Ramón y Cajal: mediation and meritocracy.∗

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Abstract

We analyze the centralized matching mechanism associated to the Ramón y Cajal Program. This Program is used in Spain to promote the hiring of top researchers in Spanish R&D centers and academic institutions. We model the process as a bilateral matching market to study if the mechanism provides the incentives to hire good researchers. We analyze the model both under complete and incomplete information. The theoretical findings and the data from the first editions of the program indicate that the model provides poor incentives to the agents involved and does not prevent from collusion between research centers and candidates. While the objective is to reward the best researchers the mechanism only guarantees “almost stable solutions”.

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1 Introduction

In this paper we analyze the mechanisms that have been used to match researchers and research centers in the Ramon y Cajal Program. The objective of this paper is to provide insights on the capacity of the mechanism to accomplish its objectives and to explain how it has evolved to its actual form.

The Ramon y Cajal Program is one of the most important actions of the Spanish's National Research, Development and Innovation Plan (R&D&I). This plan was a set of guidelines for Public R&D Policies. It assigns grants to qualified research centers to hire new researchers.

Let us summarized the main concerns that advocated for the new R&D&I Plan (see Menendez Sanz et all. (2003) for a detailed discussion). On the one hand Spain has a low number of researchers with respect to the international standards. Furthermore, there are important distortions in the recruitment process that affect negatively the level of research outputs. The R&D&I Commission have pointed out two reasons for this situation: the lack of meritocracy in the selection of researchers and, the high incidence of endogamous practices. Also a significant share of researchers where employed in poor conditions: low level salaries and temporary positions were frequent especially at entry level. Also the perspectives of employment stabilization and career advancements were worsening along time.

On the other hand there was an important growing in supply of PhDs and the demand has been detected joint with mismatches of the supplies in some areas. To overcome such problems The Ramon y Cajal Program for (co-)financing new research positions was designed.

The Ramon y Cajal Program attempt to accomplish the following objectives:
1. Create an entry point in a "research career" for PhDs with a 5 years contract.
2. Stabilize and improve the working conditions of post-doc researchers.
3. Facilitate the return of Spanish researchers working abroad.
4. Identify the best quality researchers and facilitate their employment within the Spanish R&D system.
5. Encourage R&D centers to define their strategic priorities.
6. Support the demand of researchers in priority areas of the National R&D&I Plan.
7. Establish co-responsibility of hosting institutions and the Regional Governments.
8. Support mobility of researchers.

Three main points have been made to the public to explain the novelty of the approach that lead to the design of mechanism. The first is that introduces a competitive and centralized selection of the candidates through an exhaustive and objective evaluation of their merits and potentialities realized through a governmental agency. Then it leads the research centers to better define their objectives and to develop some prioritized fields of investigation selected through the program. Finally it makes centers and authorities participate in the joint financing of the new contracts.
In this paper we focus on the mechanism that matches researches and research centers and its evolution. The matching procedure was designed and used, with no substantial modifications for the first three years of the Program. After concern publicly expressed by the participants and by the institutions about its fairness and the accomplishment of its objectives the procedure was redesigned for the fourth edition.

We present a simplified setup to describe main features of the matching process. The model provides predictions compatible with the data available. It allows an explanation of the first model in terms of lacks of incentives and the need of its redesign. We suggest that the new matching mechanism can provide a partial solution of the problems.

2 The Ramon y Cajal matching process

Let us discuss how the different tensions that lead to the design of the program interact in this model. In particular we would like to determine how the concerns about researchers stabilization of young researchers and the vetoes imposed by centers match with the objective of a competitive and meritocratic centralized selection of the candidatures.

Let us provide now an informal description of the selection process used, with minor variations, in the first triennium. A preliminary step is the selection of prioritized areas to support and of the projects to fund through the program. The evaluation was realized by a Commission at the Ministry of Science and Technology. This Commission was responsible of the program on the base of centers' requests. Finally each participating research center is endowed with a number of financed contracts. Those contracts represent the maximum number of researches they can hire on the process. The decision is made public. Any researcher who wants to participate has to contact the center(s) she would like to join and to ask for a preliminary acceptance. To obtain this acceptance each candidate must provide the center with a scientific curriculum and a detailed and extensive research statement. The proposal has to be attaining to some of the centers' investigation project and has to contain a description of the activities to be realized. The proposals of each one of the proper candidates are submitted to the Ministry.

One the application part is over the one commission in each research area is appointed to evaluate the candidates. This commission was composed by well-known specialists in the field. The researchers are ranked up to fill the number of financed contracts.

The list with the financed researchers is made public and the matching process between researches and institutions stars. Priority is given to the best aspirants according to their ranking in its area. It means that the top ranked candidate had to choose which center to join, among the ones that has previously accepted her as a candidate. Each one of the other candidates could choose among the centers that accepted them and had not all position filled with better ranked researchers. In this stage centers are "passive subjects".
Each one has to accept all candidates reaccepted in the preliminary stage up to complete the number of financed positions assigned. If at the end of this process all available contracts had been signed or if all acceptable candidates got a position the process would have ended.

Otherwise in a second round of the matching procedural the centers left with unfilled positions are asked to reconsider their acceptation policy about the ranked aspirants who got no position in the previous stage. By mutual agreement of both the parts, some or all of them are allocated to centers. This second round is somewhat informal, the rules leaves a lot of freedom to all the parts. It is remarkable that, in the first edition of the program, only few matchings were concluded in the second stage. Furthermore the unique unsuccessful matchings follow (finally the candidates decided to opt out without signing any contract).

The matching procedure resembles very much the Gale-Shapley algorithm with candidates proposing to centers and centers assuming as preferences the official ranking on the preliminary accepted candidates. The incentives were intended to make the centers internalize the objective criteria on which the evaluation was made.

We found appropriate to model the matching in a way which is equivalent to the Gale-Shapely algorithm. It is as if the Ministry had the role of assigning candidates to institutions once researchers communicated their preferences. However the matching was done in an equivalent (priorities are assigned on the base of the ranking) but decentralized way. The second stage is modeled as the first one.

The mechanism described had been in place for the first three years of the project with minor variations but always preserving the priority of the pre-accepted candidates independently on their absolute ranking. A new procedure has been in place introduced since the fourth edition (2004) coinciding with the refinancing of the program. It is simpler and eliminates the preliminary acceptation phase. Each researcher interested to participate must send a (unique) scientific curriculum and research proposal to the Ministry. The candidates are ranked and the best ones are awarded with a financed contract. In a decentralized way centers and researchers have to conclude agreements. The contracts of the candidates who signed an agreement are confirmed the other ones are withdrawn. The introduction of the new procedure was justified by concerns about the fairness of the first mechanism. The second one actually permits to any aspirant to be evaluated and ranked by the commission.

We point out on the paper the reasons to the failure of the first mechanism and the advantages of the newly adopted one. We have find out that the first mechanism was unable to conciliate the imperative of a competitive and meritocratic selection of the candidates realized by Evaluation Commission with centers corresponsability. Institutions and aspirants were able to jointly manipulate the mechanism up to make the centralized selection process on which a lot of effort has been spent almost irrelevant on the final outcome.

Let us be more precise. Distortions had been detected in researcher’s recruitment, mainly endogamous behavior. Then the problem essentially lies in
the "preferences" that research centers have. They prefer to hire the researchers they already knew, mainly researchers already working for them with short-term contracts. Such researchers would like to stabilize their positions. This makes possible that in the usual decentralized recruitment process candidates’ merit is not the main concern.

If the purpose of the mechanism was to create a system of incentives for centers to hire top researchers the coordination between centers and researches can prevent this outcome. The authority partially finances new contracts under the conditions that candidates pass a centralized selection process. The original procedure gives centers the possibility of vetoing any candidate through a preliminary acceptation phase. This acts as a philter. Centers and candidates were left with large possibility of colluding: any center can decide to accept only the candidates if with certainty such researchers want to join the center and if they are not too bad. A candidate who did not get approval by at least one institution does not enter in the selection process independently of her capacities. It is sufficient to create a large set of collusive equilibria which depends only agents preferences and on the distributions of the number of contracts among centers.

We explain it in term of stability but without the unconditionally positive accent usually given in the literature. Instead we take it as a symptom that the mechanism is not able to provide incentives other than the ones that were already present in the system itself, which are the agents’ preferences. The new procedure is a partial solution to the problem. No candidate can be excluded from the program before the public evaluation process. On the one side it prevents too low ranked candidates from getting positions. On the other one hand, given the winners’ list, the decentralized matching process does not guarantee to the best ranked researchers their favorite positions (or any position at all). Once the winning candidates are known, the final outcome will depend only on agents’ preferences and the procedure they use. It does not create incentives in hiring top researchers if for "creating incentive" we mean make the center to internalize the ranking criteria. It does not produce changes in centers’ preferences in accordance with the ranking. The mechanism does not make preferences completely responsive to the ranking. This is because it is not different for the centers to hire the best or the second best candidate. However, the system guaranties that the researches hired are the best among those that apply because only they will be financed. It is appealing for centers to participate to the program especially because it reduces the financial burden of new long-term contracts. In this sense it accomplishes part of its objectives by improving researchers’ job situation.

2.1 Structure of the paper and related literature

The study of sequential matching mechanism under perfect and complete information is not totally new in the literature even with few examples as Alcalde et all. (1998), Alcalde and Romero-Medina (2000) and (2003). In the Section 4 we study the sequential games that model the mechanism used in the first edition by the Ramon y Cajal Program. We explore the strategic incentives
of the players in the simplest situation. The structure of the game is more complex with respect to the cited papers. As both candidates and centers play twice strategy space is larger. We characterize the Subgame Perfect Nash Equilibrium outcome which results to have peculiar stability properties. What is particularly important for our concerns is that the outcome set only depends on agents preferences and not on the order in which the commission ranks the accepted candidates (Theorem 1). This results settle a bitter discussions about the some commissions and the Government on the necessity or not to provide strictly order list of admitted candidates. If we require for an outcome to be meritocratic to provide the better researches with better positions according to their preferences then it is necessary that centers order candidates as the Commission does. However, in this case, no incentives to hire good researchers are needed.

At first sight the model described share some features of the sequential mechanism described by Alcalde and Romero-Medina (2004). In the mechanism we have described candidates can apply to centers according to some priority order and the outcome is the candidates’ optimal stable matching which is independent on any initial order. In our model the preliminary acceptation phase give centers larger strategic possibilities (a sort of veto power) and this is the reason of the larger set implemented. Furthermore we deal with a more general problem in which the matching procedure is restricted by the scarcity of contracts with respect to centers aspirations. Our mechanism can be seen also as a generalization of the model analyzed by Sotomayor (2004).

We don’t model explicitly the matching procedure in use since 2004. This procedure is decentralized. We assume a minimal stability property of the outcome set and we show that a well ranked candidate cannot be excluded by the program in favor of a worst aspirant unless she not acceptable to all centers that have not all position filled with better researchers and that she would be willing to join.

We briefly consider also the possibility that centers have of strategically hiding to the authorities their real staff necessities. As observed by Sönmez (1997) in any revelation games induced by a stable mechanisms (anyway ours is not stable nor a revelation game) centers have incentives to reduce their true capacities and induce unstable outcomes. By mean of an example we show that both mechanisms suffer of the same defect. The Program not only does not give sufficient incentives to research centers to seeking for good researchers, but it might produce instabilities.

