Split-Ticket Voting: An Implicit Incentive Approach

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Abstract
Voters often split tickets, voting for candidates from different parties in simultaneous elections. In this paper, I apply a political agency framework with implicit incentives to study ticket splitting in simultaneous municipal and regional elections. I show that ticket splitting is a natural outcome of the optimal reelection scheme adopted by voters to motivate politicians’ efforts in a retrospective voting environment. I assume that an office-motivated politician (mayor or governor) prefers her counterpart to be affiliated with the same political party. This correlation of incentives leads the voters to adopt a joint performance evaluation rule, which is conditioned on the politicians belonging to the same party or different parties. The model is dynamic, generating predictions of split-ticket voting over time. I show that ticket splitting is less likely than electing candidates from the same party, but somewhat depends on ticket splitting in the previous period. Ticket splitting is also more likely in smaller municipalities, where the party affiliation of a mayor is assumed to be of less importance to the governor. These theoretical results are consistent with empirical evidence from simultaneous municipal and regional elections held in Spain.

JEL classification: D72, D82.
Keywords: Split-ticket voting; Simultaneous elections; Implicit incentive contracts; Political Agency; Retrospective voting.

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1. Introduction

Split-ticket voting, when a citizen votes for candidates from different parties in simultaneous elections, is a common feature of modern political systems. Ticket splitting has mostly been studied in the context of the US, where simultaneous presidential and congressional elections are held every four years (see Burden and Kimball 2002, Fiorina 1996, Jacobson 1990, and Zupan 1991, among many others).

The literature has produced several formal models of split-ticket voting. For example, Alesina and Rosenthal (1995, 1996) elaborated on the policy balancing argument, showing that citizens strategically split tickets to avoid the extreme policies that may arise when the executive and legislative branches are allied. Chari et al. (1997) built a model based on the budgetary externality of concentrated government spending financed by uniform taxes. They found that voters prefer a fiscally conservative president (to restrain overall spending) but fiscally liberal congressmen (to promote spending in their home districts). Bugarin (2003) showed that voters control executive corruption by splitting tickets, thereby reinforcing opposition in the legislature. Fox and Van Weelden (2010) proposed an alternative rationale for ticket splitting based on the need for effective oversight of the executive. Note that all these models explain ticket splitting in the same specific institutional setting, where the final policy outcome is determined jointly by the executive and legislative branches of government. This paper complements the aforementioned literature by analyzing ticket splitting at lower levels of government (municipal and regional) where other institutional settings are possible. As a rule, mayors and governors face distinct well-defined tasks and are elected to implement distinct specific policies.

In this paper, I apply an implicit incentive approach to explain ticket splitting within a retrospective voting model (i.e., a political agency model with moral hazard). In my framework, split-ticket voting arises as an outcome of the optimal implicit reward scheme voters use to induce politicians’ efforts. The model is dynamic, generating predictions of ticket splitting over time. This feature is original; none of the aforementioned contributions has analyzed ticket splitting in a dynamic context.

I consider a representative municipality in a region where the mayoral and gubernatorial elections are simultaneous. I work with the two settings: one with a single large city whose vote is decisive for the outcome of the regional election, and one with many cities of varying size. In the latter case, each city has a probability proportional to its population of playing a pivotal role in the regional election.

I use a political agency model of interaction between politicians and their constituency
in the presence of a moral hazard problem. The politicians want to be reelected for another term, and are held accountable for their past performance at the moment of election. The politicians therefore have incentives to satisfy the voters’ wishes. In addition, I assume that the politicians are loyal to their respective political parties: the mayor prefers the governor to be affiliated with the same political party, and vice versa.\textsuperscript{1} Hence, the incentives of the mayor and governor are correlated. The voters care about the politicians’ performances, which are observable but not contractible. The voters evaluate the incumbents’ performance and vote accordingly. More precisely, the voters employ implicit evaluation rules when deciding whether to reward (reelect) politicians. Obviously, voters can influence the politicians’ behavior through their choice of evaluation rules. I restrict the space of possible evaluation rules to linear functions of performance. The evaluation rules are also required to be sequentially rational.

I show that given the correlation between the two politicians’ incentives, voters are better off adopting a joint performance evaluation rule (conditioned on the incumbents belonging to the same party or different parties) rather than an individual politician performance evaluation rule. In particular, the voters evaluate the performance of the mayor and governor from the same party as a team. If the mayor and governor belong to different parties, then the voters compare their performances to create a competitive environment. This combination of rules implies that improved performance increases a politician’s own reelection probability, while increasing/decreasing the reelection probability of her partisan ally/rival in the other office. Politicians therefore have extra incentives to perform better, for the sake of their party as well as for themselves.

In equilibrium, the reelection outcomes of incumbents from the same party are therefore positively correlated: voters tend to reward both incumbents from a well-performing party, or punish both incumbents from a poorly-performing party. The reelection outcomes of incumbents from different parties are negatively correlated: voters tend to reward the incumbent from the better-performing party while punishing the other incumbent. This combination generates a dynamic of partisan voting: whether or not the incumbent politicians belong to the same party, ticket splitting is always less likely than electing both candidates from the same party.

Next I consider two cases of politicians’ preferences, given that all politicians prefer to

\textsuperscript{1}Fox and Van Weelden (2010) introduce a similar assumption about the partisan preferences of the legislature. In particular, in their career concerns setup the legislature ("overseer") can care about the executive’s reputation. For example, a partisan overseer may seek to damage the reputation of an executive from the other party while seeking to protect the reputation of an executive from her own party.
work with candidates from their own party. First, a mayor/governor might prefer the other office to be held by a known incumbent rather than a new politician. Allied incumbents therefore have an extra incentive to perform well so that they continue working together. This preference implies that politicians from the same party exert a higher total effort than politicians from different parties. In this case the voters adopt a joint performance evaluation rule, which results in the reelection outcomes of allied incumbents being somewhat more correlated in absolute value compared with the outcomes of incumbents from different parties. Ticket splitting is therefore more likely in elections where the incumbents belong to different parties, a consequence of ticket splitting in the previous period.

The second possibility is that politicians have a preference for new allies over incumbent allies. In this case incumbents from the same party have a somewhat weaker incentive to perform well, and the total effort of an allied government is likely to be less than that of a divided government. The joint reelection outcomes for politicians from the same party are then less correlated (in absolute value) than the outcomes for politicians from different parties. Ticket splitting is thus more likely in elections where the incumbents belong to the same party, i.e., where voters did not split tickets in the previous period.

These results rest on the assumption of politicians’ party loyalty; that is, I assumed that a mayor/governor cares about the party affiliation of the governor/mayor. The joint performance evaluation rules then give extra implicit incentives to the politicians. If I relax the assumption of party loyalty, this effect vanishes and the voters no longer evaluate incumbents jointly. Instead they use a cut-off rule that each incumbent is reappointed only when her individual performance exceeds a critical threshold. In this situation, the probability of ticket splitting goes up.

Furthermore, I assume that the mayor’s party affiliation is of less importance in smaller municipalities. This may happen for two reasons: the mayors are more likely to run as independents (or be affiliated with a minor party) and the constituencies already know the candidates very well. (Recall that one of the major roles of political parties is to provide information about unknown politicians.) Thus, a governor cares less about the party affiliation of small-town mayors. As a result, the two politicians’ incentives are less correlated. In this situation, the incumbents are more likely to be evaluated individually rather than jointly, which increases the probability of ticket splitting.

My model therefore predicts several testable patterns of voting in simultaneous two-party elections.

1. The reelection outcomes of mayors and governors from the same party are positively
correlated, while the outcomes of mayors and governors from different parties are negatively correlated.

2. Ticket splitting is less likely than a vote for candidates from the same party.

3. The probability of ticket splitting is conditional upon ticket splitting in the previous period.

4. Ticket splitting is more likely to occur in small municipalities than in large ones.

In order to illustrate the model, I estimate the mayor’s and governor’s reelection probabilities and the ticket splitting probability using panel data on simultaneous municipal and regional elections held in Spain between 1983 to 2007. The predictions outlined above are consistent with the results of this empirical analysis.

I turn now to the fundamental question of why political constitution is modeled as political agency. Firstly, in addition to a sound theoretical framework, this approach has received considerable empirical support (see, for example, Peltzman 1992 and Besley and Case 1995a, 1995b, 2003). Besley (2006) provides an excellent introduction to political agency models and "emphasizes the empirical potential of these models in explaining real world policy choices." 2

Secondly, I believe that at the municipal and regional levels politicians’ tasks require mainly managerial skills. This view is supported by the empirical work of Ferreira and Gyourko (2009), who found that in US cities the mayor’s party affiliation does not affect the size of the city government and the allocation of spending. In a recent article in the New York Times, Glaeser points out that "lack of ideology has become a major feature of big city mayors... They are... managerial mayors, appreciated by voters because they succeed in making the city work." 3 The political agency approach may therefore be appropriate to model local political constitutions. Even so, elected politicians can only be offered implicit incentive schemes; public policies are difficult to reward with explicit contracts.

The retrospective voting model I use goes back to Barro (1973). Ferejohn (1986) extended the model and studied subgame-perfect equilibria rather than Nash equilibria. Persson et al. (1997) use a retrospective voting approach to study rent extraction. In Austen-Smith and Banks (1989) voters adopt retrospective voting strategies that are conditioned on the difference between the incumbent’s performance and her initial policy platform. Banks and

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Sundaram (1993, 1996) analyze retrospective voting settings with both moral hazard and adverse selection, and with term limits respectively.

