Wage Setting in a Dispersed Information DSGE

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Abstract

I develop a general equilibrium model in which economically realistic business cycle responses to shocks result from information dispersion as opposed to multi-period price rigidities as in Calvo (1983). I introduce a labor market with wage-setting workers and a coherent definition of unemployment in a model with dispersed information. The model features persistent monetary policy non-neutrality, despite prices being set every period. Moreover, the model can explain the price puzzle and also generates a negative relationship between wage inflation and unemployment (i.e., a Phillips Curve).

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1 Introduction

Economic models featuring dispersed information have enjoyed increasing popularity lately, due to their ability of matching stylized economic facts with no need for ad-hoc exogenous frictions, above all multi-period Calvo price setting (Calvo (1983)[9]). Rather, this class of models builds on the more reasonable idea that each agent can only imperfectly assess the state of the entire economy.

In this paper, I introduce a labor market with wage-setting agents in a DSGE model in which information is dispersed and the dispersion is not resolved on a period-by-period basis. I primarily build on a set of models put forth by Lorenzoni (2009)[23] and Mendes (2007)[27]. Both restrict agents to work exclusively in their own firms, thus preventing the study of the labor market over the business cycle and imposing a strong restriction on the pricing behavior of firms, as their marginal cost is not determined by the market but by agents' preferences. Relaxing the assumption of self-production is a step toward a more realistic setup.

The introduction of a labor market, and consequently of a market wage (or set of wages) makes the treatment of firms more in line with standard representative-agent DSGE's, in which costs depend on market conditions as opposed to the disutility of working[1].

At the same time, the fact that agents set wages (and prices) under dispersed information introduces a certain degree of sluggishness in the economy, resulting in monetary policy non-neutrality, with no need for exogenously preventing changes in prices and wages for a random number of periods.

In particular, confusion between different types of shocks allows to generate the price puzzle documented in some VAR literature (e.g. Christiano, Trabandt and Walentin (2010)[12]) and also discussed in other dispersed information models like Mendes (2007)[27] or Melosi (2011)[26].

Moreover, the effect on output precedes that on inflation, as the former bottoms out earlier following an increase in the monetary policy interest rate.

On the other hand, the effect of a technology shock on inflation is immediate, in line with VAR

[1] While in a representative agent setup, the tie between marginal utilities and wages is immediate, in a model with heterogenous agents it is obviously not so, with implications that extend also to the inference problem faced by the agents.

evidence (see again Christiano, Trabandt and Walentin (2010)[12]).

As mentioned above, having a well defined labor market, also allows me to define and study variables such as participation and unemployment along the lines of Gali' (2009)[14].

While unemployment, as introduced by Gali' (2009)[14], is due to monopolistic competition in the labor market, its comovements with nominal variables over the business cycle clearly depend on the frictions considered in the model.

My simple framework allows me to address whether the negative relationship between wage inflation and unemployment (commonly known as Phillips Curve) holds in a model in which wages and prices are set under dispersed information, but are otherwise not prevented from adjusting every period. And indeed my calibration exercise produces a negative correlation between unemployment and wage inflation, in particular in response to monetary policy shocks.

Finally, from a more technical standpoint, I address the issue of wealth heterogeneity, that inevitably arises when one moves away from representative-agent and complete-market models and greatly complicates the analysis.

I do so endogenizing the agents' discount factor[2] and introducing a tax on bond-holdings.

These two elements, combined, guarantee that the cross section cross-section distribution of bond holdings is well defined. In particular, the process for bond holdings is highly persistent but stationary, so that aggregation can be safely carried out and the approximation around the steady-state employed.

The rest of the paper presents a review of the literature in Section 2, followed by the description of the model setup in Section 3 and a sketch of the solution technique in the fourth paragraph. The discussion of the main findings and the conclusions, complete the paper.

[2]In a similar fashion to that used by Schmitt-Grohe' and Uribe (2003)[29] who in turn take inspiration from Uzawa (1968) [34].

2 Related Literature

My work draws from two separate strands of the literature: New-Keynesian DSGE's and dispersed information models.

For what concerns the former, Erceg, Henderson and Levin (2000)[13] first introduced workers with wage-setting power in a macroeconomic general equilibrium model in a way that has become standard (see for example Christiano, Eichenbaum and Evans (2005)[11]). It hinges on the assumptions that workers provide differentiated labor services in an imperfectly competitive market (á la Dixit and Stiglitz (1977)[31]), which results in their ability to set wages. A key ingredient in these models is, however, a Calvo friction that allows workers to set wages only at randomly determined intervals. Recently (beginning with Woodford (2001)[37] and building on seminal works such as Lucas (1972)[19] among others), a host of scholars have been trying to propose models that generate realistically sluggish adjustments of economic variables with no need for Calvo frictions.

A number of recent papers, especially Angeletos and La'O (2008, 2009, 2009, 2011)[1][3][2][4], Mendes (2007)[27], and Lorenzoni (2009, 2009)[23][22], embodied Woodford's insight in macro-flavored general equilibrium models. While a lot of attention has been devoted to price frictions, above all by Mendes (2007)[27], the only papers that introduced a labor market are those by Angeletos and La'O. In their setup, however, the labor market is Walrasian so agents do not set wages and, more importantly, all information dispersion is resolved within each period. While this makes their models relatively easy to solve, it prevents them from generating any persistent response of economic variables to shocks as their model is basically a frictionless Walrasian framework except within the period.

A separate, although related, strand of the literature focuses on rational inattention, i.e. an endogenous information structure, to explain sluggish responses (see Maćkoviak and Wiederholt (2009, 2010)[24][25] and Paciello and Wiederholt (2011)[28] among others). Maćkoviak and Wiederholt (2010)[25] also features a non-Walrasian labor market, yet unemployment is not considered.

Also, a recent paper by Venkateswaran (2011) [35], introduces search frictions in a model with dispersed information. However, he does not study monetary policy in his model which seems more closely geared towards assessing relative volatilities of labor market variables[3].

