Strategic Vertical Market Structure with Opaque Products

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July 2012

Abstract

This paper studies the strategic introduction of an opaque channel by incumbent firms. We endow a circular city model with an intermediary that sells lotteries (opaque products) over goods produced by upstream firms. Compared to the benchmark model (Salop, 1979), opaque intermediation creates value (welfare) by increasing the intensity of price competition and expanding industry sales, but the effect on the value captured by the firms is ambiguous. We show that firms can use the opaque intermediary as a facilitating device to price discriminate and increase profits when the degree of product differentiation takes intermediate values. As an example, we consider the use of opaque intermediaries in markets exposed to seasonal demand. The value captured by the firms increases if the lower profits due to intense competition when demand is high are outweighed by the benefits of expanding the extensive margin when demand is low. **JEL Codes:** D43, L11, M31.

Keywords: Vertical market structure, Opaque products, Circular city, Intermediation, Price discrimination.

1 Introduction

Intermediation between producers and consumers is a common feature of most markets, yet the shape of the vertical market structure varies considerably. In some cases, consumers can only buy upstream products from an intermediary that takes the form of a retailer (e.g., car dealership, supermarket). In other cases, the intermediary acts as a clearing house that reduces information frictions (e.g., real estate agents, Amazon.com). In this paper, we study a new and prolific market environment where consumers have the option of buying directly from upstream firms or buying an *opaque* product from an intermediary.¹ More explicitly, we consider an intermediary that sells the goods produced upstream but hides

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¹A product is defined as opaque if one or more of its characteristics is concealed from consumers until after purchase.

the identity of the seller until after the consumer completes the purchase. The nature of the product at the time of consumption is the same regardless of whether the buyer purchased it from the upstream firm (transparent channel) or the intermediary (opaque channel). Note, however, that the product sold by the latter is ex-ante inferior. This raises the question of why opaque selling is observed some markets? We argue that it can be a strategic choice by upstream firms.

The use of opaque products has become a common practice in the travel and entertainment industries since Priceline (PCLN) introduced the Name Your Own Price (NYOP) reservation system in 1999.² The system involves a reverse auction in which a consumer commits to a price for a given product without knowing the identity of the upstream seller. For example, a customer submits a price for a three-star hotel room on a given date and in a specific city. PCLN then attempts to procure the room by running an auction among the hotels that fit the product characteristics specified by the consumer. The transaction is successfully completed if the winning bid plus PCLN's commissions is below the customer's named price.³ The consumer then learns the identity and location of the hotel she has been assigned to.

Opaque intermediation is not a practice exclusive to PCLN. The major players in the online travel market (Hotwire, Expedia and Travelocity) have launched *semi-opaque* mechanisms whereby instead of a reverse auction the intermediary posts a price and hides the identity of the provider.⁴ More generally, opaque intermediation is a selling strategy that can be applied in any industry with horizontally differentiated upstream sellers.⁵ The goal of this paper is to provide a simple general model of opaque intermediation to characterize its effects on equilibrium prices, industry profits and welfare. The main message is that upstream firms can use opaque intermediation as a facilitating mechanism to price discriminate consumers.

We model product differentiation with a two-stage circular city model, as in Salop (1979). The degree of product differentiation is captured by the endogenous density of firms, and the exogenous transportation cost faced by consumers. In our model, there is one opaque good for each pair of adjacent *transparent* products that define a sub-market segment in the circle.^{6,7} The opaque intermediary obtains the goods from participating upstream firms through a first-price auction and consumers view the opaque good as a lottery over the two

²PCLN currently uses NYOP for airfare, hotel room and rental car bookings.

³It is not exactly clear how the reverse auction is implemented or how the information flows between PCLN and the participating sellers. The customer pays the named price, and presumably PCLN allocates the customer to the lowest bidder and keeps any surplus from the transaction.

⁴As will become clear below, our model for opaque selling generates the same predictions as for semiopaque mechanisms.

⁵For example, Scorebig.com uses the NYOP system to sell seats at live concerts and sporting events.

⁶This is a simplifying assumption. In principle, an opaque product could involve any subset of two or more of the upstream products in the economy.

⁷Figure 3 in Section 3 helps to visualize the environment.

products involved. Naturally, opaque goods are inferior goods that are more attractive to consumers with high elasticity of substitution—that is, consumers who are located around the midpoint between any two firms in a sub-market.

We use the free-entry outcome in a market without opaque goods as our benchmark and consider the effects of introducing opaque intermediation. We focus on the symmetric equilibrium of the game where upstream firms decide to participate in the opaque market first and compete on prices in the last stage. Opaque intermediation reduces the importance of product differentiation and shifts the bargaining power away from upstream firms in the pricing stage. A key characteristic of our model is the strategic complementarity between prices set by producers and by the intermediary. As a result, the presence of opaque products increases competition and lowers prices across the board (i.e., transparent and opaque prices). The larger the price discount relative to transparent prices, the larger the share of consumers who choose to use the opaque channel. But the opaque intermediary requires participation by both consumers and upstream firms. The intermediary uses fixed transfer payments to induce participation by upstream firms. We show that the participation constraints for upstream firms are always satisfied when the opaque intermediary increases overall industry profits. In other words, opaque intermediation acts as a price discrimination mechanism whereby upstream firms increase profits by committing to aggressive price competition.⁸

Upstream firms will participate in the opaque channel when the lower margins are offset by the gains from the market expansion effect. We show that this happens in markets with intermediate degrees of product differentiation. For a given number of firms, low transportation cost parameter values generate too much cannibalization of transparent sales, while the demand for opaque goods disappears if the transportation cost is too high. Similarly, given the transportation cost, too many firms (little differentiation) leads to cannibalization, but there is no demand for opaque products if the number of firms is too low. We find that welfare increases relative to the benchmark model, even though opaque goods are inefficient products. The reason is that the gains from adding new customers (market expansion) are larger than the welfare loss due to the inefficient matching (excessive travel cost) associated with opaque consumption by buyers who would otherwise purchase transparent goods.

We illustrate the effect of opaque intermediation using an example of markets exposed to seasonal demand. Allowing for high and low demand seasons in the opaque model better fits the features of the travel and entertainment industries. Imagine a situation where the free entry equilibrium with transparent products (benchmark model) leaves no market gaps in high-demand periods but unserved segments in the off-peak season. Allowing for opaque selling in low-demand periods increases profits as long as the new opaque sales imply little

⁸Below we discuss the relationship to other price discrimination and market segmentation strategies addressed in the literature.

cannibalization of transparent good sales. Moreover, allowing opaque intermediation in both high- *and* low-demand seasons can still increase overall industry profits if the lower profits when demand is high are more than compensated for by the extra profits during the low-demand periods.

■ Related literature. This paper is related to different strands of the economics and marketing literature. By definition, the introduction of an opaque product segments the market and allows firms to price discriminate among consumers with different reservation values. The idea of introducing inferior or damaged goods to price discriminate was introduced by Deneckere and McAfee (1996). In their model, a monopolist finds it profitable to create a new and inferior version of an existing product by hiding or disabling some of its features at no cost (e.g., shareware vs. full-version software). An opaque product is an inferior good, but its nature implies that (at least) two transparent products are associated with it. In this paper opaque products are introduced by an intermediary in a competitive environment.

We emphasize the strategic introduction of the opaque channel by upstream firms that is, situations where the intermediary exists for the collective benefit of the firms. The intuition is related to the results in Bonanno and Vickers (1988) and Rey and Stiglitz (1995). These papers show that oligopolists can gain from vertical separation (franchising) by using two-part wholesale prices. Profits can be larger with separation than under vertical integration, due to complementarities between the prices set by upstream firms and the prices set by retailers. As a result, equilibrium prices downstream are less competitive if upstream firms choose variable wholesale prices above marginal cost. Prices are strategic complements in our model too. However, the absence of product differentiation in the wholesale market (opaque channel) implies that upstream firms can only influence opaque prices through their posted prices in the parallel transparent channel.

Consumers buying through the opaque channel commit to ignoring product differences and, making price competition by firms more intense. This idea is connected to Dana (2012), who considers situations where heterogeneous consumers commit to group-buying to reduce firms' bargaining power and profits. A consumer group that buys all quantities from the lowest-price seller generates more competition. Lower prices can thus compensate for any disutility arising from individual preference-product mismatch.⁹ The main difference with our paper is that the opaque and transparent channels coexist in equilibrium, and the introduction of the opaque product is done by the incumbent firms for their private and collective benefit. That is, participating firms commit to reducing their bargaining power in the pricing stage in exchange for a transfer payment from the intermediary. This is possible because only a fraction of consumers (determined in equilibrium) uses the opaque

⁹Similarly, Inderst and Shaffer (2007) argue that downstream firms can merge to reduce upstream firms' pricing power, even at the expense of reducing variety in their inputs.

