Demand for Slant:
The choice of news media can be influenced by personal beliefs and the desire for information that aligns with those beliefs. This phenomenon can be understood through the concept of voluntary voting.

Abstract

Many pundits believe that today’s wide array of news media allow people to live in information cocoons that isolate them from opposing points of view. This is consistent with standard theory which shows that, for purely instrumental reasons, people prefer media outlets whose biases conform to their own. On the other hand, some empirical work suggests that “cross-over” in news consumption (i.e., conservative voters consulting liberal media—and vice versa) is actually quite common. We explain this by showing that, for voters with pronounced prior beliefs, cross-over is in fact an optimal response to voluntary voting. Specifically, the option to abstain makes it optimal for voters with considerable leanings towards a particular candidate to demand information that is less biased towards that candidate than voters who are less leaning towards that candidate. Hence, the demand for slant is non-monotonic in voter ideology, and voters with different political beliefs optimally choose to consult the same media outlet. This pooling of voters generates disproportionate demand for media outlets that are either centrist or only moderately biased.

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1 Introduction

1.1 Motivation

Political pundits have argued that today’s news media, catering to every imaginable ideological bias and belief, allow voters to create “Daily Me” newspapers (Sunstein (2009)). As a result, voters self-segregate along ideological lines and rarely receive information, or listen to arguments, that go against their priors. This is considered harmful for the proper functioning of democracy (see Sunstein (2002), Sunstein (2009), Jamieson and Cappella (2010), Brooks (2010), and others). The existing theory by and large confirms this “Echo Chamber” argument. Starting with Calvert (1985) and generalized by Suen (2004), it has been shown that, for purely instrumental reasons, decision makers prefer media outlets whose biases conform to their own. This indeed suggests that every shade of conservative and liberal should exhibit a tendency to withdraw into a separate information cocoon and indulge in media consumption that rarely exposes him to opposing points of view.

Interestingly, Gentzkow and Shapiro (2011) find much less ideological segregation in media consumption than suggested by the pundits and the theory. Instead, they observe significant “cross-over” behavior, with some relatively conservative voters consulting relatively liberal news media—and vice versa. Gentzkow and Shapiro (2011) hypothesize that this phenomenon may be explained, in part, by vertical differentiation. For example, someone who does not agree with the political views of the New York Times may still read the paper, because he appreciates the quality of the reporting. In part, the cross-over effect may also be due to news junkies who consult multiple media outlets each day. Thus, a voter with a liberal bias may optimally use the Wall Street Journal as a secondary but legitimate source of news, and not just to find out “what the enemy is thinking.”

In this paper, we develop a different and perhaps more fundamental argument for the observed cross-over in media consumption. We show that strict ideological segregation is in fact an artefact of compulsory voting, which breaks down if voters can choose whether to go to the polls. In a nutshell, under voluntary voting, voters with significantly different ideological beliefs optimally choose to read the same newspaper, because they use the information provided in different ways.

Formally, we analyze a common values model where voters differ in their prior beliefs about the state of the world. While all agree that the liberal candidate is better in the liberal state and the conservative candidate in the conservative state, voters disagree as to the ex ante probability that these states pertain. The greater a voter’s prior belief that the state

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1A related argument is that people follow various media outlets simply for their consumption value. In this paper, we ignore the consumption aspect and focus on the instrumental value of information.
is liberal, the more he is said to be leaning to—or biased towards—the liberal candidate. Before casting their ballot, voters may consult a newspaper which, in the form of a voting recommendation, conveys a noisy signal as to the true state of the world. Voters can choose from a continuum of papers that span the ideological spectrum, from extremely liberal to extremely conservative.

We show that, under voluntary voting, the demand for slant is a non-monotonic and discontinuous function of voter ideology. Indeed, on both sides of the political spectrum, there are voters less leaning towards a particular candidate who optimally consult news media that are more biased towards that candidate than voters who are more leaning towards that candidate. As a consequence, voters with significantly different political beliefs end up consulting the same media outlet and demanding the same degree of slant. We show that this creates disproportionate demand for media outlets that are either centrist or only moderately biased. In other words, abstention encourages moderation.

The intuition for our results is as follows. If a voter is perfectly centrist, he optimally follows the voting recommendation of a newspaper that treats liberal and conservative candidates perfectly symmetrically. A mildly liberal-leaning voter, on the other hand, needs stronger evidence to vote for the conservative candidate than to vote for the liberal candidate. He achieves this by reading a paper that, just like himself, is also mildly liberal-leaning. Of course, in this case, a recommendation to vote for the liberal candidate does not constitute strong evidence that the state is liberal. But strong evidence in that direction is not really needed, since the voter already leans towards voting for the liberal candidate anyway. A recommendation to vote for the conservative candidate, on the other hand, does constitute strong evidence the state is conservative, and this is what a liberal-leaning voter needs if he is to vote for the conservative candidate.

Now consider a voter whose liberal leanings are more pronounced. If this voter also wants to follow his newspaper’s recommendation at the ballot box even when it tells him to vote for the conservative candidate, he needs to be reading a publication that has a strong liberal bias. However, this comes at the cost of virtually always voting for the liberal candidate even when the state is conservative, as a paper with a strong liberal bias very rarely comes out in favor of the conservative candidate. Nonetheless, if the voter had to give his support to either the liberal or the conservative candidate, this would be the best he could do. However, usually, the voter does have another option; namely, supporting neither candidate and abstaining. And for the usual reasons related to the Swing Voter’s Curse, upon receiving the signal that the state is conservative, abstaining rather than voting for the conservative candidate is indeed an attractive alternative for such a liberal-leaning voter.

What does abstaining do to this voter’s optimal choice of newspaper? That is, how biased
does he want his newspaper to be if he only follows its recommendation when the advice is to vote for the liberal candidate, but abstains when the advice is to vote for the conservative candidate? As he no longer plans on actually voting for the conservative candidate, a signal that the state is conservative now entails significantly less “risk” than under the old voting strategy. Hence, the voter can afford to read a markedly less biased paper. Indeed, he now strictly prefers to consult a more centrist outlet, because it improves his chances of not voting for the liberal candidate when the state is in fact conservative. As a result, the paper of choice of this liberal-leaning voter is less biased than that of some intrinsically more centrist voters. In principle, a liberal-leaning voter may even choose to read a more conservative paper than some conservative-leaning voters. It is this cross-over and pooling of voters with different ideologies that creates a disproportionate demand for newspapers that are either centrist or only mildly biased towards either side of the ideological spectrum. These papers benefit from serving heterogeneous constituencies that use the same information in different ways.

1.2 An example

The basic intuition for the non-monotonicity in the demand for slant is most easily understood by example. Consider a voter with the following preferences: she gains $u = 1$ if she supports the “right” candidate, but loses $c = \frac{7}{4} > u$ if she supports the “wrong” candidate. In addition, she has the option to abstain, which gives her a utility of 0. From the perspective of the voter, the “right” candidate is determined by the state of nature: in state $d$, the right candidate is $D$; in state $r$, it is $R$. The voter’s prior belief that the state is $r$ is equal to $q$.

The voter can read at most one of two newspapers, labeled $r$ and $d$. Before the election, the newspapers write editorials supporting one of the candidates. Formally, support for candidate $d$ is expressed by sending a signal $s_d$, while support for $R$ is expressed by a signal $s_r$. Newspaper $d$ supports candidate $D$ 75% of the time in state $d$ and 50% of the time in state $r$. This makes newspaper $d$ biased towards $D$. Similarly, newspaper $r$, which is biased towards $R$, supports $R$ 75% of the time in state $r$ and 50% of the time in state $d$. Let $p^k(s, q)$ denote the probability that a voter with prior $q$ assigns to newspaper $k \in \{r, d\}$ sending signal $s \in \{s_d, s_r\}$. Given the biases of each paper, we have that $p^d(s_d, q) > p^r(s_d, q)$ and $p^r(s_r, q) > p^d(s_r, q)$. Finally, let $p^k(s, q)$ denote the voter’s posterior belief that the state is $r$, upon receiving a signal $s$ from newspaper $k$, while holding prior beliefs $q$. By Bayes’ rule, $p^d(s, q) > p^r(s, q)$, for all $s \in \{s_d, s_r\}$.

Which newspaper does the voter read, and how does this depend on whether voting is voluntary? If voting is mandatory, Calvert (1985) and Suen (2004) show that the voter’s
choice of newspaper is as follows: for $q \leq \frac{1}{3}$, the voter has no use for either newspaper and always supports $D$; for $\frac{1}{3} < q \leq \frac{1}{2}$, she reads paper $d$ and follows its advice at the polls; for $\frac{1}{2} < q \leq \frac{2}{3}$, she reads and follows $r$’s advice; for $\frac{2}{3} < q$, she once again reads neither newspaper and always votes for $R$. Note that the voter only reads newspapers whose biases conform to her own. Indeed, a prior belief that the state is more likely to be $r$ induces her to read a paper, $r$, that is biased towards $R$. Similarly, a prior belief that the state is $d$, induces her to read newspaper $d$.

When voting is voluntary, the voter’s optimal choice of newspaper is more complicated—and interesting. With prior $q$, her expected utility of voting for $R$ after receiving a signal $s_r$ from newspaper $k$ is given by

$$U^k_q(R, s_r) = 1 \times \rho^k(s_r, q) - \frac{7}{4} \times (1 - \rho^k(s_r, q))$$

(1)

Similarly, her utility of voting for $D$ after a signal $s_d$ from newspaper $k$ is given by

$$U^k_q(D, s_d) = 1 \times (1 - \rho^k(s_d, q)) - \frac{7}{4} \times \rho^k(s_d, q)$$

(2)

It is easily verified that, for $k \in \{d, r\}$,

$$U^k_q(R, s_r) \geq 0 \implies U^k_q(D, s_d) < 0$$

and

$$U^k_q(D, s_d) \geq 0 \implies U^k_q(R, s_r) < 0$$

Hence, any voter $q$ who reads a paper and is responsive to its advice prefers to abstain after one signal, $s_d$ or $s_r$, and follow the paper’s advice after the other signal. At the same time, for every type of voter $q$, there is a newspaper $k \in \{d, r\}$ such that $U^k_q(D, s_d) > 0$ or $U^k_q(R, s_r) > 0$. Hence, not reading a paper and always abstaining is a dominated strategy. This leaves us with only two responsive voting strategies: 1) abstaining after $s_d$ and voting for $R$ after $s_r$, which we denote by $\emptyset R$; and 2) voting for $D$ after $s_d$ and abstaining after $s_r$, which we denote by $D\emptyset$.

For each newspaper $k$ there exists a unique cutoff $\underline{q}^k$ such that

$$EU^k(q, D\emptyset) > (\prec) EU^k(q, \emptyset R) \iff q < (\prec) \underline{q}^k$$

Simple algebra and the expected utilities (1) and (2) reveal that $\underline{q}^d = \frac{19}{41}$ and $\underline{q}^r = \frac{22}{41}$. Hence, a voter with beliefs $q \in \left[\frac{22}{41}, 1\right]$ always uses the voting strategy $\emptyset R$—even though it still remains to determine which newspaper he actually reads—while a voter with beliefs
where $q \in \left[\frac{4}{11}, \frac{27}{41}\right]$ must choose between voting $D \emptyset$ and reading newspaper $r$ on the one hand, and voting $\emptyset R$ and reading $d$ on the other. By symmetry, the interval $[0, \frac{19}{41}]$ is similarly divided, with the strategy $\emptyset R$ playing the role of $D \emptyset$.

The expected utility from voting $D \emptyset$ and reading newspaper $r$ is $U^r_q(D, s_d) \times p^r(s_d, q)$, while the utility from voting $\emptyset R$ and reading $d$ is $U^d_q(R, s_r) \times p^d(s_r, q)$. Using (1) and (2), we have

$$U^r_q(D, s_d) \times p^r(s_d, q) = \frac{1}{2} - \frac{15}{16}q$$
$$U^d_q(R, s_r) \times p^d(s_r, q) = \frac{15}{16}q - \frac{7}{16}$$

This implies that voters $q \in \left(\frac{1}{2}, \frac{27}{41}\right)$ strictly prefer reading newspaper $d$ and voting $\emptyset R$ over reading newspaper $r$ and voting $D \emptyset$, and $q \in \left(\frac{18}{41}, \frac{1}{2}\right)$ prefers the opposite. Then, all voters $q \geq \frac{1}{2}$ prefer $\emptyset R$ to any other voting strategy or always abstaining, even though they may read different newspapers. Similarly, all voters $q < \frac{1}{2}$ prefer $D \emptyset$ to any other voting strategy or always abstaining.

Since

$$U^d_q(D, s_d) \times p^d(s_d, q) = \frac{3}{4} - \frac{13}{8}q,$$

comparing the payoffs $U^r_q(D, s_d) \times p^r(s_d, q)$ with $U^d_q(D, s_d) \times p^d(s_d, q)$ reveals that a voter with prior beliefs $q \in \left[\frac{4}{11}, \frac{1}{2}\right]$ prefers to read $r$, while a voter with prior beliefs $q < \frac{4}{11}$ prefers to read $d$. Since always voting for $D$ gives an expected utility $E[U^k(q, DD)] = 1 - \frac{11}{4}q$, by symmetry, we have that a voter’s optimal strategy as a function of his prior beliefs $q$ is

$$\begin{align*}
\frac{7}{9} &< q \to RR \\
\frac{7}{11} &< q < \frac{7}{9} \to \emptyset R \ r \\
1 &< q < \frac{7}{11} \to \emptyset R \ d \\
\frac{4}{11} &< q < \frac{1}{2} \to D \emptyset \ r \\
\frac{2}{9} &< q < \frac{4}{11} \to D \emptyset \ d \\
q &< \frac{2}{9} \to DD
\end{align*}$$

Why do voters switch newspapers in this non-monotonic way? Even though all voters with $\frac{1}{2} \leq q \leq \frac{7}{9}$ plan on voting for $R$ after $s_r$ and abstaining after $s_d$, depending on the value of $q$, a voter is concerned about different attributes of that strategy. A voter whose $q$ is close to $\frac{7}{9}$ more strongly believes that the state is $r$ after any signal than a voter whose
$q$ is closer to $\frac{1}{2}$. Therefore, the former values less than the latter any additional certainty that the state is $r$ after a signal $s_r$. At the same time, in order to abstain, he needs more reassurance that the state is not $r$ after a signal $s_d$. Hence, the voter whose $q$ is close to $\frac{7}{9}$ prefers a newspaper, $r$, that provides strong evidence that the state is not $r$ when sending the signal $s_d$, while the voter whose $q$ is close to $\frac{1}{2}$ prefers a newspaper, $d$, that provides strong evidence that the state is $r$ when sending the signal $s_r$.

More formally, a voter with prior beliefs $q \geq \frac{1}{2}$ must choose between payoffs $U^r_q (R, s_r) \times p^r (s_r, q)$ and $U^d_q (R, s_r) \times p^d (s_r, q)$. Since a signal $s_r$ from newspaper $d$ provides strong evidence that the state is $r$, we have that $U^r_q (R, s_r) < U^d_q (R, s_r)$. However, since newspaper $r$ is biased towards $R$, we also have that $p^d (s_r, q) > p^r (s_r, q)$. Hence, the choice is between a strategy that generates a high payoff with relatively low probability and a strategy that generates a low payoff with relatively high probability. This trade-off is such that a voter $q$ selects newspaper $r$ if and only if $\frac{p^r (s_r, q)}{p^d (s_r, q)} > \frac{U^d_q (R, s_r)}{U^r_q (R, s_r)}$. Since $\frac{p^r (s_r, q)}{p^d (s_r, q)}$ is increasing in $q$, while $\frac{U^d_q (R, s_r)}{U^r_q (R, s_r)}$ is decreasing in $q$, voters with relatively high $q$ prefer the former strategy, while voters with relatively low $q$ prefer the latter strategy.

This intuition applies naturally to voting environments with abstention for the following reason. In the previous example, we have assumed that $\frac{u}{c} = \frac{4}{7}$; if the benefit to cost ratio is bigger than 1, that is $\frac{u}{c} > 1$, we can show that the strategies that involve abstention with information collection (that is $D\emptyset$ and $\emptyset R$) are never used in equilibrium: either voting un informatively for the ideologically preferred candidate or using the newspaper’s recommendation at face value yield a higher expected utility. Moreover, this result is independent of the number of newspapers and extends to the case where the voter can select from a continuum of media outlets as in Suen (2004). Hence, if $\frac{u}{c} > 1$, the previous set up could never deliver left leaning voters reading right leaning newspapers at the same time that it gives right leaning voters reading left leaning newspapers. Demand for slant would be monotonic in voters bias in our very simple example.

What happens when we study the decision to collect biased information in a non costly voting model with fully rational voters? First of all, in a non costly voting environment with rational voters the ratio $\frac{u}{c}$ not only is endogenous but it is also state dependent: the pivotal events that give raise to the benefit of casting a positive vote for one candidate or abstaining are equilibrium results. Second, as explained in detail in Feddersen and Pesendorfer (1996), those voters that are willing to abstain and suffer the Swing Voter’s curse believe that they are more likely to make the wrong choice if they vote for any candidate, which implies that, for them $\frac{u}{c} < 1$. That is way they decide to abstain. Now we have all the ingredients for the main point of our paper: when voting is voluntary the Swing Voter’s curse naturally gives that voters demand for slant is non monotonic on prior beliefs.
1.3 Literature Review

In recent years there has been interesting work analyzing the demand for political news and its impact on political outcomes (see, e.g., DellaVigna and Gentzkow (2010) and Prat and Stromberg (2010)). In a decision theoretic environment, Calvert (1985) and Suen (2004) show that rational agents optimally demand information that is biased towards their priors. In Oliveros and Várdy (2011), we show that this result extends to elections with compulsory voting. Gentzkow and Shapiro (2006) provide an additional rationale for voters’ apparent confirmatory bias. In their model, voters are not only uncertain about the state of the world, but also about the quality of the various media outlets. As a consequence, outlets that provide information conforming to voters’ prior beliefs are thought to be of higher quality. This gives news media an incentive to pander to the biases and prior beliefs of voters.