The results of this paper are also related to the literature on horizontal and vertical intergovernmental competition. Most analyses of horizontal competition are based on the assumption of interjurisdictional mobility of consumers, à la Tiebout (1956). In a similar vein, the literature on yardstick competition between jurisdictions started with the seminal work of Salmon (1987), to be followed by Besley and Case (1995a), Bordignon et al. (2004), Sand-Zantman (2004), Belleflamme and Hindriks (2005), Besley and Smart (2007) and others. The main assumption of this literature is that under decentralization, voters use a comparative performance evaluation between different local governments to create yardstick competition.

The vertical competition literature, on the other hand, assumes that "senior and junior governments provide similar or comparable services, and that office-holders in the government which is judged by citizens to be the more efficient supplier will increase their probability of getting the vote of these citizens" (Breton and Salmon 2001, p. 139). I follow these authors in assuming that voters compare the performance of local and regional governments, and are likely to reward the more efficient politicians with reelection. There is, however, an important difference between my research and the papers just cited. In the intergovernmental competition literature, the comparative performance evaluation result is driven by either correlated shocks or interjurisdictional spillover. In my model, the joint performance evaluation arises from the fact that the politicians’ incentives are correlated: each one cares not only about her own reelection prospects, but also about the success of other politicians affiliated with the same political party.

The remainder of the paper is organized as follows. Section 2 lays out a model of ticket splitting in one large city pivotal to the regional election. Section 3 presents a model of ticket splitting in a region with many small cities. Section 4 provides an empirical illustration with data on Spanish elections. Finally, Section 5 concludes.

2. Ticket Splitting in a Large City

In this section I study ticket splitting in simultaneous municipal and regional elections held in a large city. I assume that the city is large enough that its vote will be decisive for the regional election.

Consider a large city, with an infinite horizon, that has to elect mayor $M$ (for the municipal
government) and governor $G$ (for the region to which this city belongs). The city is inhabited by a large number (formally a continuum) of individuals. The individuals live forever. At the beginning of each period, the elections take place simultaneously and the winners are determined by majority rule. Politicians running for both offices belong to one of the two political parties, $L$ or $R$. I assume that there is exactly one candidate from each party—the incumbent and an opponent—in each election and in each period. The opponents are identical to the incumbents in all respects except party affiliation. The participation constraints of the politicians are always satisfied, and there is no term limit.

First I will describe a stage game with a stationary environment, where time subscripts can be dropped with no risk of confusion. I will then consider an infinitely repeated game.

One stage is a sequential political agency game between politicians (the mayor and governor) and their constituency (the voters). While in office, each politician $i \in \{M, G\}$ has to implement a policy determined by her unobservable effort $a_i$. The set of efforts available to each politician is taken to be a non-degenerate interval $[0, \pi] \subset \mathbb{R}$. I assume that the performance of politician $i$, $p_i$, is observed with an independent and unobservable noise $\varepsilon_i$:

$$p_i = a_i + \varepsilon_i,$$

with $\varepsilon_i \sim N(0, \sigma^2)$.6

The reward of politician $i$ is denoted by $\Pi_i(a_i)$. Effort is costly, and I assume the standard convex cost function $\frac{a^2_i}{2}$.7 The mayor and governor independently choose effort levels $a_i$ to maximize their utility, which is given by

$$\Pi_i(a_i) - \frac{a^2_i}{2}.$$

The function $\Pi_i(a_i)$ will be explicitly defined in subsection 2.1.

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5 One can add an adverse selection problem by assuming that policy outcomes are determined by effort and ability. The results are qualitatively the same if ability is modeled as a moving average (to capture the idea that a policymaker’s competence changes slowly over time).

6 I have an extended version of the model, available upon request, where the two noise terms $\varepsilon_M$ and $\varepsilon_G$ are correlated and follow a bivariate normal distribution. I want to concentrate however on the case where voters introduce joint performance evaluation due to the correlation between politicians’ incentives rather than the correlation between shocks. The latter topic has been widely studied in the context of team evaluation in contract theory (for an overview, see Bolton and Dewatripont 2005) and in the literature on yardstick competition (see the references on yardstick competition in the Introduction).

7 I have an extended version of the model, available upon request, where the cost of policy implementation for the mayor and governor from the same party is different than for the politicians from rival parties (e.g., because of synergy). The results of this extended model are qualitatively the same.
There is no cost of voting, and I assume that there are no abstainers. The individuals differ in their preferences over political parties. I assume that some individuals always prefer party $L$ to party $R$, and vote for candidates from party $L$ in both elections. Likewise, other individuals are loyal to party $R$. However, there is a large group of uncommitted individuals, whose votes are decisive for the outcome of both elections. These voters care about the politicians’ performances in each period according to a linear utility function

$$p_M + p_G.$$  

In what follows, I refer to this group of decisive voters simply as "the voters".

I assume that the voters coordinate to apply the same retrospective reappointment rules to reelect the incumbents. I follow the literature (e.g. Persson et al. 1997) in restricting the strategy space such that the voters base their reappointment decision solely on the politicians’ performances in the current period, not in any previous period. See Persson et al. (1997) for a discussion of the plausibility of this approach, and Fair (1978) and Kramer (1971) for empirical findings in its favor.

Denote by $S$ the state where mayor $M$ and governor $G$ are members of the same party (either $L$ or $R$), and by $D$ the state where $M$ and $G$ belong to different parties. State $S$ and state $D$ occur when in the previous period the voters did not split or split tickets respectively.

The timing of events in the stage game is as follows. First, state $S$ or $D$ is realized. Second, the voters decide on the reappointment rules to be used in the coming elections. Third, the politicians exert efforts $a_M$ and $a_G$. Fourth, the politicians’ performances $p_M$ and $p_G$ are observed. Finally, both elections take place simultaneously and the voters apply the selected reappointment rules to reward or punish the incumbents.

In the following subsection I describe the politicians’ preferences. I will then turn to the voters’ problem and define an equilibrium concept.

1. Politicians

The politicians’ preferences are as follows. First, once elected, mayor $M$ and governor $G$ want to be reelected in the next period. Moreover, $M$ wants to improve her party’s chances to win the governor’s office in the coming election. If $G$ and $M$ belong to the same party, then $M$ prefers $G$ to be reelected. Otherwise, $M$ wants the voters to appoint a new governor (from her own party) for the next term. Likewise, $G$ wants $M$ to be reelected if they are members of the same party, and wants the opponent to be appointed if $M$ is from the rival party. The value of holding office is normalized to 1. The values, which $M$ and $G$ associate to their parties’ holding the other office, are denoted by $\lambda_M$ and $\lambda_G$ respectively.
Furthermore, denote by $\Pr_i(\cdot)$ the probability of being reelected to office $i \in \{M, G\}$ in the coming election. Therefore, politician $i$ has the following reward function $\Pi_i : [0, \overline{a}]^2 \to \mathbb{R}$ that depends continuously on both politicians’ efforts:

$$\Pi_i(a_i, a_j) = \begin{cases} 
\Pr_i(a_i, a_j) + \lambda_i^S \Pr_j(a_i, a_j) & \text{if } S \\
\Pr_i(a_i, a_j) + \lambda_i^D (1 - \Pr_j(a_i, a_j)) & \text{if } D,
\end{cases}$$

where $i, j \in \{M, G\}$ and $j \neq i$. The preferences stated above reflect the politicians’ allegiance to their respective parties; individual politicians care about their party’s overall representation in mayor and governor offices, not just their own reelection prospects.\(^8\) Still, the reasonable assumption here is that a mayor/governor values her own office more than her party’s representation in the other office; i.e., $0 \leq \lambda_i \leq 1$.\(^9\) I call $\lambda_i$ the degree of politician $i$’s loyalty to her party (or the strength of her party alignment). I consider two cases of politicians’ preferences. If there is some preference for incumbents over unknown candidates, then $\lambda_i^S \geq \lambda_i^D$. This case reflects the idea that a mayor/governor might prefer an incumbent ally to an unknown ally for the other office. If politicians prefer newcomers, then $\lambda_i^S < \lambda_i^D$. In this case a mayor/governor would like a new ally (a newcomer) to be elected for the other office.

The model can be generalized to the case where the incumbents maximize their intertemporal utility function (as in Ferejohn 1986). At this stage, however, I want to concentrate on the interactions between politicians and voters. I therefore assume, as in Alesina and Tabellini (2008), that the incumbents are myopic: they care about reelection only in the next period, not in any subsequent period.

2.2. Voters

The politicians’ performances $p_M$ and $p_G$ (but not their composition between effort and noise) are observed at the end of each period but are not contractible. Public policies are difficult to reward with explicit contracts. It is more natural to use implicit incentive contracting in this situation. I assume that the voters coordinate on the same retrospective voting rule, and that there is no coordination failure among the voters. A coordination problem is a serious issue, but lies beyond the scope of the paper.

\(^8\)Alternatively, the stated preferences could arise because the mayor and governor have to interact while in office. Each prefers working with a member of her own party rather than a rival.

\(^9\)In other words, politician $i$ does not mind reducing her reelection chances by $1\%$ in exchange for increasing her ally’s election probability by $\frac{1}{\lambda_i}% \geq 1\%$.
The voters observe politicians’ performances \( p_M \) and \( p_G \), and in the coming elections they reward incumbents according to their performances in the current period; i.e., they reappoint incumbents who have shown "good" results in the current period. A politician thrown out of office is never reappointed. In this case an opponent from the rival party is elected.

Obviously the voters can influence the politicians’ behavior through their choice of evaluation rules. Intuitively, since politicians care about each others’ reelection chances, the reward rules should allow for joint performance evaluation. Under joint performance evaluation the voters condition politician \( i \)’s reelection on her own performance \( p_i \) (giving her an incentive to perform well since she wants to be reelected) and on the performance \( p_j \) of politician \( j \) (giving an incentive to politician \( j \) since he cares about \( i \)’s reelection chances).

