Financial Markets as a Commitment Device for the Government

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How does the presence of financial markets shape the government's ability to implement social redistribution? Individuals typically do not constrain consumption to equal their net-of-tax income every period, but instead use financial markets to allocate their resources over time. Optimal redistributive policy ought to take agents' involvement in financial markets into account. I study a two period endowment economy with heterogeneous income types that are private information where a government without commitment cannot provide any social redistribution. I show how agents' involvement in a financial market can improve the government's ability to commit at least to a partially separating allocation in the second period, enabling it to provide some redistribution across agents. In this world, agents borrow against their promised income and enter long-term consumption commitments. Changing these contracts is costly. This changes the government's ex-post incentives to renege on the promised tax schedule and fully redistribute, because some agents would have to default on their loans. I show that whenever this default cost is positive, the government is able to commit to a schedule that only pools some agents of similar type together. In other words, it serves as a commitment device in the sense that it enables the government to commit to not exploit a limited amount of information. As the default costs increase, the government is able to commit to a higher degree of separation, eventually reaching full commitment. Thus, the presence of well-functioning financial markets may in fact facilitate rather than hinder redistribution.

1 Introduction

In the presence of private information, the ability of a government to implement social redistribution depends crucially on its power to commit to future policy. This paper identifies a mechanism by which the existence of well-functioning financial markets may enhance a government's ability to commit, and thus facilitates redistribution across society. Financial markets provide an opportunity for agents to borrow against their expected future income, defaulting on such loans is costly. A government choosing policy sequentially has to take the continuation value of agents' contracts into account: Deviation from previous announcements may lead to costly default. I show that any such default costs alter the government's ex-post incentives

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to renege on the promised transfer scheme and thereby effectively provide a commitment device for the government.

In market economies, individuals typically do not constrain their consumption to equal net-of-tax income every period. Instead, the existence of financial markets provides agents with the opportunity to allocate resources over time, allowing them to make long-term consumption plans. For instance, the ability to take out a mortgage enables individuals to live in a house that reflects their life-time income rather than in a rental unit that reflects their present disposable income every period. At any point in time, when re-optimizing redistributive policy, the government needs to take agents' consumption commitments into account. If agents end up with less net income than they expected, they have to adjust their consumption plans downward. Whenever agents have entered long-term commitments, such adjustments are costly. For example, defaulting on a mortgage may trigger costs of very diverse nature: A foreclosed house often does not sell for the same amount as it was worth to the original owner. Administration of defaults is costly. But also non-pecuniary losses may occur: When agents have to move out of the house they grew attached to, they may suffer further disutility. Chetty and Szeidl (2007) report that nearly 65% of the average US household's budget is allocated to consumption commitments that cannot be adjusted costlessly. Such commitments would not be possible without access to financial markets. In this paper, I show that the ability of agents to enter consumption commitments, rather than just the plain opportunity to allocate resources over time, is what makes the existence of financial markets valuable for the government. Precisely because agents enter long-term commitments that cannot costlessly be adjusted, the government may gain the ability to commit to not changing the promised transfer scheme. Therefore, markets may in fact facilitate rather than hinder redistribution. The paper thus provides a new perspective on the concept of a social market economy, where markets play a crucial role for redistributive policy.

I consider a deterministic two-period endowment economy, where agents receive heterogeneous income. Income types are persistent and are private information. Individual income increases over time, so that every agent would like to smooth consumption by transferring resources from period 2 to period 1. Moreover, a benevolent government would like to redistribute across agents. In a seminal contribution, Mirrlees (1971) established that when the individual ability to generate income is private information, optimal redistributive policy needs to trade off allocative efficiency against information rent extraction. In the presented environment, a benevolent government with an exogenous commitment device can use a fully separating transfer scheme to implement this optimal trade-off between efficiency and equity: Analogous to Mirrlees' (1971) "efficiency at the top" result, at the constrained efficient allocation, only agents of the highest type receive perfectly smooth consumption. The government uses the degree of consumption smoothing as an incentive for agents to reveal their income type truthfully and to contribute to social redistribution. When agents are able to use a financial market to borrow against their future income, *every* agent gains the opportunity to smooth his income perfectly over time.

The government with ex-ante commitment thus finds itself unable to implement the constrained efficient allocation, where it could use the degree of smoothing as an incentive for truthful revelation. Consequently, less redistribution is implemented. In this situation, it is irrelevant whether or not agents enter long-term consumption commitments. It is only the allocative aspect of agents contracting in the financial market that constrains the set of policy instruments available to the government. When the government has an exogenous commtiment device, the existence of markets may thus hinder redistribution.[2]

On the contrary, I argue that a government that *cannot* commit ex-ante to future policy may gain from the existence of a financial market. Roberts (1984) was the first to show that lack of commitment in dynamic taxation settings with private information may lead to a government not being able to implement any social redistribution. In the presented economy, when policy can be chosen sequentially over time, the government has an incentive to use the information gathered about agents in the past to achieve a better redistributive outcome ex-post. Agents anticipate this behavior. The resulting time-consistency constraint leads to the severely inefficient outcome of no social redistribution as well as almost no smoothing of individual consumption over time.[3] In this case, the government may in fact gain from agents' involvement in a financial market. When agents pledge their income in a private contract and enter a long-term consumption commitments, deviating from past policy announcements may not be optimal for the government anymore. Depending on how many agents would have to default on their debt, the associated cost may be too high to justify any welfare gains from additional redistribution. I show that whenever default is costly, the government is effectively able to commit at least to a partially separating transfer schedule. Here, it is the consumption commitment characteristic of agents' contracts rather than the allocative aspect that has a favorable effect for the government's ability to effectively commit to future policy. The presence of financial markets and agents' involvement therein enables the implementation of social redistribution.

As the main result, I derive a simple condition that links the size of the default costs and the concavity of the utility function to the degree of separation a government is able to commit to. Intuitively, this condition equalizes the marginal benefit from additional redistribution toward the low end of the type distribution to the marginal cost due to default. The larger the default costs are, given the concavity of utility, the more separation and so the more social redistribution is possible. Conversely, for fixed default costs, a more concave utility function makes less separation possible, because it increases the ex-post welfare gain from redistribution across agents. The financial market thus effectively provides the government with a device for limited commitment, i.e. with the power to commit to not exploit a limited amount of information.

[2]Many authors have considered environments in which agents cannot only contract with a principal, but also in anonymous outside markets that make it harder to extract information from the agents truthfully. See for example Hammond (1987) for a general treatment or Golosov and Tsyvinsky (2007) for a more recent example from the dynamic public finance literature.

[3]See for example Golosov et al. (2006) for a derivation of this phenomenon in a general setting.

Moreover, the derived results provide insights into the form of partial separation that emerges at the optimum. If the government is allowed to randomize transfers between seemingly identical agents, the optimal allocation is such that agents are perfectly separated below a cutoff type and completely pooled above. The government collects perfect information about agents up to a threshold type chosen according to how much separation it can commit to, and simply pools together all higher income types. On the contrary, if the government is constrained by horizontal equity, agents are optimally pooled together in groups throughout the type distribution. The resulting transfer schedule then has a "tax bracket" structure. The government collects coarse information over the complete type distribution, higher default costs allow for more and smaller brackets, so that more detailed information is collected.

This paper identifies a mechanism by which the economic environment can provide a government with a potential commitment device that does not rely on reputational considerations or political constraints and works in a finite horizon. Thereby, my results help reconcile the observation of policies that are suggestive of governments being able to commit, even though there is no apparent commitment device. In that sense, this paper provides a new perspective on the concept of a social market economy, where the presence of well-functioning markets plays a crucial role for social redistribution. In addition, the fact that the degree of commitment can vary with the default costs introduces a rationale for why tax policy might not use all available information, but rather just condition on *coarse* private information.

Related Literature

This paper contributes to a recently growing literature on the interaction and coexistence of markets and governments. One branch of this literature attempts to identify circumstances under which markets outperform benevolent governments. Netzer and Scheuer (2010) consider a setup in which time-inconsistency arises because of an adverse selection problem. They show that markets can outperform benevolent governments even when they face the same adverse selection problem, because they provide greater incentives to exert effort. In their model, markets endogenously generate a form of commitment to refrain from full insurance and pooling. The key characteristic of markets enabling them to implement separating equilibria is competitiveness: agents are free to switch their insurance provider, just like firms can renege on the insurance contracts. This two-sided lack of commitment is what distinguishes the markets they consider from a benevolent government.

Acemoglu, Golosov, and Tsyvinski (2008a, 2008b) also compare the efficiency of markets and governments in a setting without commitment. They explore the impact of political economy constraints on optimal redistributive policy. To that end, they consider infinitely repeated games with equilibria that crucially rely on reputation effects - a channel completely abstracted from in this paper.

In contrast to these contributions, I am not comparing the performance of markets and governments, but rather ask how the presence and functioning of markets influences the government's ability to implement redistributive policy. Some characteristics of markets have been considered in the literature: Scheuer (2010) explores the impact of incomplete credit markets on optimal entrepreneurial taxation. He finds that a market friction which gives rise to cross-subsidization between different types of potential entrepreneurs may induce inefficient entry at both ends of the skill distribution, which in turn promotes an additional corrective role for type-differential, redistributive taxation, even when the government originally has no redistributive objective. Unlike in Scheuer's paper, I consider a market that is not incomplete, in the sense that it is able to provide credit without cross-subsidization.

Bisin and Rampini (2006) study a setup similar to the one considered here, but focus on the allocative role of *anonymous* markets. They find that allowing agents access to such markets is beneficial for a government without commitment, because it allows them to allocate resources over time without revealing any information, thereby increasing efficiency. However, the government's commitment problem is unchanged, no social redistribution can be implemented. In contrast, I analyze a market that does not act as a "tax haven" by enabling agents to hide information from the government. The crucial characteristic of private contracts I consider is that they constitute consumption commitments that cannot costlessly be changed. This increases the government's commitment power, enabling it to implement some social insurance.

The paper is organized as follows: I start by formulating a basic two period endowment economy with private information about income types in section 2. In section 3, I derive the constrained efficient allocations for governments with and without commitment when only the government has access to a borrowing technology as benchmarks for the following analysis. Section 4 extends the setup by introducing a financial market that allows agents to borrow against their future income. I derive the constraints that agents' involvement in a financial market imposes on the planning problem. Here I discuss in detail the characteristics of the financial market that lead to it functioning as a potential commitment device for the government. Section 5 analyzes the efficient allocations under this additional constraint for governments that can or cannot commit to future policy ex-ante. The main point of the paper is derived: due to default costs, a non-commitment government is able to implement an allocation with partial separation, and thus can provide some social redistribution. Finally, section 6 concludes.

2 Endowment Economy with Private Information

The model economy lasts for 2 periods (t=1,2) and is inhabited by a continuum of agents of unit mass. Agents derive utility from a single consumption good according to

$$U = \sum_{t=1}^{2} u(c_t).$$

Utility is time-separable, and the per period utility function $u(\cdot)$ is strictly increasing, concave, and $\lim_{c\to 0} u'(c) = \infty$. I also assume that u displays constant elasticity of intertemporal substitution. To simplify the following analysis, I assume that agents do not discount between periods.