The analysis of the mechanism under complete information and fit quite satisfactorily with the data available. In the Section 5 we introduce informational incompleteness in the old model. We would like to know if the lack of information cans alone shape centers’ preferences in a meritocratic way. With this purpose in mind we simplify the preference structure: we assume that each center only distinguishes between acceptable and not acceptable candidates. By doing this we make a charitable assumption: it is equivalent to assume that centers assume the ranking criteria on the set of their acceptable researchers. Furthermore we assume that each center, given the information each researcher
submit to it, is able to determine the relative ranking of its applicant.

Little work has been done in the analysis of matching markets under incomplete information before. Roth and Rothblum (1999) and Ehlers (2004) consider the incentives candidates have under incomplete in formation in mis representing their preferences when the Gale and Shapley mechanism is used and information is symmetric. They show that when information is symmetric they can at best have incentives to truncate their preferences list. For a game-theoretic analysis the two main references are Ordoñez de Haro and Romero-Medina (2004) who studies a sequential hiring game similar to the one analyzed in Alcalde et al. (1998) and Pais (2005) who studies a particular set of equilibrium of the Gale Shapley algorithm under incomplete and symmetric information.

On the one hand we find a light incentive effect of the uncertainty on the ranking. The reason is that accepting a low ranked candidate instead than a better one reduces the probability of matching. We classify the instability that can in two different types. One derive from to the cut of poorly ranked researchers, the other one is manipulative. We observe that both such instabilities cause inefficient outcomes in which some contracts are not assigned. The number of contract not assigned in the first round was small. Therefore either such uncertainty does not exist or it is the mechanism itself which induces agents to share the information they have in order to prevent the inefficiency.

The uncertainty seems to affect more some applicants. A limited number of candidates applied to more than one research center despite the costs of presenting different research proposals. This behavior can be explained in our model as an effect of incomplete information on other agents' preferences. While most candidates are well informed on the market structure some "outsiders" do not know the possibility they have in entering in it but they are pretty confident in their possibilities of being assigned with a contract, once pre-accepted by some center.

We use stability as reference with an accent distinct from most of matching market literature. Particular attention has been devoted to stability as natural concept: the stable set is the core of the game in the one-to-one case and it is a subset of the core in the many-to-one case. (see for instance Roth and Sotomayor (1990)). More important for market design has been the observation that markets that adopt stable mechanisms seem to be more successful than the ones that use unstable mechanisms (see Roth (1984), Roth (2002)). In our work stability has a somehow less positive flavor. On one side it prevents that the parts sign contracts that could have rearranged in a more satisfactory way. On the other one depends only on agents' preferences. Market designer' objectives is to correct a somehow collusive situation by meritocratic considerations and the stability of the final outcome is a confirmation that the mechanism by it self can not play this role.
The model

We consider a bilateral matching market with $k$ research centers (or departments) and $q \geq k$ aspirant researchers. Let $D = \{d_1, ..., d_k\}$ be the set of the research centers and let $R = \{r_1, ..., r_q\}$ be the set of aspirant researchers. Each $d \in D$ is endowed with a complete, strict, transitive preference relation, $P_d$ on $2^R$ where $2^R$ is the set of subsets of $R$, which is the set of all possible research groups. For every $S, S' \in 2^R, SP_dS'$ means that $d$ prefers to employ research group $S$ to research group $S'$. If $S = \emptyset$ then we will say that $S'$ is unacceptable to $d$, meaning that $d$ would prefer not to enroll any new researcher rather than enroll group $S'$. Given $P_d$ and $S \in 2^R$, the choice set $C_d(S)$ is the research group that $d$ would like to enroll most when $S$ is available, formally $C_d(S) = \sup_{P_{d}\subseteq S'}\{S' \subseteq S\}$. For all $r, r' \in R$ we will write $rP_dr'$ instead than $\{r\}P_dr'\{r'\}$. For each $d \in D$'s quota, $q_d$ is the maximum number of researchers that $d$ is willing to employ, formally $q_d = \max_2\{S : SP_d\emptyset\}$. Set $q_i = q_{d_i}$ and set $q = (q_1, ..., q_k)$. On the other side each researcher $r$ has complete, strict, transitive strict preferences $P_r$ on $D \cup \{r\}$. For every $d, d' \in D, dP_r d'$ means that $r$ would prefer to be employed by $d$ rather than by $d'$. Any center $d$ such that $rP_r d$ will be said to be unacceptable to $r$, which means that $r$ would prefer to stay unemployed rather than join center $d$. If $dP_r r, d$ will be said acceptable to $r$. Let $A(r, P_r) = \{d : dP_r r\}$ be the set of departments that are acceptable to $r$ according to $P_r$. In the paper we will make also use of a cardinal representation of $P_r$, $u_r : D \cup \{r\} \rightarrow \mathbb{R}$ to take in account application costs. Let $P = (P_{d_1}, ..., P_{d_k}, P_{r_1}, ..., P_{r_q})$ denote a preference profile. For each $x \in D \cup R$, $R_x$ will denote $x$'s weak preference relation.

**Definition 1** Let $D$ and $R$ be respectively the set of departments and the set of candidates. A matching on $(D, R)$ is a mapping $\mu : D \cup R \rightarrow 2^R \cup D$ such that, for every $d \in D$ and $r \in R$ (1) $\mu(d) \in 2^R$ and $\mu(r) \in D \cup \{r\}$ (2) $\mu(r) = d$ if and only if $r \in \mu(d)$. Let $M$ denote the set of matchings on $(D, R)$.

Let $\mu$ be a matching on $(D, R)$ and let $(D, R, P)$ be a matching market.

**Definition 2** The matching $\mu$ is individually rational for $d \in D$ if $rP_r \emptyset$ for each $r \in \mu(d)$ or $\mu(d) = \emptyset$ $\mu$ is individually rational for $r \in R$ if $\mu(r)P_r r$ or $\mu(r) = r$

**Definition 3** The matching $\mu$ is blocked by a pair $(d, r) \in D \times R$ if $dP_r \mu(r)$ and $r \in C_d(\mu(d)) \cup \{r\}$

**Definition 4** $\mu$ is stable in market $(D, R, P)$ if (1) $\exists \mu(d) \leq q_d$ for all $d \in D$ (2) $\mu$ is individually rational for all agents and (3) $\mu$ does not have any blocking pair.

In words a matching $\mu$ is unstable if there are a department $d$ and a candidate $r$ such that $r$ does not belong to $\mu(d)$ but $r$ would prefer to join $d$ rather than $\mu(r)$ and $d$ would choose to have $r$ in his research group if it was available together with $\mu(d)$. 

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Definition 5  Given a bilateral matching market $M = (D, R, P)$ the stable set of $M$, denoted by $\Gamma(M)$ is the set containing the matchings that are stable in market $M$.

In general the stable set may be empty. The following assumption, if satisfied by all departments’ preferences, it suffices for the stable not to be empty.

Definition 6  Let $d \in D$. The preference relation $P_d$ is said to be substitutable if for each $S \in 2^R$ and for all $r, r' \in S$ whenever $r \in C_d(S)$ then $r \in C_d(S - \{r'\})$.

$d$’s preferences are substitutable if whenever $r$ belongs to $d$’s choice set from some research group $S$ then she is contained in the choice set of any smaller research group she belongs to.

In addition to substitutability we impose an additional condition on firms’ preferences.

Definition 7  Let $d \in D$. The preference relation $P_d$ is said to be $q$-separable if, (1) for each $S \in 2^R$ such that $\not\exists S < q$ and $r \in R \setminus S \cup \{r\} P_d S$ if and only if $r P_d \emptyset$ and (2) $\not\exists S > q$ implies $\emptyset P_d S$.

$d$’ preferences are $q$-separable if it prefers to fill its positions with any acceptable researcher rather than let them empty until the number of hired researchers is $q$. All along the paper we will assume that all departments’ preferences are substitutable and separable with respect to their quota.

Another widely used assumption is responsiveness.

Definition 8  Let $P'_d$ be a profile of strict preferences on $R \cup \{d\}$. $d$’s preferences are responsive to preferences on individuals if for all $r, r' \in R \cup \{\emptyset\}$ and for all $S \subset R$ such that $\not\exists S \leq q_d - 1$

\[ S \cup \{r\} P_d S \cup \{r'\} \iff r P_d r' \]

When preferences are responsive, the preferences on set of researchers are induced (even if in not unique way), by the preferences on individuals. Responsiveness obviously implies substitutability and $q$-quota separability.

Substitutability together with quota substitutability allow for an higher degree of complementarity among departments and workers as the following the example illustrates.

Example 1  Let $r_1$ and $r_2$ be two number theorists and let $r_3$ be a statistician. Department $d$ has 2 positions to fill and it would like to fill them with researchers from different fields. Anyway, on an absolute scale it prefers $r_1$ to $r_2$ and $r_2$ to $r_3$. Such preferences can be formally represented as

$P_d = \{r_1, r_3\}, \{r_2, r_3\}, \{r_1, r_2\}, \{r_1\}, \{r_2\}, \{r_3\}, \emptyset$

They are $2$-quota separable and substitutable but not responsive.

\[^1\text{Actually all results but Proposition 6 hold also under the the assumption of substitutability.}\]
The following version of the so called Deferred Acceptance Algorithm generates a stable matching when all departments have substitutable preferences are substitutable.

**Deferred Acceptance Algorithm in which candidates propose to departments.**

**Stage 1:** Each candidate proposes to her favorite department on his preferences list of acceptable departments. Let $S_d^1$ be the candidates proposing to department $d$. Set $\mu^1(d) = C_d(S_d^1)$ and set $\mu^1(r) = d$ if $r \in \mu^1(d)$ and $\mu^1(r) = r$ otherwise.

**Stage $t+1$:** Each researcher such that $\mu^t(r) = r$ proposes to his most preferred department among those $d$ such that $r \in S_d^t$. Let $S_d^{t+1}$ be the set of candidates proposing at this point to $d$. Then set $\mu^{t+1}(d) = C_d(\mu^t(d) \cup S_d^{t+1})$, $\mu^{t+1}(r) = d$ if $r \in \mu^{t+1}(d)$ and $\mu^{t+1}(r) = r$ otherwise. The algorithm stops after any step $t$ at which $S_d^t$ is empty for all $d$ in $D$, which is when all the researchers not engaged by any department have proposed to all their acceptable departments.

The output of the deferred acceptance algorithm is a stable matching if all departments’ preferences are substitutable. The deferred acceptance algorithm produces the unique researcher’s optimal stable matching: the researchers prefer such a matching to all other stable matchings. Furthermore it is weakly Pareto optimal The version of the Deferred Acceptance Algorithm in which are the departments to propose to subsets of researchers produces the departments’ optimal matching.

When preferences are substitutable and quota separable (but not when they are only substitutable):

(a) the set of unmatched agents is the same in all stable matchings (Martinez, Massó, Neme and Oviedo (2000)).

(b) the set of stable matchings has the structure of a complete distributive lattice with the following order induced by researchers’ preferences: given two matchings $\mu$ and $\mu'$, $\mu \succeq_r \mu'$ if and only if $\mu(r) R_r \mu'(r)$ for all $r \in R$ and $\mu(r) P_r \mu'(r)$ for some $r \in R$. Under the “dual order”, the one generated by centers’ preferences the latticial structure is preserved (Martinez, Massó, Neme and Oviedo (2001))

(c) The researchers’ optimal stable matching is researchers’ strategy-proof (Martinez, Massó, Neme and Oviedo (2004))

Such results are extensions of the analogous ones obtained under responsiveness (see Roth and Sotomayor (1990) for a complete discussion). When all departments’ quota is one also the departments-optimal stable matching is weakly Pareto optimal. Otherwise it is not the case and as showed by Sönmez (1997) centers have strategic incentives to manipulate their quotas.

