same analysis for Models I and II and found similar quantitative results. Hence, these variance decompositions do not seem to depend on habit formations in the utility function or input market frictions, and should be ascribed to the evolution of the price markup (10) and other variables affecting asset prices and dividends. This is most clearly seen when we perform a similar analysis of the neoclassical growth model. In the real business cycle model of Rouwenhorst (1995) with a log utility function, we find that 90 percent of the *pd* volatility is explained by expected dividend growth. In this latter model, it takes a coefficient of risk aversion $\sigma = 10$ for fluctuations of expected asset returns to account for half of the volatility of *pd*. Therefore, traditional business cycle models cannot generate desired levels of volatility of asset prices, and such volatility is basically driven by expected dividend growth.

7 Concluding Remarks

This paper explores macroeconomic determinants of asset price volatility in a general equilibrium model with lags in technology adoption. Technologies are embedded in the production of new varieties of intermediate goods. Our analysis builds on an asset pricing equation that decomposes the market value of corporations into the value of installed capital, the value of existing technologies, and the option value of adopting new technologies. Hence, stock prices are suddenly impacted by the arrival of new technologies and other shocks affecting the economy. We show that the model has an ergodic invariant distribution. Using numerical methods we compute various second-order moments for real and financial variables and provide a variance decomposition of the price-dividend ratio.

As illustrated in some exploratory exercises with impulse response functions, the calibration of those parameters defining the evolution of the markup process becomes critical in our model. We estimate the price markup process by a simulated moments estimator. Then, this latter estimation is validated by an empirical analysis of the evolution of the markup for Compustat companies with positive R&D investment. Leverage is also an important factor to account for the volatility of stock prices. To avoid introduction of arbitrarily debt policies and dividends we consider the market value of corporations (MVC): The sum of the market value of corporate equity and the book value of net debt. Corporate equity includes both publicly and privately held companies so as to capture the effects on stock values of newly founded companies.

Overall, we get that in our model the volatility of the MVC is of the order of 18.98 percent as opposed to 23.96 percent in the data. Hence, the model can deliver about 3/4 of the observed volatility. We then carry out a battery of tests on the volatility and correlation of the stock market with various macroeconomic variables. The model performs well in terms of the autocorrelations of each variable, and the cross-correlations of output with all the other real variables. However, the model displays too little volatility for the risk-free rate, and a high correlation of the stock market with several other variables. To bypass these issues, we introduce preferences with habit formation and supply shocks; these two extensions do not affect the variability of stock prices.

We also perform a variance decomposition of the price-dividend ratio. We find that about 85 percent of the volatility of the price-dividend ratio can be explained by fluctuations of expected asset returns – leaving the remaining 15 percent to changes in expected dividend growth. These values are in line with our data estimates. In contrast, many variations of the neoclassical growth model predict low volatility for the price-dividend ratio, which is driven by dividend growth. In these models, sizable effects for the price-dividend ratio are only a possibility for close to unit-root behavior in the state variables and for rather high coefficients of risk aversion.

Our general equilibrium setting imposes severe discipline in our numerical experiments: Desired levels of volatility of asset prices usually come with pronounced changes in macroeconomic fluctuations. Thus, we find that TFP shocks and the arrival of new technologies, interest rate policies, taxes, and real and financial frictions have minor effects on the long-term volatility of asset market values under various CES formulations of the aggregate production function for labor and capital. Technological innovations can have significant effects if they come along with high markups and TFP changes. Leverage – short-term and long-term debt – appears to be an important source of stock market volatility from both additional model simulations and a simple decomposition of our data into corporate equity and debt.

In further research we are testing the influence of price markups and corporate debt for disaggregated data by industry. There is a considerable amount of variability of markups across sectors and over time; this variability seems to be highly correlated with asset market volatility as reflected in the various stock indexes under study. Also, leverage is more pronounced in industries with high fixed costs. Our model takes no account of fixed costs as sunk costs are not relevant for asset price volatility, but we find that the high levels of debt to equity ratios in these sectors are affecting asset market volatility.

8 Appendix

Data Appendix

This appendix details the variables considered in Section 2. *MVC* is the stock market value of the companies listed in Compustat. This is a proxy for the market value of US corporations. *MU* is a weighted average of company markups. Individual markups are computed as the revenues over production cost ratios. The aggregate markup is then a weighted average of company markups, using the share of company revenues. This is computed from variables REVT and COGS in Compustat. To compute *MU100*, *MU75*, and *MU50*, the companies listed in Compustat are ranked by R&D intensity. We create three subsamples comprising the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. The price markup of a company is defined from the total revenue over the cost ratio (REVT and COGS in Compustat) and the aggregate markup is then obtained as a weighted average of company markups, using the share of company revenues. *DIV* is the aggregate distribution of dividends by the companies listed in Compustat (variable DVT). *DIV2* is the Corporate value added less investment and wages. Corporate value added and wages are taken from the NIPA, Table 1.1.4, and investment is the sum of Investment in Private Nonresidential and Residential Fixed Assets of US Corporations (Standard Fixed Asset Tables 4.7 and 5.7). A similar measure has been considered by Peralta-Alva and Boldrin (2007), and McGrattan and Prescott (2005). In correspondence with our model, we do not include taxes in this cash flow definition. *GP* denotes gross profits defined as total revenue minus cost of goods sold taken from Compustat. This measure has been considered by Novy-Marx (2013). *OIADP* and *OIBDP* denote operating income after depreciation and operating income before depreciation respectively. Operating income is obtained from GP by subtracting other operating expenses. These variables are taken from Compustat. *EBE* denotes earnings before extraordinary items. It is taken from Compustat (variable IB). *EBIT* and *EBITDA* denote earnings before interest and earnings before interest and taxes respectively. These measures include extraordinary items and are taken from Compustat. NI denotes net income and is taken from Compustat.

Proof of Proposition 3.1

Recall that $d_t \equiv d_t^f + d_t^i$, where d_t^f and $d_t^i \equiv \pi_t A_t - H_t(Z_t - A_t)$ are dividends generated by the final and intermediate input sectors, respectively. Then, first order condition (19) for the household's problem jointly with the equilibrium condition $a_{t+1} = 1$ and the transversality condition $\lim_{T\to\infty} \mathbb{E}_t\left[\frac{\eta_T}{\eta_t} q_T a_{T+1}\right] = 0$ imply

$$q_t = \mathbb{E}_t\left[\sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r\right]. \tag{37}$$

Similarly, using the Euler equation for capital accumulation and the transversality condition for the aggregate firm

$$\lim_{T\to\infty} \mathbb{E}_t\left[\frac{\eta_T}{\eta_t}(p_t^k k_{T+1} - B_{T+1})\right] = 0, \tag{38}$$

we obtain

$$(1-\tau)\left(p_t^k k_{t+1} - B_t\right) = \mathbb{E}_t\left[\sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^f\right]. \tag{39}$$

Finally, we must show

$$V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t = \mathbb{E}_t\left[\sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^i\right]. \tag{40}$$

To this end, note that

$$\begin{aligned}
&V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t \\
&= \mathbb{E}_t\left[\frac{\eta_{t+1}}{\eta_t}\phi V_{t+1}\right] A_t + \mathbb{E}_t\left[\frac{\eta_{t+1}}{\eta_t}\phi[\lambda(H_t)V_{t+1} + (1-\lambda(H_t))J_{t+1}]\right](Z_t - A_t) + \\
&+\mathbb{E}_t\left[\frac{\eta_{t+1}}{\eta_t}[J_{t+1}(Z_{t+1} - \phi Z_t) + \xi_{t+1}]\right] \\
&= \mathbb{E}_t\left[\frac{\eta_{t+1}}{\eta_t}\ \ [V_{t+1}A_{t+1} + J_{t+1}(Z_{t+1} - A_{t+1}) + \xi_{t+1}]\right],
\end{aligned}$$

where the last equality comes after rearranging terms and letting $A_{t+1} = \phi\lambda(H_t)[Z_t - A_t] + \phi A_t$, and $Z_{t+1} - A_{t+1} = Z_{t+1} - \phi\lambda(H_t)[Z_t - A_t] - \phi A_t$. Hence,

$$\begin{aligned}
& V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t \\
& = \mathbb{E}_t \left[\frac{\eta_{t+1}}{\eta_t} \left[d_{t+1}^i + V_{t+1}^+ A_{t+1} + J_{t+1}^+ (Z_{t+1} - A_{t+1}) + \xi_{t+1} \right] \right].
\end{aligned}$$

Then, iterating forward this equation and ruling out bubbles in equilibrium [see Santos and Woodford (1997)] we get that (40) is satisfied.

