

# Unemployment and Rationality\*

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## I. Introduction

The explanation of persistent excess labor supply is a challenge to macro-economic theorists. In a purely competitive framework, it may be argued that unemployment is transitory since wages will react if excess supply is persistent enough for the economy to be driven towards full employment. Nevertheless, classical tatonement models do not support this view since excess demand functions are arbitrary, see Shafer and Sonnenschein (1982), and convergence to the Walrasian equilibrium cannot be guaranteed.

Fixed-price theory accounts for unemployment at the cost of total indeterminacy of equilibrium allocations. Many efforts have been made in order to narrow down this set. Roughly speaking, these attempts can be classified into two groups: (i) the conjectural approach, cf. Benassy (1976), Negishi (1974), Grandmont and Laroque (1976) and Hahn (1977, 1978); (ii) the game theoretical approach, cf. Hart (1982), Madden (1983) and Silvestre (1988). In this paper we want to contribute to the first line of thinking, by applying the rationality notion developed for a conjectural equilibrium model in Trujillo (1985) to a standard three-goods fixed price model. Both kinds of equilibria are equivalent; see John (1985).

The motivation behind our approach is that when agents set prices they have to respect the rules of the mechanism. For instance, in Hahn's rationality approach, agents may change prices even if they are not constrained. However, the assumed mechanism is such that this behavior is not allowed (see below). Therefore agents must be cheating, i.e., Hahn's notion is one of incentive compatibility and it is not surprising that he obtained negative results. In our approach, agents may change prices if they are conjecturable in the following sense: if buyers are rationed they can increase — conjecture higher — prices and if sellers are constrained

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they can decrease — conjecture lower — prices. Such a notion of price change can be traced back to Arrow (1959) and it was also the one used by Hahn. According to the above ideas, we define a fixed-price equilibrium as rational if no agent is able to conjecture another fixed-price equilibrium where his utility is strictly greater.

The main result of this paper is that under some assumptions, classical unemployment cannot be the outcome of a rational fixed-price equilibrium. Moreover, part of the so-called Keynesian zone is a rational fixed-price equilibrium. In our model, there may be a large excess supply or demand in a market but agents do not change prices because the direction of the price change signaled by the market runs against their interests. Therefore our results do not support the view that fixed-price theories apply only in a neighbourhood of Walrasian equilibria; see Malinvaud (1980).

The model presented here is extremely simple, but it suggests that the rationality approach may be a fertile way to narrow down the class of potential equilibria and, in our case, may provide some understanding of unemployment.

## II. The Model

Our model is similar to the one used by Hart (1982), Madden (1983) and Silvestre (1988).

There are three goods: output, money and labor with prices  $(p, 1, w)$ , respectively, and two consumers, a worker and a shareholder. Both consumers have identical utility functions defined on consumption of the output and holdings of money  $V(Y, M)$ . This function is assumed to be homothetic, continuously differentiable, strictly quasi-concave and strictly increasing. The worker is initially endowed with labor  $(L_w)$  and money  $(M_w)$ , while the shareholder has money  $(M_s)$  and the right to the profits of the unique firm that produces the consumption good with a production function  $Y = L^a$ ,  $0 < a < 1$ , where  $L$  is labor. The firm is assumed to be a profit maximizer. Rationing among consumers is assumed to be proportional to their demands.

It is well known that fixed-price equilibria generate three regions in the  $(p, w)$  space. To each nonnegative pair of prices correspond a unique level of output (since agents have identical and homothetic preferences) and, given the rationing scheme, a unique allocation among consumers. Then for each agent  $i = w, s$  we can define an indirect utility function  $U_i = U_i(p, w)$ .