I restrict the functional space of performance evaluation rules to linear joint evaluation rules \((\beta_M, b_M)\) and \((\beta_G, b_G)\). \( \beta_M \) and \( \beta_G \) are the slopes of the mayor and governor evaluation rules respectively, while \( b_M \) and \( b_G \) are the corresponding intercepts; \( \beta_M, \beta_G, b_M, b_G \in \mathbb{R}, \beta_M \beta_G \leq 1 \). Under rules \((\beta_i, b_i), i \in \{M, G\}\), the probability of being reelected for office \( i \) is

\[
Pr_i(a_i, a_j) = P \left( \{ p_i(a_i) + \beta_i p_j(a_j) \geq b_i \} \right)
\]

with \( i, j \in \{M, G\} \) and \( j \neq i \). Figure 2.1 depicts the possible outcomes for \( M \) and \( G \) under rules \((\beta_M, b_M)\) and \((\beta_G, b_G)\) in the two-dimensional space of observed performances \( p_M \) and \( p_G \). Note that I require \( \beta_M \beta_G \leq 1 \), so that line \( p_M + \beta_M p_G = b_M \) is steeper than line \( p_G + \beta_G p_M = b_G \). Otherwise, as one can see from Figure 2.1, a mayor and governor with poor performance would be reelected while politicians with better performance would not.

Note that under linear rules \((\beta_M, b_M)\) and \((\beta_G, b_G)\), \( M \)’s reelection is determined by random variable \( \varepsilon_M + \beta_M \varepsilon_G \sim N \left( 0, (1 + \beta_M^2) \sigma^2 \right) \), and \( G \)’s reelection is determined by random variable \( \varepsilon_G + \beta_G \varepsilon_M \sim N \left( 0, (1 + \beta_G^2) \sigma^2 \right) \). I say that the two reelection events are independent when \( \beta_M = 0 \) and \( \beta_G = 0 \), positively correlated when \( \beta_M > 0 \) and \( \beta_G > 0 \), and negatively correlated when \( \beta_M < 0 \) and \( \beta_G < 0 \).

Since differentiable functions are linear in first-order approximation, the restriction to linear rules gives an approximate fit to more general evaluation rules. Furthermore, the linear evaluation rules allow for a closed-form solution. Linear approximation methods are widely used in macroeconomics to search for time-consistent equilibria (e.g. Krusell et al. 1997). In the contract theory literature, linear contracts have been shown to be optimal under some realistic assumptions (for an overview, see Bolton and Dewatripont 2005).

As I mentioned above, in this framework split-ticket voting emerges naturally from the chosen evaluation rules. That is, following the chosen rules can result in the election of politicians from different parties. Henceforth I will find it more convenient to refer to \( S \)
and $D$ as the states characterized by the politicians belonging to the same party or different parties, keeping in mind that state $S$ or state $D$ occurs when the voters did not split tickets or split tickets respectively.

### 2.3. Equilibrium Concept

In the stage game I search for a subgame perfect equilibrium by analyzing the game backwards. First, I solve for the politicians’ efforts $a_M$ and $a_G$ under rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$. Second, I examine the voters’ choice of evaluation rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$. In what follows I introduce two definitions.

Given linear performance evaluation rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$, the equilibrium in effort strategies is a profile of efforts $(a^e_M, a^e_G)$ such that

$$\Pi_i(a^e_i, a^e_j) - \frac{a^e_i}{2} \geq \Pi_i(a_i, a^e_j) - \frac{a^2}{2}$$

for each $a_i \in [0, \bar{a}]$,

where $i, j \in \{M, G\}, \ i \neq j$.

The voters are rational, so they realize that the only alternative to reelecting incumbents is voting for opponents from rival parties. The politicians’ performances are additively separable in effort and noise, and all politicians behave in the same way irrespective of the noise. If elected, opponent $i$ will exert equilibrium effort $a^e_i$, which maximizes her expected utility. Thus, the voters compare the incumbents’ performances with their opponents’ expected
performances and vote accordingly. Formally,

\[ b_i = a_i^e + \beta_i a_j^e. \]

Thus, the linear joint performance evaluation rules are solely determined by slopes \( \beta_i \) and \( \beta_j \).

I define an equilibrium in rule strategies as the pair \((\beta_M^*, \beta_G^*)\) such that

\[
\alpha_M^e (\beta_M^*, \beta_G^*) + \alpha_G^e (\beta_M^*, \beta_G^*) = \max_{\beta_M, \beta_G \leq 1} \alpha_M^e (\beta_M, \beta_G) + \alpha_G^e (\beta_M, \beta_G),
\]

where \((\alpha_M^e (\cdot), \alpha_G^e (\cdot))\) is an equilibrium in effort strategies. Finally, the politicians’ equilibrium efforts are denoted by \( a_i^* = a_i^e (\beta_i^*, \beta_j^*), i, j \in \{M, G\}, i \neq j \).

Now consider an infinitely repeated game where each stage is the sequential political agency game presented above. First, recall that the reappointment decision depends only on the politicians’ performances in the current period and not in any previous period. Second, recall that the incumbents are "myopic": they care about reelection only in the next period and not in any subsequent period. Under these assumptions on the voters’ strategy space and the politicians’ preferences, I consider a Markov Perfect Equilibrium of the infinitely repeated game where a stage game equilibrium in rule strategies is replicated infinitely often. The payoff-relevant states are \( S \) (politicians are affiliated with the same party) and \( D \) (politicians are affiliated with different parties).

I must stress here that the evaluation rules are required to be sequentially rational; no precommitment is allowed. The model parameters are common knowledge, so the politicians know whether the voters used the evaluation rules that had been rational for them in the previous period or deviated. In the latter case, the politicians conclude that the voters reappoint incumbents randomly or use unknown rules that are not based on performance. As a result, from that period onward, the politicians will exert zero effort to minimize their costs. The voters know this, so they have no incentives to deviate and always reward incumbents according to the chosen rules.

### 2.4. Equilibrium

First, consider one stage game. Let the voters use evaluation rules \((\beta_i, b_i)\) such that \( b_i = a_i^e + \beta_i a_j^e \). Under these rules the politician \( i \)'s utility is

\[
\Pi_i (a_i, a_j) = \frac{a_i^2}{2} + \frac{a_j^2}{2}
\]
\[
\begin{align*}
&\begin{cases}
    P\left(\{p_i(a_i) + \beta_i p_j(a_j) \geq b_i\}\right) + \lambda_i^S P\left(\{p_j(a_j) + \beta_j p_i(a_i) \geq b_j\}\right) - \frac{\sigma_i^2}{2} & \text{if } S \\
P\left(\{p_i(a_i) + \beta_i p_j(a_j) \geq b_i\}\right) + \lambda_i^D \left(1 - P\left(\{p_j(a_j) + \beta_j p_i(a_i) \geq b_j\}\right)\right) - \frac{\sigma_i^2}{2} & \text{if } D.
\end{cases}
\end{align*}
\]

Politician \(i\) chooses effort \(a_i\) before observing realization of the noise, and takes the voters’ expectations as given. Figure 2.2 depicts the politicians’ best response functions in states \(S\) and \(D\) for three scenarios: independent reelection outcomes with \(\beta_i = 0\) and \(\beta_j = 0\) (black), positively correlated reelection outcomes with \(\beta_i > 0\) and \(\beta_j > 0\) (red), and negatively correlated reelection outcomes with \(\beta_i < 0\) and \(\beta_j < 0\) (blue). Note that for independent reelection outcomes (black) the best responses are flat in both states (since each politician’s reelection depends only on her own effort). For positively correlated reelection outcomes (red) the best responses shift upwards in state \(S\) and downwards in state \(D\). Intuitively, under positively correlated reelections a politician has extra incentive to exert effort in state \(S\) (to increase her ally’s reelection chances) and less incentive in state \(D\) (to avoid helping her rival get reelected). Finally, for negatively correlated reelection outcomes (blue) the best responses shift downwards in state \(S\) and upwards in state \(D\). In this scenario a politician does not want to damage her ally’s reelection prospects, so exerts a lower effort in state \(S\). However, in state \(D\) she has extra incentive to work harder and reduce her rival’s reelection chances.

Note that there is a free-riding effect under positively correlated reelection outcomes in state \(S\). Intuitively, politician \(i\) might prefer to exert a lower effort (and reduce effort cost) if her partisan ally \(j\) is performing well enough to improve her reelection prospects.

The result below establishes the existence of an equilibrium in effort strategies. Proofs of this and other propositions are given in the Appendix.

**Proposition 1.** Under linear performance evaluation rules \(\beta_M\) and \(\beta_G\) with \(\beta_M\beta_G \leq 1\), there exists an equilibrium in effort strategies \((a_M^e, a_G^e)\) given by

\[
a_i^e = \begin{cases} 
\frac{1}{\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1+\beta_i^2}} + \frac{\lambda_i^S \beta_j}{\sqrt{1+\beta_j^2}} \right) & \text{if } S \\
\frac{1}{\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1+\beta_i^2}} - \frac{\lambda_i^D \beta_j}{\sqrt{1+\beta_j^2}} \right) & \text{if } D,
\end{cases}
\]

where \(i, j \in \{M, G\}, i \neq j\).

Turn now to the voters’ choice of evaluation rules \(\beta_M\) and \(\beta_G\). Maximizing \(a_M^e + a_G^e\) with respect to \(\beta_M\) and \(\beta_G\) yields an equilibrium in rule strategies \((\beta_M^*, \beta_G^*)\). I summarize the results in the following proposition (the proof is straightforward).
Figure 2.2: Best response functions of politicians $i$ and $j$ for independent reelections (black), positively correlated reelections (red) and negatively correlated reelections (blue) in states $S$ and $D$.