 $channel.^{10}$

More recent work in the revenue management literature attempts to explicitly address the profitability of opaque selling. One strand examines the properties of different mechanisms (variants of the opaque and semi-opaque systems) when implemented by a monopolist (Fay, 2004; Fay and Xie, 2008; Jiang, 2007; Wang et al., 2009). These papers are directly related to the versioning literature mentioned above. A second group of papers study opaque intermediation in competitive environments. Jerath et al. (2010) consider a two-period model to study the last-minute pricing problem faced by capacity-constrained duopolists. In their model, the firms can dispose the leftover inventory (if any) in the last period through one selling channel: opaque or transparent. Selling through the opaque intermediary in the last period alters the traditional price-availability trade-off faced by strategic consumers, and hence pricing, in the first period. The authors find that opaque selling dominates last-minute sales when product differentiation in the market is high.

More closely related to this paper, Fay (2008) and Shapiro and Shi (2008) analyze opaque intermediation in static environments. Both papers model competition along a line (Hotelling and Salop, respectively), with consumer types defined by a location-loyalty pair and inelastic aggregate demand.¹¹ To some extent, the logic underlying the results above is similar to the mechanism at play in a direct (second-degree) price discriminating monopoly. When profitable, opaque intermediation is a new instrument that allows producers to segment the market and charge a *higher* price to loyals and a lower price to non-loyals.¹² Our model allows for elastic aggregate demand (competition with the outside good). In addition, there is no exogenous loyalty other than consumers' locations in the product space. This implies that prices for transparent products are always lower with opaque products than without them, yet profits can be higher due to a market expansion effect. One of our results is that opaque intermediation can support markets that would not otherwise exist. Another important difference between this paper and the previous literature is that we endogenize entry and study the welfare effects of opaque intermediation.

The rest of the paper is organized as follows. The next section describes the model with transparent products and no intermediation. This is the benchmark model. Section 3 introduces opaque intermediation and characterizes the pricing stage of the opaque model. Participation constraints are analyzed in Section 4, together with the effect of opaque selling on welfare and industry profits. Section 5 considers opaque intermediation in a market with high and low demand. The paper ends with conclusions and a discussion of extensions to the model. All proofs are contained in the Appendix.

¹⁰Note that our model could be extended to cover situation where entry by an opaque intermediary generates a prisoner's dilemma problem to upstream firms.

¹¹Loyal consumers in Fay (2008) are sealed to their preferred firm while Shapiro and Shi (2008) introduce loyalty through high and low values in the transportation cost parameter.

¹²The difference with the discriminating monopolist case is that competition in the non-loyal market is more intense and can make opaque selling unprofitable when the loyal segment is too small.

2 Preliminaries - The Benchmark Model

Our benchmark model is a two-stage circular city model à la Salop (1979) with L consumers uniformly distributed along the market. Firms decide entry simultaneously in the first stage and compete through prices in the second stage. We assume maximum differentiation: the i = 1, ..., n products have equidistant locations $l_i = (i - 1)/n$. Consumers have single peaked preferences over the product space and buy one unit as long as its price is below their reservation price v. We assume that a consumer with "address" x buying product i faces a quadratic disutility on the distance to the product's location:

$$u(x,i) = v - p_i - t(l_i - x)^2$$

The parameter t > 0 captures the importance of product variety for consumers' welfare.¹³ Firms face the same cost structure with entry cost F and a constant marginal cost of production normalized to zero. Additionally, only one price per firm is allowed so firms cannot price discriminate.

The demand for firm *i* is built using the critical locations of the indifferent consumers in the circle. Consider first the consumer located to the right and at a distance $_{r}x_{i}^{1} \in (0, 1/n)$ from firm *i* that is indifferent between buying product *i* and its closest neighbor *i* + 1:

$${}_{r}x_{i}^{1} = \frac{1}{2n} + \frac{n}{2t}\left(p_{i+1} - p_{i}\right).$$
(1)

We do not make the usual assumption of full market coverage. That is, v can take any value, and local monopoly (market gaps) is a possible equilibrium. A consumer located to the right of firm i that is indifferent between the outside good and product i is located at

$${}_{r}x_{i}^{0} = \left(\frac{v - p_{i}}{t}\right)^{1/2}.$$
(2)

Similarly, the consumer to the left of firm i + 1 that is indifferent between the outside good and product i + 1 has address

$${}_{l}x_{i+1}^{0} = \frac{1}{n} - \left(\frac{v - p_{i+1}}{t}\right)^{1/2}.$$
(3)

Assuming a symmetric price equilibrium p, the critical locations (1)-(3) determine firm i's perceived demand Q_i when the remaining firms charge the equilibrium prices. Two cases are possible. First, the market is fully covered, regardless of firm i's pricing, if the

¹³With free entry, the transportation cost (t) and the distance among firms (1/n) represent exogenous and endogenous product differentiation, respectively.

equilibrium price p is lower than $p = v - t/n^2$:

$$Q_i\left(p_i, p \le \underline{p}\right) = \begin{cases} 0, \ p_i > p + \frac{t}{n^2} \\ L\left[\frac{1}{n} + \frac{n}{t}\left(p - p_i\right)\right], \ p_i \in \left[p - \frac{t}{n^2}, p + \frac{t}{n^2}\right] \\ h\left(p_i\right), \ \text{if } p_i (4)$$

where $h(p_i)$ is a decreasing and weakly concave function that represents the demand when firm *i* prices low enough that it sells to consumers to the right of firm i + 1.¹⁴

The second case allows for both full and partial market coverage in equilibrium. Let $\widehat{p} = p - \frac{t}{n^2} + \frac{2}{n} \left[t \left(v - p \right) \right]^{1/2}$ represent the price above which firm *i* competes directly with the outside good (i.e., $_r x_i^0 <_l x_{i+1}^0$). The perceived demand for firm *i* becomes:

$$Q_{i}\left(p_{i}, p > \underline{p}\right) = \begin{cases} 0, p_{i} > v \\ 2L\left(\frac{v-p_{i}}{t}\right)^{1/2}, p_{i} \in \left[\widehat{p}, v\right] \\ L\left[\frac{1}{n} + \frac{n}{t}\left(p - p_{i}\right)\right], p_{i} \in \left[p - \frac{t}{n^{2}}, \widehat{p}\right] \\ h\left(p_{i}\right) \text{ if } p_{i}
$$(5)$$$$

Figure 1 presents the construction of firm i's demand from (5) when the equilibrium price p leads to full and partial market coverage. The two graphs on the top illustrate, for each case, the net utility profile given the equilibrium price p set by firms i+1 and i+2(solid curves) and different prices set by firm i (dashed curves). Assuming $l_i = 0$, the utility for consumers located to the right of i decreases at an increasing rate with the distance xand the indifferent consumers can be identified. Each firm has symmetric market shares if $p_i = p$ and firm i can increase its share by dropping prices below p. For example, a price $p_i = p - 1/n^2$ makes product *i* the preferred option for all consumers located in the [0, 1/n]interval. The bottom graphs show the corresponding normalized demand function from consumers located to the right of i. The demand faced by firm i is characterized by three regions: "monopoly", "competitive" and "supercompetitive".¹⁵ A price above \hat{p} places firm i in the monopoly region with a strictly concave demand. The location of the marginal consumer shifts to the right as p_i decreases, and firm *i* enters the competitive region in which new sales come at the expense of sales from firm i + 1. Prices below \tilde{p} lead to the "supercompetitive" region, where firm i competes directly with firm i + k, which is located farther away (k > 1). In a symmetric equilibrium, $p_i = p$ and the market can be fully covered (left panel) or partially covered (right panel). Moreover, since all firms sell positive quantities at p, the supercompetitive region is never reached.

The demand function in (5) is concave in the monopoly region and linear in the com-

¹⁴The shape of $h(p_i)$ depends on whether the market between firms i + 1 and i + 2 is fully covered at p. Additionally, $h(p_i)$ can be different for very low values of p_i if $l_{(n+1)/2} = 1/2$.

 $^{^{15}}$ The demand discontinuity shown in Salop (1979) is not present here, since we assumed quadratic instead of linear transportation costs.



Figure 1: Firm *i*'s demand. Low (left) and high (right) equilibrium prices

petitive region, with the counterintuitive property of being more elastic in the former than the latter. The rationale comes from the relative value of the alternatives to product i that are available to the marginal consumer. Due to quadratic transportation costs, the drop in price required to attract a consumer with a constant reservation price increases with x, the distance between the consumer and firm i. This is the case when the marginal consumer falls in the monopoly region where the outside good is the best alternative to product i. However, the best alternative for a consumer located in the competitive region is product i + 1, and its value increases with x. The price drop required to attract such a consumer is larger in the competitive region than in the monopoly region because it has to compensate for the increase in transportation cost and reservation price.