Figure 1 represents the three zones: classical ( $C$ ) with excess demand for goods and excess labor supply, repressed inflation ( $I$ ) with excess demand in both markets, and Keynesian ( $K$ ) with excess supply in both

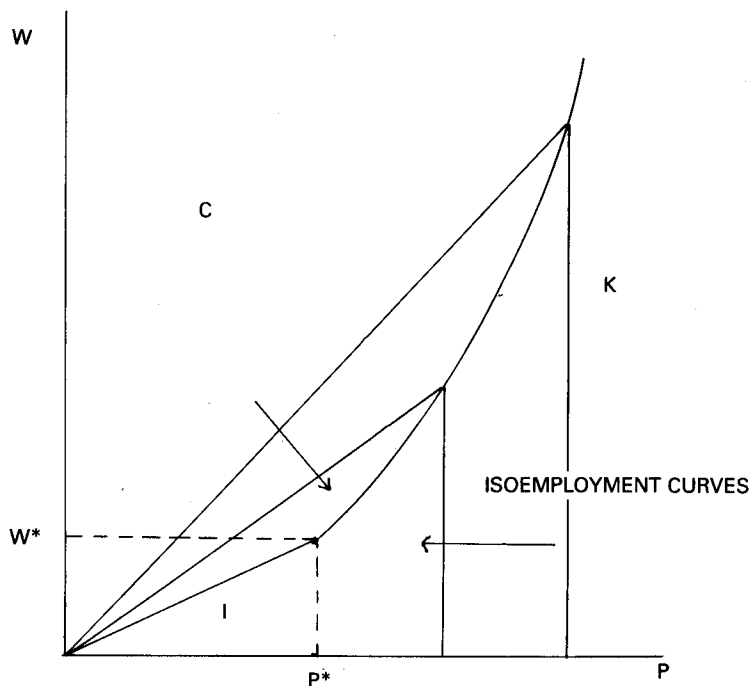


Fig. 1.

markets. The vertical dotted line represents the zone of excess supply of output and excess demand for labor. The point  $(p^*, w^*)$  is the Walrasian equilibrium.

In the Keynesian zone, consumers are unconstrained in the output market. Hence final allocation of output and money corresponds to the individual maximization of utility given prices and income. Output demand determines production, which in turn determines employment, causing excess supply in both markets.

In the classical zone, the firm is unconstrained but demand for labor leaves an excess supply in the market. Moreover, the income perceived by consumers produces excess demand for output at current prices.

In the repressed inflation zone, the firm is constrained in the labor market and there is an excess demand for output at current prices.

In Figure 1 we have drawn the isoemployment curves. In the classical zone, a unique level of employment corresponds to each real wage since the firm is unconstrained there. In the Keynesian zone, the isoemployment curves are vertical because the output price determines demand, which in

turn determines employment. Arrows indicate the direction of increasing employment.

### III. Rational Fixed-Price Equilibria

We now consider the following question: which prices can be considered rational? Since we want to avoid incentive problems, i.e., we do not want agents to engage in manipulative behavior, we first specify the rules under which agents are allowed to change prices (see Definition 1 below) and afterwards define which kind of changes can be considered rational (see Definitions 2 and 3 below).

**Definition 1.** *Prices  $(p', w')$  are conjecturable for agent  $i$  given status quo prices  $(p, w)$  if the following rules are respected. (a) If agent  $i$  is a constrained buyer (resp. seller) of output in the fixed-price equilibrium at status quo prices then  $p' \geq p$  (resp.  $p' \leq p$ ). (b) If agent  $i$  is a constrained buyer (resp. seller) of labor in the fixed-price equilibrium at status quo prices then  $w' \geq w$  (resp.  $w' \leq w$ ). (c) If agent  $i$  is unconstrained in the labor (resp. output) market at status quo prices then  $w' = w$  (resp.  $p' = p$ ).*

The idea behind Definition 1 is that agents are price-takers only when they are unconstrained. If an agent is constrained in some market he can move the corresponding price in the direction suggested by traditional tatonnement models. Such behavior has a strong competitive flavor and is consequently a natural assumption if the economy is large. However, we assume only two agents. In the final section we comment on how the introduction of several agents may affect our results.

Figure 2 shows the direction of conjecturable prices and the agent to whom the conjecture corresponds ( $s$  = shareholder,  $w$  = worker) for each possible status quo.

Next we define rational prices for agent  $i$  as those which maximize her utility for all conjecturable prices.

