Match efficiency and the cyclical behavior
of job finding rates

Petr Sedláček[*]

JOB MARKET PAPER

November 14, 2011

Abstract

The assumption of a constant matching function, typically made in the literature, implies that a given number of searching workers and employers *always* leads to the same number of matches. This is unlikely to be true, for example, if the composition of searching workers changes over the cycle. This paper relaxes the assumption of constant matching function parameters and allows match efficiency to fluctuate. Using data on the job finding rate and unemployment, an unobserved components model is estimated where both match efficiency and vacancies are unobserved. The latter deals with the poor data availability of vacancies over most of the sample. Estimated match efficiency is procyclical and can explain about 25% of job finding rate fluctuations. Drops in match efficiency account for up to 20% of unemployment rate increases during the most severe recessions. Next, the paper shows that procyclical movements in measured match efficiency are present even in a simple matching model with endogenous separations due to a countercyclical rejection rate. A simple extension of introducing firing costs results in the model performing well quantitatively.

Keywords: Matching function, estimation, endogenous separations

JEL Classification: C32, J63

1 Introduction

A popular way to model flows from unemployment to employment is by using a simple (matching) function relating aggregate labor market variables, typically unemployment and vacancies. The advantage of simple matching functions is their ability to capture the consequences of labor market heterogeneities in a parsimonious way. However, a constant matching function (commonly used in the literature) implies that a given number of searching workers and employers always leads to the same number of matches. This is unlikely to be true, for example, if the composition of searching workers changes with the business cycle.

This paper estimates a standard matching function while relaxing the assumption of constant parameters. Namely, the slope coefficient, interpreted as match efficiency, is allowed to vary. One can view time varying match efficiency as the Solow residual of the matching function. Hence, a parameter that captures fluctuations in hires that cannot be accounted for by observed unemployment and vacancies. Estimated match efficiency is found to be procyclical and it turns out to be an important driver of fluctuations in the rate at which unemployed workers find jobs. In the benchmark specification around 25% of the job finding rate variation can be explained by fluctuations in match efficiency. Furthermore, match efficiency declines account for up to 20% of the unemployment rate runups during the most severe recessions. Next, the paper shows that procyclical movements in measured match efficiency are present even in the simple matching model with endogenous separations because of a countercyclical rejection rate. A simple extension of introducing firing costs results in the model performing well also quantitatively.

Before providing intuition as to why match efficiency might be time varying, I briefly describe the estimation procedure and its caveats. Estimating the matching function on U.S. data and investigating the possibility of time variation in match efficiency is severely complicated by the lack of a good or sufficiently long data series on vacancies.[1] To tackle this problem I specify and estimate an unobserved components model where *both* match efficiency *and* vacancies are treated as unobserved. Assumptions on the underlying processes together with additional information on vacancies at the very end of the sample from the Job Openings and Labor Turnover Survey (JOLTS) facilitate identification of

---

[1]The typically used proxy for vacancies, dating back to 1951, is the Help Wanted Index. This index is constructed from help wanted ads (not number of job vacancies) in 51 newspapers across the U.S. and is therefore only a crude measure of vacancies.

the two unobserved states.[2] Robustness checks suggest that the results are not an artifact of a specific functional form, estimation procedure or sample period. Using additional information from the JOLTS database to further pin down the two unobserved states does little to the results and a Monte Carlo exercise documents that the benchmark specification can identify the unobserved states well.

One reason why match efficiency might fluctuate over the business cycle can be found in cyclical variations of labor market heterogeneity. A structural model can then shed light on the specific channel, form of heterogeneity, that drives match efficiency fluctuations. One such channel is variation in the endogenous rejection rate in the standard matching model with (a constant matching function and) endogenous separations. The mechanism is the following: in the standard endogenous separations model workers differ in their productivity levels. There exists a cut-off value for worker productivity below which employment relationships are no longer viable and thus they (endogenously) separate. Recessions are times when this cut-off increases, since a fall in aggregate productivity makes only the relatively more productive workers survive in their jobs. The same logic applies to unemployed workers who are matched with a vacancy. Those with productivity levels above the cut-off are accepted and form an employment relationship. On the other hand, those who are not productive enough are (endogenously) rejected and fall back into the unemployment pool. A positive rejection rate creates a wedge between the total unemployment pool and the part of the unemployment pool that is useful for forming employment relationships. Moreover, the countercyclical fluctuations of the rejection rate imply that in a recession the fraction of the unemployment pool useful for matching shrinks. Hence, in a downturn the aggregate job finding rate falls by more than would be implied by a constant matching function that takes into account the *total* number of unemployed and vacancies.

One can calibrate the above-mentioned model in such a way that it perfectly captures the match efficiency fluctuations observed in the data. However, such an attempt leads to the model grossly exaggerating the volatility of other endogenous variables, most significantly that of the separation rate. I document that incorporating firing costs the model explains match efficiency movements well, both qualitatively and quantitatively, while not exaggerating the volatility of other variables. The intuition is the following: firing costs drive a wedge between workers in existing employment relationships and newly hired work-

---

[2]The JOLTS database provides high quality data on vacancies, but it dates back only to December of 2000, while the sample used in this paper starts in 1948.

ers. The cut-off productivity level for workers in existing relationships is lower than in the case of no firing costs, since firms know that separations entail a cost. On the other hand, the cut-off probability for the newly hired workers is higher, since firms require a compensation for expected future firing costs. This, together with the distributional assumption of an upward sloping density in the neighborhood of the cutoff, makes the rejection rate more sensitive to aggregate fluctuations. The assumption on the distribution is not unreasonable considering that the cutoff values are in the lower tail. This simple extensions enables the model to explain about 60% of match efficiency fluctuations found in the empirical part, while not exaggerating the volatility of other variables.

This paper fits into a line of research studying and estimating the matching function. **?** provide an excellent survey of this literature. Furthermore, it is related to a strand of literature trying to understand the influence of match efficiency on unemployment and/or explain the sources of match efficiency movements. **?** undertake a steady state decomposition of the Beveridge curve using the vacancy series from **?** to estimate the contributions of firms' actions (hiring and firing), demographics and match efficiency on unemployment movements.[3] Their estimated match efficiency, however, displays a mainly counter-cyclical pattern, except for the most recent recession. **?** further study the reasons behind match efficiency fluctuations and find that the composition of the unemployment pool is responsible for most of its movements in the years 1976-2006. Over 2007-2010, however, an increase in the dispersion of labor market conditions explains almost half of the match efficiency decline. The model mechanism behind match efficiency movements presented in this paper is related to **?**, who propose that declines in the job finding rate during recessions are due to an increase in the proportion of workers with low individual probabilities of exiting unemployment. **?** presents an alternative mechanism. He builds a DSGE model combining a housing market, labor and financial frictions. His model predicts that a decline in house prices reduces geographical mobility which in turn leads to a drop in match efficiency.

This paper is organized as follows. Section 2 describes the estimation procedure and Section 3 shows the empirical results. Section 4 provides some robustness exercises. Then, Section 5 builds a matching model with endogenous separations and shows that it features procyclical match efficiency movements, but that calibrating the model to fit match efficiency fluctuations makes it exaggerate the volatility of other variables. Section 6 extends

---

[3]**?** constructs a vacancy proxy that takes into account internet posting after 1995. However, prior to 1995 the Help Wanted Index is taken as the ideal proxy for vacancies.

the model to include firing costs and shows that such a model does relatively well in explaining match efficiency fluctuations also quantitatively. Finally, Section 7 provides the conclusion.

2 Estimating match efficiency variation

The starting point of the estimation procedure is the definition of the job finding probability, $F_t = M_t/U_t$, where M_t is the number of matches (unemployed workers who find a job) and U_t is the number of unemployed in period t.[4] As noted in the introduction, the typical way to model the number of matches is using a matching function $M_t = Am(U_t, V_t)$, where A is match efficiency and V_t is the number of vacancies in period t. This paper allows A to be time-varying and at the same time deals with the problematic nature of vacancies. One can view time varying match efficiency as the Solow residual of the matching function. Hence, a parameter that capturers fluctuations in hires that cannot be accounted for by observed unemployment and vacancies. The main goal of the paper is to investigate if fluctuations in match efficiency are important for determining the aggregate job finding rate and in turn affecting the aggregate unemployment rate.

To this end, I specify a state-space model treating both match efficiency and vacancies as unobserved. I use quarterly data on the job finding rate taken from **?** and the number of unemployed published by the BLS in the period 1948Q1-2007Q1. In addition, to help identify the two unobserved states, I use information from the JOLTS vacancies series (available from December 2000) assuming that they are a noisy observation of the underlying process.[5,6]

---

[4]In the text I use the number of unemployed and unemployment interchangeably.

[5]Prior to this date the JOLTS vacancy data are treated as missing observations.

[6]In the JOLTS specification a job opening requires that "1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. The position can be full-time or part-time, and it can be permanent, short-term, or seasonal. Furthermore, active recruitment means include advertising in newspapers, on television, or on radio; posting Internet notices; posting 'help wanted' signs; networking or making 'word of mouth' announcements; accepting applications; interviewing candidates; contacting employment agencies; or soliciting employees at job fairs, state or local employment offices, or similar sources". This definition is taken from the JOLTS computer assisted telephone interview (http://www.bls.gov/jlt/jltc1.pdf). This comprehensive definition suggests that assuming the JOLTS vacancy series to be an unbiased signal of the underlying vacancies is not unreasonable.

2.1 State-space representation

In the general state space form a $m \times 1$ vector of observables, y_t, is related to a $q \times 1$ vector of unobserved states s_t via the *measurement equation*.

$$y_t = \Theta_{0,t} + \Theta_{1,t} s_t + \epsilon_t, \tag{1}$$

where $\Theta_{0,t}$ is an $m \times 1$ vector, $\Theta_{1,t}$ is an $m \times q$ matrix and ϵ_t is an $m \times 1$ vector of serially uncorrelated disturbances with mean zero and a covariance matrix R. The unobserved states are assumed to evolve according to a first-order Markov process (the *transition equation*)

$$s_t = \Phi_{0,t} + \Phi_{1,t} s_{t-1} + \eta_t, \tag{2}$$

where $\Phi_{0,t}$ is an $q \times 1$ vector, $\Phi_{1,t}$ is an $q \times q$ matrix and η_t is an $q \times 1$ vector of serially uncorrelated disturbances with mean zero and covariance matrix Q.

In the model at hand there are two unobserved states ($q = 2$): match efficiency (A_t) and vacancies (V_t). In the benchmark model vacancies are assumed to be a random walk, while match efficiency is assumed to follow a stationary AR(1) process. The choice of the random walk on vacancies is motivated by its fundamentally close relationship to unemployment, for which one cannot reject a unit root in the given sample.[7] Similarly, for the typical vacancy proxy, the Help Wanted Index (HWI), one also cannot reject a unit root.[8] This is, arguably, a less compelling argument, because of the problematic nature of the HWI as a vacancy proxy. Although the above-mentioned evidence points to a process for vacancies that is I(1), not necessarily a random walk, allowing for a richer non-stationary structure does not change the results much as is shown in Appendix D. Assuming match efficiency to be an AR(1) process then helps the identification by distinguishing it from the vacancy process. However, the appendix shows that an alternative specification where both states are random walks delivers similar results.

The two states are related to observed variables via two measurement equations ($m = 2$): one for the job finding probability (F_t) and one postulating that the JOLTS job openings series (V_t^J) is a noisy observation of the vacancy state. The former is the main

---

[7] The ADF with 4 lags and an intercept (intercept and trend) can reject a unit root at the 11.9% (12%) level. For first differenced unemployment the unit root is rejected (in all specifications: with(out) intercept and intercept with trend) at the 0% level. Although one would not, a priori, expect unemployment to be nonstationary, in the (finite) sample at hand it is a good data description.

[8] The ADF test with 4 lags and an intercept (intercept with trend) rejects the unit root at the 11.4% (40.5%) level. For first differences it rejects at the 0% level.

equation facilitating the identification of the two processes. The latter helps pin down further the properties of the vacancy state and especially their level. Remember, however, that the job openings data is available only from 2001Q1. The periods prior to that date can be conveniently handled by the Kalman filter as missing observations. Finally, I follow the literature and assume the matching function is Cobb-Douglas with constant returns to scale[9]

$$M_t = A_t U_t^{1-\mu} V_t^{\mu}. \tag{3}$$

Denoting with small letters the natural logarithm of variables one can write the state space representation of the model as

$$\begin{bmatrix} f_t \\ v_t^J \end{bmatrix} = \begin{bmatrix} -\mu u_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_t \\ v_t \end{bmatrix} + \epsilon_t, \tag{4}$$

$$\begin{bmatrix} a_t \\ v_t \end{bmatrix} = \begin{bmatrix} (1-\rho_a)\bar{a} \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ v_{t-1} \end{bmatrix} + \eta_t, \tag{5}$$

where ρ_a is the autoregressive coefficient of log match efficiency and $\bar{a}$ is its unconditional mean. Furthermore, the innovations of the state and measurement equations are assumed to be jointly normally distributed with mean zero and variance covariance matrix

$$E_t \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix} [\eta_t \quad \epsilon_t] = \begin{pmatrix} Q & C' \\ C & R \end{pmatrix}, \tag{6}$$

where C is the 2×2 cross-covariance matrix.

2.2 Estimation

Maximum likelihood (ML) is used to estimate the elasticity of vacancies in the matching function (μ), the autoregressive coefficient and unconditional mean of log match efficiency (ρ_a and $\bar{a}$) and all the elements of the variance covariance matrix of the innovations (R, C and Q).[10] The Kalman filter is then employed to obtain smoothed states[11] at the ML estimates. Furthermore, to overcome potential endogeneity problems, I use the first lag of

---

[9]**?** survey the literature and conclude that the such a functional form has large empirical support.

[10]The minimization itself is done using Chris Sims' csminwel algorithm.

[11]The term "smoothed" might be confusing later on when evaluating the volatility of the states. Note that it refers to running the Kalman filter "backwards". The estimates in period t are then based on not only past information, but also on information from observations t onwards.

the regressor as an instrument. Appendix **??** provides explicit exogeneity tests supporting this procedure.

To start the minimization routine one must pick initial values. The starting values for ρ_a, $\bar{a}$ and μ are set to 0.9, -0.6 and 0.3, respectively. The initial values for the covariance matrices are based on error variances from an auxiliary regression of the job finding probability on observed labor market tightness (using the HWI as an indicator of vacancies) using data up until 1955Q4. Denote the error variance from the trial regression by W_f. Furthermore, denote by W_v the variance of the (log) job openings series from the JOLTS database. The initial values for the covariance matrices are then $R^{init} = \begin{pmatrix} \omega_{R,f}W_f & 0 \\ 0 & \omega_{Q,v}W_v \end{pmatrix}$, $Q^{init} = \begin{pmatrix} \omega_{Q,f}W_f & 0 \\ 0 & \omega_{Q,v}W_v \end{pmatrix}$. The scaling parameters $\omega_{i,j}$, where $i = Q, R$ and $j = f, v$, are found by a grid search that maximize the log-likelihood of the model. The initial value for the cross-covariance matrix C is a 2×2 zero matrix. Robustness checks show that changing the initial values does little to the results.

Furthermore, to start the Kalman filter routine one must set the initial state vector s_0 and its covariance matrix P_0. Following **?** the former is set to the unconditional mean of the state vector, while the latter is set to a large number (10^5). This essentially means that there is large uncertainty about the initial state and the data is allowed to speak freely.

3   Estimation results

Table 1 provides the estimated parameter values as well as p-values of diagnostic tests related to the model residuals. The Cobb-Douglas elasticity on vacancies is estimated to be 0.39, which falls within the range reported in **?** and it is close to the estimates in **?** and **?**.

The diagnostic tests indicate that the model assumptions hold. It is necessary, however, to deal with 4 outliers[12] (using a single dummy variable) in order to satisfy the normality assumption. Note, however, that the other diagnostic tests still hold when the outliers are not treated. Moreover, the match efficiency and vacancy estimates are almost unchanged when the dummy variable is added. Figure 1 shows the estimated smoothed vacancies and match efficiency which I discuss in detail next.