The equilibrium properties of the pricing game in the benchmark model are well known. With constant marginal cost, the profit function for each firm is weakly concave and, the reaction functions are contractions, hence the equilibrium is unique (Economides, 1989). Following Salop (1979), we name the three possible outcomes as competitive, kinked and monopoly.¹⁶

 $^{^{16}\}mathrm{From}$ now on, we use the superscript b to indicate variables or equilibrium values of the benchmark model.

Proposition 1 The pricing game in the benchmark model has a unique equilibrium:

$$p^{b} = \begin{cases} p_{c}^{b} = t/n^{2}, \ \frac{v}{t} > \frac{5}{4n^{2}} \\ p_{k}^{b} = v - t/(4n^{2}), \ \frac{v}{t} \in \left[\frac{3}{4n^{2}}, \frac{5}{4n^{2}}\right] \\ p_{m}^{b} = 2v/3, \ \frac{v}{t} < \frac{3}{4n^{2}}. \end{cases}$$
(6)

The proof to the proposition follows Economides (1989) and is omitted. The competitive outcome takes place when consumers' valuations are sufficiently high that the market is fully covered in equilibrium and prices are proportional to the product differentiation parameters (transportation cost and distance between firms). The monopoly and kinked equilibria replace the competitive one when v is low and the outside good becomes a relevant alternative for some consumers. The equilibrium profits associated with (6) are

$$\pi^{b} = \begin{cases} \pi^{b}_{c} = \frac{tL}{n^{3}}, \ \frac{v}{t} > \frac{5}{4n^{2}} \\ \pi^{b}_{k} = \frac{L}{n} \left(v - \frac{t}{4n^{2}}\right), \ \frac{v}{t} \in \left[\frac{3}{4n^{2}}, \frac{5}{4n^{2}}\right] \\ \pi^{b}_{m} = 4tL \left(\frac{v}{3t}\right)^{3/2}, \ \frac{v}{t} < \frac{3}{4n^{2}}. \end{cases}$$
(7)

The parameters of the model affect equilibrium prices, profits and the competition regions in different ways. As Figure 2(a) shows, v/t and n are enough to characterize the equilibrium regions. For a given v/t, a higher density of firms shifts the equilibrium to more competitive regions. A similar effect occurs, holding n constant, as the willingness to pay relative to transportation cost increases. However, the parameters v and t have different effects on equilibrium prices and profits.¹⁷ First, consumers' valuations do not affect competitive prices and profits. Second, the transportation cost does not affect the monopoly price and, as shown in Figure 2(b), profits do not decrease monotonically with t. That is, conditional on equilibrium falling in the monopoly region, markets with lower transportation cost generate larger profits. But once the market is covered, lower transportation cost induces more competition among firms, hence a drop in prices and profits.



Figure 2: Competition regions and profits in the benchmark model

¹⁷Higher willingness to pay is not equivalent to lower transportation cost. The latter affects profits in all regions while the former does not change profits in the competitive region.

The free-entry equilibrium of the first stage can be obtained directly using (7). Ignoring the integer problem and assuming a constant entry cost, the number of entrants is given by

$$n = \begin{cases} n_c = \left(\frac{tL}{F}\right)^{1/3}, \text{ if } F < \overline{F}_c^b = tL \left(\frac{4v}{5t}\right)^{3/2} \\ n_k, F \in \left[\overline{F}_c^b, \overline{F}\right] \\ n_m = \left(\frac{3t}{4v}\right)^{1/2}, F = \overline{F} = 4tL \left(\frac{v}{3t}\right)^{3/2} \\ 0, F > \overline{F} \end{cases}$$
(8)

where n_k is defined implicitly by

$$n_k = \frac{L}{F} \left(v - \frac{t}{\left(2n_k\right)^2} \right).$$

It is important to note that the assumption of divisible firms leads to a unique number of entrants in the monopoly case. With a constant entry cost, firms enter up to the point where the marginal consumer is indifferent between products i and i + 1 and the aggregate demand is always inelastic. Such a property is at odds with what we observe in most markets. The extensive margin becomes a relevant dimension once we acknowledge firm indivisibilities. Alternatively, partial market coverage arises in equilibrium if entry costs are not constant. Consider the case of decreasing returns to entry: F(n) with F' > 0. In this case, the number of firms in the monopoly equilibrium is lower than n_m in (8) and, because some consumers choose the outside good, the aggregate demand can be downward sloping. We return to this point in Section 4, where we show that opaque intermediation creates value by expanding the extensive margin.

3 A Model of Opaque Intermediation

We now consider a change in the vertical market structure by introducing opaque intermediation into a benchmark model with n firms. Traditional intermediation usually involves an extra layer in the vertical chain, where the intermediary buys from producers and resells the same items to consumers with a given markup. Instead, we consider a special intermediary that buys goods produced upstream and sells them as opaque goods by hiding the identity of the seller until the consumer has paid for the good. More precisely, we assume that the intermediary can sell n new products that consist of lotteries over pairs of goods produced upstream. Thus, we define product n + i as the lottery between products i and i + 1.¹⁸ For simplicity, we assume the intermediary faces no marginal cost.

Figure 3 represents the new environment in the market segment containing goods iand i + 1. Consumers have the choice to buy directly from producer i and i + 1 at the posted prices p_i and p_{i+1} or from the intermediary. A transaction through the intermediary

¹⁸The lottery between two neighboring firms in the Salop model is analogous to the geography filter available to consumers when using Priceline's NYOP system.



Figure 3: Vertical market structure with opaque products

involves the following sequence of actions. First, a consumer commits to buying product n + i at her desired price p_{n+i} . After the order is received, the intermediary procures the good by holding an auction between producers of goods i and i + 1. The transaction is completed successfully if the lowest winning bid $\underline{w} = \min[w_i, w_{i+1}]$ plus a pre-announced intermediation fee f is less than p_{n+i} , the price named by the consumer. The latter is only known by the consumer and the intermediary.¹⁹ For simplicity, we assume that consumers who choose the opaque channel can bid only once and can use the transparent market if their bid in the opaque market is rejected.

More formally, we look for a strategy-beliefs profile for firms, the intermediary and consumers $(\sigma; \rho) = (\{E, p, w\}_i, \{T, f\}_I, \{g, p_{n+i}, g'\}_c; \rho)$ that supports a symmetric PBNE in the following game:

Participation Stage: The intermediary I makes a take-it-or-leave-it offer T (transfer) to each of the n upstream firms to participate in the opaque channel. Participation $E_i \in \{\text{In,Out}\}$ by firm i is decided simultaneously with other firms. The final outcome becomes public information at the end of the stage.

Pricing Stage: (1) Upstream firms and the intermediary simultaneously choose transparent prices $p \in \mathbb{R}^n_+$ and intermediation fees $f \in \mathbb{R}^n_+$. (2) Each consumer observes (p, f) and decides to buy good $g \in \{0, 1, ..., n, n+1, ..., 2n\}$. (3.1) If $g \leq n$, the consumer chooses the outside good or buys the good directly from the upstream firm (transparent channel). (3.2) If g > n, the consumer selects a price $p_{n+i} \in [f, v]$ and the intermediary holds the auction between upstream producers associated to the opaque good g. (3.2.1) Upstream firms with beliefs ρ about p_{n+i} and g' set their bids w simultaneously. The opaque transaction is allocated to the firm with the minimum bid \underline{w} , conditional on $p_{n+i} \geq \underline{w} + f$. (3.2.2) If the latter condition fails, the consumer can go back to the transparent market and buy $g' \in \{0, 1, ..., n\}$.

¹⁹Note that the information assumptions and the sequence of actions resemble the NYOP opaque selling system. Another (semi-opaque) way to represent intermediation is to assume that producers and the intermediary post prices (w_i , p_i and p_{n+i}) and consumers decide whether to buy from the intermediary or upstream sellers. The results in this section hold for both opaque and semi-opaque environments.

In this section we look at the equilibrium in the pricing stage and leave the analysis of the participation constraints to Section 4.²⁰ We solve the game backwards, focusing on the market segment that involves goods i and i + 1, and normalize locations in the [0, 1/n]segment.

The choices for consumers rejected in the opaque market (3.2.2) are straightforward. For each pair of transparent prices p_i and p_{i+1} there is only one product (including the outside good) that maximizes the consumer's utility at each possible location (more on this below). In fact, consumers can decide q' before participating in the opaque market. As we will see next, this branch of the game is never reached in equilibrium.