**Proposition 2.** There exists an equilibrium in rule strategies $(\beta^*_M, \beta^*_G)$ given by

$$
\beta^*_i = \begin{cases} 
\lambda^S_j & \text{if } S \\
-\lambda^D_j & \text{if } D,
\end{cases}
$$

where $i, j \in \{M, G\}, i \neq j$. The politicians' equilibrium efforts $a^*_i$ are equal to

$$
a^*_i = \begin{cases} 
\frac{1}{\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1+(\lambda^S_j)^2}} + \frac{(\lambda^S_j)^2}{\sqrt{1+(\lambda^S_j)^2}} \right) & \text{if } S \\
\frac{1}{\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1+(\lambda^D_j)^2}} + \frac{(\lambda^D_j)^2}{\sqrt{1+(\lambda^D_j)^2}} \right) & \text{if } D.
\end{cases}
$$

According to Proposition 2, if politician $j$ is loyal to her political party (i.e., $\lambda^*_j \neq 0$), the voters adopt a joint performance evaluation rule to reelect politician $i$. The probability of being reelected to office $i$ under the joint rule is equal to

$$
Pr_i (a_i, a_j) = \begin{cases} 
P \left( \{ p_i (a_i) + \lambda^S_j p_j (a_j) \geq a^*_i + \lambda^S_j a^*_j \} \right) & \text{if } S \\
P \left( \{ p_i (a_i) - \lambda^D_j p_j (a_j) \geq a^*_i - \lambda^D_j a^*_j \} \right) & \text{if } D.
\end{cases}
$$

Intuitively, the incentives of a mayor and governor are correlated, because they care about the overall representation of their party in both offices. The voters therefore reward politicians jointly rather than separately.
If the politicians belong to the same political party (state $S$), then the voters use a joint rule under which the reelection of politician $i$ is positively correlated with the performance of politician $j$ ($\beta_i > 0$). As a result, the voters evaluate the performance of the politicians from the same party as a team and tend to reward the incumbents from a well-performing party while punish the incumbents from a badly-performing party.

However, if the politicians belong to different parties (state $D$), the voters use a joint rule under which the reelection of politician $i$ is negatively correlated with the performance of politician $j$ ($\beta_i < 0$). As a result, the voters compare the performance of one politician to that of the other, creating a competitive environment between the parties. In this scenario the voters tend to reward the incumbent from the better-performing party, while punishing the incumbent from the worse-performing party.

In sum, due to the correlation between the mayor’s and governor’s incentives such that they care about their party chances of holding office, the voters are better off adopting party performance evaluation rather than individual performance evaluation.

Furthermore, the more loyal politician $j$ is to her political party (the higher $\lambda_j$ is), the more correlated the optimal reward scheme for politician $i$ is with the performance of politician $j$ (positively if $S$ or negatively if $D$). Intuitively, if the politicians care equally about their own reelection chances and their party’s election chances, then the best reward scheme would be perfectly correlated: in state $S$ the incumbents are always reelected or dismissed together, while in state $D$ reelection of one implies dismissal of the other.

The less loyal the politicians are to their political parties, the less correlated their incentives. As a result, the voters adopt the less correlated reelection rules in equilibrium. If politician $j$ is not at all loyal to her political party ($\lambda_j = 0$), then the optimal rule to reappoint politician $i$ is a simple cut-off rule: she is reappointed only if her observed performance exceeds a critical threshold given by the equilibrium effort for this office. That is, the probability of being reelected to office $i$ depends only on $a_i$:

$$P r_i (a_i) = P \left( \{ p_i (a_i) \geq a_i^* \} \right).$$

Intuitively, when politicians care only about their own reelection prospects, the voters are better off rewarding politician’s individual performance rather than the party’s performance.

Next, compare the equilibrium efforts of politicians from the same party and from different parties, as given in (2.2). If there is some preference for incumbents (i.e., $\lambda_i^S \geq \lambda_i^D$) then politicians from the same party exert a higher total effort than politicians from different parties:

$$(a_M^* + a_G^*)|_S \geq (a_M^* + a_G^*)|_D \text{ if } \lambda_M^S \geq \lambda_M^D \text{ and } \lambda_G^S \geq \lambda_G^D.$$
Intuitively, allied politicians have extra incentive to exert greater effort (thereby increasing the probability of their counterparts’ reelection).

If a mayor and governor prefer newcomers (i.e., $\lambda^S_i < \lambda^D_i$) then politicians from the same party have less incentive to perform well. So politicians from the same party exert a lower total effort than politicians from different parties:

$$(a^*_M + a^*_G) |_S < (a^*_M + a^*_G) |_D \text{ if } \lambda^S_M < \lambda^D_M \text{ and } \lambda^S_G < \lambda^D_G.$$ 

How do the equilibrium efforts $a^*_i$ in (2.2) depend on parameters’ values? First, larger variance of noise $\sigma^2$ decreases the politicians’ efforts. Intuitively, more randomness in the observed performances $p_M$ and $p_G$ makes the reelection probabilities less sensitive to effort, reducing the politicians’ incentives. Second, if politician $i$’s party alignment $\lambda_i$ is strengthened, the equilibrium effort of politician $i$, $a^*_i$ increases while that of politician $j$, $a^*_j$ decreases. The more politician $i$ cares about her ally’s appointment to office $j$, the more incentives she has to perform better. However, this weakens politician $j$’s incentives to exert effort, because his reelection becomes less sensitive to his own effort.

In the infinitely repeated game, one can show that there exists a Markov Perfect Equilibrium such that in each stage the voters’ rule strategies are given by (2.1) and the politicians’ efforts are given by (2.2).

2.5. Dynamics

In this section I calculate the equilibrium probabilities of transition between state $S$ (where the politicians are members of the same party) and state $D$ (where the politicians belong to different parties). I denote by $P_{kl}$ the probability that a city in state $k$ will shift to state $l$ in the next period, $k, l \in \{S, D\}$. I establish the following result.

**Proposition 3.** The matrix of the equilibrium one-step transition probabilities between states $S$ and $D$ is

$$
\begin{bmatrix}
P_{SS} & P_{SD} \\
P_{DS} & P_{DD}
\end{bmatrix} = 
\begin{bmatrix}
\frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda^S_M + \lambda^S_G}{1 - \lambda^S_M \lambda^S_G} & \frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda^S_M + \lambda^S_G}{1 - \lambda^S_M \lambda^S_G} \\
\frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda^D_M + \lambda^D_G}{1 - \lambda^D_M \lambda^D_G} & \frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda^D_M + \lambda^D_G}{1 - \lambda^D_M \lambda^D_G}
\end{bmatrix}
$$

where $\arctan (\cdot)$ is the arctangent function.

Note that independently of the current state, the next state is more likely to be $S$ than $D$. Indeed, the probability of the next state being a split-ticket state is never greater than $\frac{1}{2}$.
Figure 2.3: Politicians’ reelection outcomes under equilibrium rules $\beta^*_M$ and $\beta^*_G$ in states $S$ and $D$.

$P_D \in [0, \frac{1}{2}]$. The intuition for this result is as follows. If the politicians currently belong to the same party (state $S$), the voters adopt a joint rule under which the reelection outcomes are positively correlated: the incumbents are more likely to be reelected together or dismissed together than they are to receive opposite rewards. Thus, the next state is more likely to be $S$. If the politicians are currently members of different parties (state $D$), then the voters use a joint rule under which the reelection outcomes are negatively correlated. Thus, it is more likely that one incumbent will be dismissed while the other is reelected, and again the next state is more likely to be $S$ than $D$. To confirm this intuition, in Figure 2.3 I depict the politicians’ reelection outcomes under equilibrium rules $\beta^*_M$ and $\beta^*_G$ in the two-dimensional space of performances $p_M$ and $p_G$. The density function of the joint distribution of $p_M$ and $p_G$ is symmetric around $(\alpha^*_M, \alpha^*_G)$.

Furthermore, independently of the current state, the probability $P_D$ that the next state will be $D$ is decreasing in $\lambda^*_M$ and $\lambda^*_G$. This probability takes its minimal value of 0 when $\lambda^*_M = \lambda^*_G = 1$, and its maximal value of $\frac{1}{2}$ when $\lambda^*_M = \lambda^*_G = 0$. Intuitively, the more loyal politicians are to their parties, the more correlated (positively if $S$ or negatively if $D$) the optimal performance evaluation rules. The outcome $S$ is more probable for both current states, as explained above, so stronger party loyalty just increases the probability of this outcome.
How does the current state affect the probability that the next state is state \( k, k \in \{S, D\} \)?

First, if politicians prefer incumbents \((\lambda_i^S \geq \lambda_i^D)\), then \( P_{DD} \geq P_{SD} \) and \( P_{SS} \geq P_{DS} \); although \( S \) is always more likely than \( D \), the next state is more likely to be \( k \) if the current state is \( k \). Intuitively, due to the politicians’ preference for incumbents, the politicians’ incentives are more correlated in state \( S \) than in state \( D \). The voters are therefore more likely to adopt a positively correlated reelection rule in \( S \) than a negatively correlated rule in \( D \). While both states are more likely to shift to \( S \) in the next period, the \( S \) outcome is more likely to occur if the current state is \( S \). By the same logic, a \( D \) outcome is more likely if the current state is \( D \) than if the current state is \( S \).

Second, if politicians prefer newcomers \((\lambda_i^S < \lambda_i^D)\), then \( P_{DD} < P_{SD} \) and \( P_{SS} < P_{DS} \). Here the politicians’ incentives are more correlated in state \( D \) than in state \( S \). Thus, the joint reelection rules are more correlated (in absolute value) in state \( D \) than in state \( S \). State \( S \) is therefore more likely to occur if the current state is \( D \), while state \( D \) is more likely to occur if the current state is \( S \).