---

[12]The outliers are in quarters 1957Q4, 1958Q1, 1974Q4 and 1975Q1.

3.1 Match efficiency

Match efficiency varies substantially with a standard deviation of almost 5.0%. This value is the theoretical standard deviation based on the estimated parameters, hence $\sigma_\alpha = \sqrt{\frac{Q(1,1)}{1-\rho_\alpha^2}}$.[13] Furthermore, match efficiency is procyclical with respect to the business cycle. The correlation coefficients of match efficiency with (log) unemployment and output are -0.44 and 0.59, respectively.[14] This means that recessions are periods when unemployed workers on average have a harder time finding a job not only because the number of vacancies drops and there are more unemployed workers competing for a given vacancy, but also because the efficiency of the matching process declines.

Match efficiency drops, however, have different patterns across recessions. During the recessions in the late 50's, mid 70's and early 80's match efficiency experienced the sharpest declines in the range of 5-6%. Smaller, but still sizeable falls in match efficiency happened during the recessions in the early 60's and the new millenium with falls of around 4%. The 1990 recession is peculiar in that match efficiency kept on falling for a few quarters while the economy was already recovering. A similar pattern is apparent for the 2001 recession, where match efficiency picked up at the end of the recession, but showed a sharp (temporary) relapse. These developments reflect the jobless recoveries experienced after these two downturns. Even though output started to rise in the recovery phase, match efficiency remained low keeping down the job finding rate and thus dampening employment growth.

3.1.1 How important is match efficiency on average?

To answer this question, I decompose the variation of the job finding rate into contributions of match efficiency and labor market tightness. Such a decomposition is not trivial, since the two components are correlated. For a decomposition that appropriately disentangles the covariance term I follow **?**. The starting point is a log deviation of the job finding rate from its trend value (denoted by bars)

$$\ln \frac{F_t}{\bar{F}_t} = \ln \frac{A_t}{\bar{A}_t} + \mu \ln \frac{\theta_t}{\bar{\theta}_t} + \omega_t, \tag{7}$$

---

[13]Computing the standard deviation would be misleading since the Kalman filter series are an expected value, rather than a realization.

[14]The HP filter (with smoothing coefficient of 1,600) was used to extract the cyclical components of unemployment and output.

where ω_t is an error coming from the detrending procedure. In general it will not be the case that the trend components of match efficiency and labor market tightness exactly add up to that of the job finding rate. The above can be expressed generically as

$$df_t = df_t^A + df_t^\theta + df_t^\omega. \tag{8}$$

One can then show that

$$var(df_t) = cov(df_t, df_t^A) + cov(df_t, df_t^\theta) + cov(df_t, df_t^\omega), \tag{9}$$

where the term $cov(df_t, df_t^A)$ gives the amount of variation in the job finding probability due to match efficiency appropriately taking into account its covariance with labor market tightness. Expressing this variation relative to total volatility in the job finding probability gives:

$$\beta^A = \frac{cov(df_t, df_t^A)}{var(df_t)}. \tag{10}$$

From (9) it is clear that $\beta^A + \beta^\theta + \beta^\omega = 1$. Table 2 shows the respective decompositions for HP-filtered (with smoothing coefficients of $1,600$ and 10^5) and first-differenced data. The table documents how the influence of match efficiency differs depending on what frequency one focuses on. Match efficiency gains explanatory power as one focuses on higher frequencies (for first differenced data match efficiency explains up to 33%, while for HP-filtered data with smoothing coefficient 10^5 the contribution of match efficiency is 15%). Zooming in on business cycle frequencies match efficiency accounts for 24% of the variation in the job finding probability, which is a nontrivial amount.

Figure 2 shows the job finding rate and its counterfactual generated under the assumption that match efficiency is fixed at its average value. Looking at troughs of the three recessions with the largest fall in match efficiency (in 1957, 1974 and 1981), the counterfactual job finding rate is 2-3 percentage points higher. In other words, during these recessions the fall in match efficiency pushed down further the probability of finding a job by up to 3 percentage points. Given the low cyclical level in the troughs, 3 percentage points amount to almost 10% of the job finding probability. A similar effect of comparable magnitude occured also in the early 90's, but this time a few quarters *after* the recession ended. Note, however, that even the counterfactual job finding rate picks up after the recession. Hence, although match efficiency contributed to a greater drop in the job finding probability after the recession, it was not the only reason for its delayed bounce-back and hence jobless recovery.

One can use this counterfactual job finding rate to construct a counterfactual unemployment rate. To do so I use the equilibrium expression $u_t = s_t/(s_t + f_t)$, where u is the unemployment rate, s the separation rate and f the job finding rate. For the U.S. economy this "equilibrium" unemployment rate tracks the actual unemployment rate very closely. Figure 3 shows the actual and the counterfactual unemployment rate based on a job finding rate with a *fixed* match efficiency. On average the deterioration in match efficiency accounts for almost 10% of unemployment increases during recessions. However, in downturns with the largest match efficiency drops (1957, 1974 and 1982) the contribution of match efficiency to the unemployment rate runups was as high as 20.6%.

3.1.2 How important is match efficiency during different recessions?

To better understand the importance of match efficiency I decompose the cumulative fall of the job finding rate during each recession into contributions of match efficiency and labor market tightness. Log-differencing the definition of the job finding probability gives $dF_t \approx F_t(d\ln(A_t) + \mu\, d\ln(\theta_t))$.

Figure 4 shows these contributions to the cumulative drop of the (log) job finding rate during the recessions (the quarters prior to the starting dates of the recessions are indicated on the horizontal axis). It is apparent that the variance decomposition is hiding quite a bit of heterogeneity. The contribution of match efficiency to the job finding rate fall during the recessions in 1960 and 1981 is roughly half of the labor market tightness contribution. On the other hand, during the recessions in 1990 and 2001 match efficiency contributed only very slightly. This is related to the fact that match efficiency fell mostly *after* these recessions. Furthermore, the downturns with the highest contributions of match efficiency are also on average longer and deeper.[15]

It seems that match efficiency contributes to reductions in job finding rates more at the onset of recessions. Getting closer to the recovery phase match efficiency contributions slow down and in a few cases they even reverse before the end of the recession. This is related to the previous decomposition exercise where it is shown that match efficiency explains job finding rate fluctuations especially at higher frequencies. As one focuses on longer fluctuations the effect of aggregate labor market tightness gains importance.

All the above points to the fact that match efficiency is an important determinant of job finding rate fluctuations. Therefore, specifications of the matching function should not

---

[15]The recessions associated with the largest match efficiency drops are on average one quarter longer with real GDP growth falling by 1.5 percentage points more.

be such that they rule out this channel by assumption.

3.2 Vacancies

Although vacancies are not the main focus of this paper, the estimated vacancies are of separate interest, because the estimate provides information about the behavior of vacancies over several business cycles. The typically used vacancy proxy, the HWI, dates back to 1951, but is increasingly inaccurate as internet vacancy posting took over in the later part of the sample. The HWI actually stopped being published in May 2008 and was replaced by the Online HWI.[16] The more recent job openings data from the JOLTS database provide a much better indicator of vacancies, but they date back only to 2001 missing all the previous business cycles. On the contrary, the vacancy estimate in this paper enables a methodologically consistent comparison of labor market dynamics over several business cycles, including the more recent ones. For instance, studying movements of the Beveridge curve could shed some new light on the recent developments in the U.S. economy. However, the kind of analysis that this deserves is outside the scope of this paper. The following paragraphs are therefore only descriptive and a deeper investigation is left for future research.

The bottom panel of Figure 1 displays the estimated vacancy state and the HWI. At first sight, the dynamics of the two series are similar (correlation coefficient of 0.81). At the same time, the vacancy estimate is much smoother. Note, however, that the estimated vacancy series is the Kalman filter estimate (a conditional expectation) and not a realization. For a fair comparison one needs to compare the estimated theoretical standard deviation of the vacancy innovations to a suitable empirical counterpart. To this end one can assume that the HWI is also a random walk and use the standard deviation of its first difference.[17] Such a comparison shows that the benchmark vacancy series fluctuates less by approximately 10%. Alternatively, one can estimate the process for the HWI and use its innovation variance for the comparison. Based on inspecting the (partial) autocorrelation function and the Akaike and Schwarz information criteria, the HWI for

---

[16] **?** attempts to link the two indices into a composite HWI. Apart from specific assumptions on the dispersion process of internet use that Barnichon needs to make, he also assumes that prior to 1995 the HWI was the ideal characterization of vacancies.

[17] Although unit root tests do not imply that the HWI is a random walk, they show that the series is non-stationary in the given sample. ADF test with 4 lags and an intercept (intercept with trend) rejects the unit root at the 11.4% (40.5%) level. For first differences it rejects at the 0% level.

the sample at hand is estimated to be an ARIMA(1,1,0). In this case the benchmark vacancy series has an innovation variance that is roughly 10% *larger* than that of the ARIMA(1,1,0) process on the HWI. Hence, the volatility of vacancies according to the estimated model is roughly equal to estimated volatility of the observed HWI.

Going back to the bottom panel of Figure 1, after 1990 there is a clear departure of the HWI and the estimated vacancies. This is arguably due to the spur in internet posting of vacancies as was also pointed out by ? and ?.

Finally, the Beveridge curve is somewhat weaker for the estimated vacancy series. The correlation coefficient between the unemployment and vacancy rate (HP filtered with smoothing coefficient 1600) is -0.9 when using the HWI and -0.72 when using the estimated vacancies. Once again, one needs to keep in mind that the correlation can be affected by the fact that the vacancy series is a conditional expectation and not a realization.

4 Robustness checks

In this section I provide five robustness checks. First, I investigate whether alternative functional forms of the matching function result in similar match efficiency estimates. Second, an alternative estimation procedure is employed checking whether variation in match efficiency is not just a result of poor identification in the benchmark specification. Third, I estimate the model with additional information from the JOLTS database on the vacancy yield to further help pin down the unobserved states. Fourth, the sample period is extended to see whether the results hold also during the most recent severe downturn. Finally, a Monte Carlo exercise is conducted to document the ability of the benchmark procedure to identify unobserved match efficiency and vacancies. Details on the following robustness checks as well as additional robustness exercises are available in Appendix D.

4.1 Alternative functional forms

Here I repeat the benchmark estimation using two alternative matching functions also found in the literature. First, a standard CES specification $M_t = A_t(U_t^\xi + V_t^\xi)^{1/\xi}$ and second a specification proposed by ? $M_t = A_t \frac{U_t V_t}{(U_t^\zeta + V_t^\zeta)^{1/\zeta}}$. In both cases the state-space becomes non-linear in the first measurement equation. To deal with this caveat, I employ the extended Kalman filter (EKF), which essentially uses a first order approximation of the state-space system in the usual Kalman filter recursions. Admittedly, there are

13

more sophisticated non-linear filters available. For the purpose at hand, where one is interested in a first glimpse whether or not the results change substantially with different matching functions, the extended Kalman filter is a natural choice. Moreover, the degree of nonlinearity at the ML parameter estimates for these two functional forms is relatively small (Figure 6) and thus a linear approximation can be expected to perform quite well.

Figure 5 shows the benchmark match efficiency estimate and those from the two alternative functional forms. The results based on both the CES specification and the matching function proposed in ? are very similar to the benchmark estimate. If anything, using the Cobb-Douglas matching function dampens match efficiency fluctuations.

4.2 Alternative estimation procedure

One could be worried that the benchmark specification attributes variation to match efficiency only due to poor identification. To check this, I propose an alternative two-step estimation procedure. First, assume that match efficiency is *fixed* and use data on the job finding rate and unemployment to obtain an estimate for implied vacancies. Second, use the JOLTS vacancy data to decompose the implied vacancies in the first step into the "true" underlying vacancies and match efficiency fluctuations.

In this way, one is not "forcing" match efficiency to vary. If match efficiency is truly constant, then the estimated vacancies in the first step should follow the JOLTS vacancy data closely. If, on the other hand, match efficiency varies over the business cycle, the estimated vacancy series in the first step will incorporate these fluctuations and deviate from the JOLTS vacancy data. The second step will then disentangle the two states.

Figure 7 shows how the first-step vacancy estimate differs from the JOLTS vacancy data (both series are demeaned to ease comparison). Figure 8 plots the benchmark match efficiency estimate and the one from the two-step procedure. The two-step procedure yields an estimate that is very similar to the benchmark estimate.

4.3 Using information from the vacancy yield

It could be the case that the estimated match efficiency series is capturing mainly a cyclical component of the unobserved vacancies. Since both match efficiency and vacancies are procyclical, they have the same qualitative effect on the job finding rate. However, their impact on the job *filling* rate, $Q_t = M_t/V_t$, is of opposite signs. An increase in match efficiency *increases* the probability of filling a vacancy, since the overall process

of matching is more efficient. However, an increase in aggregate vacancies *decreases* the aggregate vacancy filling probability, because there are more vacancies competing for a given number of unemployed.

To make sure that the estimated match efficiency and vacancy states are consistent with job filling rate data I augment the benchmark state-space system with a third measurement equation, $i_t = \gamma_0 + \gamma_1 a_t + \gamma_2 v_t + \gamma_3 u_t + \epsilon_{i,t}$, where i_t is the vacancy yield from the JOLTS database.[18] Figure 9 compares the estimated match efficiency from the state-space system including the vacancy yield data and that from the benchmark specification. The two are very similar. Furthermore, the coefficient on match efficiency (γ_1) is 0.98, while that on vacancies (γ_2) is -0.28 and both are statistically significant. These not only have the expected signs, but also indicate that the estimation procedure does not simply ignore match efficiency fluctuations in the vacancy yield equation.

4.4 Extending the sample period

In the benchmark, the sample period ends in the first quarter of 2007. In this subsection, using a shortcut (building on **?**) explained in Appendix D, I extend the sample to the fourth quarter of 2010. The reader should, however, keep in mind that due to the way the job finding rate is calculated this subsection can serve only as a robustness check. A more careful analysis of the most recent data would require the job finding rate to be calculated by explicitly using detailed CPS data.

Figure 10 shows the benchmark match efficiency estimate together with the one based on the extended sample. The most recent recession was characterized by a sharp fall in match efficiency, almost double that of the harshest drop in past recessions. Overall, however, the picture does not change and match efficiency remains procyclical (although it leads the most recent recession slightly).

4.5 Monte Carlo experiment

This subsection checks how well the benchmark procedure can identify the unobserved match efficiency and vacancies. To this end, I use the benchmark state-space structure, the maximum likelihood parameter estimates and the estimated unobserved states to

---

[18]The vacancy yield is the flow of realized hires during the month per reported job opening at the end of the previous month; hence it is not exactly the job filling rate due to time aggregation issues. Nevertheless, **?** construct a measure of the job filling rate using the JOLTS vacancy yield data and conclude that "... *the job-filling rate exhibits the same strong patterns as the vacancy yield.*"

construct 1,000 artificial data series for the job finding rate and observed vacancies at the end of the sample (representing the JOLTS vacancy data series). Then, for each of the 1,000 replications the benchmark state-space specification is used to estimate the parameters with maximum likelihood and recover the unobserved states.

Figure 11 shows the benchmark estimate of match efficiency together with the average across the 1,000 Monte Carlo replications. The shaded area indicates the 90% confidence bands. The Monte Carlo average is very close to the true underlying state (correlation coefficient of 0.95).[19] The confidence area also clearly follows the procyclical pattern of the true state. However, it could still be that for some realizations the match efficiency estimate is much worse than the mean suggests. To check this, I count the number of Monte Carlo realizations for which the correlation of the estimate with the truth is above a certain level. 85% of the time the match efficiency estimate is correlated with the truth with a higher than 0.5 correlation coefficient and more than 99% of the time the correlation is positive.

In nine cases the correlation between the Monte Carlo estimate and the truth was negative, with a minimum of -0.17. All of these estimates were associated with one or more extreme random draws. Treating these draws with a dummy variable during the estimation always resulted in a new estimate that was positively correlated with the truth. This suggests that the procedure is able to identify the unobserved states quite well.