A consumer who participates in the opaque channel submits a price $p_{n+i} \ge f$ and triggers a reverse auction upstream. The bidding firms do not observe the identity and price submitted by this consumer, so their strategies depend on their beliefs ρ over g' and p_{n+i} .²¹ Assume for now that consumers play pure strategies when choosing p_{n+i} . The first thing to note is that, aside from their location in the circle, firms are identical and face no capacity constraints. Therefore, any strategy that places positive mass on bids that make the opaque transaction fail (above $p_{n+1} + f$) cannot exist in equilibrium. In such a case, firm j with beliefs $\Pr(q' = j) < 1$ would prefer to deviate to win the auction by setting a bid $w \leq p_{n+1} + f$. But, conditional on the opaque transaction being accepted, undercutting by firm $i \neq j$ follows, and the only equilibrium is to bid marginal cost. In other words, the reverse auction set up by the intermediary leads to the Bertrand paradox outcome (Baye and Morgan, 1999; Harrington, 1989): for any belief ρ , the unique equilibrium involves firms bidding marginal cost and, assuming a coin-toss tie-breaking rule, winning the auction half of the time. It is important to note that consumers' choice between transparent and opaque channels is never affected by a firm's decision to deviate from this equilibrium. A firm that bids higher than marginal cost loses the auction, and the customer is allocated to the remaining firm.^{22,23}

Working backwards, consider now the bidding strategy by a consumer who chooses the opaque channel (after observing the posted prices p_i, p_{i+1} and f). Let α be the consumer's belief about the probability that the reverse auction is won by firm i. The opaque good can be thought of as a virtual product that is located at $l_{n+i} = l_i + (1 - \alpha) 1/n$, since consumers with address $x = l_{n+i}$ are the ones who receive the highest expected utility from it. Note, however, that unlike the case for products i and i + 1, the consumer located at the opaque good's virtual address does not buy if $v < t\alpha (1-\alpha)/n^2$. In other words, by selling a lottery rather than the actual goods, the intermediary creates an inferior product and any

²⁰Since we look at symmetric equilibrium, we assume that all firms participate in the opaque market.

 $^{^{21}}$ A belief about q' is equivalent to a belief about the location of the customer choosing the opaque channel. 22 Submitting a losing bid by a deviating firm is not equivalent to no participation.

 $^{^{23}}$ Given that upstream firms make no profits in the opaque market, one may wonder why they choose to participate in the first place. As we show in Section 4, the intermediary can induce participation through a lump sum transfer if it increases industry profits.

equilibrium should have consumers bidding $p_{n+i} < p_i$. More importantly, rational consumers anticipate the result of the reverse auction that follows their bid: $(\alpha = 1/2, w = 0)$. Since any bid above the intermediary's fee f is accepted, the dominant strategy is to bid $p_{n+i} = f$.

We are left with the problem of finding the prices set by the upstream firms and the intermediary (p, f) that determine the consumer's choice between the opaque and transparent products. The analysis is very similar to the one in the previous section, and the equilibrium can fall in the monopoly, kinked or competitive regions of each firm's demand. Consider first a situation with very high prices so that each firm, including the intermediary, serves isolated markets. The firm demand in this case is determined by the location of the consumer who is indifferent between the outside good and the firm's product. The demand for products i and i + 1 are obtained from (2) and (3), and the demand for product n + i is given by the distance between the locations of the following indifferent consumers:

$${}_{r}x_{n+i}^{0} = \frac{1}{2n} \left(1 + \left(\frac{4n^{2}}{t} \left(v - f \right) - 1 \right)^{1/2} \right)$$

$${}_{l}x_{n+i}^{0} = \frac{1}{2n} \left(1 - \left(\frac{4n^{2}}{t} \left(v - f \right) - 1 \right)^{1/2} \right).$$
(9)

Profits for the intermediary in the monopoly region are given by

$$\pi_{m,n+i} = f \left[{}_{r} x_{n+i}^{0} - {}_{l} x_{n+i}^{0} \right] L = \frac{Lf}{n} \left(\frac{4n^{2}}{t} \left(v - f \right) - 1 \right)^{1/2}.$$
 (10)

As equilibrium prices decrease, the isolated markets expand and, as they touch, generate a kink in the demand. Let p = h(f) denote the pair of prices at which a consumer is indifferent between good *i*, the outside good, and good $n + i:^{24}$

$$h(f) = f + \frac{t}{2n^2} \left(\frac{4n^2}{t} \left(v - f\right) - 1\right)^{1/2}.$$
(11)

Equilibrium prices (p, f) where p > h(f) fall in the monopoly region and support market gaps. Otherwise, the market is fully covered. The constraint holds with equality in the kinked equilibrium.

The competitive equilibrium occurs when prices are low so that firms compete directly with each other, and the outside good is a dominated option for every consumer (p < h(f)). Let

$$_{l}x_{n+i}^{1} = \frac{1}{2n} - \frac{n}{t}\left(p_{i} - f\right)$$
(12)

²⁴That is, $\{p, f\}$ satisfies ${}_{r}x_{i}^{0} =_{l} x_{n+i}^{0}$.

be the location of the consumer to the left of firm n + i who is indifferent between product iand product n + i. Similarly, the address of the consumer indifferent between product n + iand i + 1 is

$${}_{r}x_{n+i}^{1} = \frac{1}{2n} + \frac{n}{t}\left(p_{i+1} - f\right)$$

The proposition below characterizes the equilibrium of the pricing stage. For completeness, we establish first the profits for the intermediary and firm i in the competitive region:

$$\pi_{c,n+i} = fL\left({}_{r}x_{n+i}^{1} - {}_{l}x_{n+i}^{1}\right)$$
(13)

$$\pi_{c,i} = 2p_i x_{n+i}^1 L_l. \tag{14}$$

Proposition 2 The pricing game of the opaque model with full participation has a unique symmetric equilibrium. Upstream firms and the intermediary post prices

$$(p,f) = \begin{cases} \left(\frac{t}{3n^2}, \frac{t}{6n^2}\right), & \frac{v}{t} > \frac{4}{9n^2} \\ \left(v - \frac{t}{9n^2}, v - \frac{5t}{18n^2}\right), & \frac{v}{t} \in \left[\frac{1}{3n^2}, \frac{4}{9n^2}\right] \\ \left(\frac{2v}{3}, \frac{2v}{3} - \frac{t}{6n^2}\right), & \frac{v}{t} \in \left(\frac{1}{4n^2}, \frac{1}{3n^2}\right) \\ \left(\frac{2v}{3}, 0\right), & \frac{v}{t} \le \frac{1}{4n^2}. \end{cases}$$
(15)

Consumers choose products

$$g\left(\frac{v}{t} \ge \frac{1}{3n^2}\right) = \begin{cases} i \ if \ x \le \frac{1}{3n^2} \\ i + 1if \ x \ge \frac{2}{3n^2} \\ n + i \ otherwise \end{cases} g\left(\frac{v}{t} < \frac{1}{3n^2}\right) = \begin{cases} i \ if \ x \le \ _r x_i^0 \\ i + 1if \ x \ge \ _l x_{i+1}^0 \\ n + i \ if \ x \in \left[_l x_{n+i}^0, r \ x_{n+i}^0 \right] \ and \ \frac{v}{t} \ge \frac{1}{4n^2} \\ 0 \ otherwise \end{cases}$$
$$g' = i \cdot 1_{\{x \le 1/2n\}} + (i+1) \cdot 1_{\{x > 1/2n\}}$$

and bid $p_{n+i} = f$ when using the opaque channel. Firms bid w = 0 in the reverse auction and hold beliefs consistent with (p_{n+i}, g') .

The equilibrium prices in (15) share similar features to those in the benchmark model (6). Note, however, that the introduction of the intermediary increases competition across the board. First, prices are (weakly) lower than in the benchmark model. The intermediary needs to lower prices in order to compensate consumers for their expected transportation cost. This compensation increases with the degree of product differentiation, as reflected by the spread $p - f = t/6n^2$. Upstream prices and opaque goods' prices are strategic complements. That is, a lower price set by the opaque intermediary cannibalizes upstream firms' sales, so those firms react by lowering transparent prices. Naturally, there is no cannibalization in the monopoly equilibrium because the intermediary is selling to an otherwise unserved segment.

Second, the presence of opaque products reduces the range of parameter values for which the equilibrium falls in the kinked and monopoly regions. In other words, the introduction



Figure 4: Competition regions with opaque goods

of opaque products helps to cover the market and therefore introduces competition where there was none. Figure 4 shows that the competitive, kinked, and part of the monopoly regions from the benchmark model (dashed curves) fall inside the competitive region of the opaque model. The figure also shows an additional region where the intermediary does not sell a single unit (darkest area). This happens when $v/t < 1/(4n^2)$. The elasticity of substitution across products is so low that consumers' expected utility from opaque goods becomes negative and there is no demand for good n + i.

The equilibrium quantities sold by upstream firms and the intermediary are obtained using prices from (15) into (9), (12) and (3):

$$(q_i, q_{n+i}) = \begin{cases} \left(\frac{2L}{3n}, \frac{L}{3n}\right), & \frac{v}{t} \ge \frac{1}{3n^2} \\ \left(2L\left(\frac{v}{3t}\right)^{1/2}, \max\left[\frac{L}{n}\left(\frac{4n^2v}{3t} - \frac{1}{3}\right)^{1/2}, 0\right]\right), & \frac{v}{t} < \frac{1}{3n^2}. \end{cases}$$
(16)

The market share for the opaque channel is half the share of transparent goods when the market is fully covered (kinked and competitive outcomes). Under partial coverage, the share decreases, approaching zero as the ratio v/t approaches $1/(4n^2)$. Though upstream firms sell lower quantities than in the benchmark model, the total sales (transparent plus opaque) are higher in the opaque model.

