Household Income Risk, Nominal Frictions, and Incomplete Markets*

Preliminary and incomplete.
Please do not quote!

Christian Bayer†, Ralph Lütticke,‡ Lien Pham‡ & Volker Tjaden‡

October 14, 2013

Abstract

Households face substantial idiosyncratic income uncertainty that is up to two orders of magnitude larger than total factor productivity uncertainty, very persistent and varies substantially over the business cycle. We examine the macroeconomic effects of such uncertainty shocks. We build a New Keynesian model with heterogeneous agents, where time-varying uncertainty changes precautionary savings and heightened uncertainty hence depresses aggregate activity. With sticky prices, increased precautionary savings lower aggregate demand and generate significant output losses. The decline in output is more severe, if the central bank is not active in expanding the monetary base upon an uncertainty shock. Our results imply that household income uncertainty may be an important factor in explaining the persistent decline of consumption during the Great Recession.

Keywords: Incomplete Markets, Nominal Rigidities, Uncertainty Shocks.
JEL-Codes: E22, E12, E32

1 Introduction

The Great Recession has led to a reconsideration of the role of uncertainty in business cycles. Increased uncertainty has been documented in various markets, but household income uncertainty stands out in size and importance. Households face substantial idiosyncratic income uncertainty that is up to two orders of magnitude larger than the one of total factor productivity. Shocks to household income are very persistent and their variance changed substantially over the business cycle. The seminal work by Storesletten et al. (2001) estimated that during an average NBER recession households' income uncertainty, interpreted as the standard deviation of persistent income shocks, is about 126% higher than at the peak of an expansion.

In any model with incomplete asset markets such time-varying household income uncertainty translates into business cycle variations in the propensity to consume, hence in aggregate demand. When income uncertainty increases, households demand more assets for precautionary motives and private consumption declines. To quantify these effects, we augment the incomplete markets model, as pioneered by Bewley (1980), Huggett (1993), and Aiyagari (1994), by nominal rigidities similar to Gornemann et al. (2012). In this model, uncertainty driven aggregate demand changes lead to time-varying markups and finally fluctuations in aggregate output.[1] Depending on the reaction and possibilities of monetary policy, the supply of liquidity may not increase sufficiently in order to restore the flexible-price equilibrium.

The setup we study isolates the effect of uncertainty transmitted from asset demand to aggregate demand. Other studies have emphasized the role of an increase in labor supply caused by uncertainty spikes, see e.g. Basu and Bundick (2011). Yet, this channel - at least for increases in household level income uncertainty - rests upon the assumption that households can adjust the intensive margin of their labor choice easily. If one assumes labor supply decisions to be lumpy investment-like decisions, the opposite finding should hold: households wait longer in adjusting their labor market participation.

Other authors have analyzed settings in which uncertainty shocks change aggregate productivity; either through changes in worker composition, see Takahashi (2013), through decreased reallocation, see Bloom et al. (2012), or directly through unemployment duration, see Ravn and Sterk (2013). We are agnostic about all these channels

---

[1]Storesletten et al. (2001) explore the effect of time-varying income uncertainty in a standard incomplete markets model analyzing the welfare costs of business cycles and argue that countercyclical uncertainty generates considerable welfare costs. They do not explore the business cycle implications of time-varying uncertainty. There is also a literature that models cyclical variations in factor reallocation as transmission mechanism, e.g. Bachmann and Bayer (2013) or Bloom et al. (2012)

and therefore, look at uncertainty shocks that do not impact aggregate productivity or aggregate labor supply – neither directly by changing the total labor endowment of the economy nor indirectly through households' labor supply decisions, but focus on the fluctuations in aggregate demand caused by uncertainty shocks.

For this purpose, we model an economy in which households are subject to idiosyncratic labor income risk and have access to two assets as means of self-insurance. One asset is nominal and liquid (money) the other one is an illiquid, dividend paying real asset. We model the illiquidity of the asset as limited participation in the asset market, where the real asset can only randomly be traded as an approximation to a more complex trading friction as in Kaplan and Violante (2011) who follow the tradition of Baumol (1952) and Tobin (1956). It is the liquidity of money that makes households willing to hold it even though it is a return dominated asset in our model, and when uncertainty spikes, households seek to rebalance their portfolios towards money, i.e. the nominal, liquid asset.

In this setup, we study two monetary policy regimes. First, we consider a regime, in which the central bank does not engage in stabilizing the economy. Here, the Pigou (1943) effect brings back the economy to the equilibrium and we show that in this regime, an uncertainty increase can have substantial depressing real effects. A one standard deviation increase in income uncertainty decreases aggregate activity over the first year by roughly 0.1%, roughly the same size that Fernández-Villaverde et al. (2011) find for a two standard deviation shock to fiscal uncertainty. The economy recovers from this shock only fairly sluggishly after 12 quarters. Since an uncertainty shock works effectively as a demand shock in our model, monetary policy can eliminate the aggregate effects on output. We then go on to study the distributional and welfare effects of uncertainty shocks and of systematic monetary policy response in this model. Since an uncertainty shock decreases wages and the price level (i.e. increases the price of money) in the economy, households that hold large sums of money gain from an increase in uncertainty, while households rich in human and physical capital lose most. Active monetary policy can change the overall costs of uncertainty shocks, but the general direction of distributional effects remains if we assume wasteful seignorage: Upon the uncertainty shock seignorage increases are financed by asset poor and human capital rich households that buy the freshly printed money.

The remainder of the paper is organized as follows: Section 2 reviews the related literature, Section 3 develops our model. Section 4 discusses the solution method. Section 5 presents the calibration, Section 6 our numerical results, and Section 7 concludes.

2 Related Literature

There is now a vast empirical and theoretical literature on the aggregate effects of time-varying uncertainty. The seminal paper by Bloom (2009) discusses the effects of time-varying productivity uncertainty on firms' factor demand exploring the idea and effects of time-varying real option values of investment. This paper has triggered a stream of research discussing under which conditions such variations have aggregate effects.[2]

A more recent branch of this literature focusses on the response of households to increases in uncertainty and discusses this in a framework with nominal rigidities. Basu and Bundick (2011) show that an increase in aggregate uncertainty operates in a standard New Keynesian model with capital through the labor market and the "paradox of toil". If uncertainty about aggregate productivity increases, the representative household wants to insure against the higher income uncertainty by producing more today, i.e. supplying more labor. As a result, wages and hence marginal costs for firms fall. If prices are rigid, firms' markups over marginal costs will increase and the demand for consumption and investment goods falls. Since in the New Keynesian model output is demand driven, a recession follows.

Mericle (2012) also focusses on the "precautionary labour channel" as Basu and Bundick (2011), but develops similar to us a model with incomplete markets, nominal rigidities and shocks to idiosyncratic uncertainty. Our contribution differs from Mericle (2012) by modeling strategy, solution method and focus. We explicitly model central bank policy, we solve for the full general (Krusell-Smith) equilibrium taking the dynamic evolution of heterogeneity into account, and focus on the quantitative implications of uncertainty shocks in the tradition of calibrated DSGE models of the business cycle. Mericle (2012), by contrast, keeps prices of consumption goods entirely fixed, such that households hold out-of-equilibrium price expectations. While all this leads to strong and intuitive theoretical results that highlight important propagation channels, it limits the quantitative predictions.

Moreover, we can discuss the distributional consequences of more active or more passive monetary policy upon an uncertainty shock, which relates our analysis to Gornemann et al. (2012). Since an uncertainty shock drives up the value of money but decreases wages and dividends, it implicitly transfers wealth from those households that are rich

---

[2]Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2010), Chugh (2012), Gilchrist et al. (2010), Narita (2011), Panousi and Papanikolaou (2012), Schaal (2011), and Vavra (2012) have studied the business cycle implications of a time-varying dispersion of firm-specific variables, often interpreted as and used to calibrate shocks to firm risk, propagated through various frictions: wait-and-see effects from capital adjustment frictions, financial frictions, search frictions in the labor market, nominal rigidities and agency problems.

in human and physical capital to those that hold a lot of money. In contrast to Gorne-mann et al.'s model, however, we differentiate between the effect of monetary policy on different assets and we highlight the role of money, i.e. an asset in which prices are denominated. We find that an uncertainty shock drives down the relative price of capital shares in the short-run, because households want to change their portfolio structure, holding more money for two reasons: in more uncertain times households want to hold more liquid portfolios for insurance purposes and furthermore the return on money goes up with lower inflation, while dividends fall. This portfolio readjustment effect amplifies the depressing real economic effect of uncertainty shocks.

Most closely related is the paper by Ravn and Sterk (2013), which looks at shocks to the duration of unemployment in an incomplete markets model with labor search and nominal frictions. There, a shock to unemployment duration increases the perceived income risk of households and lets these households demand more government bonds as means to self-insure. At the same time, the shock decreases labor supply as households remain unemployed longer after a job-separation. They find that the increased uncertainty might substantially propagate and amplify the effects of the shock to unemployment duration if the central bank does not stabilize inflation strongly.

