Measuring Freedom in Games
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Abstract

The paper provides freedom measures for game theoretic settings. Three core philosophical concepts of freedom are measured: positive, negative, and republican liberty. The measures solve the problem of measuring freedom in situations where agents interact. To illustrate the measure, example models of discrimination and optimal taxation are examined. The optimal government size and tax progression are strictly decreasing in the weight a policy maker attaches to freedom.

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The foundational importance of freedom may well be the most far-reaching substantive problem neglected in standard economics. (Sen, 1988)

1 Introduction

Presumably the largest difference in normative reasoning between an economist's model and a layperson's appraisal is the value of freedom. In standard economic models, the value of freedom is purely instrumental for utility satisfaction. However, philosophers have stressed the intrinsic importance of freedom (e.g. Berlin, 1958). Therefore, Sen (1988) has argued for the inclusion of freedom into economic analysis. To this end, the freedom of choice literature,[1] following the seminal contributions of Pattanaik and Xu (1990) and Jones and Sugden (1982), has attempted to provide measures which can be used to determine the freedom offered by an opportunity set. These contributions greatly enhanced our conceptual understanding of freedom but Pattanaik (1994) showed that these measures encountered problems when being applied to situations in which agents interact. The difficulty arises because in situations in which agents interact, opportunity sets from which agents can freely choose are no longer clearly defined: The choice of one agent may influence the available opportunities of another agent and vice versa. This problem has prevented the literature to provide measures even for a simple exchange economy as Pattanaik (1994) showed. Yet, it is exactly these cases when agents depend on each other to achieve their goals, when they exhibit power over each other, or when they are coerced by others that the measurement of freedom becomes interesting. The lack of freedom measures for situations where agents interact therefore creates an undesirable wedge between the normative analysis performed by economists and normative perceptions outside economic theory.

Also from a positive perspective, a microeconomic measure of freedom for interacting agents is desirable. In the macroeconomic literature on the relation of growth and freedom (e.g. Easton & Walker, 1997; de Haan & Sturm, 2000), proxies have been used such as the size of the government, price stability, or the security of property rights (Gwartney & Lawson, 2003; Gwartney, Hall, & Lawson, 2010). The development of such indices of economic freedom for cross-country comparisons and the contemporaneous development of microeconomic measures in the freedom of choice literature have been remarkably disconnected from each other. A microeconomic measure of freedom which can be applied in economic models may help bridge the gap between the economic freedom indices and the freedom of choice literature by providing microfoundations for the former.

The present paper attempts to provide a measure of freedom for interactive situations and therefore breaks with the opportunity-set based approach in favor of a game theoretic setting. The goal is to maintain the intuitions gained by the opportunity-set based measures from the freedom of choice literature and generalize them to interactive situations.

[1] For surveys of the literature, see Barberà, Bossert, and Pattanaik (2004), Baujard (2007), or Dowding and van Hees (2009).

The main issue when measuring freedom in interactive situations is the imperfect control agents have over the outcomes, which a measure must accommodate. Additionally to the number of different outcomes and their value, the degree to which an agent controls each outcome becomes relevant. It makes a difference whether an individual brings about an outcome by his own actions or whether the outcome is determined by the actions of another person. The idea of the measure is therefore that the better an agent can influence which outcome occurs, the larger the freedom. Meanwhile, it maintains the idea from the freedom of choice literature that freedom increases in the number of outcomes an agent can achieve.

An issue complicating the measurement of freedom are the diverse normative ideals people have about freedom. The paper therefore provides distinct measures for the most common philosophical concepts of freedom: Positive liberty as the degree of rational self-determination, negative liberty as the absence of interference by others, and republican liberty as the absence of potential interference by others. Moreover, the measures are only defined up to a *freedom value function* which may contain additional normative considerations of the measurer. Aside from weighting the importance of freedom over the outcomes, the freedom value function can be used in two interesting ways:

First, it may integrate a measure of well-being into the freedom measure. This problem has been extensively discussed in the literature: Sen (1985, 1988) argued that a measure of well-being should be included in a measure of freedom. Others have presented impossibility results on integrating freedom and welfare into a single measure (Puppe, 1995, 1996; Nehring & Puppe, 1996; Pattanaik & Xu, 1998; Gravel, 1998; Baharad & Nitzan, 2000), drawing a rather negative picture on the possibility to integrate welfare and freedom of choice. If one introduces utility into the measure via the freedom value function, it turns out that the freedom measure is equal to weighted expected utility, where the weights are causal influence measures representing the control an agent has over each outcome. Expected utility is only reached if the agent has perfect control over all outcomes. The less control the agent has over an outcome, the less the utility of the outcome matters for the measure. This strikes a balance between care for the well-being of an agent and his possibilities to influence his surroundings.

Second, the freedom value function can be used for the freedom measure to capture the qualitative diversity of the outcomes of the game. For the freedom of an individual, it may not only matter to have control over a large number of outcomes, but also that these outcomes are qualitatively dissimilar. If one uses the qualitative diversity weights of Nehring and Puppe (2002) as the freedom value function, the positive freedom measure is a generalization of a diversity measure proposed by Nehring and Puppe (2009) which captures both quantitative and qualitative diversity.

The properties of the freedom measures are shown in two examples. An example on labor market discrimination shows how the positive, negative, and republican liberty measures differ in their judgments of how discrimination affects freedom. In the discrimination model, positive freedom differs from negative and republican liberty in that it allows for positive discrimination to improve freedom. Negative and republican freedom both decrease in both positive and negative discrimination. Republican liberty differs from negative liberty by not requiring discrimination to actually occur. Agents only need to have the means to discriminate in order to reduce republican liberty.

Second, the example of competitive equilibrium models from Pattanaik (1994) is considered. The example shows that price stability in an economy contributes to positive freedom, which gives microfoundations for the use of price stability as a component of an index of freedom as in Gwartney et al. (2010). Moreover, optimal taxation and optimal redistribution policies are derived. It is shown that the optimal tax level/government size is decreasing in the weight a policy maker attaches to freedom. Also, the amount of redistribution (convexity of the tax) is decreasing in the weight the policy maker attaches to freedom. If policy makers care about freedom, optimal utilitarian policy recommendations are therefore normatively biased.

The paper continues as follows. First, in Section 2 a review of the literature on various measures of freedom is given, with a focus on the ones related to the measures developed here. Section 3 provides the game theoretic framework in which the measures will be developed. The three philosophical concepts of freedom are introduced in Section 4 and a measure is given for each of them. Section 5 discusses several properties of the measure and states convergence theorems of the measure to measures of the freedom of choice literature and the diversity measurement literature. The convergence results reveal that the measure is a generalization of a diverse set of measures from the literature examined in Section 2. Two applications are provided as examples: To show the differences between the three philosophical concepts and how this translates into the measures an example of labor market discrimination is given in Section 6. The application of the positive freedom measure to a production economy and the problem of optimal taxation is given in Section 7. By this example it is shown that the class of measures in this paper indeed solves the problem posed by Pattanaik (1994).

2 Freedom Measures

In the following, only few measures will be reviewed. For a more extensive survey, the reader may consider Barberà et al. (2004), Baujard (2007), or Dowding and van Hees (2009). Two very early contributions to the freedom of choice literature are Pattanaik and Xu (1990) and Jones and Sugden (1982) for which later on some convergence results will be stated. Other closely related measures to the one presented in this paper are given in Braham (2006), Suppes (1996), and Nehring and Puppe (2009). Therefore these measures will be presented here, while the rest of the literature will only be touched upon briefly. All measures will be indexed by the authors' last names. Since the measures are based on opportunity sets C, some notation needs to be introduced to state these measures: Suppose X is a set of alternatives. A freedom relation $\succsim_F$ holds between subsets C of X. $C \succsim_F C'$ with $C, C' \subseteq X$ can be interpreted as 'the opportunity set C offers weakly more freedom than the opportunity set C''. The measure of Pattanaik and Xu (1990) states that the freedom offered by an opportunity set C is its cardinality $\sharp(C)$, that is:

Definition 1. Cardinality Measure (Pattanaik & Xu, 1990)
Suppose $C, C' \subseteq X$. Then $C \succsim_{F,PX} C' \Leftrightarrow \sharp(C) \geq \sharp(C')$.

Since the interest here lies not in the axiomatization of this measure, the axioms from which the measure can be derived will not be stated. While still considered the starting point of any measure of freedom, criticism of the measure has been abundant. For example Sen (1991) demands a more intricate relation between preference and freedom. However, as shown for example by Baharad and Nitzan (2000) the joint evaluation of the welfare and the freedom provided by an opportunity set often runs into difficulties or allows only for lexicographic comparisons (Romero-Medina, 2001).

A mild introduction of preferences into a freedom measure has been made by Jones and Sugden (1982). In their measure, which has been given a formal basis by Pattanaik and Xu (1998), a set $\mathcal{R}$ of so-called "reasonable" preference relations R is introduced and freedom is measured according to the set of reasonably chosen alternatives $\{x \in C : \exists R \in \mathcal{R} : \forall y : xRy\}$. While the precise definition of "reasonable" is left open, Jones and Sugden (1982) give as an example the choice of a prisoner, who can either "stay in the cell" or "get shot". Since it would be unreasonable to prefer getting shot to staying in the cell, the set of reasonably chosen alternatives is the singleton "stay in the cell". On the basis of the ideas developed by Jones and Sugden (1982), Pattanaik and Xu (1998) axiomatize the following measure:

Definition 2. Reasonable Preference Measure (Jones & Sugden, 1982; Pattanaik & Xu, 1998)
Suppose $C, C' \subseteq X$. Then $C \succsim_{F,JS} C'$ iff

$$\sharp(\{x \in C : \exists R \in \mathcal{R} : \forall y : xRy\}) \geq \sharp(\{x \in C' : \exists R \in \mathcal{R} : \forall y : xRy\}).$$

The measure thus states that the freedom an opportunity set offers can be measured by the cardinality of the set of reasonably chosen alternatives. This does not solve the critique of Sen (1991), since it may still occur that an opportunity set C is ranked higher by the measure than C', although in terms of preference each element in C' dominates all other elements in C. The difficulties are even increased: The set C' may now have even more elements than C and each element in C' may dominate all elements in C. Still, the measure may rank C higher than C'.

The closest relatives to the measure which will be developed in this paper are given by the following measures by Braham (2006), Suppes (1996), and Nehring and Puppe (2009). Unlike the above two measures, these measures are not solely based on opportunity sets but also include probability information.