5 Matching model with endogenous separations

As noted in the introduction, match efficiency might be time-varying because of cyclical changes in labor market heterogeneity. This section documents how endogenous rejection in the standard matching model with a *constant* matching function implies procyclical fluctuations in *measured* match efficiency. One would think that the model can be calibrated such that it exactly fits observed match efficiency volatility. That is true. However, in doing so the model grossly exaggerates fluctuations in other endogenous variables, most significantly that of the separation rate. It is shown that in this setup there is a trade-off between realistic fluctuations for the separation rate and match efficiency. Next a simple extension is proposed that makes the model perform well in terms of capturing match efficiency variation while not exaggerating volatility of the separation rate.

---

[19]A very similar picture is to be seen when one plots the median of the Monte Carlo replications, instead of the average.

Before describing the model, I provide intuition as to why match efficiency fluctuates procyclically in the endogenous separations model. Workers in this model are characterized by individual specific productivity levels. In each period there is a productivity threshold, below which a given job is not viable anymore and the employment relationship is terminated. In this environment, the individual probability of finding a job depends not only on aggregate variables (unemployment and vacancies), but also on the workers' individual productivity. The aggregate job finding rate thus depends on the fraction of unemployed workers who are productive enough to form viable employment relationships. Recessions are times when the productivity threshold increases, since a fall in aggregate productivity makes some employment relationships with relatively less productive workers unsustainable. In other words, recessions are times when the part of the unemployment pool that can form employment relationships shrinks. In this environment a constant matching function taking into account *only* the total number of unemployed and vacancies *underestimates* the fall in the aggregate job finding rate during a downturn.

5.1 Model

5.1.1 Household behavior

The household consists of a continuum of workers of unit mass. Every period each worker draws a productivity level p from a constant distribution F. The productivity draws are independently and identically distributed, hence there is no persistence in individual productivity levels. This feature of the model makes it tractable as the worker productivity distribution is constant and identical for both the unemployment and employment pools. The members of the household pool their incomes from employment and non-employment activities and spend it on consumption. The model abstracts from any investment or labor force participation decisions.

Formally the household maximizes expected life-time utility by choosing aggregate consumption subject to its budget constraint

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma} - 1}{1 - \gamma} \right] \tag{11}$$

s.t.

$$c_t = \int_{\widetilde{p}_t}^{\overline{p}} (w_t(p)n_t)dF(p_t) + bu_t + \Pi_t, \tag{12}$$

17

where total c_t is aggregate consumption, $\int_{\widetilde{p}_t}^{\overline{p}}(w_t(p)n_t)dF(p_t)$ is aggregate wage income, bu_t is non-employment income and Π_t are aggregate profits. Costs of posting vacancies are assumed to be paid to the household.

5.1.2 Matching process

Matching occurs at the end of the period and matched workers are available for production in the next period. Hence, workers that separate at the beginning of period t enter the unemployment pool and are ready to be re-matched in the same period.

Let u_t be the mass of unemployed workers available for matching and let v_t be the mass of vacancies being posted by firms at the end of period t. The number of matches in period t is determined by a matching function

$$m_t = Au_t^{\mu}v_t^{1-\mu}. \tag{13}$$

The choice of the Cobb-Douglas functional form with constant returns to scale is consistent with the empirical part of the paper. Notice, however, that m_t only gives the number of matched worker-vacancy pairs. It still depends on the workers productivity in the next period, whether or not the job is created. Hence, not only workers who are not matched with a vacancy remain in the unemployment pool, but so do those who are matched with a vacancy but are not productive enough.

The probability that a worker is matched with a vacancy in period t is defined as $f_t = m_t/u_t$, while the probability that a firm with an open vacancy is matched with a worker in period t is $q_t = m_t/v_t$. Remember, that these are *not* equal to the probabilities of finding a job and filling a vacancy, which are defined below in Section 5.2.

5.1.3 Employment relationships

An employment relationship consists of a worker and firm pair. Production is given by $z_t p_{i,t}$, where z_t is the aggregate productivity shock and $p_{i,t}$ is the worker specific productivity shock.[20] The relationship can be severed exogenously before the shocks materialize

---

[20]In the specification with iid idiosyncratic productivity shocks it does not matter whether the shock is worker or job specific. Both cases would be identical in terms of the functioning of the model. The interpretation of why match efficiency varies would, however, be different. In the present environment match efficiency varies because of procyclical fluctuations in the fraction of unemployed workers that are productive enough to find jobs. If idiosyncratic productivity shocks were job specific match efficiency would fluctuate because of procyclical variation in the fraction of productive enough jobs.

and this happens with probability ρ_x. After observing the aggregate and worker specific shocks the employment relationship decides whether to continue and produce or whether to separate. In the event of (exogenous or endogenous) separation there is no production and the worker joins the unemployment pool.

5.1.4 Endogenous separations

Next I provide the value functions describing the problem of firms and workers in the matching market. Denote with $W_{i,t}$ the value at time t of being in a productive employment relationship for a worker with job specific productivity $p_{i,t}$ (measured in current consumption units). This is given by

$$W_{i,t} = w_{i,t} + E_t \left[\beta_t(1 - \rho_x) \int_{\widetilde{p}_{t+1}}^{\overline{p}} (W_{t+1} - U_{t+1})dF(p_{t+1}) + U_{t+1} \right], \tag{14}$$

where $w_{i,t}$ is the wage rate, $\beta_t = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$ is the stochastic discount factor, $\widetilde{p}_{t+1}$ is the threshold value of the worker specific shock such that employment relationships with values of $p_{i,t}$ below this threshold endogenously separate and $\overline{p}$ is the upper bound of the skill distribution. Hence, workers get a wage rate dependent on their idiosyncratic productivity levels plus the continuation value of exiting period t in an employment relationship.

The value of being in the matching pool for the worker U_t at time t is defined as

$$U_t = b + E_t \left[\beta_t f_t(1 - \rho_x) \int_{\widetilde{p}_{t+1}}^{\overline{p}} (W_{t+1} - U_{t+1})dF(p_{t+1}) + U_{t+1} \right], \tag{15}$$

where the worker enjoys leisure and the outcome of home production worth b units of consumption, the value of being in an employment relationship tomorrow if successful in the matching process or otherwise the future value of remaining unemployed.

Denote with $J_{i,t}$ the value of a productive employment relationship for the firm employing a worker with idiosyncratic productivity $p_{i,t}$. This value is given by

$$J_{i,t} = z_t p_{i,t} - w_{i,t} + E_t \left[\beta_t(1 - \rho_x) \int_{\widetilde{p}_{t+1}}^{\overline{p}} (J_{t+1} - V_{t+1})dF(p_{t+1}) + V_{t+1} \right], \tag{16}$$

where the firm gets profits from production plus the continuation value of leaving the period in an employment relationship.

The value of an unfilled vacancy V_t is driven down to zero due to the assumption of free entry of firms. This gives then the vacancy posting condition

$$\frac{\kappa}{q_t} = E_t \left[\beta_t(1 - \rho_x) \int_{\widetilde{p}_{t+1}}^{\overline{p}} J_{t+1}dF(p_{t+1}) \right], \tag{17}$$

19

where vacancies are being posted until the expected future payoffs exactly equal the effective costs (κ/q_t).

When deciding whether or not to separate, the match weighs the payoffs of staying in the relationship against the outside option. Hence, the employment relationship continues when $W_{i,t} + J_{i,t} > U_t$. In other words, the threshold value $\widetilde{p}_t$ is such that it makes the employment relationship exactly indifferent between continuing and separating

$$z_t \widetilde{p}_t - b + E_t \left[\beta_t (1 - \rho_x)(1 - f_t) \int_{\widetilde{p}_{t+1}}^{\overline{p}} (W_{t+1} - U_{t+1} + J_{t+1}) dF(p_{t+1}) \right] = 0. \qquad (18)$$

Given $\widetilde{p}_t$ the endogenous separation rate is $F(\widetilde{p}_t)$ and total separations are defined as

$$\rho_t = \rho_x + (1 - \rho_x) F(\widetilde{p}_t). \qquad (19)$$

5.1.5 Wage bargaining

Wages are assumed to be set according to Nash bargaining and are thus such that $(1 - \eta)W_{i,t} - U_t = \eta J_{i,t}$, where η is the bargaining power of workers. Using (14) to (17) one can obtain the following expression for the wage

$$w_{i,t} = \eta(z_t p_i + \kappa \theta_t) + (1 - \eta)b, \qquad (20)$$

where $\theta_t = v_t/u_t$ is labor market tightness. The wage rate is a weighted average of firms' revenues and savings on hiring costs and the foregone outside option, where the weights are determined by the relative bargaining strengths.

5.1.6 Closing the model

Let n_t be the mass of employed (producing) workers in period t. Then, the law of motion for unemployment is given by

$$u_{t+1} = (1 - f_t(1 - \rho_{t+1}))u_t + \rho_{t+1}n_t. \qquad (21)$$

Tomorrows unemployment pool thus consists of workers unsuccessful in finding a job (either because they did not match with a vacancy, or they did, but were not productive enough), plus newly separated workers employed in the previous period. Workers who were matched with a vacancy, but in the end did not start a production relationship $(\rho_t f_{t-1} u_{t-1})$ are denoted as rejected and hence ρ_t is the rejection rate. Note that in this

model the rejection rate is identical to the separation rate. Since the labor force is set to 1 $u_t = 1 - n_t$.

Finally, aggregate output is determined by

$$y_t = z_t n_t G(\widetilde{p}_t), \tag{22}$$

where $G(x) = E_t[p|p \geq x] = \int_x^{\bar{p}} p \frac{dF(p)}{1-F(x)}$ is the average productivity of workers with an idiosyncratic draw above x.

5.2 Match efficiency fluctuations

The probability that an unemployed worker is employed in the next period is given by $f_t^* = f_t(1 - \rho_{t+1})$.[21] Making the matching function explicit, one can write $f_t^* = A(1 - \rho_{t+1})u_t^{1-\mu}v_t^{\mu}$. Measured match efficiency is then defined as

$$A_t = A(1 - \rho_{t+1}). \tag{23}$$

Therefore, unless the rejection rate is constant, measured match efficiency varies over time. In other words, in the model agents who are matched with a vacancy, but are not productive enough to start working contribute to a lower job finding rate. (23) provides a direct model counterpart to the match efficiency estimates from Section 3.

5.3 Calibration

The calibration procedure follows the principle proposed in **?**, where the bargaining power and outside option of workers are set such that the model can match the wage elasticity with respect to productivity and profit share observed in the data. First, I consider a calibration, where the volatility of the separation rate is targeted. Such a calibration leads to disappointing results in terms of explaining match efficiency volatility. To highlight the basic trade-offs at play, I consider a second calibration that makes the model exactly fit match efficiency fluctuations and show how such a calibration grossly exaggerates the volatility of other endogenous variables.

To facilitate the exposition of the calibration, I divide the parameters of the model into two groups - first, a group of parameters that are fixed across both calibrations, and second, parameters which are calibrated internally to match statistics in the data. The two calibrations differ in the statistics that are being matched and therefore the second

---

[21]Similarly, the probability that a firm fills a vacancy is $q_t^* = q_t(1 - \rho_{t+1})$.

group of parameters differs between calibrations. The parameter values are summarized in Table 3 and 4 for the 1st and 2nd calibration, respectively.

5.3.1 Fixed parameters

The model period is set to be one quarter. Standard choices are made for the discount factor, $\beta = 0.99$, the coefficient of relative risk aversion, $\gamma = 1$, the standard deviation of the aggregate productivity shock, $\sigma_z = 0.007$, and its autocorrelation coefficient, $\rho_z = 0.95$. The idiosyncratic productivity distribution is assumed to be log-normal, $\log(p) \sim N(\mu_F, \sigma_F^2)$, with μ_F normalized to 0. Finally, the elasticity of unemployment in the matching function is set to the point estimate found in the empirical part, $\mu = 0.614$. As mentioned earlier, this value falls into the range reported in ?.

5.3.2 Calibrated parameters

The second group of parameters contains match efficiency, A, the flow cost of vacancies, κ, the bargaining power of workers, η, the exogenous separation rate, ρ_x, the standard deviation of the worker specific productivity distribution, σ_F and the value of leisure and home production b. These parameters are selected to match six statistics in the data.

In the case of the 1st calibration, the six targets consist of the following statistics. The mean job finding probability from ? that was also used in the empirical part (45.4%). An unemployment rate of 12% comonly used in the literature. Following ? and ? the mean vacancy filling probability of 71%. A wage elasticity with respect to productivity of 0.45 and a profit ratio of 0.03 as in ?. Finally, the 1st calibration targets the standard deviation of the separation rate equal to 0.061.

In the case of the 2nd calibration, the first five target statistics (job finding probability, unemployment rate, vacancy filling probability, wage elasticity with respect to productivity and profit ratio) remain the same. However, instead of matching separation rate volatility, the model targets match efficiency volatility equal to 0.050.

5.4 Model performance

Under both calibrations the model is solved with first-order perturbation techniques. To understand the mechanics of the model Figure 12 shows the impulse responses to a positive one-standard-deviation shock to aggregate productivity for the 1st calibration, where separation rate volatility is targeted. All workers become more productive and therefore

the idiosyncratic productivity threshold $\widetilde{p}_t$ falls. This is directly reflected in a fall of the separation rate ρ_t, which leads to a fall in unemployment (also on impact), and a rise in employment and output. At the same time labor market tightness θ_t rises, which together with a fall in the rejection rate makes the job finding probability rise which reinforces the fall in unemployment.

Table 5 compares second order moments of labor market variables from the simulated model under both calibrations and the U.S. economy. The economy, under both calibrations, is simulated $1,000$ times. Each time $1,237$ quarters are simulated and the first $1,000$ are dropped to obtain 237 quarters as in the empirical part. The simulated data are detrended with an HP filter with smoothing coefficient $1,600$ and then the standard deviations are calculated for each of the $1,000$ simulations. The reported statistics are averages over the $1,000$ simulations.

The model under the 1$^{\text{st}}$ calibration replicates the second-order moments of unemployment, vacancies and the job finding rate quite well as documented in **?**. However, it is able to explain disappointingly little of the observed match efficiency variation (only about 8%). To highlight the basic trade-offs at hand, the 2$^{\text{nd}}$ calibration targets match efficiency volatility directly. In this case the volatility of other labor market variables is grossly exaggerated. The separation rate fluctuates 7 times, the unemployment rate 3 times and vacancies 2 times more than seen in the data.

The reason why the standard calibration fails so blatantly in explaining match efficiency fluctuations and why under the 2$^{\text{nd}}$ calibration other labor market variables become enormously volatile is that the rejection rate is identical to the separation rate. Hence, calibrating the separation rate (as is done in the standard calibration) completely pins down the rejection rate properties as well. Since the average level of separations is low and the volatility moderate, the volatility of $A(1 - \rho)$ is bound to be small. The opposite reasoning holds under the 2$^{\text{nd}}$ calibration.[22]

---

[22]Another reason why the model under the standard calibration fails to generate larger match efficiency fluctuations lies in the assumption of the idiosyncratic productivity shocks being iid. Introducing persistent idiosyncratic productivity shocks would arguably strengthen the effects on match efficiency. The reason is the following. In the standard case, in a recession newly separated workers can get a new job in the next period even if aggregate productivity remains low, since they can be "lucky" and draw a high value of productivity. With persistent idiosyncratic productivity shocks such high draws would be more unlikely and thus the match efficiency fall would be stronger and more persistent.

6 Endogenous separations with firing costs

This section shows that a simple extension of the model improves its performance considerably. The introduction of firing costs drives a wedge between unemployed and employed workers. In this environment, the cut-off productivity level for workers in existing employment relationships is lower compared to the case without firing costs, since firms realize that separation entails a cost and are therefore willing to hold on to workers longer. At the same time, the cut-off for newly hired workers is higher compared to the case with no firing costs, since firms require compensation for future costs of firing. Given the distributional assumption of an upward sloping density in the neighborhood of the cut-offs, this means that a larger mass of unemployed is now affected by aggregate fluctuations Figure (14 illustrates how the productivity thresholds change when firing costs are introduced). The volatility of the rejection and separation rate depends on the mass around the cut-off points; the larger the mass, the higher the volatility. Consequently, the slope of the density around the cut-off points is crucial. I assume that the density is upward-sloping in this area which is not unreasonable considering that the cut-off points are in the lower tail of the distribution. The assumption implies that the unemployment pool is populated by a relatively larger mass of marginal workers, compared to the employment pool.