To sum up, in this simultaneous election framework, ticket splitting is less likely than voting for candidates from the same party, i.e., \( P_D \leq P_S \). (Recall that states \( S \) and \( D \) occur when the voters did not and did split tickets respectively.) Moreover, the probability of split-ticket voting depends on ticket splitting in the previous period. If politicians prefer incumbents, then the voters are more likely to split tickets if in the previous period they also split tickets:

\[
P_{DD} \geq P_{SD} \text{ if } \lambda_M^S \geq \lambda_M^D \text{ and } \lambda_G^S \geq \lambda_G^D.
\]

If politicians prefer newcomers, then the voters are more likely to split tickets if in the previous period they did not split ticket:

\[
P_{DD} < P_{SD} \text{ if } \lambda_M^S < \lambda_M^D \text{ and } \lambda_G^S < \lambda_G^D.
\]

3. Ticket Splitting in a Region with Small Municipalities

In this section I assume that a region consists of \( n \) municipalities, and that each is pivotal for the outcome of the regional election with some small probability. The main insights and intuitions of the large city case (Section 2) do not change qualitatively. Still, some novel results arise. This section stresses the novel assumptions and results, referring to the previous analysis whenever appropriate.

The model is identical to that presented in Section 2, except for the following changes. The region consists of \( n \) municipalities with population shares \( v_i \), \( \sum_{i=1}^{n} v_i = 1 \). At the
beginning of each period the voters in municipality $i$ elect mayor $M_i$ and vote for governor $G$ in the simultaneous elections. The probability that municipality $i$ is pivotal for the outcome of the regional election is equal to its population share $v_i$. While in office, mayors $M_i$ and governor $G$ implement policies which are determined by their unobservable efforts $a_i$ and $a_G$ respectively. The performances $p_i$ and $p_G$ are observed with independent and unobservable noises $\varepsilon_i$ and $\varepsilon_G$ respectively:

$$p_i = a_i + \varepsilon_i$$
$$p_G = a_G + \varepsilon_G$$

with $\varepsilon_i, \varepsilon_G \sim N(0, \sigma^2)$, $i = 1, ..., n$.

Mayor $M_i$ chooses effort $a_i$ to maximize her utility, given by

$$\Pi_i (a_i, a_G) - \frac{a_i^2}{2} = \begin{cases} 
\Pr_i (a_i, a_G) + \lambda_i^S \Pr_G (a_i, a_G) - \frac{a_i^2}{2} & \text{if } S_i \\
\Pr_i (a_i, a_G) + \lambda_i^D (1 - \Pr_G (a_i, a_G)) - \frac{a_i^2}{2} & \text{if } D_i,
\end{cases}$$

where $S_i$ and $D_i$ denote the states where mayor $M_i$ and governor $G$ are members of the same party or different parties as before. This utility function implies that the mayor is office-motivated and prefers the governor to be a politician from the same party. The results do not change if I extend the mayor’s partisan preferences to include all other mayors in the region.

Governor $G$ is also office-motivated and loyal to her political party. She prefers to see members of her own party in all the offices $M_1, ..., M_n$. However, I assume that a governor cares more about her party’s election chances in larger cities. In other words, the larger the population of the city, the more the governor wants its mayor to be a member of her own political party. This assumption reflects the idea that party affiliation might be of less importance in smaller municipalities, either because candidates are more likely to run as independents (or be affiliated with a minor party) or because the voters have personal knowledge of the candidates for mayor office. Recall that one of the major roles of political parties is to provide information about unknown politicians. Formally,

$$\Pi_G (a_1, ..., a_n, a_G) - \frac{a_G^2}{2} = \Pr_G (a_1, ..., a_n, a_G) + \lambda_G^S \sum_{i=1}^{n} v_i I_i \Pr_i (a_i, a_G) +$$

$$\lambda_G^D \sum_{i=1}^{n} v_i (1 - I_i) (1 - \Pr_i (a_i, a_G)) - \frac{a_G^2}{2}.$$ 

$I_i$ is the indicator function of state $S_i$, defined as

$$I_i = \begin{cases} 
1 & \text{if } S_i \\
0 & \text{if } D_i.
\end{cases}$$
The voters in municipality $i$ care about the politicians’ performances in each period according to a linear utility function

$$p_i + p_G,$$

where $p_i$ and $p_G$ are observable at the end of each period. The voters coordinate in choosing a linear performance evaluation rule $(\beta_i, b_i)$ to reward mayor $M_i$, and a linear rule $(\beta_i^G, b_i^G)$ to reward governor $G$. The probability that mayor $M_i$ is reelected equals

$$Pr_i(a_i, a_G) = P \{ p_i(a_i) + \beta_i p_G (a_G) \geq b_i \}.$$

As for the governor, I assume that each municipality $i$ has a probability equal to its population share $v_i$ to be pivotal in the regional election. The probability that governor $G$ is reelected is therefore additively separable, and equal to a weighted sum of the probabilities of getting a majority in each municipality. Each municipality’s term is weighted with its population share:

$$Pr_G(a_1, ..., a_n, a_G) = \sum_{i=1}^{n} v_i P \{ p_G (a_G) + \beta_i^G p_i (a_i) \geq b_i^G \}.$$

I skip the equilibrium definitions and the discussion, which are analogous to the large city model in Section 2. Next, I characterize the equilibrium in rule strategies in the case of $n$ small municipalities.

**Proposition 4.** There exists an equilibrium in rule strategies $(\beta_i^*, \beta_i^G*)$ $i = 1, ..., n$, given by

$$\beta_i^* = \begin{cases} v_i \lambda_G^S & \text{if } S_i \\ -v_i \lambda_G^D & \text{if } D_i \end{cases}$$

and

$$\beta_i^G* = \begin{cases} \lambda_i^S & \text{if } S_i \\ -\lambda_i^D & \text{if } D_i \end{cases}.$$

The politicians’ equilibrium efforts $a_i^*, a_G^*$ are equal to

$$a_i^* = \begin{cases} \frac{1}{\sqrt{2\pi \sigma}} \left( \frac{1}{\sqrt{1 + (v_i \lambda_G^S)^2}} + \frac{v_i (\lambda_i^S)^2}{\sqrt{1 + (\lambda_i^S)^2}} \right) & \text{if } S_i \\ \frac{1}{\sqrt{2\pi \sigma}} \left( \frac{1}{\sqrt{1 + (v_i \lambda_G^D)^2}} + \frac{v_i (\lambda_i^D)^2}{\sqrt{1 + (\lambda_i^D)^2}} \right) & \text{if } D_i \end{cases}$$

and

$$a_G^* = \frac{1}{\sqrt{2\pi \sigma}} \sum_{i=1}^{n} v_i \left( \frac{1}{\sqrt{1 + (\lambda_i^S)^2 I_i + (\lambda_i^D)^2 (1 - I_i)}} \right) + \frac{v_i \left( (\lambda_G^S)^2 I_i + (\lambda_G^D)^2 (1 - I_i) \right)}{\sqrt{1 + (v_i \lambda_G^S)^2 I_i + (v_i \lambda_G^D)^2 (1 - I_i)}}.$$

The equilibrium analysis and intuition for the case of $n$ small municipalities do not differ qualitatively from the large city case presented in Section 2. The only new prediction is that the correlation (positive in state $S_i$ or negative in state $D_i$) between the optimal reward rules
for mayor $M_i$ and governor $G$ is stronger in large cities than in small cities. Intuitively, the larger the municipality, the more the governor wants its mayor to belong to the same party, so the more correlated the politicians’ incentives. As a result, the more the voters correlate (positively if $S_i$ or negatively if $D_i$) the optimal reward rule for mayor $M_i$ with performance of governor $G$ to motivate the latter to exert higher effort.

I turn now to a dynamic analysis of ticket splitting in small municipalities, which reveals more novel insights. Consider a municipality that splits tickets in the current period, i.e., the voters elect a mayor from one party and vote for a governor from the other party. This does not necessarily imply the election of the city’s preferred governor, since each municipality has only a small probability of swaying the regional election. Thus, ticket splitting in municipality $i$ does not necessarily result in the state $D_i$ where mayor and governor are members of different parties. Note that in the large city model, ticket splitting always leads to state $D$.

I find the equilibrium probabilities of transition between the ticket-splitting and non-ticket-splitting states in municipality $i$. I denote by $Y_i$ the state where voters in municipality $i$ split tickets ($Y$ stands for "yes"), and by $N_i$ the state where voters in municipality $i$ do not split tickets ($N$ stands for "no"). I denote by $q_i$ the probability that the governor who wins a majority in municipality $i$ is actually elected. In other words, probability $q_i$ is equal to the probability that an incumbent governor gets the majority in municipality $i$ and is also reelected, plus the probability that an incumbent governor does not get the majority in municipality $i$ and is not reelected. I now establish the following result.