The measure by Braham (2006) relies on game forms (see also Bervoets, 2007) to account for interactions between agents. It is not necessary to further examine the formal structure of the measure, since it has a very intuitive interpretation: The measure tries to capture the degree to which an individual i can force a certain outcome x to come about in the game. With some abuse of notation the measure states:

Definition 3. Game Form Measure (Braham, 2006)
$F_B(x, i) = P(\text{outcome is } x | i \text{ chooses } x)$

where it may occur that $P(\text{outcome is } x | i \text{ chooses } x) < 1$ because the actions of the other agents may lead to another outcome, even if i chooses x. The measure therefore takes up the idea that an agent is free if he can force certain outcomes to occur. This will also be the core idea of the causal influence measure in this paper, though the degree to which an agent can force certain outcomes will be measured in a different way.

It has been argued that freedom of choice is strongly connected to diversity. Individuals are more free if they are able to make choices over a more diverse opportunity set. Two types of diversity have been identified: Quantitative diversity refers to the relative frequencies with which different objects are chosen and can for example be measured by the Shannon (1948) entropy. Suppes (1996) proposes to measure freedom as the entropy of the relative frequencies with which an agent chooses the alternatives of an opportunity set:

Definition 4. Entropy Freedom Measure (Suppes, 1996)
$F_S(C,P) = -\sum_{x\in C} P(x)\ln P(x)$ where $P(x)$ refers to the probability with which an agent chooses element x of the opportunity set C.

The entropy measure results as one special case of the measure proposed in this paper. More specifically, if agents have perfect control with their actions over outcomes and no additional normative information is relevant, the measures converge. The precise relation between the two will be given in Proposition 2 in Section 5.

Qualitative diversity instead measures how different the elements of the opportunity set are. An example is Nehring and Puppe (2002). A model of qualitative diversity has been given by Nehring and Puppe (2002). Their model supposes that objects $x \in X$ have attributes A which can be defined via the subset of objects of X which also have this attribute. Therefore, $A \in X$ is the attribute that all elements in A share. To measure the diversity value of a set, each attribute is given a weight $\lambda(A)$. Diversity of a distribution of objects P can then be measured for example by $v(\lambda,P) = \sum_{A\subseteq X:\exists x\in A:P(x)>0} \lambda_A$. The intuition is very simple: the more attributes are represented in a distribution (and the more diversity weight they have), the higher the diversity. In case of an opportunity set C one could set $P(x) = 1/|C|$ if $x \in C$ and 0 otherwise to obtain the aggregate qualitative diversity of the opportunity set.

Building on this framework in Nehring and Puppe (2009) a generalized measure is proposed which captures both quantitative and qualitative aspects of diversity at once:

Definition 5. Diversity Measure (Nehring & Puppe, 2009)

$$D_{NP}(C,P,\lambda) = -\sum_{A:A\cap C\neq\emptyset} \lambda(A) \sum_{x\in A} P(x) \ln \sum_{y\in A} P(y)$$

which is the $\lambda(A)$ weighted entropy of the attributes. While the entropy is maximized if as many objects as possible have a distribution as even as possible, the measure $D_{NP}(\lambda,P)$ changes this in two ways: First, it considers the entropy over attributes, i.e. the attributes (and not the objects) need to have a distribution as even as possible. Second, there exists an additional tradeoff between a very even distribution and higher frequencies of attributes that have a high weight $\lambda(A)$. While this diversity measure has not been explicitly formulated as a measure of freedom, it turns out that in the perfect control case the freedom measure in this paper is identical to D_{NP} if the weights $\lambda(A)$ are included as the freedom value function in the measure.

The further literature can be divided into several branches. One branch deals with the aforementioned aspect of the diversity of an opportunity set (Bavetta and del Seta (2001), Bossert, Pattanaik, and Xu (2003), and van Hees (2004)). Another considers unstable preferences to be a source of preference for freedom (Koopmans (1964), Kreps (1979), and Sugden (2007)). Freedom has been studied in game forms in Peleg (1997), Bervoets (2007), and Ahlert (2010). The idea of multiple preference relations as in Jones and Sugden (1982) has been further examined by Sugden (1998), Nehring and Puppe (1999), and Bavetta and Peragine (2006). Rosenbaum (2000) develops a measure of freedom based on underlying characteristics of the elements of the opportunity set. An important topic is also the distribution of freedom between individuals, for which a survey is given by Peragine (1999). Broader discussions are given by van Hees (1999) Bavetta (2004), Carter (2004), and Kolm (2010).

Naturally very closely related are measures for power in voting systems, where mutual information between the cast vote and the outcome of the vote has been proposed as one dimension of power (Diskin & Koppel, 2010). The present paper uses a more general form of mutual information in order to also account for normative information that becomes relevant when measuring freedom.

3 The Model

The major assumption of the way freedom will be measured is the idea that freedom involves (a) the possibility of an agent *to do otherwise* and (b) to achieve other results by doing so. This means that freedom in this paper will always involve some counterfactual such as "if the agent had acted otherwise, he would have obtained a different outcome" containing an antecedent and a consequent. Most measures in the freedom of choice literature implicitly contain such a combination of antecedent and consequent: The above given measure by Jones and Sugden (1982) has reasonable preference relations as the antecedent and the chosen elements of the opportunity set as the consequent. Freedom then involves the counterfactual: "If the agent had had reasonable preferences $\mathcal{R}$, he would have chosen element x." Similarly, in the capabilities approach, freedom is accounted for by considering the set of functionings an individual can reach from the available commodity vectors. The counterfactual notion is again very clear: "If the agent had chosen commodity vector x, he would have obtained the function vector f".

The goal of this section is therefore to find a formal framework in which one can model interactions between individuals and which can account for both (a) and (b).

Assume a normal game form $\Game = (N, A, \psi, \mathcal{P}, \mathcal{I}, \mathcal{C}, p, R)$, where $N = \{1, \ldots, n\}$ is a finite set of players, A is a finite set of nodes, and $\psi : A \backslash a_0 \to A$ is a predecessor function such that for node a, $\psi(a)$ is the immediate predecessor of a. $\mathcal{P}$ is the player partitioning of the nodes and $\mathcal{I} = \{I_0, \ldots, I_n\}$ the information partitioning with I_i being the set of information sets of player i. Let $A(I) = \{a \in A : \psi(a) \in I\}$ return the set of nodes following information set I. $\mathcal{C}$ is the set of choice sets C_I for each information set I. Further $\Delta(C_I)$ is the set of probability distributions over the choice set at I. For $b \in I$ and $b = \psi(a)$ let $c(a|b) \in C_I$ be the choice that leads at node b to node a. p is the probability distribution for moves by nature. Finally, R is the set of result functions for each player, where a result function $r_i : A_\omega \to O_i$ maps the terminal nodes A_ω into the finite set of possible outcomes for player i, O_i. This last point deviates slightly from standard definitions of game forms, where result functions are not player specific. In case all result functions are identical, we are back in the standard case. The use of player-specific outcome functions is necessary to account for (b) in a meaningful way. The difference between some outcomes of the game may simply be irrelevant to all players except for player i. If player j then gets to make a choice between these outcomes, it would not be meaningful to call this a case where j is especially free. Rather, it is a case where j has power over the outcomes relevant for i.

Due to (a) the individual must possess some nontrivial form of agency, some capacity to make choices which are not completely prescribed by a single preference relation. For example, the measure of Jones and Sugden (1982) generates this capacity via the assumption of multiple preference relations which the individual may reasonably hold. In the model here, agency is introduced for all players in the game via a *preference expansion* $\daleth = (\mathcal{J}, \mathcal{U}, \hat{p})$. The preference expansion contains $\mathcal{U} = \{U_1, \ldots, U_n\}$ as the set of the sets of utility functions $U_i = \{U_{i,1}, \ldots, U_{i,\bar{u}}\}$ over outcomes for each player $U_{i,u} : O_i \to \mathbb{R}$. $\hat{p}$ assigns each of these utility functions a probability. It will be assumed that the preferences of players are independently distributed and the marginal distribution for each player i is $\hat{p}_i \in \Delta U_i$. Similarly, the distribution of all utility profiles excluding the utility of player i is $\hat{p}_{\setminus i}$. This probability may be interpreted either as a degree of reasonableness as in Jones and Sugden (1982) or an ex-ante probability with which an individual holds a preference if randomly drawn from a population. A more precise definition on what is contained in $\mathcal{U}$ will be given when considering various concepts of freedom. $\mathcal{J}$ is the set of information partitions for each player over the reasonable utility functions. To simplify notation, it will at times be convenient to treat the sets U_i and O_i as discrete random variables instead, with realizations ranging over the elements of the original set. For example, we may want to write $P(O_i = o)$ as the probability of the outcome of the game being o but we may also want to write $\sum_{o \in O_i}$ as the sum over all possible outcomes. It will be clear from the context whether O_i refers to the set or the random variable.

A strategy $s(I) \in \Delta(C_I)$ is a probability distribution over the elements of the choice set. Define a strategy profile S as a tuple of strategies specifying behavior at each information set $S = (s(I)|_{I \in I_i}|_{i \in N})$. Further, let θ^S be the joint probability distribution over nodes resulting from strategy profile S. Finally, we need to consider that strategies may depend on preferences in the preference expansion. Let $S(\hat{u})$ be the strategy profile resulting from preference profile $\hat{u} \in \times_{i \in N} U_i$. It makes sense to enrich the probability distribution θ by the outcomes and the preferences. Define for all $i \in N$, $o \in O_i$, $u \in U_i$:

$$\theta^{S(\hat{u})}(o|\hat{u}) = \sum_{a \in A_\omega : r_i(a)=o} \theta^{S(\hat{u})}(a) \tag{1}$$

$$\theta^{\hat{p}_i,\hat{p}_{\setminus i}}(o) = \sum_{\hat{u} \in \times_j U_j} \hat{p}_i(u_i)\hat{p}_{\setminus i}(u_1 \ldots, u_{i-1}, u_{i+1} \ldots, u_n)\theta^{S(\hat{u})}(o|\hat{u}) \tag{2}$$

$$\theta^{\hat{p}_{\setminus i}}(o|u_i) = \sum_{\hat{u} \in \times_{j \neq i} U_j \times u_i} \hat{p}(\hat{u}) \frac{\theta^{S(\hat{u})}(o|\hat{u})}{\hat{p}_i(u_i)} \tag{3}$$

$$\theta^{\hat{p}_i,\hat{p}_{\setminus i}}(o, u_i) = \theta^{\hat{p}_{\setminus i}}(o|u)\hat{p}_i(u_i) \tag{4}$$

These last definitions are the central elements of the measures in this paper as they express whether it is one's own preferences or the preferences of other players that determine which outcome occurs. For notational simplicity, where unambiguous the superscripts $S(\hat{u})$, $\hat{p}_i$ or $\hat{p}_{\setminus i}$ will be omitted.