In this setup it is the firing cost that drives the threshold for the unemployed into an area of greater mass. Different mechanisms, such as on the job training, would have a similar effect. Alternatively, one could try and model two productivity distributions for the (un)employed. Such an approach, however, is too complex given the illustrative aim of this section.

6.1 Matching model with firing costs

Household behavior, the matching process and the setup of employment relationships is the same as in Sections 5.1.1 to 5.1.3. The only exception is the budget constraint, (12), where total wage income is now more complex, which will be explained in Section 6.3. In what follows I describe the rest of the model.

6.1.1 Endogenous separations

The value of being in the unemployment pool U_t at period t is given by

$$U_t = b + E_t \left[\beta_t f_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^N}^{\overline{p}} (W_{t+1}^N - U_{t+1}) dF(p_{t+1}) + U_{t+1} \right], \qquad (24)$$

24

where $\widetilde{p}_{t+1}^N$ is the productivity threshold for newly matched workers. The threshold is the only difference with (15) in the model without firing costs.

The value of a job in period t for newly matched and existing workers with idiosyncratic productivity level $p_{i,t}$ are

$$W_{i,t}^N = w_{i,t}^N + E_t \left[\beta_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^E}^{\overline{p}} (W_{t+1}^E - U_{t+1}) dF(p_{t+1}) + U_{t+1} \right], \tag{25}$$

$$W_{i,t}^E = w_{i,t}^E + E_t \left[\beta_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^E}^{\overline{p}} (W_{t+1}^E - U_{t+1}) dF(p_{t+1}) + U_{t+1} \right], \tag{26}$$

where $\widetilde{p}_{t+1}^E$ is the productivity threshold for existing relationships. The only difference between (25) and (26) is in the wage rate, which is discussed in the next subsection.

Similarly, the value for the firm of being in a productive employment relationship with a newly hired and existing worker with individual productivity level $p_{i,t}$ is, respectively

$$J_{i,t}^N = z_t p_{i,t} - w_{i,t}^N + E_t \left[\beta_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^E}^{\overline{p}} J_{t+1}^E dF(p_{t+1}) - F(\widetilde{p}_{t+1}^E) \phi \right], \tag{27}$$

$$J_{i,t}^E = z_t p_{i,t} - w_{i,t}^E + E_t \left[\beta_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^E}^{\overline{p}} J_{t+1}^E dF(p_{t+1}) - F(\widetilde{p}_{t+1}^E) \phi \right], \tag{28}$$

where ϕ is the firing cost. The firing cost is assumed to be fully paid by the firm and wasteful. It is thus not a transfer payment to the worker, but rather a tax on the match in the event of separation. Such a specification is justified by the fact that firing costs are (at least partly) of administrative and legal nature, they include for instance loss of efficiency due to disruption of regular work flow etc.

Finally, the value of an open vacancy (imposing the free entry condition) is

$$\frac{\kappa}{q_t} = E_t \left[\beta_t (1 - \rho_x) \int_{\widetilde{p}_{t+1}^N}^{\overline{p}} J_{t+1}^N dF(p_{t+1}) \right]. \tag{29}$$

The threshold for newly matched workers is such that the surplus of the new match is equal to zero

$$W^N(\widetilde{p}_t^N) + J^N(\widetilde{p}_t^N) - U_t = 0. \tag{30}$$

An analogous reasoning holds for existing employment relationships. However, in this case, one must take into account the firing cost in the outside option of the firm. Essentially, the surplus can be negative up to the value of the firing cost, since the firm saves this cost by holding onto the worker

$$W^E(\widetilde{p}_t^E) + J^E(\widetilde{p}_t^E) - U_t = -\phi \tag{31}$$

6.2 Wage bargaining

Workers coming from the unemployment pool do not posses any contract with the firm from the previous period. Therefore, if they do not come to an agreement with the firm over the wage, no firing costs need to be paid. Assuming Nash bargaining, the wage of newly matched workers is then a solution to $(1-\eta)(W_{i,t}^N - U_t) = \eta J_{i,t}^N$. On the other hand, when the firm decides to fire a worker that has been in an employment relationship in the previous period it must pay firing costs. The wage of a worker in an existing employment relationship is then a solution to $(1-\eta)(W_{i,t}^E - U_t) = \eta J_{i,t}^E + \phi)$. Using (24) to (29) one can show that the wages of newly hired workers and workers in existing relationships are, respectively

$$w_{i,t}^N = \eta(z_t p_{i,t} - \beta(1-\rho_x)\rho_{t+1}^E \phi + \kappa\theta_t) + (1-\eta)b, \tag{32}$$

$$w_{i,t}^E = \eta(z_t p_{i,t} + (1 - \beta(1-\rho_x)\rho_{t+1}^E)\phi + \kappa\theta_t) + (1-\eta)b, \tag{33}$$

where the structure is the same as in the model without firing costs. Newly hired workers, however, are penalized because of the threat of having to pay firing costs in the future. On the other hand, workers in existing employment relationships now have a higher wage compared to the case without firing costs, because their effective bargaining power increased, since firing them entails a cost for the firm.

6.3 Closing the model

Let the separation rate of existing employment relationships (ρ_t^E) and the rejection rate (ρ_t^N) be defined as, respectively

$$\rho_t^E = \rho_x + (1-\rho_x)F(\widetilde{p}_t^E), \tag{34}$$

$$\rho_t^N = \rho_x + (1-\rho_x)F(\widetilde{p}_t^N). \tag{35}$$

Then, the law of motion for unemployment is given by

$$u_{t+1} = (1 - f_t(1-\rho_{t+1}^N))u_t + \rho_{t+1}^E n_t, \tag{36}$$

where the only difference compared to (21) is that now one needs to distinguish between the separation and rejection rates. Finally, aggregate output is determined by

$$y_t = z_t n_t(\omega_t G(\widetilde{p}_t^N) + (1-\omega_t)G(\widetilde{p}_t^E)), \tag{37}$$

where $\omega_t = \frac{f_{t-1}u_{t-1}(1-\rho_t^N)}{n_t}$ is the fraction of newly employed workers in total employment. Similarly, total wage income is now given by $n_t(\omega_t w(G(\widetilde{p}_t^N), t)^N + (1 - \omega_t)w(G(\widetilde{p}_t^E), t)^E)$.

6.4 Match efficiency variation

In the case with firing costs the probability that an unemployed worker finds himself employed in the next period is given by $f_t^* = f_t(1 - \rho_{t+1}^N)$. Measured match efficiency is then defined as

$$A_t = A(1 - \rho_{t+1}^N), \tag{38}$$

where its fluctuations now depend on the rejection rate, which is not equal to the separation rate in this setup.

6.5 Calibration

The Employment Protection Legislation index (EPL) published by the OECD is a comprehensive indicator and more precise than other alternatives.[23] It is a weighted average of indicators capturing protection of regular workers against individual dismissals, requirements for collective dismissals and regulation of temporary employment. However, one needs to translate this index into a suitable model parameter. **?** provide estimates of firing cost for France, Germany, Italy and the UK in the period between 1975 and 1986. Assuming that the EPL is proportional to the estimates provided by Bentotila and Bertola, one can get an estimate of firing costs for the U.S., since EPL data is readily available for the above countries and the U.S. economy. I take the UK estimate as a benchmark assuming that its institutional environment is closest to that of the U.S. economy. The implied firing costs are equal to 4.47% of annual wage.[24] Hence, firing costs are set to $\phi = 4 * 0.0447 * \overline{w}^E = 0.179 * \overline{w}^E$ for a quarterly model, where $\overline{w}^E$ is the steady state wage for workers in existing employment relationships.

Using the above value for the firing costs, I recalibrate the model to fit the targets as in the 1st calibration which fits separation rate volatility. The resulting parameter values are summarized in Table 6.

---

[23]For instance compared to the hiring and firing costs calculated by the World Bank in its "Doing Business studies", the OECD indicator both covers a larger range of relevant aspects of LTC, and has more precise and differentiated sub-indicators.

[24]Using the "regular employment" EPL index.

6.6   Model performance

The mechanics of this model are the same as those with zero firing costs with one exception. With positive firing costs the separation and rejection rates are no longer identical. Figure 13 shows the impulse response functions of the productivity thresholds for existing and newly hired workers together with the associated separation and rejection rates. While both productivity thresholds drop by similar amounts, the separation rate fall is dampened while the rejection rate decrease is magnified because the density is upward-sloping in this part of the distribution. This difference between the magnitudes of the two responses of the separation and rejection rate is what makes the model able to better explain match efficiency fluctuations while still fitting separation rate volatility.

Table 7 compares model standard deviations with those in the US economy. The model calibrated in this way can explain 56% of the match efficiency volatility found in the data. At the same time, the volatility of other labor market variables are close to their empirical counterparts (only the volatility of unemployment is somewhat higher).

6.7   Model-based match efficiency

The previous section showed that the model can account for a sizable portion of match efficiency variation. Another way to view this is to compare the estimated match efficiency with its model-based counterpart. To this end, I use data for real GDP (logged and detrended with a HP filter with a smoothing coefficient of 1,600) and back out the implied aggregate productivity shock. This is done by inverting the policy function obtained when solving the model. I use this shock series to simulate the model. Note that the shock series is recovered *without* using labor market variables.

Figure 15 compares the estimated match efficiency and the one implied by the model using the backed-out technology shock. The model-based match efficiency series follows the estimated reasonably well (correlation coefficient 0.57), especially prior to 1990 (correlation coefficient of 0.67). In the case of the 1991 recession the economy experienced a jobless recovery and estimated match efficiency fell mostly *after* the end of the downturn. Since the only shock driving the model is backed out from real GDP, such different dynamics cannot be captured by the model. Similarly, during the recession in 2001 the fall in match efficiency as predicted by GDP growth was smaller than the estimated one, again pointing to a different character of the recession. It seems that after the onset of the Great Moderation output growth lost on importance in explaining match efficiency fluctuations.

Finally, I decompose the variance of the model-based job finding rate into contributions of match efficiency and labor market tightness as in Section 3.1.2. The model predicts that 19% of job finding rate fluctuations are driven by match efficiency. This is only slightly lower than the 24% found in the data.

7 Concluding remarks

A constant matching function, as is typically used in the literature, implicitly assumes that the labor market heterogeneity that it is aimed at capturing, is time invariant. This paper relaxes the assumption of constant parameters in the matching function. In particular, the constant, which reflects match efficiency, is allowed to vary. Using data on the job finding rate and unemployment I specify an unobserved components model, where both match efficiency and vacancies (due to the poor data availability over a longer sample) are treated as unobserved states. The JOLTS vacancy series (available only at the end of the sample) provides additional information determining vacancies in the earlier part.

Estimated match efficiency varies procyclically over the business cycle and it can explain up to 25% of job finding rate fluctuations. Hence, recessions are periods when unemployed workers have a harder time finding jobs not only because there are less vacancies and more unemployed competing for them, but also because the process of matching workers to jobs is less efficient. The results are robust to several modifications and a Monte Carlo exercise documents that the empirical model is able to identify the unobserved states quite well.

One reason for varying match efficiency can be found in changes in labor market heterogeneity which the matching function is aimed at capturing. One such form of varying heterogeneity is endogenous rejection. A positive rejection rate (not all workers that get matched with a vacancy start producing in the next period) drives a wedge between the total unemployment pool and the part useful for forming employment relationships. Moreover, countercyclical fluctuations in the rejection rate imply that in recessions the part of the unemployment pool useful for matching shrinks. The aggregate job finding rate then falls by more than would be implied by a constant matching function that takes into account the total number of unemployed and vacancies. However, calibrating this model to fit match efficiency fluctuations leads to a gross exaggeration in the volatility of other variables, most significantly that of the separation rate. Introducing firing costs helps alleviate this issue and makes the model perform well also quantitatively.

References

BARNICHON, R. (2009): "Vacancy Posting and the Matching Function over 1951-2009," Federal Reserve Board.

———— (2010): "Building a composite Help-Wanted Index," *Economics Letters*, 109(3), 175–178.

BARNICHON, R., AND A. FIGURA (2011a): "What Drives Matching Efficiency? A Tale of Composition and Dispersion," mimeo.

———— (2011b): "What Drives Movements in the Unemployment Rate? A Decomposition of the Beveridge Curve," Federal Reserve Board.

BENTOTILA, S., AND G. BERTOLA (1990): "Firing Costs and Labour Demand: How Bad is Eurosclerosis?," *The Review of Economic Studies*, 57(3), 381–402.

DARBY, M., J. HALTIWANGER, AND M. PLANT (1985): "Unemployment Rate Dynamics and Persistent Unemployment under Rational Expectations," *American Economic Review*, 75(4), 614–637.

DAVIS, S. J., R. J. FABERMAN, AND J. C. HALTIWANGER (2009): "The Establishment-level Behavior of Vacancies and Hiring," Federal Reserve Bank of Philadelphia.

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.

DURBIN, J., AND S. J. KOOPMAN (2001): *Time Series Analysis by State Space Models.* Oxford University Press.

ELSBY, M. W. L., R. MICHAELS, AND G. SOLON (2009): "The Ins and Outs of Cyclical Unemployment," *American Economic Journal: Macroeconomics*, 1(1), 84–110.

FUJITA, S., AND G. RAMEY (2009): "The Cyclicality of Separation and Job Finding Rates," *International Economic Review*, 50(2), 415–430.

HAGEDORN, M., AND I. MANOVSKII (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98(4), 1692–1706.

HORNSTEIN, A., P. KRUSELL, AND G. VIOLANTE (2005): "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Federal Reserve Bank of Richmond Economic Quarterly*, 91, 19–51.

PETRONGOLO, B., AND C. A. PISSARIDES (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

——— (2007): "Reassessing the Ins and Outs of Unemployment," NBER working paper no.13421.

STERK, V. (2010): "Home equity, Mobility and Macroeconomic Fluctuations," mimeo, University of Amsterdam.

VAN OURS, J., AND G. RIDDER (1992): "Vacancies and the Recruitment of New Employees," *Journal of Labor Economics*, 10(2), 138–155.

Appendix

A   The Kalman filter

The state-space model is summarized by (1) and (2), which I rewrite here for convenience:

$$y_t = \Theta_{0,t} + \Theta_{1,t}s_t + \Theta_{2,t}x_t + \epsilon_t, \tag{A.1}$$

$$s_t = \Phi_{0,t} + \Phi_{1,t}s_{t-1} + \eta_t. \tag{A.2}$$

The Kalman filter recursions can then be written as

$$s_{t|t-1} = \Phi_0 + \Phi_1 s_{t-1|t-1}, \tag{A.3}$$

$$P_{t|t-1} = \Phi_1 P_{t-1|t-1}\Phi_1' + Q, \tag{A.4}$$

$$Z_t = \Theta_1 P_{t|t-1}\Theta_1' + R + \Theta_1 C + C'\Theta_1', \tag{A.5}$$

$$V_t = y_t - \Theta_0 - \Theta_1 s_{t|t-1} - \Theta_2 x_t, \tag{A.6}$$

$$K_t = (P_{t|t-1}\Theta_1' + C)Z_t^{-1}, \tag{A.7}$$

$$s_{t|t} = s_{t|t-1} + K_t V_t, \tag{A.8}$$

$$P_{t|t} = P_{t|t-1} - K_t(\Theta_1 P_{t|t-1} + C'), \tag{A.9}$$

where the subscript $t|t-1$ indicates a prediction of the variable for period t, using information available in period $t-1$. Similarly, $t|t$ is the update of the period t forecast, when period t information is revealed.