Agents receive heterogeneous income at the beginning of each period. Their income types, denoted θ, are perfectly persistent over time and are private information. Across the population θ is continuously distributed over a support $\Theta = [\underline{\theta}, \bar{\theta}]$, $F(\theta)$ denotes its cdf. Apart from income heterogeneity, agents are identical.

Per period income is deterministic, and increases over time and across types. In particular, I assume that it is $t\theta$. Consequently, agents would like to smooth consumption over time and consume a constant fraction $\frac{3}{2}\theta$ of their overall income in each period.

Consider the problem of a benevolent social planner with a utilitarian objective and equal Pareto weights on all agents. He chooses an allocation $\{c_t(\theta)\}_{t,\Theta}$ that assigns a consumption level to each type $\theta \in \Theta$, for each period $t = 1, 2$.

$$\begin{aligned} \max_{\{c_t(\theta)\}_{t,\Theta}} \quad & \int_\Theta \left(\sum_{t=1}^{2} u\left(c_t(\theta)\right) \right) dF(\theta) \\ \text{s.t.} \quad & \int_\Theta \left(\sum_{t=1}^{2} c_t(\theta) \right) dF(\theta) \leq 3 \int_\Theta \theta dF(\theta) \end{aligned} \tag{1}$$

The aggregate feasibility constraint reflects the assumption that there exists a technology to costlessly transfer resources between periods. The optimal allocation solving this problem is described as follows:

Lemma 1 (First-Best Allocation)
At the first-best allocation there is full social redistribution and perfect smoothing of consumption over time. All agents consume a constant fraction $c_1(\theta) = c_2(\theta) = c = \frac{3}{2}\int_\Theta \theta dF(\theta)$ *of the economy's total endowment in each period.*

Proof: The first order condition with respect to any agent's consumption in either period satisfies

$$u'(c_t(\theta)) - \lambda = 0 \quad \forall t, \theta$$

where λ is the Lagrange multiplier on the aggregate feasibility constraint. Thus, $c_t(\theta) = c_{t'}(\theta') \quad \forall t, t', \theta, \theta'$. □

3 Government with Information Constraints

Suppose a benevolent government can borrow and save at the risk free gross interest rate $R = 1$, i.e. it can costlessly transfer resources between periods. To implement the desired allocation, it would like to institute a schedule of type specific transfers $\{T_1(\theta), T_2(\theta)\}$. However, it faces private information constraints: When conditioning the allocation on income types, the government must rely on information reported by the agents. This turns the setup into a policy game between agents, choosing which type to report, and the government, choosing the transfers to implement.

To analyze this game formally, consider first the timing of action:

1. Agents learn their income type θ.
2. The government announces a schedule $\{T_1, T_2\}$.
3. Period 1:
   a) Agents receive their first period endowment θ and send a report σ.
   b) The government implements transfers $\{T_1(\sigma)\}$.
4. Period 2:
   a) Agents receive their second period endowment 2θ.
   b) The government implements transfers $\{\hat{T}_2(\sigma)\}$, possibly different from the schedule announced before, depending on its commitment power.

As usual in such setups with private information, it is crucial whether or not the government can commit to not exploit the revealed information at a later point in time, i.e. whether or not it can commit to not changing the announced allocation to the disadvantage of some agents after information has been revealed. I assume that the government can always commit to the announced schedule at least within period 1, i.e. it will always implement transfers in period 1 according to the original announcement. The potential commitment problem that is the subject of this paper arises between periods 1 and 2. At that point the government has learned information about the agents' types. If it is not committed to the announcement made earlier, it may decide to implement transfers $\{\hat{T}_2(\sigma)\}$ that differ from the initial announcement $\{T_2(\sigma)\}$. Whether or not a government can commit is public information, and agents take it into account when choosing which type to report.

Let $\sigma : \Theta \mapsto \Sigma$ denote an agent's reporting strategy, a function that maps from the set of possible realizations of income types Θ to a set of possible reports Σ. For future reference, let σ^* denote the direct truth-telling strategy where agents simply reveal their type truthfully, i.e. $\sigma^*(\theta) = \theta$. The utility obtained from any reporting strategy σ, given the government's transfers $\{T_1, T_2, \hat{T}_2\}$ is

$$U(T(\sigma)|\theta) = u(\theta + T_1(\sigma)) + u(2\theta + \hat{T}_2(\sigma)) \quad (2)$$

For truth-telling to be optimal for an agent of type θ, it must be that

$$U(T_(\sigma^*)|\theta) \geq U(T(\sigma)|\theta) \quad \forall \sigma \quad (3)$$

The government's strategy involves choosing a set of transfer schedules $T = \{T_1, T_2, \hat{T}_2\}_M$. When the government has commitment, $\{T_2\}$ and $\{\hat{T}_2\}$ are exogenously constrained to be equal.

Definition 1
A (perfect Bayesian) equilibrium in the game between agents and the government is given by strategies σ^e and T^e and a belief system $\mathbb{B}$, such that σ^e and T^e are best responses to each other, given $\mathbb{B}$, and beliefs are derived from Bayesian updating[4].

To analyze the equilibrium of this game, I employ a general mechanism design approach (as e.g. in Bester and Strausz (2001) and Skreta (2007, 2010)) where a fictitious mechanism designer is in charge of choosing strategy sets for the agents (the set of possible reports Σ) and for the government (a set of possible transfer schedules T). While abstract, this approach has a number of advantages: The fictitious planner is always able to commit. The Revelation Principle then allows attention to be restricted to direct revealing mechanisms, i.e. agents' strategy set can without loss of generality be restricted to the set of possible types Θ. The optimal mechanism simply has to satisfy incentive compatibility for truth-telling (3). This is true even when the government (a player in this game) has no commitment, because the fictitious planner can restrict the government's strategy set as well: in particular, he can decide how much of the information agents report is revealed to the government. Formally, this amounts to the optimal mechanism specifying an *information revelation rule* $m : \Theta \mapsto M$ that maps from agents' reports to some set of possible messages the government observes. The government is then restricted to choose transfers T that condition only on these messages. The function m could be such that no information is revealed (i.e. m is constant), full information is revealed (i.e. m is the identity function), but could also allow for any form of partial information revelation (i.e. m is constant over some subset of Θ so that some agents are pooled together). Thus, this setup allows me to explicitly study situations where the government has limited commitment in the sense that it can commit not to exploit a limited amount of information. The main focus of the analysis in this paper will be on the optimal form of the information revelation rule m as a proxy for the commitment power of the government and the characteristics of the resulting allocation.

It is useful to think about the economic interpretation of the information revelation rule: In reality, when taxes and transfers are conditional on private information, the government must decide how people report this information. For example, the first step to implementing an income tax is to design a tax return form that people use to report their income. The government, knowing how much information it can commit not to exploit in the future, can choose an institutional design that asks agents only for *coarse* information. The tax return, for example, could only ask for an agent's approximate income, or an income bracket. The function m can be interpreted as this institution.

Moreover, note that since agents know whether or not the government has commitment, they correctly anticipate the government's incentive to re-optimize policy in the second period, and so condition their reporting strategy on $\{\hat{T}_2\}$ rather than on the announced $\{T_2\}$. Thus, there is no need to specify both of them separately. To summarize, the problem of the fictitious planner is to design a mechanism $\Gamma = (m, \{T_1(m), T_2(m)\})$ that satisfies incentive compatibility for all agents:

[4] In the following analysis there will be no need to explicitly derive or condition on these beliefs.

$$U(T_1(m(\theta)), T_2(m(\theta))|\theta) \geq U(T_1(m(\hat{\theta})), T_2(m(\hat{\theta}))|\theta) \quad \forall \theta, \hat{\theta}$$

It is without loss of generality to restrict attention to deterministic mechanisms[5] so that the set of possible messages is $M = \Theta$, and to assume that $m : \Theta \mapsto \Theta$ is weakly increasing. Moreover, I normalize m such that

$$m(\hat{\theta}) = \hat{\theta} \quad \text{for } \hat{\theta} = min\{\theta : m(\theta) = m\}^{6}$$

In the remainder of this section I will derive the optimal mechanisms when the government can or cannot commit, and agents do not participate in a financial market. These will serve as benchmarks. Section 4 then proceeds by deriving any additional constraints on the mechanism design problem that arise when agents can borrow individually in a financial market. Section 5 analyzes the resulting change in the optimal use of information and the implemented allocation.

3.1 Government with commitment

When the government has commitment, the optimal mechanism solves the following problem:

$$\max_{\Gamma} \int_{\Theta} \sum_{t=1}^{2} u(t\theta + T_t(m(\theta)))dF(\theta) \tag{4}$$

$$\text{s.t.} \int_{\Theta} [T_1(m(\theta)) + T_2(m(\theta))]dF(\theta) \leq 0 \tag{5}$$

$$\theta \in \arg\max_{\hat{\theta}} \sum_{t=1}^{2} u(t\theta + T_t(m(\hat{\theta}))) \quad \forall \theta, \hat{\theta} \in \Theta \tag{6}$$

$$m : \Theta \mapsto \Theta \tag{7}$$

It maximizes a utilitarian welfare function (4) with equal Pareto weights on every agent, subject to aggregate feasibility (5) and incentive compatibility (6), choosing the information revelation rule m and the transfer schedule $\{T_t(\theta)\}$ for $t = 1, 2$ optimally.

Lemma 2 (Information Revelation with Commitment)
If the government can fully commit, the optimal information revelation rule is such that complete information about types is revealed: $m(\theta) = \theta$ for all $\theta \in \Theta$.

Proof: Since the government has full commitment, Lemma 2 follows directly from the Revelation Principle[7]. □

[5] Due to the CRRA assumption, non-degenerate stochastic mechanisms are suboptimal. Since the objective function is concave, introducing risk could only improve matters if some incentive constraints were relaxed. Making payoffs for lower type agents riskier does indeed relax higher types' incentive constraints. However, since CRRA implies decreasing absolute risk aversion, the loss for the low types from facing such risk is always higher than the gain in terms of relaxing incentive constraints for higher types. See for example Fudenberg and Tirole (1991).

[6] This just means that when some types are pooled together, the message sent to the government is normalized to be equal to the lowest type in that group.

[7] See for example Myerson (1979) and Harris and Townsend (1981).

When the government is able to commit to not changing the announced transfer schedule after information is revealed, it is optimal to implement a fully separating allocation. The resulting constrained efficient allocation has the following characteristics:

Lemma 3 (Optimal Allocation with Commitment)
At the optimal allocation with commitment:

(i) There is partial social redistribution - total consumption is increasing in type, but less steeply than under autarky:

$$0 < \frac{\partial(c_1(\theta) + c_2(\theta))}{\partial\theta} < 3$$

(ii) The degree of smoothness of consumption increases with type, only the highest type smooths consumption perfectly:

$$\begin{aligned} &c_1(\bar{\theta}) = c_2(\bar{\theta}) \\ &c_1(\theta) < c_2(\theta) \quad \& \quad \frac{\partial \frac{c_1(\theta)}{c_2(\theta)}}{\partial\theta} > 0 \quad \forall\theta < \bar{\theta} \end{aligned}$$

Proof: See appendix A.1.