4 Concepts of Freedom

The concept of freedom has been heavily debated within Philosophy.[2] In a very influential article, Berlin (1958) attempted to categorize concepts of freedom in two categories: positive and negative freedom. Positive freedom refers to the actual ability to control one's own destiny while negative liberty refers to the absence of interferences of others in one's destiny. A common example (e.g. Carter, 2012) showing the difference is that of a smoker who due to his addiction does not have the ability to stop smoking. Due to this lack of control over whether he smokes or not, he does not have positive freedom. However, there is also nobody interfering with whether he smokes or not. Thus, he still has negative freedom. Only if another person was able to force him to smoke or prevent him from smoking would he lose negative freedom.

A third concept of freedom which according to its proponents falls out of the categorization by Berlin (1958) is republican liberty (e.g. Pettit, 1996). From the perspective of a republican liberal freedom is high when individuals are not subject to arbitrary power of other individuals. This idea is closely related to negative freedom. The main difference between the two is that republican freedom refers to the possibility of interference while negative freedom refers to actual interference. In the case of the smoker republican freedom is low if somebody has the possibility to prevent another from smoking (or force to smoke) even without actually preventing or forcing him.

All three concepts of liberty can be accomodated in the model. The following subsections will explore this in detail and derive a measure for each concept.

[2]For an overview, see Gaus and Courtland (2011).

4.1 Positive Liberty

Positive freedom as the degree of rational self-control is the most straightforward to measure in the model. If U_i is a variable representing the preferences of the rational self which is unrestricted by addictions, irrationalities etc., then the degree to which an individual exhibits rational self-control can be measured by the causal influence from U_i to O_i.

Players' strategies for calculating this influence should be given by empirical behavior[3] S_e to account for deficits in self-control as in the example of the smoker. That is, an individual may behave according to a behavioral theory which may limit the influence of the (non-behavioral) preferences on outcomes. Positive freedom is measured as:

$$\Phi^{pos}(\Game, \theta) = \sum_{u \in U_i} \hat{p}_i(u) \sum_{o \in O_i} \theta^{\hat{p}_{\setminus i}}(o|u) \left(c(o,u) \ln \frac{\theta^{\hat{p}_{\setminus i}}(o|u)}{\theta^{\hat{p}_i, \hat{p}_{\setminus i}}(o)} \right) \tag{5}$$

where $v : O_i \times U_i \to \mathbb{R}$ is called the freedom value function of the measure. The freedom value function controls how important the causal control of u on o is for freedom. For example, the freedom value of being able to choose one's occupation may be higher than the freedom value of owning a gun. In Section 5 it will be elaborated how these functions can be used to capture the diversity and utility of the options. The measure is a weighted expectation of the $\ln \frac{\theta(o|u)}{\theta(o)}$ terms which are large if u makes o more likely and small if u makes o less likely. Going back to the example of the smoker without self control, let the outcomes be $O_i = \{s, ns\}$ and the preferences be $U_i^h = \{u_{ps}, u_{pns}\}$ where s and ns stand for the outcome of smoking or not smoking and u_{ps} is the utility function if the player prefers to smoke and u_{nps} is the utility function if the player prefers not to smoke. Assume the freedom value function is positive and only outcome-dependent, $v(o,u) = v(o) > 0$. If the smoker has no self-control, we have $\theta(ns) = \theta(ns|u_{pns}) = 0$ and $\theta(s) = 1$. Therefore, $\Phi^{pos} = 0$ and the player has no freedom according to the measure since for each of the $\ln(\theta(o|u)/\theta(o)) = \ln(1) = 0$. If the smoker gains more self control, $\theta(ns|pns) > \theta(ns) > \theta(ns|ps)$ and thus $\ln(\theta(ns|u_{pns})/\theta(ns))$ will increase while $\ln(\theta(s|u_{pns})/\theta(s))$ will decrease. Since each utility-outcome combination is weighted by its joint probability, the former effect dominates and the overall effect will be an increase in the measure. While such self control cases are important for a proper measure of positive freedom, the present paper will mostly focus on limitations to freedom due to the structure of the game and not limits to rationality. Further research on freedom when agents are boundedly rational would be interesting, but are outside the scope of this paper.

4.2 Negative Liberty

Negative freedom is associated with two distinct aspects. The first is the idea of non-interference of others. Defining negative freedom, Berlin (1958) stated: "By being free in this sense I mean not being interfered with by others." (Berlin (1958), p.8). The second

[3]One can also use a descriptive model of behavior. The main point is that under positive freedom it is important that s_e reflects actual behavior (as opposed to theoretically optimal behavior), irrespective whether it is obtained from a descriptive model or field data.

aspect is that of not being restricted by others: "Mere incapacity to attain a goal is not a lack of political freedom. [...] It is only because I believe that my inability to get a given thing is due to the fact that other human beings have made arrangements whereby I am, whereas others are not, prevented from having enough money with which to pay for it, that I think myself a victim of coercion or slavery" (Berlin (1958), p.7). These two aspects are quite distinct, since one can be restricted by others without being interfered with and vice versa.[4]

When measuring negative freedom, these two aspects open two possibilities. One is that negative freedom decreases in the extent to which the preferences of other players determine the outcomes of a player. This emphasizes the non-interference aspect of negative freedom. The other possibility is to emphasize the aspect of not being restricted by others. This would be achieved by some maximized version of the positive freedom measure, which does not depend on the actual ability of an agent to influence his outcomes but on the potential ability given others' behavior. Since the latter can be interpreted as a variant of positive freedom, the proposed measure of negative freedom follows the idea of non-interference.

Thus, negative liberty of individual i is measured by the degree of causal influence of other individuals on the outcomes of individual i. Since negative liberty refers to actual interferences, players' strategies should be given by the empirical behavior. It is important to note that in such cases, all relevant institutions which limit an individual's freedom must be part of the model. That is, in order to measure limitations of freedom from the government, the government itself must be a player in the game.

$$\Phi^{neg}(\Game, \theta) = -\sum_{j \neq i} \sum_{u_j \in \mathcal{U}_j^e} \hat{p}_j(u_j) \sum_{o \in O_i} \theta^{\hat{p}_{\setminus j}}(o|u_j) c(o, u_j) \ln\left(\frac{\theta^{\hat{p}_{\setminus j}}(o|u_j)}{\theta^{\hat{p}_j, \hat{p}_{\setminus j}}(o)}\right) \tag{6}$$

Again the measure is a weighted expectation over the (negative) logarithmic terms measuring influence. In negative freedom we are interested however in the degree to which other agents' preferences determine one's outcomes which is why U_j is the variable causing or preventing o.

In the case of the smoker, nobody actually interferes with the decision of the smoker. Just as the smoker exercises no control over O_i, so does nobody else interfere, i.e. $\theta(ns) = \theta(ns|u_j)$. Suppose now i lives in a dictatorship where j may be a smoking-averse dictator $u_{j,pns}$ or a smoking-friendly dictator $u_{j,ps}$. If via the game played the dicator manages to influence the outcome of whether i smokes, freedom will be lower: $\theta(ns|u_{pns}) > \theta(ns) > \theta(ns|ps)$ and thus the measure decreases relative to the case where O_i and U_j are independent. In Neri and Rommeswinkel (2014) it is shown experimentally that preference for negative freedom is indeed an important driver for preference for decision rights.

[4]For example a group of the population who's vote does not count in an election is restricted in their political influence but not interfered with. A person convincing another not to go to vote may interfere with the other person's choice but does not restrict the other person's political influence.

4.3 Republicanism

Republican liberty is closely related to negative liberty but does not only consider actual interventions of others, but also potential interventions. The most prominent conception of liberty in this class has been given by Pettit (1996), "taking the antonym of freedom to be subjugation, defenseless susceptibility to interference, rather than actual interference" (p.577). When measuring potential interferences of others, one can obviously no longer rely on empirical behavior. A further difficulty in measuring this is the qualification that an agent must be "defenseless". We may consider a player i not to be defenseless if she has some means by which she may deter another player j from taking certain actions against her. This however requires some notion of rationality or at least responsiveness of j to the payoff threats that i can make against j. It therefore makes sense to use some equilibrium concept to solve the game and obtain $S^*(\hat{u})$ as the strategy profile with which to calculate $\theta^{S^*(\hat{u})}$ and the remaining probabilities defined in Section 3. Republican liberty[5] is measured as follows:

$$\Phi^{rep}(\circlearrowleft, \theta) = -\max_{\hat{p}_j} \sum_{j \neq i} \sum_{u_j \in \mathcal{U}_j} \hat{p}_j(u_j) \sum_{o \in O_i} \theta^{\hat{p}_j, \hat{p}_{\setminus j}}(o|u_j) c(o, u_j) \ln \left(\frac{\theta^{\hat{p}_i, \hat{p}_{\setminus j}}(o|u_j)}{\theta^{\hat{p}_j, \hat{p}_{\setminus j}}(o)} \right) \tag{7}$$

By maximizing with respect to probability distributions over the preference relations of each individual $j \neq i$, the measure returns the maximal interference of others into i's affairs given that individuals still act rationally. The central difference to the negative freedom measure is that even if the actual distribution $\hat{p}$ is such that nobody would interfere with i, republican freedom may still be low if other individuals (in virtue of the structure of the game and thus the conditional probabilities $\theta(o|u_j)$) have potential influence on the outcomes of i. For example, if a dictator can decide whether to allow smoking or ban smoking, republican freedom is low even if the dictator in equilibrium decides with probability 1 to allow smoking. In comparison, republican freedom is high if either there is no possibility to ban smoking or if many actors need to jointly decide to ban smoking for it to be banned. It is important to note that the measure strongly depends on the specification of U_j: a too narrow specification would bring the measure closer to negative freedom. A too wide specification may yield a very low freedom measure just by some absurd behavior. Nonetheless, the republican measure of freedom highlights how even the complex philosophical intuitions as freedom involving potential causal influence and potential subjugation can be captured in the model.

[5] One has to note though that Pettit (1996) acknowledges that for republican liberty "there are two subgoals involved. [...] One involves the reduction of subjugation, [...] the other involves the maximization of the domain of individual choice" (p. 593). This suggests that Pettit's conception of power should be measured as some aggregate of the above republican measure and a measure that is closer to the measure of positive freedom.