We recall to the reader the notion of subgame perfect implementation in matching markets framework. Let $D$, and $R$ be two disjoint and non-empty sets of departments and researchers respectively.
Definition 9 Let $\Phi$ be a class of matching markets and let $F$ be a correspondence on the set of matching on $(D,R)$. An extensive form game $(D,R,\Gamma)$ implements $F$ set in Subgame Perfect Equilibrium (SPE) if (1) for each $(D,R,P) \in \Phi$ and for each $\mu \in F(D,R,P)$ there exists a SPE of the game $(D,R,\Gamma,P)$ yielding $\mu$ as outcome (2) each SPE outcome of $(D,R,\Gamma,P)$ belongs to $F(D,R,P)$.

All along the papers we only consider equilibria in pure strategies.

4 The mechanisms

In this section we introduce formally the two matching mechanisms that we are going to be analyzed in the paper. The first one models the matching procedure which has been used in the three first editions of the program. We call it the old mechanism. Along the years it was in place minor variations in the first mechanism have been introduced in the old mechanism without affecting the characteristics relevant for our analysis of the procedure. In particular, the old mechanism requires a preliminary acceptance of the candidates from the centers. Furthermore the first mechanism requires candidates to submit one application for each center she is willing to join. The new mechanism has been used since the fourth edition. In this new procedure the aspirants apply directly to the Ministry.

4.1 The old mechanism

In a preliminary stage each research center communicates to the Ministry its scientific projects in some prioritized areas and ask for a number of contracts to be financed. The Ministry process all departments’ applications and assign to each one a maximum number of contracts to be financed through the Ramón y Cajal Program. The positions are linked to the specific projects presented by the centers. Let $n_i$ be the number of financed contracts that are assigned department $d_i$. $N = \sum_{i=1}^{k} n_i$ is the maximum number of new contracts to finance. Set $n = (n_1,...,n_k)$. The number of the positions and the project to be financed at each department are public knowledge. Then the matching takes place as a five stage game.

Stage 1: Each candidate asks to at least one department the pre-acceptance for joining it. Joint to each application the candidate must send a detailed research project related to one presented to the Ministry by the department itself. The application must specify also candidates’ preferences on the participant institutions. Then each application presents a cost for the candidate. We will introduce as linear term in her utility function. Let $\psi^d_r$ be the cost that $r$ faces in applying to $d$. We assume $0 < \psi^d_r$ for all $r \in R$, for all $d \in D$.

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2 This information is only orientative. It does not compell agents to behave conformly to their revealed preferences in the developing of the game. It seems to be used by the organization to arrange residual matchings.
and $\psi_d^r < u_r(d)$ whenever $u_r(r) < u_r(d)$. For each $r \in R$ let $D_1(r)$ be the set of centers $r$ applied to. For each $d \in D$ let $R_1(d) = \{ r : d \in D_1(r) \}$ be the set of researchers applying to department $d$. Denote $m_1^d = \{ D_1(r) \}_{r \in R}$ the action profile for researchers at stage 1. With abuse of notation $m_1^d$ will denote the applications received by departments which is $\{ R_1(d) \}_{d \in D}$.

**Stage 2:** For any demand it receives each department accepts or rejects it. For each $d \in D$ let $R_2(d) \subset R_1(d)$ the set of researchers made idoneus by department $d$. For each $r \in R$ $D_2(r) \subset D_1(r)$ be the set of departments that has accepted $r$. Set $m_2^d = \{ R_2(d) \}_{d \in D}$. Each department, $d$ which has accepted at least one researcher communicates $R_2(d)$ to the ministry.

The ministry ranks all the student accepted by at least one department. The result of this process is the ranking $T$, formally a strict order on $\bigcup_{i=1}^k R_2(d_i)$. We will denote the $i$-th ranked researcher according to $T$ by $r^i_T$ or $r^i$ when no ambiguity is possible.

**Stage 3:** The researchers who have been accepted by at least one department are assigned giving priority to better ranked aspirants until a total of $N$ positions is assigned. $r^1$ is assigned to the department she chooses among the ones in $D_2(r^1)$. For $i \leq N$, $r^i$ is assigned to the department she chooses among the ones that have some free positions in $D_2(r^i)$, if any. The other researchers stay unmatched. Each researcher accepted by some department must choose a department if at least one among the ones that accepted her has some free positions. The result of such a process is a matching $\mu_1$. If at least one researcher is unmatched and at least one university has some unfilled positions, the procedure goes to the fourth stage. Otherwise the assignment process ends and $\mu = \mu_1$ is the resulting matching.

**Stage 4:** For any department $d_i$ such that $\sharp \mu_1(d_i) < n_i$, the ministry asks to it to submit a new set $R_4(d_i)$, of acceptable researchers among the $r^i$ ($i \leq N$) not matched under $\mu_1$. Let $T'$ be the restriction of the ranking $T$ to such candidates. For each such $r^i$ let $D_4(r^i)$ be the set of departments that accepted $r^i$. For $i = 1, ..., k$ let $n'_i = n_i - \sharp \mu_1(d_i)$.

**Stage 5:** All researchers $r$ such that $D_4(r) \neq \emptyset$ are assigned to the departments following the procedure used in Stage 3. Applicant $r^i_{4'}$ is assigned to the department she chooses among the ones that have some free positions in $D_4(r^i_{4'})$, if any exists. At step $i$ if some empty position is left and if some applicant is not yet assigned to any department applicant $r^i_{4''}$ is assigned to the department she chooses among the ones that have some free positions in $D_4(r^i_{4''})$. A second matching $\mu_2$ is concluded involving the not yet assigned researchers. The process ends at this point: the researchers assigned to some
department in the first process are signed by such an institution, the other ones are assigned by $\mu_2$. For $i \leq N$ set $\mu(r^i) = \mu_1(r^i)$ if $\mu_1(r^i) \in D$, $\mu(r^i) = \mu_2(r^i)$ otherwise. Set $\mu(r) = r$ otherwise. At any point of the process each candidate can leave the game.

4.1.1 The extensive form of the game (perfect information)

We assume that $n_i \leq q_i$ for all $i$, which is consistent with the legal constraint that conditioned the design of the mechanism: departments cannot be forced to hire any researcher if they do not want to. Given $(D, R, P)$, by $(D, R, P^n)$ we will denote the matching market in which researchers’ preferences are like the original one, for all $d$, department $d$’s preference $P^n_d$ coincide with $P_d$ except for the fact that all sets of cardinality larger than $n_d$ are ranked as not acceptable. $P^n$ are the effective preferences. If $P_d$ are substitutable and $q_d$-quota separable so $P^n_d$ are substitutable and $n_d$-quota separable.

With $z_i$ we will denote the a node of the game belonging to stage $i$ for $i = 1, \ldots, 5$.

We assume that at each node, $z$ all agents such node belongs to, have complete information of what happened before.

Each node of the second stage, $z_2$ is completely characterized by the applications strategies leading to it. Let $R(z_2)$ be the set of researchers who sent at least one application and let $D(z_2)$ be the set of departments that received at least one application. $P^n(z_2)$ will denote the a profile of preferences in which departments’ preferences coincide with the ones of $P^n$ and researcher $r$’s preferences $P^n_r(z_2)$ ranks departments in the same order as $P^n$, but each department she did not applied to is considered as not acceptable. The matching belonging to $\Gamma(D(z_2), R(z_2), P^n(z_2))$ will be called $z_2$-stable.

**Definition 10** The reduced game is the extensive form game which ends at the third stage.

For each $z_4$ let $D(z_4)$ be the set of departments $d$ that filled less than $n_d$ positions. Let $R(z_4)$ be the set of idoneous researchers who did not sign for any department. If $D(z_4) = \emptyset$ or if $R(z_4) = \emptyset$ the first assignment is the definitive one. Let $\mu_1(z_4)$ be the first assignment at $z_4$. Define the profile of preferences $P^n(z_4)$ as follows: for each $d \in D(z_4)$ for all $S, S' \subset R(z_4)$, $SP^n_d(z_4)S'$ if and only if $(\mu_1(d, z_4) \cup S)P^n_d(\mu_1(d, z_4) \cup S')$, researchers’ preferences coincide with the original one. If $P^n_d(z_2)$ is substitutable and $n_d$-quota substitutable and $z_2$ follows $z_2$ then $P^n_d(z_4)$ is substitutable and $(n_d - \mu_1(d, z_4))$-quota substitutable.

The outcome matching will be the union of a first assignment matching and of a residual matching $\mu = \mu_1(z_4) \cup \mu_2(z_5)$ where $z_4$ is a node of the fourth stage and $z_5$ is a terminal node of the fifth stage following $z_4$. Let $\Gamma R(D, R, P^n(z_4))$ be the set of matchings $\mu = \mu_1(z_4) \cup \mu_2$ where $\mu_2$ is stable in $(D(z_4), R(z_4), P^n(z_4))$. Such a $\mu$ will be called $z_4$-stable.

**Definition 11** The full game is the game which ends at the fifth stage.
4.1.2 Analysis

We begin the analysis of the matching procedure by showing that it does not prevent collusion among the two sides of the market. There exists a set of SPE which only depends on the total number of contracts assigned and on agents’ preferences and not on the ministerial ranking. Such an outcome set is "stable" compatible with the restriction imposed by the number of contracts assigned to each center. In such equilibria the centers do not compete for the best candidates according to the ranking, they compete for their favorite researchers. The research centers can exclude researchers by not accepting her at the second stage. If the ranking criteria used by the Commission and the preferences of the centers differ substantially any researcher could be excluded in the assignation process because no center gave him the pre-acceptance. In this case she would never appear in the ranking compiled after the second stage of the process. We show that there are equilibrium outcomes that can result only from contracts signed at the end of the fifth stage. These equilibria are such that some center gives a “redundant” pre-acceptance to a candidate whom it is not going to hire. r will finally join a center which accepts him as “residual”. Any candidate it prefers to r either has been already hired or has been excluded from the ranking because not well ranked enough due to acceptation policies of other departments. So if a fifth stage matching is not redundant it "hurts" some department. It might explain why a low number of fifth stage contracts were signed in the first edition of the program. It could be the result of an attempt of reducing the hurting assignation on centers’ side. Such an observation leads us to consider the reduced game which gives sharper predictions on the outcome set. All SPE outcomes are stable, considering the centers’ preferences as shaped by quota assignation. Furthermore applicants’ equilibrium strategies reduce to a unique application, which resembles the data of the first application.

The result does not longer hold if one assumes zero costs of application. In such a case unstable SPE equilibria could occur due to multiple applications and to incoherent behavior of the centers. Unstable equilibria build on centers that in out of equilibrium path subgames use a different acceptation policy with respect to the equilibrium one even when they receive the same set of application. We first observe that any pre-accepted candidate has as dominant strategy, at each node he owns at the third and at the fourth stage to accept the best offer it holds. This must be her SPE strategy.

Proposition 1 Let r ∈ R. Whenever r is called to play in the third stage she has a unique dominant strategy: to join her favorite available department, which is her favorite department among the ones who made her idoneous and are left with positions to fill. It holds both for the reduced and for the full game. Whenever r is called to play in the fifth stage of the full game she a unique dominant strategy: to join her favorite available department.