The setup resembles the traditional static Mirrlees (1971) model, where the desire to smooth consumption efficiently over time corresponds to the optimal labor/leisure choice in Mirrlees' setup. The optimal allocation depicts the classic trade-off between allocative efficiency and informational rent extraction under adverse selection. Even though both forms of redistribution (across the population as well as across time) are in the government's interest, the private information constraints introduce a trade-off between the two. Since the elasticity of intertemporal substitution is constant, all types are willing to give up the same fraction of their total income for smoothing consumption over time. In absolute terms, agents with higher income types would pay more for consumption smoothing than lower income types. The government uses the degree of smoothness as an incentive for higher types to reveal themselves and agree to higher contributions to social redistribution - the ability to do so crucially depends on the government being able to commit to the allocation ex-ante. Perfect consumption smoothing for the highest type is analog to Mirrlees' (1971) "efficiency at the top" result, non-perfect smoothing for all other types refers to the distortion of efficiency for all types other than the highest.

3.2 Government without commitment

If policy is chosen sequentially and the government cannot commit to a schedule ex-ante, the before stated optimization problem becomes even more constrained. The optimal allocation can be found by solving the above planning problem subject to an additional commitment constraint.[8] It must be clear that when types are revealed, the government does not have an incentive to renege on the promised allocation at a later point in time. The problem is the same as above (equations (4) through (7)), with the following additional constraint:

$$\begin{aligned} \{T_2(m(\theta))\} \in \arg \max_{\{\hat{T}_2(m(\theta))\}} & \int_\Theta u(2\theta + \hat{T}_2(m(\theta)))dF(\theta) \\ \text{s.t.} & \int_\Theta [T_1(m(\theta)) + \hat{T}_2(m(\theta))]dF(\theta) \leq 0 \end{aligned} \tag{8}$$

This constraint requires that in period 2, the government won't change the promised transfer schedule based on information it learned in period 1. Since types are persistent, this amounts to maximizing second period welfare, only constrained by feasibility.

Lemma 4 (Information Revelation without Commitment)
If the government cannot commit, the optimal information revelation rule is such that no information about types is revealed: $m(\theta) = \underline{\theta}$ *for all* θ.

Proof: See appendix A.2.

When the government cannot commit to not exploit information about types in period 2, it is not optimal to implement any separation at all. All agents will pool with the lowest type, no information about types is revealed.

The argument of the proof is as follows. Because of the commitment constraint (8), the government loses the ability to offer any separation in period 2 consumption: Since the necessity to provide incentives for agents to reveal their type truthfully vanishes after the first period, the government would always change the announced allocation when provided with the opportunity to do so. Such deviation from the ex-ante optimal contract, though, is not beneficial for all agents. The government offering above mean type agents a worse allocation after learning their true income is known as the *ratchet effect*[9]. Agents anticipate this, so incentives for truthful revelation need to be provided through transfers in period 1. However, to achieve any separation in types, the incentive payments would have to be so high, that redistribution would go from the bottom to the top of the income distribution - inequality would rise compared to autarky. Thus, complete pooling is the optimal choice of information revelation. Consequently, no redistribution across agents (i.e. social insurance) and almost no redistribution across time (i.e. consumption smoothing) will be implemented:

Lemma 5 (Optimal Allocation without Commitment)
At the optimal allocation without commitment:

[8]While before agents were moving after the government, lack of commitment introduces a second stage to the game, where only the government can move again. The commitment constraint essentially imposes subgame-perfection on the equilibrium, as e.g. in Kydland and Prescott (1977).

[9]The insight that the only incentive compatible sequence of spot contracts is one without dynamic insurance is due to Townsend (1982).

(i) There is no social redistribution - agents consume their total endowment, total consumption increases in type as under autarky:

$$c_1(\theta) + c_2(\theta) = 3\theta \quad \forall\theta$$

(ii) Only the lowest type smooths consumption perfectly, all other types smooth only the fraction of their income equal to that of the lowest type:

$$c_1(\underline{\theta}) = c_2(\underline{\theta}) = \frac{3}{2}\underline{\theta}$$
$$c_1(\theta) = \theta + \frac{1}{2}\underline{\theta} < c_2(\theta) = 2\theta - \frac{1}{2}\underline{\theta} \quad \forall\theta > \underline{\theta}$$

Proof: When no information about types is revealed, the only instrument to increase welfare is to smooth consumption as much as possible for the lowest type. All other agents therefore smooth only the part of income equal to that of the lowest type and consume their remaining income on the spot. □

In this economy, the government's lack of commitment has dramatic implications. Not only is the government unable to implement any social redistribution, the resulting allocation is also very inefficient: Even though transferring resources across time is costless, this technology remains almost unused, because it would require the revelation of private information. Roberts' (1984) insight applies in this economy.

4 Individual Access to a Financial Market

Suppose now that agents have access to a financial market in which they can save and borrow at interest rate $R = 1$, i.e. they can use the same technology to transfer resources over time that is available to the government. Naturally, agents would use this opportunity to smooth consumption over time. Neither allocation analyzed in section 3 had full smoothing for all agents, so access to such a financial market likely imposes a binding constraint on the optimal mechanism. The purpose of this section is to derive the constraints that stem from the contracts agents may write in such a financial market.

Bisin and Rampini (2006) first showed, that in a setup similar to the one presented here, financial markets that can be used *anonymously* may be a beneficial constraint for a government without commitment. Such markets improve efficiency in the allocation of resources over time without disclosing information about types to the government. This leads to an increase in welfare. The government's commitment problem, however, remains unchanged. Still, no social redistribution would be possible.

I emphasize a different mechanism: Agents use the financial market to smooth consumption over time. To do that, they pledge future income in private contracts that resemble long-term consumption commitments (e.g. mortgages). When a government changes the announced allocation, these contracts may have to be renegotiated or even be defaulted on - a process that is usually costly. This introduces a cost to deviating from the announced allocation ex-post. In fact, it will turn out that agents' involvement in a financial market may essentially provide the government with a form of limited commitment. The presence of markets may thus facilitate rather than hinder redistribution across agents. To derive this insight formally, I will lay out the critical characteristics of the market environment and derive the structure of private contracts that emerge in the presented economy. These contracts will be treated as constraints to the mechanism design problem. Section 5 proceeds with analyzing their impact on the optimal revelation of information and the resulting allocation when the government can or cannot commit through an exogenous commitment device.

Assumptions about the financial market

The market consists of many banks that have access to unlimited outside funding. Agents and banks can write contracts

$$[(h_1(\sigma), h_2(\sigma)), (b_1(\sigma), b_2(\sigma))] \quad \text{with } h_t, b_t \geq 0$$

where the bank agrees to provide h_t units of consumption in period t for a payment of b_t by the agent who announced type σ. This structure of contracts is very general. In what follows I will discuss which of these four determinants of contracts are of relevance to the results.

I make the following assumptions that shape the type of debt contracts signed by agents in this economy.

(A1) Competition between banks ensures that they make zero profits. The gross interest rate agents face is $R = 1$.

(A2) Banks can always enforce their contracts with the individual agents. This enforcement power is never revoked, or in other words, the government is always able to commit not to shut down the market. However, banks have the first take on *net-of-tax* income only. They do not possess any power over the government to enforce bailouts.

(A3) The market is not completely anonymous as in Bisin and Rampini (2006). The government can observe the contracts agents sign in the financial market up to the precision with which it observes agents' type announcements. This means that agents who are pooled together by the information revelation rule m are still able to write differential contracts without revealing any additional information to the government. However, the government observes if the contracts are feasible given the announced type. Thus, the financial market does not act as a "tax haven". Agents who reported a lower income than they actually have are not able to secretly smooth their consumption. On the other hand, all agents who reported their type truthfully are able to smooth consumption perfectly without revealing their precise type. This restriction on observability of transactions in the financial market is introduced for expositional convenience, and I will point out its impact on the results. The main results are unchanged if contracts were completely observable.

(A4) The government is not able to restrict access to the financial market or punish agents' market involvement except when it reveals that they were lying about their type. This amounts to assuming that the government cannot announce a transfer schedule T ex-ante that conditions payments on whether agents will contract in the financial market. It does not exclude the possibility that a government without commitment implements differential transfers ex-post depending on the contracts that were signed.

Consider the following modified timing of events:

1. Agents learn their income type θ.
2. A mechanism $\Gamma = (m, \{T_1(m), T_2(m)\})$ is announced.
3. Period 1:
   a) Agents receive their first period endowment θ and send a report $\sigma(\theta)$.
   b) The government observes messages $m(\sigma)$ and implements transfers $\{T_1(m)\}$.
   c) Agents may contract in the financial market, first period payments b_1 and h_1 are executed
4. Period 2:
   a) Agents receive their second period endowment 2θ.
   b) The government implements transfers $\{\hat{T}_2(m)\}$, possibly different from the schedule announced before, depending on its commitment power. It takes the contracts agents signed into account.
   c) Second period payments b_2 and h_2 are executed

Assumption (A2) is reflected in the fact that transactions in the financial market always take place after the government implemented its transfers. Together with assumption (A4), this implies that the government cannot levy a tax on h_t directly.[10] It can only influence net income and thereby bound the possible debt payments b_t.

A government that can choose policy sequentially will take the contracts agents signed into account when re-optimizing the transfer schedule in period 2. This is the key argument: The continuation value of agents' contracts may be such that the government finds it not optimal to renege on the promised allocation and so effectively gains commitment. In case an agent of (announced) type σ defaults on his loan, i.e. in case he cannot pay the amount $b_2(\sigma)$ agreed upon, his contract is renegotiated to $[\hat{h}_2(\sigma), \hat{b}_2(\sigma)]$. It is without loss of generality to assume that the bank cuts the contracted payment h_2 to zero:

[10] In reality, Austria is one of few exceptions to this assumption. The Austrian government levies a tax of currently 0.8% of the loaned amount in any debt contract on the debitor.

$$\hat{h}_2(\sigma) = 0 \tag{9}$$

Yet, the bank remains in power to collect any outstanding balance $d_1(\sigma) = h_1(\sigma) - b_1(\sigma)$ from period 1. Such renegotiation, however, comes at a cost: The bank does not value h_2 at the same rate the agent does. From the bank's point of view, saving the second period payment is worth only $h_2 - H_B$. There are several interpretations for this cost: First, re-allocating resources to a new project is costly for a bank. Renegotiating contracts may also require costly administration. Second, notice that h_2 can be interpreted as collateral on the loan. It is only natural to assume that the bank might not be able to resell the asset for the same value it had for the particular agent. Banks, however, cannot make losses when agents default. I assume that they remain in power to collect the difference H_B from the defaulting agent's net-of-tax income. Therefore, the defaulting agent cannot consume before repaying

$$\hat{b}_2(\sigma) = min\{d_1(\sigma) + H_B, 2\sigma + \hat{T}_2(m(\sigma))\} \tag{10}$$

If his net-of-tax income is less than $d_1 + H_B$, he would have to consume zero. Moreover, an agent who has to default on his loan may suffer an additional utility loss H_A that the government also has to take into account when reneging on its promised transfer schedule.