5 Properties of the Measures

An important property of all three measures is that they do not directly refer to the structural elements of the game such as nodes or choice sets. In Thompson (1952) a set of transformations for extensive form games were proposed which leave the strategic features of the game unchanged. Later, these transformations were refined by Elmes and Reny (1994). Since the measure only refers to the conditional probabilities with which outcomes arise given a utility function, the measure is invariant to the transformations of Elmes and Reny (1994) if the strategies of players are invariant to these transformations.

There are two other important properties of the positive and negative freedom measures. The two properties show that the measure is invariant to finer partitionings of the outcome and preference type space. The derivations of these properties are given in the appendix. Let $(\partial', \theta', v') = (\partial, \theta, v)/_u^r Q$ be the game where preference type u has been replaced by types Q which are drawn with probability $\theta'(q) = \theta(u)r(u)$. Behavior remains unchanged such that $\forall \theta(o,u) = \sum_{q \in Q} \theta'(o,q) = \theta(o,u) \sum_{q \in Q} r(q)$. The value function is unchanged $\forall u, q : v'(o,q) = v(o,u)$. The $/_u^r U'$ operation therefore refines the preference type space without the additional preference types actually conveying any additional information.

Property 1. *Preference Refinement:*
$\Phi((\partial, \theta, v)/_u^r U') = \Phi(\partial, \theta, v)$

Preference refinement implies that freedom is invariant to preference information which is not behaviorally relevant. For example, suppose a particular strategy is optimal under preferences u' and u''. Then the freedom measure does not change if we treat these utility functions as a single utility function. The preference types therefore only have an impact on the measure if they impact behavior.

A similar property can be stated for the outcome space as well. Let $(\partial', \theta', v') = (\partial, \theta, v)/_o^r Q$ be the game where in all cases where the game reaches outcome o nature makes an additional move with choice set Q and probability distribution r. Behavior remains unchanged such that $\forall \theta(o,u) = \sum_{q \in Q} \theta'(q,u) = \theta(o,u) \sum_{q \in Q} r(q)$. The value function is unchanged $\forall u, q : v'(q,u) = v(o,u)$.

Property 2. *Outcome Refinement:*
$\Phi((\partial, \theta, v)/_o^r O') = \Phi(\partial, \theta, v)$

Outcome refinement implies that if an outcome o is changed into a lottery of a set of new outcomes, the measure remains unchanged. Thus, uncertainty by itself does not reduce freedom. This is important, since it distinguishes freedom from being simply a measure of risk. However, freedom could be increased by giving the individual control over the new outcomes. This means freedom is indeed a measure of the control an individual has over his outcomes.

The freedom measure is of course not the only measure which fulfills these properties. For example, the expected payoff of the game also fulfills these. For axiomatizations of mutual information as a statistical measure of dependence, see Csiszár (2008) for a survey and Frankel and Volij (2011) in the context of segregation.

5.1 Relation to Other Measures

The positive freedom measure is a generalization of several measures in the freedom of choice literature and the literature on diversity measures. This section will explore these relations. Since all the measures are based on opportunity sets, it will be useful to define $\Game^T(C)$ as the trivial game where one player faces an opportunity set C as the outcomes of the game. The freedom measure turns out to be a generalization of the freedom measure by Jones and Sugden (1982):

Proposition 1. *Suppose for all trivial games $\Game^T(C)$ a freedom value function $\forall x \in C, u \in U_i$: $v(x,u) = 1$ and rationality of player i:*

$$\forall u \in U_i : \exists x \in C : \theta(x|u) = 1 \Leftrightarrow x \in \arg\max_{x\in C} u(x). \tag{8}$$

Moreover, assume that U_i are utility representations of the reasonable preferences:

$$\forall R \in \mathcal{R} : \exists u \in U_i : \forall x, x' \in C : xRx' \Leftrightarrow u(x) \geq u(x'). \tag{9}$$

Let all outcomes with positive probability be equally probable:

$$\theta(x) > 0 \wedge \theta(x') > 0 \Rightarrow \theta(x) = \theta(x'). \tag{10}$$

Then positive freedom is a representation of the measure by Jones and Sugden (1982):

$$\Phi^{pos}(\Game^T(C), \theta) \geq \Phi^{pos}(\Game^T(C'), \theta) \Leftrightarrow C \succsim_{F,JS} C' \tag{11}$$

For the freedom measure to represent the reasonable preference measure three steps are needed: First, no additional normative information such as utility or qualitative diversity may be included. Second, individuals must maximize utility and the utility functions must be representation of the reasonable preference relations. Third, quantitative diversity must be maximal such that each outcome is equally likely.

Notice that if the set of reasonable preferences is the set of rational preferences, the measures of Jones and Sugden (1982) and Pattanaik and Xu (1990) are equivalent and freedom represents also the cardinality measure. Therefore, Φ^{pos} generalizes the measures of Jones and Sugden (1982) and Pattanaik and Xu (1990) by allowing for the game theoretic setting, a freedom value function $v(o,U)$, and the quantitative diversity of outcomes as can be seen in the next proposition.

The connection to the measure of the quantitative diversity of choices by Suppes (1996) is as follows:

Proposition 2. *Suppose for all trivial games $\Game^T(C)$ a freedom value function $\forall x \in C, u \in U_i$: $v(x,u) = 1$ and that the individual has full control over the outcomes:*

$$\forall u \in U_i : \exists x \in C : \theta(x|u) = 1. \tag{12}$$

Then positive freedom is equal to the entropy freedom measure:

$$\Phi^{pos}(\Game^T(C), \theta) = F_S(C, \theta). \tag{13}$$

Here it is only required that the individual exhibits full control over the outcomes such that for each utility function there is a single outcome that occurs with positive probability. The freedom measure therefore generalizes the entropy measure by allowing for a freedom value function $v(o,U)$ and players having only degrees of control over the outcomes of a game.

The measure can also include qualitative aspects of diversity. It turns out that by adding up the freedom of all attributes, one can obtain a generalization of $D_{NP}(C,P,\lambda)$. For this, define $\eth^T(C,A)$ as the trivial game $\eth^T(C)$ with C being the set of terminal nodes and the set of outcomes O_i replaced by a partitioning of nodes into members with attribute A and members without, $O_{i,A} = \{A, A^C\}$.

Proposition 3. *Suppose for all trivial games $\eth^T(C,A)$ the freedom value function $v(A,u) = \lambda(A)$ and $v(A^C,u) = 0$. Further, suppose full control over the outcomes*

$$\forall u \in U_i : \exists x \in X : \theta(x|u) = 1. \tag{14}$$

Then positive freedom is equal to the diversity measure by Nehring and Puppe (2009)

$$\sum_{A \subseteq X} \Phi^{pos}(\eth_A, \theta) = D_{NP}(C,\theta,\lambda) \tag{15}$$

This means that when considering a game where a player has full control over the terminal nodes of the game, we can measure aggregate diversity as the sum of positive freedom over all attributes. In situations of imperfect control, Φ^{pos} additionally controls for whether the diversity of chosen objects is due to the preferences of the individual or due to force by others, since if the realization of the attribute A is independent of U_i, i.e $\forall u \in U_i : \theta(A|u) = \theta(A)$, then $\Phi^{pos}(\eth_A,\theta) = 0$ and the added diversity from A does not increase freedom.

5.2 Freedom and Utility

An important issue in the discussion on measures of freedom has been the integration of utility into the measures such that freedom is increasing in well-being.[6] However, Puppe (1995), Nehring and Puppe (1996), Gravel (1998), Pattanaik and Xu (1998), and Baharad and Nitzan (2000) have provided impossibility results with respect to this endeavor. The freedom measure in this paper allows for the integration of utility via the freedom value function $v(o,u)$.

Suppose $v(o,u) = u(o)$, then:

$$\Phi^{pos}(\eth,\theta) = \sum_{u \in U_i^h} \hat{p}(u) \sum_{o \in O_i} \theta(o|u) u(o) \ln \frac{\theta(o|u)}{\theta(o)} \tag{16}$$

[6]E.g. Sen (1991) or Sen (1985). Although the remarks here focus on the integration of utility into a freedom measure, they also hold for other measures of well-being, e.g. value functions over capabilities as long as they are at least defined on an ordinal scale.

The advantage of integrating freedom and well-being via the function $v(\dots)$ is the complementarity between choice and well-being: There are now outcomes where without additional utility ($u(o) = 0$), more influence is irrelevant. Also, without any influence on outcomes ($\ln(\dots) = 0$), additional utility is irrelevant. Having control over the outcomes may be detrimental for one's freedom if the associated utility level is negative. It is important to realize that under this specification utility is no longer measured on a cardinal scale but instead on a ratio scale. The $u(o) = 0$ point becomes meaningful as the point where additional causal influence does not matter. For outcomes with lower utility values, additional causal influence even decreases freedom.

Suppose the individual has full control over the outcomes such that for each $u \in U_i$ there exists some $o \in O_i$ with $\theta(o|U) = 1$. Suppose further that all outcomes are equally likely: $\theta(o) = 1/|O_i|$. Then the measure becomes:

$$\Phi^{pos}(\eth, \theta) = \sum_{u \in U_i^h} \hat{p}(u) \sum_{o \in O_i} \theta(o|u) u(o) \ln(|O_i|) \tag{17}$$

Under the above given conditions, freedom is therefore expected utility times the logarithm of the number of alternatives. This captures the idea that freedom is both increasing in well-being and the number of options. Moreover, one can neither be free without well-being nor without being able to influence one's outcomes.

To integrate utility and freedom in this way especially makes sense in a behavioral context where individuals have preference for freedom. That is, individuals may not only consider the expected payoff consequences of their actions, but also the expected freedom from future subgames. Such a theory of preference for freedom making use of the above functional form has been developed in a companion paper (Neri & Rommeswinkel, 2014) to explain behavior in experiments by Fehr, Herz, and Wilkening (2013) and Bartling, Fehr, and Herz (2013).

6 Freedom and Labor Market Discrimination

In this section the central properties of the freedom measures are discussed via the example of discrimination in the labor market. A simple game is studied where wages are fixed before the employer knows the applicant and the main source of discrimination is the difference in acceptance rates of equally qualified men and women. An empirical study can be found in Oaxaca and Ransom (1994). Let there be two players, $N = \{1, 2\}$ with 1 being an applicant to an employer 2. The applicant can be either female or male, $g \in \{f, m\}$, which is determined by nature with probability 1/2. Also, the productivity ζ of the applicant is distributed uniformly over the interval $[0, 1]$. After gender and productivity have been determined, the applicant chooses whether to apply or not apply $x_1 \in \{ap, nap\}$. If the applicant does not apply, the game ends and the applicant has no job. If the applicant applies, the employer decides whether to accept or reject $x_2 \in \{acc, rej\}$ the applicant which results in the applicant having a job (job) or not ($nojob$). The outcome space for the employer is $\{noworker\} \cup [0,1] \times \{male, female\}$.[7] The last part necessary for the specification of the game is the information partition. We may consider two variants: First, $\Game_1$ is the perfect information game where each information set is a singleton. Therefore, the employer knows the gender and productivity of the applicant. Second, with $\Game_2$ we may consider a game where the gender of the applicant is not revealed before the application.