Finally, a recent work by Melosi (Melosi (2011)[26]) also adresses similar questions, in particular with reference to the price puzzle. However, his model features a competitive labor market and a Calvo lottery (for price setting) that, based on his estimation, implies that firms have about one chance in two to re-optimize prices in any given period.

In the end, the two papers that try to make the most realistic macro-model out of a dispersed information setup are Mendes (2007)[27] and Lorenzoni (2009)[23]. Lorenzoni (2009)[23], besides dispersed information, also imposes a Calvo friction in price setting so basically his results are driven by the combined effects of Calvo and information frictions[4]. While Mendes (2007)[27] dispenses from multi-period Calvo price setting, he (as well as Lorenzoni 2009[23]) assumes that workers can only work in their own firm, thus disregarding the economic interactions that occur in the labor market. This results in tying the labor disutility with price setting, which is not standard in full information models where typically firms are assumed to maximize profits given costs that are equilibrium prices on the factor markets.

An important contribution of this work lies in introducing a standard labor market setup[5] in a state-of-the-art dispersed information general equilibrium framework. By this, I mean a model in which information dispersion lasts beyond one period and agents are not assumed to disregard some information (e.g. prices) they learn in the course of their trades.

This latter aspect is particularly important in that the present analysis does not require what Angeletos and La'O (2011) [4] refer to as "schizophrenia" which usually amounts to assuming that either information is not shared within the firm or that, which is essentially equivalent, some agents

[3] The presence of capital and of complete financial markets Venkateswaran (2011)[35] make his analysis different relative to the one in this paper.

[4] By Calvo friction I mean a setting in which firms do not get a chance to update prices in every period. While in my case they have to set prices (and wages) at the beginning of the period, i.e. when the information at their disposal is relative to the previous period with the exception of their productivity, they can do so in every period. You could think of my setting as one in which the calibration of the Calvo friction assigns probability one to updating in every period. It is also worth noticing, that the non-dispersed information version of this model would have some similarities to the New Classical Model described in chapter 3 of Woodford (2003)[38] in which prices are set based on the previous period's information, the key difference being that I assume current productivity to be known.

[5] Standard in representative agent's macro DSGE's

disregard some valuable information.

For the two above conditions to hold at the same time, it has to be the case that the prices one observes do not perfectly reveal the state of the economy, in the spirit of Lorenzoni (2009). That assumption here extends to all the observables in the model, which are assumed to embed an idiosyncratic component, so that knowing, say, the wages paid out to we workers hired by the family-run firm does only imperfectly reveal the aggregate wage level.

In introducing unemployment, I tap into a growing set of papers that try to embed it into otherwise standard new Keynesian models with no information imperfections. They include Gertler, Sala and Trigari (2008)[17], Gali' (2009)[14] recently published as Gali' (2011)[15], Gali', Smets and Wouters (2011)[16], as well as Christiano, Trabandt and Walentin (2010)[12]. For its tractability, I chose to adapt Gali' (2009)[14] to my setting. Gali' (2009)[14] interprets hours worked as the number of family members hired. This makes it particularly parsimonious while still allowing one to discuss such concepts as participation and unemployment. A drawback of this approach is quantitative, in that, in absence of some preference shock, it tends to make participation in the labor market too sharply dependent on variations in consumption level. At this stage, though, this shortcoming is compensated by its ability of keeping the model tractable even in the context of dispersed information.

Finally, as mentioned above, I make the discount factor endogenous on the savings of each family. In doing so, I take inspiration from by Schmitt-Grohe and Uribe (2003)[29] which addresses a similar problem in the context of a small-open economy and indirectly on Uzawa (1968)[34]. The main difference is that in this context the variable determining the value of the discount factor is savings as opposed to consumption. One of the benefits of this assumption is that in every period the average discount factor (across agents) is equal to its unconditional mean.

This improves upon Lorenzoni (2009)[23], and also on Mendes (2007)[27] which assumes that all wealth heterogeneity is wiped out after a fixed number of periods via a taxation scheme that appears to require the government to know all the idiosyncratic shocks that hit every agent prior to that cutoff date.

3 The Model

There is a continuum of islands $h \in [0,1]$. On each island lives a family which provides specialized labor of type h and owns the firm producing good h. Families can save in a non-state-contingent claim B_{ht}. Hence, families make three decisions each period:

- Consumption-Saving
- Price Setting
- Wage Setting, in the spirit of Erceg, Henderson, Levin (2000)[13], except they are free to re-set their wages in every period.

Decision making is at the household level but each household is made up of a continuum of agents that differ only in their disutility of providing labor effort, along the lines of Gali' (2009)[14]. This enables me to treat what would otherwise be a number of hours worked as the number of family members that worked, thus allowing the discussion of unemployment and participation.

3.1 Households

On island h lives a family that maximizes the following utility function:

$$E_{ht} \sum_{s=0}^{\infty} \beta_{h,t+s}^{s} \left(\frac{C_{h,t+s}^{1-\gamma}}{1-\gamma} - \chi_{h,t+s} \frac{N_{h,t+s}^{1+\sigma}}{1+\sigma} \right) \tag{1}$$

where $C_{h,t+s}$ is the consumption aggregator and $N_{h,t+s}$ represents the number of family members employed, while $\beta_{h,t+s}$ is the endogenous discount factor and $\chi_{h,t+s}$ a purely exogenous and idiosyncratic preference shock[6].

[6] In log-linear terms it is a simple idiosyncratic AR(1) process:

$$\widehat{\chi}_{ht} = \varepsilon_{ht}^{\chi}$$

$$\varepsilon_{ht}^{\chi} = \rho_{\varepsilon^{\chi}} \varepsilon_{h,t-1}^{\chi} + u_{ht}^{\varepsilon^{\chi}}$$

The consumption aggregator follows a standard definition:

$$C_{ht} \equiv \left[\int_{J_{ht}} C_{jht}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

Where C_{jht} is the quantity of good j consumed on island h and ϵ governs the elasticity of substitution across goods. The only difference, relative to the standard Dixit-Stiglitz case, is that agents are assumed to only consume a subset J_{ht} of the goods available on the market to prevent full revelation in the spirit of Lorenzoni (2009)[23].