The default costs $H_A + H_B = H$ summarize the key characteristic of contracts in private financial markets that I want to emphasize in this paper: Contracts are not easily reversible, nor is it costless to renegotiate or default. The costs may be of very diverse nature: On the one hand, one might think of pure resource costs for administering the renegotiation. Re-allocating funds from one loan to another is also costly. A bank might not be able to resell an asset for the same value agreed upon previously with the now defaulting agent. Such costs are summarized by H_B. On the other hand, agents might suffer a loss in utility when they have to default on their loan in addition to the resource costs of the bank. They may have made life plans contingent on this loan that require further costly alteration. For example, they might have grown attached to their house, which they financed with a mortgage, and lose utility when they have to move. Such costs are summarized by H_A.

In short, agents who have access to financial markets may tie up their resources in a contract whose continuation value needs to be taken into account when redistributing across the population. Redistributing across agents more than initially announced becomes costly ex-post, and so alters the government's optimization problem.

Structure of debt contracts

Banks set borrowing limits per announced type σ that reflect total net-of-tax income. Assumption (A1) implies that all contracts will be such that

$$h_1(\sigma) + h_2(\sigma) = b_1(\sigma) + b_2(\sigma) \tag{11}$$

Conditioning the contracts on agents' reports σ rather than their types θ eludes to the fact that banks must rely on the information agents reveal about themselves. In the first period all agents are net-borrowers. This opens up the possibility that an agent reports a much higher type to take advantage of a high borrowing limit and plans a sure default. To avoid such adverse selection, banks would like to verify that agents are at least of the type they claimed. While banks cannot verify an agent's type directly, notice that they can offer contracts that require a *down payment* of

$$b_1(\sigma) = \sigma + T_1(m(\sigma)) \tag{12}$$

This proof of solvency acts as a screening device, i.e. it signals to the bank that the agent is indeed at least of the type he claimed he was.[11] Competition then ensures that each agent can find a bank offering a contract with a borrowing limit that reflects the exact net income of the type he announced. Banks cannot gain by offering contracts that don't require down payments, since only agents who misreported their type would sort into those.

Notice that because of these down payments, default is only possible if the government deviates from the announced transfer schedule. For the remainder of the analysis, it is enough to keep track of the net debt obligation each agent carries over to period 2:

$$d_1(\sigma) = h_1(\sigma) - b_1(\sigma)$$

Agents in this economy use the market to borrow against their second period income, to smooth consumption and consume more in period 1 than they are endowed with. Contracts will thus typically have $h_1 > 0$. How much agents can borrow depends on the type they reported. Banks will set borrowing limits that reflect total net income:

$$h_1(\sigma) + h_2(\sigma) \leq 3(\sigma) + T_1(m(\sigma)) + T_2(m(\sigma))$$

Even though the market and information structure impose some constraints, agents can still choose between a variety of contracts. Given a schedule of transfers and his report σ, an agent chooses to contract in the financial market to maximize his life-time utility:

$$\begin{aligned}
\max_{h_t, b_t} \sum_{t=1}^{2} u(t\theta + T_t(m(\sigma)) + h_t(\sigma) - b_t(\sigma)) & \\
\text{s.t. } h_1(\sigma) + h_2(\sigma) &= b_1(\sigma) + b_2(\sigma) \\
b_1(\sigma) &\in \{0, \sigma + T_1(m(\sigma))\} \\
b_2(\sigma) &\begin{cases} \leq 2\sigma + T_2(m(\sigma)) & \text{if } b_1(\sigma) > 0 \\ = 0 & \text{if } b_1(\sigma) = 0 \end{cases}
\end{aligned}$$

Whenever transfers are such that the agent's net income is not smooth over time,

[11] This setup allows me to abstract from any additional adverse selection problem the financial market may face. Scheuer (2010) considers the impact of a financial market with adverse selection on optimal policy.

the optimal solution to this problem is such that

$$
\begin{aligned}
b_1 &= \sigma + T_1(m(\sigma)) \\
h_1 &= \frac{1}{2}(3\sigma + T_1(m(\sigma)) + T_2(m(\sigma))) \\
\to d_1 = h_1 - b_1 = &= \frac{1}{2}(\sigma - T_1(m(\sigma)) + T_2(m(\sigma))
\end{aligned} \tag{13}
$$

Optimal contracts are not uniquely pinned down. In the second period, payments could be as low as

$$b_2(\sigma) = d_1(\sigma) \quad \text{and} \quad h_2(\sigma) = 0 \tag{14}$$

(in this case, the agent would simply repay his outstanding debt), or as high as

$$b_2(\sigma) = 2\sigma + T_2(m(\sigma)) \quad \text{and} \quad h_2 = h_1 = \frac{1}{2}(3\sigma + T_1(m(\sigma)) + T_2(m(\sigma))) \tag{15}$$

I refer to these possibilities as *net* or *gross* contracts respectively. All contracts in between these extremes leave the agent with the same consumption allocation. However, when signing a gross contract, agents enter a consumption commitment for period 2 beyond the repayment of their net balance. While both types of contracts serve to allocate resources over time and to smooth consumption, a gross contract does that in the form of a long-term commitment. In other words, a gross contract may be interpreted as a mortgage, where the agent constrains his consumption of housing to a particular house not only in period 1, but also in period 2 with the help of a financial contract. If the government has ex-ante commitment, agents are indifferent between signing net and gross contracts. However, if the government has no ex-ante commitment, it will turn out to be individually optimal for agents to sign gross contracts.

The financial market provides agents with the opportunity to smooth their consumption perfectly over time. Moreover, agents may sign contracts that resemble long-term consumption commitments (e.g. mortgages). While this additional characteristic of private contracts in a financial market is irrelevant for the government with commitment, it will turn out to be crucial in determining the de facto commitment power of a government that chooses policy sequentially.

5 Commitment Through the Financial Market

When agents have access to a financial market, the private contracts described by equations (13) through (15) constrain the choice of the optimal mechanism $\Gamma = (m, \{T_1(m), T_2(m)\})$.

5.1 Government with commitment

When the government can commit to a schedule $\{T_1(m), T_2(m)\}$ ex-ante, the optimal mechanism now solves problem (4) subject to feasibility (5) and a set of *modified* incentive compatibility constraints:

$$\theta \in \arg\max_{\hat{\theta}} \sum_{t=1}^{2} u(t\theta + T_t(m(\hat{\theta})) + h_t(\hat{\theta}) - b_t(\hat{\theta})) \quad \forall \theta, \hat{\theta} \in \Theta \tag{16}$$

taking the contracts agents sign as given. With access to the financial market, all agents who revealed their true type are able to perfectly smooth consumption themselves.

Lemma 6 (Information Revelation with Commitment)
If the government can fully commit, the optimal information revelation rule is such that complete information about types is revealed: $m(\theta) = \theta$ for all $\theta \in \Theta$.

Proof: Since the government has full commitment, Lemma 6 follows directly from the Revelation Principle.[12] □

The government still implements a fully separating allocation. However, the set of *modified* incentive constraints (16) implies that it cannot achieve the allocation outlined in Lemma 3. Due to the fact that it cannot use the smoothing of consumption as an incentive anymore, less redistribution across the population is implemented at the optimum.

Lemma 7 (Optimal Allocation with Commitment and Financial Market)
At the optimal allocation with commitment, when agents have access to a financial market:

(i) There is partial social insurance, but less than without the financial market: Total consumption is increasing in type, more than in Lemma 3, but less than under autarky:

$$0 < \frac{\partial(c_1^N(\theta) + c_2^N(\theta))}{\partial\theta} < \frac{\partial(c_1(\theta) + c_2(\theta))}{\partial\theta} < 3$$

where N denotes the allocation derived in Lemma 3 without the market.

(ii) All agents smooth consumption perfectly over time: $c_1(\theta) = c_2(\theta)$ $\forall\theta$.

Proof: Given how agents contract in the financial market, the government essentially faces the extra constraints

$$\theta + T_1(\theta) = 2\theta + T_2(\theta) \quad \forall\theta$$

Except for the highest type $\bar{\theta}$, this constraint changes the allocation the government would have liked to implement. Incentives for truthful revelation can now only be given by higher total consumption, using the degree of smoothness as incentive is

[12] If contracts were perfectly observable, i.e. the restriction in assumption (A3) would not apply, the simple revelation principle would not apply. Then, agents who are pooled together by the function m would also pool on the financial market so to not reveal any additional information. Thus, by pooling agents together, the government could influence how agents can use the financial market, and implement allocations with non-smooth consumption for pooled agents. However, even in that case it turns out to be optimal for the government to choose full separation over any partial pooling arrangement, so the result is unchanged.

not an option anymore. Redistributing across agents thus becomes more expensive, less social insurance is possible. □

Notice that the equilibrium is not unique: Since the government can commit to the allocation, all agents will be indifferent as to whether or not they borrow in the financial market, as long as they receive smooth net-of-tax income. Because of the ex-ante commitment, it is irrelevant for the government, whether agents have entered long-term consumption commitments. It is only the allocative aspect of agents' contracts that impacts the governments ability to redistribute across society. The implemented consumption allocation, however, is the same in all equilibria. I mark the resulting allocation with superscript c for future reference.

This leads to the following proposition:

Proposition 1 (Government with Commitment and Financial Market)
A government that can commit to an allocation ex-ante does not benefit from agents having access to a financial market.

Proof: Without individual access to financial markets, the government still had the technology to provide perfectly smooth consumption for all agents. Yet, it optimally chose not to do so. Thus, the extra constraint reduces overall welfare, the allocation is inferior to the allocation of Lemma 3 from the government's point of view. Assumption (A4) implies that the government cannot deter agents from using the market by announcing punishments for doing to.□

The presence of markets reduces the set of policy instruments available to the government. When agents can costlessly take care of individual consumption smoothing, the resulting allocation does not allow the government to implement the desired trade-off between redistribution over time and redistribution across the population. Instead, perfect smoothing over time, but less social insurance will be realized.