Another closely related paper is Den Haan et al. (2013). In their setup, agents face imperfectly insured unemployment risk and a search friction in the labor market à la Diamond-Mortensen-Pissarides. As in our model, agents can hold two types of assets to save and insure themselves: money and equity. But unlike in our model, equity is not physical capital in the sense of the neoclassical standard model but is equated with vacancy-ownership. The asset market clears, when the demand for money equates the constant money supply and the demand for firm ownership equates the number of firms. While in our model the nominal friction is a price rigidity for consumption goods, their model imposes a wage setting function which potentially allows for nominal and real wage rigidity. When wages are sticky the precautionary money demand induces a strong deflation which pushes up real wages and strongly decreases the incentive for firm creation which is governed by a standard free-entry condition. In that case, they also find that the downturn is amplified through precautionary savings.

3 Model

We model an economy inhabited by two types of agents: workers and entrepreneurs. Workers supply capital and labor and are subject to idiosyncratic shocks to their labor productivity. Workers face idiosyncratic persistent labor income risk that is time

varying. They self insure in a liquid nominal asset (money) and a less liquid physical asset (capital), which they rent out to the intermediate goods producing sector on a perfectly competitive rental market. This sector combines labor services, a fixed capital stock, and final goods into intermediate goods (in roundabout production). Risk neutral entrepreneurs then differentiate these intermediate goods into final consumption goods and set prices for these final goods monopolistically competitive. They are subject to a pricing friction à la Calvo (1983) and adjust their prices with some positive probability.

We model the liquidity of money in the general spirit of Kaplan and Violante's (2011) model of wealthy hand-to-mouth consumers, where households hold claims on the economy's stock of capital, but trading these claims is subject to a trading friction. We model this trading friction as limited participation in the asset market. Every period a fraction of households is randomly selected to be able to trade shares in the physical stock of capital. All other households can only adjust their money holdings. We keep the economy's stock of capital fixed in the aggregate such that changes in demand for physical assets only translate into asset price changes.

3.1   Worker Households

There is a continuum of ex-ante identical worker-households (in short households) of measure one. Households are infinitely lived, derive felicity from consumption c_t and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}\}} \sum_{t=0}^{\infty} \beta_W^t u\left(c_{it}\right) \tag{1}$$

The felicity function is twice continuously differentiable, increasing and concave in c_t, and takes CRRA form with risk aversion ξ:

$$u(c_{it}) = \frac{1}{1-\xi} c_{it}^{1-\xi}, \xi > 0.$$

c_{it} is household i's demand of the bundled consumption good obtained from bundling varieties j of differentiated consumption goods according to a Dixit-Stiglitz aggregator

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} \, dj \right)^{\frac{\eta}{\eta-1}}.$$

Each of these differentiated goods is offered at price p_{jt} such that the demand for each of the varieties is given by

$$c_{ijt} = \left(\frac{p_{jt}}{P_t}\right)^{-\eta} c_{it},$$

where $P_t = \left(\int p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$ is the average price level.

Workers derive income from supplying labor and from renting out physical capital. A household is endowed in each period with h_{it} efficiency units of labor, which evolves according to an AR(1)-process.

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N\left(\mu, \sigma_{ht}\right) \tag{2}$$

Since we abstract from a labor leisure trade-off to isolate the precautionary savings effect of income uncertainty, households provide all of their hours of labor and thus total labor input supplied is given by[3]

$$N_t^S = \int h_j dj.$$

We assume that asset markets are incomplete. Households can only trade in nominal money, $\tilde{m}_{it}$, that does not bear any interest and in physical capital to smooth their consumption. Households can moreover only hold non-negative amounts of both assets.[4] Real money holdings of a household at the end of period t are denoted by $m_{it+1} := \frac{\tilde{m}_{it+1}}{P_t}$.

Every period a fraction of ν households is randomly selected to participate in the asset market and to trade their physical capital. All other households only obtain their dividends and may adjust their money holdings. For those households participating in the asset market, the budget constraint reads

$$c_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + w_t h_{it}, \quad \tilde{m}_{it+1}, k_{it+1} \geq 0, \tag{3}$$

where m_{it} are real money holdings, k_{it} are physical capital holdings, h_{it} is the stochastic endowment with efficiency units of human capital, q_t is the price of a capital share, w_t is the wage rate, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate.

For those households that cannot trade in the market for physical capital the budget

---

[3]Time varying dispersions of log-human capital endowments introduce small variations in labor supply. We solve the model taking these into account - hence the time index N_t^S - but the actual variations are small.

[4]The non-negativity requirement on money holdings reflects the natural borrowing limit. Any other borrowing limit might lead in case of a sufficiently large deflation to a violation of the requirement that households need to be able to repay their debt.

constraint simplifies to

$$c_{it} + m_{it+1} = \frac{m_{it}}{\pi_t} + r_t k_{it} + w_t h_{it} \quad \tilde{m}_{it} \geq 0. \tag{4}$$

Since households' saving decisions will be some non-linear function of a household's wealth and productivity, the price level P_t and therefore aggregate real money $M_{t+1} = \frac{\tilde{M}_{t+1}}{P_t}$ will be functions of the entire joint distribution Θ_t of (m_t, k_t, h_t). This makes Θ_t a state variable of the household's planning problem. This distribution evolves as result of the economy's reaction to shocks to uncertainty, which we model as time variations in the variance of idiosyncratic income shocks. We assume as process for the stochastic income shock volatility, σ_{ht}^2,

$$\sigma_{ht}^2 = \bar{\sigma}^2 s_t, \quad \log s_t = \rho_s \log s_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_s), \tag{5}$$

where $\bar{\sigma}^2$ is the steady state labor risk of the households and s shifts this uncertainty.

With this setup, the dynamic planning problem of a household is then characterized by two Bellman equations, V_a in case the household can adjust its capital holdings and V_n otherwise

$$\begin{aligned}
V_a(m, k, h; \Theta, s) =& max_{k', m_a'} u[c(m, m_a', k, k', h)] \\
&+ \beta_W \left[\nu EV^a(m_a', k', h', \Theta', s') + (1-\nu)EV^n(m_a', k', h', \Theta', s') \right] \\
V_n(m, k, h; \Theta, s) =& max_{m_n'} u[c(m, m_n', k, k, h)] \\
&+ \beta_W \left[\nu EV^a(m_n', k, h', \Theta', s') + (1-\nu)EV^n(m_n', k, h', \Theta', s') \right] \quad (6)
\end{aligned}$$

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as c_a^* and c_n^*, the money holding policies as m_a^* and m_n^* and the capital investment policy as k^*. Details on the properties of the value and policy functions, the first order conditions, and the algorithm we employ to calculate the policy functions can be found in Appendix A.

3.2 Intermediate Goods Producers

Since we abstract from a household's labor-leisure choice and treat aggregate capital as fixed, we need to introduce another mechanism through which aggregate output can vary in the economy.[5] We follow Basu (1995) and assume that the intermediate goods

---

[5]Another way would be to model capacity utilization. Varying utilization, however, has consequences for the variability of the marginal productivity of capital, which is substantially dampened.

producing sector operates a gross production function with constant returns to scale instead of a value added production function, which combines pre-products X_t acquired on the final consumption goods market at price P_t, labor $N_t = N_t^S$ and capital $K_t = \bar{K}$. Hence, total gross output of the intermediate goods sector is

$$Y_t = X_t^\gamma N_t^{\alpha(1-\gamma)} \bar{K}^{(1-\alpha)(1-\gamma)}.$$

Let MC_t be the relative price at which intermediate goods are sold to final goods producers. The intermediate goods producers seek to maximize profits through their choice of the extend of pre-products used in production.

$$MC_t Y_t - X_t = MC_t X_t^\gamma N_t^{\alpha(1-\gamma)} \bar{K}^{(1-\alpha)(1-\gamma)} - X_t$$

The optimal amount of pre-products is then given by

$$X_t^* = \gamma MC_t Y_t = (\gamma MC_t)^{\frac{1}{1-\gamma}} N_t^\alpha \bar{K}^{1-\alpha}. \tag{7}$$

Once the optimal amount of pre-products used in production is determined, we can express GDP, which is equal to consumption in this setting, as

$$C_t = Y_t - X_t^* = \left[(\gamma MC_t)^{\frac{\gamma}{1-\gamma}} - (\gamma MC_t)^{\frac{1}{1-\gamma}} \right] N_t^\alpha \bar{K}^{1-\alpha}. \tag{8}$$

It moreover implies that the intensity in which pre-products are used in production is pro-cyclical, which is in line with the data, as:

$$X_t/Y_t = \gamma MC_t.$$

The real wage and the user costs of capital are given by the marginal products of labor and capital.

$$w_t = \alpha(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} MC_t^{\frac{1}{1-\gamma}} \left(\bar{K}/N_t \right)^{1-\alpha} \tag{9}$$

$$r_t + \delta = (1-\alpha)(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} MC_t^{\frac{1}{1-\gamma}} \left(N_t/\bar{K} \right)^\alpha \tag{10}$$

3.3 Entrepreneurs

Entrepreneurs produce final goods by differentiating intermediate goods and set prices. We assume that entrepreneurs are risk neutral and have the same discount factor as the worker-households. We assume that only the central bank can issue money such

that the entrepreneurs do not participate in the money market. We make this assumption for tractability reasons in order to separate the price setting problem from the worker-household's saving problem, as it enables us to determine the price setting of entrepreneurs without having to take into account intertemporal decision making of the workers.[6] Under these assumptions, the consumption of an entrepreneur is given by

$$c_t^E = \Pi_{jt},$$

where Π_{jt} is the current profit of the j-th final goods producer. Given their preferences,

$$V = E_0 \sum_{t=0}^{\infty} \beta_E^t \Pi_{jt} \tag{11}$$

entrepreneurs maximize over prices of final goods.