B   The extended Kalman filter

Let the non-linear state space be described by the following measurement and transition equation:

$$y_t = h(s_t, x_t) + \epsilon_t, \tag{B.1}$$

$$s_t = f(s_{t-1}) + \eta_t, \tag{B.2}$$

where f and h are non-linear functions. Note that in Section 4.1 f is in fact linear. For the state-space system given in (B.1) and (B.2) the extended Kalman filter recursions are the following:

$$s_{t|t-1} = f(s_{t-1|t-1}), \tag{B.3}$$

$$P_{t|t-1} = F_t P_{t-1|t-1} F_t' + Q, \tag{B.4}$$

$$Z_t = H_t P_{t|t-1} H_t' + R + H_t C + C' H_t', \tag{B.5}$$

$$V_t = y_t - h(s_{t|t-1}, x_t), \tag{B.6}$$

$$K_t = (P_{t|t-1} H_t' + C) Z_t^{-1}, \tag{B.7}$$

$$s_{t|t} = s_{t|t-1} + K_t V_t, \tag{B.8}$$

$$P_{t|t} = P_{t|t-1} - K_t (H_t P_{t|t-1} + C'), \tag{B.9}$$

where F_t and H_t are the Jacobian matrices of the transition and measurement equations, respectively:

$$F_t = \frac{\partial f}{\partial s} |_{s_{t-1|t-1}}, \tag{B.10}$$

$$H_t = \frac{\partial h}{\partial s} |_{s_{t|t-1}, x_t}. \tag{B.11}$$

C Diagnostic tests

The assumptions underlying the specified model is that the residuals are normally distributed with constant variance and no serial correlation. Following **?** one can apply diagnostic tests of these properties to the *standardized prediction errors* defined as:

$$e_t = V_t Z_t^{-1}, \tag{C.1}$$

where it then follows that the standard deviation of e_t is 1.

C.1 Serial correlation

One can use the Ljung-Box test to investigate the presence of serial correlation in the residuals. Denote the residual autocorrelation of order k as

$$r_k = \frac{\sum_{t=1}^{n-k}(e_t - \bar{e})(e_{t+k} - \bar{e})}{\sum_{t=1}^{n}(e_t - \bar{e})^2}, \tag{C.2}$$

where $\bar{e}$ is the mean of the residuals. The Ljung-Box statistic is then

$$Q(k) = n(n+2) \sum_{l=1}^{k} \frac{r_l^2}{n-l}, \tag{C.3}$$

which is $\chi^2(k-w+1)$ distributed, with w being the number of estimated hyperparameters (elements in the disturbance variance matrix).

C.2 Homoscedasticity

The assumption of constant variance can be tested with the following test statistic:

$$H(h) = \frac{\sum_{t=n-h+1}^{n} e_t^2}{\sum_{t=1}^{h} e_t^2}, \tag{C.4}$$

where h is typically set to the nearest integer to $n/3$. The statistic then tests whether the variance in the first third of the sample is equal to that in the last third of the sample. This statistic is then $F(h,h)$ distributed.

C.3 Normality

The assumption that the standardized prediction errors are normally distributed can be readily tested using the Jarcque-Berra test. The test statistic is defined as

$$JB = n \left(\frac{S^2}{6} + \frac{(K-3)^2}{24} \right), \tag{C.5}$$

where S denotes the skewness and K the kurtosis of the standardized prediction errors. The test statistic is $\chi^2(2)$ distributed.

D More robustness checks and further details

D.1 Different state processes

In the benchmark specification match efficiency was assumed to be an AR(1) process, while the process for vacancies was postulated to be a random walk. In this section I check the robustness of the results against two alternative specifications for the underlying states. First, I estimate the model assuming match efficiency follows a random walk, while keeping the specification of vacancies as in the benchmark model.[25] Second, I retain the AR(1) assumption on match efficiency, but I allow for a richer non-stationary structure for vacancies. Namely, I assume that the first difference of vacancies follows an AR(2) process. The level of vacancies can then be written as

$$v_t = (1 + \rho_1)v_{t-1} + (\rho_2 - \rho_1)v_{t-2} - \rho_2 v_{t-3} + \eta_t^v. \tag{D.1}$$

Table 8 shows the estimated parameters for the benchmark model and the two alternative specifications. All specifications deliver very similar results. Figure 16 shows the Kalman smoothed states for the three specifications. As with the model parameters, the smoothed states are also very close to each other.

D.2 Estimating on subsamples and with different frequencies

Here I use two different subsamples to check whether the results are not driven just by a certain part of the data. The first subsample uses data after 1970 and the second data after 1985. Figure 17 shows the Kalman smoothed states for the subsamples together with the benchmark. Table 9 then shows the estimated parameter values. There are slight differences in the parameters, but they are also estimated with less precision as one discards more data points. Overall, the dynamics of the states are quite robust over the different samples. Furthermore, virtually identical results are obtained using monthly frequencies.

---

[25]This specification makes it harder to identify the two states, because both have the same process. To help with this issue I use information from the benchmark for the starting values of the Kalman filter.

D.3 Alternative functional forms

The two alternative functional forms considered in the main text change the measurement equation related to the job finding rate (the rest of the state-space system remains the same). In the case of the first alternative matching function, the measurement equation becomes

$$f_t = \alpha_{1,t} + 1/\xi \log(\exp(u_t)^\xi + \exp(v_{1,t})^\xi) - u_t + \epsilon_{1,t}, \tag{D.2}$$

and the case of the second specification it is

$$f_t = \alpha_{2,t} + v_{2,t} - 1/\zeta \log(\exp(u_t)^\zeta + \exp(v_{2,t})^\zeta) + \epsilon_{2,t}. \tag{D.3}$$

D.4 Alternative estimation procedure

The first step of the alternative estimation procedure assumes a constant matching function and extracts a vacancy series implied by only data on the job finding rate and unemployment. The state-space of this system is given by

$$f_t = \alpha + \mu(v_t^* - u_t) + \epsilon_t, \tag{D.4}$$

$$v_t^* = v_{t-1}^* + \eta_t. \tag{D.5}$$

The second step then takes v_t^* and decomposes it into match efficiency (with mean zero) and the underlying vacancy state. The state-space system (allowing still for some measurement error) is

$$v_t^* = v_t + \frac{1}{\mu}\alpha_t + \epsilon_{v^*,t}, \tag{D.6}$$

$$\begin{bmatrix} \alpha_t \\ v_t \end{bmatrix} = \begin{bmatrix} \rho_a & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ v_{t-1} \end{bmatrix} + \eta_{v^*,t}. \tag{D.7}$$

D.4.1 Checking the degree of non-linearity

To check the nonlinearity I fix match efficiency and unemployment at their average values and construct an artificial job finding rate using the state-space specification and the respective ML estimates. Then a scatter plot between this counterfactual (log) job finding rate and (log) vacancies indicates the degree of non-linearity in the model (in the benchmark case the result would be a straight line with the slope of μ). Figure 6 shows the scatter plots for the two matching functions. In both cases the curves are very close to

being linear indicating that the degree of non-linearity is not large and a first order Taylor expansion can be expected to perform quite well. Repeating the exercise for different (fixed) values of unemployment and match efficiency delivers practically identical results.

D.5 Using information from the vacancy yield

The benchmark state-space system is augmented by a third measurement equation that reads

$$i_t = \gamma_0 + \gamma_1 a_t + \gamma_2 v_t + \gamma_3 u_t + \epsilon_{i,t}, \tag{D.8}$$

where i_t is the log of the vacancy yield. Once again, data points prior to 2001Q1 when the JOLTS data were not available, are treated as missing observations.

Although the job filling rate based on a Cobb-Douglas matching would imply that $\gamma_0 = 0$, $\gamma_1 = 1$ and $\gamma_2 = \gamma_3$, here I do not impose such restrictions since the estimation uses vacancy yield data instead. Table 10 shows the parameter estimates and shows that although all four parameters are close to satisfying the above restriction, they do violate them from a statistical significance point of view.

D.6 Extending the sample period

In calculating the job finding rate according to **?** one needs short term unemployment data. In 1994 there is a discontinuity in this series due to a methodological change. Shimer proposes to deal with this by multiplying the official count of unemployment by the short-term share in only the first and fifth rotation groups in the CPS panel (these groups are measured in the same way, even after the methodological change, as the full sample prior to 1994). Here, instead of using the CPS groups directly I follow **?**. The authors propose to multiply the official count of unemployment with the era's average of the ratio of the short-term share for the first and fifth rotation groups relative to the full sample's short-term share. In their case this ratio is 1.155 with the sample ending in 2005Q1. I take the shortcut of assuming that this ratio has not changed dramatically in the last five years and use it to calculate the job finding rate up until 2010Q4. The reader should keep in mind, however, that this subsection is only illustrative and a more careful analysis would require the job finding rate to be calculated using the actual CPS data. Finally, the estimation over this longer period uses the JOLTS vacancies data after the revision in March 2011.[26]

---

[26]There are two more dummy variables included in the quarters 2009Q1 to 2010Q2. This is likely due to the inaccuracy of the job finding rate estimates in this period.

D.7 Monte Carlo experiment

The artificial data series are constructed using the benchmark state-space system, the maximum likelihood parameter estimates and the estimated unobserved states. For convenience I repeat the measurement equations below

$$
\begin{bmatrix} f_t^{MC} \\ v_t^{MC} \end{bmatrix} = \begin{bmatrix} -\mu^{ML} u_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & \mu^{ML} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_t \\ v_t \end{bmatrix} + \epsilon_t^{MC}, \tag{D.9}
$$

where the superscript MC indicates that the respective data series is one of the artificial series generated in the Monte Carlo exercise while the superscript ML indicates the benchmark maximum likelihood parameter estimate. Finally, u_t is period t U.S. unemployment and a_t and v_t are the benchmark Kalman smoothed estimates of match efficiency and vacancies, respectively, and are fixed across Monte Carlo replications. The variance-covariance matrix of ϵ_t^{MC} is equal to benchmark ML estimate.

The estimation procedure is exactly as in the benchmark, where the initial mean of the Kalman states is set to the respective expected value and its variance is set to 10^5. The initial conditions for the maximization are set to the true values, to ease the computational burden. The actual ML estimates in each Monte Carlo replication step will, however, be different from the true parameter values due to the small sample at hand.

E   Endogeneity

A valid concern is that there are endogeneity problems in the first observation equation. Therefore, the model in the main text is estimated using lagged values of the regressor as an instrument, which is typically done in the literature. Such an instrument is valid only if there is no serial correlation in the residual. As was shown in the main text, the model satisfies the assumption of no serial correlation in the residuals. In addition, the Hausmann test on exogeneity of instruments cannot reject the null hypothesis of exogenous instruments at the 40% level when the instrument is the first lag of unemployment. In the case when the model is estimated on monthly data, the exogeneity tests suggest 4 lags as the appropriate instrument, which is consistent with the quarterly tests.

F   Effects of firing costs

In the case of zero firing costs the separation rate exactly equals the rejection rate. However, introducing positive firing costs drives a wedge between the two, making the rejection rate larger than the separation rate. This section shows analytically how firing costs increase the idiosyncratic productivity threshold for newly matched workers, while reducing the threshold for workers in existing employment relationships.

The two equations defining the threshold values are (31) and (30). First note that one can write the following

$$J^E(p_{t+1}) = J^E(p_{t+1}) - (J^E(\widetilde{p}_{t+1}^E) + \phi) = z_{t+1}(1-\eta)(p - \widetilde{p}_{t+1}^E) - \phi, \qquad \text{(F.1)}$$

where the first equality follows from the threshold condition (31) and the fact that with Nash bargaining the job value (J_t^E) is proportional to total surplus. The second equality comes from observing that both $J^E(p_{t+1})$ and $J^E(\widetilde{p}_{t+1})$ have all terms common apart from the value of idiosyncratic productivity. Substituting (F.1) into (30) and (31) one can obtain analytical expressions for the thresholds.

$$\widetilde{p}_t^N = \frac{1}{z_t} \left[\begin{array}{c} b + \frac{\eta}{1-\eta}\kappa\theta_t - \beta_t(1-\rho_x)(G(\widetilde{p}_{t+1}^E) - \widetilde{p}_{t+1}^E) + \\[2mm] \phi\frac{\beta_t(1-\rho_x)(1+F(\widetilde{p}^E)-\eta)}{1-\eta} \end{array} \right], \qquad \text{(F.2)}$$

$$\widetilde{p}_t^E = \frac{1}{z_t} \left[\begin{array}{c} b + \frac{\eta}{1-\eta}\kappa\theta_t - \beta_t(1-\rho_x)(G(\widetilde{p}_{t+1}^E) - \widetilde{p}_{t+1}^E) - \\[2mm] \phi\left(1 - \beta_t(1-\rho_x)\left(1 + \frac{F(\widetilde{p}_{t+1}^E)}{1-\eta}\right)\right) \end{array} \right], \qquad \text{(F.3)}$$

where $1 + F(\widetilde{p}_{t+1}^E) - \eta > 0$ for any non-negative value of endogenous separations and $1 - \beta(1-\rho_x)\left(1 + \frac{F(\widetilde{p}_{t+1}^E)}{1-\eta}\right) > 0$ for low enough values of $F(\widetilde{p}_{t+1}^E)$. The steady state effect of firing costs on the threshold for new matches is directly evident from (F.2). Firing costs make the firm demand higher productivity of new matches as a compensation for expected future separations. The opposite reasoning holds for existing matches, where the firm settles for lower productivity levels, because separations now entail a cost. Obtaining an analytical expression for the steady state threshold for existing employment relationships is, however, impeded by the assumption of the log-normal distribution. The following subsection shows this steady state effect analytically under the assumption of a uniform distribution for idiosyncratic productivity. Nevertheless, in all the analysis it was always the case that the threshold for existing employment relationships fell with higher values of firing costs.

F.1 The case of a uniform distribution

Assuming a uniform distribution over idiosyncratic productivity levels p and normalizing its lower bound to 0, the steady state threshold level for existing matches can be shown to be

$$\widetilde{p}^E = \frac{b + \frac{\eta}{1-\eta}\kappa\theta - \beta(1-\rho_x)\frac{\overline{p}}{2} - \phi(1-\beta(1-\rho_x))}{1 - \beta(1-\rho_x)(1/2 + \frac{\phi}{(1-\eta)\overline{p}})}, \tag{F.4}$$

where $\overline{p}$ is the upper bound of the uniform distribution. Since $1 - \beta(1-\rho_x) > 0$ then for the threshold to fall with higher firing costs it must be that $1 - \beta(1-\rho_x)(1/2 + \frac{\phi}{(1-\eta)\overline{p}}) > 0$. This depends not only on the extent of the firing costs, but also on the width of the uniform distribution. It holds true as long as $\frac{\phi}{a} < \frac{2-1}{2\beta(1-\rho_x)}(1-\eta)$. For example, assuming a tight distribution, where the upper bound is 1, then firing costs need to be smaller than 0.194. For comparison with the benchmark model, one needs to multiply this value by 2, since average idiosyncratic productivity is half of what it is in the main text.

Tables and Figures

Table 1: Parameter estimates

α		-0.639
		(0.025)
β		0.386
		(0.032)
ρ_α		0.696
		(0.065)
$10^3\ R$	1.15	-0.15
	-0.15	0.95
$10^3\ Q$	1.28	-0.38
	-0.38	3.11
serial independence		0.628
homoscedasticity		0.275
normality		0.447

Notes: The reported values of diagnostic tests are p-values, where the null hypothesis is a satisfaction of the assumption. Details on the tests used are in the appendix. The result of the serial correlation diagnostic test is based on 4 lags (1, 2 and 8 lags were also tested and could not reject the null). The homoscedasticity test is based on the first and last third of the sample (the fist and last quarters were also used and could not reject the null). Standard errors are in brackets.

Table 2: Contributions to job finding probability volatility

	β^A	β^θ
1st-differenced	0.326	0.674
HP-filtered (1600)	0.241	0.759
HP-filtered (10^5)	0.153	0.847

Notes: β^A and β^θ indicate contributions of match efficiency and vacancies, respectively, in a variance decomposition of the job finding rate.