Proof of Theorem 3.2

The proof of Theorem 3.2 can be divided into two parts. First, we need to establish the existence of a general equilibrium. For this proof we need to assume convexity of preferences, which rules out some forms of habit formation. Then, under standard assumptions of continuity and convexity of preferences the existence proof goes through using well-known methods [Mas-Colell and Zame (1991)]. Initially, we truncate the economy over finite horizons. Then, an equilibrium is established as the limit of a sequence of finite-horizon equilibria. We should point out that the assumption of monopolistic behavior does not complicate the analysis because we have a unit mass of agents who maximize static problems. Hence, the optimal strategy of an intermediate good producer can be derived from the inverse of the aggregate demand function.

Second, the continuity of the production and utility functions implies that the equilibrium correspondence is upper-semicontinuous. Moreover, this equilibrium admits a recursive form under a suitable expansion of the state-space [Duffie *et al*. (1994)]. Then, all required elements are in place to apply Theorem 3.2 of Santos and Peralta-Alva (2011) which proves the existence of an invariant distribution. Hence, by the ergodic decomposition theorem (cf. *op. cit.*) there is an ergodic invariant distribution. Following their method of proof one can show that for any initial condition the moments computed from arbitrarily long simulations approach the set of population moments of an invariant distribution of the model.

Linearization Procedure

By a first-order Taylor expansion of the model's policy function, it follows that

$$\begin{aligned} \widetilde{\varrho}_t &\approx g_\varrho^T \widetilde{s}_t, \\ \widetilde{d}_t &\approx g_d^T \widetilde{s}_t, \\ \widetilde{q}_t &\approx g_q^T \widetilde{s}_t, \end{aligned}$$

where for each variable x_t we define $\widetilde{x}_t \equiv ln\left(\frac{x_t}{x_{ss}}\right)$, and x_{ss} is a deterministic steady-state value, s_t is a $(n_s \times 1)$ vector of state variables, $\varrho_t \equiv \frac{\eta_t}{\eta_{t-1}}$ is the model's discount factor between $t-1$ and t, and g_ϱ, g_d, g_q are $(n_s \times 1)$ vectors. The vector of state variables $\widetilde{s}_t \equiv [\widetilde{k}_t, \widetilde{A}_t, \widetilde{Z}_t, \widetilde{\theta}_t, \widetilde{x}_t, \widetilde{\vartheta}_t, \widetilde{\Omega}_t, \widetilde{c}_{t-1}]^T$ follows the law of motion

$$\widetilde{s}_t = h\widetilde{s}_{t-1} + \epsilon_t,$$

where $\epsilon_t \overset{iid}{\sim} (\mu_\epsilon, \Sigma_\epsilon)$, and h is a $(n_s \times n_s)$ constant matrix. For these linear approximations, we calculate the ratios $NCVAR_{1,r}$ and $NCVAR_{1,d}$ in (33)-(34). These ratios are evaluated over the non-linear motion of the state variables.

REFERENCES

Abel, A. B., and J. C. Eberly (2005): "Investment, Valuation, and Growth Options," Unpublished paper.

Ang, A. (2002): "Characterizing the Ability of Dividend Yields to Predict Future Dividends in Log-Linear Present Value Models," Unpublished paper.

Ang, A., and G. Bekaert (2007): "Stock Return Predictability: Is it There?," *Review of Financial Studies,* 20, 651–707.

Auerbach, A. J. (2009): "Implementing the New Fiscal Policy Activism," *American Economic Review,* 99, 543-549.

Bansal, R., and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance,* 59, 1481–1509.

Boldrin, M., L. J. Christiano, and J. D. M. Fisher (2001): "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review,* 91, 149–166.

Boudoukh, J., R. Michaely, M. Richardson, and M. R. Roberts (2007): "On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing," *Journal of Finance,* 62, 877–915.

Broda, C., and D. E. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review,* 100, 691–723.

Campbell, J. Y. (1999): "Asset Prices, Consumption, and the Business Cycle," in J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics,* Amsterdam: North-Holland.

Campbell, J. Y., and R. J. Shiller (1988): "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance,* 43, 661–676.

Campbell, J. Y., and R. J. Shiller (1989): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies,* 1, 195–228.

Christiano, L. J., and J. D. M. Fisher (2003): "Stock Market and Investment Good Prices: Implications for Macroeconomics," NBER Working Paper No. 10031.

Christiano, L. J., and T. J. Fitzgerald (2003): "The Band Pass Filter," *International Economic Review,* 44, 435–465.

Cochrane, J. H. (1992): "Explaining the Variance of Price-Dividend Ratios," *Review of Financial Studies,* 2, 243–280.

Cochrane, J. H. (2008): "The Dog That Did Not Bark: A Defense of Return Predictability," *Review of Financial Studies,* 21, 1533–1575.

Comin, D., and M. Gertler (2006): "Medium-Term Business Cycles," *American Economic Review,* 96, 523–551.

Comin, D., M. Gertler, and A. M. Santacreu (2009): "Technology Innovation and Diffusion as Sources of Output and Asset Price Fluctuations," Unpublished paper.

Cooley, T. F., and E. C. Prescott (1995): "Economic Growth and Business Cycles," in T. F. Cooley (ed.), *Frontiers of Business Cycle Research,* N.J.: Princeton University Press.

Danthine, J. P., and J. B. Donaldson (2002): "Labor Relations and Asset Returns," *Review of Economic Studies,* 69, 41–64.

Dvir, E., and K. Rogoff (2009): "Three Epochs of Oil," NBER Working Paper No. 14927.

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. McLennan (1994): "Stationary Markov Equilibria," *Econometrica,* 62, 745–781.

Fama, E. F., and R. F. French (2006): "Profitability, Investment and Average Returns," *Journal of Financial Economics,* 82, 491–518.

Gârleanu, N. B., S. Panageas, and J. Yu (2009): "Technological Growth and Asset Pricing," NBER Working Paper No. 15340.

Gilles, C., and S. LeRoy (1991): "Econometric Aspects of the Variance-Bounds Tests: A Survey," *Review of Financial Studies,* 4, 753-791.

Gomme, P., B. Ravikumar, and P. Rupert (2011): "The Return to Capital and the Business Cycle," *Review of Economic Dynamics,* 14, 262-278.

Guvenen, F. (2009): "A Parsimonious Macroeconomic Model for Asset Pricing," *Econometrica,* 77, 1711–1750.

Hall, B. H. (2007): "Measuring the Returns to R&D: The Depreciation Problem," NBER Working Paper No. 13473.

HALL, R. E. (1988): "The Relation between Price and Marginal Cost in US Industry," *Journal of Political Economy,* 96, 921–947.

HALL, R. E. (2001): "The Stock Market and Capital Accumulation," *American Economic Review,* 91, 1185–1202.

JAIMOVICH, N., AND M. FLOETOTTO (2008): "Firm Dynamics, Markup Variations and the Business Cycle," *Journal of Monetary Economics,* 55, 1238-1252.

JERMANN, U. J. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics,* 41, 257–275.

JINNAI, R. (2009): "Intangible Capital, Asset Prices, and Business Cycles," Unpublished paper.

JOVANOVIC, B., AND P. L. ROUSSEAU (2001): "Vintage Organization Capital," NBER Working Paper No. 8166.

KEHOE, T. J., AND D. K. LEVINE (2001): "Liquidity Constrained Markets versus Debt Constrained Markets," *Econometrica,* 69, 575–598.

KLEIDON, A. W. (1986): "Variance Bounds Tests and Stock Price Valuation Models," *Journal of Political Economy,* 94, 953–1001.

KRUSELL, P., L. E. OHANIAN, J. V. RIOS–RULL, AND G. L. VIOLANTE (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica,* 68, 1029–1054.

KUNG, H., AND L. SCHMID (2011): "Innovation, Growth and Asset Prices," Unpublished paper.

KYDLAND, F. E. (1995): "Business Cycles and Aggregate Labor Market Fluctuations," in T. F. Cooley (ed.), *Frontiers of Business Cycle Research,* N.J.: Princeton University Press.

Larrain, B., and M. Yogo (2008): "Does Firm Value Move Too Much to be Justified by Subsequent Changes in Cash Flow," *Journal of Financial Economics,* 87, 200–226.

Marsh, T. A., and R. C. Merton (1987): "Dividend Behavior for the Aggregate Stock Market," *Journal of Business,* 60, 1–40.

Mas-Colell, A., and W. R. Zame (1991): "Equilibrium Theory in Infinite Dimensional Spaces," in W. Hildenbrand and H. Sonnenschein (eds.), *Handbook of Mathematical Economics,* Amsterdam: North-Holland.