Final goods producers buy intermediate goods at a price equalling the nominal marginal costs $MC_t P_t$, where MC_t are the real marginal costs at which the intermediate goods are traded due to perfect competition, and differentiate them without the need of additional input factors. Final goods come in varieties uniformly distributed on the unit interval and each indexed by $j \in [0, 1]$. Resellers are monopolistic competitors and therefore can charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction and can only update their prices with probability θ. They maximize the expected value of future discounted profits by setting today's price p_{jt} taking into account the price setting friction:

$$\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} (\theta \beta_E)^s E \Pi_{jt,t+s} = \sum_{s=0}^{\infty} (\theta \beta_E)^s E Y_{jt,t+s} (p_{jt} - MC_{t+s} P_{t+s}) \tag{12}$$

$$s.t. : Y_{jt,t+s} = \left(\frac{p_{jt}}{P_{t+s}} \right)^{-\eta} Y_{t+s}$$

where $\Pi_{jt,t+s}$ are the profits and $Y_{jt,t+s}$ is the production level in $t + s$ of a firm j whose last price reset was in period t.

We obtain the following first order condition with respect to p_{jt}:

---

[6]Alternatively, we could have assumed Rotemberg (1982) pricing at the expense of a less straightforward calibration.

$$\sum_{s=0}^{\infty}(\theta\beta_E)^s EY_{jt,t+s}\left(\frac{p_{jt}^*}{P_{t-1}} - \underbrace{\frac{\eta}{\eta-1}}_{\mu} MC_{t+s}\frac{P_{t+s}}{P_{t-1}}\right) = 0 \tag{13}$$

where μ is the static optimal markup.

Since individual resellers are risk neutral, we can solve the resellers' planning problem by log-linearizing around the zero inflation steady state without having to know the solution of the worker-households' problem as entrepreneurs and worker-households do not interact in any inter-temporal trades.[7] This yields after some tedious algebra, see e.g. Galí (2008), the new Keynesian Phillips curve.

$$\log \pi_t = \beta_E E_t(\log \pi_{t+1}) + \kappa(\log MC_t + \mu) \tag{14}$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

3.4 Goods, Money, Asset and Labor Market Clearing

The labor market clears at the competitive wage given in (9); so does the market for capital services if (10) holds. The goods market then clears, whenever the money and asset markets clear. We assume that money supply is given by a monetary policy rule that adjusts the growth rate of money in order to stabilize inflation, i.e.

$$\frac{M_{t+1}}{M_t} = (\theta_1/\pi_t)^{1+\theta_2}. \tag{15}$$

Here M_{t+1} are real balances at the end of period t (with the timing aligned to our notation for the household's budget constraint), $\theta_1 \geq 1$ determines steady state inflation and $\theta_2 \geq 0$ the extent to which the central bank tries to stabilize inflation around its steady state value – the larger θ_2 the more does the central bank react to current inflation, as $\theta_2 \to \infty$ inflation is perfectly stabilized at the steady state value. We assume the central bank to waste any seignorage buying final goods and choose the above functional form for simplicity in order to guarantee a (weakly) positive value of seignorage.

---

[7]Importantly we study the effect of idiosyncratic, not aggregate, uncertainty shocks. Therefore, the shocks that entrepreneurs face are small and approximately homoscedastic, such that a first order approximation is sufficiently precise.

The money market clears, whenever

$$(\theta_1/\pi)^{1+\theta_2} M_t = \int \left[\nu m_a^*(m,k,h;q,\pi) + (1-\nu)m_n^*(m,k,h;q,\pi) \right] \Theta_t(m,k,h)dmdkdh. \tag{16}$$

with last end-of-period's real money holdings

$$M_t := \int m_t \Theta_t(m_t,h_t)dm_t dh_t$$

In addition, we need that the market for capital shares clears

$$\nu \bar{K} = \nu \int k^*(m,k,h;q,\pi)\Theta_t(m,k,h)dmdkdh. \tag{17}$$

The goods market clears, whenever both, money and asset, markets clear due to Walras' law.

3.5   Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{c_a^*, c_n^*, m_a^*, m_n^*, k^*\}$, value functions V_a, V_n, pricing functions $\{r, w, \pi, q\}$, aggregate capital and labor supply functions $\{N, K\}$, distribution Θ over individual asset holdings and productivity, and a perceived law of motion Γ, such that

1. Given V, Γ, prices, and distributions, the policy functions $\{c_a^*, c_n^*, m_a^*, m_n^*, k^*\}$ solve the household's problem and given the policy functions $\{c_a^*, c_n^*, m_a^*, m_n^*, k^*\}$, prices and distributions, the value functions V_a, V_n are a solution to the Bellman equations (6).

2. The labor, money, capital and intermediate and final good markets clear, i.e. (9), (14), (16), and (17) hold.

3. The actual law of motion and the perceived law of motion Γ coincide, i.e. $\Theta' = \Gamma(\Theta, s')$.

4   Numerical Implementation

Of course the dynamic program (6) and hence the recursive equilibrium is not computable as it involves the infinite dimensional object Θ.

4.1 Krusell-Smith equilibrium

In order to turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments as in Krusell and Smith (1997, 1998). Specifically, we make the assumption that households condition their expectations only on last period's aggregate real money holdings M_t, the realized variance of idiosyncratic productivity $var(h_{it})_t$, and the uncertainty state s_t. The idea of this assumption is that (16) determines inflation, which obviously depends on the current money stock. Once inflation is determined, the Phillips curve (14) determines markups and hence wages and dividends. In turn, this will pin down asset prices by making the marginal investor indifferent between money and physical assets. If the optimal money demand function $m_{a,n}^*$ and k^* are sufficiently close to a second order polynomial in h and linear in non-human wealth where the mass of Θ is, then we can expect approximate aggregation to hold with s_t, $var(h_{it})_t$, and M_t.

While the laws of motion for s_t and $var(h_{it})_t$ are pinned down by (2) and (5), households use the following log-linear forecasting rule for future inflation and asset prices where the coefficients may depend on the uncertainty state.

$$
\log \pi_t = \beta_\pi^1(s_t) + \beta_\pi^2(s_t) \log M_t + \beta_\pi^3(s_t) var(h_{it})_t \tag{18}
$$

$$
\log q_t = \beta_q^1(s_t) + \beta_q^2(s_t) \log M_t + \beta_q^3(s) var(h_{it})_t. \tag{19}
$$

The law of motion for real money holdings M_t then follows from the monetary policy rule and sequential market clearing and is given by

$$
\log M_{t+1} = \log M_t + (1 + \theta_2)(\log \theta_1 - \log \pi_t)
$$

Fluctuations in q and π come from two sources: The self-insurance services that workers receive from the capital good fluctuate as uncertainty varies and the rental rate of capital fluctuates as firms' markup is changing. When making their investment decisions, workers need to predict the capital price q' in the next period in order to predict returns on their investment. Since the amount of physical capital is fixed and share prices are not linked intertemporarily (unlike goods prices i.e. there is no pricing friction in the asset market), last periods value of physical assets is no state variable. Since all other prices are known functions of the markup, only π and q need to be predicted.

Technically, finding the equilibrium inflation rate and asset prices is similar to Krusell and Smith (1997), as we need to find market clearing prices in each period. Concretely, this means the posited rules (18) and (19) are used to solve for the household's policy

functions. Having solved for the policy functions of the household conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \ldots, T$ periods, keeping track of the actual distribution Θ_t. The initial distribution Θ_1 in each simulation equals the stationary one from a model without aggregate risk. We then calculate in each period t the optimal household policies for market clearing inflation rates and asset prices assuming that the household resorts to the policy functions derived under rule (19) from period $t+1$ onwards. After determining the market clearing inflation rate, we obtain next period's distribution Θ_{t+1}. In doing so, we obtain n sequences of equilibria. The first 150 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (19) from the simulated data and update the parameters accordingly. By using $n = 10$ and $T = 350$, it is possible to make use of parallel computing resources and obtain 2500 equilibrium observations. Subsequently, we re-calculate policy functions and iterate until convergence in the forecasting rules.

The quality of approximation from (18) and (19) is relatively high. The minimal within sample R^2 is 99.88%. Also the out-of-sample performance, see Den Haan (2010a), of the forecasting rule is good, see the Appendix for details.

4.2 Solving the household planning problem

In solving for the household's policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We approximate the productivity process by a discrete Markov chain with 11 states and time-varying transition probabilities, using the method proposed by Tauchen (1986). The stochastic volatility process is approximated in the same vein using 5 states.[8] Details on the algorithm can be found in Appendix A.4.

5 Calibration

We calibrate the model to the U.S. economy, identifying all parameters from steady state model behavior where we set uncertainty fluctuations to zero.[9] In the simulation of the model we find that time-averages of aggregate variables in the model are very close to

---

[8]We solve the household policies for 50 points on the grid for money and 80 points on the grid for capital.

[9]We check whether average asset demand is different in the model with uncertainty fluctuations and find little difference.

their steady state values. The aggregate data used for calibration spans 1984:Q1 to 2008:Q4. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 1 and 2 is explained next. We present the parameters as if they were individually changed in order to match a specific data moment, but of course all parameters are calibrated jointly.