Just as in Gali' (2009)[14], family members differ only in their disutility of working. The family meets the demand for labor of type h with those family members whose disutility of working is lower, resulting in the following definition for what is commonly thought of as hours worked:

$$\frac{N_{ht}^{1+\sigma}}{1+\sigma} = \int_0^{N_{ht}} i^{\sigma}\ di \tag{3}$$

Where i is the index of different family members of household h.

Based on this assumption, I can define participation simply as:

$$E_{ht}\left[\frac{W_{ht}}{P_{ht}} C_{ht}^{-\gamma}\right] = E_{ht}\left[\chi_{ht} {N_{ht}^{*}}^{\sigma}\right] \tag{4}$$

Where the marginal worker N_{ht}^{*} is the one for which the marginal benefit (for the family) of working (left-hand side) equals the marginal disutility of doing so (right-hand side). All the workers for which the first term is larger than the second, are considered part of the labor force. Because of the wage-setting power, however, it will generally be the case that $N_{ht} < N_{ht}^{*}$, so that unemployment arises.

The other key element of the utility function is the endogenous discount factor, which deserves closer attention in that it is crucial to guarantee stationarity of the wealth distribution in a context with heterogeneous agents and incomplete markets such as this.

In general, this class of models, tend to produce a non-stationary autoregressive coefficient in the log-linear equation governing the process for savings, which results from the fact that the steady-state interest rate equals $1/\beta > 1$. This results in an explosive distribution of savings which is problematic for at least two reasons.

Firstly, it makes the discussion of unconditional moments of unreliable. Secondly, it makes the log-linear approximation a very poor benchmark because it is evident that once shocks start to hit the economy the is no force that keeps the economy from wandering away from the original steady-state, one in which savings are zero for all agents.

This problem is somewhat similar to one addressed in international economics when it comes to small open economies in which each country takes the world interest rate as given. Schmitt-Grohe' and Uribe (2003)[29] propose a handful of different methods to work around this issue and show that they deliver quantitatively similar results.

Here, I adapt one of them that is particularly parsimonious and yet produces well behaved bond holdings in a heterogeneous-agent setting.

It hinges on endogenizing the the discount factor, in particular making it dependent on bond holdings. I assume the following simple functional form:

$$\beta_{ht} = \beta_0 e^{\psi_1 B_{h,t-1}} \quad \psi_1 < 0 \tag{5}$$

where β_0 is to be thought of as the usual discount factor, while the second term makes the discount factor decrease when agents are savers and increase when agents borrow ($\psi_1 < 0$).

Making savers a bit more impatient and borrowers more willing to sacrifice present consumption reduces the dispersion of wealth to the point of making it stationary.

Please note that, looking at (5), it is clear that β_0 is the "average" discount factor in every period, and also the one that prevails in steady state, while parameter ψ_1 guarantees the required curvature. In particular, if $\psi_1 = 0$ we are in the standard case. Here, I will set ψ_1 to a small negative value so that it is not a big departure from the baseline case while at the same time reducing the incentives of big savers to keep increasing their saving, the opposite holding for large borrowers.

Notice that if this incentive is to be effective, island-level bonds have to be in the argument of the function determining the discount factor. In fact, if there were aggregate bonds, the dispersion problem would not be addressed because the discount factor would be independent of the agents' decisions.

Because of this curvature, a tax/subsidy on bonds holdings is required to guarantee that savings are zero in steady state[7].

To gain some intuition into this process, consider for a second that under my calibrated parameter values $\psi_1 = -.0011$ and, as a result, the implied tax/subsidy on bond holdings is of the order of .1 percent: $\tau_B = .0012$. Fortunately such small deviations from the baseline case are enough to ensure a stationary process for savings.

Figure 1, illustrates this point by showing the impulse response of savings for family h after it is hit by a positive idiosyncratic productivity shock. While savings evolve slowly, as one would expect, they are also clearly stationary in that they start to revert back towards their unconditional mean after about a dozen quarters.

The definition of the budget constraint completes the description of households in this model:

$$B_{ht} + P_{ht}C_{ht} = (1-\tau_B)R_{h,t-1}B_{h,t-1} + W_{ht}N_{ht} + \Pi_{ht} + T_{ht} \tag{8}$$

The resources that each family consumes come from either past savings, wage earnings or profits from firm h. T_{ht} is a lump-sum transfer to finance the production subsidy which includes an idiosyncratic component[8], while τ_B is the tax/subsidy on bond holdings discussed above.

The interest rate deserves a few more comments in that it is the sum of two components: the aggregate economy-wide interest rate set by the Central Bank and an idiosyncratic component

[7] Note that, in a sense I have one degree of freedom here. Either I allow the steady-state interest rate to differ from the value it usually takes on $(\frac{1}{\beta_0})$ or I constrain the tax to take on a specific value. I follow this second alternative because it helps intuition. In particular, I set the tax on bond holdings τ_B as follows:

$$\tau_B = -\frac{\beta_0\psi_1 N_s^{1-\alpha}\left((\gamma-1)\left(N_s^{1-\alpha}\right)^{\gamma} N_s^{\alpha+\sigma} + \sigma + 1\right)}{(\gamma-1)(\sigma+1)} \tag{6}$$

Where N_s is the steady state value of N_{ht} which takes value:

$$N_s = \left(\frac{(1-\alpha)(\eta-1)}{\eta}\right)^{\frac{1}{\alpha+\gamma(1-\alpha)+\sigma}} \tag{7}$$

[8] In log-linear terms it reads $t_{ht} = \frac{1}{-\tau P_s Y_s}(y_t + p_t) + \varepsilon_{ht}^{tax}$, where the idiosyncratic component could be thought as some sort of family-specific tax deduction/increase which prevents full revelation of the underlying state.

which could be thought of as being a simple indicator of credit worthiness or special conditions guaranteed by a bank to some of its customers.