The finding of rational confirmation bias has been used extensively in reduced-form modelling of voters’ demand for information. Examples are Mullainathan and Shleifer (2005) and Baron (2006), who study competition in the market for news; Chan and Suen (2008) and Krasa et al. (2008), who study political competition; and Duggan and Martinelli (2010) who look at economic policy selection.

Papers such as Strömberg (2004), DellaVigna and Kaplan (2007), Chiang and Knight (2008), Gerber et al. (2007), Gerber et al. (2009), and Ruben Enikolopov and Zhuravskaya (2011) study whether news media affect voting behavior. They find some evidence to that effect. However, the ideological leanings of media outlets and their customers are not as closely correlated as one might expect. Indeed, Gentzkow and Shapiro (2011) show that news consumption is less “segregated” than would be predicted by existing theories. They argue that vertical differentiation between media outlets and the fact that some voters consult multiple outlets might explain this empirical finding. The contribution of this paper is to show that, even in the absence of vertical differentiation and even if voters read only one newspaper, the option to abstain induces cross-over in media consumption and less than perfect political segregation.

Our paper is closely related to the literature on abstention and information acquisition. Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999) have shown that voters who are (close to) indifferent between supporting either one of two candidates suffer from a “Swing Voter’s Curse” which gives these voters a strict incentive to abstain. Oliveros (2011) studies the incentives to abstain when information acquisition is endogenous and costly. Martinelli (2006) and Martinelli (2007) also allow for endogenous information acquisition and study the information aggregation properties of elections. However, in these papers, voters cannot abstain.
The rest of the paper is organized as follows. In Section 2, we introduce the model, which we solve in Section 3. Section 3 also contains our results regarding individual and aggregate demand for information. Section 4 discusses some of the implications of our model and Section 5 concludes. Most proofs are relegated to the Appendix.

2 The model

2.1 Set up

There are two alternatives \( j \in \{ R, D \} \) and two states of the world \( \omega \in \{ r, d \} \). We follow closely Feddersen and Pesendorfer (1999) assuming a Poisson environment (see Myerson (1998) and Myerson (2000)). Nature selects a number of players according to the Poisson distribution \( \Pr (n) = \frac{e^{-\nu} \nu^n}{n!} \) where \( \nu \) is the average number of voters. If a player is selected he can be either a behavioral voter or a responsive voter. Behavioral voters support \( R \) or \( D \) independently of the state while responsive voters have contingent preferences described. We assume that a voter that is selected is behavioral with probability \( \xi \); a behavioral voter supports \( R \) with probability \( \eta \) and supports \( D \) with probability \( 1 - \eta \).\(^2\) We assume that most voters are responsive so \( \xi \) is small.\(^3\) We assume that responsive voters preferences are described by

\[
U(R | \omega = r) = U(D | \omega = d) = 0
\]
\[
U(D | \omega = r) = U(R | \omega = d) = -1
\]

Responsive voters differ in their prior beliefs over the states. Let \( \theta_r \in [0, 1] \) be the prior probability assigned to state \( r \) by voter type \( \theta_r \) and \( \theta_d = 1 - \theta_r \) be the prior probability assigned to the state \( d \) by the same voter.\(^4\) We say that the larger \( \theta_r \) the larger the bias in favor of candidate \( R \). We refer to responsive voter \( i \)'s prior beliefs as his type, and to a

\(^2\)Chan and Suen (2008) (see page 719) assumes that behavioral voters can be understood as voters that choose not to listen or consume any political news and base their vote on other rules. Healy et al. (2010) provide evidence of rules that are not associated with political news; in particular they show that irrelevant information might drive some of the voting decisions.

\(^3\)In particular we need that \( \min \left\{ \frac{1}{2\eta}, \frac{1}{2(1-\eta)} \right\} > \xi \) and a sufficient condition is \( \frac{1}{2} > \xi \).

\(^4\)Clearly our assumption about the loss functions is without loss of generality. To see this note that if

\[
U(R | \omega = r) = U(D | \omega = d) = 0
\]
\[
U(D | \omega = r) = -l, U(R | \omega = d) = -(1-l)
\]

we could redefine priors beliefs as \( \theta'_d = \frac{(1-l)\times \theta_d}{(1-l)\times \theta_d + l \times \theta_r} \) and \( \theta'_r = \frac{l \times \theta_r}{(1-l)\times \theta_d + l \times \theta_r} \) and perform the whole analysis in the same way.
"responsive voter type $\theta$" simply as a "type $\theta$". A responsive voter’s prior beliefs are private information and are drawn independently from a distribution with cumulative distribution function $F$ on $[0,1]$ with no mass points that admits a well behaved density function $f$. We assume that $F$ and $f$ are common knowledge. We refer to responsive voters generically as voters since they are the only ones that are actually strategic.

Before casting a ballot voters can select to listen to one and only one media outlet from the available media outlets. A media outlet sends a signal that is correlated with the true state of nature and media outlets differ in the correlation they offer between the signals and the states. We discuss the information technology (available media outlets) below. The set of possible actions for a voter is $X = \{D, \emptyset, R\}$ where $D$ ($R$) is a vote for candidate $D$ ($R$) and $\emptyset$ stands for abstention.

The timing of the game is as follows: 1) nature selects the number of voters, the profile of types and the state, 2) each player $i$ observes his own type, 3) player $i$ privately decides the media outlet he wants to use by selecting $p_r$ (which is also private information), 4) each voter draws a private signal from the selected media outlet, 5) players vote after signals are observed, and 6) the winner is elected according to majority rule. We define below $p_r$ as a sufficient description of the chosen media outlet.

### 2.2 Information technology

Before casting a ballot each voter can collect a signal $S \in \{s_d, s_r\}$ from a media outlet. Each media outlet is characterized by a pair $(p_r, p_d)$ such that $\Pr(s_d | d) = p_d$ and $\Pr(s_r | r) = p_r$ where $\Pr(S | \omega)$ is the probability of signal $S \in \{s_d, s_r\}$ in state $\omega \in \{d, r\}$\footnote{Calvert (1985) uses a similar information set up to understand a decision theoretic problem.}. We assume that there is a continuum of media outlets and the available media outlets are described by the information technology $p_d = G(p_r)$\footnote{We can also assume that every $p_d < G(p_r)$ is available but no voter would select one such media outlet. $G$ is the efficient frontier of the available media outlets.}, such that for a given media outlet that provides $p_r$ we must have that $p_d = G(p_r)$. The information technology verifies the following assumptions:

**Assumption 1**

1. $G$ is strictly decreasing and strictly concave
2. For each $\omega \in \{r, d\}$ there is a pair $\left\{p_\omega, \overline{p}_\omega\right\}$ such that 1) $1 > \overline{p}_\omega > \frac{1}{2} > p_\omega > 0$,
3. $1 < p_r + G(p_r)$ for all $p \in \left(\overline{p}_r, \overline{p}_r\right)$ and $1 = p_r + G(p_r)$ for $p \in \left(p_r, \overline{p}_r\right)$.

Let's discuss our assumptions about the information technology\footnote{This technology was used in Oliveros and Várdy (2011) were there is a more extensive discussion of the microfoundations and its properties. It can be derived as the result of coarsening of information from the media outlets as in Suen (2004) and Duggan and Martinelli (2010)}.
is if the state is $\omega$ and the less likely the other signal is in the other state. Voters can select accuracies of these signals by trading higher accuracy in one state with lower accuracy in the other state. Because $G$ is concave the voter’s information selection problem is clearly defined and does not rule out that some voters might want information that is fair: $p_r \approx G(p_r)$. The last two assumptions allow for media outlets that will always give the same signal with higher probability in every state ($\min\{p_d, p_r\} < \frac{1}{2}$); it also assures that there are always media outlets that provide the "right" signals in every state ($\min\{p_d, p_r\} > \frac{1}{2}$). Given the assumptions about $G$ we have that

Lemma 1 There is some $\tilde{p} > \frac{1}{2}$ such that $\tilde{p} = G(\tilde{p})$ and $G'(p_r) > -1 > G'(p_r)$.

If a voter selects the media outlet with $\tilde{p}$ then the accuracy of the signal is the same in both states. We are going to say that an information source is biased towards $R$ ($D$) if $p_r > (<) p_d$. By definition of $G$ it must be that an information source is biased towards $R$ ($D$) iff $p_r > (<) \tilde{p}$. It is clear that our definition of biased information source is not in the statistical sense but in a more intuitive way and is defined with respect to the parameter $\tilde{p}$. The intuition for our definition of bias is fairly simple: given information sources $k$ and $k'$, the information source $k$ is more biased towards $R$ ($D$) if the the probability of the signal $s_r$ ($s_d$) is higher when $k$ is the information source than when it is $k'$. Therefore, collecting biased information increases the probability that the voter will actually support the candidate that he is biased to.\(^8\)

In terms of the information conveyed by each signal is important to understand the likelihood ratios after each signal. Using our definition of $G$ we define the likelihood ratio of the states $r$ and $d$ with a media outlet $p_r$ as $l_{p_r}(s) = \frac{\Pr(r|s)}{\Pr(d|s)}$ which gives $l_{p_r}(s_d) = \frac{1 - p_r}{G(p_r)} \theta_d$ and $l_{p_r}(s_r) = \frac{p_r}{1 - G(p_r)} \theta_d$.\(^9\) Since voters can select different media outlets it matters how these

\(^8\)It is worth noticing that in terms of Blackwell’s informativeness criteria our information sources cannot be ranked: it is never the case that $(p_a, p_q)$ is sufficient for another pair $(p'_a, p'_q)$ where $p_a > p'_a$ and $p'_q > p_q$(see Crémer (1982)). Using a less stringent condition like accuracy (see Persico (2000) or as defined by Lehmann (1988): more effective) neither provides a straight answer since the information source $(p_a, p_q)$ and the information source $(p'_a, p'_q)$ cannot be orderd according to accuracy..

\(^9\)Note that the posteriors beliefs after the different signal $\Pr(d | s_d)$ and $\Pr(r | s_r)$ are equivalent to

$$
\Pr(d | s_d) = \left(1 + \frac{1}{l_{p_r}(s_d)}\right)^{-1}
$$

$$
\Pr(r | s_r) = \left(1 + \frac{1}{l_{p_r}(s_r)}\right)^{-1}
$$

so the posteriors inherit all the basic analytic properties of the likelihood ratios.
likelihood ratios change with the different media outlets which are given by
\[
\begin{align*}
\frac{\partial l_{p_r}(s_d)}{\partial p_r} &= -\frac{G(p_r) + G'(p_r)(1 - p_r)}{(G(p_r))^2} \times \frac{\theta_r}{\theta_d} \\
\frac{\partial l_{p_d}(s_r)}{\partial p_d} &= \frac{(1 - G(p_r)) + G'(p_r)p_r}{(1 - G(p_r))^2} \times \frac{\theta_r}{\theta_d}
\end{align*}
\]

Hence the following functions are particularly useful to understand investment behavior.

**Definition 1** Let \( H_d(p) \) and \( H_r(p) \) be
\[
\begin{align*}
H_r(p) &= (1 - G(p)) + p \times G'(p) \\
H_d(p) &= G(p) + (1 - p) \times G'(p)
\end{align*}
\]
and define \( \lim_{p \to p_r} H_j(p) = H_j(p_r) \) and \( \lim_{p \to p_d} H_j(p) = H_j(p_r) \) for \( j = 1, 2 \).

These functions were introduced in Oliveros and Várdy (2011) and are also related to the slope of the posteriors beliefs when the investment in information changes. Indeed, the likelihood ratios are maximized where the posteriors of the relevant state given the relevant signal are also maximized. That is if \( p_r' \) is such that \( l_{p_r}(s_r) \) is maximized (\( l_{p_r}(s_d) \) is minimized), then \( \Pr(r \mid s_r) \) (\( \Pr(d \mid s_d) \)) is also maximized at \( p_r' \). Given the information technology \( G \) here are some useful properties of these \( H_d \) and \( H_r \) functions:

**Lemma 2** If \( G \) verifies Assumption (1), then
\[
\begin{align*}
1. & \quad H_j(p) \text{ is decreasing in } p \in [p_r, p_r] \text{ for every } j = 1, 2 \\
2. & \quad H_r(p) < H_d(p) \text{ for every } p \in [p_r, p_r] \\
3. & \quad H_r(p_r) < 0 < (=) H_r(p_r) \text{ if } p_r > (=) 0 \text{ and } H_d(p_r) < (=) 0 < H_d(p_r) \text{ if } p_r < (=) 1.
\end{align*}
\]

Given the properties in the previous Lemma it follows that \( 1 - l_{p_r}(s_d) \) achieves a maximum for some \( p \in (p_d, p_r) \) if \( p_d, p_r \subset (0, 1) \) while \( l_{p_r}(s_r) \) achieves a maximum for some other \( p' \in (p_d, p_r) \) if \( p_d, p_r \subset (0, 1) \). Since more certainty after \( s_r \) is associated with a higher posterior about the state \( r \) (higher \( l_{p_r}(s_r) \)) and more certainty after \( s_d \) is associated with a lower posterior about the state \( r \) (higher \( 1 - l_{p_r}(s_d) \)) it follows that some media outlets will never receive customers. Indeed, voters will not demand media outlets that are too biased since increasing (if \( p_r \) is low) or decreasing (if \( p_r \) is high) the bias of the media outlet will lead to higher certainty after both signals.
2.3 Strategies and equilibrium

We study symmetric equilibria in pure strategies. A strategy for voter $i$ is a pair of measurable functions $\sigma^i : [0, 1] \times \{ s_r, s_d \} \rightarrow \{ D, \emptyset, R \}$ and $p^i_r : [0, 1] \rightarrow \left[ p_d, p_r \right]$ where $p^i_r (\theta) \in \left[ p_d, p_r \right]$ is the media outlet that voter $i$ type $\theta$ decides to collect information from, and $\sigma^i_x (\theta, S) \in \{ D, \emptyset, R \}$ is the probability that a voter type $\theta$, with a signal $S$, takes action $x \in \{ D, \emptyset, R \}$. Let $\sigma = \{ \sigma^i_x (\theta, S) \}_{x \in \{ D, \emptyset, R \}}$ describe an strategy or, when it bears no confusion, an equilibrium.

Let $n^\sigma_x (\omega)$ be the random variable that describes the number of voters taken action $x \in \{ D, \emptyset, R \}$ in state $\omega$ when the equilibrium is $\sigma$. Using the Poisson environment the expected number of agents who take action $x$ in state $\omega$ under $\sigma$ is just $n^\sigma_x (\omega) = vt^\sigma_x (\omega)$ where $t^\sigma_x (\omega)$ is the probability that a voter takes action $x$ in state $\omega$ under $\sigma$. From the perspective of a given voter, $n^\sigma_x (\omega)$ also describes the number of voters taking action $x$ in state $\omega$ excluding himself. As proven in (Myerson (2000)), $n^\sigma_R (\omega)$ and $n^\sigma_D (\omega)$ are independent Poisson random variables with parameters $vt^\sigma_R (\omega)$ and $vt^\sigma_D (\omega)$. Given the equilibrium $\sigma$, the ex ante probability that a voter takes action $x \in \mathcal{X}$ in state $\omega$ is just:

$$
t^\sigma_x (\omega) = \xi \left( 2I (x = R) \left( \eta - \frac{1}{2} \right) + (1 - \eta) \right) + (1 - \xi) \int_0^1 \left[ \sum_{S \in \{s_r, s_d\}} \mathbf{I} (\sigma (\theta, S) = x) \Pr (S | \omega, \theta) \right] f (\theta) \, d\theta
$$

We use the simple majority rule so $D \ (R)$ wins if $n^\sigma_D (\omega) > (\prec) n^\sigma_R (\omega)$ and the winner is randomly selected with 50% probability to each candidate if there is a tie.