Proof. It is trivial for the reduced game and the fifth stage. For the third stage of the full game it suffices to observe that if r is called to play at most once between the third and the fifth stage.
4.1.3 The lack of meritocratic incentives

First of all we show that the procedure is not immune from agents' manipulation. Actually we proof that the set of SPE includes a “stable set”, which is independent on the ranking elaborated by the Ministry. It follows that the mechanism does not provide enough incentives for hiring the best aspirants. Furthermore it may foster the collusion among centers in order to make the game end after the first stage. The SPE outcome sets both of the full and of the reduced game contains the “stable set” \( \Gamma(D,R,P^n) \), which is independent on any ranking, \( T \) of the applicants. It depends only on agents’ preferences once the number of financed contracts, \( n \) is known. The result does not build on application costs.

**Proposition 2** Let \( n \) be given, let \( P \) be a profile of preferences and let \( \mu \in \Gamma(D,R,P^n) \). Then there exists a SPE of full game yielding \( \mu \) as outcome, in which all assignments are concluded at Stage 3. Such strategies are SPE of the reduced game that yield \( \mu \) as outcome, too

**Proof.** Let \( \mu \in \Gamma(D,R,P^n) \). Consider the following strategy profile.

Stage 1: Each \( r \) applies \( \mu(r) \)
Stage 2: At \( z_2 \in Z_2 \) department \( d \) accepts the applicants \( r \in \mu^R(z_2)(d) \).
Stage 3: Each researcher plays her dominant strategy.
Stage 4: At each \( z_4 \in Z_4 \) department \( d \) accepts the applicants \( r \in \mu^R(z_4)(d) \).
Stage 5: Each researcher plays her dominant strategy. Given such a profile of strategies, any subgame \( z_2 \in Z_2 \) no student is accepted by more than one college in any subgame. Then the stability of \( \mu^R(z_2) \) in \( (D(z_2), R(z_2), P^n(z_2)) \) and of \( \mu^R(z_4) \) in \( (D(z_4), R(z_4), P^n(z_4)) \), for all \( z_2 \) and for all \( z_4 \) implies subgame perfection. The same strategy profile, “cut” after the third stage constitutes a SPE of the reduced game and both yield \( \mu \) as outcome.

**Remark 1** The strategy profile employed in the proof of Proposition 2 are quite consistent with the evidence of the first edition of the program. Most of the aspirants presented applications to few departments, most institutions made idoneous only the researchers that finally they would have hired, with a unique relevant exception, which can be seen as an “institutional player”, because it is directly controlled by the government itself. Furthermore almost all matchings were agreed at the end of the third stage. Of the few matchings concluded at the fifth stage some were not finally ratified because applicants opted out. It seems that agents deliberately choose not to use stage 4 and 5.

The equilibrium constructed in the proof of Proposition 2 ends with the first assignment. Anyway not all equilibria conclude with a first definitive assignment. More important there are equilibrium outcome that cannot be result of strategies end with the first assignment if costs are strictly positive.

**Example 2** Assume Let \( k = 3 \) and let \( q = 4 \). Let \( n_1 = n_2 = n_3 = 1 \) and \( N = 3 \). Let \( 0 < \psi^d_r < u_r(d) \) for all \( d \) and for all \( r \) \( T: r_1, r_2, r_3, r_4 \) \( P_{t_1} = r_1 \)
\( P_{t_2} = r_2, r_3 \) \( P_{t_3} = r_4, r_3 \) \( P_{t_4} = d_1 \) \( P_{t_5} = d_2 \) \( P_{t_6} = d_2, d_3 \) \( P_{t_7} = d_3 \) Consider
the following strategy profile: First Stage: $D_1(r_1) = \{d_1\}$, $D_1(r_2) = \{d_2\}$, $D_1(r_3) = \{d_3\}$. Second Stage: Let $d_1$ accepting only $r_1$ and let $d_2$ accepting only $r_2$ and $r_3$. Let $d_3$ always accepting $r_4$. Let $d_3$ accepting only $r_4$ whereas $r_3$ did not apply to any other university at the first stage. Let $d_3$ not accepting any applicant different from $r_3$ and $r_4$. Third Stage: Let researchers play their dominant strategies. Fourth Stage: Let departments conforming to the strategies described in Proposition 2. Fifth stage: Let researchers play their dominant strategies. The described strategies constitute an SPE yielding the following matching, $\mu$ as outcome.

$$
\begin{array}{cccc}
d_1 & d_2 & d_3 \\
r_1 & r_2 & r_3 & r_4 \\
\end{array}
$$

By contradiction assume there exists a SPE yielding $\mu$ in which all the agreements conclude with the first assignment. Then it must be the case that $r_4$ did not apply to any department, otherwise she would pay positive costs. Furthermore $r_3$ applied only to $d_3$. Otherwise consider the following deviation for $r_3$, $D_4(r_3) = \{d_3\}$. In the subgame induced by such deviation, by subgame perfection $d_3$ accepts $r_3$ because it is the unique acceptable application it receives. Then such a deviation would be profitable for $r_3$ because she would save the costs of applying to different universities.

**Remark 2** In the example the redundant acceptance of $r_3$ by $d_2$ force $d_4$ to be excluded by the process. This hurts $d_4$, which would prefer to hire $d_4$ rather than $d_3$ but it cannot. It would not been happened if the game had end after the first assignment. It reveals the interest that departments may have in terminate with the reduced game.

The next result characterizes the set of equilibria of each one of the subgames beginning at stage 4. The equilibria of $z_4$ are “ex-post stable”. Me. It implies that the residual matching of each subgame is independent on the ranking.

**Proposition 3** Let $z_4$ be a fourth stage node. The game starting at $z_4$ implements the set of $z_4$-stable matchings, $\Gamma R(D, R, P''(z_4))$ in SPE.

**Proof.** Let $\mu_2$ be stable in $(D(z_4), R(z_4), P''(z_4))$. It suffices to consider only the agents in $(D(z_4), R(z_4))$. We first prove that $\mu = \mu_1(z_4) \cup \mu_2$ is a SPE outcome. Let agents’ strategies at each stage be the following. Stage 4: $d \in D(z_4)$ admits the applicants in $\mu_2(d)$. Stage 5: each $r \in R(z_4)$ plays his strictly dominant strategy. To show that such strategy profile constitute an SPE we have only to show that no $d \in D$ can profitably deviate. Let $R' \subset R(z_4)$ be the set of applicants $d$ such that $r \in R' \not\in R_0\mu_2(r)$. By deviating $d$ can get only the applicants in $R'$. From the stability of $\mu_2$ it follows that $\mu_1(z_4, d) \cup \mu_2(d) \cup R_0\mu_1(z_4, d) \cup R''$ for all $R'' \subset R'$ so no deviation is profitable for $d$. On the other side let $\mu_2$ be an SPE outcome matching $z_4$. Let $4 r \in R(z_4)$ if $d_{\mu_2}r$ it means that $d$ has not accepted $r$ in the fourth stage. If $r \in Ch_d(\mu_2(d) \cup \{r\})$ then $d$ can profitably deviate by accepting only the applicants in $Ch_d(\mu_1(d) \cup \{r\}) \setminus \mu_1(z_4, d)$, a contradiction. So $\mu_2$ belongs to $\Gamma R(D(z_4), R(z_4), P''(z_4))$. ■
Remark 3 When $z_4$ is such that $\mu_1(z_4,d) = \emptyset$ for all $d$, then Proposition 3 generalizes the result obtained by Sotomayor (2004) to the many-to-one case.

4.1.4 The reduced game

For an assignment to be realized in the fifth stage it is necessary that some applicant have been accepted by a centre which will fill all its position with better ranked candidates at the first assignment. The acception is redundant and it might hurt some centers. Furthermore the behavior appears to be quite strange especially when the information is complete. Why should a center give the pre-acceptance to a candidate it is sure it will not hire? So it could be a deliberate collusive choice of the departments to play only a one assignment game. It would prevents unwanted matchings as the one presented in Example 2. This is the rationale to study the reduced game whose equilibrium set is smaller and more appealing, at least for the centers. The reduced game has no equilibrium outcomes outside the stable set. So they does not depend on the ranking criteria used by the Ministry, but only on the number of financed contracts. So the ranking cannot “force” better researchers to better placements at equilibrium.

These intuitions lead us to analyze the game which ends with the first assignment, which we will call the first assignment game. The first assignment game implements the stable set in SPE, if costs are strictly positive. Furthermore each applicant apply to a unique department.

Theorem 1 For each $n$ the first assignment game implements $\Gamma(D,R,P^n)$ in SPE. In each SPE no aspirant apply to more than one center.

To proof Theorem 1 we need some preliminary Lemmas.

Lemma 1 Let $z_2$ be second stage node where at most $N$ researchers $(D(z_2) \leq N)$ has applied. The game beginning at $z_2$ implements the set of $z_2$-stable matchings, $\Gamma(D(z_2), R(z_2), P^n(z_2))$ in SPE.

Proof. Without loss of generality assume $D(z_2) = D$ and $R(z_2) = R$. Let $\mu$ be stable in $(D, R, P^n(z_2))$. Consider the following strategies. For each $d \in D$ let $R_2(d) = \mu(d)$. Let each $r \in R$ playing her dominant strategy at each node of the game. No researcher can profitably deviate. If some $d$ deviates let $\mu'(d)$ be the outcome matching, keeping fixed other agents’ strategies. Let $r \in \mu(d)) \setminus \mu'(d)$. Then $d\mu'(z_2)\mu(r)$ as no aspirant has been accepted by more than one center different from $d$. $\mu$ is stable in $(D, R, P(z_2))$ so $r \notin Ch_\mu(d) \cup \{r\}$. In particular $\mu(d)P(z_2)\mu'(d)$. On the other side, as $\mu$ is stable in $(D, R, P(z_2))$, it cannot be the case that $d$ can profitably deviate simply by not accepting some applicants in $\mu(d)$. So no deviation is profitable for $d$. On the other side let $\mu$ be an SPE outcome matching of $z_2$. Let $d \in D$ and assume that $d\mu'(z_2)\mu(r)$ for some $r \in R$. Consider the following deviation for $d$: $R_2(d) = Ch_\mu(\mu(d) \cup \{r\})$,
where, as $d$ preferences we consider $P_d(z_2)$. It must be the case that such deviation produces a matching $\mu'$ such that $\mu'(d) = Ch_d((\mu(d) \cup \{r\})$ because the aspirants decide sequentially according to ranking $T$, and at any SPE each one selects her best available offer at her turn. Then if $r \in Ch_d(\mu(d) \cup \{r\})$ the deviation would be profitable for $d$. Then $\mu$ is stable in $(D, R, P(z_2))$. ■

The next result, show that in any SPE in which each researcher applies to at most one department is stable under the effective preferences.

**Lemma 2** All SPE equilibria outcomes of the first assignment game in which each matched researcher applies to exactly one department are stable in $(D, R, P^n)$

**Proof.** Let $\mu$ be an SPE outcome in which each researcher applies to exactly one department, and by contradiction assume that $(d, r)$ blocks $\mu$ in $(D, R, P^n)$. Consider the following deviation for $r$: applies only to $d$ and then conform to SPE strategies. In the subgame induced by such a deviation $d$ receives the applications of the agents in $\mu(d) \cup \{r\}$. Given subgame perfection it must be the case that the deviation matches $r$ with $d$, yielding a contradiction. ■

Then the main result follows

**Proof of Theorem 1.** From Lemma 2 it suffices to show that, at equilibrium, no agent applies to more than one department. At equilibrium no unmatched agent applies to any center because of the costs. So, at equilibrium no more than $n$ researchers apply. On the other side let $m^*$ be an SPE yielding a matching $\mu^*$ as outcome. Let $\{D^*_r(r)\}_{r \in R}$ be the equilibrium application profile. Let $r^* \in R$ and let $d^* \in D$ such that $\mu^*(r) = d$. Let $z_2$ be the node in which each $r \neq r^*$ has applied to $D^*_r(r)$ for all $r \neq r^*$ and $r^*$ has applied to only $d^*$. $\mu^*$ is stable in $(D(z_2), R(z_2), P^n(z_2))$ and $d^*$ is $r^*$’s unique stable partner in $(D(z_2), R(z_2), P^n(z_2))$. So by Lemma 1 if $D^*_r(r^*) \supseteq \{d^*\}$ the deviation $D^*_r(r^*) = \{d^*\}$ would be profitable to $r^*$, because it would produce the same matching as $m^*$ with a lower number of application so with lower costs. Then it must be the case that $D^*_r(r) = \{\mu^*(r)\}$ for all $r \in R$. ■

In any case the system perform correctly if the preferences of the centers coincide with the ranking criteria.