In log-linear terms it is defined as:

$$r_{ht} = r_t + \varepsilon^R_{ht} \tag{9}$$

$$\varepsilon^R_{ht} = \rho_{\varepsilon^R}\varepsilon^R_{h,t-1} + u^{\varepsilon^R}_{ht} \tag{10}$$

Where r_t is the Taylor rule interest rate defined below.

3.2 Firms

As already mentioned, on each island operates a firm exclusively owned by the family living on that same island. Each firm produces a single good in a monopolistically competitive market which guarantees price-setting power.

Contrary to self-production economies, however, in this model firm h is required to hire different worker types on the labor market.

The firm operates the following technology:

$$Y_{ht} = A_{ht}\check{N}_{ht}^{1-\alpha} \tag{11}$$

Where $\check{N}_{ht}$ is the aggregate of worker types hired by firm h:

$$\check{N}_{ht} = \left[\int_{\check{L}_{ht}} N_{jht}^{\frac{\eta-1}{\eta}}\,dj\right]^{\frac{\eta}{\eta-1}} \tag{12}$$

At this point, it essential to note that throughout this paper, in cases where confusion can arise, I will use the $\check{}$ superscript to indicate variables that pertain to the firm owned by family h. For example, $\check{P}_{ht}$ is the price set by family h at which they offer the good their firm produces, while P_{ht} is the composite price of the bundle of goods they consume. The same convention holds for other variables, such as $\check{N}_{ht}$ in this case.

The other element of the production function is the island-specific productivity[9] component which is made up of an aggregate and a purely idiosyncratic component which in log-linear term read:

$$a_{ht} = a_t + \varepsilon_{ht}^A \tag{13}$$

$$a_t = \rho a_{t-1} + u_t^A \tag{14}$$

$$\varepsilon_{ht}^A = \rho_{\varepsilon^A} \varepsilon_{h,t-1} + u_{ht}^{\varepsilon^A} \tag{15}$$

Both components are allowed to be autocorrelated and both are supposed to be stationary.

3.3 Government and Central Bank

In this model the government simply levies lump-sum taxes to finance the production subsidy (which is set at a value equal to the goods-market markup) and redistributes resources through the tax/subsidy scheme on bonds described above.

The Central Bank sets the aggregate interest rate following a simple Taylor rule which, in log-linear form, reads:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\iota_\pi \pi_t + u_t^R \tag{16}$$

While this is the Taylor rule I use for my baseline calibration I also considered calibrations in which the Central Bank responds to past output growth as well. While I leave a more thorough analysis on the implications of different Taylor rule setups for future work[10], I can preliminarily say that including past output growth does not appear to produce huge differences.

3.4 Information

Island heterogeneity results from two different aspects of the model. Firstly, technology, the preference shock on labor effort, the transfer and the interest rate charged by the bank have exogenous island-specific components. Secondly, by its economic interactions each family is assumed to get

[9] I use productivity and technology as synonymous through this paper.

[10] In this context, a similar exercise includes not only the coefficients of the Taylor rule but the noise that might affect the Central Bank's information set.

an imprecise indication of the underlying state of the aggregate economy, in the spirit of Lorenzoni (2009)[23].

I assume each family's expectations on the state of the economy to depend on the history of all the signals they collect each period.

This is an improvement relative to some other imperfect information models which assume what Angeletos and La'O (2011) call "schizophrenia", which usually boils down to separating firms into compartments that do not communicate crucial information to each other.

Here pricing is made taking full advantage of all the information that the family, who runs the firm, gathers from all its economic interactions.

Moreover, I allow idiosyncratic components to be autocorrelated overtime. This complicates the analysis relative to Lorenzoni (2009)[23] because the state vector now includes island-specific variables, but makes the sampling analogy more compelling[11].

In fact, if one thinks of the idiosyncratic components as being a reduced-form representation of some type of sampling, it makes sense to think that the sampling error is not iid over time. Think, for instance, to the fact that the labor types a firm needs tend to remain the same over time, so if a firm needs carpenters to operate their business, chances are it will not sample the wage of bakers any time soon, thus making the noisy component in their wage payments correlated over time.

It is convenient to define the information set recursively as follows:

$$\Omega_{ht} = \left\{ a_{ht}, b_{h,t-1}, \widehat{\chi}_{h,t-1}, \widehat{\Lambda}_{h,t-1}, \widehat{\Xi}_{h,t-1}, p_{h,t-1}, \check{w}_{h,t-1}, t_{h,t-1}, r_{h,t-1}, s_{t-1} \right\} \cup \Omega_{h,t-1} \tag{17}$$

Where the productivity component a_{ht}, the interest rate $r_{h,t-1}$, as well as transfers $t_{h,t-1}$ and the preference shock $\widehat{\chi}_{h,t-1}$ have been defined above.

The other terms result from each family observing their savings $b_{h,t-1}$, the number of family members hired on the labor market $\widehat{\Lambda}_{h,t-1}$, the quantity of goods sold $\widehat{\Xi}_{h,t-1}$ and the wages $\check{w}_{h,t-1}$ paid out by their firm as well as the price of the bundle of goods they consumed $p_{h,t-1}$ and a common

[11] Sampling is not exactly the proper word when dealing with a continuum of type. Because it is a very intuitive concept and I could actually rewrite this exact model with countable types and get the exact same dynamics (in log linear form they are identical), I sometimes refer to the sampling analogy.

signal on output growth.

Before I turn to describing these elements in detail, it has to be noted that when the family make their decisions they sit at the beginning of the period. I assume that the only relevant piece of information concerning the current period they observe is the island-specific technology shock. All the information originating from the interaction with other islands is relative to the previous period because all the trades with the rest of the world have yet to take place in period t.