The expected utility of a voter type $\theta$ that votes $V$ conditional on a given event $E$ (the realization of a vote tally excluding voter $i$) and a signal $S$ obtained from a media outlet $p$ is

$$
EU \ (V \ | \ S, E, \theta, p) = \sum_{\omega \in \{d,r\}} U (W \ | \ \omega) \Pr (W \ | \ V, E) \Pr (\omega \ | \ S, E, \theta)
$$

while the expected utility for voting $V$ conditional on a signal $S$ after using media outlet $p$ is just

$$
EU \ (V \ | \ S, \theta, p) = \sum_{E \in \left\{ n^\sigma_B (\omega), n^\sigma_R (\omega) \right\}} EU \ (V \ | \ S, E, \theta, p) \Pr (E \ | \ S, p, \theta)
$$

We also define $U^i (p^i_r (\theta), \sigma^i (\theta), S | \theta)$ as the ex antee expected utility of using the information strategy $p^i_r (\theta)$ and the voting strategy $\sigma^i (\theta, S) \in \{ D, \emptyset, R \}$ for $S \in \{ s_r, s_d \}$ which implies
that

\[ U_i^i (p^i_r (\theta), \sigma^i (\theta, S) | \theta) \equiv \sum_{x \in \{d,x\}} EU (\sigma^i (\theta, s_x) | s_x, \theta, p^i_r (\theta)) \Pr (s_x | p^i_r (\theta), \theta) \]  

(7)

Hence we have

**Definition 2** A symmetric Bayesian equilibrium for the voting game is a strategy \((p^*_r (\theta), \sigma^* (\theta, S))\) such that: 1) for all \(i\) and for every \(\theta \in [0, 1]\), \(\sigma^i (\theta, S) = \sigma^* (\theta, S)\) and \(p^i_r (\theta) = p^*_r (\theta)\), and 2) for all \(i\) and for every \(\theta \in [0, 1]\), and for any other feasible \(\sigma^i (\theta, S)\) and \(p^i_r\), the strategy \((p^*_r (\theta), \sigma^* (\theta, S))\) satisfies

\[ U_i^i (p^*_r (\theta), \sigma^* (\theta, S) | \theta) \geq U_i^i (p^i_r, \sigma^i | \theta) \]  

(8)

The next Lemma is standard in statistical decision theory (see DeGroot (2004)) so its proof is omitted. It states that maximization of the ex ante expected utility is sufficient for voters to follow the signal once they have received. That is, voters do not suffer any dynamic inconsistency in which they collect information planning on following a particular strategy and then, when they receive the signal, they prefer not to follow the strategy as planned:

**Lemma 3** If \(U_i^i (p^*_r (\theta), \sigma^* (\theta, S) | \theta) \geq U_i^i (p^i_r, \sigma^i | \theta)\) then for any \(s \in \{s_a, s_q\}\) and any \(p^i_r \in [p^i_r, \overline{p}_r]\) we have that \(EU (\sigma^i (\theta, s) | s, \theta, p^i_r (\theta)) \geq EU (\sigma' (\theta, s) | s, \theta, p^i_r (\theta))\) for all other \(\sigma' \neq \sigma^* (\theta, s)\)

There are three events that are relevant for the voter: when he can create a tie or when he can break one. Creating a tie occurs when \(n_D + 1 = n_R\) and when \(n_D - 1 = n_R\) while breaking a tie occurs when \(n_D = n_R\). Using that \(n^R_R (\omega)\) and \(n^D_D (\omega)\) are independent, we have that

\[ \Pr (n_R, n_D | \omega) = \frac{e^{-v(t_R (\omega) + t_D (\omega))} (vt_R (\omega))^{n_R} (vt_D (\omega))^{n_D}}{n_R! n_D!} \]  

(9)

\[ = \frac{(v)^{n_R+D}(t_R (\omega))^{n_R} (t_D (\omega))^{n_D} e^{-v(t_R (\omega) + t_D (\omega))}}{n_R! n_D!} \]

The existence of partisan voters that vote for \(R\) or \(D\) with some probability is enough for \(\Pr (n_D, n_R | \sigma, \omega) > 0\) for every \(|n_D - n_R| \leq 1\). Moreover, this induces voters to use strategies
that are not weakly dominated. Using (6) we have that

\[ EU(V | S, \theta) = \tilde{V}(S, \theta) + \sum_{\omega \in \{d, r\}} \sum_{n_R = 0}^{\infty} \sum_{n_D = n_R}^{n_R + 1} U(W | \omega) \Pr(W | V, n_R, n_D) \Pr(\omega, n_R, n_D | S, \theta) \]

(10)

where

\[ \tilde{V}(S, \theta) = \sum_{\omega \in \{d, r\}} \sum_{n_R = 0}^{\infty} \sum_{n_D = 0}^{\infty} U(R | \omega) \Pr(\omega, n_R, n_D | S, \theta) \]

\[ + \sum_{\omega \in \{d, r\}} \sum_{n_R = 0}^{\infty} \sum_{n_D = n_R + 2}^{\infty} U(D | \omega) \Pr(\omega, n_R, n_D | S, \theta) \]

and we applied Bayes rule. Hence we have

\[ EU(R | S, \theta) = \tilde{V}(S, \theta) - \left( \sum_{x=0}^{\infty} P^x(d) \frac{P_1(d)}{2} \right) \Pr(d | S, \theta) - \frac{P_1(r)}{2} \Pr(r | S, \theta) \]

(11)

\[ EU(D | S, \theta) = \tilde{V}(S, \theta) - \left( \sum_{x=0}^{\infty} P^x(r) \frac{P_1^{-1}(r)}{2} \right) \Pr(r | S, \theta) - \frac{P_1^{-1}(d)}{2} \Pr(d | S, \theta) \]

\[ EU(\emptyset | S, \theta) = \tilde{V}(S, \theta) - \left( P_1^{-1}(d) \frac{P_0(d)}{2} \right) \Pr(d | S, \theta) - \left( P_1(r) + \frac{P_0(r)}{2} \right) \Pr(r | S, \theta) \]

(13)

where we used Bayes rule and define \( P^x(\omega) = \sum_{n_R = 0}^{\infty} \Pr(n_R, n_R + x | \omega) \) so \( P^x(\omega) \) is the probability that \( D \) has \( x \) ahead of \( R \) and \( \tilde{P}^x(\omega) = P_0(\omega) + P^x(\omega) \) for \( x \in \{-1, 1\} \) and any \( \omega \in \{d, r\} \).

3 Solving the Model

Using (11) we have that a voter is willing to vote for \( R \) instead of \( D \) or \( \emptyset \) if

\[ \frac{\theta_r \Pr(S | r)}{\theta_d \Pr(S | d)} \geq \max \left\{ \frac{\tilde{P}^1(d)}{P_1(r)}, \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{P_1^{-1}(r) + P_1(r)} \right\} \]

(12)

and to vote for \( D \) instead of \( R \) or \( \emptyset \) if

\[ \frac{\theta_r \Pr(S | r)}{\theta_d \Pr(S | d)} \leq \min \left\{ \frac{\tilde{P}^{-1}(d)}{P^{-1}(r)}, \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{P^{-1}(r) + P_1(r)} \right\} \]

(13)
while abstaining is preferred if
\[
\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} \leq \frac{\theta_r \Pr(S \mid r)}{\theta_d \Pr(S \mid d)} \leq \frac{\tilde{P}^1(d)}{\tilde{P}^1(r)}
\] (14)

We need to identify those types that are willing to collect and use information. Let a voting strategy be \((v_d, v_r)\) where \(v_x\) is the vote after signal \(s_x\). Since information is only useful when it changes behavior we have 6 possible informative strategies: \((D, R)\), \((D, \emptyset)\), and \((\emptyset, R)\), and \((R, D)\), \((\emptyset, D)\), and \((R, \emptyset)\). On the other hand there are three strategies that do not use information: \((D, D)\), \((\emptyset, \emptyset)\) and \((R, R)\). It is immediate to see that our assumptions about the information technology assures that signals are read actually a face value or ignored; hence \((R, D)\), \((R, \emptyset)\), and \((\emptyset, D)\) are not feasible strategies

**Lemma 4** The only informative strategies that are used with positive probability in equilibrium are \((D, R)\), \((D, \emptyset)\) and \((\emptyset, R)\).

The intuition for the previous result is rather simple. Given that signals are informative, a voter will never make the wrong inference and vote against the information received after both signals.

### 3.1 Information acquisition

Now we can concentrate on the strategies in which the information is used "properly": \((D, R)\), \((D, \emptyset)\) and \((\emptyset, R)\). Using (11) we have that the expected utilities for each one of these strategies are

\[
EU(DR, p_r \mid \theta) = \tilde{V}(\theta) + \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{2} \theta_d + \frac{\tilde{P}^{-1}(r) + \tilde{P}^1(r)}{2} p_r \theta_r
\] (15)

\[
EU(\emptyset R, p_d \mid \theta) = \tilde{V}(\theta) + \frac{\tilde{P}^{-1}(r)}{2} \theta_r + \frac{\tilde{P}^1(d)}{2} G(p_r) \theta_d + \frac{\tilde{P}^1(r)}{2} p_r \theta_r
\] (16)

\[
EU(D\emptyset, p_d \mid \theta) = \tilde{V}(\theta) + \frac{\tilde{P}^1(d)\theta_d}{2} + \frac{\tilde{P}^{-1}(d)}{2} G(p_r) \theta_d + \frac{\tilde{P}^{-1}(r)}{2} p_r \theta_r
\] (17)

where

\[
\tilde{V}(\theta) = \tilde{V}(s_r, \theta) + \tilde{V}(s_d, \theta) - \theta_d \frac{P^{-1}(d)}{2} - \theta_r \frac{P^1(r)}{2} - \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{2} \theta_d - \frac{\tilde{P}^{-1}(r) + \tilde{P}^1(r)}{2} \theta_r
\]

Let \(p_{\theta}^{\text{univr}}(\theta)\) be the information collected by type \(\theta\) that wants to use the strategy \(v_d v_r\) where we set \(p_{\theta}^{\text{univr}}(\theta) = \frac{1}{2}\) for the uninformative strategies. Then we have
Lemma 5 If $0 < p_r < \frac{1}{2} < \bar{p}_r < 1$, then $p_r^{v_dv_r} (\theta) \in (p_r, \bar{p}_r)$ for every $v_dv_r \in \{D, \emptyset, R\}^2$ and $\theta \in [0, 1]$, and $p_r^{v_dv_r} (\theta)$ are described by

$$
G' (p_r^{DR} (\theta)) = -\frac{\theta}{1 - \theta} \frac{\bar{P}^1 (r)}{\bar{P}^{-1} (d)} + \frac{\bar{P}^{-1} (r)}{\bar{P}^{-1} (d)}
$$

$$
G' (p_r^{P\emptyset} (\theta)) = -\frac{\theta}{1 - \theta} \frac{\bar{P}^{-1} (r)}{\bar{P}^{-1} (d)}
$$

$$
G' (p_r^{\emptyset R} (\theta)) = -\frac{\theta}{1 - \theta} \frac{\bar{P}^1 (r)}{\bar{P}^{-1} (d)}
$$

First of all note that $p_r^{v_dv_r} (\theta)$ increases with $\theta_r$ for any $v_dv_r$ by the implicit function theorem and the fact that $G$ is concave. Using (15)- (17) we define $EU^* (v_dv_r \mid \theta) = EU (v_dv_r, p_r^{v_dv_r} (\theta) \mid \theta)$ which is the expected utility for playing the strategy $v_dv_r \in \{D, \emptyset, R\}^2$ when the information collected is $p_r^{v_dv_r} (\theta)$; by the previous continuity argument it is easy to see that $EU^* (v_dv_r \mid \theta) - EU^* (v_d'v'_r \mid \theta)$ for $v_dv_r \neq v_d'v'_r$ is continuos. Let

$$
\Theta_{v_dv_R}^* = \{\theta \in [0, 1] : V (v_dv_r \mid \theta) \geq V (v_d'v'_r \mid \theta) \text{ for } v_dv_r \neq v_d'v'_r\}
$$

be the set of voters that receive the highest expected utility when using $v_dv_R$ and the fact that $V (v_dv_r \mid \theta) - V (v_d'v'_r \mid \theta)$ for $v_dv_r \neq v_d'v'_r$ is continuos implies that $\Theta_{v_dv_R}^*$ for every strategy $v_dv_R$, is the union of closed intervals almost surely (there might be some isolated points). This result will turn out to be important when we discuss existence.

At this point ranking the different media outlets selected by each voter is not possible since we need to understand how $\frac{\bar{P}^{-1} (d)}{P^{-1} (r)}$, $\frac{\bar{P}^1 (d)}{P^1 (r)}$, and $\frac{\bar{P}^1 (d) + \bar{P}^{-1} (d)}{P^1 (r) + P^{-1} (r)}$ are ordered. Hence the relation between the different investments are linked to the equilibrium behavior of voters by the order of these ratios. The following Lemma state that order contingent on the different possibilities.

Lemma 6 If $\frac{\bar{P}^{-1} (d)}{P^{-1} (r)} > (\leq) \frac{\bar{P}^1 (d)}{P^1 (r)}$ then $p_r^{P\emptyset} (\theta) < (>) p_r^{DR} (\theta) < (>) p_r^{\emptyset R} (\theta)$.

3.2 Existence and characterization

We are now ready to show that an equilibrium exists. The proof follows the lines in Oliveros (2011) where instead of looking for a fixed point in the space of best response functions we find a fixed point in the space of pivotal probabilities, in this case $\tilde{P}^* (\omega)$.

Theorem 1 There exists a voting equilibrium.
In proving existence of equilibrium a partial characterization is needed in order to create a continuous mapping from the space of pivotal probabilities into itself. It is important to note that we have shown that an equilibrium exists but we have not showed anything regarding of which strategies are used. In particular we have not shown whether \( \Theta_{\emptyset \emptyset}^*, \Theta_{\emptyset R}^* \) or \( \Theta_{D \emptyset}^* \) are actually empty or not. We now proceed to characterize the equilibrium and discuss some of its most interesting properties.

**Proposition 1** Every equilibrium is characterized by the following conditions:

1. The set \( \Theta_{\emptyset \emptyset}^* \) is empty
2. There is some \( \nu \) such that for any \( \nu > \nu \) there is abstention in any equilibrium.

Therefore the space of types in equilibrium is

1. Divided in potentially five groups: \( \Theta_{RR}^* \), \( \Theta_{DR}^* \), \( \Theta_{D \emptyset}^* \) and \( \Theta_{DD}^* \)
2. All are non empty except for \( \Theta_{DR}^* \) that might be empty
3. They verify

\[
\min \Theta_{DD}^* = \min \Theta_{D \emptyset}^* < \min \Theta_{DR}^* < \max \Theta_{DR}^* < \max \Theta_{\emptyset R}^* = \max \Theta_{RR}^* \quad (19)
\]

The proof of this proposition is rather cumbersome although it is not conceptually difficult. First we show that among those voters that use an informed strategy, if the strategies \( D \emptyset \) and \( \emptyset R \) are preferred to both uninformative strategies \( DD \) or \( RR \), then there is an open subset of voters that prefer the strategies \( D \emptyset \) and \( \emptyset R \) to any other strategy or prefers to abstain without collecting information.

We then proceed to derive necessary and sufficient conditions for abstention in equilibrium which boils down to \( \frac{\tilde{p}^{-1}(d)}{\tilde{p}^{-1}(r)} < \frac{\tilde{p}^1(d)}{\tilde{p}^1(r)} \). Intuitively, \( \frac{\tilde{p}^1(d)}{\tilde{p}^1(r)} \) is the ration of the probability the voter can create or break a tie in favor of \( R \) in the state \( d \) and in the state \( r \); analogously \( \frac{\tilde{p}^{-1}(d)}{\tilde{p}^{-1}(r)} \) is the ration of the probability the voter can create or break a tie in favor of \( D \) in the state \( d \) and in the state \( r \). \( \frac{\tilde{p}^{-1}(d)}{\tilde{p}^{-1}(r)} < \frac{\tilde{p}^1(d)}{\tilde{p}^1(r)} \) only states that it is more likely to create or break a tie in favor of \( R \) in the wrong state \( (d) \) than to create or break a tie in favor of \( D \) in the right state. Since \( \frac{\tilde{p}^{-1}(d)}{\tilde{p}^{-1}(r)} < \frac{\tilde{p}^1(d)}{\tilde{p}^1(r)} \) is equivalent to \( \frac{\tilde{p}^1(r)}{\tilde{p}^1(d)} < \frac{\tilde{p}^{-1}(r)}{\tilde{p}^{-1}(d)} \) it is also more likely to create or break a tie in favor of \( D \) in the wrong state \( (r) \) than to create or break a tie in favor of \( R \) in the right state. This is the swing voter’s curse. As a corollary of this result we get that if there is abstention in equilibrium the order of sets presented in the proposition must hold.
With these results we focus on the case $\hat{p}^{-1}(d) < \hat{p}^{1}(d)$ and we show that nobody abstains without collecting information. We do that by partitioning the space of types in five subsets. The first two are just the subset of types that will use $RR$ and $DD$; the remaining three are selected so that each one of the informed strategies are preferred to $RR$ and $DD$. Then we show that in each of these subsets of types the relevant strategy actually yields higher utility than $\emptyset \emptyset$. The intuition is quite simply. On the sides we have that uninformed strategies $RR$ and $DD$ which are characterized by strong ideological preferences by each one of the voters so it cannot be that they are indifferent between each candidate. For the informed strategies it is important to recall that information is not costly so voters can freely select the optimal amount of bias they desire. If the voter is still biased (those that belong to the sets $\Theta^*_{\emptyset R}$ and $\Theta^*_{D\emptyset}$) this seems rather obvious. For the case of $DR$ and $\emptyset \emptyset$ they intuition hinges on the fact that information is not costly at all and if, the available media outlets are relatively bad, this will also be reflected in equilibrium reducing the amount of swing voter’s curse that we should observe.

Finally we take the average number of voters to infinity and we make use of the Poisson environment and the fact that $\frac{\hat{p}^{-1}(d)}{\hat{p}^{-1}(r)} < \frac{\hat{p}^{1}(d)}{\hat{p}^{1}(r)}$ is equivalent to $\frac{\hat{p}^{1}(r)}{\hat{p}^{-1}(r)} < \frac{\hat{p}^{1}(d)}{\hat{p}^{-1}(d)}$. The right hand side is the ratio of the probability between an extra vote for $D$ or a tie in state $d$ and in state $r$, and the left hand side is the same ratio but in state $r$. In order to reach a contradiction we use that $\frac{\hat{p}^{1}(r)}{\hat{p}^{-1}(r)} > \frac{\hat{p}^{1}(d)}{\hat{p}^{-1}(d)}$ implies that $\Theta^*_{\emptyset R} = \emptyset$ and $\Theta^*_{D\emptyset} = \emptyset$ which, as proven initially, means that the set of voters that prefer $DR$ to any strategy (now with abstention not being feasible) is not empty. This means that, in equilibrium, an average vote has some informational content and it must be more likely to cast a vote for the right candidate. Note that $\frac{\hat{p}^{1}(r)}{\hat{p}^{-1}(r)} > \frac{\hat{p}^{1}(d)}{\hat{p}^{-1}(d)}$ actually says the opposite: is more likely that $D$ is winning by 1 vote or tying than losing by 1 vote or tying in the wrong state ($r$). This proves the contradiction and leads to swing voter’s curse in equilibrium for sufficiently large average number of voters.