5.1 Income Process

We calibrate the income process and hence uncertainty faced by households to established estimates for the U.S. We adopt the conventional AR(1) process for idiosyncratic productivity, as the dynamics of individual earnings in the Panel Study of Income Dynamics (PSID) is quite well replicated by an autoregressive process. The algorithm by Tauchen (1986) is used to discretize the AR(1) process for the log of individual productivity with mean zero, persistence parameter ρ_h and a variance of the innovation of σ_t^2. The autocorrelation of annual earnings is chosen to be 0.95, which is within the range of existing empirical estimates (0.9 to 1) and close to the number reported in Bayer and Juessen (2009). In line with their estimates, we set the quarterly variance of persistent income innovations to $\sigma_h^2 = 0.0873$.

Bayer and Juessen also find countercyclical variations of the variance of persistent shocks, which negatively co-moves with deviations from HP-filtered GDP. They find a coefficient of variation of roughly 50%. The persistence of uncertainty shocks is arguably the hardest item to measure as it would require extremely long panel data and we can only estimate it to be in line with aggregate fluctuations and hence set the quarterly autocorrelation to $\rho_s = .9$. In Appendix D, we provide robustness checks. Table 1 summarizes the parameter choices.

Table 1: Income Process

Parameter	Value	Description
ρ_h	$0.95^{1/4}$	Persistence of income
σ_h^2	0.09	Average variance of innovations to income
ρ_s	0.90	Persistence of the income-innovation variance, σ_h^2
σ_s^2	0.02	Conditional variance the income-innovation variance, σ_h^2

5.2 Preferences and Technology

Table 2 summarizes our calibration for the parameters that cannot be directly estimated as an exogenous shocks process. In detail we calibrate them as follows.

5.2.1 Households

The period utility function is of the constant relative risk aversion form, twice continuously differentiable as well as increasing and concave in c_t. It takes the form:

$$u = \frac{1}{1-\xi} c^{1-\xi}$$

where we set $\xi = 4$ as in Kaplan and Violante (2011). The time-discount factor, β_W, and the asset market participation frequency are calibrated, to match a ratio of high powered money (St. Louis Ajdusted Monetary Base from the Federal Reserve Bank of St. Louis) to annual output of 6.5% and a capital to output ratio of 3.5. For simplicity, we set the entrepreneurs' discount factor equal to the workers' discount factor. It is important to note that we need to calibrate to base money and not higher money aggregates as higher money aggregates do not constitute claims of the private sector, as they are created by issuing private (nominal) debt. Hence, their net value does not change with inflation other than through the change of the value of the monetary base. We consider a robustness check, where we equate the total amount of the liquid asset to the total amount of government liabilities.

5.2.2 Intermediate Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy, total amount of pre-products used in production, the intermediate consumption, makes up roughly 45% of gross output. Hence, we set $\alpha = 0.45$. The labor and capital share (2/3 and 1/3) align with standard macroeconomic calibrations.

5.2.3 Final Goods Producers

Final good producers differentiate intermediate goods and set final goods prices. We calibrate the price setting behavior to match the standard markup and price stickiness employed in the New Keynesian literature. The Calvo parameter κ implied by the New Keynesian Phillips curve is chosen in such a way to yield an average price duration of

Table 2: Calibrated Parameters

Parameter	Value	Description	Target
Households			
β_W, β_E	0.97	Discount factor	Capital to Output Ratio of 3.5
ν	63.7%	Participation frequency	M/Y Ratio 6.5% annual (Basemoney)
alternative:	8.6%		M/Y Ratio 50% (Gov. liabilities)
ξ	4	Coefficient of rel. risk av.	Standard value
Intermediate Goods			
γ	0.45	Share of pre-products	Ratio of Gross Output/GDP
α	2/3	Share of labor	Income share of labor
$\bar{K}$	1.14	Aggregate capital stock	Annual capital to output is 3.5
δ	2.5%	Depreciation rate	NIPA data
Final Goods			
κ	0.09	Price stickiness	Average price duration 4 quarters
μ	10%	Markup	Standard value
Monetary policy			
θ_1	1.005	Money Growth	2% p.a. ($\approx$Average US Inflation 1980-2008)
θ_2	0	Inflation Stabilization	No stabilization
alternative:	10^5		Perfect stabilization

4 quarters. The steady state marginal costs $exp(-\mu) = 0.90$ and imply a markup of roughly 10%.

5.2.4 Central Bank

We set the average growth rate of money θ_1 such that our model produces an average annual inflation rate of 2% in line with the usual estimates of the inflation target of the Fed and in line with the average inflation between 1980 and 2008. We do not have a good estimate of the reaction to inflation θ_2 and set this either zero for inactive policy or to 10^5 to capture a central bank behavior with strong inflation stabilization.

6 Quantitative Results

6.1 Aggregate Impact of Uncertainty Shocks

Figure 1 displays the impulse responses of output, consumption, real balances, and asset prices under the assumption of a monetary policy that follows a strict money growth rule and no inflation stabilization for our baseline calibration of money holdings being 50% of annual GDP. After a one standard deviation increase in the variance of idiosyncratic productivity shocks, output drops on impact by 0.25% and returns after roughly 12 quarters to the normal growth path. This effect is roughly of a similar size that Fernández-Villaverde et al. (2011) find for a two standard deviation shock to fiscal uncertainty.

This output drop is a result of households trying to increase their wealth for consumption smoothing and readjusting their portfolios. As households thrive for higher savings, aggregate demand and in consequence also output falls, since output is demand determined in our model. Notwithstanding the increase in the propensity to save for precautionary motives, on impact the price of capital decreases by 2.5% and only real balances grow (by 0.25%).

While the overall drop in total wealth (roughly equal to the drop in the capital price) is driven by a paradox of thrift in our model, the strong decline in asset prices also comes from a change in the optimal portfolio composition. In times of high uncertainty, households dislike illiquidity, as it disrupts their ability to smooth consumption and therefore the real money holdings increase after a shock to uncertainty. The flight to liquidity is particularly pronounced for households with little current money holdings. So not only the asset poor shift resources into the liquid asset, but also wealthy but liquidity constrained households will, if possible, liquidate part of their capital. As the

Figure 1: IRFs: money and capital, constant money growth
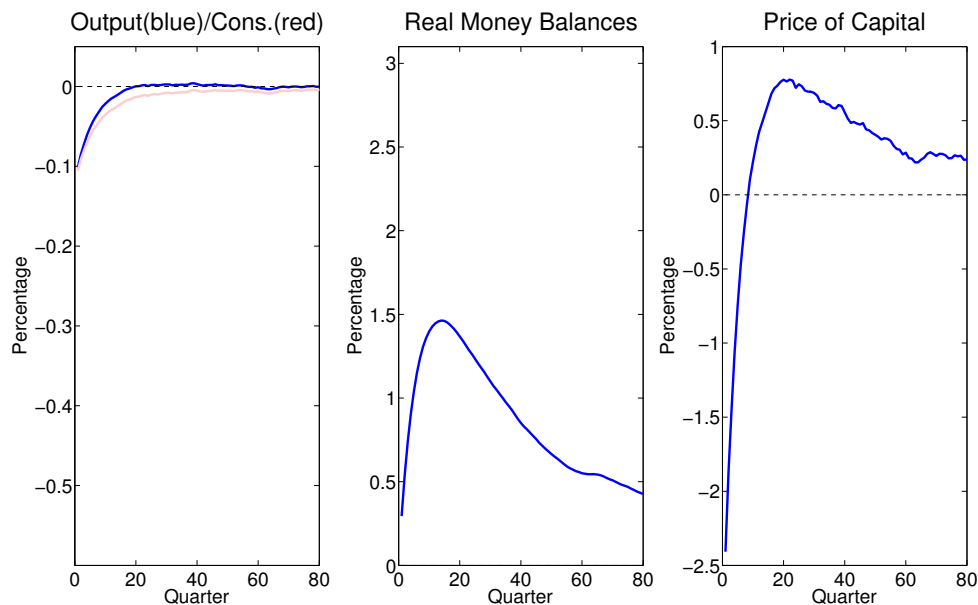
aggregate demand for capital falls, its price decreases on impact.

Yet, not only this portfolio adjustment effect drives down capital prices. Also the disinflation itselfs shifts the relative returns of money and capital, making money holdings more attractive. Disinflation decreases the inflation tax on money holdings and dividend payments on capital fall when mark-ups of final goods producers rise above their steady state levels. We can isolate the portfolio adjustment effect, when we look at a monetary policy that is perfectly stabilizing below. The corresponding impulse responses are displayed in Figure 2. In case of strict inflation targeting the price of capital falls less, as there is no deflation and thus no change in the relative returns. Yet, still it falls by roughly .5%, while money demand now jumps up on impact by .5%. Households would like to dissave as they know they will face lower than steady state consumption in the next periods as the central bank continuously issues money and creates wasteful seignorage. This wasteful seignorage makes the costs of stabilization in terms of consumption also higher in the model with inflation stabilization. Yet, although the initial drop in consumption is more pronounced, consumption recovers faster than under the money growth rule.

This changes after 2 years roughly when real money balances peek. Now households

18

are totally more wealthy and hence also want to hold more physical capital. Furthermore, in the model without stabilization, the relative return advantage reverses. As liquid wealth has become abundant, households expect inflation in the future and money becomes an unappealing asset and households start investing relatively more in capital. This effect can again be seen when comparing inflation stabilization to money growth rule. In case of strict inflation targeting, the capital price increases slightly less, as no inflation is expected.