**Definition 12** Let $d \in D$. Let $T$ be a ranking on $R$. Let $P_d$ be a profile of preferences on $2^R$. $P_d$ is $T$-meritocratic if it is responsive to $T$.

When preferences are $T$-meritocratic there exists a unique stable matching in $(D, R, P^n)$, $\mu^T$, $\mu^T$ matches the best researcher to her favorite department, the second is to her favorite center among the ones having free positions and so on.

**Proposition 4** Let all departments’ preferences be $T$-meritocratic then the reduced game implements $\mu^T$ in SPE.

**Proof.** Observe that, if researchers plays their SPE strategy the best each department can do in any $z_2 \in Z_2$ is to accept all applicants regardless of other centers’ moves. So in any SPE of $z_2$ it must be as well as playing such
strategy. So the SPE outcome of $z_2$ must be the unique stable matching of $(D(z_2), R(z_2), P(z_2))$. Let then $\mu^*$ be an SPE outcome of the game. Without loss of generality let $T = r_1, \ldots, r_n$. By contradiction assume that there exists $r_i$ such that $\mu^*(r_i) \neq \mu^T(r_i)$. Let $r_i$ be the best ranked of such candidates which means $\mu^*(r_j) \neq \mu^T(r_j)$ for $j < i$ and $\mu^*(r_i) \neq \mu^T(r_i)$. If $d \in P \mu^T(r_i)$ then $\mu^T(d) = \{r_1, \ldots, r_{i-1}\}$ and $\mu^*(d) = n_d$. Then $\mu^T(d) = \mu^*(d)$ by hypothesis. So $r_i$ can attain any any department in $\{d : \mu^T(d) < n_d\}$ by applying. But $r_i$’s most preferred department in this set is exactly $\mu^T(d)$. ■

**Remark 4** The result is independent on any assumption on costs.

When there are no application costs and preferences are not $T$ meritocratic the mechanism might produce instable matchings. The intuition for the result is that the application strategy may create opportunities for the applicants to collude. An unstable matching may be sustained as equilibrium if some agents apply to more than one college.

**Example 3** Let $k = q = 3$ and let $\psi_d^r = 0$ for all $r$ and for all $d$. Let $n_1 = n_2 = n_3 = 1$. Let $P_{r_1} = d_1, d_2, d_3$, $P_{r_2} = d_2, d_1, d_3$, $P_{r_3} = d_2, d_3$. Let $P_{d_1} = r_2, r_1, r_3$, $P_{d_2} = r_1, r_3, r_2$, $P_{d_3} = r_3, r_1, r_2$. Consider the following matching, $\mu$.

\[
\begin{array}{ccc}
  d_1 & d_2 & d_3 \\
  r_1 & r_2 & r_3 
\end{array}
\]

It is blocked by $(d_2, r_3)$ in $(D, R, P^n)$, but it is an SPE outcome of the first assignment game. Consider the following strategies. $r_1$ and $r_2$: apply to $\{d_1, d_2\}$ at the first stage and and use the strict dominant strategies at the third stage. $r_3$: apply to $\{d_3\}$ at the first stage and and use the strict dominant strategies at the third stage. At any $z_2$ of the second stage $d$ accepts only the applicants in $\mu^T(z_2)(d)$. It is easy to verify that such strategies constitute an SPE: $r_1$ and $r_2$ have no profitable deviation as $\mu$ assign them to their first choice. If $r_3$ deviates she is matched to $r_3$ if she includes it in her application, otherwise she is left unmatched. On the other side the stability of $\mu^T(z_2)$ assures that no center can profitably deviate from the described strategies at $z_2$.

$\mu$ can be sustained as an equilibrium of the full game without costs, too.

It is not clear which kind of SPE outcomes can be sustained without costs: it is larger than the set of stable matchings, but it is strictly contained in the set of IR matchings$^3$.

**Example 4** Let $k = q = 2$. Let $n_1 = n_2 = 1$ and let $N = 2$. Let $P_{r_1} = d_1, d_2$, $P_{r_2} = d_2, d_1$, $P_{d_1} = r_1, r_2$ and $P_{d_2} = r_2, r_1$. The following matching is IR (but not stable)

\[
\begin{array}{ccc}
  d_1 & d_2 \\
  r_2 & r_1 
\end{array}
\]

$^3$The point has been made by Jordi Massó.
Anyway it cannot be sustained by any SPE of the reduced game, for any $T$. It cannot be sustained by single application strategies because is not stable (Lemma 2). It cannot be sustained by multiple application strategies: each department who receives more than one application would be better off by accepting its favorite researcher.

When costs are positive and information is complete no researcher will apply to more than one center at equilibrium. This fact makes impossible situations like the ones described in the example above. On the other side the example relies on an “incoherent” behavior on centers’ side. In the subgames in which $r_3$ applies to $d_2$ while the other researchers behave as in equilibrium $d_1$ accepts only $r_2$ even if it receives the same applications as on the equilibrium path (by $r_1$ and $r_3$). A convenient behavioral assumption on departments’ strategy would extend the stability result to the case of zero costs.

4.1.5 Some remarks on capacities

A well known result is that, in many-to-one matching markets is that the department-optimal stable matching is not weakly Pareto optimal. Sönmez (1997) has shown that the incentives in hiding capacities may result in unstable matchings, in stable revelation mechanisms.

Our mechanism is not a revelation one but the same perverse effect may take place. We consider the game in which, in a preliminary stage centers must ask Ministry a number of financed position. We assume that the Ministry is willing to confirm departments requests. Then the agents play the reduced game. The mechanism will be called the extended assignment game. We show that the result of Theorem 1 does not extend. Therefore hiding capacities might be not only in departments’ interest, it can also emerge as equilibrium behavior. This deeps Sönmez (1997) result.

Proposition 5 Assume there are at least two departments and three admissible researchers. The extended assignment game does not implement the stable correspondence in Subgame Perfect Equilibrium.

Proof. The proof is by means of an example, based on Sönmez (1997).

Let:

- $S = \{s_1, s_2, s_3\}$, $D = \{d_1, d_2\}$,
- $c_1 = c_2 = 2$, $c_1' = c_2' = 1$, the possible capacities
- $P(d_1) = \{s_1, s_2\}\{s_1, s_3\}\{s_2, s_3\}\{s_2\}\{s_3\}$
- $P(d_2) = \{s_2, s_3\}\{s_1, s_3\}\{s_1, s_2\}\{s_2\}\{s_1\}$
- $P(s_1) = d_2d_1$
- $P(s_2) = d_1d_2$
- $P(s_3) = d_1d_2$

Let

$\mu_1 = \begin{pmatrix} d_1 \\ s_2, s_3 \\ s_1 \end{pmatrix}$, $\mu_2 = \begin{pmatrix} d_1 \\ s_1, s_2 \end{pmatrix}$, $\mu_5 = \begin{pmatrix} d_1 \\ s_1, s_2, s_3 \end{pmatrix}$

20
Then the stable sets $\Gamma$ are $\Gamma(c_1,c_2) = \{\mu_1\}, \Gamma(c_1,c_2') = \{\mu_1,\mu_2\}, \Gamma(c_1,c_2) = \{\mu_3\}, \Gamma(c_1',c_2) = \{\mu_4,\mu_5\}$. In particular when both capacities are equal to 2 it should implement $\mu_1$. Assume by contradiction that the mechanism implements in Subgame Perfect Equilibrium stable set. The second part of the game as already shown implements in Subgame Perfect Equilibrium the stable matching with respect to the quotas the departments ask for. So: if the complete mechanism implemented the stable matching in Subgame Perfect Equilibrium it should be the case that $\mu_1$ is the NE outcome of the following game when capacities are 2 for both the departments, or the game has no NE.

$$
\begin{array}{ccc}
d_1 \setminus d_3 & 1 & 2 \\
1 & s_1, s_3 & s_1, \{s_2, s_3\} \\
2 & \{s_2, s_3\}, s_1 & \{s_2, s_3\}, s_1 \\
\end{array}
$$

But it is easily seen that when capacities are $(2, 2)$ the game has $(1, 2)$ as NE and $\mu_2$ as equilibrium outcome which is not stable when capacities are $(2, 2)$.

4.2 The new mechanism: trade-off efficiency-stability

The new mechanism designs only part of the matching procedure: it prescribes how the candidate researchers can enter the selection process. It does not prevent anyone from being ranked, if he chooses to participate introducing, in our view, the most important novelty with respect to the old mechanism. Observe that, in any case, the application are heavily reduced A unique research proposal is needed to participate to the selection.

Stage 1: Aspirants simultaneously send their scientific CV and a research proposal to the Ministry. Let $R_1$ be the set of agents who apply.

The Ministry ranks all applicants The first $N = \sum n_i$ ranked researchers have the right to see their contract (eventually) financed through the program. We will call them idoneous. The other students are definitively out of the program.

Stage 2: In a decentralized way the departments and the idoneous applicants sign contracts. Each department $d_i$ cannot sign more than $n_i$ contracts with idoneous researchers. A matching $\mu$ is agreed.

We assume that the decentralized assignation takes place as in the following way. Once the set of idoneous researchers is known, each department reveals the aspirants it is willing to hire. Then, sequentially each researcher decide which center to join among the ones that admitted him and are left with vacancies. The mechanism constitutes an extension of Sotomayor (2004) to the many-to-one case. The process can be formally described as follows. Let $N^0 \leq N$ be the number of idoneous researchers. Let $r_{i_1}, ..., r_{i_{N^0}}$ the order in which researchers

\footnote{Anyway the main result of this section holds under a more general assumption on the decentralized assignment process (see the Appendix).}
can choose which department to sign for. We do not assume that such order agrees with $T$.

**Stage 2.1** Each department $d$ selects a subset $R(d) \subset \{r_{i_1}, ..., r_{i_N}\}$.

Let $D(r)$ be the set of departments that accepted $r$.

**Stage 2.1.1** $r_{i_1}$ chooses mate $x_{i_1}$ in $D(r_{i_1}) \cup \{r_{i_1}\}$. Set $\mu_1(r_{i_1}) = x_{i_1}$, $\mu_1(x_{i_1}) = \{r_{i_1}\}$ if $x_{i_1} \in D$, $\mu_1(r) = r$ for all $r \neq r_{i_1}$, and $\mu_1(d) = \emptyset$ for all $d \neq x_{i_1}$.