As I move on to spelling out the definition of all the signals, it is essential to bear in mind that all the formulas are in log-linear terms and expressed as linear function of elements of the state of the economy which is described below.

As just mentioned $\widehat{\Lambda}_{h,t-1}$ is the signal family h receives from working on a number of islands. Observing how many family members are required to work on different islands and given the imperfect substitutability of different labor types, they can gauge something about the aggregate wage and the total hours worked in the economy. It can be written as[12]:

$$\widehat{\Lambda}_{h,t-1} = \frac{y_{t-1} - a_{t-1}}{1-\alpha} + \eta w_{t-1} + \zeta^{\Lambda}_{h,t-1} \tag{18}$$

Similarly, $\widehat{\Xi}_{h,t-1}$ represents the information collected by firm h from the demand for the good they produce. Observing how many units they sell at the price they set, they can make some inference on the aggregate demand in the economy. It boils down to the following:

$$\widehat{\Xi}_{h,t-1} = y_{t-1} + \epsilon p_{t-1} + \zeta^{\Xi}_{h,t-1} \tag{19}$$

$p_{h,t-1}$ is the average price of the bundle of goods purchased by family h and also serves as a noisy indicator of the underlying aggregate price index:

$$p_{h,t-1} = p_{t-1} + \zeta^{P}_{h,t-1} \tag{20}$$

[12]This signal follows from the Dixit-Stiglitz demand for labor of type h once one consider that both the wage and number of family members hired are known. Note also, that in this model, $n_t = \frac{y_{t-1}-a_{t-1}}{1-\alpha}$ so I could equivalently rewrite the signal including hours worked or number of family members hired.

By hiring a subset of workers in the firm they run, family h gathers information on the aggregate wage as follows:

$$\breve{w}_{h,t-1} = w_{t-1} + \zeta^{W}_{h,t-1} \tag{21}$$

The signal s_{t-1} is a common indicator on output growth. It can be thought of as representing early releases of GDP growth figure by government agencies, as studied among other by Aruoba (2008)[6]:

$$\begin{aligned} s_{t-1} &= y_{t-1} - y_{t-2} + v_{t-1} & (22)\\ v_t &= \rho_v v_{t-1} + u^v_t & (23) \end{aligned}$$

Finally bond holdings can be described as follows:

$$b_{ht} = \varrho_{61} b_{h,t-1} + \varrho_{62}\zeta^{P}_{ht} + \varrho_{63}\varepsilon^{T}_{ht} + \varrho_{64}\zeta^{W}_{ht} + \varrho_{65}\zeta^{\Lambda}_{ht} + \varrho_{66}\zeta^{\Xi}_{ht} + \varrho_{67}\varepsilon^{A}_{ht} + \underline{\varrho}_{68}\underline{\zeta}^{Z_{ht|ht}}_{ht} \tag{24}$$

The expression above highlights how the savings on island h at the end of period t are a linear function of past bond holdings[13], a host of idiosyncratic shocks[14] and a term ($\underline{\zeta}^{Z_{ht|ht}}_{ht}$) capturing the difference between average expectations and family h's expectations[15]:

$$\underline{\zeta}^{Z_{ht|ht}}_{ht} \equiv \mathbf{Z}_{ht|ht} - \int_{j\in[0,1]} \mathbf{Z}_{jt|jt} dj \tag{25}$$

This term highlights a crucial informational role of bonds within this framework. In fact, because $\mathbf{Z}_{ht|ht}$ is known to family h, the savings represent an important signal on the average expectations. In other words, when a family observe their wealth change they attach some probability to fact that this might be because their expectations differed from that of other agents. This has potentially important implications, especially in telling whether a shock hit the entire economy or only family h.

[13] The ϱ's are functions of parameters, determined by the log-linearization and guess-and-verify processes.

[14] Note that while the idiosyncratic interest-rate component does not show up directly in equation (24), it enters through the expectational term, so savings are indeed a function of the idiosyncratic interest-rate component.

[15] Note how all these terms, including the difference in expectations are idiosyncratic in nature, thus guaranteeing zero aggregate savings in each period.

4 Solution

I will sketch the different steps of the solution process below trying to emphasize intuition while leaving the details to the appendix and to a number of files where algebra computations are carried out and which can be requested to the the author.

4.1 Decision Rules

Solving the single family problem yields first-order conditions that define optimal price setting and wage setting, as well as an Euler equation. Once log-linearized around the non-stochastic symmetric steady state, they boil down to intuitive linear decision rules.

The Euler equation in this model departs from its most standard form because of the endogenous discount rate and the tax on bonds. In log-linear terms the Euler Equation takes on the following form:

$$c_{ht} = (1-\tau_B)E_{ht}c_{h,t+1} - \frac{1}{\gamma}E_{ht}\left[(1-\tau_B)r_{ht} - ((1-\tau_B)p_{h,t+1} - p_{ht})\right] + \Psi E_{ht}b_{ht} \tag{26}$$

Where Ψ is a coefficient depending on underlying parameters[16] which takes on a positive but small value of the order 10^{-3}. Its economic impact is small but helps make agents who expect to be saving a lot to actually consume a bit more than they would absent the endogenous discount rate and the tax on savings. This is what it takes to keep the cross section of wealth under control in the sense illustrated above.

Except for this difference the usual logic applies: the higher the interest rate, the more one wishes to save, while the more one expects consumption prices to increase the more he or she will tend to consume in the present. The only twist relative to the vanilla case is in the tax on savings which obviously impacts the real interest rate.

[16] In particular $\Psi \equiv \frac{\psi_1\left(2(\tau_B-1)\lambda_s+\beta_0\psi_1\left(\frac{C_s^{1-\gamma}}{\gamma-1}+\frac{\chi_s\left(\Lambda_s W_s^{-\eta}\right)^{\sigma+1}}{\sigma+1}\right)\right)}{\gamma\lambda_s}$ which takes value .0011 in my standard calibration. Subscript s indicates steady state values which are in turn simple functions of the parameters.