We now discuss in detail some of the more salient characteristics of the equilibrium that are more related to those voters that abstain.

### 3.2.1 The individual choice of information

As we show later in Proposition (1) we have that for sufficiently large $v$ it must be that $\frac{\hat{p}^{-1}(d)}{\hat{p}^{-1}(r)} < \frac{\hat{p}^{1}(d)}{\hat{p}^{1}(r)}$ (this is indeed what leads to abstention in equilibrium as described in (14)). Using now Lemma (6) it follows that

$$p^D(\emptyset) > p^D(\emptyset) > p^D(\emptyset)$$
Recalling that the lower boundary of set $\Theta^*_{DR}$ shares (at least) the upper boundary of the set $\Theta^*_{D0}$, and the upper boundary of the set $\Theta^*_{DR}$ shares (at least) the lower boundary of the set $\Theta^*_{0R}$, we must have that around the lower boundary of $\Theta^*_{DR}$ some types in $\Theta^*_{D0}$ collect information from a media outlet that is more biased towards $R$ than the media outlet some types in $\Theta^*_{DR}$ collect from; but since $\Theta^*_{D0} < \Theta^*_{DR}$ around the boundary of $\Theta^*_{DR}$ we have that some types that are more leaning towards $D$ collected information that is more centrist that some other types that are less leaning towards $D$. Analogously around the upper boundary of $\Theta^*_{DR}$ some types in $\Theta^*_{0R}$ collect information from a media outlet that is less biased towards $R$ than the media outlet some types in $\Theta^*_{DR}$ collect from. Figure (1) present the demand of bias in an equilibrium of the game where $F$ is uniform, $\xi = 0$ and the information technology verifies $G(p) = \sqrt{0.9 - p^2}$.

Figure 1: In Black we have $p^{DR}(\theta_r)$ while the lower red is $p^{0R}(\theta_r)$ and the higher red is $p^{D0}(\theta_r)$ that emerge in equilibrium for a uniform distribution of types $F$, $\xi = 0$ and an information technology that verifies $G(p) = \sqrt{\beta - p^2}$ (symmetric). Note that there is a set of $p_r$ where demand comes from both voters using $DR$ and voting using one of the other strategies.

Let’s discuss the intuition using Figure (1). Let’s focus on the type $\theta = 0.25$ which is the type in the first vertical thin solid red line. This type is biased towards $D$ since his beliefs about the state being $r$ are low. This type is very particular since it is the lowest type that would have collected information, had voting been mandatory; the level of slant collected on that hypothetical case by this voter is represented by the black dashed line (which is equivalent to 0.3). This type also verifies that, given the information collected, if
the signal is $s_r$ he is indifferent between voting for $D$ and voting for $R$.\footnote{A detailed explanation of this statement can be found in Oliveros and Várdy (2011); suffices to say that this type selects investment such that $l_{pa}(s_d)$ is maximized.} As Feddersen and Pesendorfer (1996) explain this type suffers the swing voter’s curse: after receiving the signal $s_r$ he strictly prefers to abstain. When abstention is allowed this voter will use abstention instead of voting for $R$ after the signal $s_r$.

This voter now knows that after the signals $s_r$ she takes an action that entails relatively low cost (abstention) but the action after the signal $s_d$ is associated with a higher "risk". Following this new situation she would like to be more sure that the state is $d$ before supporting the liberal candidate, which is what the signal $s_d$ calls for. In order to improve her certainty in the state where she cast a positive vote she would like to demand information from a relatively more conservative media outlet. This is done by increasing $p_r$.

It remains to explain why the demand for information has that particular shape. Because of continuity of the utility functions around this type and because $\theta$ strictly prefers to abstain after $s_r$ we have that the expected utility for using $D\emptyset$ is strictly higher than the expected utility for using $DR$. Therefore, there should be a subset of other voters with lower bias than his own bias that also prefer to use $D\emptyset$ instead of $DR$. This is why we see the solid red line (that maps types to $p_r^{D\emptyset}(\theta)$) above the dashed black line meaning that, when abstention is allowed, those types prefer to use the strategy that calls for abstention after $s_r$ than the strategy that calls to vote after every signal. On the other hand, we have that $\theta$ is also the type that is indifferent between $DR$ and $RR$ when abstention is not allowed.\footnote{Again, see Oliveros and Várdy (2011) for a detailed explanation.} But since now $D\emptyset$ does strictly better than $DR$ it also does strictly better than $RR$ and hence, types with stronger liberal bias than $\theta$ would start using $D\emptyset$ instead of $DR$.

It is now clear that the voter will choose $p_r^{DR}(\theta_1) < p_r^{D\emptyset}(\theta_1)$ to increase the degree of certainty in the state where a positive vote is cast. It is useful to think of abstention as, in fact, an action that is associated with a lower likelihood that the voter will be decisive in making $R$ the winner than the action of casting a positive vote supporting $R$. Because, the voter is less decisive (less pivotal) by abstaining he does not need strong evidence that the state is $r$ to decide to abstain and she is relatively more concerned with the state being $d$ when casting a positive vote in favor of candidate $D$.

4 Discussion

While outcomes of two-candidate elections are necessarily binary, voters’ choices are not: in addition to supporting either candidate, usually, voters have the option of supporting neither
candidate and, instead, abstaining. As we have shown, this seemingly innocuous fact has interesting and perhaps surprising implications for the kind of news media voters choose to consult. In particular, the option to abstain encourages moderation and makes voters’ preferred levels of media bias a non-monotone function of their ideology. That is, voters with relatively pronounced leanings toward either side of the political spectrum may optimally read more centrist newspapers than some intrinsically more centrist voters.

4.1 Abstention and Polarization

Because news media must report binary signals, voluntary voting drives a wedge between the interests of the voter and those of the media outlet that is ideologically closest to him when the voter is planning on abstaining after one signal. Specifically, if the voter plans on abstaining rather than voting against his bias, he is more relaxed about receiving a signal that goes against his prior belief than if his strategy were to always follow his signal. The reason is simple: by abstaining he avoids the risk associated with voting against his prior. Hence, he no longer requires such a high level of certainty about the state truly being the a priori less likely one but demands a relatively higher certainty about the state when casting a positive vote in the direction of her ideologically preferred candidate.

This moderation effect of abstention is generated at the moment of acquiring information. We want to discuss also the effect of abstention on polarization. Note that those voters that are using a strategy that abstains only vote when the signal confirm their bias. Hence, any measure of polarization after the election that only considers voters that actually voted is bound to overestimate the polarization level in the society. On one hand, abstention encourages listening to the other camp, but allow for votes that are expressed to be more extreme.

Comparing voluntary voting and mandatory voting (see Oliveros and Várdy (2011)) we have that abstention encourages the acquisition of information but not necessarily its use. Moreover, while mandatory voting makes even voters with signals that are contrarian to their bias vote, abstention might allow them not to express their preference/ information. In a sense, the comparison between these two institutions seems to be trading off the extensive use of information (voluntary) with the intensive use of information (mandatory). It is clear that abstention allows for a bigger exchange of ideas so, if a cohesive society is valuable, abstention should be associated with a higher social welfare in that dimension at least.
4.2 The aggregate demand for slant

The implications of this particular use of information go beyond a purely decision theoretic effect. In Figure (1) we present the equilibrium demand for slant by a particular symmetric distribution of types and symmetric information technology $G$. This non-monotonicity in the demand for slant boosts the customer base of relatively centrist newspapers. A conservative voter who would rather abstain than vote for the liberal candidate prefers his paper to be more liberal than he is himself. Similarly, an abstaining liberal prefers his paper to be more conservative. This makes the demand for centrist—or only mildly biased—media disproportionate to the number of centrist voters in the population. In other words, centrist news media benefit from their ability to serve a wider ideological spectrum of voters, who end up using the information provided in different ways.

In order to be able to describe in more detail the aggregate implications of the non monotonicity of the demand for slant we will now focus on a particular symmetric case. Let’s define symmetric technologies as:

**Assumption 2** The information technology $G$ is symmetric if for any $p \in [\bar{p}_\omega, \overline{p}_\omega]$ we have that $G(G(p)) = p$ and $G(\overline{p}_\omega) = \bar{p}_\omega$.

and symmetric electorates:

**Assumption 3** An electorate is symmetric if $\eta = \frac{1}{2}$ and $F(\theta) = 1 - F(1 - (\theta))$.

The next Theorem characterizes the equilibrium under symmetric conditions where some certain monotonic conditions emerge.

**Theorem 2** If the electorate and the information technology are symmetric, there exists a symmetric equilibrium in which:

1. $t_D(d) = t_R(r)$ and $t_D(r) = t_R(d)$, for every $v$, and the following relations hold for every $v$,
   \begin{align*}
   \theta \in \Theta_{\partial D}^* \iff (1 - \theta) \in \Theta_{\partial R}^* \\
   \theta \in \Theta_{\partial DR}^* \iff (1 - \theta) \in \Theta_{\partial RD}^* \\
   \theta \in \Theta_{\partial DR}^* \iff (1 - \theta) \in \Theta_{\partial RD}^*
   \end{align*}

2. $\lim_{v \to \infty} \frac{\hat{p}_1(d)}{\hat{p}_1(r)} = \lim_{v \to \infty} \frac{\hat{p}_1^{-1}(d)}{\hat{p}_1^{-1}(r)} = \sqrt{\frac{t_D(d)}{t_R(d)}} = \sqrt{\frac{t_R(r)}{t_D(r)}}$

3. $\Theta_{\partial DR}^*$ has a non empty interior and $\frac{1}{2} \in \Theta_{\partial DR}^*$
Take a voter type $\theta_1$ that is indifferent between using $DR$ and $D\emptyset$. Using that $p_{r}^{DR}(\theta_1) < p_{r}^{D\emptyset}(\theta_1)$ we have that, even though voters have different ideologies and, in a sense because of that they are using different strategies, they will collect information from the same media outlet as discussed above. Let’s define

$$P_{v_d,v_r}^{\theta_d} = \left\{ p \in \{p_d, p_r\} : p_{v_d,v_r}^{\theta_d}(\theta) = p \right\}$$

as the set of media outlets that are demanded by some voter that uses the (informative) strategy $v_d, v_r$ and we have that $P^{\emptyset R} \cap P^{DR} \neq \emptyset$ and $P^{D\emptyset} \cap P^{DR} \neq \emptyset$ which implies that demand for information is non monotonic: some voters that are less biased towards candidate $D$ ($R$), demand information from a media outlet that is more biased towards candidate $R$ ($D$), than voters that are more biased towards candidate $D$ ($R$).

**Example 1** Let $G(p) = \sqrt{\beta - p^2}$ with $p = \frac{1}{2} - \sqrt{\frac{\beta - \frac{1}{2}}{2}}$ and $\frac{1}{2} + \sqrt{\frac{\beta - \frac{1}{2}}{2}} = \overline{p}$ so (18) gives

$$
p_{r}^{DR}(\theta) = \sqrt{\frac{\beta}{1 + (\frac{q_d}{q_r})^2}}$$

$$
p_{r}^{D\emptyset}(\theta) = \sqrt{\frac{\beta}{(\frac{q_d}{q_r})^2 \frac{t_R(d)}{t_R(r)} + 1}}$$

$$
p_{r}^{\emptyset R}(\theta) = \sqrt{\frac{\beta}{(\frac{q_d}{q_r})^2 \frac{t_R(r)}{t_R(d)} + 1}}$$

The case for $\beta = 0.9$ is plotted in Figure (1) with

$$P^{D\emptyset} = [0.3 \ , \ 0.46175]$$

$$P^{DR} = [0.41230 \ , \ 0.85441]$$

$$P^{\emptyset R} = [0.82873 \ , \ 0.9]$$

which implies that $P^{D\emptyset} \cap P^{DR} = [0.41230 \ , \ 0.46175]$ and $P^{\emptyset R} \cap P^{DR} = [0.82873 \ , \ 0.85441]$.

There are some media outlets that have more market share than other media outlets or, in other terms, there is more demand for a particular slant. If the sets $\{P^{\emptyset R} \cap P^{DR}\}$ and $\{P^{D\emptyset} \cap P^{DR}\}$ do not share any element that means that in equilibrium aggregate demand for news is higher for media outlets that are slightly biased. Hence competition and entry in these markets will generate more media outlets in these regions than in other regions.

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12This will, in general, depend on the particular distribution of types.
or media outlets with higher market share. In particular, the media outlet that is totally unbiased (\(\bar{p}\)) is not highly demanded.

Note that we could not prove that \(\{P^{\emptyset R} \cap P^{DR} \cap P^{\emptyset}\} \neq \emptyset\). If there is some \(p\) such that \(p \in P^{v_d,v_r}\) for all informative strategies \(v_d, v_r\), we have that there is a subset of media outlets that are fairly unbiased (including the media outlet \(\bar{p}\)) that are highly demanded because voters using all informative strategies select this slant. In particular this media outlet will serve centrists, democrats and republicans. Now we will have that competition and entry will lead to high concentration of relatively unbiased media outlets (or higher market share), with a second concentration of media outlets around the centered and fewer media outlets on the extremes. The next Proposition states that in a formal way:

**Proposition 2** Every media outlet that belongs to \([P^{DR}_{\emptyset R}(\bar{\theta}_1), P^{\emptyset}_{DR}(\bar{\theta}_1)] \cup [P^{\emptyset R}_{\emptyset}(1 - \bar{\theta}_1), P^{DR}_{\emptyset}(1 - \bar{\theta}_1)]\) receives more than one customer.

Theorem (2) is somewhat unsatisfying. In Proposition (1) we showed that \(\min \Theta^{\emptyset}_{DR} < \min \Theta^{\emptyset}_{DR} < \max \Theta^{\emptyset}_{DR} < \max \Theta^{\emptyset}_{\emptyset R}\): Theorem (2) improves on it by showing that \(\Theta^{\emptyset}_{DR}\) is non empty and indeed it is centered around the unbiased voter. It should be noted that Theorem (2) does not rule an important case that is ruled out in Example (1): a situation \(\Theta^{\emptyset}_{DR}\) is the union of two disjoint closed intervals. In this case, some media outlets that are fairly unbiased are not demanded by any type. This is because if some voters using \(DR\) are surrounded by voters with higher bias towards \(R\) but using \(D\emptyset\) then these voters will demand a higher \(p_r\) discontinuously.

We believe that this case is somewhat pathological and most likely will emerge in asymmetric environments. Unfortunately we’ve been unable to rule it out with the extra assumptions in (2). In Proposition (1) we have not been able to rule out a situation where \(\Theta^{\emptyset}_{DR} = \emptyset\). In this case we have that voters do not vote after every signal and every informed voter abstains after one signal. In this case aggregate demand for slant will be concentrated around the unbiased media outlet and we should observe very different ideologies demanding from the same media outlet. The exact conditions under which this to cases happen, whether they happen or not, and the impact on the aggregate demand for information remains an open question.

### 4.3 Consumption value of information

One could think that voters demand for information is not instrumental in the sense that they derive direct utility for receiving a particular signal. In this subsection we evaluate the merits of this argument as a potential explanation for the cross over that we obtained when
news are used to decide the vote. To simplify the exposition we assume symmetric electorate and information technology. There are two crucial questions that we need to answer: is the consumption demand for news monotonic in ideology or not, and is the voter using the information rationally?

For the first question we believe that it is interesting to assume that it is actually monotonic. In a sense we believe that assuming confirmatory bias is the most appropriate set up given the question we want to answer and the evidence suggesting that voters actually seek to hear from the same kind. This is, indeed, the basic assumption underlying most of the popular reports in the press (see Sunstein (2002), Sunstein (2009), Jamieson and Cappella (2010), Brooks (2010)). The second question is more subtle. In particular we need to answer the following question: does the voter use information efficiently at the voting stage? That is, if the voter has biased $\theta_r$ and received a signal $s$ from the media outlet $p_r$, will she vote for $D$ if $Pr(d \mid \theta_r, p_r, s)$ or not? Note that if we answer that the voter should not be ex post efficient we will end up with dynamic inconsistent voters or with voters that are smart enough to balance consumption value and instrumental value. We believe that the second avenue is better.

Assumption 4 Every voter consumption value for information is given by the terms $c_{s_r}(\theta_r)$, consumption value for signal $s_r$, and $c_{s_d}(\theta_d)$, consumption value for signal $s_d$, where $c_{s_\omega}(\theta_\omega)$ is increasing in $\theta_\omega$ and $c_{s\omega}(\frac{1}{2}) = 0$; voters use the information efficiently in the sense that select who to support in a Bayesian way.