**Definition 2.** *A pair of prices  $(p, w)$  is rational for agent  $i$  if  $U_i(p, w) \geq U_i(p', w')$  for all conjecturable  $(p', w')$  from the status quo  $(p, w)$ .*

We now define our main equilibrium concept.

**Definition 3.** *A pair of prices  $(p^0, w^0)$  is a Rational Fixed-Price Equilibrium (R.F.P.E.) if they are rational for all agents.*

Note that Definition 3 implies that Walrasian prices  $(p^*, w^*)$  are a R.F.P.E. since, at the Walrasian allocation, no agent is constrained and therefore there are no conjecturable prices other than  $(p^*, w^*)$ .

A possible criticism of the above definition is that the concept of a fixed-price equilibrium implies profit maximization. However, the

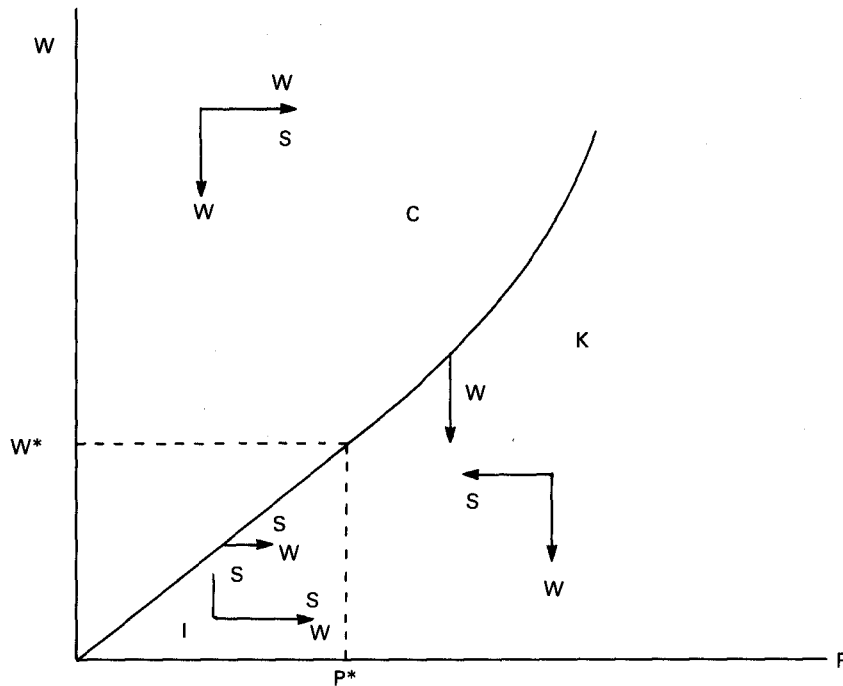


Fig. 2.

behavior of the shareholder assumed above does not necessarily lead to profit maximization. For instance, in the  $K$  zone, a constrained firm should reduce  $p$  if and only if profits are increased and not if the utility of the shareholder is greater. However if the economy is large in an appropriate sense — as in Hart (1982) — both concepts can be expected to give approximately the same outcome. Again the reader is referred to the final section where a sketch of the model with many agents is offered.

#### IV. Results

Let  $C^0$ ,  $K^0$  and  $I^0$  be the interior of  $C$ ,  $K$  and  $I$ , respectively.

**Proposition 1.** *If  $(p, w) \in C^0$  they are not a R.F.P.E.*

*Proof:* Since the worker is constrained,  $(p, w')$ ,  $w' < w$  is a conjecturable pair of prices. Moreover, the utility of the worker is higher when the wage is  $w'$  because the elasticity of demand for labor is greater than one and  $p$  is constant. Hence her real income increases, as does her consumption demand. Since rationing is proportional to demand, a revealed preference

argument can be used to show that she is now better off since both her ration and her real income have increased.

**Proposition 2.** *If  $(p, w) \in C \cap K$  they are a R.F.P.E.*

*Proof:* In this zone, the worker is the unique rationed agent. However, a decrease in  $w$ , holding  $p$  constant, does not affect employment, as is shown in Figure 1. Hence her real income decreases and so does her utility.