**Proposition 5.** The matrix of the equilibrium one-step transition probabilities between states $N_i$ and $Y_i$ is

$$
\begin{bmatrix}
P_{N_iN_i} & P_{N_iY_i} \\
P_{Y_iN_i} & P_{Y_iY_i}
\end{bmatrix}
$$

where

$$
P_{N_iY_i} = \frac{1}{2} - \frac{1}{\pi} \left( q_i \arctan \frac{\lambda_i^S + v_i \lambda_G^S}{1 - v_i \lambda_i^S \lambda_G^S} + (1 - q_i) \arctan \frac{\lambda_i^D + v_i \lambda_G^D}{1 - v_i \lambda_i^D \lambda_G^D} \right)
$$

$$
P_{Y_iY_i} = \frac{1}{2} - \frac{1}{\pi} \left( q_i \arctan \frac{\lambda_i^D + v_i \lambda_G^D}{1 - v_i \lambda_i^D \lambda_G^D} + (1 - q_i) \arctan \frac{\lambda_i^S + v_i \lambda_G^S}{1 - v_i \lambda_i^S \lambda_G^S} \right)
$$

with

$$
q_i = 1 - \frac{1}{\pi} \sum_{j \neq i} v_j \arctan \sqrt{\left( \beta_i^G \beta_j^G \right)^2 + \left( \beta_i^G \beta_j^G \right)^2 + \left( \beta_i^G \beta_j^G \right)^2}.
$$

Proposition 5 generalizes the dynamic predictions of the large city case studied in Section 2. In analogy with previous results, the next state is more likely to be non ticket-splitting state
$N_i$ than ticket-splitting state $Y_i$ regardless of the current state, since $P_{Y_i} \leq \frac{1}{2}$. Furthermore, if politicians prefer incumbents ($\lambda^S_i \geq \lambda^D_i$ and $\lambda^S_G \geq \lambda^D_G$), then the municipality is more likely to split tickets if in the previous period it also split tickets ($P_{Y_iY_i} \geq P_{N_iY_i}$). If politicians prefer newcomers ($\lambda^S_i < \lambda^D_i$ and $\lambda^S_G < \lambda^D_G$), then the municipality is more likely to split tickets if in the previous period it did not split tickets ($P_{Y_iY_i} < P_{N_iY_i}$).

The following result arises only in the small-city model. According to Proposition 5, ticket splitting is more likely to happen in small municipalities than in large ones regardless of the current state. The probabilities $P_{N_iY_i}$ and $P_{Y_iY_i}$ are decreasing functions of the population share $v_i$. Intuitively, a governor cares less about the party affiliation of small-town mayors. As the politicians’ incentives are less correlated in small municipalities than in large ones, the voters adopt less correlated joint performance evaluation rules. As a result, the incumbents are more likely to be evaluated according to their individual performance, which increases the probability of ticket splitting.

4. Empirical Illustration

The goal of this section is to illustrate my theoretical model with empirical data on ticket splitting in Spain. I use an unbalanced panel data set on simultaneous municipal and regional elections held in Spain in 1983, 1987, 1991, 1995, 1999, 2003 and 2007.\(^{10}\) I find the following voting patterns in these data.\(^{11}\)

1. The reelection outcomes of a mayor and governor from the same party are positively correlated. The reelection outcomes of a mayor and governor from different parties are negatively correlated.

2. Ticket splitting is less likely than voting for candidates from the same party.

3. Voters are more likely to split tickets if in the previous period they also split tickets.

4. Ticket splitting is more likely in small municipalities than in large ones.

\(^{10}\)One might be concerned whether the majoritarian model presented here is applicable to the empirical context, since Spanish regional elections use a proportional representation system. In response, I stress that in Spain, the regional leader of the party that gets the most seats in the regional Legislative Assembly is usually elected as a president of the corresponding autonomous community. As such, ordinary citizens often regard the regional election as a way of determining the president of their autonomous community, rather than an election of their representatives.

\(^{11}\)To conduct a complete empirical analysis of my framework, I would also need data shedding light on the politicians’ party loyalty and perceived performance. To the best of my knowledge, available polls do not provide such data.
The predictions of my theoretical model are consistent with these findings. I describe the empirical model and its estimation below.

In order to demonstrate the first point, I use bivariate probit analysis to jointly estimate the mayor’s and governor’s reelection probabilities. I define $g_{it}$, a binary variable that takes the value 1 if the governor is reelected in municipality $i$ in period $t$, and 0 otherwise. By analogy, $m_{it}$ is a binary variable that takes the value 1 if the mayor is reelected in municipality $i$ in period $t$, and 0 otherwise. $v_i$ stands for municipality $i$’s time invariant population share. The control variables are $x_{it}$, regional effects are $\xi_r$ ($r$ is the region to which municipality $i$ belongs), and year effects are $\xi_t$. A detailed description of the data and precise definitions of these variables can be found in the Appendix. The theoretical model suggests the following estimating equation:

$$P(g_{it} = 1, m_{it} = 1 | v_i, x_{it}, \xi_r, \xi_t) =\Phi_2(\gamma_0 + \gamma_1 v_i + \gamma_2 x_{it} + \xi_r + \xi_t, \mu_0 + \mu_1 v_i + \mu_2 x_{it} + \xi_r + \xi_t, \rho),$$

where $\Phi_2(\cdot)$ denotes the cumulative function of the bivariate normal distribution, and $\rho$ is the tetrachoric correlation between $g_{it}$ and $m_{it}$. The region effects and year effects are included as dummies. Since the specification does not include any time invariant unobserved municipality effects that could be correlated with the explanatory variables, I can estimate the model by pooling all the cross sections.\textsuperscript{12}

To properly apply my framework, I condition equation (4.1) to two subsamples: municipalities whose mayor and governor are affiliated with the same political party at the beginning of period $t$ (i.e., before election $t$), and municipalities where the politicians are from different parties. I split the sample and estimate model (4.1) for these two subsamples.

Table 4.1 summarizes the estimated tetrachoric correlations between $g_{it}$ and $m_{it}$. The first regression includes only region dummies. The second regression includes both region and year dummies. The third regression includes only year dummies.

In the subsample where mayor and governor are affiliated with the same political party, the correlation $\rho$ is significantly positive in all specifications. In the subsample where politicians are affiliated with different political parties, $\rho$ is significantly negative in all specifications. I conclude that in agreement with my theoretical results, the reelection outcomes of politicians from the same party are positively correlated while the reelection outcomes of politicians from different parties are negatively correlated.

\textsuperscript{12}Fixed time invariant effects at the municipality level do not seem realistic over such a long period.
The politicians are affiliated with the same party before election t.

<table>
<thead>
<tr>
<th>Correlation, ρ</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.783***</td>
<td>0.790***</td>
<td>0.766***</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.057)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Observations</td>
<td>2434</td>
<td>2434</td>
<td>2434</td>
</tr>
</tbody>
</table>

The politicians are affiliated with different parties before election t.

<table>
<thead>
<tr>
<th>Correlation, ρ</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.790***</td>
<td>-0.791***</td>
<td>-0.780***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.023)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Observations</td>
<td>1749</td>
<td>1749</td>
<td>1749</td>
</tr>
</tbody>
</table>

Robust standard errors clustered by region (in (1) and (2)) and by year (in (3)) in parentheses.

Significant at 10% *, 5% **, 1% ***

(1) - only region dummies; (2) - region dummies and year dummies; (3) - only year dummies.

Table 4.1: Tetrachoric correlations ρ between g_{it} and m_{it}.

To demonstrate points 2-4 on page 22, I estimate the probability of ticket splitting. Here I apply the binary response model, which employs a probit link function. A logit model yields the same qualitative results. However, the probit model is more consistent with my theoretical framework, where re-elections are determined by normally distributed noise.

I denote by $y_{it}$ a binary variable that takes the value 1 if municipality $i$ splits tickets in period $t$, and 0 otherwise. As before, $v_i$ stands for municipality $i$’s population share, $x_{it}$ for the control variables, $\xi_r$ for region effects, and $\xi_t$ for year effects. The data and variables are described in more detail in the Appendix. I estimate the following equation:

$$ P(y_{it} = 1|v_i, y_{it-1}, x_{it}, \xi_r, \xi_t) = \Phi (\varphi_0 + \varphi_1 v_i + \varphi_2 y_{it-1} + \varphi_3 x_{it} + \xi_r + \xi_t), \quad (4.2) $$

where $\Phi (\cdot)$ denotes the cumulative function of the standard normal distribution. The region effects and year effects are included as dummies. I capture the dynamic nature of ticket splitting by introducing the lagged dependent variable, and estimate the model by pooling all the cross sections.

Table 4.2 presents the coefficients of interest in the panel regression (4.2), which estimates the probability of ticket splitting. The first regression includes only region dummies. The second regression includes both region and year dummies. In the third regression, the region
\[ P(\text{ticket splitting in } i \text{ at } t) = P(y_{it} = 1) \]

| Pop. share, \( \varphi_1 \) | (1) \(-2.544^*\) | (2) \(-2.616^{**}\) | (3) \(-2.793^*\) |
| TS in \( t-1 \), \( \varphi_2 \) | (1.332) | (1.309) | (1.456) |
| (0.043) | (0.043) | (0.04) |

Observations | 4183 | 4183 | 4177 |

Robust standard errors clustered by region in parentheses. Significant at 10\% \(^*\); 5\% \(^{**}\); 1\% \(^{***}\).

(1) – only region dummies; (2) – region dummies and year dummies; (3) – region-year dummies.

Table 4.2: Ticket splitting (TS) and municipality population share.

<table>
<thead>
<tr>
<th>( \hat{P}(y_{it} = 1) )</th>
<th>obs.</th>
<th>mean</th>
<th>std. dev.</th>
<th>min</th>
<th>max</th>
<th>std. err.</th>
<th>95% conf. interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4183</td>
<td>0.193</td>
<td>0.106</td>
<td>0.006</td>
<td>0.487</td>
<td>0.002</td>
<td>0.190</td>
</tr>
</tbody>
</table>

Table 4.3: Summary statistics of the predicted probability of ticket splitting \( \hat{P}(y_{it} = 1) \).

dummies are interacted with each year dummy.

First, consider coefficient \( \varphi_1 \) for the population share. It is significantly negative in all specifications. I conclude that ticket splitting is more likely to occur in smaller municipalities, as predicted. Second, coefficient \( \varphi_2 \) for the effect of ticket splitting in the previous period is significantly positive in all specifications. This result shows that in the sample ticket splitting is more likely in municipalities where the voters split tickets in the previous period.

Finally, Table 4.3 presents summary statistics of the predicted probability of ticket splitting \( \hat{P}(y_{it} = 1) \) in the regression (4.2), which includes both region and year dummies. The maximal predicted probability does not exceed 0.5 and is equal to 0.487. So I conclude that ticket splitting is less likely than partisan voting.