McGrattan, E. R., and E. C. Prescott (2003): "Average Debt and Equity Returns: Puzzling?," *American Economic Review,* 93, 392–397.

McGrattan, E. R., and E. C. Prescott (2005): "Taxes, Regulations, and the Value of U.S. and U.K. Corporations," *Review of Economic Studies,* 72, 767–796.

Nicholas, T. (2008): "Does Innovation Cause Stock Market Runups? Evidence from the Great Crash," *American Economic Review,* 98, 1370–1396.

Novy-Marx, R. (2013): "The Other Side of Value: The Gross Profitability Premium," *Journal of Financial Economics,* 108, 1–28.

Pástor, L., and P. Veronesi (2009): "Technological Revolutions and Stock Prices," *American Economic Review,* 99, 1451–1483.

Peralta-Alva, A. (2007): "The Information Technology Revolution and the Puzzling Trends in Tobin's Average q," *International Economic Review,* 48, 929–951.

Peralta-Alva, A. and M. Boldrin (2009): "What Happened to the U.S. Stock Market? Accounting for the Past 50 Years," *Federal Reserve Bank of St. Louis,*, , 627–646.

Poterba, J. (2004): "Taxation and Corporate Payout Policy," *American Economic Review,* 94, 171–175.

ROMER, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy,* 98, 71–102.

ROTEMBERG, J. J., AND M. WOODFORD (1995): "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets," in T. F. Cooley (ed.), *Frontiers of Business Cycle Research,* N.J.: Princeton University Press.

ROUWENHORST, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in T. F. Cooley (ed.), *Frontiers of Business Cycle Research,* N.J.: Princeton University Press.

SANTOS, M. S. (1999): "Numerical Solution of Dynamic Stochastic Models," in J. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics,* Amsterdam: North-Holland.

SANTOS, M. S. (2010): "Consistency Properties of a Simulation-Based Estimator for Dynamic Processes," *Annals of Applied Probability,* 20, 196–213.

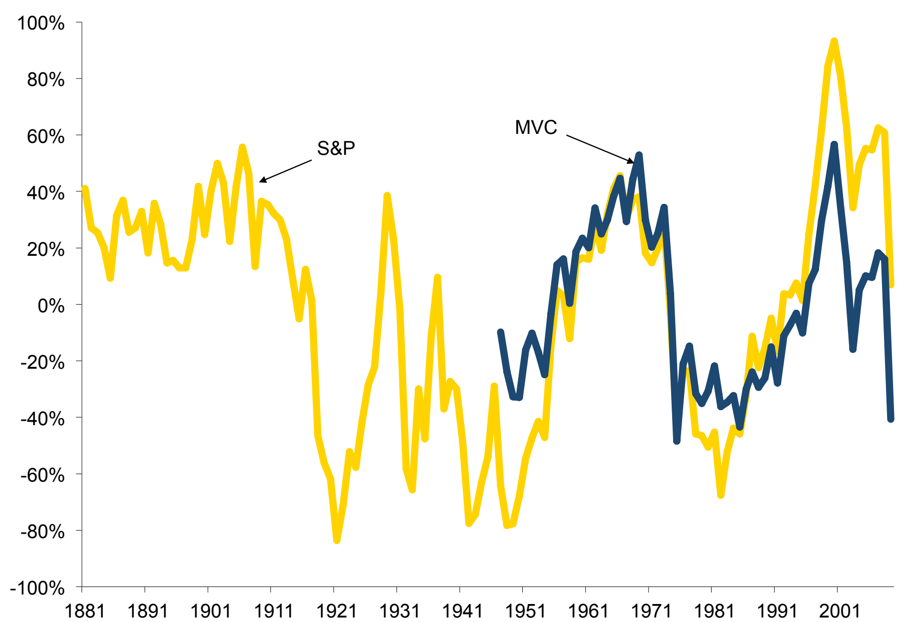
SANTOS, M. S., AND M. WOODFORD (1997): "Rational Asset Pricing Bubbles," *Econometrica,* 65, 19–57.

SANTOS, M. S., AND A. PERALTA-ALVA (2011): "Existence of Ergodic Invariant Distributions for a General Class of Dynamic Economies," Unpublished paper.

SCHMITT-GROHÉ, S., AND M. URIBE (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control,* 28, 755–775.

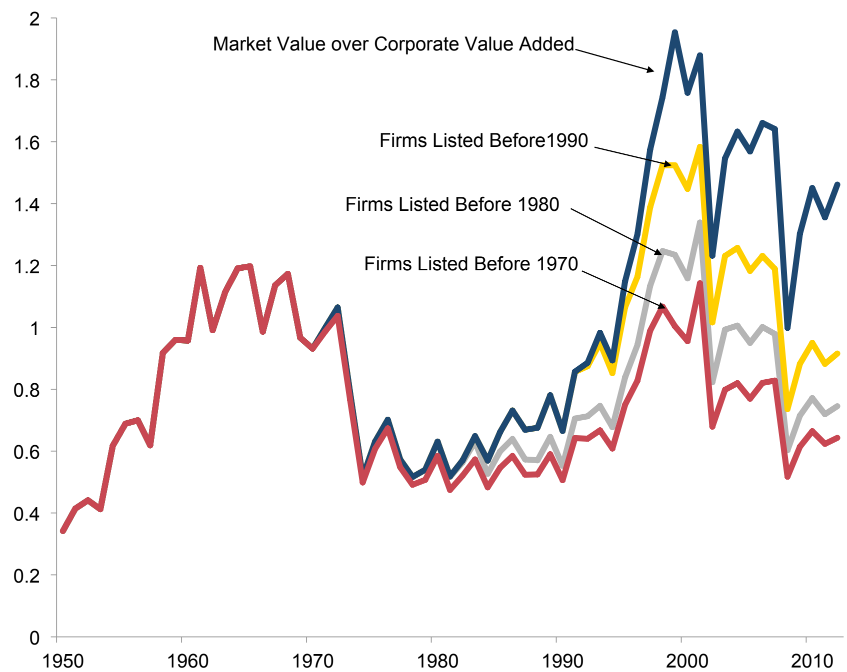
SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review,* 97, 586–606.

Figure 1: Evolution of $S\&P$ and MVC



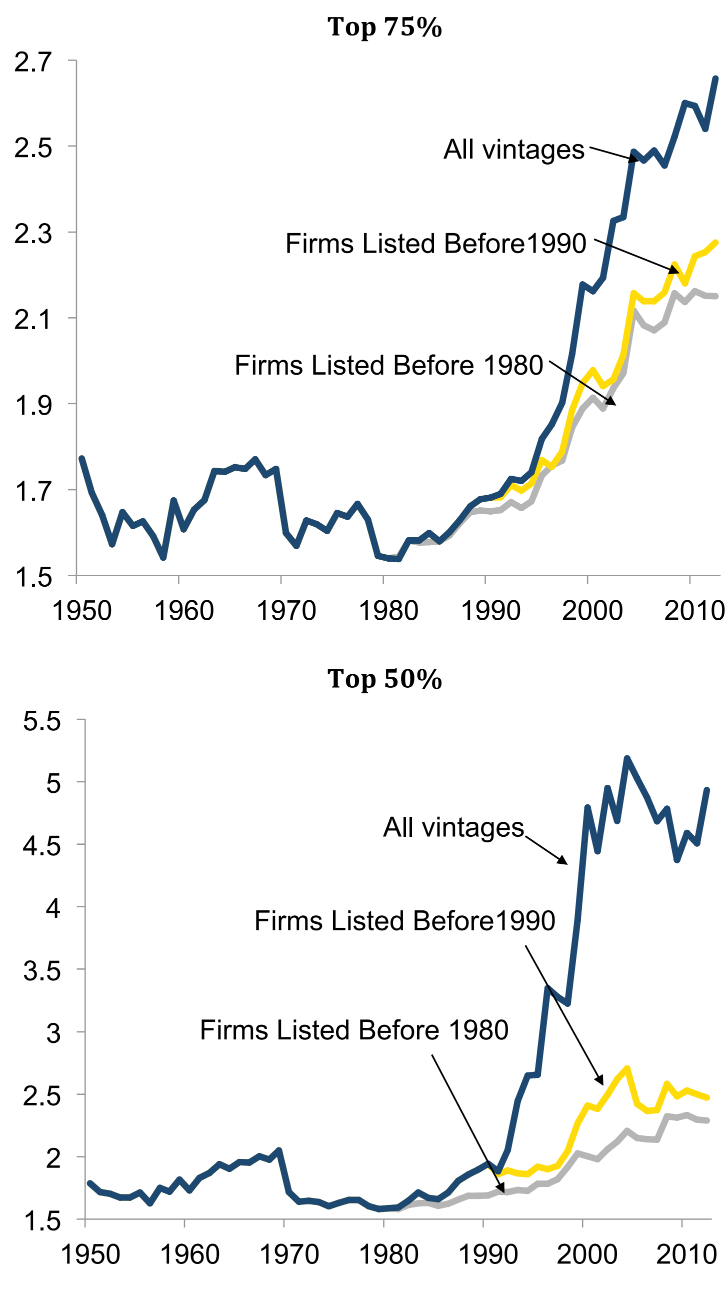
Source: Robert Shiller's web site: $http://www.econ.yale.edu/\ shiller/data.htm$ and Flow of Funds Accounts of the United States.
Note: Detrended real $S\&P$ 500 price index and real market value of corporations (MVC). The market value of corporations is defined as the sum of corporate's market value of equity and book value of net debt.