Finally and important question is related to when the consumption value is realized. If the consumption value is realized independently of thee voting behavior, that is the expected utility of the voter is separable in instrumental reasons and consumption reasons in the limit all demand fro information will be consumption driven. The intuition is similar to Morgan and Várdy (2011): when the electorate becomes large the probability of being pivotal goes to 0 and the voter only cares about consuming news, hence every voter will have the same demand for information albeit they will use it differently. The following proposition summarizes the previous discussion:

Proposition 3 Let $C(s_r, s_d; \theta) = c_{s_r}(\theta_r)(p_r\theta_r + (1 - G(p_r))\theta_d) + c_{s_d}(\theta_d)((1 - p_r)\theta_r + G(p_r)\theta_d)$. If voter consumption value for formation is separable from the instrumental value, i.e. $\bar{EU}(v_dv_r, p_r \mid \theta) = \beta C(s_r, s_d; \theta) + (1 - \beta) EU(v_dv_r, p_r \mid \theta)$ where $EU(v_dv_r, p_r \mid \theta)$ is defined as in (16), (16) and (17), then $p^{DR}_r(\theta_r) \rightarrow p^{D\theta}_r(\theta_r) \rightarrow p^{DR}_r(\theta_r)$ when $v \rightarrow \infty$.

Although people use information efficiently all voters demand for information is due to
consumption value and it is fully monotonic... but in the opposite direction. Note that

\[ C(s_r, s_d; \theta) = (c_{sr}(\theta_r) - c_{sd}(\theta_d)) (p_r \theta_r + (1 - G(p_r)) \theta_d) + c_{sd}(\theta_d) \]

\[ C(s_r, s_d; \theta) = c_{sr}(\theta_r) + (c_{sd}(\theta_d) - c_{sr}(\theta_r)) ((1 - p_r) \theta_r + G(p_r) \theta_d) \]

If \( c_{sr}(\theta_r) > c_{sd}(\theta_d) \) (or equivalently \( \theta_r > \frac{1}{2} \)) the voter wants to select \( p_r \) such that \( p_r \theta_r + (1 - G(p_r)) \theta_d \) is maximized (see first line) and since \( p_r \theta_r + (1 - G(p_r)) \theta_d \) is strictly increasing she will select \( p_r \). On the other hand if \( c_{sr}(\theta_r) < c_{sd}(\theta_d) \) (or equivalently \( \theta_r < \frac{1}{2} \)) the voter wants to maximize \( (1 - p_r) \theta_r + G(p_r) \theta_d \) (see second line) but since it is strictly decreasing the voter will select \( p_r \). The intuition is obvious: when the voter receives utility for the signal \( s_r \) and disutility for the signal \( s_d \) she will select the most extreme media outlet that sends the signal \( s_r \) with the highest probability. The fact that being pivotal disappear on the incentives is due to the large electorate.

A model where voters enjoy the signal per the signal itself becomes a model where voters do not care about the instrumental value of information. It predicts that only three media outlets will serve the political spectrum of ideologies. The unbiased voter will demand information from the only unbiased media outlet, while all the voters to her left will concentrate on the most left media outlet and all voters to her right will concentrate on the most extreme right media outlet.\(^\text{13}\) The main three strategies described in our equilibrium will be present but voters will not demand information in the non monotonic way that we presented.

Alternatively one could think that voters derive consumption value from the signals that they actually use. In this case, voters still condition their choice of information on the fact that they are pivotal. We have then that

\[ \widehat{EU}(v_d v_r, p_r | \theta) = EU(v_d v_r, p_r | \theta) + (c_{sd}(\theta_d) G(p_r) + c_{sr}(\theta_r) (1 - G(p_r))) (1 - \theta) \sum_{x=-1}^{1} P^x(d) \]

\[ + (c_{sd}(\theta_d) (1 - p_r) + c_{sr}(\theta_r) p_r) \theta \sum_{x=-1}^{1} P^x(r) \]

where \( EU(v_d v_r, p_r | \theta) \) is defined as in (16), (16) and (17). First order conditions for infor-

\(^\text{13}\)Strictly speaking, for finite but large \( \nu \), voters that are fairly unbiased will demand information from fairly unbiased media outlets and their demand will be monotonic. There will be a fairly moderate leaning towards \( D \) cutoff voter and everybody to her left will concentrate on the most biased media outlet in favor of \( D \). Similarly there will be a type leaning towards \( R \). These types will approach the most unbiased voters when \( \nu \) grows large.
0 = \frac{\partial EU(v_dv_r, p_r | \theta)}{\partial p_r} + (c_{sd}(\theta_d) - c_{sr}(\theta_r)) \left( G'(p_r) (1 - \theta) \sum_{x=1}^{1} P^x(d) - \theta \sum_{x=1}^{1} P^x(r) \right)

which implies that the optimal choice of media outlet is given by

\[ -G'(\hat{p}^{DR}_r(\theta)) = \frac{\theta}{(1 - \theta)} \left( 1 + \frac{P^0(d)}{P^1(d) + P^{-1}(d)} - 2 (c_{sd}(\theta_d) - c_{sr}(\theta_r)) \right) \]

\[ -G'(\hat{p}^{D0}_r(\theta)) = \frac{\theta}{(1 - \theta)} \left( 1 + \frac{P^0(d)}{P^1(d) + P^{-1}(d)} + 2 (c_{sd}(\theta_d) - c_{sr}(\theta_r)) \right) \]

\[ G'(\hat{p}^{0R}_r(\theta)) = \frac{\theta}{(1 - \theta)} \left( 1 + \frac{P^{-1}(d)}{P^1(d) + P^{-1}(d)} + 2 (c_{sd}(\theta_d) - c_{sr}(\theta_r)) \right) \]

**Proposition 4** If voter consumption value for formation is non separable from the instrumental value, i.e. the consumption value is only relevant when the voter is pivotal, and the electorate and information technology is symmetric, then \( \hat{p}^{D0}_r(\theta) > \hat{p}^{DR}_r(\theta) > \hat{p}^{0R}_r(\theta) \) when \( v \to \infty \), and demand for slant is discontinuous and non monotonic.

The equilibrium demand for news is qualitatively indistinguishable from the description in the previous sections. The main difference is that now we have a mass of extremely biased voters demanding the most extreme media outlets in both ends.

## 5 Conclusions

What do our findings imply for the risk that voters withdraw into information cocoons and fall victim to echo chamber effects? Our results suggest that strict segregation of media consumers along ideological lines only occurs at the fringes of the political spectrum. By contrast, centrist voters naturally demand centrist news, while more moderately biased voters have a natural tendency to consult news media whose ideological beliefs are different—i.e., more centrist—than their own. Hence, instrumental demand for political news does not lead to strict ideological segregation and, therefore, makes information cocoons and echo chamber effects perhaps less of a problem than previously thought. Interestingly, our theoretical

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\[ ^{14} \text{Our basic intuition for the result can be extended to non symmetric environments.} \]
results are consistent with recent empirical evidence, which suggests that there is far more “cross-over” in media consumption than commonly believed (see Gentzkow and Shapiro (2011)).

Let us conclude by pointing out some of the limitations of our model. First, the model only deals with instrumental demand for news. It could be, however, that the demand for news is primarily driven by its consumption value. In that case, the primary purpose of news is not to serve as a voting guide but, rather, to entertain. While this kind of news consumption may create echo chambers, it not clear to what extend they would constitute a political problem, since exposing these voters to different sources of news might not affect their behavior at the polls. Second, we have assumed that voters perfectly know the biases of all media outlets. This raises the question of how uncertainty about these biases would affect voter behavior. We leave this for future research.
References


A Appendix: Proofs

Proof of Lemma (1). Let $K_1(p) = p - G(p)$ and note that $K_1$ is continuous, increasing in $p$. Since $G\left(\frac{1}{2}\right) > \frac{1}{2}$ it follows that $K_1\left(\frac{1}{2}\right) < 0 < K_2(\bar{p}_d)$ so there is some $\bar{p} \in \left(\frac{1}{2},\bar{p}_d\right)$ such that $\bar{p} = G(\bar{p})$ and since $\bar{p} > \frac{1}{2}$ we have the first result.

For the second part note that $K_2(p) = p + G(p)$ is strictly concave and achieves a maximum for some $\bar{p}$ where $G'(\bar{p}) = -1$; since $K_2(p) > 1$ for all $p \in \left(p_r, \bar{p}_r\right)$ and $K_2(p) = 1$ for $p \in \left(p_r, \bar{p}_r\right)$ it must be that the maximum is interior giving that $\bar{p} \in \left(p_r, \bar{p}_r\right)$. Because $G$ is decreasing and $G'(\bar{p}) = -1$ we must have that $G'(p_r) > G'(\bar{p}) = -1 > G'(\bar{p}_r)$ yielding the second part. ■

Proof of Lemma (2). The fact that $H_j(p)$ is decreasing follows by using (18) and the assumption $G''(p) < 0$.

For the second part note that $\frac{H_d(p) - H_r(p)}{2} = G(p) - \frac{1}{2} + \left(\frac{1}{2} - p\right)G'(p)$ is decreasing for every $p < \frac{1}{2}$ and increasing for every $p > \frac{1}{2}$ which gives that

$$\frac{H_d(p) - H_r(p)}{2} \geq \frac{H_d\left(\frac{1}{2}\right) - H_r\left(\frac{1}{2}\right)}{2} = G\left(\frac{1}{2}\right) - \frac{1}{2} > 0$$

which implies that $H_d(p) > H_r(p)$.

Finally, using (4) and the assumption that $1 = p_r + G(p_r)$ for $p \in \{p_r, \bar{p}_r\}$ we have that

$$H_r\left(p_r\right) = p_r \times \left(1 + G'(p_r)\right)$$
$$H_d\left(\bar{p}_r\right) = \left(1 - \bar{p}_r\right) \times \left(1 + G'(\bar{p}_r)\right)$$

Using Lemma (1) so $G'(p_r) > -1 > G'(\bar{p}_r)$ we have that $\bar{p}_r < 1$ implies $H_r(\bar{p}_d) < H_d(\bar{p}_r) < 0$ and if $\bar{p}_r = 1$ then $H_r(\bar{p}_r) < H_d(\bar{p}_r) = 0$ while if $\bar{p}_r > 0$ we must have $H_d(p_r) > H_r(p_r) > 0$ and if $\bar{p}_r = 0$ we must have $H_d(p_r) > H_r(p_r) = 0$. ■

Proof of Lemma (5). The case for uninformative strategies is trivial so we focus on informative strategies. Note that the expressions in (18) follow directly from (15), (16) and (17) by taking first order conditions. We are going to show that if a voter decides to collect information the voter will select a media outlet that belongs to the interior of $\left(p_r, \bar{p}_r\right)$ so (18) are necessary and sufficient.

First we need to think about the situations where there is abstention and where there is no abstention. First assume that some informative voter abstains after some signal so we need (14) to hold which implies that $\frac{\tilde{P}^{-1}(d)}{\tilde{P}^{-1}(r)} \leq \tilde{P}^1(d) \tilde{P}^1(r)$ and Lemma (7) implies that $\frac{\tilde{P}^{-1}(d)}{\tilde{P}^{-1}(r)} + \tilde{P}^1(d) \tilde{P}^1(r) \leq \tilde{P}^1(d) \tilde{P}^1(r)$.

Using Lemma (8) we have that $EU\left(\sigma^1(\theta, s) \mid s, \theta, p_r^{\text{opt}}(\theta)\right) \geq EU\left(\sigma^1(\theta, s, \theta, p_r^{\text{opt}}(\theta))\right)$ which implies that for any strategy that involves $v_d = D$ we need that (13) holds so

$$\frac{\theta_r\left(1 - p_r^{\text{opt}}(\theta)\right)}{\theta_d G\left(p_r^{\text{opt}}(\theta)\right)} \leq \left\{\frac{\tilde{P}^{-1}(d) \tilde{P}^{-1}(d) \tilde{P}^1(d)}{\tilde{P}^{-1}(r) \tilde{P}^{-1}(r) + \tilde{P}^1(r)}\right\}$$
and in the case of interest it implies that

\[
\frac{\theta_r 1 - p_r^{D0} (\theta)}{\theta_d G (p_r^{D0} (\theta))} \leq \frac{\tilde{P}^{-1} (d)}{\tilde{P}^{-1} (r)}
\]

\[
\frac{\theta_r 1 - p_r^{DR} (\theta)}{\theta_d G (p_r^{DR} (\theta))} \leq \frac{\tilde{P}^{-1} (d) + \tilde{P}^{1} (d)}{\tilde{P}^{-1} (r) + \tilde{P}^{1} (r)}
\]

Using (18) we have that

\[
0 \leq G (p_r^{D0} (\theta)) + G'(p_r^{D0} (\theta)) \left( 1 - p_r^{D0} (\theta) \right) = H_d \left( p_r^{D0} (\theta) \right)
\]

\[
0 \leq G (p_r^{DR} (\theta)) + G'(p_r^{DR} (\theta)) \left( 1 - p_r^{DR} (\theta) \right) = H_d \left( p_r^{DR} (\theta) \right)
\]

where we used (1). Analogously for a strategy using \( v_r = R \) we have that (12) yields

\[
H_r \left( p_r^{DR} (\theta) \right) \geq 0
\]

\[
H_r \left( p_r^{0R} (\theta) \right) \geq 0
\]

Since the informative strategies that abstain after one signal use different signals we have that (14) yields

\[
\frac{\tilde{P}^{-1} (d)}{\tilde{P}^{-1} (r)} \leq \frac{\theta_r}{\theta_d} \frac{1 - G (p_r^{D0} (\theta))}{1 - G (p_r^{DR} (\theta))} \leq \frac{\tilde{P}^{1} (d)}{\tilde{P}^{1} (r)}
\]

\[
\frac{\tilde{P}^{-1} (d)}{\tilde{P}^{-1} (r)} \leq \frac{\theta_r 1 - p_r^{0R} (\theta)}{\theta_d G (p_r^{0R} (\theta))} \leq \frac{\tilde{P}^{1} (d)}{\tilde{P}^{1} (r)}
\]

and (18) gives

\[
H_d \left( p_r^{0R} (\theta) \right) \leq 0 \leq H_r \left( p_r^{D0} (\theta) \right)
\]

Therefore for every \( p_r^{v_d v_r} (\theta) \) where \( v_d v_r \) is an informative strategy we have that \( H_d \left( p_r^{v_d v_r} (\theta) \right) \leq 0 \leq H_r \left( p_r^{v_d v_r} (\theta) \right) \) and if there is no abstention the inequalities are strict; Lemma (2) gives the result.

**Proof of Lemma (6).** The result follows directly by using (18), the fact that \( G \) is deceasing, and the following Lemma which will be used later:

**Lemma 7** \( \frac{\tilde{P}^{-1} (d)}{\tilde{P}^{-1} (r)} > (\leq) \frac{\tilde{P}^{-1} (d) + \tilde{P}^{1} (d)}{\tilde{P}^{-1} (r) + \tilde{P}^{1} (r)} \) \( \iff \frac{\tilde{P}^{-1} (d)}{\tilde{P}^{-1} (r) + \tilde{P}^{1} (r)} > (\leq) \frac{\tilde{P}^{1} (d)}{\tilde{P}^{1} (r)} \)
\textbf{Proof.} If \( \frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} > (\leq) \frac{\tilde{P}^{-1}(d) + \tilde{P}^{1}(d)}{P^{-1}(r) + P^{1}(r)} \) we have that

\[
\frac{\tilde{P}^{-1}(r) + \tilde{P}^{1}(r)}{P^{-1}(r)} > (\leq) \frac{\tilde{P}^{-1}(d) + \tilde{P}^{1}(d)}{P^{-1}(d)} \geq (\leq) \frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} \geq (\leq) \frac{\tilde{P}^{-1}(d) + 1}{P^{-1}(r) + P^{1}(r)} > (\leq) \frac{\tilde{P}^{1}(d)}{P^{1}(r)}
\]

\[\]

\textbf{Proof of Theorem (1).} First note that (15), (16) and (17) and (18) depend on the terms \( \tilde{P}^{1}(\omega) \) and \( \tilde{P}^{-1}(\omega) \) for \( \omega \in \{d, r\} \) so using the arguments in (Oliveros (2011)) the existence will follow. Let the set of relevant pivotal probabilities be \( P = \{\tilde{P}^{1}(d), \tilde{P}^{-1}(d), \tilde{P}^{1}(r), \tilde{P}^{-1}(r)\} \).

Since \( \min\{t_{\text{R}}^{\text{r}}(\omega), t_{\text{D}}^{\text{r}}(\omega)\} \geq \xi \min\{\eta, 1 - \eta\} \) we have that (9) is strictly positive for each \( n_{D} \) and \( n_{R} \) which implies that there is some \( \delta \in (0, 1) \) such that \( P \in [\delta, 1 - \delta]^4 \).

Using (18) we define

\[
G'(p_{\text{DR}}^{1}(\theta; P)) = -\frac{\theta}{1 - \theta} \frac{\tilde{P}^{1}(r) + \tilde{P}^{-1}(r)}{\tilde{P}^{1}(d) + \tilde{P}^{-1}(d)}
\]

\[
G'(p_{\text{D}0}^{1}(\theta; P)) = -\frac{\theta}{1 - \theta} \frac{\tilde{P}^{-1}(r)}{\tilde{P}^{-1}(d)}
\]

\[
G'(p_{\text{R}}^{0}(\theta; P)) = -\frac{\theta}{1 - \theta} \frac{\tilde{P}^{1}(r)}{\tilde{P}^{1}(d)}
\]

Using (11) we define \( EU^{*}(v_{d}v_{r}; P \mid \theta) = EU(v_{d}v_{r}, p_{\text{DR}}^{1}(\theta; P); P \mid \theta) \) as the expected utility for using the strategy \( v_{d}v_{r} \) and collecting information according to \( p_{\text{DR}}^{1}(\theta; P) \) when the set of relevant pivotal probabilities is \( P \). Now let

\[
\Theta_{v_{d}v_{r}}^{*}(P) = \{\theta \in [0, 1] : EU^{*}(v_{d}v_{r}; P \mid \theta) \geq EU^{*}(v_{d}v_{r}'; P \mid \theta) \text{ for any } v_{d}v_{r}' \neq v_{d}v_{r}\}
\]

and since (18) is necessary and sufficient we have that the sets \( \Theta_{v_{d}v_{r}}^{*}(P) \) are well defined (they partition the interval \([0, 1]\)); moreover, an application of the envelope theorem gives that the boundaries of \( \Theta_{v_{d}v_{r}}^{*}(P) \) change smoothly with smooth changes in each one of the components of \( P \).