5.2 Government without commitment

For a government that cannot commit to a second period schedule $\{T_2(m)\}$ ex-ante, individual access to a financial market interferes with its optimization problem in the same way as if it had commitment. But there is an additional effect: the lack of commitment constraint (8) is modified as well. When deciding about redistributive policy after information has been revealed, the government has to take into account the continuation value of the debt contracts agents hold. Redistributing away from an agent who pledged all his income results in a costly default. Thus, for a government without commitment, the optimal mechanism solves the same problem as above, but subject also to the *modified* commitment constraint

$$
\begin{aligned}
\{T_2(m)\} \in \arg\max_{\{\hat{T}_2\}} \int_\Theta & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (17)\\
& u(2\theta + \hat{T}_2(m(\theta)) + h_2(\theta) - b_2(\theta)) \quad \mathbb{I}\{2\theta + \hat{T}_2(m(\theta)) \geq b_2(\theta)\} \quad + \\
& u(2\theta + \hat{T}_2(m(\theta)) - (d_1(\theta) + H_B) - H_A) \quad \mathbb{I}\{2\theta + \hat{T}_2(m(\theta)) < b_2(\theta)\} \quad dF(\theta) \\
\text{s.t.} & \int_\Theta T_1(m(\theta)) + \hat{T}_2(m(\theta)) dF(\theta) \leq 0
\end{aligned}
$$

taking the contracts agents can sign as given. Notice that both forms of default costs enter in the same way into the consideration: They both determine the continuation value of the contract the agent signed, and the government has to take potential losses that may result from default into account.

Suppose all agents sign a *gross* debt contract, i.e. $b_2(\theta) = 2\theta + T_2(m(\theta))$ for all types. In this case, any deviation from the previously announced allocation will lead to default. Given the enforcement power of the financial market, agents who are forced to default will have to be at least provided with a payment that covers $d_1 + H$ to avoid zero consumption. The above problem de-facto reduces to redistributing based on what agents planned to consume in the second period, h_2:

$$
\begin{aligned}
& \max_{\{\hat{x}_2(\theta)\}} \int_\Theta u(\hat{x}_2(m(\theta))) dF(\theta) \\
& \text{s.t.} \int_\Theta \hat{x}_2(m(\theta)) dF(\theta) \leq \int_\Theta [h_2(m(\theta)) - H \mathbb{I}\{\hat{x}_2(m(\theta)) < h_2(m(\theta))\}] dF(\theta)
\end{aligned} \tag{18}
$$

This formulation of the government's problem at the beginning of period 2 nicely depicts the main point: the government is still free to redistribute, but doing so is costly. The default cost $H = H_A + H_B$ conceptually enters only on the resource side of the feasibility constraint.[13]

From here it is also immediate that a large enough H would prevent any deviation from the promised schedule:

Proposition 2 (Limit Case: Full Commitment)
When all agents pledge their complete income in the financial market and if $H \geq \frac{1}{2}(3\bar{\theta} + T_1^c(\bar{\theta}) + T_2^c(\bar{\theta}))$, the government can implement the same allocation as if it had full commitment.

Proof: Suppose the government had promised the full commitment schedule $\{T_2^c(\theta)\}_\Theta$. The highest type $\bar{\theta}$ will accordingly plan to consume $h_2^c(\bar{\theta}) = \frac{1}{2}(3\bar{\theta} + T_1^c(\bar{\theta}) + T_2^c(\bar{\theta}))$ in period 2. In case he has to default, the default cost is more than what he actually planned to consume. The government would gain no resources for redistribution from letting even the highest type default, and thus would never attempt any redistribution ex-post. It can therefore implement the same allocation as if it had full

[13] Farhi and Werning (2008) consider a government that faces an exogenous cost of reform. The analysis in the present paper can be interpreted as providing one possible micro-foundation for such a reform cost and showing how it leads to limited commitment.

commitment ex-ante. □

The result of Proposition 2 should be understood as a limit result: If default costs are so high that they leave no value after renegotiation, it obviously serves as a device for full commitment. Such high default costs are unrealistic, they could be interpreted as not offering default as an option. It is interesting, however, that a finite default cost would be enough to induce full commitment. The more relevant case, though, is one where default costs are too low to offer full commitment:

Proposition 3 (Information Revelation: Limited Commitment)
If the government has no ex-ante commitment, but all agents pledge their complete income in the financial market and the default costs H are positive, the optimal information revelation rule is such that it pools agents above a cutoff type $\tilde{\theta}$ together but separates all other types:

$$\begin{aligned} m(\theta) &= \theta && \forall \theta \leq \tilde{\theta} \\ m(\theta) &= \tilde{\theta} && \forall \theta > \tilde{\theta} \end{aligned}$$

The cutoff $\tilde{\theta}$ and transfers T must be such that

$$u'(h_2(\underline{\theta}))(h_2(\tilde{\theta}) - h_2(\underline{\theta}) - H) = u(h_2(\tilde{\theta})) - u(h_2(\underline{\theta})) \tag{19}$$

where $h_2(\theta) = \frac{1}{2}(\theta + T_1(m(\theta)) + T_2(m(\theta)))$.

Proof: See appendix A.3.

The proposition states that a government without commitment gains the power to commit to a partially separating allocation if agents hold gross financial contracts, as long as they face positive default costs. How much separation is possible, or in other words how many types at the top of the distribution will pool, depends on the default cost H and on the concavity of the utility function. The intuition for the constraint is simple: the marginal benefit from deviating from the promised allocation (on the left hand side) is measured by the marginal utility of the lowest type (since he is the one distributed toward) times the amount of resources available for redistribution. The marginal cost of such deviation (on the right hand side) is the utility loss of the highest type: his consumption is equalized with that of the lowest type.

More separation of types, i.e. a higher $\tilde{\theta}$, leads to a larger differentiation in period 2 consumption $h_2(\tilde{\theta}) - h_2(\underline{\theta})$ (due to the incentive constraints in the ex-ante optimization problem), and in turn to a tightening of the constraint. A higher default cost H on the other hand relaxes the constraint, so that more separation is possible. In fact, when $H \geq h_2^c(\bar{\theta})$, the government will be able to commit to the same allocation as the full commitment government (Proposition 2). Proposition 3 states, however, that any positive default cost, even a very small one, allows the government at least some commitment. Notice also that the degree of possible separation is negatively linked to the concavity of the utility function. If $u(\cdot)$ is more concave, the ex-post gain from redistribution increases.[14] In order to be able to withstand the higher temptation to let a fraction of agents at the top default, more agents have to be pooled together, the cutoff $\tilde{\theta}$ has to be lower.

When $H = 0$, the case of no commitment is recovered: As long as the utility function is strictly concave, condition (19) is only satisfied when $h_2(\tilde{\theta}) = h_2(\underline{\theta})$, i.e. when there is no separation at all. As in in the benchmark case without a financial market, when the government is not able to implement any separation in the second period, it cannot provide enough incentives through first period transfers to implement any social redistribution from the top to the bottom of the distribution.

The default costs essentially serve as a commitment device for the government. With any positive default costs, the government can gain limited commitment: It can credibly commit to not exploit a limited amount of information. By pooling agents at the top of the distribution together, only limited information is revealed: For agents of type $\theta \leq \tilde{\theta}$ the true type is revealed, while for all agents of type $\theta > \tilde{\theta}$ the government only learns that they are part of the high income group, but not their exact type.

The following lemma summarizes the characteristics of the best allocation the government is able to commit to:

Lemma 8 (Optimal Allocation without Commitment and Financial Market) *At the optimal allocation without commitment, when agents have access to a financial market, the default cost is $0 < H < h_2^c(\bar{\theta})$ and the conditions in Proposition 3 are met:*

(i) There is partial social insurance - total consumption increases in type, but less steeply than under autarky:

$$0 < \frac{\partial(c_1(\theta) + c_2(\theta))}{\partial\theta} < 3$$

(ii) All agents smooth consumption perfectly over time: $c_1(\theta) = c_2(\theta)\ \forall\theta$.

Proof: All agents who report their type truthfully are able to use the financial market for perfect consumption smoothing. This provides incentives for agents to reveal themselves. As in the Lemma 3, the government uses the information gained about types to implement some social insurance. □

The government, even though per se not able to commit to an allocation ex-ante, is able to implement some redistribution across agents, i.e. it can provide some level of social insurance. This leads directly to the following proposition:

[14] Because I study the problem of a government with a utilitarian objective with equal Pareto weights on all agents, the concavity of the individual utility function also measures the potential welfare gain from redistribution. More generally, the form of the government's objective function is the crucial characteristic to determine the optimal cutoff $\tilde{\theta}$.

Proposition 4 (Government without Commitment and Financial Market) *If default costs H are positive, a government that cannot commit to an allocation ex-ante always benefits from agents having access to financial markets.*

Proof: Proposition 3 establishes that with positive default costs H the government is able to commit to a partially separating allocation that provides some social insurance - an improvement over the pooling allocation in Lemma 5 without financial markets. However, contrary to the case in which the government has commitment ex-ante, here the commitment power hinges critically on agents actually borrowing in the financial market. It remains to be shown that agents will indeed sign *gross* contracts in the financial market. Since agents are small, non-strategic players in this policy game, it would be a stretch to assume that they coordinate on signing such contracts in order to provide a commitment device for the government. Notice, however, that once a schedule is announced and types have been reported, i.e. at the stage of choosing a debt contract, it is individually optimal to sign a gross contract and enter a consumption commitment for period 2:

At the beginning of period 2, the last stage of the game, the government without an exogenous commitment device chooses the transfer schedule to solve problem (17), knowing what contract each agent has signed. The government would like to equalize consumption as much as possible. If an agents has signed a contract such that

$$b_2(\theta) < m(\theta) + T_2(m(\theta))$$

it is costless and thus optimal for the government to redistribute the unpledged portion of promised after-tax income away from that agent, and redistribute it toward lower types. This leaves the agents worse off than if he had signed a gross contract. However, regardless of whether or not all other agents signed gross contracts, the government will not find it optimal to let those who did default. Thus, it is a dominant strategy for agents to sign such gross contracts. $\square$

The role of the financial market is to give agents the opportunity to pledge their expected resources in debt contracts which in turn influences the government's ability to commit at least to a partially separating allocation. Therefore, it also facilitates redistribution across the population. This mechanism is the central insight of this paper. The crucial characteristic of contracts in the financial market is not that agents are free to allocate resources. It is that in order to allocate resources, agents are able to enter consumption commitments that cannot costlessly be changed.

5.2.1 Additional Assumption: Horizontal Equity

The key insight of this paper is that the prospect of agents defaulting on their loans enables the government to commit to some separation even after it learned agents' private information. In the previous section I derived the specific form of separation at the optimal allocation: Agents below a certain cutoff are perfectly separated, while agents above that cutoff pool. Thus, in this specific setup, the commitment comes from the fact that agents at the top of the income distribution would default on their loans, if the government decided to redistribute across the population more than it announced ex-ante.

In this section, I derive the form of partial pooling at the optimal allocation under the additional assumption of horizontal equity. It means that the government is bound to treat equal agents equally, i.e. it cannot randomize transfers between seemingly equal agents. While there is nothing in the model that necessitates this assumption, it has some realistic appeal.

Proposition 5 (Information Revelation under Horizontal Equity)
If the government has no ex-ante commitment, all agents pledge their complete income in the financial market and the default cost is $0 < H < h_2^c(\bar{\theta})$, the optimal information revelation rule is such that it pools agents into finitely many groups throughout the type distribution. Any $H > 0$ allows separation into at least two groups.

Proof: See appendix A.4.