Besides their consumption-saving decision, each household sets the price of the good produced by the firm they own according to the following profit-maximizing formula:

$$\check{p}_{ht} = \frac{1}{1+\alpha(\epsilon-1)} \left(-a_{ht} + \alpha E_{ht} \widehat{\Xi}_{ht} + (1-\alpha) E_{ht} \check{w}_{ht} \right) \tag{27}$$

The optimal price level for firm h depends on the technology level (which is known at the time the decision is made), the expectations of the average wage payments[17] to the workers to be hired in the production process and on the expected level of the demand $\widehat{\Xi}_{ht}$ as described in equation (19), the key parameters being the curvature of the production function α and the elasticity of substitution between goods ϵ.

Finally, this model is characterized by agents setting the wage level at which they are willing to work. They do so according to this formula:

$$w_{ht} = \frac{1}{1+\sigma\eta} \left(\sigma E_{ht} \widehat{\Lambda}_{ht} + E_{ht} \widehat{\chi}_{ht} + \gamma c_{ht} + E_{ht} p_{ht} \right) \tag{28}$$

The key here is to note that $\widehat{\Lambda}_{ht}$, defined in equation (18), is to be thought of as the labor market conditions on the subset of islands where members of the family h will be called to work, $\widehat{\chi}_{ht}$ is the preference shock on labor, while γc_{ht} represents the marginal utility of consumption. Finally, it is important to bear in mind that the p_{ht} is the price index for consumption of family h not the price set by their firm - that is way it is not yet known at the beginning of the period. It shows up in the wage-setting equation because it is obviously key in determining the purchasing power of labor earnings.

At this point, one can see that an essential component of price and wage setting decisions is the expectation of what other agents are doing - namely aggregate prices and wages which enter the definitions of $\widehat{\Lambda}_{ht}$ and $\widehat{\Xi}_{ht}$ - which are a crucial driver of persistent responses of economic variables to shocks, see Angeletos and Pavan (2004)[5].

Also, looking at equations (27) and (28) one can notice how the wealth of the family does not enter

[17] Average within the set of workers hired, not to be confused with the overall aggregate wage rate.

the log-linear price-setting function, while it does the wage-setting function.

In models in which each family work exclusively for the firm they own, the pricing decision would directly impact the number of hours they are called to work, here, instead, it primarily affects wage setting[18].

On the other hand, the decisions made by the workers (wage setting in this case) are obviously affected by their utility function, as the wage level impacts the number of family members that will be called to work.

Besides the decision-making aspect, in a dispersed information setting this difference has one more dimension to it, which is of an informational nature. In fact, one would definitely expect agents to have a better knowledge of their own preferences than they have for market conditions.

4.2 Guesses

The conditions above describe how agents make their decisions optimally, yet to solve the model one has to guess a solution for the endogenous variables. In a model with heterogeneous agents and information they are in general not trivial and deserve special attention relative to a model with a representative agent.

Prior to discussing the guesses themselves, it is essential to define the state of the economy:

$$\begin{aligned} z_{ht} &\equiv \begin{bmatrix} \zeta_{ht}^{P} & \zeta_{ht}^{W} & \zeta_{ht}^{\Lambda} & \zeta_{ht}^{\Xi} & \varepsilon_{ht}^{R} & \varepsilon_{ht}^{\chi} & \varepsilon_{ht}^{T} & \varepsilon_{ht}^{A} & b_{h,t-1} & a_t & y_t & p_t & w_t & r_t & v_t \end{bmatrix}' \\ \mathbf{Z}_{ht} &\equiv \begin{bmatrix} z_{ht} \\ z_{h,t-1} \\ \ldots \\ z_{h,t-T} \end{bmatrix} \end{aligned} \tag{29}$$

[18]If one was to consider the non-linear version of the price-setting condition, a covariance term involving the Lagrange multiplier on the budget constraint would show up, which obviously depends on wealth. As usual, these covariance terms drop out when equations are log-linearized to solve the model. In this sense the impact of wealth on price setting is second-order, while in the case of self-production the marginal disutility of working directly determines the marginal cost of production.

I will refer to z_{ht} as the period-by-period state of economy because it includes the fifteen variables that are required in each period to fully characterize the economy[19].

Because, in general, the number of lags of each variables to be included in the solution process is essentially a question of numerical accuracy, I stack up T lags of the period-by-period state.

This structure is similar to that in Lorenzoni (2009), whose solution method I adapt. However, one key difference should be noticed. While a key feature of Lorenzoni's setup is that the state of the economy is the same for every family, here I have to introduce a number of idiosyncratic components in the state vector due to the fact that they are not i.i.d. over time. Hence, the solution method is to be adapted. In particular, the endogenous variables, i.e. those determined through the numerical fixed-point exercise, are not only of the aggregate type (namely, output, wage and price level) but include island-level bond holdings.

Given the definition of the state, one can show[20] that the following guesses for consumption, wage and price[21] setting are verified and thus provide a description of the stochastic process of endogenous variables:

$$c_{ht} = \phi_1 b_{h,t-1} + \phi_2 a_{ht} + \underline{\phi}(A) E_{ht} \mathbf{Z}_{ht} \tag{30}$$

$$w_{ht} = \theta_1 b_{h,t-1} + \theta_2 a_{ht} + \underline{\theta}(A) E_{ht} \mathbf{Z}_{ht} \tag{31}$$

$$\check{p}_{ht} = -\frac{a_{ht}}{1+\alpha(\epsilon-1)} + \underline{\pi} E_{ht} \mathbf{Z}_{ht} \tag{32}$$

The coefficients are in general a complicated and nonlinear function of the underlying parameters but they can be solved for analytically, albeit with one caveat. The vectors $\underline{\phi}(A)$ and $\underline{\theta}(A)$ turn out to be functions of matrix A which describes the autoregressive evolution of the state and some elements of which are determined by a numerical fixed point. While the functions can be solved analytically, the actual values of the vectors are only pinned down once convergence is achieved.