Figure 2: IRFs: money and capital, inflation targeting
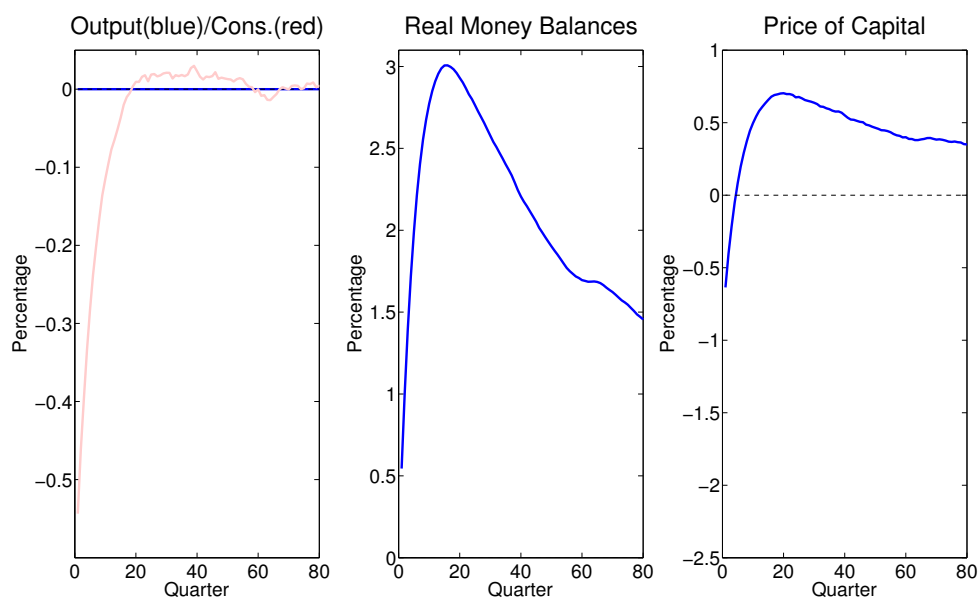
Table 3 displays standard aggregate unconditional business cycle statistics for the cycles produced by uncertainty shocks. Compared to roughly 1%-1.5% output volatility in actual GDP data for developed countries, uncertainty shocks can produce sizable business cycles of roughly 0.1% of what we find in the data. The uncertainty shocks produce too little persistence.

6.2 Redistributive effects

Since uncertainty shocks affect the price level, asset prices, dividends and wages differently, our model predicts that not all agents (equally) lose from the decline in consumption upon an uncertainty shock.

19

Table 3: Simulated business cycle statistics

Variable	Narrow Money STD	Narrow Money AC	Broad Money STD	Broad Money AC
Output			0.07	0.66
Consumption			0.07	0.66
Real money balances			0.63	0.94
Inflation			0.20	0.66
Dividend			2.64	0.66
Wage			0.79	0.66
Realized real return on capital			1.51	-0.10
Capital price			1.83	0.66

STD: Standard deviation after log-HP(1600)-filtering.
AC: First order autocorrelation.

To understand the relative welfare consequences, we simulate two sets of economies. One where the uncertainty state simply evolves according to its Markov chain properties and another set where we exogenously set the uncertainty state in T to the highest uncertainty state and let the economies evolve stochastically from then on. We trace agents over the next s periods for both sets of economies and calculate their period-felicity u_{iT+t} and then calculate for each agent with individual state h, m, k in period T the discounted expected felicity stream over the next S periods as

$$v_S(h, m, k) = E\left[\sum_{t=0}^{S} \beta^t u_{T+t} \middle| (h_T, m_T, k_T) = (h, m, k)\right]$$

where u_{T+t} is the felicity stream in period in period $T+t$ under the household's optimal saving policy. We then calculate an equivalent consumption tax households would be willing to face over the next S quarters in order to eliminate the uncertainty shock in T as
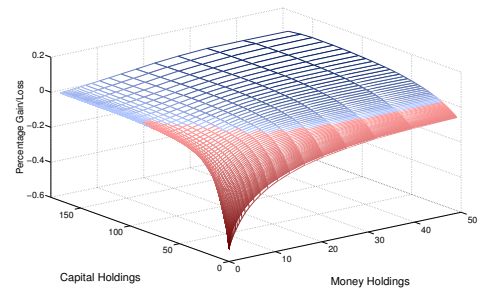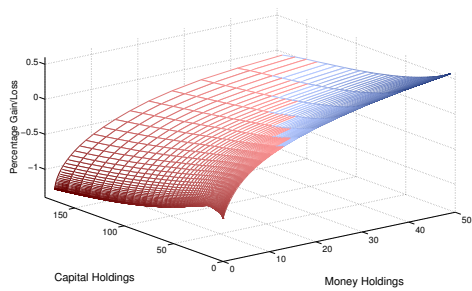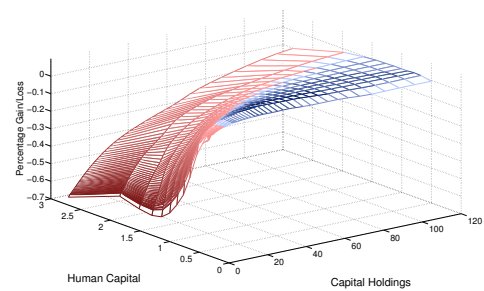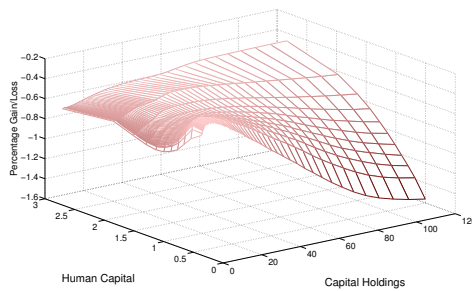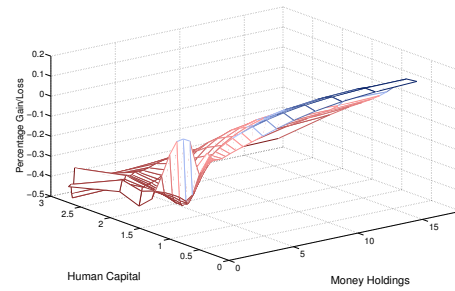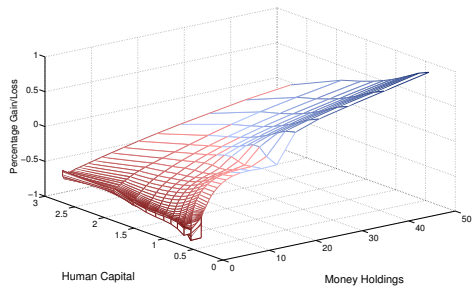
$$\tau = -\left(\frac{v_S^{\text{shock}}}{v_S^{\text{no shock}}}\right)^{1/\xi} + 1 \tag{20}$$

Figure 3 displays the relative differences in v_S for $S = 20$ quarters in terms of con-

Figure 3: Welfare effects after 5 years, broad money definition

Constant money growth

Inflation targeting

Notes:
Welfare costs in terms of consumption equivalents τ, as defined in (20)
The graphs refer to the conditional expectations of τ with respect to the two displayed
dimensions, respectively. The missing dimension has been integrated out.

Table 4: Welfare after 20 quarters

	Policy Regime: Money Growth Rule									
	Quintiles of Money Holdings					Quintiles of Capital Holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.64	-0.56	-0.48	-0.39	-0.25	-0.58	-0.49	-0.45	-0.44	-0.51
Median	-0.66	-0.58	-0.49	-0.37	-0.17	-0.60	-0.53	-0.52	-0.54	-0.62
	Quintiles of Human Capital									
Conditional	-0.31	-0.42	-0.49	-0.51	-0.52					
Median	-0.46	-0.49	-0.53	-0.54	-0.62					

	Policy Regime: Inflation Targeting									
	Quintiles of Money Holdings					Quintiles of Capital Holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.21	-0.26	-0.23	-0.16	-0.08	-0.41	-0.31	-0.21	-0.12	-0.05
Median	-0.22	-0.23	-0.19	-0.14	-0.07	-0.41	-0.28	-0.20	-0.09	-0.00
	Quintiles of Human Capital									
Conditional	-0.03	-0.12	-0.19	-0.23	-0.31					
Median	-0.03	-0.13	-0.23	-0.30	-0.47					

sumption equivalents τ between the two sets of simulations of the economy. After 20 quarters, the initial position before the uncertainty shock hit has washed out mostly and any remaining differences are discounted substantially. Of course in the long-run there are no differences between the two sets of economies. On average households would be willing to forgo roughly 0.5% of their consumption over 4 years to eliminate the uncertainty shock, but the welfare effects are heterogeneous depending on the asset positions of households and their human capital. Moreover, monetary policy can substantially shift the burden of the shock between various households.