To summarize, compared to the benchmark model, opaque intermediation reduces upstream firms' profits through both prices and quantities. There are two reasons for that. First, it expands the set of parameters that support the competitive outcome. Second, it increases (weakly) the intensity of competition among firms. Note, however, that upstream firms might still prefer a market structure with opaque intermediation if the lower profit from intense competition in the pricing stage is offset with transfer payments from the intermediary in the participation stage.

4 Participation Constraints, Industry Profits and Welfare

We now turn to the first stage of the opaque model, where we consider the entry by an intermediary into a mature (long-run equilibrium) industry. To induce participation in the opaque channel, the intermediary offers a fixed transfer payment to each upstream firm. Alternatively, one can think of the intermediary offering partial ownership to upstream firms in exchange for their participation.^{25,26} In this section, we focus on the symmetric opaque equilibrium with full participation and analyze how the introduction of opaque intermediation affects profits and welfare.

As we have seen in the previous section, a firm that chooses to participate in the opaque channel is committing to aggressive price competition. Notice that in our model, an opaque good cannot exist without the participation of the two upstream firms associated with it. Hence, unlike in the pricing stage, upstream firms enjoy significant bargaining power in the participation stage. We construct an equilibrium where opaque intermediation leads to higher industry profits—that is, an equilibrium where upstream firms strategically allow the intermediary to set up the opaque channel.²⁷

Let the industry profits without and with opaque goods be represented by

$$\Pi^b = n\pi^b \tag{17}$$

$$\Pi = n \left[\pi_i + \pi_{n+i} \right], \tag{18}$$

where π^b corresponds to profits in the benchmark model of Section 2 and *n* is determined by (8). The expected profits for upstream firms and the intermediary in the opaque model are obtained from (15) and (16):

$$\pi_{i} = \begin{cases} \frac{2tL}{9n^{3}}, \frac{v}{t} > \frac{4}{9n^{2}} \\ \left(v - \frac{t}{9n^{2}}\right)\frac{2L}{3n}, \frac{v}{t} \in \left[\frac{1}{3n^{2}}, \frac{4}{9n^{2}}\right] \\ 4tL\left(\frac{v}{3t}\right)^{3/2}, \frac{v}{t} < \frac{1}{3n^{2}} \end{cases} \pi_{n+i} \begin{cases} \frac{tL}{18n^{3}}, \frac{v}{t} > \frac{4}{9n^{2}} \\ \frac{L}{54n}\left(18v - \frac{5t}{n^{2}}\right), \frac{v}{t} \in \left[\frac{1}{3n^{2}}, \frac{4}{9n^{2}}\right] \\ \frac{tL}{2n^{3}}\left(\frac{4vn^{2}}{3t} - \frac{1}{3}\right), \frac{v}{t} \in \left[\frac{1}{4n^{2}}, \frac{1}{3n^{2}}\right] \\ 0, \frac{v}{t} < \frac{1}{4n^{2}}. \end{cases}$$
(19)

From (6) and (15) we can see that a necessary condition for $\Pi \ge \Pi^b$ is that the equilibrium in the benchmark model does not have full market coverage. If it did, the introduction of opaque products would drive overall prices down while the total quantity sold would not

 $^{^{25}}$ This appears to be the approach taken by PCLN in the travel industry. According to Forbes.com, "Delta was the first airline to partner with Priceline when it launched, and in exchange for letting Priceline sell the airline's seats, Delta received a 5% stake in the site" (http://www.forbes.com/2001/02/09/0209priceline.html, accessed July 23, 2012).

²⁶We emphasize that we assume no collusion or pricing coordination between the intermediary and upstream provider.

²⁷Note that there is always an equilibrium where firms choose not to participate. This is because a single deviation is never enough to create an opaque good.

change. Thus, the intermediary can only increase industry profits when the equilibrium in the benchmark model leaves room for it to operate on the extensive margin. We assume this is the case from now on. That is, we assume $\frac{v}{t} < \frac{3}{4n^2}$ so that the equilibrium without opaque goods falls in the monopoly region in Figure 2. As discussed in Section 2, free entry in the benchmark model leads to market gaps if we allow for entry cost to vary with the number of entrants or firm indivisibilities. Another reason could be demand volatility; we leave the analysis of that case for next section.

The intermediary offers a transfer $T \in \{0, \pi_{n+1}\}$ to induce participation. We can see that, assuming full participation, a transfer $T = \pi_{n+1}$ makes the upstream firms better off if $\Pi \ge \Pi^b$. However, we need to consider single-firm deviations from such an equilibrium. Since an opaque good is a lottery over two transparent goods, a firm j that chooses to deviate from full participation makes opaque goods n + j and n + j - 1 disappear. Let π^d represent the profits for the deviating firm. If n = 2, a deviation by one firm leaves the market without opaque products and the profits are exactly those of the benchmark model ($\pi^d = \pi^b$). When n > 2, a deviation by firm j leaves n - 2 opaque markets active. Given that prices in these opaque markets are determined together with the prices for the n transparent products, the equilibrium prices are asymmetric. An explicit solution becomes involved, even when $n = 3.^{28}$ However, we can characterize the pattern of these prices and put bounds on profits for the deviating firm.

Lemma 1 Higher industry profits with opaque goods is a sufficient condition to support full participation.

Full participation is guaranteed because when $\frac{v}{t} < \frac{3}{4n^2}$, the deviating firm can never get more than its profits in the benchmark model. But note that partial market coverage in the benchmark model is a necessary but not sufficient condition for higher industry profits. Introducing opaque intermediation can generate too much competition (cannibalization effect), even when the equilibrium without it leaves unserved segments to cover (market expansion effect). Naturally, industry profits are higher if the opaque equilibrium falls in the monopoly region, because there is no cannibalization. But opaque sales compete with transparent sales if the *new* equilibrium falls in the competitive or kinked regions of Figure 4. The cannibalization effect implies lower market share *and* prices for transparent goods. Thus, there must exist a set of parameters supporting a competitive opaque equilibrium such that the market expansion effect dominates the cannibalization effect.

Proposition 3 Opaque intermediation increases industry profits and welfare when $4vn^2/t \in [1, (25/3)^{1/3}]$.

²⁸Given that opaque and transparent prices are strategic complements, we expect them to decrease with the distance to the deviating firm j.

Together, the lemma and the proposition above establish the sufficient conditions for the intermediary to implement transfer payments that support participation by all firms.²⁹ Put differently, allowing for opaque selling is a profitable equilibrium strategy for upstream firms. The left panel of Figure 5 illustrates the proposition (shaded area), together with the boundaries for each equilibrium region (dashed curves) in each model. For a given number of firms, the introduction of opaque products generates too much price competition if v/t is high. On the other hand, when v/t is too low, consumers' willingness to pay for opaque goods is negative and the opaque market disappears. Similarly, given v/t, profits with opaque intermediation are larger when the number of firms does not take extreme (low or high) values. In other words, opaque selling is profitable for upstream firms when the degree of product differentiation takes intermediate values.³⁰



Figure 5: Competition regions and industry profits with and without opaque intermediation

This can also be seen in the right panel of Figure 5. The profits with opaque intermediation (red) are larger and drop below the profits in the benchmark model (black) when there are too many firms in the market. That is, when the number of firms is such that there is full market coverage in the opaque model (n > n') but partial coverage in the benchmark model $(n < n'^b)$. The figure also shows the industry profits captured by upstream firms' sales (dashed red) in the pricing game of the opaque model. The contribution by the intermediary is large and increases with the number of firms when the equilibrium is not competitive (n < n'). A corollary of Proposition 3 is that opaque intermediation can support industries with high entry costs. More formally, there is no entry in the benchmark model when entry costs are larger than π_m^b in (8). With opaque goods, the intermediary can pay transfers to upstream firms to cover larger entry costs.

Corollary 1 Opaque intermediation supports new products.

Opaque products are inferior goods because, compared to transparent products, they generate excessive "transport cost". That is, the buyer of an opaque good is not matched

²⁹Note that the set of parameter values that support full participation with opaque products is expected to be larger than that in the proposition.