5. Conclusion

This paper applies an implicit incentive approach to study split-ticket voting in simultaneous municipal and regional elections. In a political agency model with moral hazard, ticket splitting is a natural outcome of the optimal implicit reward schemes that voters use to motivate the politicians’ efforts.

I assume that the incentives of local and regional politicians are correlated, as a mayor/governor
prefers her counterpart (the governor/mayor) to be affiliated with the same political party. Voters are therefore better off adopting a joint performance evaluation rule rather than an individual performance evaluation rule when deciding whether to reward the incumbents. Under a joint rule, I have shown that the reelection outcomes of politicians from the same party should be positively correlated while the reelection outcomes of politicians from different parties are negatively correlated. Furthermore, the adoption of a joint rule affects the dynamics of split-ticket voting. In particular, the model suggests that ticket splitting is less likely than voting for candidates from the same party. Moreover, the probability of split-ticket voting depends on ticket splitting in the previous period. Finally, ticket splitting is more likely in smaller municipalities, where the party affiliation of a mayor is assumed to be less important to the governor. These theoretical results are consistent with empirical evidence from simultaneous municipal and regional elections held in Spain.

I have focused on single task policies. However, in reality public policies pursue many goals. So it is of interest to study split-ticket voting under a more realistic assumption of multiple-task policy where the problem of effort allocation among tasks can create policy trade-offs. To refine the empirical results, one could apply other estimation methods such as a Markov switching model. It would also be interesting to examine data from other countries where municipal and regional elections are held simultaneously. I leave these tasks for future research.

Appendix

Throughout the Appendix, I use $F$ to denote the normal distribution function and $f$ for the corresponding density.

A. Proof of Proposition 1

Under linear performance evaluation rules $(\beta_i, b_i)$ the probability of being reelected for office $i$ is

$$Pr_i(a_i, a_j) = P(\{\varepsilon_i + \beta_i\varepsilon_j \geq b_i - a_i - \beta_i a_j\}) = 1 - F_{\varepsilon_i + \beta_i\varepsilon_j}(b_i - a_i - \beta_i a_j),$$

where noises $\varepsilon_i$ and $\varepsilon_j$ ($i, j \in \{M, G\}$, $i \neq j$) are independent normally distributed random variables, so by the convolution formula $\varepsilon_i + \beta_i\varepsilon_j \sim N(0, (1 + \beta_i^2) \sigma^2)$. Politician $i$’s utility is

$$\Pi_i(a_i, a_j) - \frac{\sigma^2}{2} =$$
The reelection of incumbent $i$ is determined by random variable $\varepsilon_i + \beta_i \varepsilon_j \sim N (0, (1 + \beta^2) \sigma^2)$, $i, j \in \{M, S\}$, $i \neq j$. The density function of bivariate normal distribution of random variables $\varepsilon_M + \beta_M \varepsilon_G$ and $\varepsilon_G + \beta_G \varepsilon_M$, denoted by $f_{\varepsilon_M + \beta_M \varepsilon_G, \varepsilon_G + \beta_G \varepsilon_M}(x, y)$, is

$$f_{\varepsilon_M + \beta_M \varepsilon_G, \varepsilon_G + \beta_G \varepsilon_M}(x, y) = \frac{1}{2\pi \sigma^2 \sqrt{(\beta_M \beta_G - 1)^2}} \exp \left\{ -\frac{(x - y \beta_M)^2 + (y - x \beta_G)^2}{2\sigma^2 (\beta_M \beta_G - 1)^2} \right\}.$$ 

The transition from state $k$ back to state $k$, $k \in \{S, D\}$, occurs either when both incumbents are reappointed or when none of them is reappointed (so, opponents from rival parties are elected). Denote by $p^*_i = a^*_i + \varepsilon_i$ the performance of politician $i$ in equilibrium. The equilibrium transition probabilities are

$$P_{SS} = P \left( \{ p^*_M + \lambda^S_G p^*_G \geq a^*_M + \lambda^S_G a^*_G \} \cap \{ p^*_G + \lambda^S_M p^*_M \geq a^*_G + \lambda^S_M a^*_M \} \right) + P \left( \{ p^*_M + \lambda^S_G p^*_G < a^*_M + \lambda^S_G a^*_G \} \cap \{ p^*_G + \lambda^S_M p^*_M < a^*_G + \lambda^S_M a^*_M \} \right) +$$

$$+ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{\varepsilon_M + \lambda^S_G \varepsilon_G, \varepsilon_G + \lambda^S_M \varepsilon_M}(x, y) \, dx \, dy + 0 \int_{-\infty}^{0} \int_{0}^{0} f_{\varepsilon_M + \lambda^S_G \varepsilon_G, \varepsilon_G + \lambda^S_M \varepsilon_M}(x, y) \, dx \, dy =$$
\[
\frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda_p^G + \lambda_a^G}{1 - \lambda_M^P \lambda_C^G} \\

P_{DD} = P \left( \left\{ p_m^* - \lambda_G^P p_G^* \geq a_M^* - \lambda_M^D a_M^* \right\} \cap \left\{ p_G^* - \lambda_M^P p_m^* \geq a_G^* - \lambda_M^D a_M^* \right\} \right) + \\
P \left( \left\{ p_m^* - \lambda_G^P p_G^* < a_M^* - \lambda_M^D a_M^* \right\} \cap \left\{ p_G^* - \lambda_M^P p_m^* < a_G^* - \lambda_M^D a_M^* \right\} \right) = \\
P \left( \{ \varepsilon_M - \lambda_G^P \varepsilon_G \geq 0 \} \cap \{ \varepsilon_G - \lambda_M^D \varepsilon_M \geq 0 \} \right) + \\
P \left( \{ \varepsilon_M - \lambda_G^P \varepsilon_G < 0 \} \cap \{ \varepsilon_G - \lambda_M^D \varepsilon_M < 0 \} \right) = \\
\int_0^{+\infty} \int_0^{+\infty} f_{\varepsilon_M, \varepsilon_G} \varepsilon_G - \lambda_M^P \varepsilon_M (x, y) \, dx \, dy + \int_0^{-\infty} \int_0^{-\infty} f_{\varepsilon_M, \varepsilon_G} \varepsilon_G - \lambda_M^P \varepsilon_M (x, y) \, dx \, dy = \\
\frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda_p^G + \lambda_a^G}{1 - \lambda_M^P \lambda_C^G} \\

P_{SD} = 1 - P_{SS} = \frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda_p^G + \lambda_a^G}{1 - \lambda_M^P \lambda_C^G} \\

P_{DS} = 1 - P_{DD} = \frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda_p^G + \lambda_a^G}{1 - \lambda_M^P \lambda_C^G},
\]

where \( \arctan(x) \) is an arctangent function.

**C. Proof of Proposition 4**

Under linear performance evaluation rules \((\beta_i, b_i)\) and \((\beta_i^G, b_i^G)\), \(i = 1, ..., n\), mayor \(M_i\)'s utility is

\[
\Pi_i(a_i, a_G) = \frac{a_i^2}{2} = \\
\left\{ \begin{array}{ll}
1 - F_{\varepsilon_i + \beta_i, \varepsilon_G} (b_i - a_i - \beta_i a_G) + \lambda_i^G \sum_{j=1}^{n} v_j \left( 1 - F_{\varepsilon_G + \beta_j^G, \varepsilon_j} \left( b_j^G - a_G - \beta_j^G a_j \right) \right) - \frac{a_i^2}{2} & \text{if } S_i \\
1 - F_{\varepsilon_i + \beta_i, \varepsilon_G} (b_i - a_i - \beta_i a_G) + \lambda_i^p \sum_{j=1}^{n} v_j F_{\varepsilon_G + \beta_j^G, \varepsilon_j} \left( b_j^G - a_G - \beta_j^G a_j \right) - \frac{a_i^2}{2} & \text{if } D_i.
\end{array} \right.
\]

The first-order conditions with respect to actual effort \(a_i\), taking \(b_i = a_i^e + \beta_i a_G^e\) and \(b_j^G = a_G^e + \beta_j^G a_j^e\) as given, are

\[
\left\{ \begin{array}{l}
f_{\varepsilon_i + \beta_i, \varepsilon_G} (b_i - a_i - \beta_i a_G) + \lambda_i^S v_i \beta_i^G f_{\varepsilon_G + \beta_i^G, \varepsilon_i} \left( b_j^G - a_G - \beta_j^G a_j \right) - a_i = 0 & \text{if } S_i \\
f_{\varepsilon_i + \beta_i, \varepsilon_G} (b_i - a_i - \beta_i a_G) + \lambda_i^P v_i \beta_i^G f_{\varepsilon_G + \beta_i^G, \varepsilon_i} \left( b_j^G - a_G - \beta_j^G a_j \right) - a_i = 0 & \text{if } D_i.
\end{array} \right.
\]

I impose the equilibrium requirements \(a_i = a_i^e\) and \(a_G = a_G^e\) to get mayor \(M_i\)'s equilibrium effort strategy

\[
a_i^e = \left\{ \begin{array}{ll}
\frac{1}{\sqrt{2\pi\sigma}} \left( \frac{1}{\sqrt{1 + \beta_i^e \beta_i^G}} + \frac{\lambda_i^S v_i \beta_i^G}{\sqrt{1 + \beta_i^e \beta_i^G}} \right) & \text{if } S_i \\
\frac{1}{\sqrt{2\pi\sigma}} \left( \frac{1}{\sqrt{1 + \beta_i^e \beta_i^G}} - \frac{\lambda_i^P v_i \beta_i^G}{\sqrt{1 + \beta_i^e \beta_i^G}} \right) & \text{if } D_i.
\end{array} \right.
\]
Next, consider governor $G$’s utility:

$$\Pi_G(a_1, ..., a_n, a_G) - \frac{a_G^2}{2} = \sum_{j=1}^{n} v_j \left(1 - F_{\xi_G + \beta_j \epsilon_j} (b_j^G - a_G - \beta_j a_G)\right) + \lambda_G^S \sum_{j=1}^{n} I_j v_j \left(1 - F_{\xi_j + \beta_j \epsilon_j} (b_j - a_j - \beta_j a_G)\right) + \lambda_G^D \sum_{j=1}^{n} (1 - I_j) v_j F_{\xi_j + \beta_j \epsilon_G} (b_j - a_j - \beta_j a_G) - \frac{a_G^2}{2}.$$ 