Without stabilization, money rich and physical asset poor households win. The gradient in human capital is relatively flat. Human capital rich households suffer from lower wages, but are as savers partly compensated as they can acquire physical capital at

Figure 4: Welfare effects of uncertainty shock by quintiles of money and capital at various horizons S (up to 100 quarters), broad money definition, constant money growth
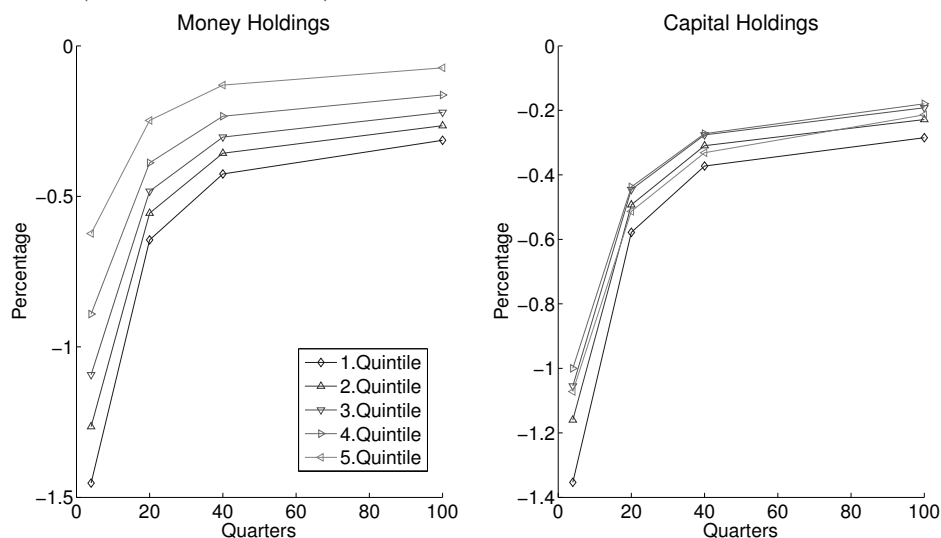
lower prices. At median incomes and median money holdings also the gradient in capital holdings is relatively flat. Median income households with sufficient liquid wealth hardly want to change their physical assets and hence are little affected by the decrease in asset prices, they only suffer from decreased dividend incomes. Table 4 summarizes the figures numerically, conditioning on just one dimension of the households' portfolio, and displays the average relative welfare gains depending on the household's relative position in the distribution of human/non-human wealth. Yet by looking at only the marginal distribution we lose some interesting information on the interaction of human and non-human wealth for the welfare effects.

Stabilization policy shifts the burden of the uncertainty shock substantially. The wealthy agents with low human capital profit the most from this policy, the asset poor lose the most and the gradient in human capital becomes steeper. Now the asset poor households with a lot of human capital finance the increased seignorage by accumulating money and by later buying physical capital that quickly reaches above steady state prices. However, on average the welfare loss is smaller with stabilization policy than without and the median household would only be willing to decrease consumption by 0.2% to avoid the uncertainty shock.

Finally, Figure 4 displays the consumption equivalents as a function of the time horizon S. Since the effect of the uncertainty shock dies out after 12 quarters, the willingness to forgo consumption at longer horizons in exchange for avoiding the shock (effects at longer horizons) decreases relatively quickly.

23

7 Conclusion

This paper examines how changes in the uncertainty of household income affect the macroeconomy. We start out with a setup that merges the standard New Keynesian and the standard incomplete markets model and produces real effects of demand swing via countercyclical markups arising through sticky prices. Calibrating the model to match income uncertainty movements in the US, we find that an increase in income uncertainty can lead to substantive output drops which may help to understand the slow recovery of the U.S. economy during the Great Recession. An increase in precautionary savings and a flight to liquidity in reaction to an increase in idiosyncratic income uncertainty can generate output losses in the environment we are study that are roughly twice as large as the output losses generated by an increase in aggregate policy uncertainty documented in Fernández-Villaverde et al. (2011).

Moreover, the welfare effects of such uncertainty shock depend crucially on a households asset position and the stance of monetary policy. Unconventional monetary policy that increases money supply drastically in times of uncertainty hikes may limit substantially the negative welfare effects of uncertainty shocks.

References

Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Arellano, C., Bai, Y., and Kehoe, P. (2012). Financial markets and fluctuations in uncertainty. *Federal Reserve Bank of Minneapolis Research Department Staff Report 466*.

Bachmann, R. and Bayer, C. (2013). Wait-and-see' business cycles? *Jounal of Monetary Economics*, forthcoming.

Basu, S. and Bundick, B. (2011). Uncertainty Shocks in a Model of Effective Demand. *Boston College Working Papers in Economics*.

Baumol, W. J. (1952). The transactions demand for cash: An inventory theoretic approach. *The Quarterly Journal of Economics*, 66(4):pp. 545–556.

Bayer, C. and Juessen, F. (2009). The life-cycle and the business-cycle of wage risk: A cross-country comparison. (4402).

Bewley, T. (1980). Interest bearing money and the equilibrium stock of capital. *mimeo.*

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 77(3):623–685.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2012). Really uncertain business cycles.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Carroll, C. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.

Christiano, L., Motto, R., and Rostagno, M. (2010). Financial factors in economic fluctuations. *ECB Working Paper 1192.*

Chugh, S. (2012). Firm risk and leverage-based business cycles. *mimeo, University of Maryland.*

Den Haan, W. J. (2010a). Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents. *Journal of Economic Dynamics and Control*, 34(1):79–99.

Den Haan, W. J. (2010b). Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents. *Journal of Economic Dynamics and Control*, 34(1):79–99.

Fernández-Villaverde, J., Guerrón-Quintana, P. A., Kuester, K., and Rubio-Ramírez, J. (2011). Fiscal volatility shocks and economic activity. Technical report, National Bureau of Economic Research.

Galí, J. (2008). *Monetary Policy, inflation, and the Business Cycle: An introduction to the new Keynesian Framework*. Princeton University Press.

Gilchrist, S., Sim, J., and Zakrajšek, E. (2010). Uncertainty, financial frictions, and investment dynamics. *mimeo, Boston University.*

Gornemann, N., Kuester, K., and Nakajima, M. (2012). Monetary policy with heterogeneous agents.

Hintermaier, T. and Koeniger, W. (2010). The method of endogenous gridpoints with occasionally binding constraints among endogenous variables. *Journal of Economic Dynamics and Control*, 34(10):2074–2088.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969.

Kaplan, G. and Violante, G. L. (2011). A model of the consumption response to fiscal stimulus payments.

Krusell, P. and Smith, A. A. (1997). Income And Wealth Heterogeneity, Portfolio Choice, And Equilibrium Asset Returns. *Macroeconomic Dynamics*, 1(02):387–422.

Krusell, P. and Smith, A. A. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Mericle, D. (2012). Uncertainty, the liquidity trap, and social insurance.

Nancy L. Stokey, Robert E. Lucas, E. C. P. (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.

Narita, F. (2011). Hidden actions, risk-taking and uncertainty shocks. *mimeo, University of Minnesota*.

Panousi, V. and Papanikolaou, D. (2012). Investment, idiosyncratic risk, and ownership. *Journal of Finance*, 67(3):1113–1148.

Pigou, A. C. (1943). The classical stationary state. *The Economic Journal*, 53(212):pp. 343–351.

Ravn, M. O. and Sterk, V. (2013). Job uncertainty and deep recessions. *mimeo*.

Santos, M. S. (1991). Smoothness of the policy function in discrete time economic models. *Econometrica*, 59(5):1365–1382.

Schaal, E. (2011). Uncertainty, productivity and unemployment in the great recession. *mimeo, Princeton University*.

Storesletten, K., Telmer, C. I., and Yaron, A. (2001). The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk. *European Economic Review*, 45(7):1311–1339.

Takahashi, S. (2013). Time varying wage risk, incomplete markets, and business cycles. *mimeo*.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177 – 181.

Tobin, J. (1956). The interest-elasticity of transactions demand for cash. *The Review of Economics and Statistics*, 38(3):pp. 241–247.

Vavra, J. (2012). Inflation dynamics and time-varying uncertainty: New evidence and an ss interpretation.

A Dynamic Planning Problem with Two Assets

The dynamic planning problem of a household in the model is characterized by two Bellman equations, V_a in case the household can adjust its capital holdings and V_n otherwise

$$
\begin{aligned}
V_a(m,k,h;\Theta,s) =& max_{k',m'_a \in \Gamma_a} u[c(m,m'_a,k,k',h)] \\
& + \beta_W \left[\nu EV^a(m'_a,k',h',\Theta',s') + (1-\nu)EV^n(m'_a,k',h',\Theta',s') \right] \\
V_n(m,k,h;\Theta,s) =& max_{m'_n \in \Gamma_n} u[c(m,m'_n,k,k,h)] \\
& + \beta_W \left[\nu EV^a(m'_n,k,h',\Theta',s') + (1-\nu)EV^n(m'_n,k,h',\Theta',s') \right]
\end{aligned}
\tag{21}
$$

where the budget sets are given by

$$
\Gamma_a(m,k,h;\Theta,s) = \{m',k' \geq 0 | q(\Theta,s)(k'-k) + m' \leq w(\Theta,s)h + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} \}
\tag{22}
$$

$$
\Gamma_n(m,k,h;\Theta,s) = \{m' \geq 0 | m' \leq w(\Theta,s)h + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} \}
\tag{23}
$$

$$
c(m,m',k,k',h) = w(\Theta,s)h + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} - q(\Theta,s)(k'-k) - m'
\tag{24}
$$