**Stage 2.1.2** $(2 \leq t \leq N)$ $r_{i_t}$ chooses a mate $x_{i_t}$ in the set $D(r_{i_t}) \setminus \{d : \mu_t(d) \geq q_t\} \cup \{r_{i_t}\}$. Set $\mu_t(r_{i_t}) = x_{i_t}$, $\mu_t(x_{i_t}) = \mu_{t-1}(r_{i_t}) \cup \{r_{i_t}\}$, and $\mu_t(x) = \mu_{t-1}(x)$ for all $x \neq r_{i_t}, x_{i_t}$.

Set $\mu = \mu_N$.

Let $z$ be an initial node of the second stage and let $R(z)$ be the set of idoneous researchers at $z$. Let $(D, R(z), P^n(z))$ be the matching markets in which researchers’ preferences are the original ones and let each department $d$’s preferences to coincide with $P^n$ on $2^D \setminus \emptyset$ and ranks as not acceptable all subsets containing agents in $R \setminus R(z)$.

We will call a matching belonging to $\Gamma(D, R(z), P^n(z))$, $z$-stable.

The next result characterizes the SPE outcomes of the subgames beginning at the seconds extending the result of Sotomayor (2004), to the many-to-one case.

**Corollary 1** Let $z$ be an initial node of the second stage. Then the game starting at $z$ implements the $z$-stable set, $\Gamma(D, R(z), P^n(z))$ in SPE stable matching.

**Proof.** Observe that the subgame beginning at $z$ is a subgame beginning at the fourth stage of the full game with the empty and agents choosing in the order $r_{i_1}, ..., r_{i_N}$. Then the claim follows from Proposition 3.

We finally have

**Proposition 6** Let $T = r_{i_1}, ..., r_{i_N}, ..., r_{i_N'}$. Let $\nu \in \Gamma(D, R, P^n)$.

(i) In any SPE equilibrium at most $\nu(D)$ are employed.

(ii) Let $\nu(D) \geq N$ and let $\mu$ be a matching in which exactly $N$ researchers are employed. Let $j^*$ be the worst ranked employed researcher. Let $j^* > N$.

Then $\mu$ is an SPE outcome of the new mechanism if and only if $\mu$ is stable in the market in which all non employed researchers are excluded $(D, \mu(D), P_D, P_{\mu(D)})$ and for all $j < j^*$ such that $\mu(r_j) = r_j, r_j$ is single in market $(D, \mu(D) \setminus \{r_j\} \cup \{r_j\}, P_D, P_{\mu(D)\setminus \{r_j\}}).$

(iii) Let $\mu$ be a matching in which $\nu(D) < N$. Then $\mu$ is an SPE outcome of the new mechanism if and only if $\mu$ is stable for $(D, \mu(D), P_D, P_{\mu(D)})$ and for all $j$ such that $\mu(r_j) = r_j, r_j$ is single in market $(D, \mu(D) \cup \{r_j\}, P_D, P_{\mu(D)\cup \{r_j\}}).$

(iv) Let $\nu(D) > N$ and let participation costs to be strictly positive for each agent. At any SPE the same set of $N$ agents is matched. The set of SPE outcomes coincides with the set of matchings described in (ii).

(v) Let $\nu(D) \leq N$ and let participation costs to be strictly positive for each agent. The set of SPE outcomes of the new mechanism is $\Gamma(D, R, P^n)$.
Proof. See the appendix.

Consider a full employment equilibrium in which a poorly ranked researcher signs a financed contract and some better researcher did not apply for the grant. From Proposition 6, it is because had the smarter researcher participated to the selection he would not get any acceptable position and a contracted would have been wasted. Wasting contracts is possible at equilibrium if application costs are null, from (ii) or if the stable set of the market has less than \( N \) researchers employed. This eliminates a number of equilibria of the reduced game even if does not completely prevent agent collusion: it could exclude some IR matchings in which a well ranked researcher is employed, because of incompatibilities.

Example 5 Assume Let \( k = 3 \) and let \( q = 4 \). Let \( n_1 = n_2 = n_3 = 1 \) and \( N = 3 \). \( T : r_1, r_2, r_3, r_4 \) \( P_{d_1} = r_2 \) \( P_{d_2} = r_1, r_3 \) \( P_{d_3} = r_4 \) \( P_{r_1} = d_2 \) \( P_{r_2} = d_1 \) \( P_{r_3} = d_2, d_3 \) \( P_{r_4} = d_3 \) The unique stable matching of \((D, R, P) = (D, R, P_0)\) is \( \mu \):

\[
\begin{array}{cccc}
d_1 & d_2 & d_3 \\
r_2 & r_1 & r_4 & r_3
\end{array}
\]

By Theorem 1 \( \mu \) is the unique equilibrium of the reduced game. Only \( r_1, r_2, r_4 \) are employed. By Proposition 6 \( \mu \) cannot be sustained as a SPE outcome of the new mechanism. Actually the unique SPE outcome of the new mechanism is

\[
\begin{array}{cccc}
d_1 & d_2 & d_3 \\
r_2 & r_1 & r_3 & r_4
\end{array}
\]

Through the new mechanism we recover some efficiency, with respect to \( T \). This was one of the designers’ concerns. The new mechanism may act exactly by preventing the formation of inefficient stable markets, preventing the entry of poor researchers’ Anyway the mechanism cannot accomplish completely the task if preferences are too endogamic: researchers are excluded by the market itself (from (ii) and (iii)), whatever is their position in \( T \).

5 More general information structure

In this paragraph we relax the assumption of perfect and complete information and we allow for a more general structure.

5.1 The setup

Let \( S \) be the (finite) set of state of the world. A state of the world, \( s \) is characterized by a ranking \( T(s) \) and a profile of preferences, \( U(s) = (U_x(s) \mid s) \mid x \in R \cup D \). In addition each agent, \( x \in R \cup D \) has a fixed prior distribution \( \pi^x \) on \( S \). An allocation is a function \( M \) from \( S \) to the set of possible matchings.

The agent play one of the games described in the previous section. A path in the game is a pair \((s, h)\), where \( s \in S \) and \( h \) is an history of the actions

\[
23
\]
taken by the players. To any terminal path is associated an outcome matching \( \mu = \mu(h, s) \).

An information set for a player \( x \) is a set, \( I^x \) of histories identifying those paths player \( v \) is not able to distinguish. \( \mathcal{I}^v \) will denote \( v \)'s collection of information set. Let \( I^x(s) \) to denote \( x \)'s information set at the beginning of the game. Behavioral strategies for \( x \in R \cup D \), \( \sigma^x \), specify the actions taken by agents at each information set. Let \( \sigma = (\sigma^x)_{x \in R \cup D} \) be a profile of strategies.

A profile of (behavioral) strategies, \( \sigma^* \) is sequentially rational if, for all \( x \in R \cup D \), for all \( s \in S \) and for every information set \( I^x \in \mathcal{I}^x \)

\[
E_{\pi^x}[U_x(\sigma^x(s))] \mid I^x \geq E_{\pi^x}[U_x(\sigma^{-v^*}, \sigma^v, s)] \mid I^x
\]

where \( E_{\pi^x}[\cdot, s] \) denotes the expectation operator with respect to to \( \pi^x \). Expectations are determined using Bayes’ Rule on the information sets that are reached with positive probability. It is however necessary to model beliefs outside the equilibrium path, which is if \( I^v \) has probability 0 under \( (\sigma^*, \pi) \), then an appropriate conditional distribution has to be assigned. We will use as equilibrium concept Perfect Bayesian Equilibrium (PBE) and Sequential Equilibrium (SE). Observe that in the second case the information at any information set must be consistent with initial information through Bayesian updating.

5.2 The reduced old mechanism

Let \( T \) be the set of possible rankings on \( R \). So agent \( x \)'s information determines a probability distribution \( \rho^x \) on \( T \), where \( \rho^x(T) = \sum_{T = T(s)} \pi^x(s \mid I^x(s)) \). Each department \( d \), once it receives the applications from the researchers in some subset \( R_1 \) of \( R \), is able to determine the relative ranking among the members of \( R_1 \). Its information set at \( (s, h) \) can be then identified by \( I^d = (R_1, T|_{R_1}) \) where \( T|_{R_1} \) is the restriction of \( T \) to \( R_1 \). Then \( d \)'s beliefs are a probability distribution \( \pi^d(\cdot \mid I^d) \). We assume that the induced probability distribution on \( T \), \( \rho^d(\cdot \mid I^d) \) is consistent with \( \rho^x \) for all \( I^d \). Let \( T^d(I^d) = \{T : T \mid R_1(d) = T(I^d)|_{R_2(d)} \} \), the set of rankings consistent with \( d \)'s information at \( I^d \)’s. This induces a differential information structure. Departments are better informed than applicants on the final ranking. Each department knows only the ranking of its applicants but does not have detailed information about other participants’ positions on ranking. Pooling of all departments’ informations would reveal the full ranking.

Remark 5 In the final stage of the game, at which researchers choose which department to join all relevant information has already been disclosed. Then like in Proposition 1 each researchers has a dominant strategy to accept the best offer she holds whenever she is called to choose. The result holds at any equilibrium consistent with sequential rationality.

If \( \mu \) is the outcome function, and \( (h, s) \) is a path let \( R^d_1(h, s) \) be the set applying to \( d \) along the path. For all \( (h, s), (h', s') \) belonging to the same information set \( I^d \), we will have \( R^d_1(h, s) = R^d_1(h', s') = R^d_1(I^d) \). \( R^d_2(h, s) = \)
$R^d_2(I^d) \subset R^d_1(I^d)$ will denote the researchers accepted by department $d$ at $I^d$. Let $T^d(I^d)$ be the profile of preferences for $d$, responsive to the following order on individuals.

$$r T^d(I^d) d \Leftrightarrow r \in R^d_2(h, s) \text{ for some } (h, s) \in I^d$$

$$r T^d(I^d) r' \Leftrightarrow r T(s) r'$$

Such preferences will be said to be meritocratic at $I^d$.

Let $M(I^d)$ be the matching market in which researchers have their original preferences as in state $s$ and each department $d$ has preferences $T^d(I^d)$. Let $\mu^T(I^d)$ be the unique stable matching in $M(I^d)$. Then the following result holds.

**Lemma 3** Let $I^d$ be an information set belonging to the second stage and let researchers playing conforming to Proposition 1. Then for any $(h, s)$ belonging to $I^d$ the outcome of $(h, s)$ is $\mu^T(I^d)$ at any SE and at any PBE.

**Proof.** Let $r_1 T \ldots T r_s$, $s \leq n$ be the applicants who receive a financed contract according in path $(h, s)$. Let $d_1 = \max_{P^r_j(s)} \{ d : r_j \in R^d_2(h, s) \}$, $d_{j+1} = \max_{P^r_j+1(s)} \{ d : r_j \in R^d_2(h, s), \# \{ 0 \leq k \leq j - 1 : d = d_{j-k} \} < n^d \}$, for $j \leq s - 1$ with the convention $\max_{P^r_j(s)} \emptyset = r$. $d_j$ is $r_j$’s partner at the end of $h$ at state $s$. Executing the deferred acceptance algorithm with departments applying, where each department $d$ is substituted by $n^d$ replicas in the order $(d_1, \ldots, d_s)$ one obtain the same matching. Then the first claim follows taking in account that the outcome of such procedure is the departments’ optimal stable matching in $M(h, s)$ for any order of application. The second claim follows by executing the deferred acceptance algorithm with aspirants applying the order defined by the ranking.

Then departments’ strategy amounts in “revealing” a profile of preferences from a restricted set. If the application stage does not convey more information than the initial one to departments and if preferences are public knowledge the conclusions of previous section applies. In particular.