Figure 2: Market Value of Different Vintages



Sources: Compustat and NIPA.
Note: Market value of corporate equity over corporate value added for different vintages.

Figure 3: Markup of different Vintages



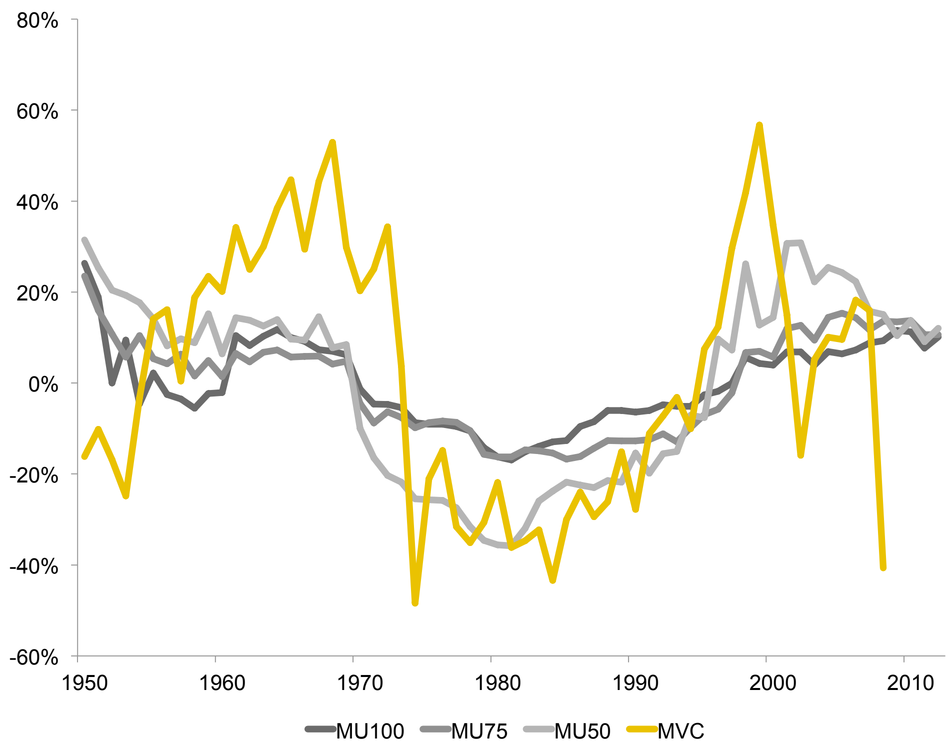
Sources: Compustat.
Note: Average markup for the top 75% and 50% companies with the highest ratio of R&D expenditure over total revenue. Average markups are computed as weighted averages using the share of company revenues. The price markup of a company is defined from the revenue over cost ratio.

Table 1: Markups and Vintage Shares for Financial Measures

Year	90–94			95–99				00–04			
Cohort	70	70–80	80-90	70	70–80	80–90	90–00	70	70–80	80–90	90–00
MVC	71.86	7.65	18.61	60.86	9.56	16.69	12.87	54.42	11.50	15.19	18.20
MU	94.51	106.60	121.25	97.43	110.84	113.50	91.45	98.98	114.66	122.37	83.84
MU100	97.83	90.26	123.32	98.38	89.18	109.55	116.33	97.95	95.35	86.57	108.71
MU75	98.87	88.31	121.68	94.30	83.67	104.18	168.16	88.70	74.98	101.35	165.62
MU50	80.44	73.42	158.92	57.69	56.42	87.07	164.71	43.26	41.93	70.07	121.47
DIV	69.93	12.23	15.43	65.52	11.44	12.04	10.98	60.20	9.84	12.11	14.58
GP	67.52	11.94	17.18	57.37	11.74	15.33	15.54	50.73	10.80	16.50	20.03
OIADP	63.91	14.94	17.95	55.99	14.75	16.22	13.01	51.79	13.72	18.16	14.77
OIBDP	65.15	13.75	18.23	56.50	14.30	16.36	12.81	50.37	13.12	17.91	16.69
EBE	69.35	14.94	14.43	64.57	13.81	15.42	6.18	102.29	25.39	25.26	-51.63
EBIT	62.29	15.48	19.12	54.11	15.21	16.80	13.86	49.92	13.76	18.63	15.65
EBITDA	65.15	13.75	18.23	56.50	14.30	16.36	12.81	50.37	13.12	17.91	16.69
NI	66.28	18.01	13.18	64.44	13.77	13.98	7.80	189.96	47.07	20.37	-150.38

Note: Percentages over the aggregate value considering all the vintages. Reported values are averages for different time ranges.

Figure 4: MVC and Markup Measures



Sources: Flow of Funds Accounts of the United States and Compustat.
Note: Detrended real market value of corporations (MVC) and detrended markup measures. *MU100* is the average markup for the companies in Compustat that present positive R&D expenditures, and *MU75* and *MU50* are the average markup for the top 75% and 50% companies with the highest ratio of R&D expenditure over total revenue. Average markups are computed as weighted averages using the share of company revenues. The price markup of a company is defined from the revenue over cost ratio.

Table 2: Growth Rate Adjusted for Inflation and Growth

Window	1950-1960	1960-1970	1970-1980	1980-1990	1990-2000	2000-2010
MVC	1.03	-0.03	-0.39	0.05	0.97	-0.19
MU	0.03	0.03	0.16	-0.03	0.15	0.03
MU100	-0.23	0.04	-0.12	0.12	0.14	0.13
MU75	-0.10	-0.01	-0.04	0.09	0.25	0.18
MU50	-0.03	-0.01	-0.08	0.20	0.90	-0.04
DIV	0.28	0.09	0.09	-0.06	-0.10	0.16
DIV2	-0.18	-0.15	-0.18	0.12	-0.14	0.52
GP	0.39	0.58	0.21	-0.03	0.17	0.06
OIADP	-0.05	0.39	0.41	-0.11	0.22	0.09
OIBDP	0.08	0.42	0.32	-0.07	0.23	0.04
EBIT	-0.08	0.33	0.32	0.00	0.27	0.12
EBITDA	0.08	0.42	0.32	-0.07	0.23	0.04
EBE	0.05	0.12	0.36	-0.53	0.25	0.40
NI	-0.07	0.26	0.40	-0.53	0.24	0.42

Note: All the variables except the markups have been scaled by corporate value added. For each variable x_t, reported growth rates are computed as $log(x_{t+10}/x_t)$.

Table 3: Correlations with MVC 1960-2012

Window	1	3	5	7	10	20	30
MU	0.08	0.08	0.00	0.02	0.03	0.06	0.05
	(0.14)	(0.11)	(0.14)	(0.21)	(0.22)	(0.10)	(0.07)
MU100	0.33	0.36	0.53	0.64	0.76	0.92	0.85
	(0.09)	(0.15)	(0.17)	(0.17)	(0.15)	(0.07)	(0.10)
MU75	0.12	0.31	0.43	0.54	0.64	0.83	0.88
	(0.13)	(0.15)	(0.19)	(0.19)	(0.20)	(0.14)	(0.08)
MU50	-0.03	0.39	0.62	0.74	0.81	0.92	0.89
	(0.11)	(0.15)	(0.14)	(0.14)	(0.11)	(0.07)	(0.05)
DIV	0.13	0.08	-0.18	-0.40	-0.44	-0.57	0.26
	(0.14)	(0.12)	(0.13)	(0.16)	(0.19)	(0.21)	(0.14)
DIV2	0.24	0.09	-0.08	0.00	0.11	0.62	0.70
	(0.21)	(0.18)	(0.17)	(0.24)	(0.29)	(0.21)	(0.17)
GP	0.23	0.07	-0.02	-0.17	-0.27	-0.56	-0.82
	(0.18)	(0.21)	(0.20)	(0.20)	(0.17)	(0.08)	(0.04)
OIADP	0.23	0.19	0.04	-0.18	-0.19	-0.37	-0.47
	(0.17)	(0.20)	(0.24)	(0.25)	(0.25)	(0.17)	(0.15)
OIBDP	0.28	0.20	0.07	-0.11	-0.18	-0.47	-0.61
	(0.17)	(0.22)	(0.26)	(0.28)	(0.26)	(0.11)	(0.13)
EBIT	0.22	0.22	0.11	-0.06	0.01	0.06	-0.19
	(0.17)	(0.20)	(0.22)	(0.25)	(0.24)	(0.14)	(0.12)
EBITDA	0.28	0.20	0.07	-0.11	-0.18	-0.47	-0.61
	(0.17)	(0.22)	(0.26)	(0.28)	(0.26)	(0.11)	(0.13)
NI	0.48	0.37	0.19	0.04	0.00	-0.10	0.12
	(0.07)	(0.07)	(0.10)	(0.13)	(0.12)	(0.13)	(0.07)