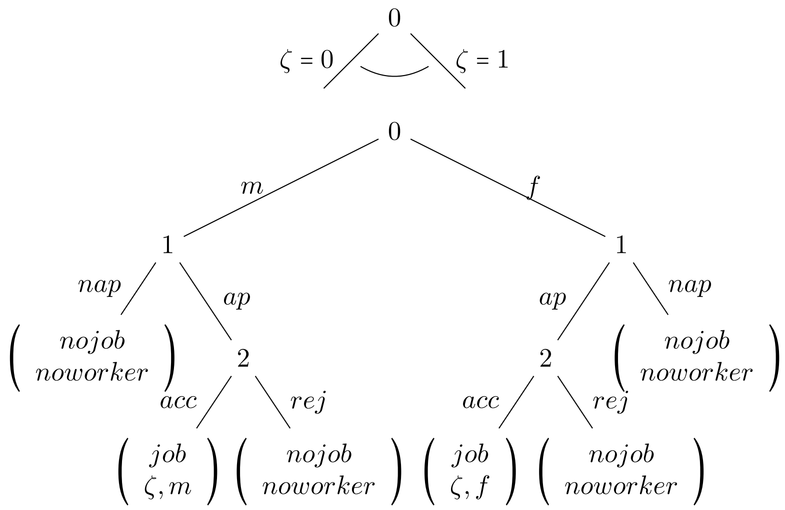

Figure 1: Discrimination Game $\Game_1$

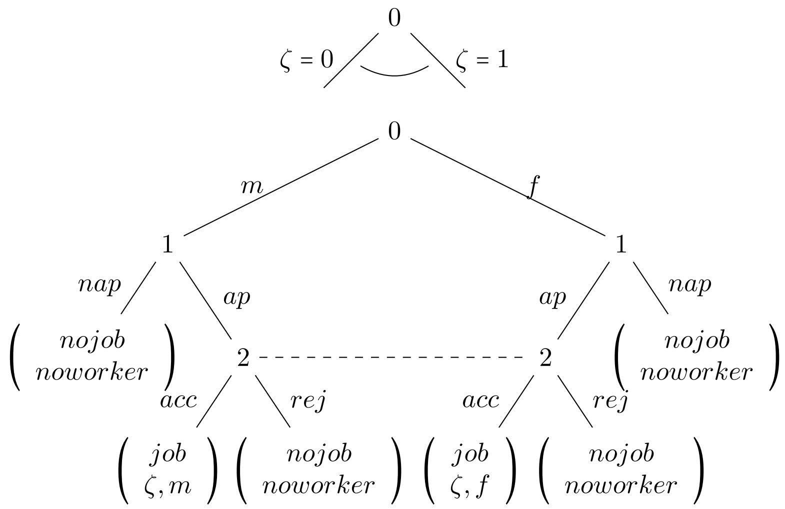

Figure 2: Discrimination Game $\Game_2$

Given the extensive game form, we can measure the freedom if we supplement it with

[7] One may hold the normative conviction that the employer's freedom on whom to employ is also normatively relevant. In this case, one would need to find a way to aggregate the two. Since this would require a further analysis of interpersonal comparisons of freedom, this is outside the scope of this paper.

a preference expansion. Let the utility functions of the applicant be $U_1 = \{u_1^A, u_1^B\}$ with

$$u_1^A(job) = 1 > 0 = u_1^A(nojob) \tag{18}$$

$$u_1^B(job) = 0 < 1 = u_1^B(nojob). \tag{19}$$

Therefore, type A likes to take the job while type B does not. Both types occur with probability $\hat{p}(u_1^A) = \hat{p}(u_1^B) = 1/2$. The employer can either be discriminatory or nondiscriminatory: $U_2 = \{u_2^A, u_2^B\}$ with

$$\begin{aligned} u_2^A(noworker) = u_2^B(noworker) &= 1 \\ u_2^A(\zeta) &= 2\zeta \\ u_2^B(\zeta, male) &= 2(\zeta + d) \\ u_2^B(\zeta, female) &= 2(\zeta - d). \end{aligned}$$

The extent to which the discriminatory employer has preferences against women is therefore measured by $d \in [0, 0.5]$. Let the fraction of discriminatory types B be δ. Finally, $\mathcal{J} = \{\{\{u_1^A, u_2^A, u_2^B\}, \{u_1^B, u_2^A, u_2^B\}\}, \{\{u_2^A, u_1^A, u_1^B\}, \{u_2^B, u_1^A, u_1^B\}\}\}$ is the information partitioning over the preferences such that each individual knows his own preferences.

The derivations in this section are given in more detail to give a feeling for the usage of the measure. It is straightforward to calculate the Perfect Bayesian Nash equilibrium strategies of each game:

$$\begin{aligned} x_1^{\Game_1} = x_1^{\Game_2} &= \begin{cases} ap, & U_1 = u_1^A \\ nap, & \text{else} \end{cases} \\ x_2^{\Game_1} &= \begin{cases} acc, & U_2 = u_2^A \ \wedge \ \zeta \geq 1/2 \\ acc, & U_2 = u_2^B \ \wedge \ \zeta + d \geq 1/2 \ \wedge \ g = m \\ acc, & U_2 = u_2^B \ \wedge \ \zeta - d \geq 1/2 \ \wedge \ g = f \\ rej, & \text{else} \end{cases} \\ x_2^{\Game_2} &= \begin{cases} acc, & \zeta \geq 1/2 \\ rej, & \text{else} \end{cases} \end{aligned} \tag{20}$$

Assume that empirical strategies are identical with the above given Bayesian Nash equilibrium strategies. Then the above strategies give us the probability distribution $\theta(O_1|U_1)$ for the measure as shown in Tables 1 and 2. Aside from the freedom value function, these tables contain all the information relevant for calculating each of the freedom measures.

Positive freedom for each game and gender is given by the following calculations:

$$\Phi_f^{pos}(\Game_1, \theta, v) = \frac{1 - 2\delta d}{4} \ln(2) + \frac{1 + 2\delta d}{4} \ln\left(\frac{2 + 4\delta d}{3 + 2\delta d}\right) + \frac{1}{2} \ln\left(\frac{4}{3 + 2\delta d}\right) \tag{21}$$

$$\Phi_m^{pos}(\Game_1, \theta, v) = \frac{1 + 2\delta d}{4} \ln(2) + \frac{1 - 2\delta d}{4} \ln\left(\frac{2 - 4\delta d}{3 - 2\delta d}\right) + \frac{1}{2} \ln\left(\frac{4}{3 - 2\delta d}\right) \tag{22}$$

$$\Phi^{pos}(\Game_2, \theta, v) = (3/4) \ln(4/3), \tag{23}$$

$\Game_1$	job	$nojob$
u_1^A	$1/2 + \delta d(1 - 2 \cdot \mathbb{1}(g = f))$	$1/2 - \delta d(1 - 2 \cdot \mathbb{1}(g = f))$
u_1^B	0	1
u_2^A	$1/4$	$3/4$
u_2^B	$1/4 + d(1 - 2 \cdot \mathbb{1}(g = f))/2$	$3/4 - d(1 - 2 \cdot \mathbb{1}(g = f))/2$
marginal	$1/4 + \delta d(1 - 2 \cdot \mathbb{1}(g = f))/2$	$3/4 - \delta d(1 - 2 \cdot \mathbb{1}(g = f))/2$

Table 1: $\theta(O|U)$ in $\Game_1$

$\Game_2$	job	$nojob$
u_1^A	$1/2$	$1/2$
u_1^B	0	1
u_2^A	$1/4$	$3/4$
u_2^B	$1/4$	$3/4$
marginal	$1/4$	$3/4$

Table 2: $\theta(O|U)$ in $\Game_2$

where for simplicity $v(o, U) = 1$. Moreover, in $\Game_2$ freedom does not depend on gender. The comparative statics are very intuitive as can be seen in Figure 3: Men's freedom increases in δ and d since the more likely it is that they will be employed if they apply, the greater their control over their employment status. For women the opposite is the case: If they apply, their employment chances are negatively affected by δ and d, which translates into lower freedom. If $\delta d = 0$ in $\Game_1$ or if $\Game_2$ is played, their freedom is maximal.

Negative freedom measures the degree to which other players interfere with a player's outcomes:

$$\begin{aligned}
\Phi_f^{neg}(\Game_1, \theta, v) =& -\frac{\delta(1-2d)}{4}\ln\left(\frac{1-2d}{1-2\delta d}\right) - \frac{\delta(3+2d)}{4}\ln\left(\frac{3+2d}{3+2\delta d}\right) \\
& - \frac{(1-\delta)}{4}\ln\left(\frac{1}{1-2\delta d}\right) - \frac{(1-\delta)3}{4}\ln\left(\frac{3}{3+2\delta d}\right) \qquad (24) \\
\Phi_m^{neg}(\Game_1, \theta, v) =& -\frac{\delta(1+2d)}{4}\ln\left(\frac{1+2d}{1+2\delta d}\right) - \frac{\delta(3-2d)}{4}\ln\left(\frac{3-2d}{3-2\delta d}\right) \\
& - \frac{(1-\delta)}{4}\ln\left(\frac{1}{1-2\delta d}\right) - \frac{(1-\delta)3}{4}\ln\left(\frac{3}{3+2\delta d}\right) \qquad (25) \\
\Phi^{neg}(\Game_2, \theta, v) =& 0, \qquad (26)
\end{aligned}$$

where again the freedom value function has been set to $v(o, u) = 1$. The central difference of negative freedom is that freedom for men is now also decreasing in d. This is because the positive discrimination of discriminating employers towards men constitutes just as much of an interference as their negative discrimination against women. This is a central conceptual difference: under negative freedom, affirmative action or other types of positive discrimination restrict freedom and are thus undesirable. No matter whether an individual is improved or worsened in its well-being by an interference of another player, in both cases