[19] The variables are: the eight innovations to all the idiosyncratic components, bond holdings, aggregate technology, output, the price level, the wage level, the economy-wide interest rate and the noisy component in the common signal on output respectively.

[20] See the appendix

[21] When it comes to price setting, given the nature of the optimal decision rule there is no need to have undetermined coefficients so in this case the guess amounts to a simple re-definition of the log-linear condition defined above.

Once solved for the coefficients, the guesses are, as one would expect, a linear function of variables in the agent's information set.

In particular, consumption depends on the household's wealth, the state of technology which is a critical indicator of future profits and the expectations on the state of the economy.

This last term is crucial and summarizes all the expectational terms in the decision rules of the previous section. It is important to keep in mind that $E_{ht}\mathbf{Z}_{ht}$ is known (for a given matrix A) as it is the output of some Kalman filtering the agents are doing based on the infinite history of signal vectors. In fact, this is just a compact way to express what one could write out as an infinite sum of the vectors of signals whose weights are determined optimally by the Riccati equation governing the Kalman filter. In light of this, one can see how the guesses are a linear function of every piece of information known to the agents.

The wage setting function takes on the same form as that of consumption, although with different weights, while price setting is independent of bond holdings in line with the above-mentioned discussion regarding the importance of separating profit maximizing, from the leisure/labor decision.

Finally note that a crucial aspect of this problem is that, while obviously conditional moments are different on each different island, based on the history of signals, the stochastic processes governing the evolution of all the variables are the same across the entire economy, so once I have pinned down the matrices governing the state-space representation of the economy I have solved the model.

4.3 Fixed Point

The solution strategy is in the spirit of Lorenzoni (2009)[23] adapted to this particular framework. I will briefly sketch the main assumptions trying to highlight the differences, leaving further details to the appendix.

Once I have characterized the agents' behavior as in equations (30), (57) and (32), computing the expectations becomes the main focus.

For this, I need to have a state-space representation for $\mathbf{Z}_{ht}$ and all the informative signals described above. This in turn requires some initialization for the process of endogenous variables (i.e. $y_t,\ p_t,\ w_t$ and $b_{h,t-1}$) which will be the object of updating.

Given the state-space representation, the Kalman filter delivers the Kalman gain matrix C for expectations at the island level:

$$E_{ht}\left[\mathbf{Z}_t\right] = E_{ht-1}\left[\mathbf{Z}_t\right] + C\left(\underline{s}_{ht} - E_{h,t-1}\left[\underline{s}_{ht}\right]\right) \tag{33}$$

Where $\underline{s}_{ht}$ is the vector collecting all the signals described above.

The next step entails an approximation of expectations, so that they can be expressed as a linear function of the state:

$$\mathbf{Z}_{ht|ht} = \Theta \mathbf{Z}_{ht} \tag{34}$$

In particular, I consider a simple projection of the true process for expectations onto a subset of state variables (selected to mimic the information set as much as possible and to avoid collinearity). This method does not require the assumption that state variables after a certain number of periods are set to zero, a strong assumption for typically highly correlated macro variables. Once I have expectations expressed as a function of the state I can impose the restrictions delivered by the decision-making functions (the guesses) and compare the process implied by these to that governed by the state-space specification I started with. In case the two differ I will update the elements of matrices governing the state-space representation until convergence is achieved.

4.4 Calibration

To solve the model I need to calibrate a number of parameters. In the following section I discuss those I consider more economically relevant, leaving the complete list, which includes correlations and standard deviations for all the idiosyncratic components to the appendix.

Starting with parameters that affect the utility function, one can see that the intertemporal preference parameter β_0 is set to .99 which implies an annual real interest rate of about 4% in steady state, while γ, which governs the substitutability of consumption, is set to 2.

The curvature of the disutility of labor σ takes on value 2 which, in a standard macro model, is traditionally thought of as the inverse Frisch elasticity. Here things get a bit more complicated

α	.3	γ	2	σ	2	β_0	.99
ρ	.86	ρ_r	.7	ι_π	1.5	η	6
ϵ	11	ψ_1	-.0011	τ_B	.0012		

Table 1: Some Calibrated Prameter Values

because labor is, strictly speaking, indivisible as each individual family-member is either working or not. However, since decisions are made at the family level, it turns out that σ governs the intratemporal substitution of labor and consumption in the usual sense. A level of 2 sets what would be the Frisch elasticity to .5 which is in the lower range of micro estimations, so it is a rather conservative parametrization.

Finally α determines the curvature of the production function. In a model with capital, $1-\alpha$ would correspond to the labor share which is about 2/3 of the output.

The autocorrelation in the aggregate technology process is determined by ρ which is set to .86, a value taken from Mendes (2007)[27]. I set ρ_r, which governs the interest-rate smoothing in the Taylor rule, to .7, while the response to inflation is determined by $\iota_\pi = 1.5$.

The substitutability of goods in the Dixit-Stiglitz framework I employ depends on ϵ which I set to 11, a value that implies a mark-up of 10% which is considered in the acceptable range. By the same token, η determines the substitutability of labor types. Setting its value to 6, I assume that labor types are harder to substitute for one another than good types, which seems realistic given the specialization involved in most labor tasks. As it is customary, the production subsidy τ is assigned value equal to the godds-market mark-up, i.e. .1.

Finally, I set ψ_1, which governs the responsiveness of the discount factor to changes in savings to a value small enough not to generate a big departure from the standard representative agent case and yet able to deliver a well behaved wealth distribution. Strictly speaking, τ_B is not calibrated but set according to equation (6). The good news is that the value for the tax/subsidy on bonds implied by my calibrated values is small, at just around a tenth of a percentage point.

5 Results and Discussion

I will divide the discussion in three main parts. I will first assess how my model fares in generating realistic impulse responses to shocks, then I will focus on unemployment and its correlation with wage inflation.

5.1 Output and Inflation Behavior

One of the key feature of this model is that monetary policiy produces long-lasting real effects despite the absence of multi-period Calvo frictions.