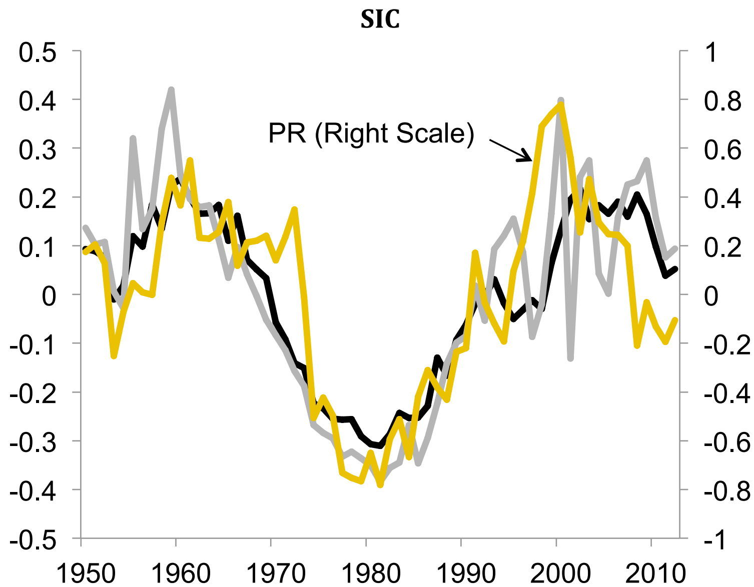
Note: All the variables except markups have been scaled by corporate value added. Reported values are contemporaneous correlations with MVC of growth rates for different time lapses. Figures in parentheses are Newey-West standard errors.

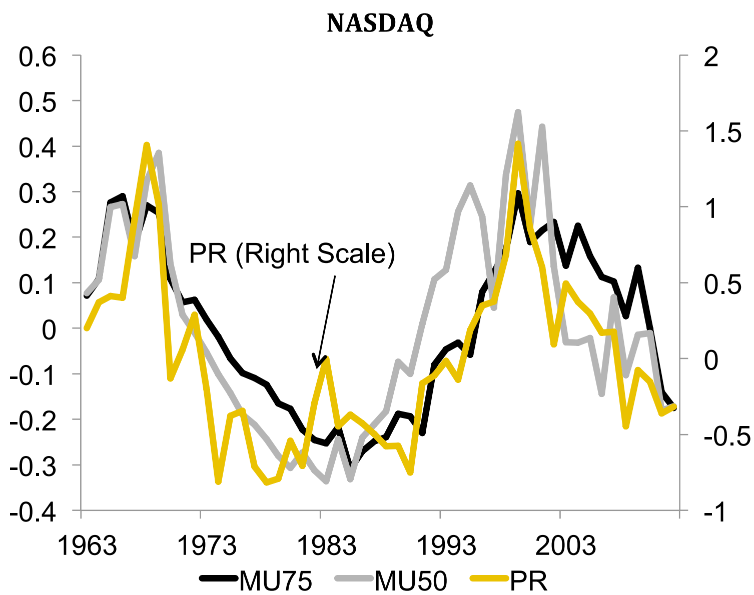
Table 4: Cross-Correlation with MVC Filtered data: 5-50

Lead	0	1	3	5	7	10	20	30
MU	-0.11	-0.04	-0.03	0.02	0.05	0.07	-0.02	0.45
	(0.14)	(0.15)	(0.20)	(0.17)	(0.25)	(0.39)	(0.24)	(0.08)
MU100	0.35	0.71	0.76	0.74	0.65	0.37	-0.77	-0.75
	(0.15)	(0.07)	(0.10)	(0.13)	(0.12)	(0.26)	(0.10)	(0.12)
MU75	0.35	0.57	0.77	0.88	0.83	0.58	-0.61	-0.72
	(0.13)	(0.12)	(0.12)	(0.11)	(0.11)	(0.21)	(0.21)	(0.13)
MU50	0.74	0.85	0.87	0.75	0.56	0.13	-0.82	-0.09
	(0.10)	(0.10)	(0.09)	(0.15)	(0.22)	(0.28)	(0.15)	(0.27)
DIV	-0.03	-0.21	-0.37	-0.27	-0.11	-0.05	0.59	-0.41
	(0.18)	(0.19)	(0.23)	(0.21)	(0.28)	(0.22)	(0.24)	(0.10)
DIV2	-0.21	-0.30	-0.23	0.07	0.07	0.12	0.25	-0.44
	(0.18)	(0.20)	(0.19)	(0.15)	(0.13)	(0.16)	(0.19)	(0.18)
GP	0.36	0.55	0.68	0.78	0.83	0.67	-0.74	-0.46
	(0.15)	(0.13)	(0.11)	(0.10)	(0.08)	(0.11)	(0.09)	(0.12)
OIADP	-0.03	0.10	0.18	0.37	0.54	0.44	-0.29	-0.29
	(0.16)	(0.16)	(0.20)	(0.17)	(0.14)	(0.26)	(0.16)	(0.15)
OIBDP	0.11	0.27	0.35	0.48	0.60	0.48	-0.45	-0.26
	(0.17)	(0.16)	(0.17)	(0.15)	(0.13)	(0.24)	(0.14)	(0.17)
EBIT	0.03	0.17	0.27	0.45	0.60	0.41	-0.48	-0.30
	(0.17)	(0.18)	(0.20)	(0.16)	(0.12)	(0.21)	(0.16)	(0.15)
EBITDA	0.11	0.27	0.35	0.48	0.60	0.48	-0.45	-0.26
	(0.17)	(0.16)	(0.17)	(0.15)	(0.13)	(0.24)	(0.14)	(0.17)
NI	0.00	-0.13	-0.31	-0.09	0.03	-0.06	0.11	-0.05
	(0.14)	(0.14)	(0.14)	(0.11)	(0.09)	(0.12)	(0.13)	(0.11)

Note: All the variables except markups have been scaled by corporate value added and then filtered with a band-pass filter that extracts fluctuations with a periodicity of 5 to 50 years. Reported values are estimates of correlations with *MVC* for different time leads. Figures in parentheses are Newey-West standard errors.

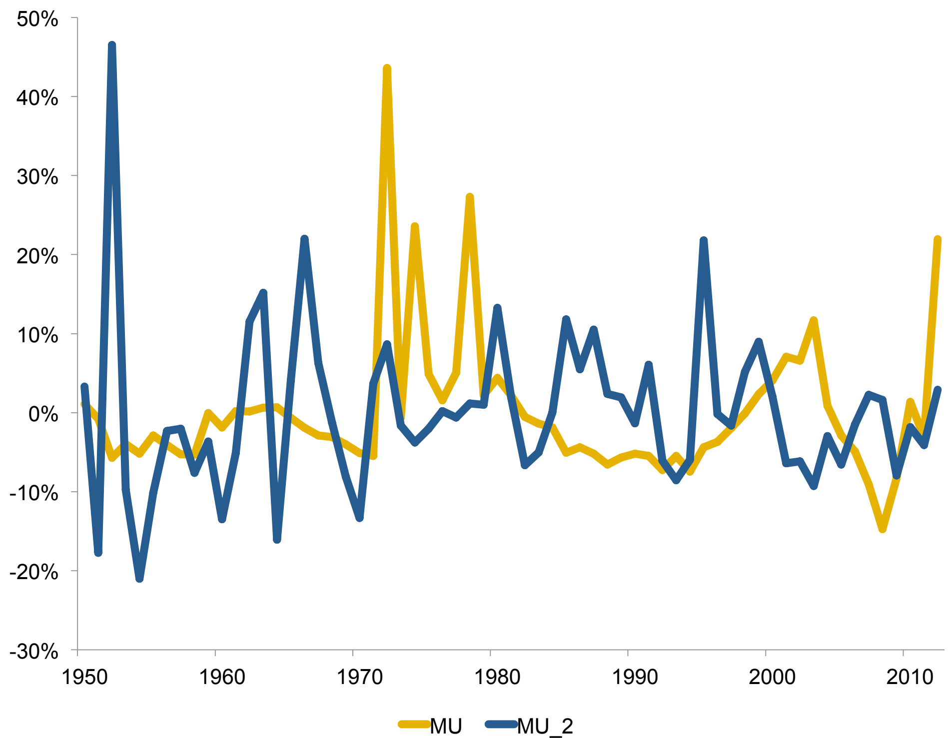
Figure 5: Market Value over Revenue and Markup Measures: High-Tech Companies