 $^{^{30}}$ Equivalently, when the demand heterogeneity, represented by the range of consumers' reservation prices, is intermediate.

with her preferred product half of the time. Nevertheless, the second part of Proposition 3 establishes that welfare improves—together with industry profits—relative to the benchmark model. Two opposing forces are at play. On the one hand, opaque intermediation increases welfare because it generates additional sales (extensive margin). On the other hand, some of the opaque sales replace what would be transparent sales in the benchmark model and, due to the inefficient transport cost, reduce welfare. The proof of the proposition shows that the former effect dominates the latter.³¹

In this section, we showed that introducing opaque intermediation in a market that is not fully served can be a full participation equilibrium that increases welfare and industry profits. Partial market coverage is a common feature of most markets, and we can expect firms to introduce an opaque intermediary for their own benefit when product differentiation takes intermediate values. A drawback of using a circular city model with homogeneous unit demands is that it only captures some of the possible reasons for partial coverage in real markets. As argued before, one could think of firm indivisibilities or increasing entry costs. In the next section, we consider the role of demand volatility, possibly a more relevant ground underlying partial market coverage.

5 Seasonal Demand

In this section, we provide an example where opaque products are introduced in a market exposed to demand volatility. Based on the results from the previous section, to support full participation we require the opaque channel to operate through the extensive margin and increase industry profits. Market gaps in the free-entry equilibrium of the benchmark model will now arise from seasonal shifts in the demand, and we ignore indivisibilities or variable entry costs. As we will see, this requires that we look at the range of entry costs that support market gaps during low seasons.

Let the demand take two possible states: high with frequency ρ and low with frequency $(1 - \rho)$.³² High and low demand differ in the number of potential consumers $(\overline{L} > \underline{L})$ as well as in their valuations $(\overline{v}/\underline{v} > 2)$.³³ The number of firms \hat{n} in the market is determined from the zero expected profit condition:

$$\pi^{b}\left(\widehat{n}\right) = \rho\overline{\pi}^{b} + (1-\rho)\underline{\pi}^{b} = F,$$
(20)

where $\overline{\pi}^{b}$ and $\underline{\pi}^{b}$ represent the profits in the benchmark model for high and low seasons respectively. From (7), $\overline{\pi}^{b} > \underline{\pi}^{b}$.

³¹This trade-off is only present when parameter values support a monopoly equilibrium in the benchmark model and a competitive or kinked equilibrium in the opaque model.

 $^{^{32}\}mathrm{We}$ use upper and lower bars to relabel variables and parameters accordingly.

³³The assumption regarding valuation is done for tractability purposes.

Figure 6 illustrates the relationship between expected profits (red) and the number of firms in the market when $\rho = 0.5$. The profits in high and low seasons (black) decrease with the number of firms at different rates and converge as the equilibrium in both seasons falls in the competitive region.³⁴ The critical values \overline{n}_m^b and \underline{n}_m^b represent the largest number of firms that support monopoly pricing equilibrium in the high and low season, respectively.³⁵ Entry above \overline{n}' leads to competitive equilibrium in the high season, and profits are determined by the production cost and the degree of product differentiation rather than consumers' valuations.



Figure 6: Expected profits (red) with seasonal demand ($\rho = 0.5$)

Assume that the entry cost takes the largest value supported by the market: $F = \pi^b(\overline{n}_m^b)$. In such a case, firms enter up to the point where the market in the high season is fully covered. As F drops, more entry leads to a competitive equilibrium in the high season $(\widehat{n} > \overline{n}')$ and full market coverage in the low season $(\widehat{n} \ge \underline{n}_m^b)$. Now consider the entry cost F_0 that supports \widehat{n}_0 firms in Figure 6. The equilibrium in the high season is competitive $(\widehat{n}_0 > \overline{n}')$ and a monopoly in the low season $(\widehat{n}_0 < \underline{n}_m^b)$. Such a market outcome satisfies the necessary condition for profitable opaque intermediation: market gaps. A lower entry cost F_1 would generate too much entry and lead to a competitive equilibrium in both seasons. The number of entrants solves $\rho \overline{\pi}_c^b(\widehat{n}) + (1 - \rho) \underline{\pi}_m^b = F$:

$$\widehat{n} = \left[\frac{\rho t \overline{L}}{F - 4(1 - \rho) t \underline{L} \left(\frac{\underline{v}}{3t}\right)^{3/2}}\right]^{1/3}.$$
(21)

The lemma below specifies the range of entry costs that support market gaps: $\overline{n}_m^b < \widehat{n} < \underline{n}_m^b$. Lemma 2 For any $\rho \in (0,1)$, the benchmark model with entry costs $F \in [F^l, F^u]$ has a

³⁴Full convergence with large n occurs only if $\overline{L} = \underline{L}$.

³⁵Using (7), $\overline{n}_m^b = \sqrt{3t/4\overline{v}} < \sqrt{3t/4\underline{v}} = \underline{n}_m^b$.

long-run equilibrium with partial market coverage in low seasons.

$$F^{l} = 4t \left(\frac{\underline{v}}{3t}\right)^{3/2} \left[2\rho \overline{L} + (1-\rho)\underline{L}\right].$$
(22)

$$F^{u} = \left(\frac{16}{27t}\right)^{1/2} \left[\rho \overline{L} \overline{v}^{3/2} + (1-\rho) \underline{L} v^{3/2}\right].$$
(23)

The upper limit for F is the maximum expected profits that a firm can make in this industry $(\rho \overline{\pi}_m^b + (1 - \rho) \underline{\pi}_m^b)$. Firms can cover such a high entry cost as long as consumers' valuations are high and the equilibrium prices lead to no market contact.³⁶ Since $\overline{\pi}^b > \underline{\pi}^b$, the upper bound increases with ρ . At the same time, as ρ increases, the lower bound in (22) rises so that entry is deterred and the monopoly equilibrium in the low season keeps the market partially covered.³⁷

We now turn to the profits after opaque goods have been introduced to the industry. It is natural to think of two alternative scenarios for opaque intermediation: one where opaque products are sold only during the low season, and one where the intermediary operates in both seasons. The relevance of each scenario is likely to depend on institutional characteristics that are not captured by our simple framework. We consider both possibilities.³⁸ The equivalent to industry profits is the profit per market segment:

$$\pi_r\left(\widehat{n}\right) = \rho\overline{\pi}^b + (1-\rho)\left(\underline{\pi}_i + \underline{\pi}_{n+i}\right),\tag{24}$$

when opaque products are allowed in low demand periods only, and

$$\pi\left(\widehat{n}\right) = \rho\left(\overline{\pi}_{i} + \overline{\pi}_{n+i}\right) + (1 - \rho)\left(\underline{\pi}_{i} + \underline{\pi}_{n+i}\right) \tag{25}$$

when the intermediary operates in every season. We compare (24) and (25) with (20), taking into account that the number of firms \hat{n} involves different equilibrium regions in each case and season.

Opaque intermediation is expected to reduce industry profits when demand is high because, absent firm indivisibilities, $\widehat{n} \geq \overline{n}_m^b$. But it can still increase profits when demand is low because $\overline{n}_m^b < \underline{n}_m^b$. Thus, when feasible, opaque intermediation restricted to low demand periods is desired by upstream firms. It allows the industry to avoid the competition between upstream firms and the intermediary in the high season and benefit from the possible gains in the low season. In other words, the range of (F, ρ) for which opaque intermediation increases industry profits is larger when intermediation is restricted to the low season than when it is not. The next proposition establishes the sufficient conditions for the case $\pi_r(\widehat{n}) > \pi^b(\widehat{n})$.

³⁶Note that if the ratio $\overline{v}/\underline{v}$ is too high, the number of entrants is so low that opaque products have negative value to consumers in low seasons and the opaque intermediary does not sell.

³⁷The assumption $\frac{\overline{v}}{\underline{v}} > 2$ implies that, as *n* increases, the shift in the low season equilibrium from monopoly to kinked occurs *after* the shift from kinked to competitive in the high season equilibrium $(\underline{n}_m^b > \overline{n}')$.

³⁸In addition, it matters whether the demand state is random or deterministic.

Proposition 4 Opaque intermediation in the low season only is a profitable strategy for established industries when $F \in [F_r^l, \min\{F^u, F_r^u\}]$

$$F_r^l = \frac{4t}{5} \left(\frac{\underline{v}}{3t}\right)^{3/2} \left[18\rho \overline{L} + 5\left(1-\rho\right)\underline{L}\right]$$
(26)

$$F_r^u = 4t \left(\frac{\underline{v}}{3t}\right)^{3/2} \left[6\sqrt{3}\rho \overline{L} + (1-\rho)\underline{L}\right].$$
⁽²⁷⁾

Figure 7 illustrates the proposition.³⁹ The large empty area represents the parameter space that supports market gaps in the low season, provided by Lemma 2. Opaque intermediation allows upstream firms to expand market coverage in the low season but is not sufficient to increase the industry profits. Thus, the number of firms associated with each entry cost in (21) should satisfy the conditions in Proposition 3 (shaded area).



Figure 7: Conditions for profitable intermediation in low seasons only

The conditions for profitable opaque selling when the intermediary operates in both seasons are harder to satisfy. Instead of finding the set of (ρ, F) such that opaque intermediation increases industry profits, we provide an existence result.