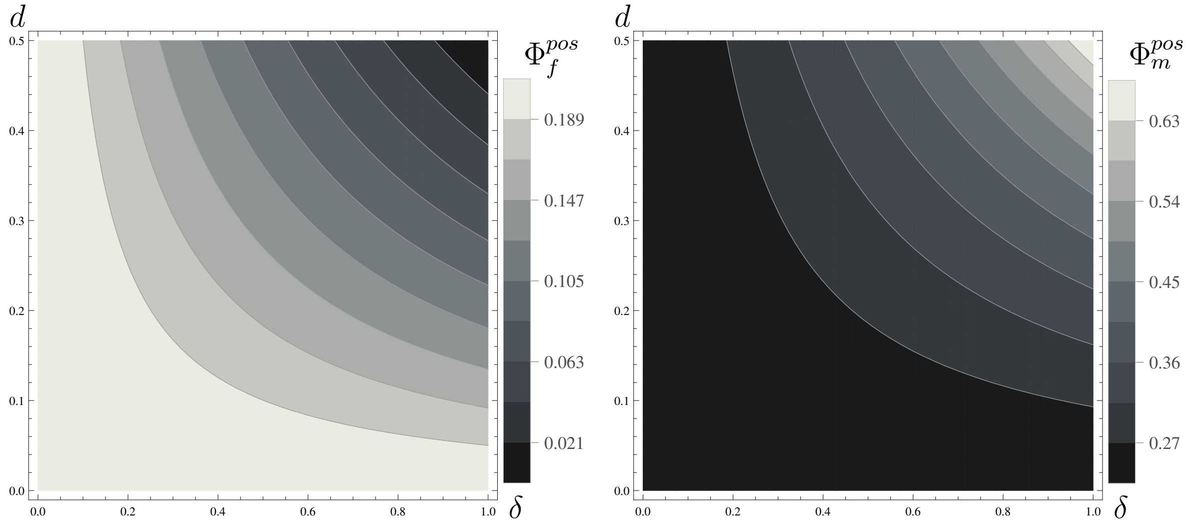

Figure 3: Positive Freedom of Women and Men in $\Game_1$
δ: Fraction of discriminating employers,
d: Extent of discrimination

negative freedom decreases in the extent and intensity of discrimination. What negative freedom shares with positive freedom is that $\delta d = 0$ yields the same freedom as $\Game_2$: The fact that nobody actually discriminates men and women is equivalent with a setting in which the employer cannot discriminate due to the structure of the game.

What appears counterintuitive at first is the fact that the negative freedom measure is not monotone in δ. This is related to the fact that it is a measure of actual interference but not restriction. If all employers discriminate women, the employment status of an individual no longer depends on the variations in the preferences of others. Naturally, one could move away from using the actual distribution of preferences to a hypothetical distribution to account for the fact that the employer still has power over the employee. However, then one would move from actual to hypothetical interference and at a republican liberty measure:

$$\Phi_f^{rep}(\Game_1, \theta) = \min_{\delta, d} \Phi_f^{neg}(\Game_1, \theta) \tag{27}$$

$$\Phi_m^{rep}(\Game_1, \theta) = \min_{\delta, d} \Phi_m^{neg}(\Game_1, \theta) \tag{28}$$

$$\Phi^{rep}(\Game_2, \theta) = 0 \tag{29}$$

Republican freedom in this model simply maximizes negative freedom with respect to the fraction of discriminating employers and the intensity of discrimination. The minimal freedom of women is reached at $d = 1$ and $\delta \approx 0.61$ as marked with the dots in Figure 4. Notice that in practice one may want to put restrictions on d for this measure, since extreme values for d may no longer be economically feasible for the employer. The republican freedom measure shares with the negative freedom measure the impossibility of positive

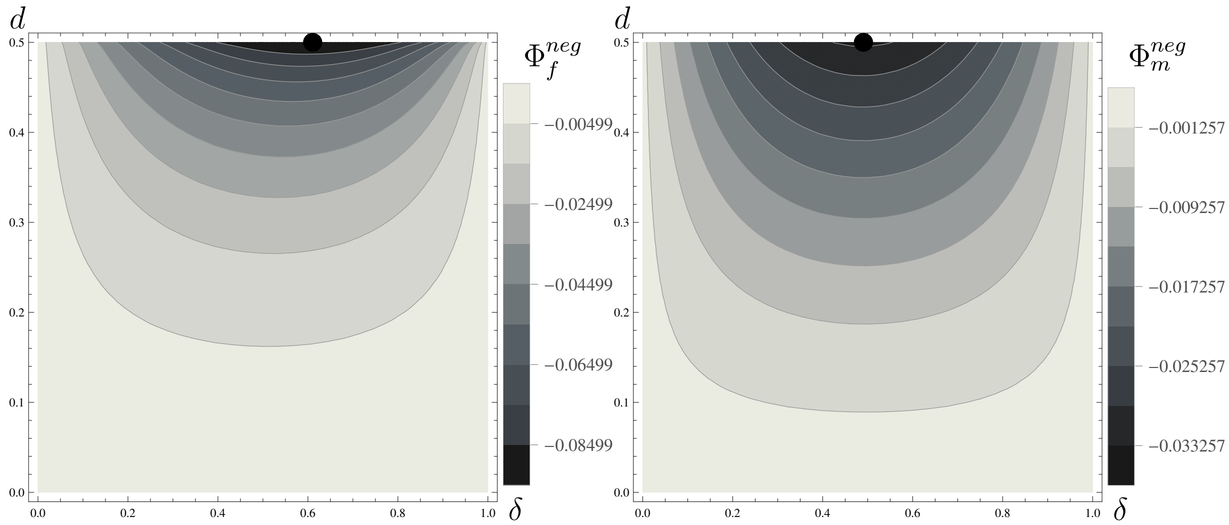

Figure 4: Republican and Negative Freedom of Women and Men in $\eth_1$
δ: Fraction of discriminating employers; d: Extent of discrimination; •: Republican liberty

discrimination increasing freedom. The main difference is that one does not need to observe actual interference to conclude that a person is unfree. Just by the information structure given in $\eth_1$ the applicant's freedom is jeopardized. This fits with current debates on privacy rights: From the perspective of a republican libertarian a corporation or government does not need to actually interfere with the lives of individuals to violate their freedom. It already constitutes a violation of freedom if it has acquired the information necessary to interfere. Having his information, even if it is not used to interfere in equilibrium, gives nontheless the potential to interfere which decreases republican freedom.

7 Freedom in a Production Economy

To further illustrate the measure for positive freedom, it will be employed to analyze freedom in a production economy, a similar problem to the one posted in Pattanaik (1994). According to Pattanaik (1994), the problem of measuring freedom in an exchange economy is that prices and therefore also opportunity sets change both with one's own preferences and preferences of the other agents. Since most measures of freedom are based on opportunity sets, they fail to give a satisfying answer to the problem, as Pattanaik (1994) concludes.

This section applies the freedom measure to a simple production economy. It is analyzed how freedom is influenced by two examples of government policies. The first example is a linear income tax with endogenous government consumption. It is shown that freedom is decreasing in taxation. Therefore, governments which put a higher weight on freedom will tax less and have lower government expenditures. The second example is a nonlinear income tax with fixed government size. It is shown that freedom is decreasing in the convexity of the tax. Governments with higher emphasis on freedom will therefore choose a more concave tax, i.e. redistribute less income to the poor.

Assume a population of individuals i distributed on the unit interval $[0,1]$. Let the utility function of each consumer i be

$$u_i(c(i), l(i), g) = \alpha(i) \ln c(i) + l(i) + \ln g. \tag{30}$$

The preference parameter α is distributed log-normally, i.e., $\alpha(i) \sim \ln \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$ with density θ_α. Individuals face a budget constraint, $w(i)(1 - l(i)) = c(i)p$. Let there be a firm with production function $C = \int_i (1 - l(i))\beta di$. The efficiency parameter, which is not individual specific, is also distributed log-normally with $\beta \sim \ln \mathcal{N}(\mu_\beta, \sigma_\beta^2)$ with density θ_β. The firm's profit is given by $pC - \int w(i)(1 - l(i))di$ where $C = \int c(i)di$.

The optimal consumption of an individual is then given by $c^*(i, \beta) = \beta\alpha(i)$ and $l^* = 1 - \alpha(i)$. The freedom measure measures the weighted mutual information between the preference parameter α and the outcomes c^* and l^*. Since l^* is a deterministic function of α, the interesting component of freedom is the freedom over consumption, c^*. We are therefore interested in:

$$\Phi = \int \int v(c^*, \alpha)\theta(\alpha, c^*) \ln \frac{\theta(\alpha, c^*)}{\theta(\alpha)\theta(c^*)} dc^* d\alpha \tag{31}$$

Note that since individuals are symmetric in their distribution of α, freedom equals the freedom of each individual. Note also that the continuous version of mutual information is invariant to scaling of the variables. That is, a strong (weak) control over small changes in c^* will be equivalent to a strong (weak) control over large changes in c^*. It therefore makes sense to adjust for the scale of c^* via the freedom value function $v(\alpha, c^*)$. One possibility would be to use qualitative distance metrics as explained above. To keep the analysis tractable, it will be assumed that v is an increasing function of $E[c^*]$ which is independent of c^* and α. This is intuitive as long as one does not value extreme consumption choices as more important for freedom than consumption choices closer to the mean.

Proposition 4. *The positive freedom in the described production economy is given by*

$$\Phi = -v\left(e^{\mu_\beta + \mu_\alpha + (\sigma_\beta^2 + \sigma_\alpha^2)/2}\right) \frac{1}{2} \ln \left(1 - \frac{\sigma_\alpha^2}{(\sigma_\alpha^2 + \sigma_\beta^2)^{1/2}}\right) \tag{32}$$

Several properties are noteworthy: First, freedom is increasing in the mean of α, and the variance of α, which are both increasing functions of μ_α and σ_α^2. This reflects that the larger the range of reasonable preferences, the larger the potential freedom. For very small variance and expectation of α, the individual is almost completely determined in his preferences over consumption and leisure. If the variance of α is large, however, the individual may prefer either high consumption and low leisure, or vice versa. This effect could also be observed in the opportunity set based measure of Jones and Sugden (1982). For a given opportunity set, the measure increases when adding additional preference relations with new maximal elements.

Second, $MI(\alpha, c^*)$ is decreasing in σ_β^2 given a fixed $E[\beta]$. Holding constant the expectation of β (by a decrease in μ_β) while increasing the variance of β thus yields a decrease in freedom. This is intuitive since stochastic production possibilities limit the extent to which an individual's preferences control consumption. An individual is less free, if his consumption is strongly dependent on fluctuating production conditions. Since the technology directly determines consumer prices here, this argument extends to price stability: The more stable the prices are, the greater the freedom of an agent. The simple model of this section therefore provides microfoundations for the use of price stability as a macroeconomic indicator of freedom as for example done in Gwartney et al. (2010).[8]

Third, freedom is increasing in μ_β. Since μ_β is the geometric mean of the technology parameter, this means the greater the efficiency of the individuals, the greater their freedom of consumption. This is consistent with the idea of Sen (1991) as freedom being increasing in well-being.