Using (5) we define \( t_{\text{R}}^{\text{r}}(\omega; P) \) for \( x = D, R \) by setting

\[
t_{\text{R}}^{\text{r}}(\omega; P) = \sum_{v_{d}v_{r} \in \{D, \emptyset, R\}^2} \left[ \int_{\theta \in \Theta_{v_{d}v_{r}}^{*}(P)} \left( \sum_{x \in \{s_{e}, s_{d}\}} \Pr(\sigma(\theta, S) = x \mid \omega, \theta) \Pr(S \mid \omega, \theta) \right) f(\theta) d\theta \right]
\]

where \( \Pr(\sigma(\theta, S) = x \mid \omega, \theta) \) is the probability that a given type \( \theta \) (belonging to a set \( \Theta_{v_{d}v_{r}}^{*}(P) \)) votes for \( x \) in state \( \omega \).
Let $\Gamma: [\delta, 1 - \delta]^4 \rightarrow [\delta, 1 - \delta]^4$ be constructed in the following way. First, for a given element $z \in [\delta, 1 - \delta]^4$ we define $p_r^{v_dv_r}(\theta; z)$, $EU^*(v_dv_r; z | \theta)$ for each strategy $v_dv_r$ and hence the set of types that use these strategies $\Theta^*_{v_dv_r}(z)$, which are all continuous in the elements of $z$. Now we use these objects to define $t^*_{\omega}(\omega; z)$ for $x = D, R$ and $\omega \in d, r$ which is also continuous in $z$. Using (9) we let

$$\Gamma_\omega(z) = \left\{ \frac{1}{\sum_{h=0}^{\infty} \sum_{n_r=0}^{\infty} \Pr(n_R, n_R + h | \omega)} + \sum_{h=0}^{\infty} \sum_{n_r=0}^{\infty} \Pr(n_R, n_R + h | \omega) \right\}$$

and define $\Gamma(z) = \{\Gamma_d(z), \Gamma_r(z)\}$. Brouwer’s fixed point theorem gives that there is some $z^*$ such that $\Gamma(z^*) = z^*$. □

**Proof of Proposition (1).** We state the result in a series of Lemmas. First, for each strategy $(v_dv_r)$ that uses information let’s define the set

$$\Theta^{v_dv_r} = \{\theta : EU(v_dv_r, p_r^{v_dv_r}(\theta) | \theta) \geq \max\{EU(DD | \theta), EU(RR | \theta)\}\}$$

which is the set of types that prefer to use the strategy $v_dv_r$ (that requires use of information) over any non informative strategy that does not use abstention (that is $DD$ and $RR$). Using (11) we have that the expected utility for relevant uninformed strategies is given by

$$EU(DD | \theta) = \tilde{V}(\theta) + \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{2} \theta_d$$

$$EU(RR | \theta) = \tilde{V}(\theta) + \frac{\tilde{P}^{-1}(r) + \tilde{P}^1(r)}{2} \theta_r$$

where $\tilde{V}(\theta)$ is defined as in the sequence (15)-(17). Take the strategy $\emptyset R$ and note that type $\theta$ prefers it over the strategies $RR$ and $DD$ if

$$\theta_d \left( \frac{\tilde{P}^{-1}(d) + \tilde{P}^1(d)}{2} \right) \geq \theta_d \left( \frac{\tilde{P}^{-1}(d)}{2} + \frac{\tilde{P}^1(d)}{2} (1 - G(p_r)) \right) + \theta_r \frac{\tilde{P}^1(r)}{2} (1 - p_r)$$

$$\theta_r \left( \frac{\tilde{P}^{-1}(r)}{2} + \frac{\tilde{P}^1(r)}{2} \right) \geq \theta_d \left( \frac{\tilde{P}^{-1}(d)}{2} + \frac{\tilde{P}^1(d)}{2} (1 - G(p_r)) \right) + \theta_r \frac{\tilde{P}^1(r)}{2} (1 - p_r)$$

or

$$H_d \left( p_r^{\emptyset R}(\theta) \right) \geq 0$$

$$0 \geq \frac{\tilde{P}^{-1}(d)}{\tilde{P}^1(d)} + H_r \left( p_r^{\emptyset R}(\theta) \right) + G' \left( p_r^{\emptyset R}(\theta) \right) \frac{\tilde{P}^{-1}(r)}{\tilde{P}^1(r)}$$

Note that if we let $p_r^{\emptyset R}(\theta) = \bar{p}$ the first inequality holds and the second one is equivalent to

$$0 \geq \frac{\tilde{P}^{-1}(d)}{\tilde{P}^1(d)} - \frac{\tilde{P}^{-1}(r)}{\tilde{P}^1(r)} + (1 - 2\bar{p})$$

so it is sufficient for it to hold that $\frac{\tilde{P}^{-1}(d)}{\tilde{P}^1(r)} \leq \frac{\tilde{P}^1(d)}{\tilde{P}^1(r)}$. Analogously we have that the strategy $D\emptyset$ is
preferred to the strategies \(RR\) and \(DD\) by type \(\theta\) if

\[
H_r \left( p^D_{r \theta}(\theta) \right) \leq 0
\]

\[
\frac{\tilde{p}^1(d)}{P^{-1}(d)} + H_d \left( p^D_{r \theta}(\theta) \right) + G' \left( p^D_{r \theta}(\theta) \right) \frac{\tilde{p}^1(r)}{P^{-1}(r)} \geq 0
\]

The same condition \(\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} \leq \frac{\tilde{P}^1(d)}{P^1(d)}\) gives that the type that selects \(p^D_{r \theta}(\theta) = \tilde{p}\) prefers \(D\) over the strategies \(RR\) and \(DD\). Finally let’s focus on the strategy \(DR\) and its expected utility (15) and we must have that \(DR\) is preferred as long as

\[
H_d \left( p^{DR}_{r \theta}(\theta) \right) \geq 0 \geq H_r \left( p^{DR}_{r \theta}(\theta) \right)
\]

where we used (18). In this case we have if one holds with equality the other one holds strictly.

Let \(\bar{p}\) and \(\bar{p}\) be defined as \(H_r \left( \bar{p} \right) = 0 = H_d \left( \bar{p} \right)\) and since \(H_r < H_d\) we have that \(H_d \left( \bar{p} \right) = 0 = H_r \left( \bar{p} \right) < H_d \left( \bar{p} \right)\) and since \(H_d\) is decreasing \(\bar{p} > \bar{p}\); define now

\[
H_d \left( \bar{p}^D \right) + G' \left( \bar{p}^D \right) \frac{\tilde{p}^1(r)}{P^{-1}(r)} = - \frac{\tilde{p}^1(d)}{P^{-1}(d)}
\]

\[
H_r \left( \bar{p}^{DR} \right) + G' \left( \bar{p}^{DR} \right) \frac{\tilde{p}^{-1}(r)}{P^1(r)} = - \frac{\tilde{p}^{-1}(d)}{P^1(d)}
\]

and define \(\bar{\theta}^{p_r,v_r}\) and \(\bar{\theta}^{v_r,v_r}\) for each strategy in the following way:

\[
\frac{\bar{\theta}^{DR}}{1 - \bar{\theta}^{DR}} \frac{\tilde{p}^1(r) + \tilde{p}^{-1}(r)}{\tilde{P}^1(d) + \tilde{P}^{-1}(d)} = \frac{\bar{\theta}^{D0}}{1 - \bar{\theta}^{D0}} \frac{\tilde{p}^{-1}(r)}{\tilde{P}^{-1}(r)} = -G' \left( \bar{p}^D \right)
\]

\[
\frac{\bar{\theta}^{DR}}{1 - \bar{\theta}^{DR}} \frac{\tilde{p}^1(r) + \tilde{p}^{-1}(r)}{\tilde{P}^1(d) + \tilde{P}^{-1}(d)} = \frac{\bar{\theta}^{D0}}{1 - \bar{\theta}^{D0}} \frac{\tilde{p}^1(r)}{\tilde{P}^1(r)} = -G' \left( \bar{p}^{D0} \right)
\]

Therefore we have that \(\text{int} \Theta^{p_r,v_r} = \left( \bar{\theta}^{p_r,v_r}, \bar{\theta}^{v_r,v_r} \right)\) and even though \(\text{int} \Theta^{D0} \neq \emptyset\) we still do not know whether \(\text{int} \Theta^{D0}\) and \(\text{int} \Theta^{D0}\) are empty or not. By the previous discussion and the definition of \(\bar{\theta}^{D0}\) and \(\bar{\theta}^{D0}\) (and \(\bar{\theta}^{D0}\)) it follows that \(\text{int} \Theta^{D0} \neq \emptyset\) and \(\text{int} \Theta^{D0} \neq \emptyset\) if \(\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} \leq \frac{\tilde{P}^1(d)}{P^1(r)}\).

The next Lemma states this more formally and it is useful in its own terms:

**Lemma 8** In equilibrium 1) \(\bar{\theta}^{D0} < \bar{\theta}^{D0}\), and if \(\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} < \frac{\tilde{P}^1(d)}{P^1(r)}\) then 2) \(\bar{\theta}^{D0} < \bar{\theta}^{D0} < \bar{\theta}^{D0}\) if \(\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} \leq \frac{\tilde{P}^1(d)}{P^1(r)}\).

3) \(\bar{\theta}^{D0} < \bar{\theta}^{D0}\) and \(\bar{\theta}^{D0} > \bar{\theta}^{D0}\).

**Proof.** Note that the third part was proven before when showing that \(\frac{\tilde{P}^{-1}(d)}{P^{-1}(r)} < \frac{\tilde{P}^1(d)}{P^1(r)}\) was sufficient.
for the type $\bar{p}$ preferred the informative strategies that involved abstention over the uninformed strategies.

For the first part note that, since $p$ and $\bar{p}$ are defined by $H_d(p) = 0 = H_r(\bar{p})$ we have that $p > \bar{p}$ is independent of the order $\frac{P^{1}(d)}{P^{1}(r)}$. Since $-G'(p) < -G'(\bar{p})$ we have that the first and forth line of (25) give that $\tilde{\theta}_{DR} < \tilde{\theta}_{DR}^D$. 

For the second part use that $\frac{\tilde{P}^{-1}(d)}{P^{1}(r)} < \frac{\tilde{P}^{-1}(d)}{P^{1}(r)}$ implies $\frac{\tilde{P}^{-1}(d)}{P^{1}(r)} < \frac{\tilde{P}^{-1}(d)P^{-1}(d)}{P^{1}(r)P^{-1}(r)} < \frac{\tilde{P}^{-1}(d)}{P^{1}(r)}$ by Lemma (7), so the first line of (25) gives $\tilde{\theta}_{DR}^{-1} > \tilde{\theta}_{DR}^{-1}$, and the third line of (25) gives 

$$\frac{\tilde{P}^{-1}(d)}{P^{1}(r)} < \frac{\tilde{P}^{-1}(d)}{P^{1}(r)}$$

We now provide necessary and sufficient conditions for abstention.

**Lemma 9** $\frac{P^{-1}(d)}{P^{1}(r)} < \frac{P^{-1}(d)}{P^{1}(r)}$ is necessary and sufficient for abstention to be part of some voting strategy

**Proof. Necessity**

Using (14) we have that a voter $\theta$ that is planning on abstaining after some information collected $p(\theta)$ must verify $\frac{P^{-1}(d)}{P^{1}(r)} = \frac{\theta_r Pr(S|\theta) \leq \frac{P^{1}(d)}{P^{1}(r)}}{Pr(S|\theta) \leq \frac{P^{1}(d)}{P^{1}(r)}}$. If $\frac{P^{-1}(d)}{P^{1}(r)} = \frac{P^{1}(d)}{P^{1}(r)}$ we must have that $\theta_r Pr(S|\theta) = \frac{P^{1}(d)}{P^{1}(r)}$ and the probability of a type abstaining is nil.

**Sufficiency**

It follows by Lemma (8) that the set $\{ \theta : \theta \in \text{int} \{ \Theta^{D0} \cup \Theta^{0R} \} \neq \emptyset$ since $\tilde{\theta}_{DR} < \tilde{\theta}_{DR} < \tilde{\theta}_{DR}$. In that set every type prefers to use either $D0$ or $0R$ instead of the strategies $RR$ and $DD$ and, since $\theta \notin \Theta^{DR}$ the voter type $\theta$ prefers at least one of the strategies $RR$ and $DD$ to the strategy $DR$. Therefore the set of types $\{ \theta : \theta \in \text{int} \{ \Theta^{D0} \cup \Theta^{0R} \} \neq \emptyset$ must involve types that abstain either because they prefer $D0$ or $0R$ or they prefer $\emptyset$.

Now we are ready to prove the first part of the Proposition; namely, the strategy $\emptyset \emptyset$ is never used in equilibrium. We are going to prove this result in steps. First note that if $\frac{P^{-1}(d)}{P^{1}(r)} \geq \frac{P^{1}(d)}{P^{1}(r)}$ abstention cannot be part of an equilibrium and we are done. Therefore, assume that $\frac{P^{-1}(d)}{P^{1}(r)} < \frac{P^{1}(d)}{P^{1}(r)}$ which implies the orders in Lemma (8).

**Remark 1** Every $\theta \in [\tilde{\theta}_{DR}, 1]$ prefers $RR$ to $\emptyset \emptyset$

**Proof.** First assume that $\theta \in [\tilde{\theta}_{DR}, 1]$ and recall that $\tilde{\theta}_{DR}$ is defined as $\frac{\tilde{\theta}_{DR}^{1}(r)}{1-\tilde{\theta}_{DR}^{1}(r)} = -G'(\bar{p})$ for $H_{d}(\bar{p}) = 0$. Let $\Delta_{RR-\emptyset \emptyset}(\theta) = EU(RR \mid \theta) - EU(\emptyset \emptyset \mid \theta)$ which implies

$$\Delta_{RR-\emptyset \emptyset}(\theta) = \frac{P^{-1}(d)}{2} - \frac{P^{1}(d)}{2}(1 + \theta_r) + \left( P^{1}(r) + \frac{P^{1}(d)}{2} \right) \theta_r$$

and verifies $\lim_{\theta \to 1} \Delta_{RR-\emptyset \emptyset}(\theta) > 0$ and $\Delta_{RR-\emptyset \emptyset}(\theta) = \left( P^{1}(r) + \frac{P^{1}(d)}{2} \right) - \left( \frac{P^{-1}(d)}{2} - \frac{P^{1}(d)}{2} \right)$ which implies that is monotonic. If $\left( P^{1}(r) + \frac{P^{1}(d)}{2} \right) \leq \left( \frac{P^{-1}(d)}{2} - \frac{P^{1}(d)}{2} \right)$ it follows that $\frac{\partial \Delta_{RR-\emptyset \emptyset}(\theta)}{\partial \theta} \leq 0$ and since $\lim_{\theta \to 1} \Delta_{RR-\emptyset \emptyset}(\theta) > 0$ we know that $\Delta_{RR-\emptyset \emptyset}(\theta) > 0$ for every $\theta$ and the strategy $\emptyset \emptyset$ is never
used, so assume that

\[ P^1(r) + \frac{P^0(r)}{2} > \frac{P^{-1}(d)}{2} - \frac{\bar{P}^1(\bar{d})}{2} \]

\[ P^1(r) + \bar{P}^1(\bar{r}) > P^{-1}(d) - \bar{P}^1(\bar{d}) \]

Using now the definition of \( \bar{\theta}^{DR} \) in (25) we have \( \frac{\bar{\theta}^{DR}(r)}{P^1(d)} = -G'(\bar{\theta}) \)

\[
\frac{\Delta_{RR-\emptyset}(\theta)}{(1 - \theta_r)} \frac{\bar{P}^1(r)}{P^1(d)} = \left( \frac{P^{-1}(d) - \bar{P}^1(\bar{d})}{2} \right) \frac{\bar{P}^1(\bar{r})}{P^1(d)} - G'(\bar{\theta}) \left( P^1(r) + \frac{P^0(r)}{2} \right)
\]

\[ = \frac{P^{-1}(d) \bar{P}^1(\bar{r})}{2 \bar{P}^1(d)} - G'(\bar{\theta}) \frac{P^1(r)}{2} - (1 + G'(\bar{\theta})) \frac{\bar{P}^1(\bar{r})}{2} \]

so it is sufficient if \( 1 + G'(\bar{\theta}) \leq 0 \). First we are going to show that \( \bar{p} < \bar{\theta} \); to see that note that \( H_d(\bar{\theta}) = 1 - 2\bar{p} < 0 \) and since \( H_d(\bar{\theta}) = 0 \) the result follows by noting that \( H_d \) is decreasing. Since \( \bar{p} < \bar{\theta} \) and \( G' \) is decreasing we have

\[ G'(\bar{\theta}) < G'(\bar{\theta}) = -1 \]

which is the desired result. ■

**Remark 2** Every \( \theta \in \left[ \vartheta^{DR}, \bar{\theta}^{DR} \right] \) prefers DD to \( \emptyset \)

**Proof.** It is analogous to the previous case ■

**Remark 3** Every \( \theta \in \left[ \vartheta^{DR}, \bar{\theta}^{DR} \right] \) prefers DR to \( \emptyset \)

**Proof.** Take then \( \Delta_{DR-\emptyset}(\theta) = EU(DR \mid \theta) - EU(\emptyset \mid \theta) \) which implies

\[
\Delta_{DR-\emptyset}(\theta) = \left( \frac{P^{-1}(d) + \bar{P}^{-1}(\bar{d})}{2} \right) \theta_d + \left( \frac{P^1(r) + \bar{P}^1(\bar{r})}{2} \right) \theta_r - \bar{P}^{-1}(\bar{r}) + \bar{P}^1(\bar{r}) (1 - p_r^{DR}(\theta)) \theta_d
\]

\[ - \bar{P}^{-1}(\bar{r}) + \bar{P}^1(\bar{r}) \left( 1 - G(p_r^{DR}(\theta)) \right) \theta_r \]

\[ \frac{\Delta_{DR-\emptyset}(\theta)}{\theta_r} = \left( \frac{P^{-1}(d) + \bar{P}^{-1}(\bar{d})}{2} \right) \theta_d \theta_r + \left( \frac{P^1(r) + \bar{P}^1(\bar{r})}{2} \right) H_r(p_r^{DR}(\theta)) \]

since we must have that \( H_r(p_r^{DR}(\theta)) \geq 0 \) for every \( \theta \in \left[ \vartheta^{DR}, \bar{\theta}^{DR} \right] \) it follows that \( \frac{\Delta_{DR-\emptyset}(\theta)}{\theta_r} > 0 \) for all \( \theta \in \left[ \vartheta^{DR}, \bar{\theta}^{DR} \right] \).