To save on notation, let X be the set of possible idiosyncratic state variables controlled by the household, let Z be the set of potential aggregate states, $\Gamma_i : X \to X$ be the correspondence describing the feasibility constraints, and $A_i(z) = \{(x,y) \in X \times X : y \in \Gamma_i(x,z)\}$ be the graph of Γ_i. Hence the states and controls of the household problem can be defined as

$$
X = \{x = (m,k) \in R_+^3 : m,k \leq \infty\}
\tag{25}
$$

$$
z = \{h,\Theta,s\}
\tag{26}
$$

$$
\tag{27}
$$

and the return function $F : A \to R$ reads

$$F(\Gamma_i(x,z), x; z) = \frac{c_i^{1-\gamma}}{1-\gamma} \tag{28}$$

Define the value before the adjustment /no-adjustment shock realizes as

$$v(x,z) := \nu V_a(x,z) + (1-\nu)V_n(x,z).$$

Now we can rewrite the optimization problem of the household in terms of the definitions above in a compact form:

$$V_a(x,z) = \max_{y \in \Gamma_a(x,z)} [F(x,y;z) + \beta_w Ev(y,z')] \tag{29}$$

$$V_n(x,z) = \max_{y \in \Gamma_n(x,z)} [F(x,y;z) + \beta_w Ev(y,z')]. \tag{30}$$

Finally we define the mapping $T : C(X) \to C(X))$, where $C(X)$ is the space of bounded, continuous and weakly concave functions.

$$(Tv)(x,z) = \nu V_a(x,z) + (1-\nu)V_n(x,z) \tag{31}$$
$$V_a(x,z) = \max_{y \in \Gamma_a(x,z)} [F(x,y;z) + \beta_w Ev(y,z')]$$
$$V_n(x,z) = \max_{y \in \Gamma_n(x,z)} [F(x,y;z) + \beta_w Ev(y,z')].$$

A.1   Properties of Primitives

The following properties of the primitives of the problem obviously hold:

P 1. *Properties of sets $X, \Gamma_a(x,z), \Gamma_n(x,z)$*

1. *X is a convex subset of R^3.*

2. *$\Gamma_i(\cdot, z) : X \to X$ is non-empty, compact-valued, continuous, monotone and convex for all z.*

P 2. *Properties of return function F*

F is bounded, continuous, strongly concave, C^2 differentiable on the interior of A, and strictly increasing in each of its first two arguments.

A.2   Properties of the Value and Policy Functions

Lemma 1. *The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous and weakly concave functions $C(X)$.*

 a) *It satisfies discounting.*

 b) *It is monotonic.*

 c) *preserves boundedness (assuming an arbitrary maximum consumption level).*

 d) *It preserves strict concavity.*

Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in m, k, and so is optimal consumption.

Proof. The proof proceeds item by item and closely follows Nancy L. Stokey (1989) taking into account that the household problem in the extended model consists of two Bellman equations.

 a) Discounting
Let $a \in R_+$ and the rest be defined as above. Then it holds that

$$
\begin{aligned}
(T(v+a))(x,z) =& \nu \max_{y \in \Gamma_a(x,z)} [F(x,y,z) + \beta_w Ev(y,z') + a] \\
& + (1-\nu) \max_{y \in \Gamma_n(x,z)} [F(x,y,z) + \beta_w Ev(y,z') + a] \\
=& (Tv)(x,z) + \beta_w a
\end{aligned}
$$

Accordingly, T fulfills discounting.

 b) Monotonicity
Let $g : X \times Z \to R^2$, $f : X \times Z \to R^2$ and $g(x,z) \geq f(x,z) \ \forall x, z \in X \times Z$, then it

follows that

$$
\begin{aligned}
(Tg)(x,z) =& \nu \max_{y \in \Gamma_a(x,z)}[F(x,y,z) + \beta_w Eg(y,z')] \\
& + (1-\nu) \max_{y \in \Gamma_n(x)}[F(x,y,z) + \beta_w Eg(y,z')] \\
\geq& \nu \max_{y \in \Gamma_a(x,z)}[F(x,y,z) + \beta_w Ef(y,z')] \\
& + (1-\nu) \max_{y \in \Gamma_n(x)}[F(x,y,z) + \beta_w Ef(y,z')] \\
=& Tf(x,z)
\end{aligned}
$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.

c) Boundedness

From properties **P1** it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(x,y)+\beta_w Ev(y,z'))]$ over the compact sets $\Gamma_i(x,z)$ for $i = (a,n)$. Hence the maximum is attained. Since F and v are bounded, Tv is also bounded.

d) Strict Concavity

Let $f \in C''(X)$, where C'' is the set of bounded, continuous, strictly concave functions on X. Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(X)] \subseteq C''(X)$, where T_i is defined by

$$
T_i v = \max_{y \in \Gamma_i(x,z)}[F(x,y,z) + \beta_w Ev(y,z')], i \in a, n
$$

Let $x_0 \neq x_1, \theta \in (0,1), x_\theta = \theta x_0 + (1-\theta)x_1$.
Let $y_j \in \Gamma_i(x_j,z)$ be the maximizer of $(T_i f)(x_j)$ for $j = 0,1$ and $i = a, n$, $y_\theta = \theta y_0 + (1-\theta)y_1$.

$$
\begin{aligned}
(T_i f)(x_\theta, z) \geq& [F(x_\theta, y_\theta, z) + \beta_w Ef(y_\theta, z')] \\
>& \theta[F(x_0, y_0) + \beta_w Ef(y_0, z'))] + (1-\theta)[F(x_1, y_1) + \beta_w Ef(y_0, z')] \\
=& \theta(Tf)(x_0, z) + (1-\theta)(Tf)(x_1, z)
\end{aligned}
$$

The first inequality follows from y_θ being feasible because of convex budget sets.

The second inequality follows from the strict concavity of f. Since x_0, x_1 were arbitrary, it follows that $T_i f$ is strictly concave, and since f was arbitrary that $T[C''(X)] \subseteq C''(X)$.

$\square$

Lemma 2. *The value function is C^2 and the policy function C^1 differentiable.*

Proof. The properties of the choice set **P1**, of the return function **P2**, and the properties of the value function proven in (1) fulfill the assumptions of Santos (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 differentiable.

Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. $\square$

Lemma 3. *The total savings $S_i^* := m_i^*(x, z) + q(z)k_i^*(x, z)$ and consumption $c_i^*, i \in a, n$ are increasing in x if $r(z)$ is positive. In the adjustment case total savings and consumption are also increasing in total resources $R = [q(z) + r(z)]k + m/\pi(z)$.*

Proof. Define $\tilde{v}(S, z) := \max_{\{m,k \mid m+q(z)k \leq S\}} Ev(m, k; z')$. Since v is strictly concave and increasing, so is $\tilde{v}$ by the line of the proof of Lemma 1.d). Now we can (re)write the planning problem as

$$V_a(m, k; z) = \max_{S \leq w(z)h + [q(z)+r(z)]k + m/\pi(z)} [u(w(z)h + [q(z)+r(z)]k + m/\pi(z) - S) + \beta_W \tilde{v}(S, z)]$$

$$V_n(m, k; z) = \max_{m' \leq w(z)h + r(z)k + m/\pi(z)} [u(w(z)h + r(z)k + m/\pi(z) - m') + \beta_W Ev(m', k; z')].$$

Due to differentiability we obtain the following (sufficient) first order conditions

$$\frac{\partial u(w(z)h + [q(z) + r(z)]k + m/\pi(z) - S)}{\partial c} = \beta_W \frac{\partial \tilde{v}(S, z)}{\partial S}]$$

$$\frac{\partial u(w(z)h + r(z)k + m/\pi(z) - m')}{\partial c} = \beta_W \frac{\partial v(m', k; z)}{\partial m'}]. \tag{32}$$

Since the left-hand sides are decreasing in $x = (m, k)$, and increasing in S (respectively m'), and the right-hand side is decreasing in S (respectively m'), $S_i^* = \begin{cases} qk' + m' & \text{if } i = a \\ qk + m' & \text{if } i = n \end{cases}$ must be increasing in x.

Since the right-hand side of (32) is hence decreasing in x, so must be the left-hand side of (32). Hence consumption must be increasing in x.