These properties depend to some extent on the choice of the freedom value function. For example, a very convex distance metric would lead to a much greater emphasis on qualitative differences in consumption, even if they would come at a loss in control. Then freedom may be increasing in the variance of the technology parameter since it leads to more qualitatively diverse outcomes.

7.1 Optimal Government Size

The above analysis is now extended by a government which decides on a (consumption) tax rate t and the amount of a public good g to produce. The budget restriction of the individual thus changes to $w(i)(1 - l(i)) = (1 - t)pc(i)$. gp^* must equal the tax revenue in expectation, $\int\int t(1 - l^*(\alpha, \beta, g))w^*(\beta)\theta_\alpha(\alpha)\theta_\beta(\beta)did\beta = gp^*$ where p^* and $w^*(i)$ are equilibrium price and wage. Note that equilibrium leisure l^* does not depend on g or β due to the assumption of quasilinear preferences. For simplicity, the consumer freedom in this economy will simply be denoted $\Phi(g)$.

[8]It has of course to be noted that the price stability in this model refers to relative price stability and not absolute price stability. However, it is clear how an extension to an intertemporal framework with real imperfections would yield the same result for absolute price stability over time. If unstable prices influence consumption, freedom will decrease as there is less room for individual preferences to influence consumption.

The government maximizes a linear combination of expected total welfare and freedom $\lambda \int \int \theta_\alpha(\alpha)\theta_\beta(\beta)(\alpha \ln c^*(\alpha,\beta,g) + l^*(\alpha) + \ln(g))d\alpha d\beta + (1-\lambda)\Phi(g)$ with $\lambda \in (0,1)$.

Proposition 5. *The optimal government consumption,*

$$g^* = \arg\max_g \lambda \int \int \theta_\alpha(\alpha)\theta_\beta(\beta)(\alpha \ln c^*(\alpha,\beta,g) + l^*(\alpha) + \ln(g))d\alpha d\beta + (1-\lambda)\Phi(g), \quad (33)$$

is strictly increasing in λ.

The optimal government size is therefore increasing in the weight the policy maker attaches to utility maximization. Conversely, this means that optimal utilitarian taxes and expenditures will be too high if the policy maker attaches positive value to freedom. The extent to which the utilitarian government size is too large is increasing in $MI(\alpha, c^*)$. This means especially in cases where individual preferences have a strong control over consumption, i.e. in stable economic environments, it is desirable to tax less. Although this result has been derived here for the (explicitly solvable) case of lognormally distributed parameters, the intuition carries over to more general cases: the larger the mutual information between preferences and consumption, the stronger the loss in freedom from higher taxation.

7.2 Optimal Libertarian Redistribution

The following section extends the analysis by allowing for personal differences in ability and nonlinear income taxation. The utilitarian case of a similar model has been considered by Mirrlees (1971) and Diamond (1998). Each individual has an individual-specific efficiency parameter $\gamma(i) \sim \ln\mathcal{N}(\mu_\gamma, \sigma_\gamma^2)$, which is independent of α and β. Instead of considering the optimal government size we may be interested in the optimal tax scheme given a fixed government size $\bar{g}$. Tax rates are therefore allowed to depend on income, such that the budget constraint of the individual becomes $c(i)p = (w(i)(1-l(i)))^x r(x)$ if firms's profits are zero. x determines the curvature of the tax and $r(x)$ determines the level of the tax given the curvature. $x < 1$ means that the tax rate is progressive in income, i.e. the degree of income redistribution is decreasing in x. We can therefore interpret $-x$ as the degree of tax progression. Ex ante it is unclear whether high or low redistribution is optimal. The reason is that redistributive income taxes have two effects on freedom. On the one hand they reduce the influence of γ on consumption. This is desirable for freedom since it reduces the control of outside forces on consumption. On the other hand redistributive taxes also reduce the influence of α on consumption. This is undesirable, since it means that who chooses to work more due to his preferences will be adversely affected by the taxes. As it turns out, the latter effect dominates.

Proposition 6. *The progression of the income tax,*

$$-x^* = -\arg\max_x \lambda \int \int \int \theta_\alpha(\alpha)\theta_\beta(\beta)\theta_\gamma(\gamma)(\alpha \ln c^*(\alpha,\beta,\gamma,x) + l^*(\alpha) + \ln(\bar{g}))d\alpha d\beta d\gamma \quad (34)$$

$$+ (1-\lambda)\Phi(x), \quad (35)$$

is strictly increasing in λ*.*

It should be noted that the above result depends to some extent on the utility functions and parameter distributions involved. Since in the above case c^* is a function of the product of the efficiency and preference parameter, one cannot redistribute to the less efficient (and thereby reduce this limitation to freedom), without also limiting the extent to which preferences determine outcomes. Under different assumptions on preferences and parameter distributions, the redistributive effect of higher progression may dominate if the incentive effects are small.

8 Concluding Remarks

The paper has proposed a class of freedom measures for extensive form games. It has shown that such measures can be applied in a wide range of economic models, where freedom is normatively relevant. In many of these applications there already exist measures for normatively relevant phenomena such as option diversity, discrimination, price stability or government size. The measure presented in this paper however provides a unified normative framework according to which one can evaluate all these cases.

From a political economy perspective, the possibility to analyze freedom in economic models may be useful to explain deviations from optimal (utilitarian) policies. In the example of the exchange economy it was shown that optimal utilitarian taxation is too high and redistributes too little if politicians care about freedom. The paper hopefully contributes to bridging this gap between the normative reasoning within and outside of economics.

A Derivations of Properties

The derivations are only provided for the positive freedom measure. The derivations for the negative measure are largely analogous.

Let $(\Game', \theta', v') = (\Game, \theta, v)/^r_u Q$, then:

$$\begin{aligned}
\Phi(\Game', \theta', v') =& \sum_o \sum_{\bar{u} \in U \setminus u} v'(o, \bar{u}) \hat{p}'(\bar{u}) \theta'(o|\bar{u}) \ln \frac{\theta'(o|\bar{u})}{\theta'(o)} \\
&+ \sum_o \sum_{q \in Q} v'(o, q) \hat{p}'(q) \theta'(o|q) \ln \frac{\theta'(o|q)}{\theta'(o)} \\
=& \sum_o \sum_{\bar{u} \in U \setminus u} v(o, \bar{u}) \hat{p}(\bar{u}) \theta(o|\bar{u}) \ln \frac{\theta(o|\bar{u})}{\theta(o)} \\
&+ \sum_o \sum_{q \in Q} v(o, u) r(q) \hat{p}(u) \theta(o|u) \ln \frac{\theta(o|u)}{\theta(o)} \\
=& \sum_o \sum_{\bar{u} \in U \setminus u} v(o, \bar{u}) \hat{p}(\bar{u}) \theta(o|\bar{u}) \ln \frac{\theta(o|\bar{u})}{\theta(o)} \\
&+ \sum_o v(o, u) \hat{p}(u) \theta(o|u) \ln \frac{\theta(o|u)}{\theta(o)} \\
=& \Phi(\Game, \theta, v)
\end{aligned} \tag{36}$$

Let $(\Game', \theta', v') = (\Game, \theta, v)/^r_o Q$, then:

$$\begin{aligned}
\Phi(\Game', \theta', v') =& \sum_u \sum_{\bar{o} \in O \setminus o} v'(\bar{o}, u) \hat{p}'(u) \theta'(\bar{o}|u) \ln \frac{\theta'(\bar{o}|u)}{\theta'(\bar{o})} \\
&+ \sum_u \sum_{q \in Q} v'(q, u) \hat{p}'(u) \theta'(q|u) \ln \frac{\theta'(q|u)}{\theta'(q)} \\
=& \sum_u \sum_{\bar{o} \in O \setminus o} v(\bar{o}, u) \hat{p}(u) \theta(\bar{o}|u) \ln \frac{\theta(\bar{o}|u)}{\theta(\bar{o})} \\
&+ \sum_u \sum_{q \in Q} v(o, u) \hat{p}(u) r(q) \theta(o|u) \ln \frac{r(q)\theta(o|u)}{r(q)\theta(o)} \\
=& \sum_u \sum_{\bar{o} \in O \setminus o} v(\bar{o}, u) \hat{p}(u) \theta(\bar{o}|u) \ln \frac{\theta(\bar{o}|u)}{\theta(\bar{o})} \\
&+ \sum_u v(o, u) \hat{p}(u) \theta(o|u) \ln \frac{\theta(o|u)}{\theta(o)} \\
=& \Phi(\Game, \theta, v)
\end{aligned} \tag{37}$$

B Proof of Proposition 1

Proof. Setting $v(x,u) = 1$ gives:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \sum_{u \in U_i} \hat{p}(u) \sum_{x \in C} \theta(x|u) \left(\ln \frac{\theta(x|u)}{\theta(x)} \right) \tag{38}$$

Since for each u there exists x: $\theta(x|u) = 1$ it holds for all other x' that $\theta(x'|u) = 0$ and thus $\theta(x'|u)\ln(\theta(x'|u)/\theta(x')) = 0$. Define $U(x) = \{u \in U_i : \theta(x|u) = 1\}$. We then get after rearranging terms:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \sum_{x \in C} \left(\ln \frac{1}{\theta(x)} \right) \sum_{u \in U(x)} \hat{p}(u)\theta(x|u) \tag{39}$$

Moreover, $\theta(o) = \sum_{u \in U(x)} \hat{p}(u)\theta(x|u) + \sum_{u \in U_i \setminus U(x)} \hat{p}(u)\theta(x|u) = \sum_{u \in U(x)} \hat{p}(u)\theta(x|u)$:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \sum_{x \in C} \theta(o) \left(\ln \frac{1}{\theta(o)} \right) = F_S(C, \theta) \tag{40}$$

By rationality and the fact that the members of U_i represent the members of $\mathcal{R}$:

$$\theta(x) > 0 \Leftrightarrow x \in \{x \in C : \exists R \in \mathcal{R} : \forall y : xRy\} \tag{41}$$

Since all outcomes with strictly positive probability have equal probability:

$$\theta(x) > 0 \Rightarrow \theta(x) = \frac{1}{\sharp\{x \in C : \exists R \in \mathcal{R} : \forall y : xRy\}} \tag{42}$$