If the government is bound to horizontal equity, it is not optimal anymore to pool agents just at the top of the distribution. In fact, since the government cannot let just a fraction of any pooled group of agents default, less pooling at the top is necessary, more information about the highest types can be revealed and used to provide social insurance.

At the optimum, agents will be pooled throughout the distribution in groups of varying size - a structure that can be interpreted as tax brackets. The effective commitment power of the government then stems from the concern of agents defaulting on their loan throughout the distribution, not just at the top.

6 Discussion

This paper uncovers a mechanism by which the presence of a financial market may effectively provide the government with a (limited) commitment device, thereby enabling the implementation of commitment-type policies. It thus helps reconcile the observation of policies that are suggestive of governments being able to commit, even though there is no apparent commitment device. Moreover, the model provides a rational for why governments do not implement policies contingent on complete information: When they have no ex-ante commitment power, a reasonable default cost provides them with *limited* commitment, i.e. with the power to commit to not exploit a limited amount of information.

In the presence of private information, the ability of a government to implement social redistribution crucially depends on its power to commit to future policy. In reality, there is little reason to believe that governments possess some exogenous commitment device. Instead, commitment must stem from the environment the government operates in. The literature has focused on political economy constraints as mechanisms for commitment. In contrast to that, the presented paper highlights the fact that also the *economic* environment might enhance the commitment power of the government. In that sense, the paper establishes a theoretical foundation for what can be referred to as a social market economy, where the presence of well functioning competitive markets that allow agents to enter consumption commitments plays a crucial role for social redistribution.

People typically do not just rely on the government and simply consume their net-of-tax income every period. Instead, they use private financial markets to allocate their resources over time. Redistributive policy ought to take that into account. To address the question how the presence of a financial market shapes the government's ability to implement redistributive policy, I studied a standard Mirrlees framework. In the presented economy agents receive heterogeneous income, and a benevolent government might attempt two forms of redistribution: Smoothing of individual consumption over time and social redistribution. Private information about income types, however, introduces a trade-off between the two.

If the government can commit to future policy ex-ante, it is able to implement a fully separating allocation. In this case, agents having access to a financial market reduces the set of policy instruments available to the government: It loses the ability to provide incentives for truthful revelation through the degree of consumption smoothness. Agents can use the financial market to smooth consumption by themselves. The government is deprived of the power to discriminate along this dimension, extracting private information from the agents gets harder. Consequently, a government with full commitment cannot gain from agents' involvement in financial markets. It finds itself unable to implement the constrained optimal trade-off between allocative efficiency and equity. In such a setup, financial markets hinder redistribution across the population.

However, if the government cannot commit to an allocation ex-ante and is thus unable to implement social redistribution, it might gain from agents' involvement in financial markets. In fact, it might gain the power to commit at least to a partially separating allocation, making some social redistribution possible. The reason is that in order to smooth their income over time, agents pledge future income in the financial market in contracts that induce long-term consumption commitments. Such private contracts are typically not easily reversible. A surprise redistribution, after agents have revealed their type and signed individual debt contracts, will lead to some agents having to default on their debt. Such default, however, is costly. These default costs mitigate the government's desire to exploit information and implement full social insurance ex-post.

The costs may be of very diverse nature: On the one hand, one might think of pure resource costs for administering the default on a loan. For banks, re-allocating funds from one loan to another is also costly. A bank might not be able to resell an asset for the same value agreed upon previously with the now defaulting agent. On the other hand, agents might suffer a loss in utility when they have to default on their loan in addition to the resource costs of the bank. They may have made life plans contingent on this loan that require further costly alteration. They might, for example, have grown attached to their house, which they financed with a mortgage, and lose utility when they have to move. I argued that any such costs alter the government's ex-post optimization problem, since it has to take the continuation value of agents' contracts into account. Agents' involvement in a financial market thus effectively provides a commitment device for the government: Even though it has the ability to re-optimize its policy over time, it does not find it useful to do so at any point.

How much commitment is possible depends on the size of the default costs. I derived a simple condition that links the size of the default costs and the concavity of the utility function to the degree of separation a government is able to commit to. I show that whenever the default costs are positive, some separation can persist after information is revealed: The government will optimally pool agents at the top of the type distribution together and separate all other types perfectly. This allows for some social insurance to be implemented and is thus a strict improvement on the no-commitment equilibrium with complete pooling and no social insurance.

The intuition for the constraint is simple: it equates the marginal benefit from deviating from the promised allocation (as measured by the marginal utility of the lowest type who would be distributed toward times the amount of resources available for redistribution) with the marginal cost of such deviation (the utility loss of the highest type who would have to default). For a given functional form of utility, the higher the default costs, the more separation can be implemented. Conversely, for given default costs, the more concave the utility function is, the higher would be the ex-post welfare gain from redistribution. The government would be more tempted to deviate from announced policy ex-post, and so is able to commit only to less separation ex-ante.

The particular form of optimal information revelation changes when the government is restricted to horizontal equity, i.e. if it cannot randomize transfers between seemingly equal agents. This constraint exogenously mitigates the commitment problem, and while the allocation with pooling above a threshold is still feasible, it is not optimal anymore. Instead, it turns out optimal for the government to pool agents into finitely many groups throughout the type distribution. The resulting transfer schedule then has a "tax bracket" structure. The government collects coarse information about agents' types over the complete distribution. The higher the default costs are, the more and the smaller these brackets can be, so that finer information can be collected.

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A Appendix

A.1 Proof of Lemma 3

First, notice that the first-best allocation is not incentive compatible: From an agent's point of view, his consumption allocation $x_1 = x_2 = x$ is fixed, no matter what type he reports. He then chooses to report type $\hat{\theta}$ to solve

$$\max_{\hat{\theta}} u(x + (\theta - \hat{\theta})) + u(x + 2(\theta - \hat{\theta}))$$

Since utility is time-separable and per period utility is strictly increasing, first and second period consumption are not perfect complements. Thus, every type has an incentive to hide income from the government, thereby receiving the same allocation of consumption as under truth-telling x plus the extra hidden income $t(\theta - \hat{\theta})$. Each agent then optimally chooses tho report the lowest possible type $\underline{\theta}$. Full social insurance *and* perfect smoothing cannot be implemented.

Consider next the allocation with perfect smoothing over time for all types and no redistribution across agents, i.e. $x_1(\theta) = x_2(\theta) = \frac{3}{2}\theta$. This allocation is incentive compatible: The agent solves

$$\max_{\hat{\theta}} u(\theta + \frac{1}{2}\hat{\theta}) + u(2\theta - \frac{1}{2}\hat{\theta})$$

Optimality requires

$$\frac{\partial}{\partial \hat{\theta}} = \frac{1}{2}(u'(\theta + \frac{1}{2}\hat{\theta}) - u'(2\theta - \frac{1}{2}\hat{\theta})) = 0 \tag{20}$$

$$u'(\theta + \frac{1}{2}\hat{\theta}) = u'(2\theta - \frac{1}{2}\hat{\theta}) \tag{21}$$

$$\rightarrow \hat{\theta} = \theta \tag{22}$$

The last step follows because $u(\cdot)$ is strictly concave. At this allocation, per period consumption $x_t(\theta) = t\theta + T_t(\theta)$ increases with slope $\frac{3}{2}$. The concavity of the utility function implies that it is strictly optimal for all agents report the true type. This means that the incentive constraints are not binding for any type. Thus, there is room for welfare increasing redistribution across agents. It follows directly that total consumption will be increasing less than under autarky, i.e.

$$\frac{\partial(x_1(\theta) + x_2(\theta))}{\partial \theta} < 3$$

Next, I will derive the properties of the optimal allocation that result from such redistribution. Redistributing across agents from top to bottom requires that the sum of transfers $T_1(\theta) + T_2(\theta)$ should be decreasing in type, i.e.

$$\frac{\partial(T_1(\theta) + T_2(\theta))}{\partial\theta} = T_1'(\theta) + T_2'(\theta) < 0 \leftrightarrow -\frac{T_2'(\theta)}{T_1'(\theta)} > 1 \tag{23}$$

for all types $\theta < \bar{\theta}$. Just at the highest type, the contribution to the social redistribution system need not be increasing, i.e. $T_1(\bar{\theta}) = -T_2(\bar{\theta})$.

When agents choose which type to report, they solve

$$\max_{\hat{\theta}} u(\theta + T_1(\hat{\theta})) + u(2\theta + T_2(\hat{\theta}))$$

A necessary condition for incentive compatibility thus is that the first order condition of this problem be zero at $\hat{\theta} = \theta$:

$$\frac{u'(\theta + T_1(\theta))}{u'(2\theta + T_2(\theta))} = \frac{-T_2'(\theta)}{T_1'(\theta)} \tag{24}$$

First, notice that (23) together with (24) and concavity of $u(\cdot)$ implies that $x_1(\theta) < x_2(\theta)$ for all types $\theta < \bar{\theta}$, but $x_1(\bar{\theta}) = x_2(\bar{\theta})$. That is, perfect smoothing for the highest type is optimal and smoothing is distorted for all other types.

For (24) to also be sufficient for incentive compatibility, it must be the case that the second order condition for optimality is also satisfied at $\hat{\theta} = \theta$

$$u''(\theta+T_1(\theta))(T_1')^2+u'(\theta+T_1(\theta))T_1''+u''(2\theta+T_2(\theta))(T_2')^2+u'(2\theta+T_2(\hat{\theta}))T_2'' < 0 \tag{25}$$

Further differentiating (24) yields

$$u''(\theta+T_1(\theta))T_1'x_1'+u'(\theta+T_1(\theta))T_1''+u''(2\theta+T_2(\theta))T_2'x_2'+u'(2\theta+T_2(\hat{\theta}))T_2'' = 0 \tag{26}$$

where $x_t(\theta) = t\theta + T_t(\theta)$ and so $x_t'(\theta) = t + T_t'(\theta)$.

Combining (25) and (26) gives the following monotonicity requirement

$$u''(\theta+T_1(\theta))T_1'x_1'+u''(2\theta+T_2(\theta))T_2'x_2' > u''(\theta+T_1(\theta))(T_1')^2+u''(2\theta+T_2(\theta))(T_2')^2 \tag{27}$$

which simplifies to

$$u''(\theta + T_1(\theta))T_1' + 2u''(2\theta + T_2(\theta))T_2' > 0 \tag{28}$$

A sufficient condition for this to hold is that

$$2 > \frac{u''(x_1)}{u''(x_2)} \tag{29}$$

which due to CRRA implies

$$x_1 > \frac{1}{2}x_2 \tag{30}$$

Autarky implies $x_1 = \frac{1}{2}x_2$, so that this condition is met when smoothness of consumption is increased for all agents. Thus, the full set of IC constraints can be replaced by the local incentive constraints (24) and the requirement that $x_1 > \frac{1}{2}x_2$.