The last statement follows directly from the same proof. $\square$

31

A.3 Euler Equations

Denote the optimal policies for consumption, for money holdings and capital as $c_i^*, m_i^*, k^*, i \in \{a, n\}$ respectively. The first order conditions for an inner solution in the (no-)adjustment case read

$$k^* : \frac{\partial u(c_a^*)}{\partial c} q \qquad = \beta_W E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial k} + (1 - \nu) \frac{\partial V_n(m_a', k'; z')}{\partial k} \right] \qquad (33)$$

$$m_a^* : \frac{\partial u(c_a^*)}{\partial c} \qquad = \beta_W E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial m} + (1 - \nu) \frac{\partial V_n(m_a^*, k^*; z')}{\partial m} \right] \qquad (34)$$

$$m_n^* : \frac{\partial u(c_n^*)}{\partial c} \qquad = \beta_W E \left[\nu \frac{\partial V_a(m_n^*, k; z')}{\partial m} + (1 - \nu) \frac{\partial V_n(m_n^*, k; z')}{\partial m} \right] \qquad (35)$$

Note the subtle difference between (34) and (35), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain

$$\frac{\partial V_a(m, k; z)}{\partial k} = \frac{\partial u[c_a^*(m, k; z)]}{\partial c} (q + r) \qquad (36)$$

$$\frac{\partial V_a(m, k; z)}{\partial m} = \frac{\partial u[c_a^*(m, k; z)]}{\partial c} \pi^{-1} \qquad (37)$$

$$\frac{\partial V_n(m, k; z)}{\partial m} = \frac{\partial u[c_n^*(m, k; z)]}{\partial c} \pi^{-1} \qquad (38)$$

$$\frac{\partial V_n(m, k; z)}{\partial k} = r(z) \frac{\partial u[c_n^*(m, k; z)]}{\partial c} \qquad (39)$$
$$+ \beta E \left[\nu \frac{\partial V_a[m_n^*(m, k; z), k; z']}{\partial k} + (1 - \nu) \frac{\partial V^n[m_n^*(m, k; z), k; z']}{\partial k} \right]$$
$$= r(z) \frac{\partial u[c_n^*(m, k; z)]}{\partial c} + \beta_W \nu E \frac{\partial u\{c_a^*[m_n^*(m, k; z), k; z], k; z'\}}{\partial c} (q(z') + r(z'))$$
$$+ \beta_W (1 - \nu) E \frac{\partial V_n\{[m_n^*(m, k; z), k; z], k; z'\}}{\partial k}$$

Such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug in the second set of equations into the first set of equations and

obtain the following Euler equations (in slightly shortened notation)

$$\frac{\partial u[c_a^*(m,k;z)]}{\partial c} q(z) = \beta_W E\left[\nu\frac{\partial u[c_a^*(m_a^*,k^*;z')]}{\partial c}[q(z') + r(z')] + (1-\nu)\frac{\partial V^n(m_a^*,k';z')}{\partial k'}\right]$$
(40)

$$\frac{\partial u[c_a^*(m,k;z)]}{\partial c} = \beta_W E\pi'(z')^{-1}\left[\nu\frac{\partial u[c_a^*(m_a^*,k^*;z')]}{\partial c} + (1-\nu)\frac{\partial u[c^n(m_a^*,k';z')]}{\partial c}\right]$$
(41)

$$\frac{\partial u[c_n^*(m,k,;z)]}{\partial c} = \beta_W E\pi'(z')^{-1}\left[\nu\frac{\partial u[c_a^*(m_n',k;z')]}{\partial c} + (1-\nu)\frac{\partial u[c_n^*(m_n^*,k;z')]}{\partial c}\right]$$
(42)

A.4 Algorithm

The algorithm we use to solve for optimal policies given the Krussel-Smith forecasting rules is a version of Hintermaier and Koeniger's (2010) extension of the Endogenous Grid Method, originally developed by Carroll (2006).

It works iteratively (until convergence of policies) as follows: Start with some guess for the policy functions c_a^* and c_n^* on a given grid $(m,k) \in M \times K$. Define the shadow value of capital

$$
\begin{aligned}
\beta^{-1}\psi(m,k;z) :=& \nu E\left\{\frac{\partial u\{c_a^*[m_n^*(m,k,z),k;z']\}}{\partial c}[q(z') + r(z')]\right\} \\
& + (1-\nu)E\frac{\partial V_n[m_n^*(m,k,z),k;z']}{\partial k} \\
=& \nu E\left\{\frac{\partial u\{c_a^*[m_n^*(m,k,z),k;z']\}}{\partial c}[q(z') + r(z')]\right\} \\
& + (1-\nu)E\left\{\frac{\partial u\{c_n^*(m_n^*(m,k,z),k;z')]}{\partial c}r(z')\right\} \\
& + (1-\nu)E\left\{\psi[m_n^*(m,k,z),k;z']\right\}.
\end{aligned}
$$
(43)

Guess initially $\psi = 0$. Then

1. Solve for an update for c_n^* using standard endogenous-grid methods using equation (42), denote $m_n^*(m,k;z)$ the optimal money holdings without capital adjustment.

2. Find for every k' on-grid some (off-grid) value of $\tilde{m}_a^*(k';z)$ such that - combining

(41) and (40) -

$$0 = \nu E \left\{ \frac{\partial u[c_a^*(\tilde{m}_a^*(k',z),k';z')]}{\partial c} \left[\frac{q(z')+r(z')}{q(z)} - \pi(z')^{-1} \right] \right\} \tag{44}$$

$$+ (1-\nu)E \left\{ \frac{\partial u[c_n^*(\tilde{m}_a^*(k',z),k';z')]}{\partial c} \left[\frac{r(z')}{q(z)} - \pi(z')^{-1} \right] \right\} + (1-\nu)E \left[\frac{\psi(\tilde{m}_a^*(k',z),k';z')}{q(z)} \right]$$

N.B. that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ. If no solution exists, set $\tilde{m}_a^* = 0$. Uniqueness (conditional on existence) of $\tilde{m}_a^*$ follows from the strict concavity of v.

3. Solve for total initial resources, by solving the Euler equation (41) for $\tilde{c}^*(k',z)$, such that

$$\tilde{c}^*(k',z)]$$
$$= \frac{\partial u}{\partial c}^{-1} \left\{ \beta E \pi(z')^{-1} \left[\nu \frac{\partial u\{c_a^*[m_a^*(k',z),k';z']\}}{\partial c} + (1-\nu) \frac{\partial u\{c_n^*[m_a^*(k',z),k';z']\}}{\partial c} \right] \right\} \tag{45}$$

where the right-hand side expressions are obtained by interpolating $c_a^*(m_a^*(k',z),k',z')$ from the on-grid guesses $c_a^*(m,k;z)$ and taking expected values with respect to z'.

This way we obtain total non-human resources $\tilde{R}(k',z)$ that are compatible with plans $(m^*(k'),k')$ and a consumption policy $\tilde{\tilde{c}}_a^*(\tilde{R}(k',z),z)$ in total resources.

4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows: Calculate total resources for each (m,k) pair $R(m,k) = (q+r)k+m/\pi$ and use the before obtained consumption policy to update $c_a^*(m,k,z)$ by interpolating at $R(m,k)$ from the set $\left\{ (\tilde{\tilde{c}}_a^*(\tilde{R}(k',z),z), R(k',z)) \middle| k' \in K \right\}$.[10]

5. Update ψ: Calculate a new value of ψ using (39), such that

$$\psi^{new}(m,k,z) = \beta_W \nu E \left\{ \frac{\partial u\{c_a^*[m_n^*(m,k,z),k;z']\}}{\partial c} [q(z')+r(z')] \right\}$$
$$+ \beta(1-\nu)E \left\{ \frac{\partial u\{c_n^*(m_n^*(m,k,z),k;z')]}{\partial c} r(z') \right\}$$
$$+ \beta(1-\nu)E \left\{ \psi^{old}[m_n^*(m,k,z),k;z'] \right\}. \tag{46}$$

making use of the **updated** consumption policies.

---

[10]If a boundary solution $\tilde{m}^*(0) > 0$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{m}^*(0)$.

B   Equilibrium Forecasting Rules

C   Quality of the Numerical Solution

The following Figures 5 and 6 provide Den Haan (2010b) tests and show that the quality of the numerical approximation of the equilibrium by the Krussel Smith rules is relatively good. There is no trend of divergence between forecasts and actual equilibria over time.

Figure 5: Den Haan Test of the Quality of Approximation, money growth rule, broad money
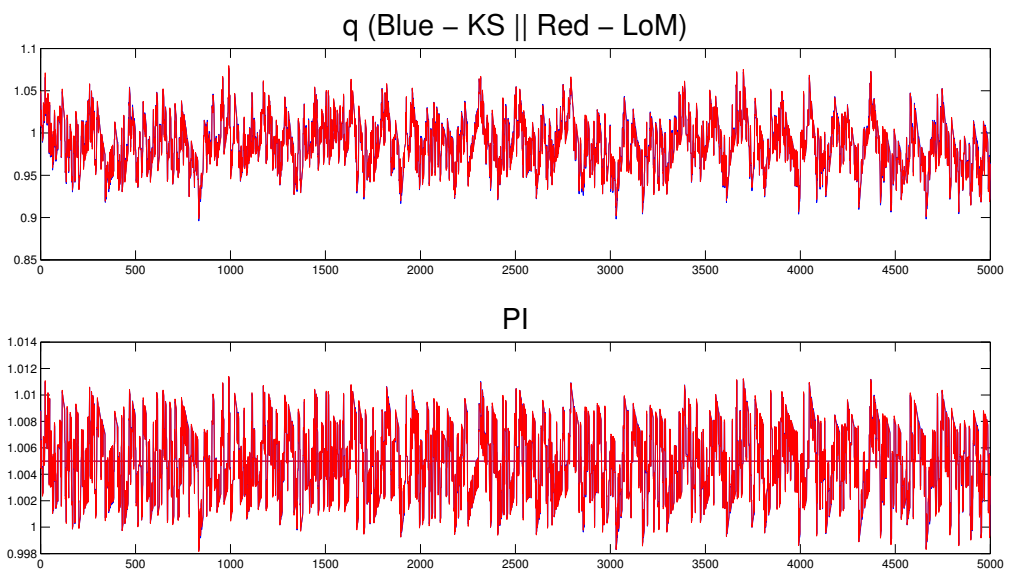
D   Robustness Checks

- Different Calibrations of ρ_s

- Skewness Shocks

- Timing of uncertainty realizations.

- Price Stickiness

Figure 6: Den Haan Test of the Quality of Approximation, inflation targeting, broad money