Table 3: Parameter values: 1$^{\text{st}}$ calibration

Parameter		Value	Target/Source
Discount factor	β	0.99	
Relative risk aversion	γ	1	
Agg. shock persistence	ρ_z	0.95	
Agg. shock st. dev.	σ_z	0.007	
Idio. shock mean	μ_F	0	
Match elasticity	μ	0.614	empirical part
Match efficiency	A	0.574	$f = 0.45$, Shimer (2007)
Exogenous destruction	ρ_x	0.059	$u = 0.12$
Vacancy posting costs	κ	0.307	$q = 0.71$, den Haan et al. (2000)
Worker bargaining power	η	0.097	$\epsilon_{W,\widehat{p}} = 0.45$
Worker outside option	b	0.972	$W/\widehat{p} = 0.97$
Idio. shock st. dev.	σ_F	0.221	$\sigma(\rho) = 0.061$

Notes: This calibration targets separation rate volatility. $\widehat{p}$ is the average productivity of the employment relationships, consisting of aggregate productivity z and the average worker productivity $G(\widetilde{p})$. $\epsilon_{W,\widehat{p}}$ is the elasticity of wages with respect to productivity and $W/\widehat{p}$ is the wage share.

Table 4: Parameter values: 2$^{\text{nd}}$ calibration

Parameter		Value	Target/Source
Discount factor	β	0.99	
Relative risk aversion	γ	1	
Agg. shock persistence	ρ_z	0.95	
Agg. shock st. dev.	σ_z	0.007	
Idio. shock mean	μ_F	0	
Match elasticity	μ	0.614	empirical part
Match efficiency	A	0.574	$f = 0.45$, Shimer (2007)
Exogenous destruction	ρ_x	0.004	$u = 0.12$
Vacancy posting costs	κ	0.282	$q = 0.71$, den Haan et al. (2000)
Worker bargaining power	η	0.113	$\epsilon_{W,\widehat{p}} = 0.45$
Worker outside option	b	1.020	$W/\widehat{p} = 0.97$
Idio. shock st. dev.	σ_F	0.295	$\sigma(A(1 - \rho)) = 0.05$

Notes: This calibration targets match efficiency volatility. $\widehat{p}$ is the average productivity of the employment relationships, consisting of aggregate productivity z and the average worker productivity $G(\widetilde{p})$. $\epsilon_{W,\widehat{p}}$ is the elasticity of wages with respect to productivity and $W/\widehat{p}$ is the wage share.

Table 5: Standard deviations of variables: US data and different calibrations

	Data	Model 1st	2nd
A	0.050	0.004	0.050
ρ	0.061	0.061	0.425
u	0.122	0.146	0.366
v	0.138	0.104	0.256
f	0.077	0.076	0.062

Notes: The empirical standard deviations are based on U.S. data in the period between 1951Q1 and 2007Q1. The data were seasonally adjusted, logged and detrended with an HP filter with smoothing coefficient 1,600. Unemployment is taken from the Current Population Survey published by the BLS, vacancies are the Help Wanted Index published by the Conference Board and the job finding rate and separation rates are taken from Shimer (2007). "1st" refers to the case with zero firing costs $\phi = 0$ and when the volatility of the separation rate $\sigma(\rho)$ is targeted. "2nd" refers to the case with zero firing costs $\phi = 0$ and when the volatility of match efficiency $\sigma(A(1 - \rho))$ is targeted.

Table 6: Parameter values: 1st calibration with firing costs

Parameter		Value	Target/Source
Discount factor	β	0.99	
Relative risk aversion	γ	1	
Agg. shock persistence	ρ_z	0.95	
Agg. shock st. dev.	σ_z	0.007	
Idio. shock mean	μ_F	0	
Match elasticity	μ	0.614	empirical part
Firing costs	ϕ	0.179	Bentotila and Bertola (1990), OECD
Match efficiency	A	0.605	$f = 0.45$, Shimer (2007)
Exogenous destruction	ρ_x	0.058	$u = 0.12$
Vacancy posting costs	κ	0.423	$q = 0.71$, den Haan et al. (2000)
Worker bargaining power	η	0.079	$\epsilon_{W,\widehat{p}} = 0.45$
Worker outside option	b	1.004	$W/\widehat{p} = 0.97$
Idio. shock st. dev.	σ_F	0.386	$\sigma(\rho) = 0.061$

Notes: This calibration targets separation rate volatility and introduces positive firing costs. $\widehat{p}$ is the average productivity of the employment relationships, consisting of aggregate productivity z and the average worker productivity $G(\widetilde{p})$. $\epsilon_{W,\widehat{p}}$ is the elasticity of wages with respect to productivity and $W/\widehat{p}$ is the wage share.

Table 7: Standard deviations of variables: US data and model with firing costs

	Data	Model
A	0.050	0.028
ρ	0.061	0.061
u	0.122	0.149
v	0.138	0.102
f	0.077	0.077

Notes: The empirical standard deviations are based on U.S. data in the period between 1951Q1 and 2007Q1. The data were seasonally adjusted, logged and detrended with an HP filter with smoothing coefficient 1,600. Unemployment is taken from the Current Population Survey published by the BLS, vacancies are the Help Wanted Index published by the Conference Board and the job finding rate and separation rates are taken from Shimer (2007). The model is calibrated with firing costs $\phi = 0.179$ and it targets separation rate volatility $\sigma(\rho)$.

Table 8: Parameter estimates: different state space representations

	α AR(1) v RW	α RW v RW	α AR(1) Δv AR(2)
α	-0.671		-0.660
	(0.016)		(0.033)
β	0.354	0.321	0.352
	(0.009)	(0.050)	(0.033)
ρ_α	0.719		0.790
	(0.006)		(0.023)
σ_α	0.042	0.038	0.031
σ_v	0.058	0.060	0.060

Notes: Standard errors in brackets. $AR(p)$ and RW indicate that the given process is an autoregressive process with p lags or a random walk, respectively.

Table 9: Parameter estimates: different subsamples

parameters/sample	Full	after 1970	after 1985
α	-0.639	-0.577	-0.659
	(0.025)	(0.041)	(0.127)
β	0.386	0.497	0.327
	(0.032)	(0.053)	(0.133)
ρ_α	0.696	0.897	0.791
	(0.065)	(0.106)	(0.125)

Notes: Standard errors in brackets.

Table 10: Parameter estimates in vacancy yield equation

γ_0	-0.006	γ_2	-0.284
	(0.003)		(0.014)
γ_1	0.984	γ_3	0.365
	(0.002)		(0.012)

Notes: Standard errors in brackets. In the regression where the dependent variable is the vacancy yield, γ_0, γ_1, γ_2 and γ_3 are the coefficients on the constant, match efficiency, vacancies and unemployment, respectively.

Table 11: Monte Carlo summary statistics

correlation	percentage
> 0.50	0.848
> 0.25	0.953
> 0	0.991
minimum corr.	-0.172

Notes: The table shows the percentage of Monte Carlo estimates that are correlated with the truth with a higher correlation coefficient than is given. "minimum corr." gives the minimum correlation coefficient with the truth over all Monte Carlo estimates.

Figure 1: Kalman smoothed states: benchmark
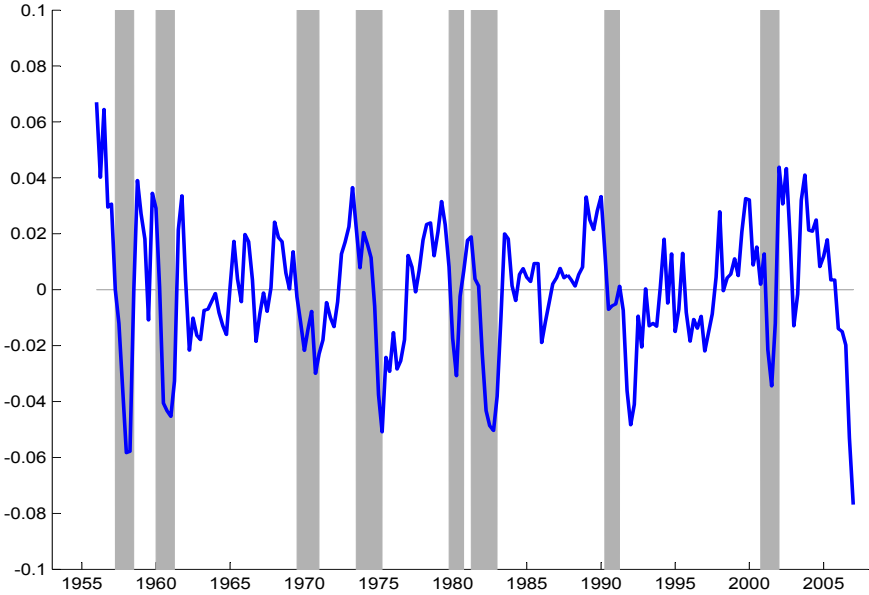
(a) Match efficiency
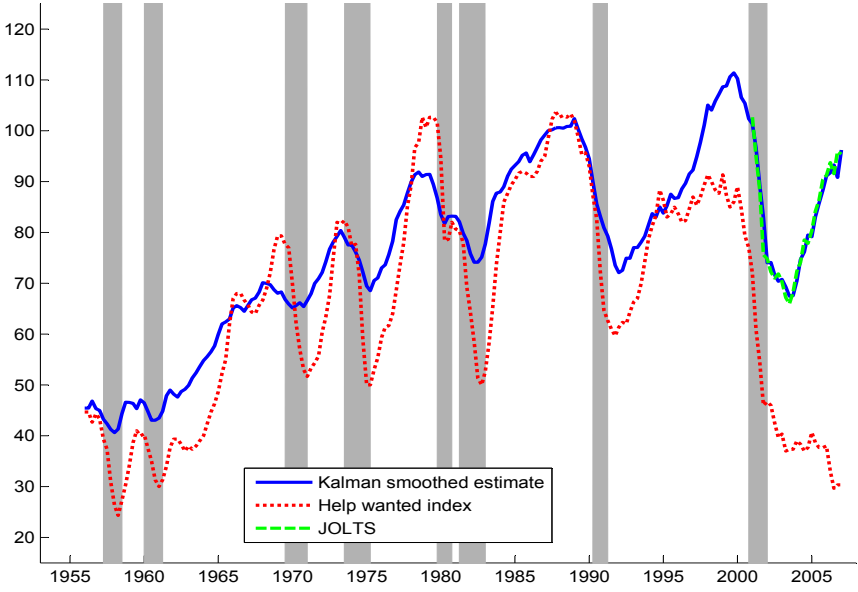
(b) Vacancies

Notes: The Kalman smoothed match efficiency and vacancy estimates together with the JOLTS vacancy data and HWI. The vacancy estimate and the JOLTS data are appropriately scaled to ease comparison with the HWI. Shaded areas are NBER recessions.

Figure 2: Job finding probability
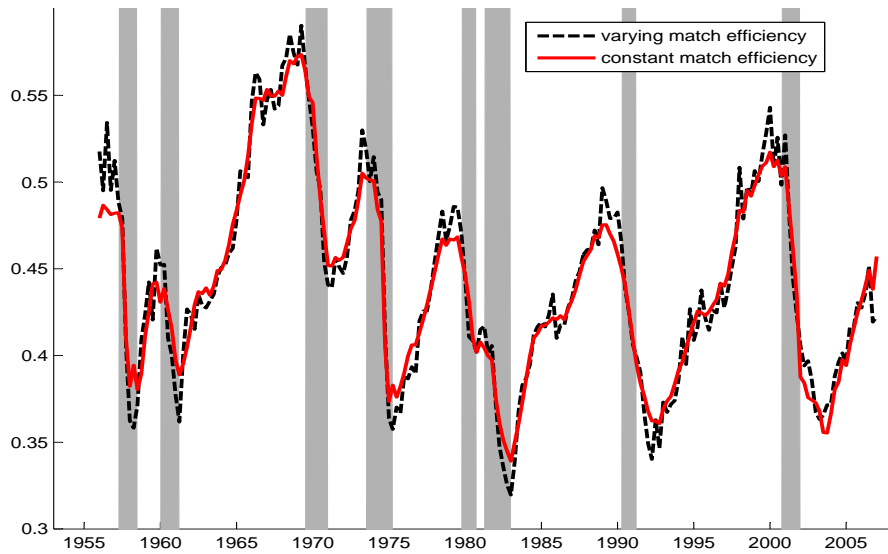
Notes: "Varying match efficiency" refers to the actual job finding rate and "constant match efficiency" is a counterfactual job finding rate where match efficiency was fixed at its average value. Shaded areas are NBER recessions.

Figure 3: Unemployment rate
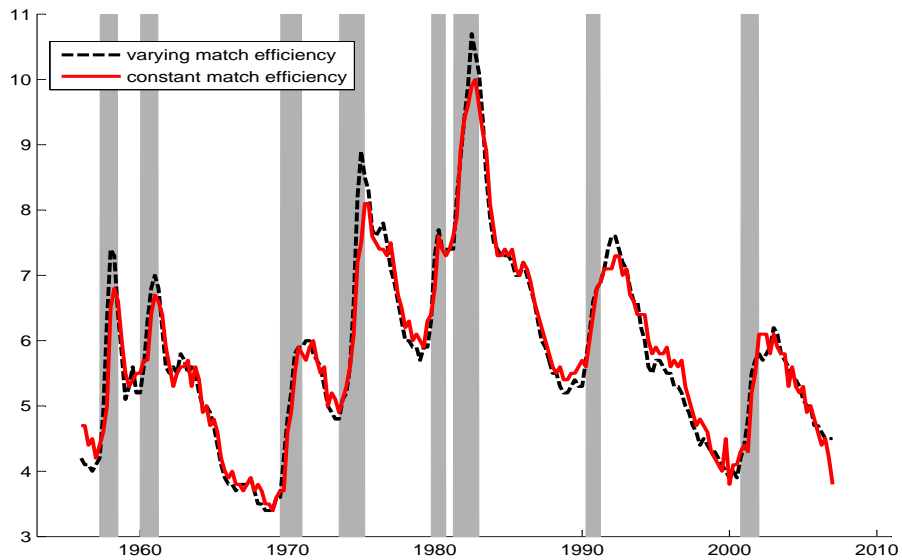
Notes: "Varying match efficiency" refers to the actual unemployment rate and "constant match efficiency" is a counterfactual unemployment rate where match efficiency was fixed at its average value. Shaded areas are NBER recessions.

Figure 4: Decomposition of the cummulative drop in the job finding rate
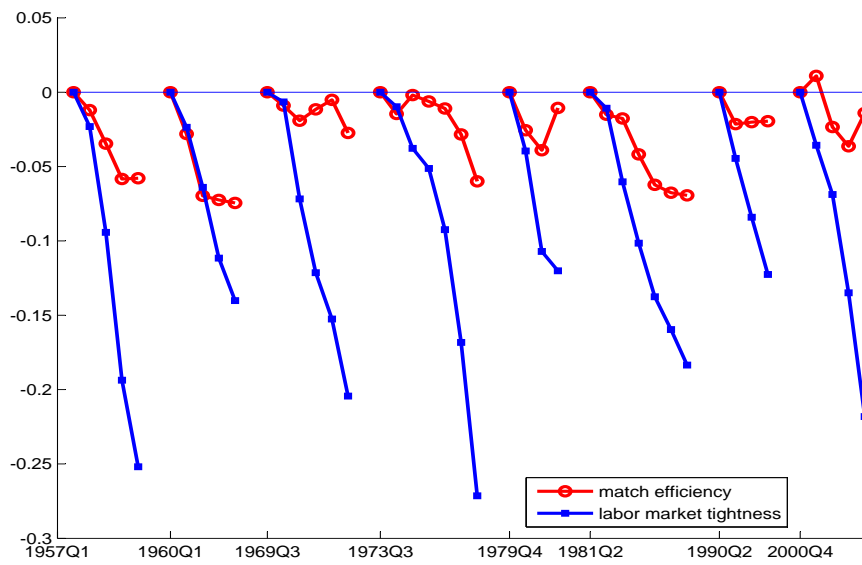
Notes: The cumulative (log) drops in the job finding rate for each recession are decomposed into the contributions of match efficiency and labor market tightness. Adding the two contributions together gives the total cumulative (log) job finding rate drop.