**Corollary 2** Assume information is complete on agents’ preferences and the informational structure is the same than in Section 4.1

(i) the full game weakly implements the stable set in PBE and in SE

(ii) the reduced game implements the stable set in PBE and in SE.

Then information asymmetries may have a different impact on the analysis if and only it is affect agents’ preferences and not only the ranking.

Now, to make the formal analysis simpler we consider only preference profiles in which departments can only differentiate between acceptable and not acceptable aspirants. They only care to fill their positions with acceptable researchers. They assume the official ranking on the acceptable, but it does not mean that their preferences are meritocratic.

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5 Analogous result holds in the case of complete information
Let $u^d_A(s)$ be $d$'s utility from an acceptable researcher at $s \in S$ and let $u^d_{NA}(s)$ be $d$'s utility from an unacceptable researcher at $s \in S$. Let $u^d(\emptyset, s)$ be the utility of hiring non researchers at $s \in S$. With no loss of generality we assume $u^d(\emptyset, s) = 0$. Let $u^d_A(s) < 0 < u^d_{NA}(s)$. Let $A^d = A^d(s)$ be the set of acceptable researchers for department $d$ at $s \in S$. Let $u^d(\emptyset, s) = 0$. Let $u^d_{NA}(s) < u^d_A(s)$. Let $A^d = A^d(s)$ be the set of acceptable researchers for department $d$ at $s \in S$. Set

$$u^d(R', s) = |A^d(s) \cap R'| u^d_A(s) + |R' \setminus A(d, s)| u^d_{NA}(s), R' \neq \emptyset, |R'| \leq q_d$$

$$u^d(R', s) < u^d(\emptyset, s) \text{ if } |R'| > q_d$$

We will say that a department strategy is essentially dominant if it is dominant in the reduced game in which researchers play according to the strategy in proposition 1. In Appendix B we prove that, in this case it essentially dominant, for all departments to accept all acceptable researchers.

Proposition 7 Accepting all acceptable applicants is the unique essentially dominant departments’ strategy. Furthermore it is the unique departments’ strategy which resists to the iterated elimination dominated strategies.

It follows that accepting all acceptable researchers are the unique strategies used in stable (a la Mertens) equilibria.

5.3 Core stability and efficiency

Now we consider the issue of core stability.

Let $\mathcal{M} = (D, R, P) = (D, R, P^D, P^R)$ be a matching market where $P^D$ is of the same form as in 1. Let $n = (n^d)_{d \in D}$ an assignation of financed contracts.

Definition 13 A matching $\mu$ on $\mathcal{M}$ is $n$–stable in $\mathcal{M}$ if it is individually rational and if there exists no $d \in D$ and no $\tilde{R} \subset A^d$ such that $n^d \geq |\tilde{R}| > |\mu(d)|$ and $dP^R \mu(r)$ for all $r \in \tilde{R} \setminus \mu(d)$.

The definition adapts to our definition of. In words a matching is stable with respect to a given assignation of contracts if is individually rational and no department could increase the number of hired acceptable researchers with candidates who prefer to join it rather than their current employer.

Incomplete information mitigates the clear-cut results obtained under the hypothesis of full information. This framework. In contrast with Theorem ?? a researcher could apply to some department(s) even she is not sure to join it. Consider the case in which a candidate who beliefs that if accepted by some departments she will be ranked in a good position. If she does not know exactly if she is acceptable to some departments she would like to join particularly she would apply to more departments if she cares enough of the profession (i.e. if application costs are low enough). It is similar the case in which a candidate applies to some department despite the risk of ending unemployed. She is not sure of passing the cut but, if she did she would join such department.

On departments’ side it is possible that for some realizations a department has accepted more candidates than the ones it finally hires. Even if the strategy
is ex-post redundant it can be strictly optimal at departments’ information set. This behavior might cause an idoneous candidate not to be employed. Actually any unstable equilibrium matching results in not assigned financed position/s.

There are two causes of such instabilities. The first one we could call **risk-induced** is due to the fact that the applicants who would like to join a department which otherwise would not fill all its vacancies are too low ranked with respect to the other idoneous candidates. It comes from the fact that departments because of lack of information and risk aversion over-accept candidates. The other form of instability is called **manipulative** and it is due to the fact that a good ranked candidate is by some department, not to loose lower ranked candidates. This form of instability requires more precise information as rejecting a well ranked candidate in favor of a lower ranked one reduced the probability of a matching with an acceptable researcher. It can emerge in a structure in which departments’ preferences are strict. From Proposition 7 such kind of strategies are dominated.

**Proposition 8** Only risk-induced or manipulative instabilities can emerge at equilibrium. In both cases at least one pre-accepted and idoneous researcher is unemployed. If departments play their essentially dominant strategy only meritocratic instabilities can emerge at equilibrium otherwise. So an outcome matching is stable iff the it employs $N$ researchers.

Looking at the data instabilities seems not very likely to occur. All contracts in budget (and even more) have gone assigned. An explanation is that ex-ante information is quite precise. It is also likely that the mechanism itself induces agents, mainly departments, to share the information they have on applicants and on the ranking. The argument is that the lack of information may induce redundant acceptations and instabilities that harm departments by reducing the number of the researchers assigned to some centers. Voluntary disclosure of the information can prevent such effects. Given the dimension and the structure of the market communication costs are probably lower than the ones in which departments would incur loosing financed positions.

### 6 Data and results

The data set we use can from the applications of the researchers on the first year of the Ramón y Cajal Program provided by the Direccion General de Investigacion of the Minestry of Science and Tecnology. In this process were involved 151 research centers that asked for a total of 2064 contracts. A total of 2807 apply to at least one research center and got a pre-acceptance that allows then to be evaluated. The researches present a total of 2939 different applications and where pre-accepted to 3974 positions. Finally 802 candidates were selected and 782 signed contracts. The applicants were evaluated by 24 commissions created by the "Agencia Nacional de Evaluacion y Prospectiva" (ANEP). A total of 341 experts participate on the evaluation process.
The application process allowed the researchers to choose the research centers that have pre-accepted then allowing also simultaneous applications to more than one research institution. 76.3% of the researches presented only one pre-acceptance.

**Table 1: Pre-acceptances**

<table>
<thead>
<tr>
<th>Number of pre-acceptances</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>More than 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Researches</td>
<td>229</td>
<td>486</td>
<td>124</td>
<td>45</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>(percentaje)</td>
<td>76.3%</td>
<td>16.6%</td>
<td>4.2%</td>
<td>1.5%</td>
<td>0.8%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Grants</td>
<td>562</td>
<td>150</td>
<td>31</td>
<td>11</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>(percentaje)</td>
<td>72.6%</td>
<td>19.4%</td>
<td>4.0%</td>
<td>1.4%</td>
<td>1.6%</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

The 17.31% of the applicants have obtain their PhD outside of Spain and 82.13% of then were residents in Spain at the moment of the application. Out of the 2229 applicants with only one pre-acceptance 616 of them were pre-accepted by the research center where their obtain their PhD. a total of 998 researches were in fact pre-accepted by the center where they obtain their PhD. From the total of 151 research centers 91 of then pre-accepted more researches than positions they have available. More details about the applicants on the first Ramon y Cajal can be found at Sanz Menéndez L. et all (2003).

Using the data of the applications realized by the researches to the research centers we test several of the insights provided by the model analyzed. First we regress using an order probit the relation between the number of pre-acceptances and the information the the researcher has.

In the data set we have several variables that we might think as proxies of the information available to the researches. We ca assume that a Spanish resident should have more information than a non-Spanish resident. Also the fact that a researcher has been previously in the Spanish system should provide then with contacts and information about the needs and perspectives they have in research center that are in some case already involved with.

On Table 2 we can observe how information (PhD obtained in the research center, Spanish resident) and availability of positions in the targeted research center (Max number of available positions) decrease the number of Acceptances solicited by the researchers. On the other hand the quality of the candidate (R&C Score) proxy by the score obtained in the Ramon y Cajal evaluation process increase the number of acceptances obtained.

These results seem to confirm the insights provided by the model both in complete and incomplete information.

If we restrict the analysis to those agents that are pre-accepted by only one research center the results are confirmed. The probability of been pre-accepted by only one research center increase the information (PhD obtained in the research center, Spanish resident) and decrease with the quality of the candidate.
Table 2: The Model of Pre-acceptance

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PhD obtained in the center</td>
<td>0.6018913</td>
<td>0.000</td>
</tr>
<tr>
<td>R&amp;C Score</td>
<td>0.0063939</td>
<td>0.000</td>
</tr>
<tr>
<td>Spanish resident</td>
<td>-0.3024992</td>
<td>0.000</td>
</tr>
<tr>
<td>Max number of available positions</td>
<td>0.0033858</td>
<td>0.000</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-2141.898</td>
<td></td>
</tr>
<tr>
<td>pseudo-R²</td>
<td>0.0881</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>2807</td>
<td></td>
</tr>
<tr>
<td>LR tests for joint significance of slopes</td>
<td>414.07</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(1) Maximum Likelihood estimates for order probit estimates. The pseudo-R² is the scaled value of the likelihood function whereby 100 is perfect fit and 0 is the constant model.

(2) p-values

7 Conclusions

Economic Theory suggests that successful mechanism design should take in account agent’s motives in order to provide them with the right incentives to accomplish its goals. The Ramón y Cajal program was designed to improve Spanish scientific researchs base by promoting the recruiting of top-researchers while preserving the autonomy of research institutions. The model we have presented provides insights on the way these objectives have interacted in the design of the matching mechanisms.

As a result we obtained a design failure. The limits on the recruitment of Spanish scientists were attributed to lack of meritocracy caused mainly by endogamous behavior. The objective of the mechanism was to affect agent’s preferences to a point that the center will accept high quality researchs base on meritocratic criteria.

So the meritocratic concerns gave rise to a candidate’s selection, which apparently had been take far from concerned centers, and gave best candidates apparent priority in their choices. On the other hand in order to preserve centers’ independence a preliminary acceptance phase was introduced. This interaction of this two stages produced large possibility of colluding between candidates and centers. Only large information asymmetry would seem able to prevent such collusion.

The analysis of the data seems to confirm the theoretical findings. Most of candidates presented a very low number pre-acceptance. Of them many took their PhD in the same center they finally were hired from. On the informational side insiders seem likely to present less application than non-Spanish resident, which confirms the results obtained in Section 5. Furthermore the quality of the candidate seems to be directly correlated with the number of applications made.

To these failures could be attributed the redesign of the mechanism done on
the fourth edition. In this redesign the pre-acceptance edition has been elimi-
nated. Therefore all candidates are considered by the evaluation commission. 
Also, the matching has been completely decentralized and any priority has been 
eliminated.

The new design should produce an improvement in the level of candidates 
awarded. However the mechanism does not guarantee that centers are compet-
ing for the best researchers it only can assure the overall quality of the candidates 
selected.

An interesting point for future research both theoretical and empirical, the 
latter conditioned by the availability of data, is the evolution of the agents’ 
behavior through the different editions of the Program. Changes due to learn-
ing but especially shifts in the informational environment would probably be 
evident. An interesting roles should have the previously rejected candidates, in 
their decision of participating (or not) once more to the process.

The data regarding the other editions could also help in clarify the role of the 
awarded candidates that finally decided quit the Program without having signed 
with any center. Their low number in the first edition seems to support the 
hypothesis of a reasonably complete information environment, but it precludes 
testing this hypothesis.
References


[22] Roth A. E., Sotomayor M. (1992), Two-sided matching, Handbook of Game Theory, Volume 1, Elsevier Science Publisher B.V.