**Remark 4** Every \( \theta \in \left[ \vartheta^{D\emptyset}, \vartheta^{DR} \right] \) prefers D\emptyset to \( \emptyset \)

**Proof.** Now assume that \( \theta \in \left[ \vartheta^{D\emptyset}, \vartheta^{DR} \right] \) and we have that 1) type \( \vartheta^{DR} \) prefers DR to \( \emptyset \) by the previous argument, 2) \( \vartheta^{D\emptyset} \) is indifferent between DD and DR, 3) \( \vartheta^{DR} \) strictly prefers D\emptyset to DD (since the type \( \vartheta^{D\emptyset} \) is indifferent between DD and D\emptyset, and all \( \theta, > \vartheta^{D\emptyset} \) prefer D\emptyset to DD). Therefore, it follows that \( \vartheta^{DR} \) prefers D\emptyset to \( \emptyset \).
Let $\Delta_{D\emptyset - \emptyset\emptyset} (\theta) = EU (D\emptyset | \theta) - EU (\emptyset\emptyset | \theta)$ which implies

$$\Delta_{D\emptyset - \emptyset\emptyset} (\theta) = \frac{\hat{P}^{-1} (d)}{2} G \left( \frac{p_r^{\emptyset \emptyset} (\theta)}{p_r^{D\emptyset} (\theta)} \right) \theta_d - \frac{\hat{P}^{-1} (r)}{2} \left( 1 - p_r^{D\emptyset} (\theta) \right) \theta_r$$

and using the implicit function theorem we get

$$\frac{\partial \Delta_{D\emptyset - \emptyset\emptyset} (\theta)}{\partial \theta} = - \frac{\hat{P}^{-1} (r)}{2} \left( 1 - p_r^{D\emptyset} (\theta) \right) - \frac{\hat{P}^{-1} (d)}{2} G \left( \frac{p_r^{\emptyset \emptyset} (\theta)}{p_r^{D\emptyset} (\theta)} \right) < 0$$

$$\frac{\partial^2 \Delta_{D\emptyset - \emptyset\emptyset} (\theta)}{\partial \theta^2} = \left( \frac{\hat{P}^{-1} (r)}{2} G' \left( \frac{p_r^{\emptyset \emptyset} (\theta)}{p_r^{D\emptyset} (\theta)} \right) - \frac{\hat{P}^{-1} (d)}{2} \right) < 0$$

and since $\Delta_{D\emptyset - \emptyset\emptyset} (\theta^{DR}) > 0$ we have that $\Delta_{\emptyset R - \emptyset\emptyset} (\theta) > 0$ for all $\theta \in [\theta^{DR}, \theta^{DR}]$. ■

**Remark 5** Every $\theta \in [\theta^{DR}, \theta^{DR}]$ prefers $\emptyset R$ to $\emptyset\emptyset$

**Proof.** It is analogous to the previous case. ■

This finishes the first part of the Proposition. For the second part recall that, combining the fact that $\emptyset\emptyset$ is not used in equilibrium and Lemma (8), we have $[\theta^{DR}, \theta^{DR}] \subset \Theta_{D\emptyset}$ and $\min \Theta^*_{D\emptyset} = \hat{\theta}^{D\emptyset}$, $[\theta^{DR}, \theta^{DR}] \subset \Theta_{\emptyset R}$ and $\max \Theta^*_{\emptyset R} = \hat{\theta}^{\emptyset R}$, $\Theta_{DR} \subset [\theta^{DR}, \theta^{DR}]$, and $[\theta^{\emptyset R}, 1] = \Theta^*_{DD}$ and $[0, \hat{\theta}^{D\emptyset}] = \Theta^*_{RR}$.

We are ready to show that there is abstention in equilibrium for sufficiently large $v$. In order to do that we only need to show that $\frac{\hat{P}^{-1} (d)}{P^{-1} (r)} \geq \frac{\hat{P}^{-1} (d)}{P^{-1} (r)}$ leads to a contradiction. Assume that $\frac{\hat{P}^{-1} (d)}{P^{-1} (r)} \geq \frac{\hat{P}^{-1} (d)}{P^{-1} (r)}$ and using Lemma (9) we have that there is no abstention in equilibrium and (7) gives that $\frac{\hat{P}^{-1} (d)}{P^{-1} (r)} \geq \frac{\hat{P}^{-1} (d) + \hat{P}^{-1} (d)}{P^{-1} (r)} \geq \frac{\hat{P}^{-1} (d)}{P^{-1} (r)}$ must hold. Recalling the definition of $P^x (\omega) = \sum_{n_R=0}^{\infty} \Pr (n_R, n_R + x | \omega)$ we have that $\frac{\hat{P}^{-1} (d)}{P^{-1} (r)} \geq \frac{\hat{P}^{-1} (d)}{P^{-1} (r)}$ translates into

$$\sum_{x=-1}^{0} \sum_{n_R=0}^{\infty} \Pr (n_R, n_R + x | d) \geq \sum_{x=0}^{1} \sum_{n_R=0}^{\infty} \Pr (n_R, n_R + x | d)$$

$$\sum_{x=-1}^{0} \sum_{n_R=0}^{\infty} \Pr (n_R, n_R + x | r) \geq \sum_{x=0}^{1} \sum_{n_R=0}^{\infty} \Pr (n_R, n_R + x | r)$$

(27)

$$\tilde{P}^1 (\omega) = e^{-v (t_R (\omega) + t_D (\omega))} \left( \sum_{i=0}^{\infty} \frac{(vt_R (\omega))^i}{i!} \frac{(vt_D (\omega))^{i+1}}{i+1} \right) + \sum_{i=0}^{\infty} \frac{(vt_R (\omega))^i}{i!} \frac{(vt_D (\omega))^{i}}{i!}$$

$$\tilde{P}^{-1} (\omega) = e^{-v (t_R (\omega) + t_D (\omega))} \left( \sum_{i=0}^{\infty} \frac{(vt_R (\omega))^{i+1}}{i!} \frac{(vt_D (\omega))^i}{i!} \right) + \sum_{i=0}^{\infty} \frac{(vt_R (\omega))^i}{i!} \frac{(vt_D (\omega))^{i}}{i!}$$

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Following Feddersen and Pesendorfer (1999) we used the modified Bessel function $I_z(x) = \left(\frac{1}{2}\right)x^z \sum_{i=0}^{\infty} \frac{(\frac{x}{2})^i}{i!}$ to get that

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R \mid \omega) = e^{-v(t_R(\omega) + t_D(\omega))} \sum_{i=0}^{\infty} \frac{(vt_R(\omega))^i}{i!} \frac{(vt_D(\omega))^i}{i!}
$$

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R + 1 \mid \omega) = e^{-v(t_R(\omega) + t_D(\omega))} \sum_{i=0}^{\infty} \frac{(vt_R(\omega))^i}{i!} \frac{(vt_D(\omega))^{i+1}}{i+1!}
$$

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R - 1 \mid \omega) = e^{-v(t_R(\omega) + t_D(\omega))} \sum_{i=0}^{\infty} \frac{(vt_R(\omega))^{i+1}}{i+1!} \frac{(vt_D(\omega))^i}{i!}
$$

We need to study the behavior of $e^{-v(t_R(\omega) + t_D(\omega))} I_0 \left(2v \sqrt{t_D(\omega) t_R(\omega)}\right)$. Recall that we are interested in the case where $\nu \to \infty$ so we are going to use that $\lim_{x \to \infty} \frac{x^z}{I_z(x)} = 1$ for $z \in (0, 1)$ which gives that

$$
\frac{e^{-v(t_R(\omega) + t_D(\omega))} I_0 \left(2v \sqrt{t_D(\omega) t_R(\omega)}\right)}{\sqrt{4\pi v t_D(\omega) \sqrt{vt_R(\omega)}}} \approx \frac{e^{-v(t_D(\omega) - t_R(\omega))^2}}{\sqrt{4\pi v t_D(\omega) \sqrt{vt_R(\omega)}}}
$$

which implies that

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R \mid \omega) = \frac{e^{-v(t_D(\omega) - t_R(\omega))^2}}{\sqrt{4\pi v t_D(\omega) \sqrt{vt_R(\omega)}}}
$$

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R + 1 \mid \omega) = \sqrt{\frac{t_D(\omega)}{t_R(\omega)}} \sum_{n_R=0}^{\infty} \Pr(n_R, n_R \mid \omega)
$$

$$
\sum_{n_R=0}^{\infty} \Pr(n_R, n_R - 1 \mid \omega) = \sqrt{\frac{t_R(\omega)}{t_D(\omega)}} \sum_{n_R=0}^{\infty} \Pr(n_R, n_R \mid \omega)
$$

\[\text{(28)}\]
Therefore (27) turns into

\[
\frac{1 + \sqrt{\frac{t_D(d)}{t_D(r)}}}{1 + \sqrt{\frac{t_R(d)}{t_R(r)}}} \geq \frac{1 + \sqrt{\frac{t_D(d)}{t_D(r)}}}{1 + \sqrt{\frac{t_R(d)}{t_R(r)}}}
\]

(29)

Recalling that (5) we have that

\[
t^\sigma_R(\omega) = \xi \eta + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} \Pr(s_r | \omega) f(\theta) d\theta + \int_{\theta \in \Theta_{R,R}} f(\theta) d\theta \right)
\]

\[
t^\sigma_D(\omega) = \xi (1 - \eta) + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} \Pr(s_d | \omega) f(\theta) d\theta + \int_{\theta \in \Theta_{R,D}} f(\theta) d\theta \right)
\]

where \( \Theta^{X,X} \) is the set of voters that supports \( X \in \{R, D\} \) always and \( \Theta^{D,R} \) is defined as in Lemma (9). Note that the fact that \( \text{int} \Theta^{D,R} \neq \emptyset \) in Lemma (9) is independent of the order \( \frac{\tilde{P}^{-1}(d)}{\tilde{P}^{-1}(r)} \geq \frac{\tilde{P}^{-1}(d) + \tilde{P}^{t}(d)}{\tilde{P}^{-1}(r) + \tilde{P}^{t}(r)} \). Using \( p^{DR}_r(\theta) \) as defined in (18) for voter \( \theta \in \Theta_{D,R} \) we have

\[
t^\sigma_R(r) = \xi \eta + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} p^{DR}_r(\theta) f(\theta) d\theta + \int_{\theta \in \Theta_{R,R}} f(\theta) d\theta \right)
\]

\[
t^\sigma_R(d) = \xi \eta + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} (1 - G(p^{DR}_r(\theta))) f(\theta) d\theta + \int_{\theta \in \Theta_{R,R}} f(\theta) d\theta \right)
\]

\[
t^\sigma_D(r) = \xi (1 - \eta) + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} (1 - p^{DR}_r(\theta)) f(\theta) d\theta + \int_{\theta \in \Theta_{D,D}} f(\theta) d\theta \right)
\]

\[
t^\sigma_D(d) = \xi (1 - \eta) + (1 - \xi) \left( \int_{\theta \in \Theta_{D,R}} G(p^{DR}_r(\theta)) f(\theta) d\theta + \int_{\theta \in \Theta_{R,R}} f(\theta) d\theta \right)
\]

and since we assume that \( p^{DR}_r(\theta) \geq 1 - G(p^{DR}_r(\theta)) \) we must have that \( t^\sigma_R(r) > t^\sigma_R(d) \) and \( t^\sigma_D(d) > t^\sigma_D(r) \) when \( \Theta^{D,R} \neq \emptyset \) (since types are collecting different information one must verify the inequality \( G(p_d) \geq (1 - p_d) \) strictly) which contradicts the last line of (29).

Since \( \frac{\tilde{P}^{-1}(d)}{\tilde{P}^{-1}(r)} < \frac{\tilde{P}^{t}(d)}{\tilde{P}^{t}(r)} \), Lemma (9) imply that there is abstention by either informed voters using \( D\emptyset \) or \( \emptyset R \) or rationally ignorant voters that abstain (strategy \( \emptyset \emptyset \)).
Proof of Theorem (2). The proof is structured in a series of Lemmas. First note that existence of "an" equilibrium follows as a corollary of (1) so we have that in equilibrium we must have that \( \frac{P^{-1}(d)}{P^{-1}(r)} < \frac{P^1(d)}{P^1(r)} \). The first Lemma shows that condition 1) and 2) in the Theorem are equivalent. Once that equivalence is established we show that the fixed point problem in Theorem (1) can be reduced from a simplex of four elements to a simplex of 2 elements; existence of an equilibrium with the described characterization follows by a slight modification of Theorem (1).

First we show an equivalence result between pivotal probabilities and the probabilities of a vote

**Lemma 10** If the electorate and the information technology are symmetric, for every \( v \), the following two conditions are equivalent

1) \( t_D(d) = t_R(r) \), and \( t_D(r) = t_R(d) \)
2) \( P^0(d) = P^0(r) \), \( P^1(r) = P^{-1}(d) \), and \( P^1(d) = P^{-1}(r) \)

**Proof.** We first prove the direction 1) implies 2). In this case symmetry of the electorate and the information technology is not used.

Using (9) we have that

\[
\Pr(x, y \mid d) = \frac{(v)^{x+y}}{x!y!} (t_R(d))^x (t_D(d))^y e^{-v(x+y)}
\]

\[
= \frac{(v)^{x+y}}{x!y!} (t_D(r))^x (t_R(r))^y e^{-v(x+y)} = \Pr(y, x \mid d)
\]

where the second line follows by the assumption \( t_R(d) = t_D(r) \) and \( t_D(d) = t_R(r) \). Using that \( P^x(\omega) = \sum_{n_R=0}^{\infty} \Pr(n_R, n_R + x \mid \omega) \) it follows that \( P^0(d) = P^0(r) \) and

\[
P^1(\omega) = \sum_{n_R=0}^{\infty} \Pr(n_R, n_R + 1 \mid \omega)
\]

\[
= \sum_{n_R=0}^{\infty} \Pr(n_R + 1, n_R \mid -\omega) = P^{-1}(\omega)
\]

Now we prove the direction 2) implies 1). Here is where symmetry matters.

First note that \( P^0(d) = P^0(r) \), \( P^1(r) = P^{-1}(d) \) and \( P^1(d) = P^{-1}(r) \) implies that \( \frac{P^1(d)+P^{-1}(d)}{P^1(r)+P^{-1}(r)} = 1 \), \( \frac{P^{-1}(d)}{P^{-1}(r)} = \frac{P^1(r)}{P^1(d)} \) which gives, using (18), that

\[
G'(P_r^{DR}(\theta)) = -\frac{\theta_r}{\theta_d}
\]

\[
G'(P_r^{P0}(\theta)) = -\frac{\theta_r}{\theta_d} \frac{P^{-1}(r)}{P^{-1}(d)} = -\frac{\theta_r}{\theta_d} \frac{P^1(d)}{P^1(r)}
\]

Using now that the information technology is symmetric we have that \( G'(G(p)) G'(p) = 1 \) we have
that

\[
G' \left( p_r^{\theta \theta} (\theta) \right) = -\frac{1}{\frac{1-\theta}{\theta} p_1 (d) p_1 (r)}
= -\frac{1}{\frac{1-\theta}{\theta} p_1 (r) p_1 (d)} = G' \left( p_r^{\theta R} (1 - \theta) \right)
\]

so symmetry of the information technology gives that \( G' \left( p_r^{\theta R} (1 - \theta) \right) G' \left( p_r^{\theta \theta} (\theta) \right) = 1 \) and since \( G' (p) G' (p) = 1 \) we must have that

\[
\begin{align*}
p_r^{\theta R} (\theta) &= G \left( p_r^{\theta \theta} (1 - \theta) \right) \\
p_r^{\theta \theta} (\theta) &= G \left( p_r^{\theta R} (1 - \theta) \right) \\
p_r^{\theta D} (\theta) &= G \left( p_r^{\theta D} (1 - \theta) \right) 
\end{align*}
\]

where the last line follows by noting that \( G' \left( p_r^{\theta D} (\theta) \right) = -\frac{1}{\frac{1-\theta}{\theta} p_1 (r) p_1 (d)} = \frac{1}{G' (p_r^{\theta D} (1 - \theta))} \) when we use

that \( \frac{p_1 (d) + r_1 (d)}{p_1 (r) + r_1 (r)} = 1 \).