Proposition 5 There exists (ρ, F) such that opaque intermediation in both seasons is preferred to no intermediation.

Formally, Proposition 5 establishes that there exists a pair (ρ, F) supporting an opaque equilibrium with $\hat{n} \in [\overline{n}_m^b, \underline{n}_m^b]$ and $\pi(\hat{n}) > \pi^b(\hat{n})$. The expected profit function in (25) is a weighted average of the profits in high and low seasons and therefore increases in ρ . Note that, for each season, industry profits with opaque goods increase with n, peak above the benchmark profits at n', and decrease when n takes larger values (see Figure 5b). Depending on ρ 's value, the expected profits resemble the high or low season and thus peak near $\overline{n'}$ or $\underline{n'}$. The proof of the proposition looks at the expected profits with \hat{n} firms such that: (i) \hat{n} implies full market coverage in high season and partial coverage in low season (Lemma 2), and (ii) expected profits are larger with $\underline{n'}$ than $\overline{n'}$. Figure 8 shows an example.

³⁹The parameter values used in the figure are: $\overline{v}/\underline{v} = 5$, $\overline{L}/\underline{L} = 2$ and t = 4. $F^u < F^u_r$ when $\overline{v}/\underline{v} < 4.7622$.



Figure 8: Opaque (red) and transparent (black) industry profits with seasonal demand $(\rho = 2.5\%)$

6 Conclusions

We developed a model of opaque selling and showed that incumbent firms in a given industry can strategically use an opaque intermediary as a facilitating device to engage in price discrimination. Opaque products, by definition, involve differentiated goods. We formally study opaque intermediation with a simple, generic two-stage model where products differ in only one dimension and consumer heterogeneity is limited to the address in the product space. The model generates several insights on opaque intermediation. It makes price competition between upstream firms and the intermediary more intense. The opaque channel cannibalizes transparent sales but also expands market coverage. Higher industry profits in the pricing stage are enough to guarantee an equilibrium with full participation by incumbent firms. Moreover, the opaque channel increases welfare, since the inefficiencies that characterize the consumption of opaque products are outweighed by the gains through the extensive margin.

The model chosen in this paper does not pretend to simulate any industry in particular. On the contrary, the objective is to present a simple framework to think about opaque products. Its simplicity comes at the expense of ignoring some features that could be highly relevant in specific industries. Among the assumptions currently debated in the literature are richer consumer types, capacity constraints, opacity level, and competition among intermediaries.⁴⁰ We conjecture that our model allows us to think about the effects of each one on the strategic vertical market structure result. Intuitively, loyal customers, capacity constrained firms, and competition in intermediation increase the bargaining power of upstream firms in the pricing stage. That is, the reverse auction would generate outcomes away from the Bertrand trap and make the participation constraints by upstream firms easier to satisfy. On the other hand, allowing for opaque products that involve more than two transparent goods can change the bargaining power and can then create a prisoner dilemma situation where the industry is worse off with opaque goods than without them.

 $^{^{40}}$ See the related literature description in the Introduction.

We leave the formal analysis for future research.

7 APPENDIX

Proof of Proposition 2. We derive first the equilibrium posted prices in (15). Let $(p^j, f^j), j \in \{m, k, c\}$ denote the monopoly, kinked and competitive equilibrium prices. The monopoly prices for transparent goods $i \leq n$ are the same as those in Proposition 1. The intermediary fee f^m is obtained directly from the first order conditions of (10). The bounds on the parameter space that supports the monopoly equilibrium arise from the constraints $f^m \geq 0$ and $p^m > h(f^m)$ in 11. Note that the upper bound guarantees ${}_l x_{n+i}^0 - {}_r x_i^0 > 0$ when evaluated at the equilibrium prices.

The competitive equilibrium prices are obtained from maximizing (13) and (14). The reaction functions are $p_i = \frac{t}{4n^2} + \frac{f}{2}$ and $f = \frac{p_i + p_{i+1}}{4}$. Using $p_i = p_{i+1} = p$ and solving for (p, f) leads to $\{p^c, f^c\}$ in (15). The parameter domain is obtained from $p^c < h(f^c)$. Last, the kinked equilibrium prices are obtained from

$${}_{r}x_{i}^{0}(p^{k}) = {}_{l}x_{n+i}^{1}(p^{c}, f^{c}) = 1/(3n)$$

$${}_{l}x_{n+i}^{0}(f^{k}) = {}_{l}x_{n+i}^{1}(p^{c}, f^{c}) = 1/(3n).$$

Kinked prices converge to the competitive or monopoly outcomes as v/t approaches the corresponding parameter bounds.

The text before Proposition 2 discusses the unique bidding equilibrium in the reverse auction and relates it to the Bertrand Paradox. Any firm bidding above the equilibrium strategy w = 0 loses the auction with probability one, regardless of the beliefs about consumers' types (p_{n+i}, g') . Any strategy that assigns positive probability to bids above marginal costs cannot be an equilibrium, because a firm can reduce this probability by epsilon and increase its expected profits. The equilibrium in the reverse auction makes p_{n+i} a (weakly) dominant strategy for consumers, and their choice between the transparent and opaque market is straightforward, given (15). *Q.E.D.*

Proof of Lemma 1. We need to show that $\pi_i + T = \pi_i + \pi_{n+i} \ge \pi^d$ when $\Pi \ge \Pi^b$ and all (n > 2) firms participate in the opaque market. Under full participation, $\Pi \ge \Pi^b$ implies $\pi_i + \pi_{n+i} \ge \pi^b$. Therefore, it is sufficient to show that $\pi^d \le \pi^b$. Consider the deviation by firm j and note that, from the pricing first-order conditions, all prices go up after removing the opaque goods n+j+1 and n+j-1. In fact, the price for good j increases more than the prices for j+1 and j-1. Since $v/t < 1/3n^2$ (no market contact in the benchmark model) is a necessary condition for $\Pi \ge \Pi^b$, prices can potentially reach monopoly levels. There are two possible outcomes regarding market contact between j, j+1 and j-1: (i) partial market coverage (monopoly) and (ii) full market coverage (kinked and competitive). A deviation in the first case leads directly to $\pi^d = \pi^b = \pi_i$ because the pricing by firm j after deviation is not affected by the prices of competitors (opaque or transparent). In case (ii), eliminating

goods n+j+1 and n-j+1 alters the pricing strategies for firm j. Given that there is market contact, prices do not reach the monopoly level. Since prices are strategic complements, firm j cannot increase its price as much as it would in a market without opaque goods and makes $\pi^d < \pi^b$. Even if it did, the lower prices of firms j + 1 and j - 1 make the quantities sold by j lower than in the benchmark model and $\pi^d < \pi^b$. Q.E.D.

Proof of Proposition 3. We first show the set of parameter values that support higher industry profits. The welfare result follows. Note first that Lemma 1 allows us to ignore participation constraints if opaque goods increase industry profits. The mapping between v/t and the opaque equilibrium (monopoly, kinked and competitive) is established in Proposition 2. The lower bound in Proposition 3 is determined by the fact that the intermediary does not enter a market when $v/t \leq 1/(4n^2)$. To obtain the upper bound on v/t we compare (17) with (18), conditional on the benchmark model equilibrium being a monopoly $(v/t < 3/(4n^2))$. Parameter values $v/t \in [1/(4n^2), 1/(3n^2)]$ correspond to monopoly opaque equilibrium and $\Pi > \Pi^b$, since $\pi_{n+i} > 0$ and $\pi_i = \pi^b$ in (18). When $v/t \in [1/(3n^2), 4/(9n^2)]$, the equilibrium with opaque products falls in the kinked equilibrium region. Using (19) in Π , the difference in industry profits is

$$\Delta_k(v/t) = \Pi - \Pi^b = Lt \left[\frac{v}{t} - \frac{1}{6n^2} - 4n \left(\frac{v}{3t} \right)^{3/2} \right],$$

and increases with v/t for $\frac{v}{t} < \frac{3}{4n^2}$. Additionally, $\Delta_k > 0$ since $\Delta_k(4/9n^2)$ and $\Delta_k(1/3n^2)$ are positive. Last, we need to show the effects on industry profits when the new equilibrium falls in the competitive region: $v/t > 4/(9n^2)$. As before, using (19), the difference in industry profits becomes

$$\Delta_c(v/t) = \Pi - \Pi^b = \frac{5Lt}{18n^2} - 4ntL\left(\frac{v}{3t}\right)^{3/2}$$

and $\Delta_c(v/t) > 0$ as long as $v/t < \frac{1}{4n^2} \left(\frac{25}{3}\right)^{1/3}$.