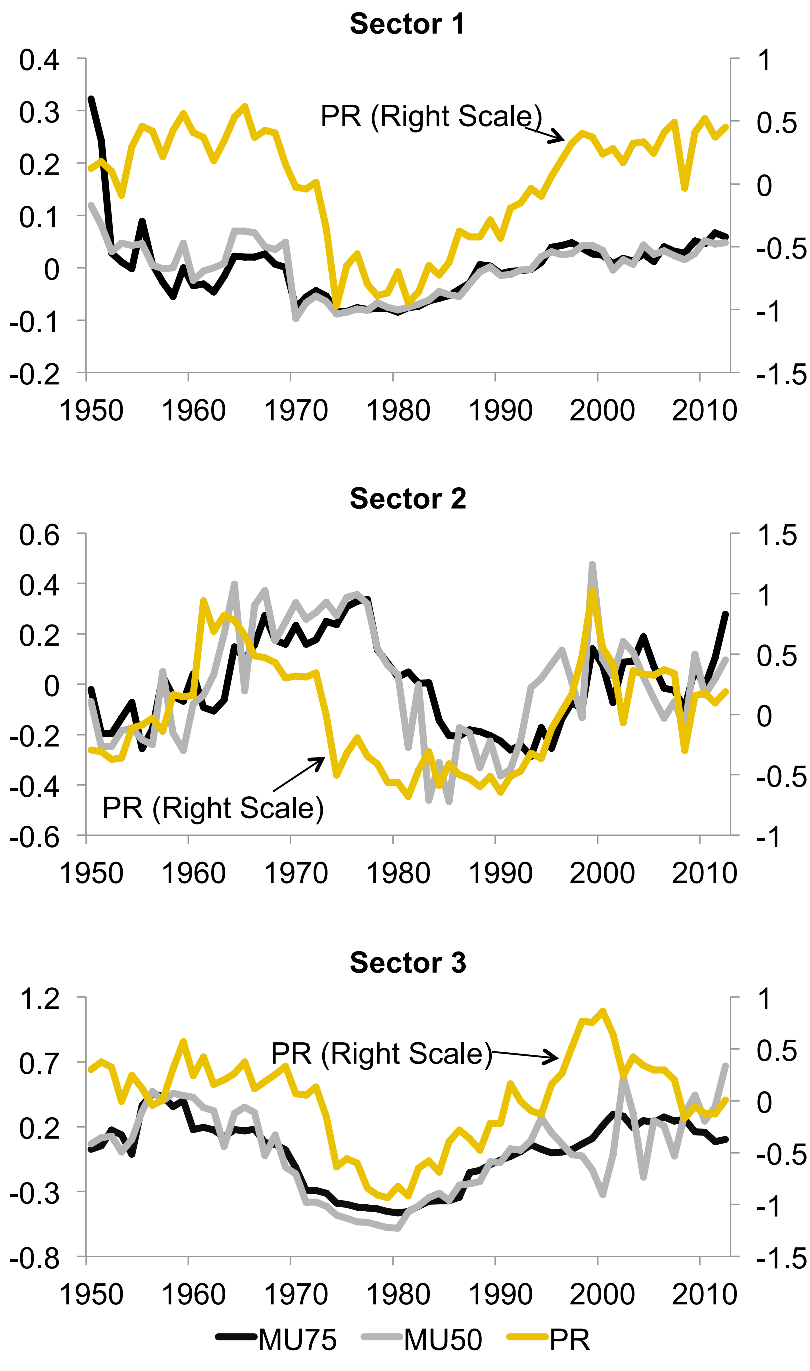
Sources: Compustat.
Note: All variables have been detrended. Markup measures are defined as in the previous figure. *PR* denotes the market value over revenue ratio for the considered sample of companies. The top figure is a subset of high-tech companies defined by their SIC codes: 281, 283, 284, 289, 357, 367, 381, and 384. The bottom figure considers the companies listed in NASDAQ.

Figure 6: Aggregate Markup Measures



Sources: Compustat.
Note: Detrended markup measures. *MU* is a weighted average markup for all the companies in Compustat. The weight factor is the share of company revenues. *MU_2* is the aggregate markup for all the companies in Compustat. It is computed as the aggregate revenue over cost ratio.

Figure 7: MVC and Markup Measures



Sources: Flow of Funds Accounts of the United States and Compustat.
Note: All variables have been detrended. Markup measures are defined as in the previous figure.

Table 5: Correlations with PR 1960-2012: Sectorial

	Sector 1		Sector 2		Sector 3		Nasdaq		SIC	
Window	5	10	5	10	5	10	5	10	5	10
MU100	0.76	0.89	0.16	0.29	0.41	0.60	0.72	0.79	0.40	0.62
	(0.15)	(0.11)	(0.17)	(0.16)	(0.22)	(0.21)	(0.10)	(0.09)	(0.21)	(0.20)
MU75	0.72	0.90	0.21	0.30	0.60	0.80	0.67	0.76	0.52	0.74
	(0.09)	(0.03)	(0.24)	(0.24)	(0.12)	(0.11)	(0.16)	(0.11)	(0.14)	(0.13)
MU50	0.71	0.89	0.27	0.43	0.08	0.47	0.67	0.77	0.39	0.67
	(0.11)	(0.04)	(0.21)	(0.22)	(0.27)	(0.22)	(0.15)	(0.12)	(0.17)	(0.13)

Note: Reported values are contemporaneous correlations with *PR* of growth rates for different time lapses. Figures in parentheses are Newey-West standard errors.

Figure 8: Impulse–Response Function of MVC to a Perturbation in θ

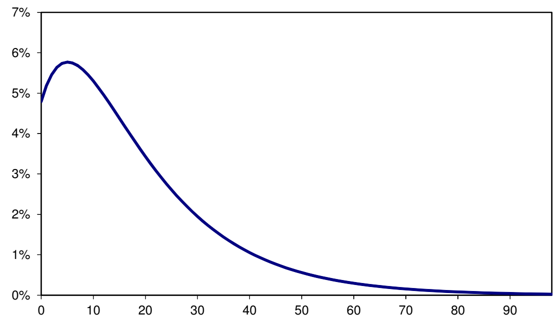


Figure 9: Impulse–Response Function of MVC to a Perturbation in Z

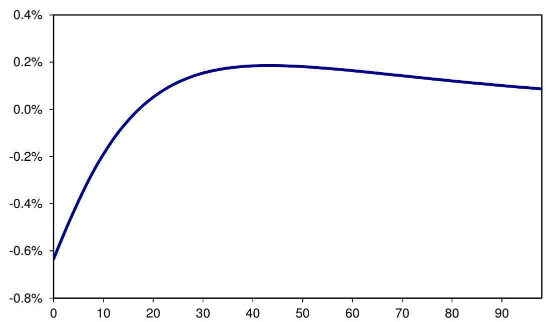
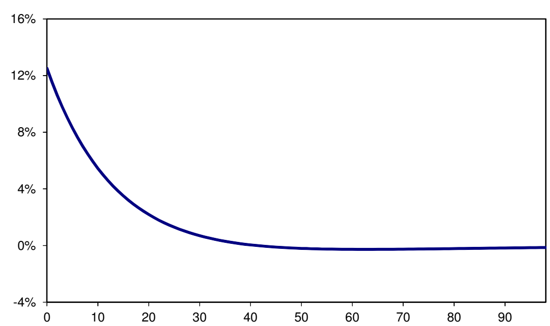
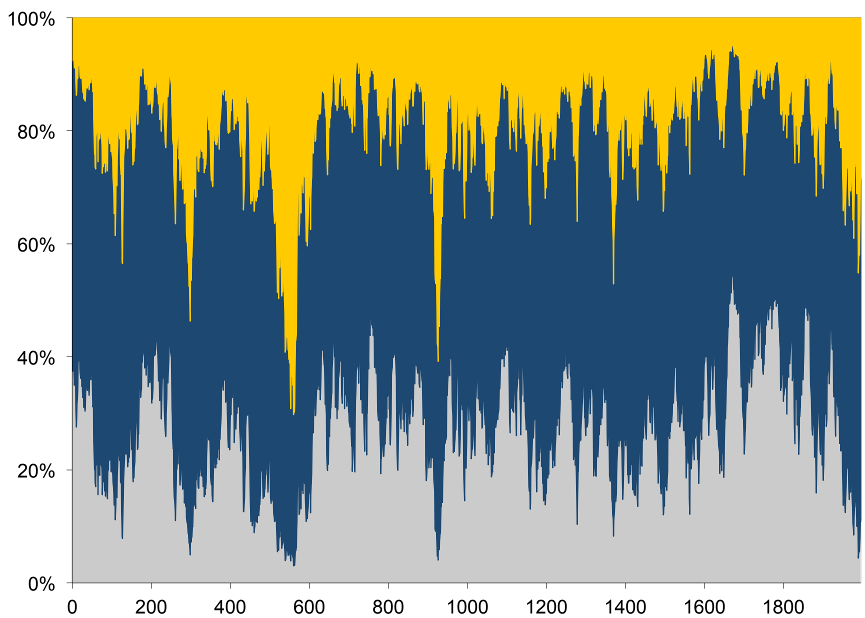