Let \( p_1 \) be such that \( H_d (p_1) = 0 \) so we have that

\[
\begin{align*}
0 &= H_d (p_1) \\
0 &= G (p_1) + (1 - p_1) G' (p_1)
\end{align*}
\]

and let \( G (p_1) = p_2 \) so we have that \( G' (p_1) G' (p_2) = 1 \) and \( p_1 = G (p_2) \) which implies that

\[
\begin{align*}
0 &= p_2 + (1 - G (p_2)) \frac{1}{G' (p_2)} \\
0 &= G' (p_2) p_2 + (1 - G (p_2)) \\
0 &= H_r (p_2)
\end{align*}
\]

Using the definition of \( p \) and \( \bar{p} \) (\( H_r (p) = 0 = H_d (\bar{p}) \)) we have that \( G (p) = p \), \( G' (\bar{p}) = \frac{1}{G' (p)} \) and \( \bar{p} = G (p) \).

Recalling now the definition of cutoffs (25) we have that

\[
\begin{align*}
-G' (p) &= \theta^{D \theta} \frac{\bar{p} - 1 (r)}{1 - \theta^{D \theta}} \\
\frac{1}{G' (\bar{p})} &= \theta^{D \theta} \frac{\bar{p} - 1 (r)}{1 - \theta^{D \theta}} \\
-G' (\bar{p}) &= \theta^{D \theta} \frac{\bar{p} - 1 (r)}{1 - \theta^{D \theta}}
\end{align*}
\]

and since \( \frac{\theta^{D \theta}}{1 - \theta^{D \theta}} \frac{\bar{p}^{1} (r)}{\bar{p}^{1} (d)} = G' (\bar{p}) \) we have that \( \theta^{D \theta} = 1 - \theta^{D \theta} \). Using now that \( G' \left( p_r^{\theta \theta} (\theta) \right) = \)
Using (17) and (16) we have that
\[
\frac{1}{\sigma'(\rho_0^R(1-\theta))} \text{ we have that } \frac{\bar{\sigma}^0 \bar{P}^{-1}(r)}{1-\bar{\sigma}^0 \bar{P}^{-1}(d)} = -G'(\rho^D) \text{ implies }
\]
\[
\frac{\bar{\sigma}^0 \bar{P}^{-1}(r)}{1-\bar{\sigma}^0 \bar{P}^{-1}(d)} = -G'(\rho^0 R (\bar{\sigma}^0))
\]
\[
\frac{\bar{\sigma}^D \bar{P}^{-1}(r)}{1-\bar{\sigma}^D \bar{P}^{-1}(d)} = -G'(\rho^0 R (1-\bar{\sigma}^D))
\]
\[
\frac{\bar{\sigma}^D \bar{P}^{-1}(r)}{1-\bar{\sigma}^D \bar{P}^{-1}(d)} = -G'(\rho^0 R (1-\bar{\sigma}^D))
\]
so \(1-\bar{\theta}^D = \bar{\theta}^D \) by (25). Using again that \(G'(\bar{\rho}) = \frac{1}{G'(\rho)} \) and that we must have that \(\frac{\bar{P}^{-1}(d)+\bar{P}^{-1}(d)}{\bar{P}^{-1}(d)+\bar{P}^{-1}(d)} = 1 \) it follows that
\[
\frac{\bar{\sigma}^D}{1-\bar{\sigma}^D} = -G'(\rho)
\]
\[
\frac{\bar{\sigma}^D}{1-\bar{\sigma}^D} = -G'(\rho)
\]
and since \(\frac{\bar{\sigma}^D}{1-\bar{\sigma}^D} = -G'(\rho) \) by (25) we have that \(1-\bar{\sigma}^D = \bar{\theta}^D \). Using Lemma (8) we have that
\[
\bar{\theta}^D < \frac{1}{2} < \bar{\theta}^D
\]
and
\[
1-\bar{\sigma}^D < \frac{1}{2} < \bar{\sigma}^D
\]
Note that for \(\bar{\theta}^D = \frac{1}{2} \) we have that (16) gives
\[
EU \left( \theta R, \rho^0 R \left( \frac{1}{2} \right) | \frac{1}{2} \right) = \tilde{V} \left( \frac{1}{2} \right) + \frac{\bar{P}^{-1}(r) + \bar{P}^{-1}(d) \rho^0 R \left( \frac{1}{2} \right)}{2} - \frac{\bar{P}^{-1}(d) \left( 1 - G \left( \rho^0 R \left( \frac{1}{2} \right) \right) \right)}{2}
\]
where the second line follows by (30), the third line follows by the assumption \(P^0(d) = P^0(r)\), \(P^1(r) = P^{-1}(d)\) and \(P^1(d) = P^{-1}(r)\), and the final equivalence follows by definition of (17). Using (17) and (16) we have that
\[
\frac{\partial \tilde{V} \left( \theta R, \rho^0 R \left( \frac{1}{2} \right) \right)}{\partial \theta} - \frac{\partial \tilde{V} \left( \theta \right)}{\partial \theta} < 0 \text{ and } \frac{\partial \tilde{V} \left( \theta R, \rho^0 R \left( \frac{1}{2} \right) \right)}{\partial \theta} - \frac{\partial \tilde{V} \left( \theta \right)}{\partial \theta} > 0
\]
which implies that $\frac{\partial \Delta_{\theta_{DR-\theta} R} (\theta_r)}{\partial \theta_r} > 0$. Therefore, for $\theta_r < \frac{1}{2}$ the relevant comparison with $DR$ is $D\emptyset$ and for $\theta_r > \frac{1}{2}$ the relevant comparison with $DR$ is $\emptyset R$.

Using (15), (16) and (16) we have

$$
\Delta_{DR-\emptyset R} (\theta) = \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} G (p_{r}^{DR}(\theta)) \theta_d - \frac{\bar{P}^{-1}(r) + \bar{P}^{1}(r)}{2} (1 - p_{r}^{DR}(\theta)) \theta_r
$$

$$
\Delta_{DR-D\emptyset} (\theta) = \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} p_{r}^{DR}(\theta) \theta_d - \frac{\bar{P}^{-1}(r) + \bar{P}^{1}(r)}{2} (1 - G (p_{r}^{DR}(\theta))) \theta_r
$$

and using (30) we get

$$
\Delta_{DR-\emptyset R} (1-\theta) = \left( \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} p_{r}^{DR}(\theta) - \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} p_{r}^{D\emptyset}(\theta) \right) \theta
$$

$$
+ \left( \frac{\bar{P}^{1}(r)}{2} (1 - G (p_{r}^{D\emptyset}(\theta))) \theta - \frac{\bar{P}^{-1}(r) + \bar{P}^{1}(r)}{2} (1 - G (p_{r}^{DR}(\theta))) \theta \right) (1 - \theta)
$$

$$
= \left( \frac{\bar{P}^{-1}(r) + \bar{P}^{1}(r)}{2} p_{r}^{DR}(\theta) - \frac{\bar{P}^{-1}(r) + \bar{P}^{1}(r)}{2} p_{r}^{D\emptyset}(\theta) \right) \theta
$$

$$
+ \left( \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} (1 - G (p_{r}^{D\emptyset}(\theta))) \theta - \frac{\bar{P}^{-1}(d) + \bar{P}^{1}(d)}{2} (1 - G (p_{r}^{DR}(\theta))) \theta \right) (1 - \theta)
$$

$$
= \Delta_{DR-D\emptyset} (\theta)
$$

so we have $\Delta_{DR-D\emptyset} (\theta) > (\theta) > (\theta) > 0$ which is useful since we know that there are some voters that strictly prefer $DR$. Using that $\Theta_{DR}^* \subset \left[ \theta_{D\emptyset}^*, \theta_{D\emptyset}^* \right]$ we have that

$$
\theta \in \Theta_{D\emptyset}^* \iff (1-\theta) \in \Theta_{\emptyset R}^*
$$

$$
\theta \in \Theta_{DR}^* \iff (1-\theta) \in \Theta_{DR}^*
$$

and by symmetry of $F$ we have that $\theta \in \Theta_{DD}^* \iff (1-\theta) \in \Theta_{RR}^*$. Now we are ready to calculate
which are all continuous in the elements of \( t \).

First note that using (22) we have that \( t^D_D (r) \) is equivalent to

\[
\frac{t^D_D (r) - \frac{1}{2} \eta}{1 - \eta} = \int_{\theta \in \Theta_{D_D}} f (\theta) + \int_{\theta \in \Theta_{D_R}} \left( 1 - p^{DR}_D (\theta) \right) f (\theta) + \int_{\theta \in \Theta_{D_R}} \left( 1 - p^{D_R}_D (\theta) \right) f (\theta)
\]

where the first line follows by symmetry of \( F \), the second follows since \( \Delta_{DR-D_R} (\theta) > (\leq) 0 \) iff \( \Delta_{DR-D_R} (1 - \theta) > (\leq) 0 \), and the fourth line follows by information equivalence and (22); \( t^D_D (d) = t^D_R (d) \) follows analogously by noting

\[
\frac{t^D_D (d) - \frac{1}{2} \eta}{1 - \eta} = \int_{\theta \in \Theta_{D_D}} f (\theta) + \int_{\theta \in \Theta_{D_R}} p^{DR}_D (\theta) f (\theta) + \int_{\theta \in \Theta_{D_R}} p^{D_R}_D (\theta) f (\theta)
\]

This concludes the proof of the Lemma. Now using Theorem (1) we need to focus only on a pair \( \tilde{P} = \left\{ \tilde{P}^1 (d), \tilde{P}^{-1} (d) \right\} \) since we know that \( \left( \tilde{P}^1 (d), \tilde{P}^{-1} (d) \right) = \left( \tilde{P}^1 (r), \tilde{P}^{-1} (r) \right) \). Since \( \min \{ x^D_D (d), x^D_D (d) \} \geq \frac{1}{2} \min \{ \eta, 1 - \eta \} \) we have that (9) is strictly positive for each \( n_D \) and \( n_R \) which implies that there is some \( \delta \in (0, 1) \) such that \( \tilde{P} \in [\delta, 1 - \delta]^2 \). Let \( \tilde{\Gamma} : [\delta, 1 - \delta]^2 \to [\delta, 1 - \delta]^2 \) be constructed in the following way. First, for a given element \( z \in [\delta, 1 - \delta]^2 \) we define \( p^u_{d,vv} (\theta; z) \), \( EU^* (u_{d,vv}; z | \theta) \) for each strategy \( u_{d,vv} \) and hence the set of types that use these strategies \( \Theta^*_{u_{d,vv}} (z) \), which are all continuous in the elements of \( z \). Now we use these objects, (33) and (34) to define \( x^D (d; z) \) for \( x = D, R \) which is also continuous in \( z \). Using (9) we let

\[
\tilde{\Gamma} (z) = \left\{ \sum_{n_R = 0}^{\infty} \sum_{n_R = 0}^\infty \Pr (n_R, n_R + h | d), \sum_{n_R = 0}^{\infty} \sum_{n_R = 0}^\infty \Pr (n_R, n_R + h | d) \right\}
\]

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and Brouwer’s fixed point theorem gives that there is some $z^*$ such that $\hat{\Gamma} (z^*) = z^*$.

This concludes the proof of the first part and the second part. For the third part we use the properties of the Poisson distribution which gives again that

$$\frac{\tilde{P}^1 (d)}{P^1 (r)} = \frac{\sum_{x=0}^{\infty} \Pr (n_R, n_R + x \mid d)}{\sum_{x=0}^{\infty} \Pr (n_R, n_R + x \mid r)}$$

and (28) we get

$$\lim_{v \to \infty} \frac{\tilde{P}^1 (d)}{P^1 (r)} = \frac{e^{-(\sqrt{t_D (d)} - \sqrt{t_R (d)})^2}}{4\pi \sqrt{t_D (d) t_R (d)}} \left( 1 + \sqrt{\frac{t_D (d)}{t_R (d)}} \right)$$

$$= \frac{1 + \sqrt{t_D (d)}}{1 + \sqrt{t_R (d)}} = \sqrt{\frac{t_D (d)}{t_R (d)}}$$

where the second line follows since $t_R (d) = t_D (r)$ and $t_R (r) = t_D (d)$.

For the final part we need to show that $\Delta_{DR-\emptyset R} \left( \frac{1}{2} \right) > 0$ and $\Delta_{DR-D\emptyset} \left( \frac{1}{2} \right) > 0$. Using (31) we have

$$\Delta_{DR-\emptyset R} (\theta) = \frac{\tilde{P}^{-1} (d) + \tilde{P}^1 (d)}{2} G (p^{DR}_r (\theta)) \theta_d - \frac{\tilde{P}^{-1} (r) + \tilde{P}^1 (r)}{2} \left( 1 - p^{DR}_r (\theta) \right) \theta_r$$

$$\Delta_{DR-D\emptyset} (\theta) = \frac{\tilde{P}^{-1} (r) + \tilde{P}^1 (r)}{2} p^{DR}_r (\theta) \theta_r - \frac{\tilde{P}^{-1} (d) + \tilde{P}^1 (d)}{2} \left( 1 - G (p^{DR}_r (\theta)) \right) \theta_d$$

and using (30) we get

$$\Delta_{DR-\emptyset R} (\theta) + \Delta_{DR-D\emptyset} (\theta) = \frac{\tilde{P}^{-1} (d) + \tilde{P}^1 (d)}{2} \left( p^{DR}_r (1 - \theta_r) - \frac{1}{2} \right) \theta_d$$

$$+ 2 \frac{\tilde{P}^{-1} (r) + \tilde{P}^1 (r)}{2} \left( p^{DR}_r (\theta_r) - \frac{1}{2} \right) \theta_r$$

$$- \frac{\tilde{P}^1 (d)}{2} p^{D\emptyset}_r (1 - \theta) \theta_d - \frac{\tilde{P}^{-1} (r)}{2} p^{D\emptyset}_r (\theta) \theta_r$$

$$+ \frac{\tilde{P}^1 (r)}{2} \left( 1 - G (p^{D\emptyset}_r (1 - \theta)) \right) \theta_r + \frac{\tilde{P}^{-1} (d)}{2} \left( 1 - G (p^{D\emptyset}_r (\theta)) \right) \theta_d$$

and

$$\Delta_{DR-\emptyset R} (\theta) + \Delta_{DR-D\emptyset} (\theta) = \frac{\tilde{P}^{-1} (d) + \tilde{P}^1 (d)}{2} \left( p^{DR}_r (1 - \theta_r) - \frac{1}{2} \right) \theta_d$$

$$+ 2 \frac{\tilde{P}^{-1} (r) + \tilde{P}^1 (r)}{2} \left( p^{DR}_r (\theta_r) - \frac{1}{2} \right) \theta_r$$

$$- \frac{\tilde{P}^1 (d)}{2} p^{D\emptyset}_r (1 - \theta) \theta_d - \frac{\tilde{P}^{-1} (r)}{2} p^{D\emptyset}_r (\theta) \theta_r$$

$$+ \frac{\tilde{P}^1 (r)}{2} \left( 1 - G (p^{D\emptyset}_r (1 - \theta)) \right) \theta_r + \frac{\tilde{P}^{-1} (d)}{2} \left( 1 - G (p^{D\emptyset}_r (\theta)) \right) \theta_d$$

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and when \( \theta_r = \frac{1}{2} \) we have

\[
2 \frac{\Delta_{DR-D0}(\frac{1}{2})}{P^1(r) + P^{-1}(d)} = 2 \frac{\Delta_{DR-\theta R}(\frac{1}{2})}{P^1(r) + P^{-1}(d)} = 2p_r^{DR} \left( \frac{1}{2} \right) - \frac{1}{2} - \frac{p_r^{D0} \left( \frac{1}{2} \right) + G \left( p_r^{D0} \left( \frac{1}{2} \right) \right)}{2}
\]

since \( \Delta_{DR-D0}(\frac{1}{2}) = \Delta_{DR-\theta R}(\frac{1}{2}) \). Note that by concavity of \( G \) we have that

\[
G \left( p_r^{D0} \left( \frac{1}{2} \right) \right) > p_r^{DR} \left( \frac{1}{2} \right) + G' \left( p_r^{D0} \left( \frac{1}{2} \right) \right) \left( p_r^{D0} \left( \frac{1}{2} \right) - p_r^{DR} \left( \frac{1}{2} \right) \right)
\]

\[
G \left( p_r^{D0} \left( \frac{1}{2} \right) \right) < p_r^{DR} \left( \frac{1}{2} \right) - \left( p_r^{D0} \left( \frac{1}{2} \right) - p_r^{DR} \left( \frac{1}{2} \right) \right)
\]

so we have that

\[
2 \frac{\Delta_{DR-\theta R}(\frac{1}{2})}{P^1(r) + P^{-1}(d)} > p_r^{DR} \left( \frac{1}{2} \right) - \frac{1}{2} > 0
\]

and by continuity of \( \Delta_{DR-\theta R}(\theta_r) \) we get the non empty interior. ■

**Proof of Proposition (2).** Since \( p_d^{\theta R} > p_d^{DR} \) we have that \( p_d^{\theta R} (\theta_1) > p_d^{DR} (\theta_1) \) and since \( p_d^{D0} < p_d^{DR} \) we have \( p_d^{D0} (\bar{\theta}_2) < p_d^{DR} (\bar{\theta}_2) \). Since \( p_d^{DR} (\bar{\theta}_1) < p_d^{DR} (\bar{\theta}_2) \) we know that there is overlap. ■