To show the welfare result, we calculate welfare under the benchmark and opaque models for parameter values that satisfy $4vn^2/t \in [1, (25/3)^{1/3}]$. The equilibrium in the benchmark model falls in the monopoly region for all parameter values but not in the opaque model. Using equations (2), (6) and (15), the welfare under monopoly equilibrium in the benchmark and opaque models is given by

$$W_m^b = 2Ln \int_0^{(v/3t)^{1/2}} (v - tx^2) dx = \frac{16}{9} nvL \left(\frac{v}{3t}\right)^{1/2}$$
$$W_m = W_m^b + 2Ln \int_{1/(2n)}^{rx_{n+i}^o} v - \frac{t}{2} \left(x^2 + \left(\frac{1}{n} - x\right)^2\right) dx,$$

where $_{r}x_{n+i}^{o} = \frac{1}{2n} \left[1 + \left(\frac{4vn^{2}}{3t} - \frac{1}{3} \right)^{1/2} \right]$. Thus, the welfare change when $\frac{v}{t} \in \left[\frac{1}{4n^{2}}, \frac{1}{3n^{2}} \right]$ is $W_{m} - W_{m}^{b} = \frac{2L \left[4vn^{2} - t \right]^{3/2}}{9n^{2} \left(3t \right)^{1/2}} > 0.$

The opaque equilibrium for $\frac{v}{t} \ge \frac{1}{3n^2}$ is kinked or competitive. The welfare is given by

$$W_c = 2Ln\left[\int_0^{1/(3n)} \left(v - tx^2\right) dx + \int_{1/(2n)}^{2/(3n)} v - \frac{t}{2}\left(x^2 + \left(\frac{1}{n} - x\right)^2\right) dx\right] = L\left(v - \frac{t}{9n^2}\right)$$

and the welfare change becomes

$$W_c - W_m^b = Lv \left[1 - \frac{16n}{9} \left(\frac{v}{3t} \right)^{1/2} - \frac{t}{9vn^2} \right].$$

This expression increases with v/t if $\frac{v}{t} < \frac{1}{4n^2} \left(\frac{25}{3}\right)^{\frac{1}{3}}$. Thus, $W_c - W_m^b \left(\frac{v}{t} = \frac{1}{3n^2}\right) = \frac{2Lv}{27} > 0$ guarantees $W_c - W_m^b > 0$. Q.E.D.

Proof of Lemma 2. To find the upper and lower bounds for the entry cost, we look for the largest number of firms that support a monopoly equilibrium in high and low seasons. We start with the upper bound. From (8), the largest (smallest) number of firms that support a monopoly (kinked) equilibrium in the high season is $\overline{n}_m^b = L/(2_r x_i^0) = (\frac{3t}{4\overline{v}})^{1/2}$. Since $\overline{v} > \underline{v}$, $\overline{n}_m^b > \underline{n}_m^b$ and $\widehat{n} = \overline{n}_m^b$ guarantees a monopoly outcome in the low season. Using (7),

$$F^{u} = \pi^{b} \left(\overline{n}_{m}^{b}\right) = \rho \overline{\pi}_{m}^{b} + (1-\rho) \underline{\pi}_{m}^{b} = \left(\frac{16}{27t}\right)^{1/2} \left[\rho \overline{L} \overline{v}^{3/2} + (1-\rho) \underline{L} \underline{v}^{3/2}\right].$$

The lower bound is obtained by evaluating (20), where \hat{n} takes the largest value that supports a monopoly outcome in the low season: $\hat{n} = \underline{n}_m^b = \left(\frac{3t}{4\underline{v}}\right)^{1/2}$. Two cases need to be considered, depending on whether \underline{n}_m^b implies a competitive or a kinked equilibrium in the high season. From (8), the competitive outcome occurs when $\underline{n}_m^b > \left(\frac{5t}{4\overline{v}}\right)^{1/2}$. If $\frac{\overline{v}}{\underline{v}} > 5/3$, the equilibrium in the high season is competitive. Since we assumed $\frac{\overline{v}}{\underline{v}} > 2$, the lower bound is

$$F^{l} = \Pi^{b}\left(\underline{\underline{n}}_{m}^{b}\right) = \rho \overline{\pi}_{c}^{b} + (1-\rho) \underline{\underline{\pi}}_{m}^{b} = 4t \left(\underline{\underline{v}}_{\overline{t}}\right)^{3/2} \left[2\rho \overline{L} + (1-\rho) \underline{L}\right].$$

Alternatively, we can use (21) and solve for F when $\widehat{n} = \overline{n}_m^b$ and $\widehat{n} = \underline{n}_m^b$. Q.E.D.

Proof of Proposition 4. From (24) and (20), $\pi_r(\widehat{n}) > \pi^b(\widehat{n})$ if $\underline{\pi}_i + \underline{\pi}_{n+i} > \underline{\pi}^b$. Proposition 3 established the set of parameter values for which opaque intermediation is an equilibrium and increases industry profits. The bounds are obtained by replacing n with \widehat{n} in Proposition 3. In addition, the conditions on Lemma 2 need to be satisfied. $F_r^l \ge F^l$ is always true but $F_r^u \le F^u$ depends on the relative size of $\frac{\overline{v}}{v}$. Q.E.D.

Proof of Proposition 5. To show that $\pi(\widehat{n}) > \pi^b(\widehat{n})$ with $\widehat{n} \in [\overline{n}_m^b, \underline{n}_m^b]$ is possible, we find

 ρ and F that support \underline{n}' such that a) $\underline{n}' = \sqrt{4t/9\underline{v}} > \sqrt{3t/4\overline{v}} = \overline{n}_m^b$ and b) $\pi(\underline{n}') > \pi^b(\underline{n}')$. The first condition is satisfied when $\overline{v}/\underline{v} \ge 2$. To check the second condition, we first need to calculate the profits in each model and season, considering the equilibrium type associated with \underline{n}' . The profits with opaque intermediation are straightforward:

$$\pi(\underline{n}') = \rho \overline{\pi}_c(\underline{n}') + (1-\rho) \underline{\pi}_k(\underline{n}') = \frac{15\underline{v}^{3/2}}{16t^{1/2}} \left[\rho \overline{L} + (1-\rho) \underline{L}\right]$$

By definition of \underline{n}' , $\underline{\pi}_k(\underline{n}') = \underline{\pi}_c(\underline{n}')$. Also, since $\underline{n}' > \overline{n}'$, the equilibrium when demand is high is competitive. The second equality is obtained by replacing $\underline{n}' = \sqrt{4t/9\underline{v}}$ in (15) and simplifying.

To calculate $\pi^b(\underline{n}')$, we know that $\underline{n}' < \underline{n}_m^b$ but need to establish first whether $\underline{n}' \leq \overline{n}'^b$, that is, whether the equilibrium type in the high season of the benchmark model is competitive or kinked. The former case occurs when $\overline{v}/\underline{v} > 45/16$.

Case 1 $(\overline{v}/\underline{v} > 45/16)$: $\underline{n}' > \overline{n}'^{b}$.

$$\pi^{b}\left(\underline{n}'\right) = \rho \overline{\pi}_{c}^{b}\left(\underline{n}'\right) + (1-\rho) \underline{\pi}_{m}^{b} = \frac{\underline{v}^{3/2}}{72t^{1/2}} \left[243\rho \overline{L} + 32\sqrt{3}\left(1-\rho\right)\underline{L}\right].$$

We can see that $\pi(\underline{n}') > \pi^b(\underline{n}')$ as long as $\rho < \widehat{\rho}_1$:

$$\widehat{\rho}_1 = \frac{\left(135 - 64\sqrt{3}\right)\underline{L}}{351\overline{L} + \left(135 - 64\sqrt{3}\right)\underline{L}}$$

Case 2 ($2 \leq \overline{v}/\underline{v} < 45/16$): $\underline{n}' < \overline{n}'^b$.

$$\pi^{b}\left(\underline{n}'\right) = \rho \overline{\pi}_{c}^{b}\left(\underline{n}'\right) + (1-\rho) \underline{\pi}_{k}^{b}\left(n'\right) = \frac{1}{288} \left(\frac{\underline{v}}{t}\right)^{1/2} \left[27\rho \left(16\overline{v} - 9\underline{v}\right)\overline{L} + 128\sqrt{3}\underline{v}\left(1-\rho\right)\underline{L}\right].$$

and $\pi(\underline{n}') > \pi^b(\underline{n}')$ as long as $\rho < \widehat{\rho}_2$

$$\widehat{\rho}_{2} = \frac{2\underline{v}\left(135 - 64\sqrt{3}\right)\underline{L}}{\overline{L}\left(432\overline{v} - 513\underline{v}\right) + 2\underline{v}\left(135 - 64\sqrt{3}\right)\underline{L}}$$

From Lemma 2, there is a range of entry costs for $\rho < \widehat{\rho}_i$ that supports entry $\widehat{n} \in [\overline{n}_m^b, \underline{n}_m^b]$ in the benchmark model. Thus, there exists (F, ρ) that supports $\widehat{n} = \underline{n}'$ and so $\pi(\widehat{n}) > \pi^b(\widehat{n})$. *Q.E.D.*

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