The government's problem then is to solve

$$\max_{\{T_1,T_2\}} \int_{\underline{\theta}}^{\bar{\theta}} u(\theta + T_1(\theta)) + u(2\theta + T_2(\theta))$$

$$\text{s.t.} \int_{\Theta} T_1(\theta) + T_2(\theta) \leq 0$$

$$u'(\theta + T_1(\theta))T_1'(\theta) + u'(2\theta + T_2(\theta))T_2'(\theta) = 0 \quad \forall\theta$$

The first order conditions to this problem yield the following optimality condition:

$$u'(\theta + T_1(\theta)) - u'(2\theta + T_2(\theta)) = \gamma(\theta)(u''(\theta + T_1(\theta))T_1'(\theta) + u''(2\theta + T_2(\theta))T_2'(\theta))$$

where $\gamma(\theta)$ are the Lagrange multipliers on the incentive compatibility constraints. From this condition it follows that when $x_1(\theta) < x_2(\theta)$

$$u''(\theta + T_1(\theta))T_1'(\theta) + u''(2\theta + T_2(\theta))T_2'(\theta) < 0 \tag{31}$$

CRRA implies that

$$\frac{x_2}{x_1} = \frac{u''(x_1)}{u''(x_2)}\frac{u'(x_2)}{u'(x_1)} \tag{32}$$

so that

$$x_1(\theta) < x_2(\theta) \rightarrow \frac{x_2(\theta)}{x_1(\theta)} > \frac{u'(x_2(\theta))}{u'(x_1(\theta))} \tag{33}$$

Moreover, note that

$$\frac{x_2(\theta)}{x_1(\theta)} = \frac{2\theta + T_2(\theta)}{\theta + T_1(\theta)} \leftrightarrow \frac{x_2'(\theta)}{x_1'(\theta)} = \frac{2 + T_2'(\theta)}{1 + T_1'(\theta)} \tag{34}$$

We would like to show that the degree of smoothness as measured by the ratio $\frac{x_1}{x_2}$ is increasing in type, i.e

$$\frac{\partial \frac{x_1(\theta)}{x_2(\theta)}}{\partial\theta} = \frac{x_1'(\theta)x_2(\theta) - x_1(\theta)x_2'(\theta)}{(x_2(\theta))^2} > 0 \leftrightarrow \frac{x_2(\theta)}{x_1(\theta)} > \frac{x_2'(\theta)}{x_1'(\theta)} \tag{35}$$

Combining optimality (31), CRRA (33), and (34) with (24) and (28) implies that (35) holds, and thus the degree of consumption smoothness increases with type. This concludes the proof.

A.2 Proof of Lemma 4

First, suppose the information revelation rule was such that all information reported by the agents would be revealed to the government, i.e. $m(\theta) = \theta$ for all types θ. Constraint (8) implies that if the government possesses any information about types at the beginning of the second period, it will exploit it so to equalize consumption as much as possible. To see that, consider the first order conditions of the government's problem (8) at t=2:

$$u'(2\theta + \hat{T}_2(\theta)) - \lambda = 0 \quad \forall\theta$$

These conditions imply that the government will choose $\{\hat{T}_2\}$ so to equalize consumption across all agents, $x_2(\theta) = x_2 \ \forall\theta$. From the agent's point of view then the consumption allocation in period 2 is fixed, and he solves:

$$\max_{\hat{\theta}} u(x_1(\hat{\theta}) + \theta - \hat{\theta}) + u(x_2 + 2(\theta - \hat{\theta}))$$

For truth-telling to be optimal, it is necessary that the first and second order conditions are satisfied at $\hat{\theta} = \theta$, i.e. $\forall\theta$:

$$(x_1'(\theta) - 1)u'(x_1(\theta)) - 2u'(x_2) = 0 \tag{36}$$

$$(x_1'(\theta) - 1)^2 u''(x_1(\theta)) + x_1''(\theta)u'(x_1(\theta)) + 4u''(x_2) < 0 \tag{37}$$

Further differentiating (36) yields

$$x_1'(\theta)(x_1'(\theta) - 1)u''(x_1(\theta)) + x_1''(\theta)u'(x_1(\theta)) = 0$$

which reduces (37) to

$$-(x_1'(\theta) - 1)u''(x_1(\theta)) + 4u''(x_2) < 0 \tag{38}$$

This, together with (36) implies that for the allocation to be incentive compatible, it must be such that $\forall\theta$

$$\begin{aligned} -\frac{u''(x_1(\theta))}{u'(x_1(\theta))} &< -2\frac{u''(x_2)}{u'(x_2)} \\ \leftrightarrow -\frac{u''(x_1(\theta))}{u'(x_1(\theta))}x_1(\theta)x_2 &< -2\frac{u''(x_2)}{u'(x_2)}x_2x_1(\theta) \\ \leftrightarrow x_2\frac{1}{\epsilon} &< 2x_1(\theta)\frac{1}{\epsilon} \end{aligned}$$

where ϵ is the elasticity of intertemporal substitution, which is constant by assumption. Thus, it must be true for all types that

$$x_1(\theta) > \frac{1}{2}x_2 \tag{39}$$

Moreover, (36) can be rearranged as

$$x_1'(\theta) = 2\frac{u'(x_2)}{u'(x_1(\theta))} + 1$$

This differential equation determines the shape of the consumption schedule in period 1. Two properties are important: $x_1(\theta)$ is *increasing* in type, with a slope strictly larger than 1, and with *increasing slope*. The lowest type, $\underline{\theta}$ will receive the lowest period 1 consumption. To relax incentive constraints for the higher types, it is optimal to start from the lowest possible $x_1(\underline{\theta})$. A lower bound is $x_1(\underline{\theta}) = \frac{1}{2}x_2$. What is x_2?

$$x_2 = 2\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} x_1(\theta)dF(\theta) \tag{40}$$

The second summand cannot be solved without further assumptions on the utility function. But we can use a conservative lower bound to see what the government would at most be able to achieve with a fully separating allocation. To that end, suppose we ignore that $x_1(\theta)$ has to be increasing with *increasing* slope, and rather assume that it will increase with *constant* slope $x_1'(\underline{\theta}) \approx 2$. This is not a bad approximation, since constant elasticity of intertemporal substitution implies $u'''(\cdot) < 0$ and so $\frac{u'(x_2)}{u'(\frac{1}{2}x_2)} \geq \frac{1}{2}$ is not a terrible assumption. This lower bound allows to compute an upper bound on x_2:

$$x_2 \leq 2\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta) - \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{2}x_2 + 2(\theta - \underline{\theta})dF(\theta) \tag{41}$$

$$\leftrightarrow x_2 \leq \frac{4}{3}\underline{\theta} \tag{42}$$

This leaves the lowest type at best with the consumption allocation $[\frac{2}{3}\underline{\theta}, \frac{4}{3}\underline{\theta}]$. Notice that this means he is distributed *away* from in the aggregate and also doesn't gain any smoothness. This cannot be optimal from a social welfare point of view. It means that the only separating allocation that can be implemented is one that increases inequality and lowers welfare compared to autarky, and thus it is not optimal.

Notice that the argument does not change when the government learns only partial information about types. Since the second period allocation is fixed, providing incentives for any separation through first period transfers is so costly that it is not optimal to do so. Thus, the optimal information revelation rule is one where no information is revealed, i.e.

$$m(\theta) = \underline{\theta} \quad \forall \theta$$

This concludes the proof.

A.3 Proof of Proposition 3

The proposition states that a government without commitment is able to implement an allocation with at least partial separation, if all agents pledge their complete income in the financial market. Strictly positive default costs alter the ex-post problem of the government: it might even after the revelation of information not have an incentive to redistribute fully, because this would lead to costly default by agents who are redistributed away from.

The proof proceeds as follows: In a first step I will establish the optimal form of the information revelation rule. It turns out to be optimal that agents with income types above a threshold $\tilde{\theta}$ are pooled together, while all agents below the cutoff are completely separated. The second step derives the optimal cutoff type, dependent on the the size of the default costs H and the concavity of the utility function.

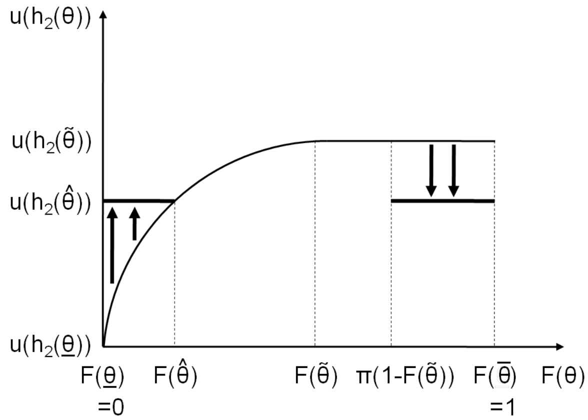
First notice that the following Lemma holds:

Lemma 9
If the government wants to default, it will always default on the highest (observed) type first. Even if the density of highest types is large (e.g. due to pooling at the top), it prefers to randomize and default on some of the highest types rather than on lower types.

Proof: Since ex-ante incentive compatibility implies that the promised allocation in $t = 2$ is increasing in type, the gains from redistributing ex-post are highest when letting the highest types default. The default costs H, on the other hand, are constant per default. □

Suppose agents above some cutoff $\tilde{\theta}$ are pooled together. Even if it is not optimal to default on all of them, it might still be profitable for the government to default only on a fraction π of them. The reason is that neither the gained resources nor the gain in welfare from redistributing these resources are are linear in π. The resources saved are optimally distributed toward the lowest types. Thus, the gain is the highest for the first redistributed dollars and decreases thereafter.

Let $\hat{\theta}$ denote the type below which agents get extra resources when the government lets a fraction π of agents above the cutoff $\tilde{\theta}$ default. The following graph clarifies notation:



The resources gained for redistribution are

$$\pi(1 - F(\tilde{\theta}))(h_2(\tilde{\theta}) - H - x_2(\hat{\theta})) \tag{43}$$

This is because the types who are forced to default will receive the same allocation $x_2(hat\theta)$ as the agents at the bottom of the distribution who are distributed toward. $\hat{\theta}$ is a function of the resources gained, and so a function of π so that the gain is generally not linear in π.

The redistribution causes a loss of utility for the defaulting agents at the top, which for the same argument is nonlinear in π:

$$\pi(1 - F(\tilde{\theta}))(u(h_2(\tilde{\theta})) - u(x_2(\hat{\theta}))) \tag{44}$$

The gain in welfare comes from the utility gain for the types at the low end of the distribution, below $\hat{\theta}$:

$$F(\hat{\theta})u(x_2(\hat{\theta})) - \int_{\underline{\theta}}^{\hat{\theta}} h_2(\theta)dF(\theta) \tag{45}$$

The new consumption level $x_2(\hat{\theta})$ is derived by distributing resources equally between the defaulting high type group and the low type group:

$$x_2(\hat{\theta}) = \frac{\int_{\underline{\theta}}^{\hat{\theta}} h_2(\theta)dF(\theta) - \pi(1 - F(\tilde{\theta}))(h_2(\tilde{\theta}) - H)}{F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))} \tag{46}$$

The net gain from letting a fraction π of agents in the pooled group at the top of the distribution default thus is nonlinear in π. This makes it possible that the government might optimally choose to randomize between seemingly equal agents instead of defaulting on all of them.