Figure 5: Match efficiency, robustness: alternative functional forms

Legend:
$M_t = A_t (U_t V_t)/(U_t^\zeta + V_t^\zeta)^{1/\zeta}$
$M_t = A_t (U_t^\xi + V_t^\xi)^{1/\xi}$
benchmark
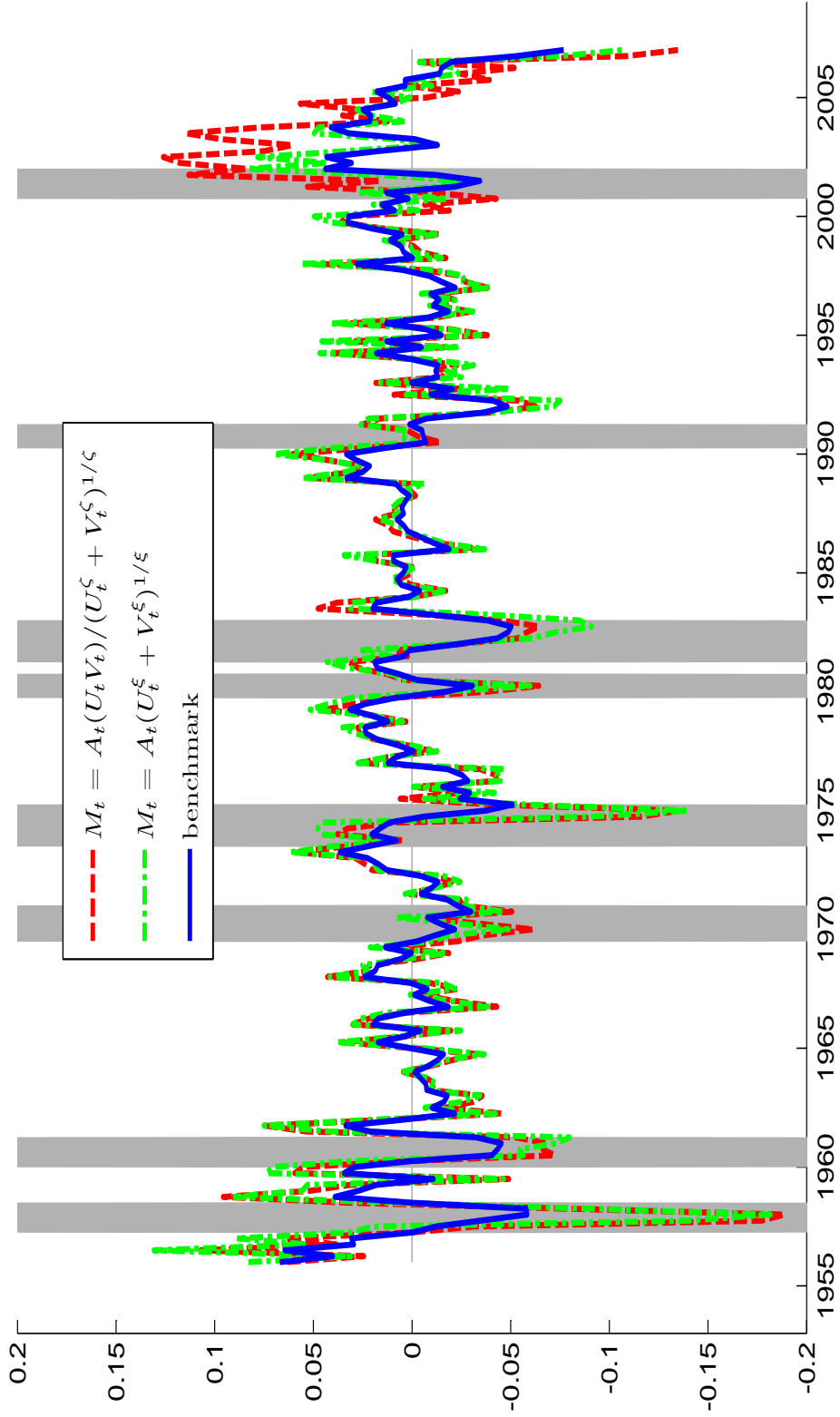
Notes: "Benchmark" refers to the Cobb-Douglas specification. The other two functional forms are estimated using the Extended Kalman filter due to the non-linearity of the state-space. Shaded areas are NBER recessions.

49

Figure 6: Indication of the degree of non-linearity
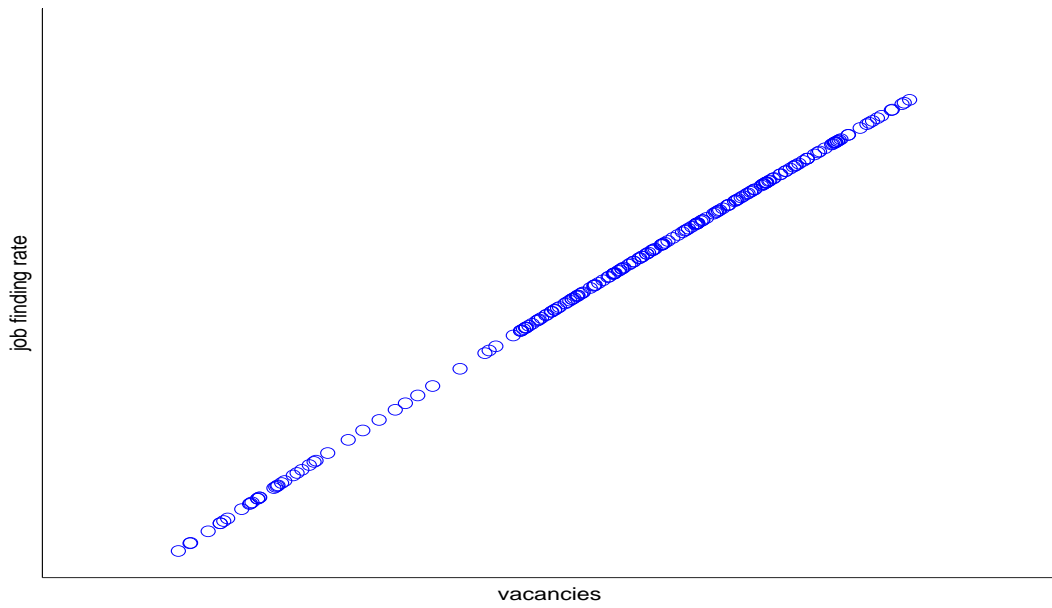
(a) CES matching function
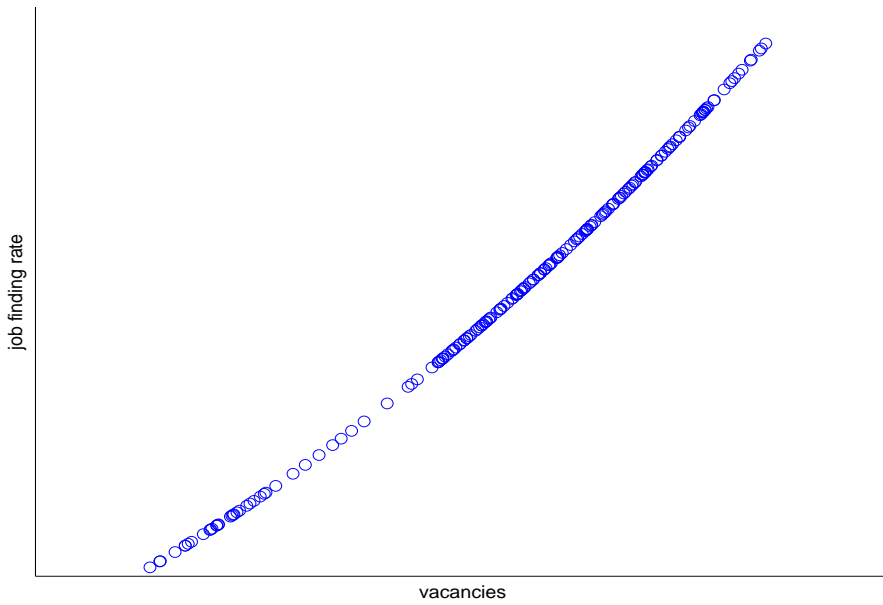
(b) matching function according to Den Haan et. al. 2000

Notes: The figure scatter-plots the estimated (log) vacancy series and an artificial (log) job finding rate series created by using the average value of match efficiency and unemployment and the ML parameter estimates. In the benchmark specification the scatter-plot would be exactly linear with a slope of μ.

Figure 7: Vacancies, robustness: alternative estimation
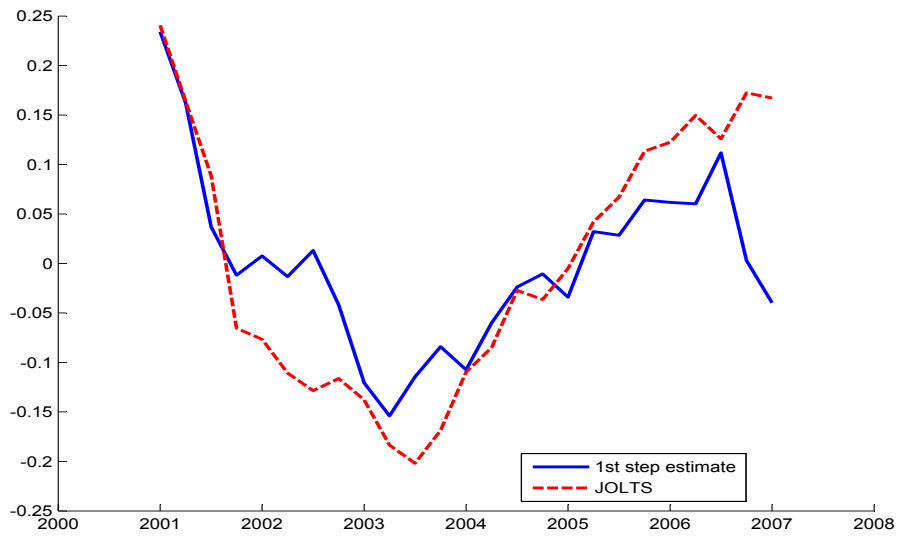
Notes: In the first step of the alternative estimation procedure match efficiency is forced to be constant and thus all of its (potential) variation is captured by the vacancy estimate.

Figure 8: Match efficiency, robustness: alternative estimation
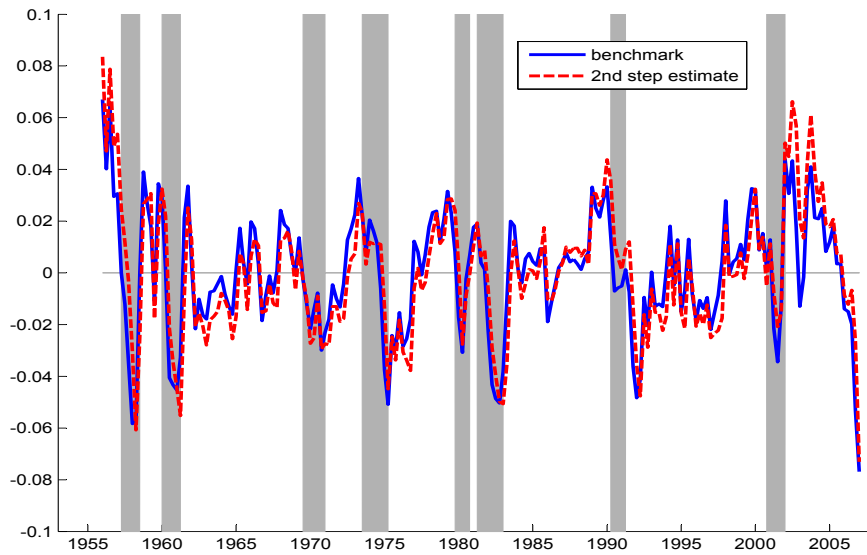
Notes: In the second step of the alternative estimation procedure the vacancy estimate from the first step is decomposed into match efficiency and vacancies by using the JOLTS vacancy data. Shaded areas are NBER recessions.

Figure 9: Match efficiency, robustness: using vacancy yield data
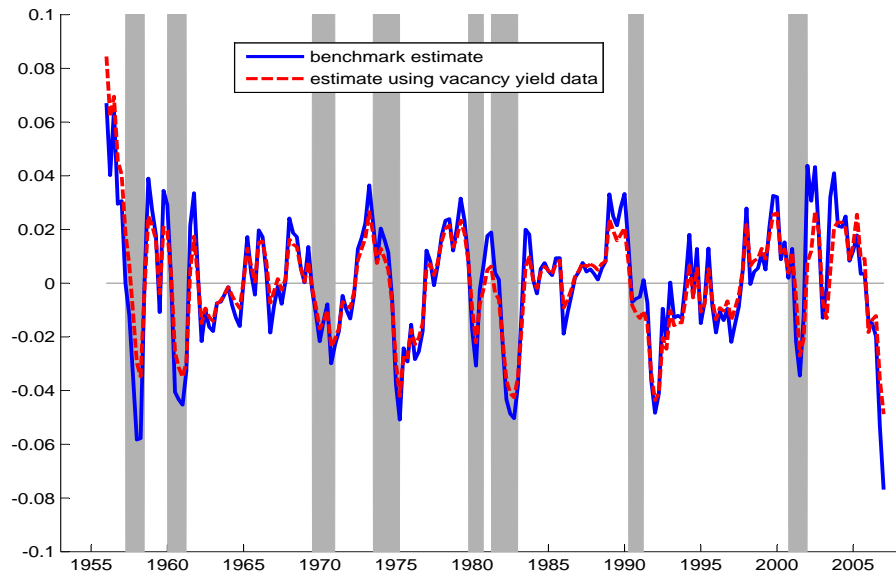
Notes: Benchmark estimate and estimate based on a state-space system augmented with a third measurement equation using vacancy yield data. Shaded areas are NBER recessions.

Figure 10: Match efficiency, robustness: extended sample
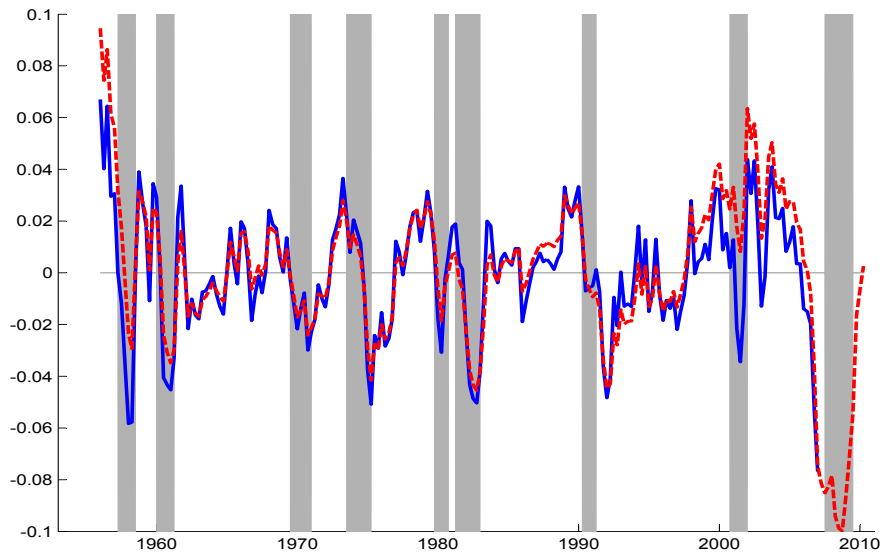
Notes: The benchmark estimate and an estimate based on an extended sample. Shaded areas are NBER recessions.