The first-order condition with respect to actual effort $a_G$, taking $b_j = a_j^e + \beta_j a_G^e$ and $b_j^G = a_G^e + \beta_j^G a_j^e$ as given, yields

$$\sum_{j=1}^{n} v_j f_{\xi_G + \beta_j^G \epsilon_j} (b_j^G - a_G - \beta_j^G a_j) + \lambda_G^S \sum_{j=1}^{n} I_j v_j \beta_j f_{\xi_j + \beta_j \epsilon_G} (b_j - a_j - \beta_j a_G) - \lambda_G^D \sum_{j=1}^{n} (1 - I_j) v_j \beta_j f_{\xi_j + \beta_j \epsilon_G} (b_j - a_j - \beta_j a_G) - a_G = 0.$$ 

Imposing the equilibrium requirements $a_j = a_j^e$ ($j = 1, ..., n$) and $a_G = a_G^e$ yields governor $G$’s equilibrium effort strategy

$$a_G^e = \frac{1}{\sqrt{2\pi} \sigma} \sum_{j=1}^{n} v_j \left(\frac{1}{\sqrt{1 + \beta_j^G \sigma^2}} + \frac{\beta_j (\lambda_G^S I_j - \lambda_G^D (1 - I_j))}{\sqrt{1 + \beta_j^G \sigma^2}}\right).$$

Finally, maximizing $a_i^e + a_G^e$ with respect to $\beta_i$ and $\beta_i^G$ yields an equilibrium in rule strategies $(\beta_i^*, \beta_i^G^*)$ and politicians’ equilibrium efforts $a_i^* = a_i^e (\beta_i^*, \beta_i^G^*)$ and $a_G^e = a_G^e (\beta_i^*, \beta_i^G^*)$, which completes the proof.

**D. Proof of Proposition 5**

The matrix of the equilibrium one-step transition probabilities between states $N_i$ and $Y_i$ equals

$$\begin{bmatrix} P_{N_i N_i} & P_{N_i Y_i} \\ P_{Y_i N_i} & P_{Y_i Y_i} \end{bmatrix} = \begin{bmatrix} q_i P_{S_i N_i} + (1 - q_i) P_{D_i N_i} & q_i P_{S_i Y_i} + (1 - q_i) P_{D_i Y_i} \\ q_i P_{D_i N_i} + (1 - q_i) P_{S_i N_i} & q_i P_{D_i Y_i} + (1 - q_i) P_{S_i Y_i} \end{bmatrix}.$$ 

Refer to the proof of Proposition 3 to find transition probabilities to states $N_i$ and $Y_i$ from states $S_i$ and $D_i$:

$$P_{S_i N_i} = \frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda_G^S + v_i \lambda_Y^S}{1 - v_i \lambda_Y^S \lambda_G^S} \quad \text{and} \quad P_{S_i Y_i} = \frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda_G^S + v_i \lambda_Y^S}{1 - v_i \lambda_Y^S \lambda_G^S}.$$ 

$$P_{D_i N_i} = \frac{1}{2} + \frac{1}{\pi} \arctan \frac{\lambda_G^D + v_i \lambda_Y^D}{1 - v_i \lambda_Y^D \lambda_G^D} \quad \text{and} \quad P_{D_i Y_i} = \frac{1}{2} - \frac{1}{\pi} \arctan \frac{\lambda_G^D + v_i \lambda_Y^D}{1 - v_i \lambda_Y^D \lambda_G^D}.$$
Next, 
\[ q_i = P(\{G \text{ gets majority in } i\} \cap \{G \text{ is reelected}\}) + \]
\[ P(\{G \text{ does not get majority in } i\} \cap \{G \text{ is not reelected}\}) = \]
\[ 2 \sum_{j=1}^{n} v_j P(\{p_G^* + p_i^* \geq a_G^* + a_i^*\} \cap \{p_G^* + p_j^* \geq a_G^* + a_j^*\}) = \]
\[ 2 \sum_{j=1}^{n} v_j P(\{\varepsilon_G + \beta_i^* \varepsilon_i \geq 0\} \cap \{\varepsilon_G + \beta_j^* \varepsilon_j \geq 0\}) = \]
\[ v_i + 2 \sum_{j \neq i} v_j \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{\varepsilon_G + \beta_i^* \varepsilon_i, \varepsilon_G + \beta_j^* \varepsilon_j}(x, y) \, dx \, dy, \]
where \( f_{\varepsilon_G + \beta_i^* \varepsilon_i, \varepsilon_G + \beta_j^* \varepsilon_j}(x, y) \) is the density function of bivariate normal distribution of random variables \( \varepsilon_G + \beta_i^* \varepsilon_i \) and \( \varepsilon_G + \beta_j^* \varepsilon_j \). Finally,
\[ q_i = v_i + \sum_{j \neq i} v_j \left( 1 - \frac{1}{\pi} \arctan \sqrt{\frac{1}{\beta_i^*} + \frac{1}{\beta_j^*}} \right) = \]
\[ 1 - \frac{1}{\pi} \sum_{j \neq i} v_j \arctan \sqrt{\frac{1}{\beta_i^*} + \frac{1}{\beta_j^*} + \frac{1}{\beta_i^* \beta_j^*}}, \]
which completes the proof.

E. Data Description

In Spain, local municipal and regional elections occur simultaneously every four years in 13 out of 17 regions (the so-called autonomous communities). The two leading parties are Partido Popular (PP) and Partido Socialista Obrero Español (PSOE). To build the data set I use the aggregate election results of 10 Spanish regions, which are partially available online at the official websites of the regional governments and the Spanish Ministry of the Interior.

13Municipal and regional elections take place simultaneously in Aragon, Principality of Asturias, Balearic Islands, Canary Islands, Cantabria, Castile-La Mancha, Castile and León, Extremadura, La Rioja, Community of Madrid, Region of Murcia, Foral Community of Navarre and Valencian Community. In Andalusia, Basque Country, Catalonia and Galicia, municipal and regional elections are held on different dates.
14There are also several minor parties; for example, Izquierda Unida has considerable support in some regions.
15Some data were kindly provided by the statistical institutes of the corresponding regions, and are available upon request. The community of Castile and León is not included in my analysis because the data on regional elections in this community are not available. The Canary Islands and the Foral Community of Navarre are not included in the data set, because local parties apart from PP and PSOE enjoy widespread support in these regions and the theoretical model assumes just two political parties.
The sample consists of 3218 municipalities, and depending on the region, covers from 4 to 7 election years from 1983 to 2007. Initially, each observation (of municipality $i$ in election year $t$) includes a census, the number of abstainers, the votes for PP, the votes for PSOE, and the votes for other parties in both municipal and regional elections.

In the theoretical analysis I assumed that all voters participate in both municipal and regional elections. To meet this requirement, from the initial sample I discard all observations where the number of voters in municipal elections differs significantly from the number of voters in regional elections (the maximum allowable difference is 5%).$^{16}$ This ensures that almost the same electorate participated in both elections. Next, I exclude all observations where a third party obtained more votes than either PP or PSOE, in either the municipal or regional elections. All observations thus have the same two leading parties, in line with the theoretical model.

Then I define the binary variable $g_{it}$ such that $g_{it} = 1$ if a candidate who obtained the largest number of votes in the regional elections in municipality $i$ in period $t$ is affiliated with the same political party as the governor of the corresponding region in period $t - 1$, and $g_{it} = 0$ otherwise. By analogy, the binary variable $m_{it}$ is defined as follows: $m_{it} = 1$ if in municipality $i$ a mayor elected in period $t$ is affiliated with the same political party as the mayor in period $t - 1$, and $m_{it} = 0$ otherwise.$^{17}$ Moreover, I define the binary variable $y_{it}$ such that $y_{it} = 1$ if different parties obtained the largest number of votes in the municipal and regional elections in municipality $i$ in period $t$ (ticket splitting) and $y_{it} = 0$ if the same party received the largest number of votes in both elections (no ticket splitting). I use the census share of municipality $i$ in a region during the last observed election year as a proxy for the population share $v_i$. The per capita GDP by province (in thousands of euros) serves as the control variable $x_{it}$.$^{18}$ Table E.1 and Figure E.1 provide descriptive statistics and characteristics of the final sample.

$^{16}$The results for other thresholds of allowable difference, available upon request, are qualitatively the same.

$^{17}$Note that, in line with the theoretical model, the binary variables $g_{it}$ and $m_{it}$ take value 1 when a party (rather than a particular politician) is reelected in the corresponding elections. Moreover, the data set only includes the candidates’ party labels (and not their names).

$^{18}$In Spain, provinces are administrative subdivisions of autonomous communities. In turn, municipalities are subdivisions of provinces.
<table>
<thead>
<tr>
<th>Region</th>
<th>mun.</th>
<th>years</th>
<th>obs.</th>
<th>mean</th>
<th>std.</th>
<th>dev.</th>
<th>obs.</th>
<th>mean</th>
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<th>mean</th>
<th>std.</th>
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Table E.1: Sample characteristics and descriptive statistics.
Figure E.1: Percentage of split-ticket voting (ST) in the regions included in the sample.

References