In order to prove the next Proposition, we define a reaction correspondence for the shareholder.

**Definition 4.** *The reaction correspondence of the shareholder denoted  $r(w)$  is defined as  $r(w) = \{p > 0 : U_s(p, w) \geq U_s(p', w) \text{ for all } p' > 0\}$ .*

In words,  $r(w)$  is the set of prices which, for given  $w$ , maximize the indirect utility function of the shareholder. It can be proved that  $r(w)$  lies in  $K$ ; see Silvestre (1988). In Figure 3 we have represented  $r(w)$  in the case where it is single valued.

Let  $r'(w)$  denote the left boundary of  $r(w)$ , i.e.,  $r'(w) = \inf r(w)$  for given  $w$ .

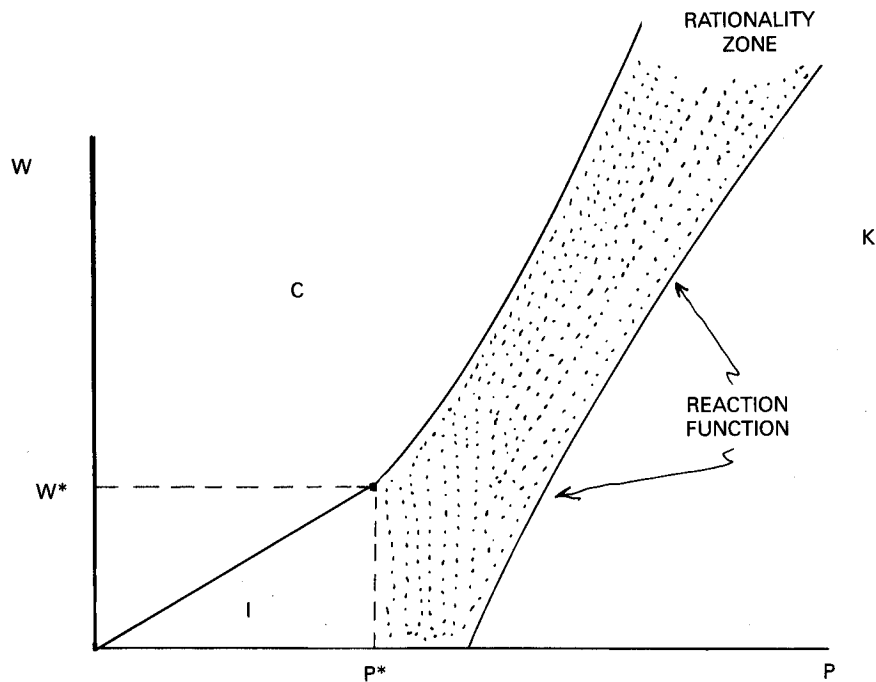


Fig. 3.

**Proposition 3.** *If  $(p, w) \in K$  and  $p > r'(w)$ , but  $p \notin r(w)$ , then  $(p, w)$  are not a R.F.P.E.*

*Proof:* Since the shareholder is a constrained seller of output,  $r'(w)$  is a conjecturable price which by definition gives her greater utility.

We also have the following:

**Proposition 4.** *If  $(p, w)$  is such that  $p \in r(w)$ , then they are a R.F.P.E.*

*Proof:* First note that in the above situation  $(p, w)$  are in the  $K$  zone and that, by definition,  $p$  maximizes  $U_s(p, w)$  for given  $w$ . Therefore no price change can be profitable. The worker is in the same situation described in Proposition 2.

We now study the zone enclosed between  $r'(w)$  and  $K \cap C$ .

**Proposition 5.** *If  $(p, w) \in K$  with  $p < r'(w)$  and  $U_s(p, w)$  strictly quasi-concave in  $p$ , then  $(p, w)$  is a R.F.P.E.*

*Proof:* In this case there are no conjecturable  $p$  which give the shareholder a greater utility since by decreasing  $p$ , the quasi-concavity assumption implies that her utility will not increase. The worker is in the same situation described in Proposition 2.