Figure 11: Match efficiency, robustness: Monte Carlo experiment
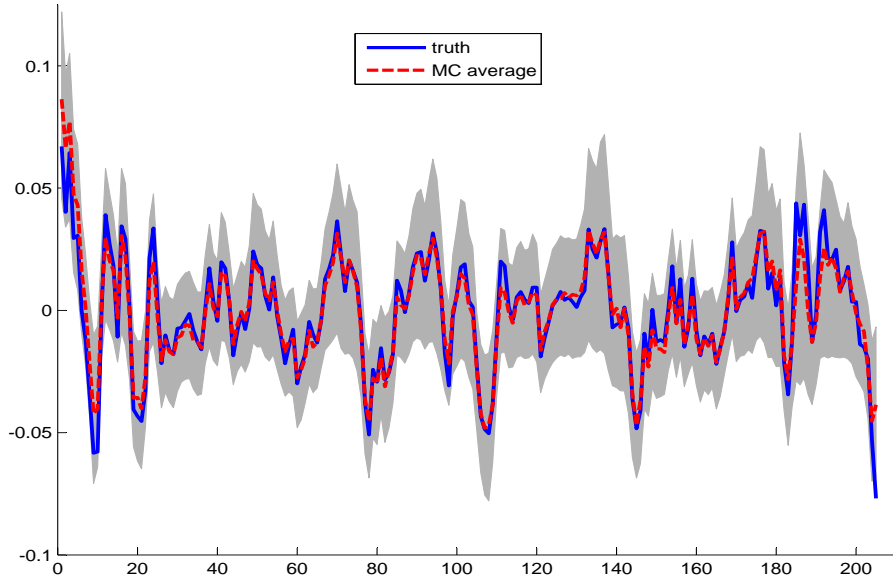
Notes: "Truth" refers to the underlying time series used in the data generating process. "MC average" refers to the average of the estimates from the 1,000 Monte Carlo replications. The shaded area indicates the 90% confidence intervals.

Figure 12: IRFs to a positive one-standard-deviation technology shock, 1st calibration
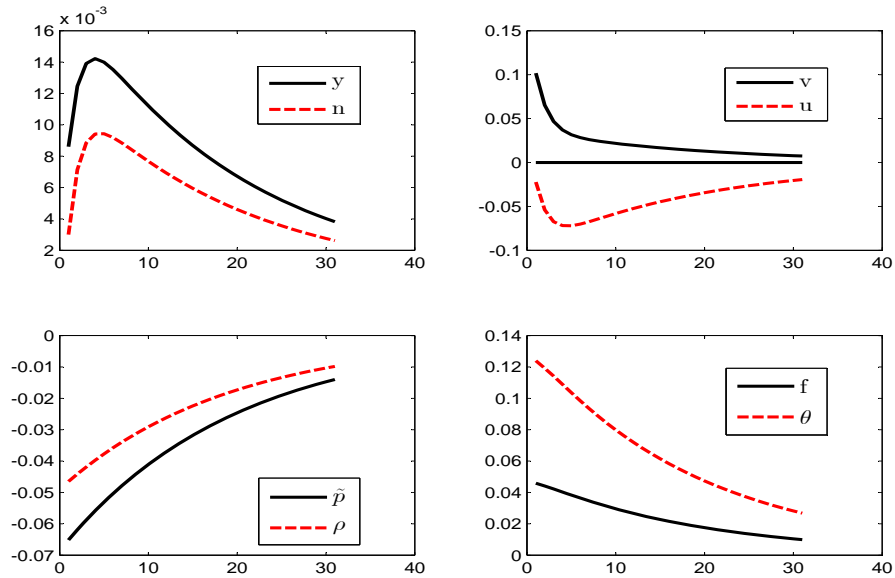
Notes: The model is calibrated with zero firing costs and targets the volatility of the separation rate.

Figure 13: IRFs to a positive one-standard-deviation technology shock, model with firing costs
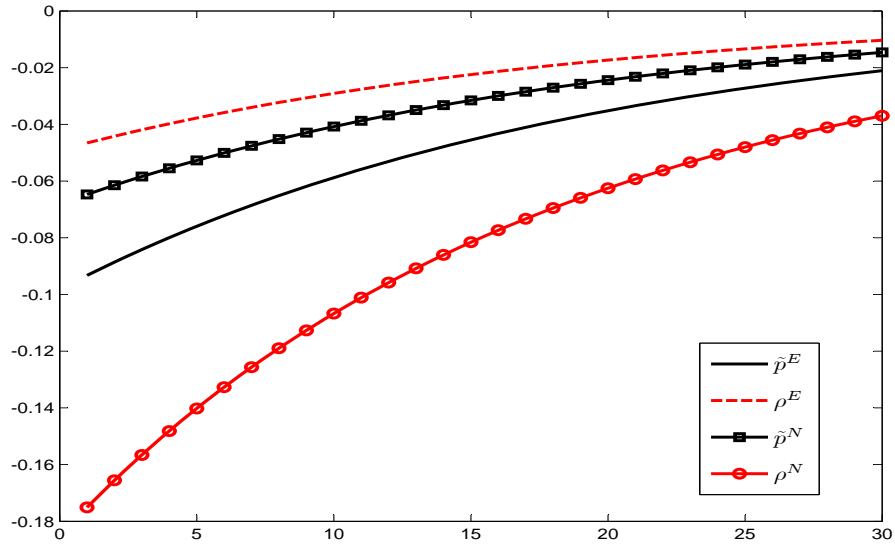
Notes: The model is calibrated with firing costs $\phi = 0.179$ and targets the volatility of the separation rate. $\widetilde{p}^E$ and ρ^E are the idiosyncratic productivity cut-off and the separation rate for workers in existing employment relationships, respectively. $\widetilde{p}^N$ and ρ^N are the idiosyncratic productivity cut-off and the rejection rate for unemployed workers, respectively.

Figure 14: Idiosyncratic productivity cut-offs, cases with and without firing costs
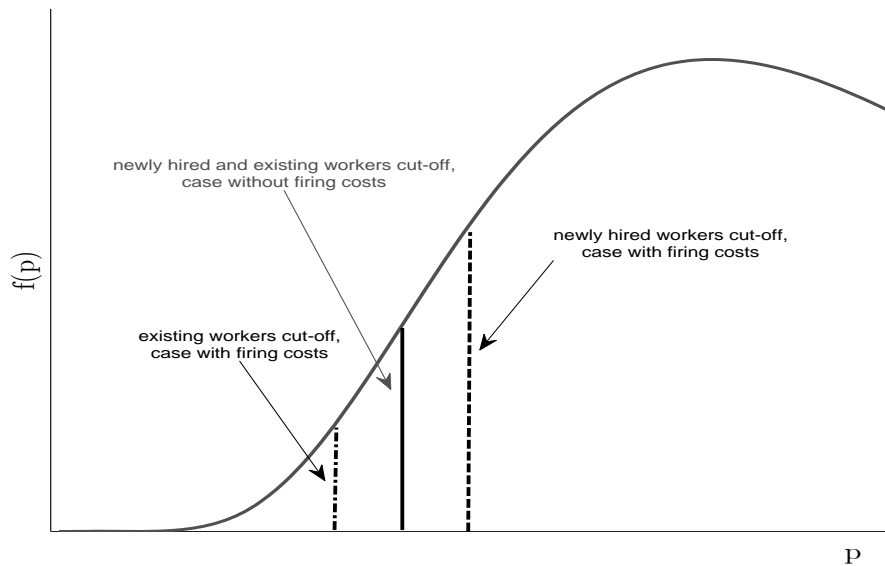
Notes: The figure shows part of the log-normal distribution and illustrates the effect of firing costs on the idiosyncratic productivity cut-offs.

Figure 15: Estimated match efficiency and its model prediction
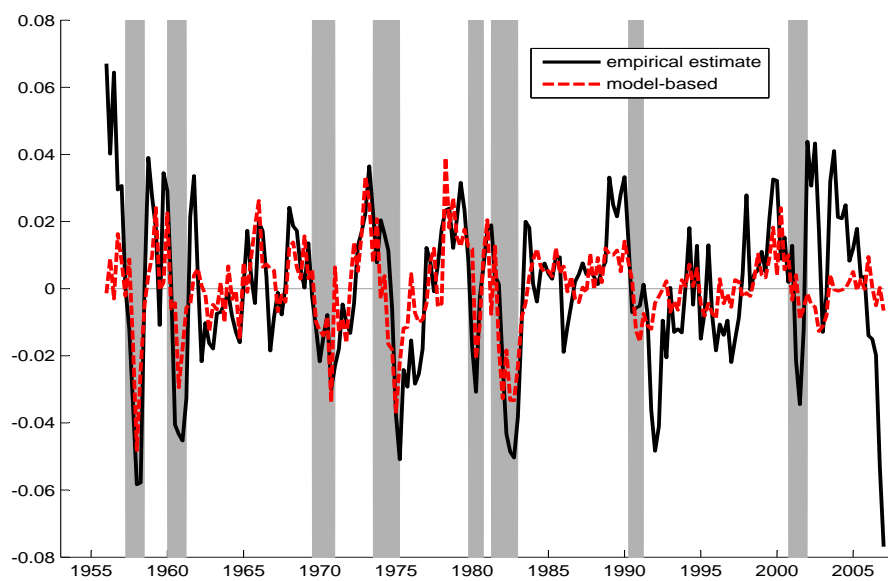
Notes: The model based match efficiency is constructed in the following way. Using detrended real GDP data for the U.S. and the inverted policy rules of the model with firing costs one can back out the implied shock series consistent with the real GDP data. These shocks are then fed through the model which gives the "model-based" match efficiency. The "empirical estimate" series is the benchmark match efficiency estimate.

Figure 16: Kalman smoothed states: benchmark and other specifications
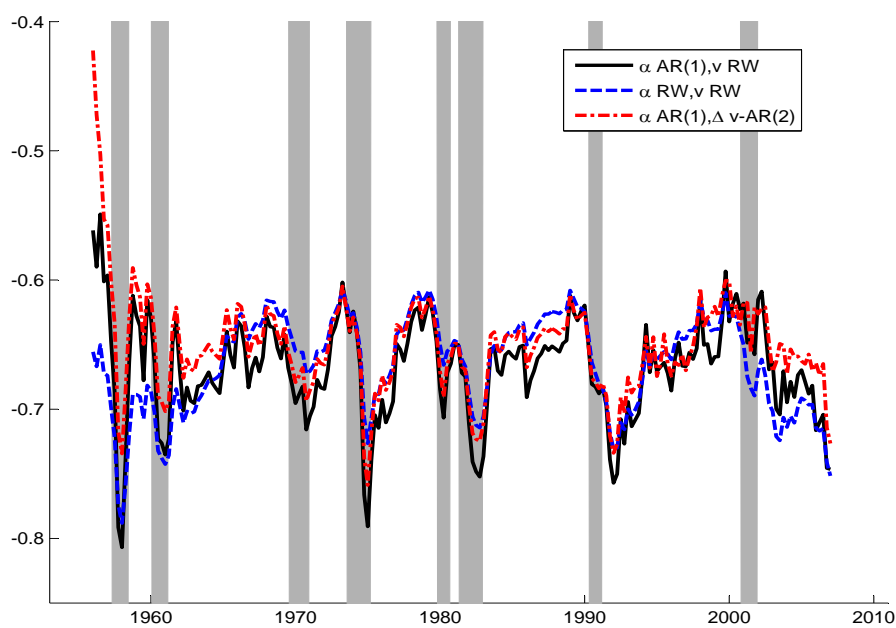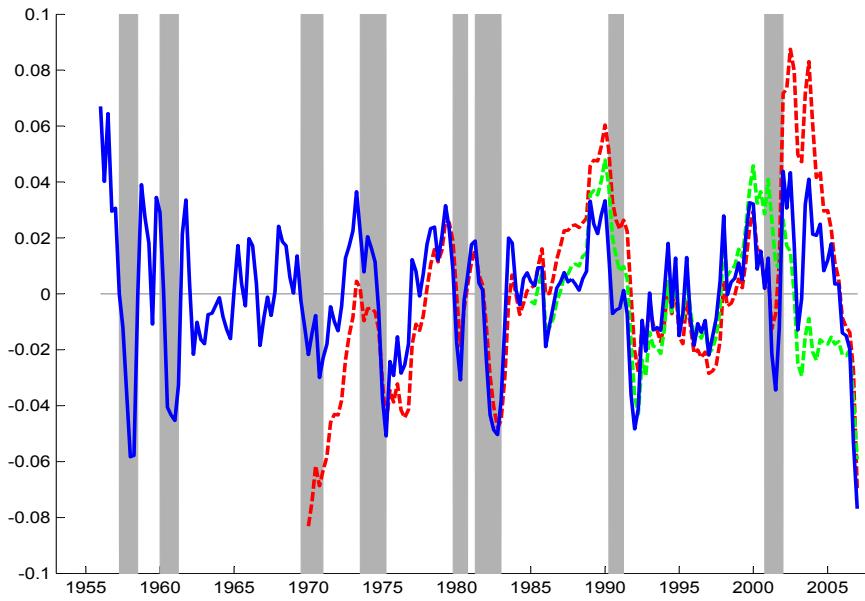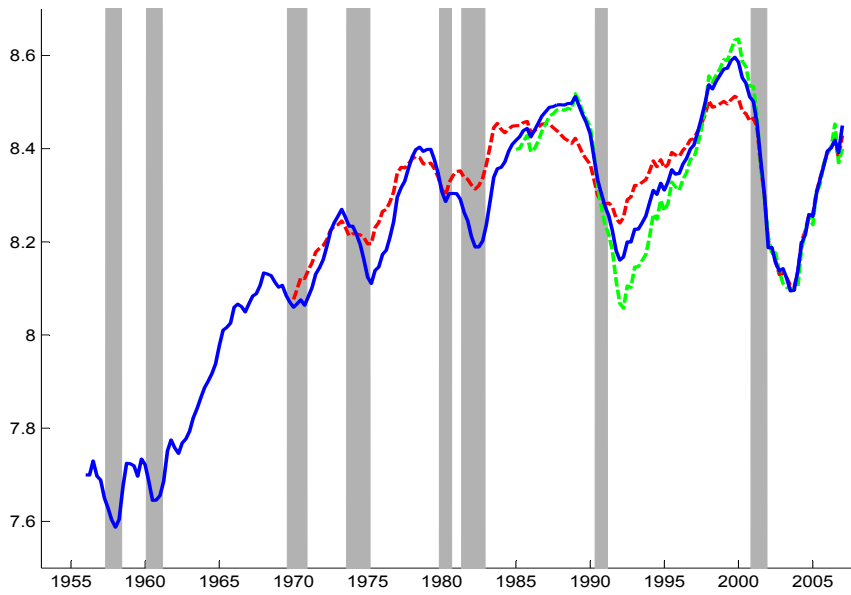

(a) Match efficiency
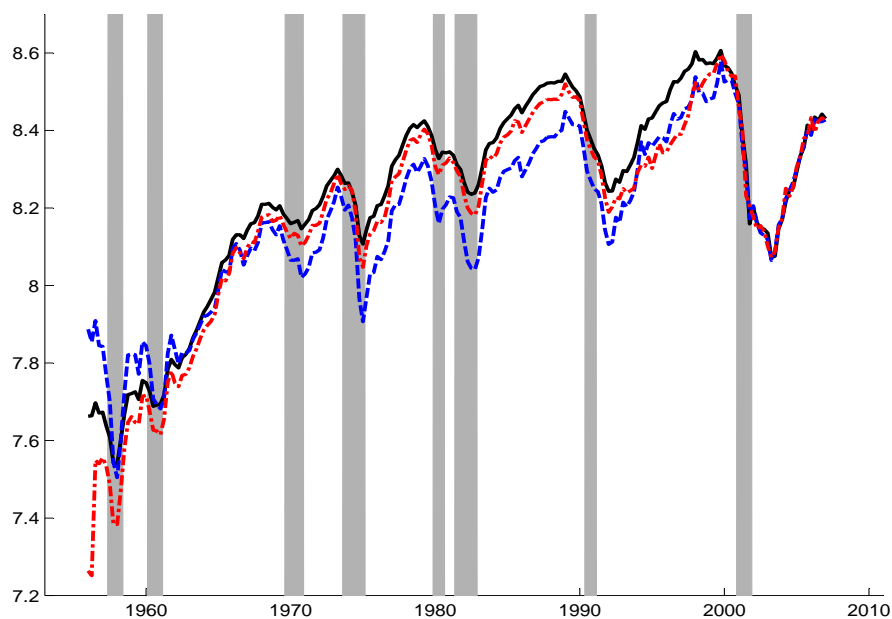
(b) Vacancies

Notes: $AR(p)$ and RW indicate that the given process is an autoregressive process with p lags or a random walk, respectively. Shaded areas are NBER recessions.

Figure 17: Kalman smoothed states: benchmark and subsamples

(a) Match efficiency

(b) Vacancies

Notes: The benchmark estimates and those based on shorter subsamples. Shaded areas are NBER recessions.