Since all agents can smooth their consumption perfectly regardless of whether they are pooled together with other types, the government will choose as much separation as possible to gain as much information as it can commit not to exploit. Thus, it will choose to separate all agents below the cutoff. This establishes the optimal form of the information revelation rule:

$$\begin{aligned} m(\theta) &= \theta \qquad && \forall \theta \leq \tilde{\theta} \\ m(\theta) &= \tilde{\theta} \qquad && \forall \theta > \tilde{\theta} \end{aligned}$$

All agents below the cutoff are asked for precise information, all types above the cutoff can truthfully only report the same income.

The second step of the proof involves finding the optimal pooling cutoff $\tilde{\theta}$ so that the government will not find it optimal ex-post to let even a few of the pooled agents default, and so consequently does not find it optimal to let anyone default.

Given a promised allocation with pooling at the top, the government will choose the optimal fraction π of default on the pooled group of agents according to

$$\max_{\pi} [F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))]u(h_2(\hat{\theta})) - \int_{\underline{\theta}}^{\hat{\theta}} u(h_2(\theta))dF(\theta) - \pi(1 - F(\tilde{\theta}))u(h_2(\tilde{\theta}))$$

$$\text{s.t. } h_2(\hat{\theta}) = \frac{\left[\int_{\underline{\theta}}^{\hat{\theta}} (h_2(\theta))dF(\theta) + \pi(1 - F(\tilde{\theta}))(h_2(\tilde{\theta}) - H)\right]}{(F(\hat{\theta}) + \pi(1 - F(\tilde{\theta})))} \tag{47}$$

$$0 \leq \pi \leq 1 \tag{48}$$

The problem states that the government maximizes the welfare gain from defaulting on a fraction π of the pooled agents subject to how many agents can be provided with higher consumption depending on the resources saved due to not paying out the promised high income to the high types. As introduced earlier, I denote with $\hat{\theta}$ the cutoff below which agents are better off after the redistribution. The government wants to distribute the saved resources to the low types such that it makes optimal use of the highest marginal utilities of more consumption. As a result, all agents up to type $\hat{\theta}$ will get the same consumption as the type $\hat{\theta}$ was promised ex-ante, i.e $x_2(\hat{\theta}) = h_2(\hat{\theta})$. Of course the government will choose to provide the same level of consumption to the agents who were just forced to default. The cutoff $\hat{\theta}$ is obviously endogenous to the choice of π - the constraint (47) defines the optimal cutoff implicitly.

The first order condition to this optimization problem, disregarding constraint (48) for the moment, is:

$$[F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))]u'(h_2(\hat{\theta}))\frac{\partial h_2(\hat{\theta})}{\partial \pi} - (1 - F(\tilde{\theta}))[u(h_2(\tilde{\theta})) - u(h_2(\hat{\theta}))] = 0 \tag{49}$$

where

$$\begin{aligned}\frac{\partial h_2(\hat{\theta})}{\partial \pi} = & \frac{(1 - F(\tilde{\theta}))}{[F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))]^2}((h_2(\tilde{\theta}) - H)[F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))] \\ & - [\int_{\underline{\theta}}^{\hat{\theta}} h_2(\theta)dF(\theta) + \pi(1 - F(\tilde{\theta}))(h_2(\tilde{\theta} - H))])\end{aligned} \tag{50}$$

Note that since $\hat{\theta}$ is always chosen optimally depending on π, by the Envelope Theorem the derivative of $\hat{\theta}$ with respect to π need not be taken into account. First note that since

$$\int_{\underline{\theta}}^{\hat{\theta}} h_2(\theta)dF(\theta) = F(\hat{\theta})h_2(\hat{\theta}) - \pi(1 - F(\tilde{\theta}))(h_2(\tilde{\theta}) - h_2(\hat{\theta}) - H)$$

we can rewrite

$$\frac{\partial h_2(\hat{\theta})}{\partial \pi} = \frac{(1 - F(\tilde{\theta}))(h_2(\tilde{\theta}) - h_2(\hat{\theta}) - H)}{F(\hat{\theta}) + \pi(1 - F(\tilde{\theta}))} > 0 \tag{51}$$

and so the first derivative simplifies to:

$$\frac{d}{d\pi} = (1 - F(\tilde{\theta}))\left[(h_2(\tilde{\theta}) - h_2(\hat{\theta}) - H) - (u(\tilde{\theta}) - u(h_2(\hat{\theta})))\right] \tag{52}$$

The second order condition to this problem is always negative:

$$\frac{d^2}{d\pi^2} = (1 - F(\tilde{\theta}))u''(h_2(\hat{\theta}))(h_2(\tilde{\theta}) - h_2(\hat{\theta}) - H)\frac{\partial h_2(\hat{\theta})}{\partial \pi} < 0$$

Thus, there is only one optimal default probability π^*. Next, I will derive a condition under which the government will find it optimal to choose $\pi^* = 0$. For $\pi^* = 0$ to be optimal, we need the first derivative (52) to be less or equal to zero at $\pi = 0$. Less than zero makes $\pi = 0$ optimal because of the non-negativity constraint (48) disregarded before. Setting $\pi = 0$ leads to $\hat{\theta} = \underline{\theta}$. Then evaluating (52) at $\pi = 0$, gives the following final condition:

$$u'(h_2(\underline{\theta}))(h_2(\tilde{\theta}) - h_2(\underline{\theta}) - H) \leq u(h_2(\tilde{\theta})) - u(h_2(\underline{\theta})) \tag{53}$$

Given H and the functional form of $u(\cdot)$, the government can commit to a schedule $\{h_2(\theta)\}_\Theta$ that pools agents above $\tilde{\theta}$ and satisfies constraint (53). In fact, should this condition not bind, less agents can be pooled together, which is preferable for the government. Thus it will always choose $\tilde{\theta}$ such that the condition holds with equality.

It remains to be shown that for any positive H some separation is possible, i.e. there exists a $\tilde{\theta} > \underline{\theta}$ such that condition (53) is satisfied. Notice that when there is no separation ($\tilde{\theta} = \underline{\theta}$) and $H > 0$, the condition is always slack:

$$u'(h_2(\underline{\theta}))(-H) < 0 \tag{54}$$

Thus, there is room for separation until the constraint binds, as long as $H > 0$. This concludes the proof of Proposition 3.

A.4 Proof of Proposition 5

This proof proceeds by analyzing how the additional constraint of horizontal equity changes the optimization problem in Proof A.3.

First, notice that Lemma 9 (in Proof A.3) does not hold anymore. Since the government cannot randomize transfers, once cannot conclude that in case any default is profitable ex-post, it is optimal to let the highest types default. If enough agents at the top are pooled together, it might be that defaulting on all of them is suboptimal, while defaulting on some of them might have given a welfare improvement, but is not allowed anymore. In that case, the government might decide to default on lower type agents, simply because they are not so many. In what follows, I will show when that can happen, and how the optimal pooling is chosen so to prevent the desirability of any default ex-post.

Starting from an allocation where all agents are perfectly separated, it is still optimal to default on the highest types first (unless H is high enough to give full commitment). Thus, to be able to commit, some agents at the top need to be pooled. The cutoff $\tilde{\tilde{\theta}}$ has to be such that

$$[F(\hat{\theta}) + (1 - F(\tilde{\tilde{\theta}}))]u(h_2(\hat{\theta})) \leq \int_{\underline{\theta}}^{\hat{\theta}} u(h_2(\theta))dF(\theta) + (1 - F(\tilde{\tilde{\theta}}))u(h_2(\tilde{\tilde{\theta}})) \quad (55)$$

$$\text{where } h_2(\hat{\theta}) = \frac{\int_{\underline{\theta}}^{\hat{\theta}} (h_2(\theta))dF(\theta) + (1 - F(\tilde{\tilde{\theta}}))(h_2(\tilde{\tilde{\theta}}) - H)}{(F(\hat{\theta}) + (1 - F(\tilde{\tilde{\theta}})))} \quad (56)$$

This ensures that the gain from defaulting on *all* agents above $\tilde{\tilde{\theta}}$ (left side of equation (55)) is smaller than the associated loss (right side of equation (55)). Notice that if the cutoff $\tilde{\tilde{\theta}}$ is chosen such that the government is exactly indifferent (i.e. equation (55) holds with equality), then $\tilde{\tilde{\theta}}$ is larger than the cutoff $\tilde{\theta}$ derived by solving the problem without the additional constraint of horizontal equity (problem (47) in proof A.3). The reason is simply that without the horizontal equity requirement, the government had to be deterred from defaulting with any positive probability - a stronger requirement than indifference for default probability $\pi = 1$.

Next, suppose $\tilde{\tilde{\theta}}$ is chosen as high as possible, such that (55) binds, and all types below this cutoff are perfectly separated. Then, the government will always find it optimal to let a few agents just below the cutoff default. In other words, one can always find a positive ϵ, so that

$$[F(\hat{\theta}) + (F(\tilde{\tilde{\theta}}) - F(\tilde{\tilde{\theta}} - \epsilon))]u(h_2(\hat{\theta})) > \int_{\underline{\theta}}^{\hat{\theta}} u(h_2(\theta))dF(\theta) + \int_{\tilde{\tilde{\theta}}-\epsilon}^{\tilde{\tilde{\theta}}} u(h_2(\theta))dF(\theta) \quad (57)$$

$$\text{where } h_2(\hat{\theta}) = \frac{\int_{\underline{\theta}}^{\hat{\theta}} (h_2(\theta))dF(\theta) + \int_{\tilde{\tilde{\theta}}-\epsilon}^{\tilde{\tilde{\theta}}} (h_2(\theta) - H)dF(\theta)}{(F(\hat{\theta}) + (F(\tilde{\tilde{\theta}}) - F(\tilde{\tilde{\theta}} - \epsilon)))} \quad (58)$$

To prevent default on types below the cutoff $\tilde{\tilde{\theta}}$, the government can either choose $\tilde{\tilde{\theta}} = \tilde{\theta}$ (then since default is not even optimal with probability zero, so it cannot be optimal for any lower type, since less resources would be gained) or it can pool agents below $\tilde{\tilde{\theta}}$ so that it won't find it optimal to default on them. In other words, the government is still able to implement the same allocation with pooling at the top only as before. However, it may find it optimal to let fewer agents pool at the top, but implement pooling throughout the distribution. In fact, it is easy to verify that whenever agents in some range are perfectly separated, it is optimal to pool them together and in return achieve more separation higher up the type distribution. The reason is simply that more precise information about agents with higher income types is more valuable, since more redistribution across agents can be achieved.

Thus, at the optimal allocation, agents will be pooled into finitely many groups throughout the type distribution. As the default costs increase, the number of groups increases. Only when default costs are high enough to give full commitment, perfect separation (i.e. “pooling” into infinitely many groups) can be sustained. Since any positive default cost allowed some separation in the case without horizontal equity, any positive default costs always allows separation into at least 2 groups of agents with the horizontal equity requirement. This concludes the proof of Proposition 5.