In Figure 3, the dotted area corresponds to R.F.P.E. in the case where the quasi-concavity assumption holds.

This Proposition relies on a nonprimitive assumption on the shape of  $U_s(\cdot)$  but it is a useful starting point for discussing the case where quasi-concavity does not hold. In this case the area of R.F.P.E. will no longer be solid and the exact shape cannot be predicted. However it is possible to show that if  $U_s(\cdot)$  is continuous in  $p$ , there must be R.F.P.E. arbitrarily close to the left of  $r'(w)$  since  $U_s(\cdot)$  must be decreasing on  $p$  there. Also, if  $p'$  is a local maximum of  $U_s(p, w')$  with  $(p', w') \in K$  and  $p'$  is less than any other local maximum of  $U_s(\cdot)$ , then a simple graphical argument shows that any  $(p^0, w') \in K$  with  $p^0 \leq p'$  must be a R.F.P.E.

Propositions 1–5 above can be summarized as follows:

**Theorem 1.** *In the model described above, unemployment can be generated by a R.F.P.E. only if the corresponding prices belong to  $K$  (including its boundary with  $C$ ). Moreover some prices in  $K$  (including the whole boundary of  $K$  and  $C$ ) are indeed R.F.P.E.*

Finally, note that no point in  $I$  (including its boundary with  $C$ ) can be a R.F.P.E. since in this zone, the shareholder is always interested in a higher price level, and this new price is conjecturable.

## V. Discussion

We end our paper with some comments on the nature of our results.

(i) In our model, classical unemployment is not found to be rational because a decrease in nominal wages benefits workers. This result, however, depends on the specific form of the production function. It is not difficult to produce examples, using alternative production functions, where workers are not interested in a reduction in nominal wages in  $C$ .

However, there is an alternative reason for the nonrationality of  $C$ . As Silvestre (1988, p. 175) has shown, shareholder's utility is increasing in prices in  $C$ . Since higher prices are conjecturable there, he will be interested in bidding prices up. Therefore no pair of prices in  $C$  can be rational.

(ii) If we had a large number of agents and firms, under a wide class of assumptions concerning competition in the output market, we would expect  $r'(w)$  to be very close to  $K \cap C$ , i.e., firms will approximately be nonrationed and the output market will (approximately) balance. Therefore, we are left with the Walrasian equilibrium,  $K \cap C$  and, possibly, some allocations in the lower boundary of  $K$  with  $w=0$  as unique candidates for R.F.P.E. ( $K \cap C$  corresponds to the usual presentation of Keynes' ideas, and  $w=0$  represents some kind of classical unemployment.)

In the labor market, everything depends on how individual rationing varies with prices, i.e., on the kind of contracts we have there. Under Bertrand-like contracts, i.e., workers sell their labor at some predetermined price, competition among workers will drive wages down, so no point in  $K \cap C$  can be a R.F.P.E. (This is in the same spirit as Benassy (1986), where only the Walrasian allocation is left as an equilibrium.)

However, we may have Cournot-like contracts in the labor market, i.e., employed workers might allow their own wages to fluctuate in order to keep their jobs. Intuition suggests that this is a plausible possibility since employed workers would prefer a cut in their wages (under certain limits) rather than the dole. In this case no wage cut can be profitable for an unemployed worker since labor's demand is totally inelastic and employed workers will accept the new wage. Therefore, any point in  $K \cap C$  will be a R.F.P.E. In any case further research is required here, perhaps incorporating ideas from principal-agent models (but with endogenous reservation utility).

(iii) Finally, we compare our results with those obtained by Silvestre (1988) in a cooperative framework. He finds that for a family of CES utility functions with sufficiently small elasticity of substitution, there are Pareto-undominated allocations on the boundary of  $K \cap C$  (see his Theorem 5). Moreover all pairs of prices in the interior of  $C$  and  $K$  are Pareto dominated. Our results point out that those allocations in the boundary of  $K \cap C$  will persist as an equilibrium in the case where agents may set prices in a noncooperative way.

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