Inserting this yields:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \ln\left(\sharp\{x \in C : \exists R \in \mathcal{R} : \forall y : xRy\}\right) \tag{43}$$

Since ln() is monotonically increasing, it follows that Φ^{pos} represents $\succsim_{F,JS}$. □

C Proof of Proposition 2

Proof. Setting $v(x,u) = 1$ gives:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \sum_{u \in U_i} \hat{p}(u) \sum_{x \in C} \theta(x|u) \left(\ln \frac{\theta(x|u)}{\theta(x)} \right) \tag{44}$$

Since for each u there exists x: $\theta(x|u) = 1$ it holds for all other x' that $\theta(x'|u) = 0$ and thus $\theta(x'|u)\ln(\theta(x'|u)/\theta(x')) = 0$. Define $U(x) = \{u \in U_i : \theta(x|u) = 1\}$. We then get after rearranging terms:

$$\Phi^{pos}(\eth^T(C), \theta, v) = \sum_{x \in C} \left(\ln \frac{1}{\theta(x)} \right) \sum_{u \in U(x)} \hat{p}(u)\theta(x|u) \tag{45}$$

Finally $\theta(o) = \sum_{u \in U(x)} \hat{p}(u)\theta(x|u) = \sum_{u \in U_i} \hat{p}(u)\theta(o|u)$.

$$\Phi^{pos}(\partial^T(C), \theta, v) = \sum_{x \in C} \theta(o) \left(\ln \frac{1}{\theta(o)} \right) = F_S(C, \theta) \tag{46}$$

□

D Proof of Proposition 3

Proof. Setting $v(A, u) = \lambda(A)$ and $v(A^C, u) = 0$ gives:

$$\Phi^{pos}(\partial^T(C, A), \theta, v) = \sum_{u \in U_i} \hat{p}(u)\lambda(A)\theta(A|u) \ln \frac{\theta(A|u)}{\theta(A)} \tag{47}$$

Since for each u there exists x: $\theta(x|u) = 1$ it holds for outcome A that $\theta(A|u) = 1$ or $\theta(A|u) = 0$. Define $U(A) = \{u \in U_i : \theta(A|u) = 1\}$. It follows that:

$$\Phi^{pos}(\partial^T(C, A), \theta, v) = \sum_{u \in U(A)} \hat{p}(u)\lambda(A) \ln \frac{1}{\theta(A)} \tag{48}$$

Since $\theta(A) = \sum_{u \in U(A)} \hat{p}(u)\theta(A|u) = \sum_{u \in U(A)} \hat{p}(u)$, we have:

$$\Phi^{pos}(\partial^T(C, A), \theta, v) = \lambda(A)\theta(A) \ln \frac{1}{\theta(A)} \tag{49}$$

Summing over A:

$$\sum_{A \subseteq X} \Phi^{pos}(\partial^T(C, A), \theta, v) = \sum_{A: A \cap C \neq \varnothing} \Phi^{pos}(\partial^T(C, A), \theta, v) = -\sum_{A: A \cap C \neq \varnothing} \lambda(A)\theta(A) \ln \theta(A) \tag{50}$$

where the first step follows from the fact that if $A \cap C = \varnothing$ then $\theta(A) = 0$. Since with the aforementioned (Section 3) abuse of notation $\theta(A) = \sum_{x \in A} \theta(x)$, we get:

$$D_{NP}(C, \theta, \lambda) = -\sum_{A: A \cap C \neq \varnothing} \lambda(A) \sum_{x \in A} \theta(x) \ln \sum_{y \in A} \theta(y) = \sum_{A \subseteq X} \Phi^{pos}(\partial^T(C, A), \theta, v) \tag{51}$$

□

E Proof of Proposition 4

Proof. Given a constant freedom value function $v = E[c^*]$, the measure is proportional to the mutual information of α and c^*:

$$\Phi = E[c^*] MI(\alpha, c^*) = E[c^*] \int \int \theta(c^*, \alpha) \ln \frac{\theta(c^*, \alpha)}{\theta(\alpha)\theta(c^*)} dc^* d\alpha \tag{52}$$

Mutual information is invariant under homeomorphisms of the variables (Kraskov, Stögbauer, & Grassberger, 2004). Thus, $MI(\alpha, c^*) = MI(\ln\alpha, \ln c^*)$. Taking logarithms, we get: $\ln c^* = \ln\beta + \ln\alpha$. Since β and α are lognormally distributed, c^* is lognormally distributed with $c^* \sim \ln\mathcal{N}(\mu_\alpha + \mu_\beta, \sigma_\alpha^2 + \sigma_\beta^2)$. Therefore, $\ln c^*$, $\ln\alpha$, and $\ln\beta$ are all normally distributed. For jointly normally distributed variables, $MI = -1/2\ln(1-\rho)$ where ρ is the correlation coefficient. Since $\Phi = v(E[c^*])MI(\alpha, c^*) = v(E[c^*])MI(\ln\alpha, \ln c^*)$ it then follows:

$$\Phi = -v\left(e^{\mu_\beta+\mu_\alpha+(\sigma_\beta^2+\sigma_\alpha^2)/2}\right)\frac{1}{2}\ln\left(1-\frac{\sigma_\alpha^2}{(\sigma_\alpha^2+\sigma_\beta^2)^{1/2}}\right) \tag{53}$$

since $Cov[\ln\alpha, \ln c^*] = \sigma_\alpha^2$, $Var[\ln\alpha] = \sigma_\alpha^2$ and $Var[\ln c^*] = \sigma_\alpha^2 + \sigma_\beta^2$. Since c^* is lognormally distributed, we moreover have $E[c^*] = e^{\mu_\beta+\mu_\alpha+(\sigma_\beta^2+\sigma_\alpha^2)/2}$. □

F Proof of Proposition 5

Proof. It can easily be shown that the new equilibrium consumption is $c^*(i) = \alpha(i)\beta(1-t)$. c^* is therfore distributed according to $c^* \sim \ln\mathcal{N}(\mu_\alpha + \mu_\beta + \ln(1-t), \sigma_\alpha^2 + \sigma_\beta^2)$. The government therefore maximizes:

$$\max_g \lambda \int \alpha\ln(\alpha\beta(1-t(g))) + \ln(g)dF(\alpha,\beta) + (1-\lambda)E[c^*]MI(\alpha, c^*) \tag{54}$$

with $t(g) = g/(\int \alpha(i)\beta di)$.

The first order condition is:

$$\lambda\int \alpha\frac{-t'(g^*)}{1-t(g^*)} + 1/g^* dF(\alpha,\beta) - (1-\lambda)t'(g^*)MI(\alpha,c^*) = 0 \tag{55}$$

since MI is independent of g^*.[9] Notice also that $t'(g^*)$ is a constant.

The first order condition is a necessary and sufficient condition for optimality due to the continuity and concavity of the problem. Rearranging yields:

$$\lambda\int \alpha\frac{-t'(g^*)}{1-t(g^*)} + 1/g^* dF(\alpha,\beta) = (1-\lambda)t'(g^*)MI(\alpha,c^*) > 0 \tag{56}$$

where the inequality follows from the fact that MI is strictly positive. We can use the implicit function theorem to find $\frac{\partial g*}{\partial\lambda}$:

$$\frac{\partial g*}{\partial\lambda} = \frac{-\lambda\int \alpha\frac{-(t'(g^*))^2}{(1-t(g^*))^2} - 1/(g^*)^2 dF(\alpha,\beta)}{\int \alpha\frac{-t'(g^*)}{1-t(g^*)} + 1/g^* dF(\alpha,\beta) + t'(g^*)MI(\alpha,c^*)} \tag{57}$$

From (56) follows that the first term of the denominator is strictly positive. Due to positivity of MI the denominator is then positive. Moreover, since $\alpha > 0$ the numerator is strictly positive. Thus, $\frac{\partial g^*}{\partial\lambda}$ is positive. □

[9] This can easily seen from the fact that the covariance of $\ln\alpha$ and $\ln c^*$ does not depend on g^*. Repeating the steps from the previous section with the new equilibrium condition also yields this result.

G Proof of Proposition 6

Proof. In equilibrium, $c^*(i) = r(x)(\beta\alpha(i)\gamma(i)x)^x$. Thus, $c^* \sim \ln\mathcal{N}((\mu_\beta + \mu_\gamma + \mu_\alpha)x + \ln x^x r(x), x^2(\sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\beta^2))$. $r(x)$ needs to be chosen such that $\bar{g} = \int \alpha\beta\gamma x - r(x)(\alpha\beta\gamma x)^x dF(\alpha,\gamma,\beta)$. Thus, $r(x) = \frac{E[\alpha\gamma\beta]x - \bar{g}}{x^x E[(\alpha\gamma\beta)^x]}$. The government maximizes:

$$\max_x \int \alpha \ln(r(x)(\alpha\beta\gamma x)^x + 1 - \alpha x) + \ln(\bar{g}) dF(\alpha,\beta,\gamma) \tag{58}$$

$$+ v(E[c^*])MI(c^*,\alpha) \tag{59}$$

It is sufficient to show that both $v(\ldots)$ and $MI(c^*,\alpha)$ are increasing in x. Note that $E[c^*] = E[\alpha\beta\gamma]x - \bar{g}$ is strictly increasing in x since $E[\alpha\beta\gamma] > 0$. Since $v(\ldots)$ is an increasing function, v is increasing in x. Due to invariance of MI to homomorphisms of the variables,

$$MI(c^*,\alpha) = MI(\ln c^*, \ln\alpha) = -0.5\ln(1 - \frac{COV[\ln c^*, \alpha]}{(Var[\ln c^*] + Var[\ln\alpha])^{1/2}}) \tag{60}$$

We have $COV[\ln c^*, \ln\alpha] = x\sigma_\alpha^2$, $Var[\ln\alpha] = \sigma_\alpha^2$ and $Var[c^*] = x^2(\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\gamma^2)$, therefore:

$$MI(c^*,\alpha) = -0.5\ln(1 - \frac{x\sigma_\alpha^2}{(x^2(\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\gamma^2) + \sigma_\alpha^2)^{1/2}}) \tag{61}$$

The sign of the first derivative with respect to x is determined by the expression:

$$\frac{\partial MI(c^*,\alpha)}{\partial x} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\partial \frac{x\sigma_\alpha^2}{x^2(\sigma_\alpha^2+\sigma_\beta^2+\sigma_\gamma^2)+\sigma_\alpha^2)^{1/2}}}{\partial x} \gtreqless 0 \tag{62}$$

The latter simplifies to $\sigma_\alpha^2 \gtreqless 0$ and thus MI is strictly increasing in x. $\square$

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