Figure 10: Impulse–Response Function of MVC to a Perturbation in ϑ



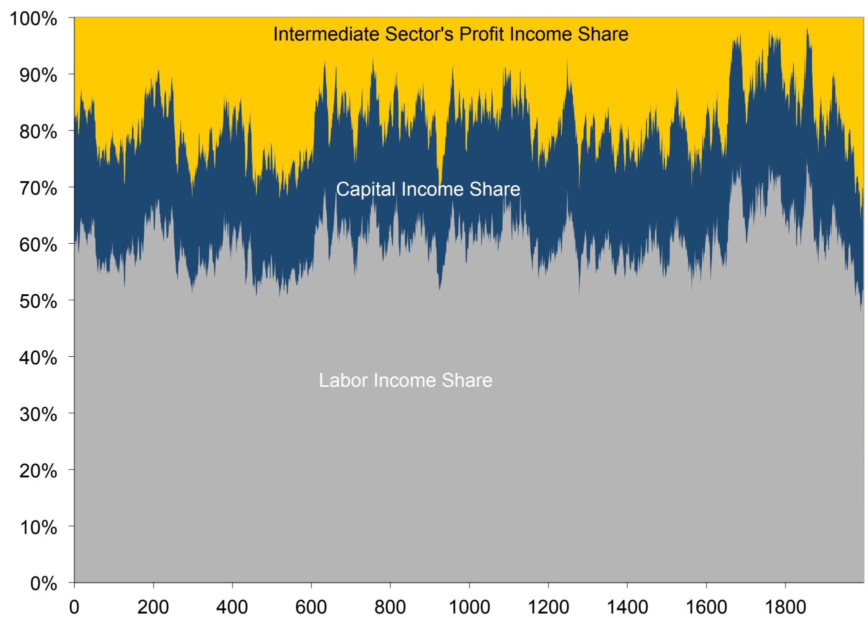
Note: Response of MVC to a positive perturbation to each shock (θ, Z, and ϑ) by one standard deviation. The y-axis measures percentage deviation from the deterministic steady-state value and the x-axis measures time in years.

Figure 11: Components of the Stock Market Value



Note: The bottom area is the relative value of physical capital (i.e., $p_t^k k_{t+1}$), the middle area is the relative value of adopted technologies (i.e., $V_t^+ A_t$), and the top area is the relative value of unadopted technologies (i.e., $J_t^+ (Z_t - A_t) + \xi_t$.

Figure 12: Factor Income Shares



Note: The bottom area is the labor income share, the middle area is the capital income share, and the top area is the share of intermediate sector's profits.

Table 6: Calibration of Parameter Values

Parameter	MI	MII	MIII
β	0.95	0.95	0.95
σ	5	5	5
h	0	0.5	0.5
α	0.26	0.26	0.26
γ	0.70	0.70	0.70
δ	0.09	0.09	0.09
ρ	-0.60	-0.60	-0.60
ς	8	8	8
κ	0.80	0.80	0.80
λ	0.166	0.166	0.166
ϕ	0.98	0.98	0.98
R_0	-0.0038	-0.0038	-0.0038
R_1	0.222	0.222	0.222
ψ^θ	0.87	0.91	0.91
ψ_0^ϑ	-0.145	-0.145	-0.145
ψ_1^ϑ	0.968	0.968	0.968
ψ_2^ϑ	0.1503	0.1503	0.1503
ψ^x	0.40	0.40	0.40
ψ^Ω	–	–	0.79
σ_θ	0.0133	0.013	0.0123
σ_ϑ	0.2135	0.1820	0.15
σ_x	0.21	0.21	0.20
σ_Ω	–	–	0.045
$Corr\left\{\varepsilon^\theta, ln(\varepsilon^\vartheta)\right\}$	0.81	0.81	0.81
$Corr\left\{\varepsilon^\theta, \varepsilon^x\right\}$	0.45	0.45	0.75
$Corr\left\{\varepsilon^x, ln(\varepsilon^\vartheta)\right\}$	0.70	0.70	0.70
$Corr\left\{\varepsilon^\Omega, ln(\varepsilon^\vartheta)\right\}$	–	–	0.55

Table 7: Point-Estimates of the Markup Process

Parameter	100%	90%	80%	70%	60%	50%
$\widehat{\psi}_1^{\vartheta}$	0.9643	0.9434	0.9827	0.9718	0.9648	0.9491
$\widehat{\sigma}_{\vartheta}$	0.1413	0.1831	0.1713	0.2334	0.3485	0.3862

Table 8: Standard Deviations

	2-50				2-8				8-50			
	Data	MI	MII	MIII	Data	MI	MII	MIII	Data	MI	MII	MIII
YN	3.41	3.01	3.02	2.78	1.68	1.12	1.13	1.03	2.96	2.79	2.80	2.58
	(2.77, 4.06)				(1.30, 2.07)				(2.21, 3.71)			
C	1.88	2.29	2.21	1.64	0.74	0.76	0.57	0.38	1.73	2.16	2.13	1.60
	(1.44, 2.31)				(0.59, 0.88)				(1.28, 2.19)			
I	9.71	11.27	9.57	11.65	4.29	4.87	5.28	5.48	8.71	10.15	7.98	10.27
	(7.76, 11.66)				(3.55, 5.03)				(6.86, 10.56)			
SR	2.48	2.57	2.74	2.51	1.06	1.16	1.17	1.08	2.21	2.30	2.48	2.27
	(2.04, 2.93)				(0.73, 1.40)				(1.88, 2.53)			
PT	7.50	8.00	8.61	8.83	2.80	3.07	3.24	3.18	6.96	7.39	7.98	8.24
	(6.04, 8.97)				(1.98, 3.63)				(5.49, 8.44)			
MVC	23.96	16.61	19.37	18.98	9.81	6.41	8.56	8.08	21.78	15.32	17.38	17.16
	(19.34, 28.58)				(6.93, 12.70)				(17.80, 25.76)			
RC	14.80	9.92	13.03	12.18	13.23	8.71	11.75	10.83	6.45	4.73	5.64	5.56
	(10.19, 19.41)				(9.36, 17.09)				(4.86, 8.05)			
D	8.08	10.59	9.32	12.21	4.34	4.15	3.41	6.31	6.68	9.74	8.67	10.45
	(5.96, 10.21)				(3.45, 5.23)				(5.10, 8.26)			
PD	23.17	8.13	13.53	24.00	9.32	3.19	7.07	12.38	21.12	7.48	11.53	20.56
	(17.85, 28.48)				(7.62, 11.03)				(16.59, 25.64)			
R	2.48	0.78	2.16	2.56	1.32	0.30	1.49	1.39	2.06	0.73	1.56	2.15
	(1.69, 3.26)				(0.94, 1.69)				(1.44, 2.68)			

Note: Both data and model's simulations have been filtered for various frequency bands: 2-50 years, 2-8 years, and 8-50 years. YN: corporate value added; C: consumption; I: investment; SR: Solow residual; MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. MI refers to Model I; MII refers to Model II; and MIII refers to Model III. 95-percent confidence intervals appear beneath sample statistics.