8 Appendix A: Proof of Proposition 6

We prove Proposition 6 under a more general assumption on the decentralized mechanism beginning in the second stage.

**Condition 2** Let \( z \) be an initial node of the second stage. Let \( R' = R(z_2) \) be the set of idoneous researchers. Any SPE outcome of the decentralized assignment starting at \( z \) is stable in \((D, R(z), P^n(z), P_{R'})\) students.

We now prove Proposition 6 in a more general form, changing the claim to take in account Condition 2

**Proposition 9** Assume Condition 2 holds. Let \( T = r_1, \ldots, r_N, \ldots r_q \). and let \( \nu \) be stable in \((D, R, P^n)\).

(i) At any SPE of the new mechanism no more than \( \nu(D) \) researchers are employed.

(ii) If \( \nu(D) > N \) and let \( \mu \) be a matching in which exactly \( N \) researchers are employed. Let \( r_j^* \) be the worst ranked employed researcher. Let \( j^* > N \). Then \( \mu \) is an SPE outcome of the new mechanism if and only if \( \mu \) is an SPE outcome at \( z_\mu \) where \( z_\mu \) is the subgame induced by each department \( d \) accepting only agents in \( \mu(d) \) and if for all \( j < j^* \) such that \( \mu(r_j) = r_j \), \( r_j \) is single in the stable set of \((D, R(z_\mu) \setminus \{r_j^*\}) \cup \{r_j\}, P^n_\mu(z_\mu \cup \{r_j^*\}) \).

(iii) Let \( \mu \) be a matching in which \( N' < N \) researchers are employed. Then \( \mu \) is a SPE outcome at \( z_\mu \) where \( z_\mu \) is the subgame induced by each department \( d \) accepting only agents in \( \mu(d) \) for \((D, R(z_\mu) \cup \{r_j\}, P^n_\mu(z_\mu \cup \{r_j\}) \).

(iv) If \( \nu(D) > N \) and each researcher has positive application costs, exactly \( N \) researchers are employed at equilibrium. The set of SPE coincides with the set of matchings described in (ii)

(v) Let \( \nu(D) < N \) and assume that each researcher has positive application costs. Then the set of SPE at \((D, R, P)\) is the set of SPE outcomes of the decentralized assignment process characterized starting at \( R_1(z) = \nu(D) \).

**Proof.** (i) is trivial

(ii) Let \( \mu \) and \( j^* \) as in the claim and set \( R' = \mu(D) \). Let \( j < j^* \) such that \( \mu \) is an SPE outcome of the new mechanism if and only by participating to the selection, no such \( r_j \) can get a position. By deciding to participate \( j < j^* \) would result idoneous. Let \( z(j) \) be the subgame induced by such a deviation. \( R'(j) = (R' \setminus \{r_j^*\}) \cup \{r_j\} \). \( \mu \) is an SPE if and only if \( r_j \) is single as result of such a deviation. From Condition 2 and Martinez and al (2000) follows (ii) part of the claim.

(iii) is proven as (ii).

(v) follows from (ii) once observed that with positive application costs, at equilibrium, no unmatched agent applies.

(iv) follows from (ii) once observed that with positive application costs, at equilibrium, no unmatched agent applies. ■
9 Appendix B: Proofs of Proposition 7 and of Proposition 8

Let \((h, s)\) be a path in the game and let \(\sigma^{-d}\) be a profile of strategies for players other than \(d\).

Let \(R^d(h, s, \sigma^{-d}) = A(d, s) \cap R^d_1(h, s, \sigma^{-d})\) be the set of acceptable applications received by \(d\).

Let \(R^{-d}(s, \sigma^{-d}) = R^d(h, s, \sigma^{-d}) \cup \bigcup_{d \neq d} R^d(h, s, \sigma^{-d}) = \{r_1, ..., r_l\}, \) where \(r_1T...T r_l\). \(R^{-d}(s, \sigma^{-d})\) is the set of researchers that would be pre-accepted at state \(s\) if \(d\) accepted only its acceptable applicants.

Consider the following algorithm.

Let \(d_1 = \max_{\sigma^{-d}} \{d : r_1 \in R^{-d}(h, s, \sigma^{-d})\} \)

\(d_{j+1} = \max_{\sigma^{-d}} \{d : r_j \in R_2(d, h, s), \{0 < k < j : d = d_{j-k}\} < n^d\}\).

Let \(\mu(h, s, \sigma^{-d})(r_j) = d_j\) and set \(\mu(h, s, \sigma^{-d})(r) = r\) for all other \(r\).

Let \(\sigma(h, s, \sigma^{-d})(d) = \mu(h, s, \sigma^{-d})(d)\) for all \((h, s)\) and set \(R^d_2(I^d, \sigma^{-d})(d) = \bigcup_{\pi^d(h, s, I^d, \sigma^{-d})(d) > 0} \sigma(h, s, \sigma^{-d})(d)\).

We will call \(\sigma(I^d, \sigma^{-d})(d)\) the secure strategy against \(\sigma^{-d}\) at \(I^d\). The secure strategy against \(\sigma^{-d}\) amounts in accepting acceptable researchers that with positive probability will join \(d\) at the following stage. Playing secure strategy assure the best ranked ones.

Then it is the best each department can do if it only cares to fill all its position, whatever information it owns. Formally

**Lemma 4** Let \(d \in D\). Let \(\sigma^{-d}\) be a profile of strategies for players other than \(d\), in which researchers play according to the strategies described in Proposition 1. Using the secure strategy profile at \(I^d\) against is \(\sigma^{-d}\) sequentially rational for \(d\) at \(I^d\). And the secure strategy maximizes the number of researchers that can get at each state.

**Proof.** Let \(\pi^d(h, s \mid I^d, \sigma^{-d})(d) > 0\). Accepting any \(R_2 \supset \sigma^d_2(h, s, \sigma^{-d})(d)\) will match \(d\) with all the agents in \(\mu(h, s, \sigma^{-d})(d) = \sigma(h, s, \sigma^{-d})(d)\). So we can consider subsets of \(R_1(d, I^d)\) such that \(\sigma(I^d, \sigma^{-d}) \setminus R_2 \neq \emptyset\). In particular for some \((h, s)\) such that \(\pi^d(h, s \mid I^d, \sigma^{-d})(d) > 0 \sigma(h, s, \sigma^{-d})(d) \setminus R_2 \neq \emptyset\). It suffices to show that for any such \((h, s)\) accepting such \(R_2\) does not increase the cardinality of \(\mu(h, s)(d)\) where \(\mu(h, s)\) denotes the outcome function at \((h, s)\), fixed \(\sigma^{-d}\). We can assume also that \(R_2 \setminus \sigma(I^d, \sigma^{-d}) \neq \emptyset\) and that \(R_2\) contains only acceptable researchers. Let \(\mu_0\) be the outcome matching after \((h, s)\) when \(d\) plays her secure strategy and let \(\mu\) be the outcome matching after \((h, s)\) when \(d\) accepts the researchers in \(R_2\). \(\mu_0(d) = \sigma(h, s, \sigma^{-d})(d)\). As the outcome for \(d\) after \((h, s)\) is the same if \(d\) accepts \(R_2\) and if it accepts \(\mu(d)\) we can assume that \(\mu(d) = R_2(d)\). Let \(\mu_0(d) \setminus R_2 = \{r_1, ..., r_l\} = \{\hat{r}_1, ..., \hat{r}_l\}\) and let \(\mu_1(d) \setminus \mu_0(d) = \{\tau_1, ..., \tau_p\} = \{\hat{\tau}_1, ..., \hat{\tau}_p\}\). Observe that it must be the case that

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by construction of \( \sigma(I^d, \sigma^{-d}) \). Consider the following algorithm. Let \( \mu^0 \) be the outcome matching when \( d \) accepts the researchers in \( \mu_0(d) \cup R_2 \). For 
\[ 1 \leq j \leq s; \textbf{Step j: } R^j = \mu^{j-1}(d) \cup R_2 \setminus \{r_{s-j+1}\}. \]
Let \( \mu^j \) be the outcome matching when \( d \) accepts the researchers in \( R^j \). We have \( \mu^s = \mu \). Accepting \( \mu_0(d) \cup R_2 \) matches \( d \) with \( \mu_0(d) \). We now prove that \( |\mu^{j+1}(d)| \leq |\mu^j(d)| \) for all \( 0 \leq j \leq s \). First of all observe that \( \mu^{j-1}(r_{s-j+1}) = d \). Rejecting \( r_{s-j+1} \) at step \( j \) does not affect the choices of candidates ranked above \( r_{s-j+1} \). This rejection can create at most one cycle of rejections interesting only lower ranked candidates. Then \( d \) is matched at most with one candidate different than \( r_{s-j+1} \) at each step. So \( |\mu^{j+1}(d)| \leq |\mu^j(d)| \) and \( |\mu(d)| = |\mu^s(d)| \leq |\mu^0(d)| \).

In general not only supersets of the secure strategy are best responses with respect to a given strategy. From the proof of Lemma 4 follows that are someway robust to changes in the information setup. Playing different strategies means to believe in the possibility of a matching with low ranked candidates. But lower ranked candidates have in general lower probability of passing the “cut”.

We will say that a department strategy is \textbf{essentially dominant} if it is dominant in the reduced game in which researchers play according to the strategy in proposition 1.

\textbf{Proof of Proposition 7.} It follows from Lemma 4 and Remark 5.

\textbf{Lemma 5 (Proof of Lemma ??)} Let \( \sigma^* \) be an equilibrium and let \( \mu^* = \mu^*(s, T) \) be the equilibrium outcome matching at state \( (s, T) \). Let \( d \in D \) and let \( \hat{R} \subset A(d, s) \) such that \( n^{d*} \geq \hat{R} > |\mu^*(d)| \) and \( dP^r(s)\mu^*(r) \) for all \( r \in \hat{R} \mu^*(d) \). At equilibrium \( |\mu^*(d)| \) is the maximum number of researchers \( d \) can get at \( (s, T) \) against \( \sigma^{-d} \) (from Lemma 4). Let \( r \in \hat{R} \mu^*(d) \) such that \( \left| \left\{ r' \in \bigcup_{d'} A_2(d', s, I^{s-d}): r'Tr \right\} \right| < N \). Consider first the case in which \( r \) applied to \( d \). By contradiction suppose \( r'Tr \) for all \( r' \in \mu^*(d) \). Consider the following deviation for \( d \): accepts \( \mu^*(d) \cup \{r\} \).

It would match \( d \) with \( \mu^*(d) \cup \{r\} \) at \( (T, s) \). But it contradicts Lemma 4: it would follow that \( d \) could obtain at least \( |\mu^*(d)| + 1 \) researchers by playing its secure strategy against \( \sigma^{-d} \). Now consider the case in which \( r \) did not applied to \( d \). Consider the following deviation for \( r \): applies to the same departments as in \( \sigma^{-d} \) and to \( d \). It changes the information set of no department but \( d \). Starting from such path \( d \) can get at least \( |\mu^*(d)| \) researchers at \( (T, s) \). As costs are small it must be the case that \( d \) is not matched with \( r \) as result of such deviation. If it was not the case applying also to \( d \) would increase \( r \)’s probability of better matchings. Then \( d \) can get at most \( |\mu^*(d)| \). Let by contradiction be \( r'Tr \) for all \( r' \in \mu^*(d) \). Accepting only \( \mu^*(d) \cup \{r\} \) would procure \( |\mu^*(d)| + 1 \) researchers which, via Lemma 4, constitutes a contradiction. The last part of the claim follows by observing that \( d \) has a vacancy at \( (s, T) \).