Table 9: Autocorrelations

	2-50				2-8				8-50			
	Data	MI	MII	MIII	Data	MI	MII	MIII	Data	MI	MII	MIII
YN	0.71 (0.54, 0.87)	0.83	0.83	0.83	0.18 (0.03, 0.32)	0.05	0.07	0.01	0.86 (0.75, 0.97)	0.95	0.95	0.96
C	0.80 (0.65, 0.95)	0.86	0.91	0.92	0.25 (0.11, 0.39)	0.07	0.24	0.26	0.89 (0.79, 0.99)	0.96	0.96	0.96
I	0.77 (0.63, 0.92)	0.78	0.64	0.73	0.25 (0.10, 0.40)	0.06	-0.02	-0.03	0.90 (0.78, 1)	0.94	0.93	0.95
SR	0.80 (0.68, 0.91)	0.76	0.78	0.77	0.18 (-0.01, 0.37)	0.04	0.05	0.00	0.92 (0.84, 1)	0.94	0.94	0.95
PT	0.85 (0.71, 0.99)	0.83	0.84	0.85	0.03 (-0.19, 0.26)	0.24	0.26	0.29	0.97 (0.86, 1)	0.94	0.94	0.94
MVC	0.79 (0.63, 0.95)	0.82	0.77	0.78	0.00 (-0.13, 0.13)	0.06	0.02	0.06	0.95 (0.86, 1)	0.95	0.95	0.95
RC	0.01 (-0.24, 0.25)	-0.02	-0.09	-0.04	-0.21 (-0.39, -0.04)	-0.28	-0.31	-0.29	0.89 (0.75, 1)	0.87	0.87	0.87
D	0.66 (0.48, 0.85)	0.82	0.84	0.68	0.09 (-0.05, 0.24)	0.04	0.08	0.00	0.90 (0.76, 1)	0.96	0.96	0.94
PD	0.78 (0.60, 0.95)	0.81	0.68	0.69	-0.03 (-0.17, 0.10)	0.07	-0.01	0.03	0.94 (0.82, 1)	0.95	0.94	0.93
R	0.67 (0.54, 0.80)	0.82	0.44	0.67	0.10 (-0.18, 0.39)	0.01	-0.07	0.04	0.91 (0.77, 1)	0.96	0.91	0.93

Note: Both data and model's simulations have been filtered for various frequency bands: 2-50 years, 2-8 years, and 8-50 years. YN: corporate value added; C: consumption; I: investment; SR: Solow residual; MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. MI refers to Model I; MII refers to Model II; and MIII refers to Model III. 95-percent confidence intervals appear beneath sample statistics.

Table 10: Correlations with YN

	2-50				2-8				8-50			
	Data	MI	MII	MIII	Data	MI	MII	MIII	Data	MI	MII	MIII
C	0.72	0.96	0.96	0.81	0.81	0.95	0.90	0.66	0.71	0.96	0.97	0.84
	(0.46, 0.98)				(0.68, 0.93)				(0.39, 1)			
I	0.75	0.82	0.89	0.85	0.78	0.85	0.94	0.93	0.75	0.82	0.90	0.84
	(0.55, 0.95)				(0.64, 0.92)				(0.56, 0.95)			
SR	0.52	0.94	0.97	0.92	0.70	1.00	1.00	1.00	0.46	0.94	0.96	0.91
	(0.20, 0.85)				(0.49, 0.90)				(0.10, 0.82)			
PT	0.17	0.28	0.38	0.53	0.06	0.32	0.34	0.36	0.21	0.27	0.39	0.56
	(-0.07, 0.42)				(-0.21, 0.35)				(-0.07, 0.49)			
MVC	0.17	0.59	0.82	0.58	0.03	0.57	0.83	0.42	0.20	0.59	0.83	0.61
	(-0.25, 0.60)				(-0.24, 0.30)				(-0.25, 0.67)			
RC	-0.26	0.29	0.19	0.11	-0.46	0.42	0.50	0.25	-0.19	0.35	0.05	0.05
	(-0.50, -0.03)				(-0.72, -0.21)				(-0.48, 0.09)			
D	0.02	0.34	0.50	0.16	0.09	0.33	0.23	0.00	-0.02	0.34	0.54	0.20
	(-0.29, 0.34)				(-0.12, 0.30)				(-0.38, 0.33)			
PD	0.17	0.70	0.56	0.29	-0.01	0.12	-0.14	-0.05	0.22	0.80	0.74	0.37
	(-0.30, 0.64)				(-0.29, 0.27)				(-0.27, 0.71)			
R	-0.09	-0.52	-0.74	-0.10	-0.11	-0.33	-0.83	-0.07	-0.08	-0.55	-0.79	-0.11
	(-0.35, 0.16)				(-0.47, 0.23)				(-0.33, 0.16)			

Note: Both data and model's simulations have been filtered for various frequency bands: 2-50 years, 2-8 years, and 8-50 years. YN: corporate value added; C: consumption; I: investment; SR: Solow residual; MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. MI refers to Model I; MII refers to Model II; and MIII refers to Model III. 95-percent confidence intervals appear beneath sample statistics.

Table 11: Correlations with PD

	2-50				2-8				8-50			
	Data	MI	MII	MIII	Data	MI	MII	MIII	Data	MI	MII	MIII
YN	0.17	0.70	0.56	0.29	-0.01	0.12	-0.14	-0.05	0.22	0.80	0.74	0.37
	(-0.30, 0.64)				(-0.29, 0.27)				(-0.27, 0.71)			
C	-0.03	0.77	0.47	0.39	0.13	0.10	-0.33	-0.21	-0.06	0.87	0.62	0.50
	(-0.42, 0.35)				(-0.11, 0.38)				(-0.47, 0.35)			
I	0.06	0.39	0.58	0.09	-0.01	0.15	0.02	0.04	0.07	0.44	0.81	0.10
	(-0.36, 0.49)				(-0.34, 0.33)				(-0.39, 0.53)			
SR	0.04	0.63	0.59	0.26	0.25	0.13	-0.09	-0.04	0.01	0.74	0.79	0.35
	(-0.39, 0.47)				(-0.02, 0.53)				(-0.53, 0.54)			
PT	0.39	0.34	0.29	0.30	0.03	-0.07	-0.12	-0.26	0.45	0.42	0.40	0.44
	(0.02, 0.76)				(-0.25, 0.30)				(0.02, 0.88)			
MVC	0.94	0.66	0.60	0.60	0.89	0.04	-0.07	-0.01	0.95	0.77	0.80	0.78
	(0.79, 1.00)				(0.72, 1.00)				(0.79, 1.00)			
RC	0.17	0.28	0.28	0.28	0.60	0.26	0.21	0.25	-0.11	0.44	0.48	0.41
	(-0.09, 0.43)				(0.36, 0.83)				(-0.51, 0.28)			
D	-0.07	0.41	0.26	-0.42	-0.11	0.01	-0.15	-0.07	-0.05	0.49	0.36	-0.55
	(-0.44, 0.29)				(-0.62, 0.39)				(-0.52, 0.41)			
R	0.03	-0.61	-0.56	-0.61	-0.35	-0.03	-0.05	-0.05	0.12	-0.71	-0.88	-0.83
	(-0.37, 0.44)				(-0.68, -0.02)				(-0.38, 0.63)			

Note: Both data and model's simulations have been filtered for various frequency bands: 2-50 years, 2-8 years, and 8-50 years. YN: corporate value added; C: consumption; I: investment; SR: Solow residual; MVC: market value of corporations; RC: return of MVC; D: dividends; R: risk-free interest rate; and PD: price-dividend ratio. MI refers to Model I; MII refers to Model II; and MIII refers to Model III. 95-percent confidence intervals appear beneath sample statistics.

Table 12: Variance Decomposition of PD

	$CVAR_{N,r}$		$CVAR_{N,d}$		$CVAR_{N,pd}$		Total	
Time Horizon	Data	Model	Data	Model	Data	Model	Data	Model
$N = 1$	19.37	6.72	-1.37	4.33	81.22	88.49	99.22	99.55
	(7.78, 30.95)		(-8.42, 5.68)		(67.5, 94.94)			
$N = 5$	58.70	26.18	6.81	10.87	31.94	60.85	97.45	97.91
	(43.70, 73.69)		(-8.18, 21.80)		(18.90, 44.97)			
$N = 10$	95.13	41.11	-10.17	12.31	10.15	42.81	95.10	96.24
	(62.28, 127.97)		(-22.71, 2.37)		(-11.25, 31.55)			
$N = 20$	128.58	59.76	-16.41	12.42	-11.19	21.99	100.98	94.18
	(108.94, 148.21)		(-34.52, 1.70)		(-16.97, 5.40)			
$N = 30$	118.27	69.28	-16.86	12.42	0.59	11.53	101.99	93.24
	(98.18, 138.36)		(-35.83, 2.11)		(-1.37, 2.55)			
$N = 100$		80.32		12.30		0.08		92.71

Note: Variance decomposition for different horizons N. 95-percent confidence intervals appear beneath sample statistics.