As one can see in Figure 2, output drops following an increase in the Taylor-rule interest rate and stays below normal for a number of quarters.

Just as important is to note how inflation responds to what is a "disinflationary" monetary policy shock.

In particular, in the first period after the shock, inflation will increase, (price puzzle) before going negative for some quarters.

The size of the responses stacks up reasonably well with VAR evidence from Christiano, Trabandt and Walentin (2010)[12]. At the peak the response of inflation is slightly bigger than in Christiano, Trabandt and Walentin (2010[12][22] estimates while that of output is in the same range. Consistent with empirical evidence is also the fact that inflation peaks prior to output. Both of them occur at an earlier stage relative to the above mentioned VAR evidence, which might be due to the absence of investment and capital in this framework. On top of that, it is also to be noticed that Smets and Wouters (2007)[30] in estimating their DSGE find that the response of inflation peaks earlier than found in Christiano, Trabandt and Walentin (2010)[12].

To understand where these results stem from it is essential to highlight the two primary sources of confusion, on the part of the agents, that generate them.

At the end of the first period agents realize that the interest rate at which they can save or borrow has increased. What they are unsure about is whether this interest rate hike is specific to their

[22] When one considers that they report annualized percent rates while I have quarterly rates and that they study a drop in the interest rate.

family or hit the entire economy.

Independent of the nature of the interest rate increase, the natural response is to cut consumption to take advantage of the opportunity to save while high returns on savings last.

On the other hand, the implications for price setting are different whether the interest rate increase affects the only on their island or not.

In the former case, they would have no reason to expect any change in the demand for the good they sell or in the cost of the labor they hire, hence they would not change the price for the good they produce. Notice how with self production this would not necessarily be the case as, agents would not consider market wages but their marginal utilities which are bound to be affected by a change in the family savings' plan.

If, however, the increase originates in the Taylor rule, it would have to be related to either a monetary policy shock or a rise in inflation which is generally associated with a fall in aggregate technology.

The fact that the central bank responds to current inflation, makes agents suspect that the cause of the interest rate hike might indeed be driven by a price increase[23] which would lead price setters to increase prices as they expect other to have done so. Figure 3 reports the approximated expectations of the level of aggregate technology over time and it shows how it is well below zero which explains the price puzzle effect in the first period after the shock[24].

By the time period two (the first after the shock becomes known) is over agents will get a number of signals suggesting that output has fallen. In particular, bond holdings are presumably the most informative of them all.

In fact, if the interest rate shock was idiosyncratic their savings plans should have realized and their checking account report should show lower debts or higher savings than in the previous period.

If, however, the shock was aggregate, because every family's consumption would have fallen the

[23]The movement in prices could also be caused by an aggregate noise shock but because technology shocks are on average much bigger they would be considered the most likely culprit.

[24]Obviously the expectation of the idiosyncratic component is the mirror image around zero. This is so because agents observe the sum of the sum and know it is zero. They are unsure how to break this sum into its two components. Because aggregate technology has pricing implication for *all the other firms* while, idiosyncratic technology does not, the former is the one driving the prize puzzle.

aggregate demand would be lower to the point where nobody would end up saving[25].

While it could be that a combination of idiosyncratic shocks produced unvaried savings despite the change in the interest rate being purely idiosyncratic, this is pretty unlikely and, in fact, Figure 4 shows how by the start of period 3 it is quite obvious that an aggregate monetary policy shock occurred in period 1. Figure 4 reports the evolution expectations of the innovation to the Taylor rule (solid line) and to the island-specific interest rate component (dotted line) in period 1. In other words, it answers the question: what do you think was the innovation to the Taylor rule and to the idiosyncratic component of your interest rate in period 1? And this question is asked at the start of every subsequent period. While in period 1 agents have no reason to expect anything happened to their interest rate (which they have not seen yet), in period two they clearly sense something happened in the previous period but they are still very confused as to the nature of the shock because what they observe is the sum of the aggregate and the idiosyncratic component of the interest rate[26].

By period 3 they are fairly confident, although not sure, it was not an idiosyncratic shock[27]. An intuitive way to address the confidence in the beliefs is to consider the dispersion in the distribution around the conditional mean.

The lower section of figure 4 reports the same curves shown in the top pane, although on separate graph and with what we could call confidence bands. They are a simple indicator of information dispersion, as they indicate that 95 percent of the probability mass falls between the dashed lines. The closer they get to the expected value for the variable at hand, the stronger the consensus across different islands and the closer we are to a full information setting.

Focusing on the Taylor rule shock at time one u_1^R, it is easy to see that as periods go by the in-

[25]The resource constraint $c_t = y_t$ holds in every period.

[26]Note that in period two, on average, they attribute a bigger portion of the interest-rate increase to a jump in the innovarion their idiosyncratic component. This is because the variance of the innovation is higher for this component. However, when one considers the overall variance of island-level interest-rates, only about 19 percent of it is explained by the idiosyncratic component. This is because the Taylor rule interest-rate does not only depend on the its own innovation but also on the response to inflation.

[27]Please note the key difference between Figure 4 and Figure 3, besides the obvious fact that they refer to different variables. In fact, Figure 3 which reports the contemporaneous approximated expectations ($\tilde{E}_{ht}[a_t]$, $\tilde{E}_{ht}[\varepsilon^A_{ht}]$), as opposed to the expected innovation which occurred at time 1, which is the focus of Figure 4 ($E_{ht}[u_1^R]$, $E_{ht}[u_{h1}^{\varepsilon^R}]$). In other words the expectations reported in Figure 3 also incorporate the expected innovations from periods 2 onwards. Both figures are interesting and insightful in their own respect but they should not be confused.

formation gets more precise with respect to the mean and also more concentrated around the true value $u_1^R = .19$ percent.

In the first period, the bands limit the interval $\pm 1.96\sigma_{u^R}$ so they are simply determined by the unconditional variance. Over time the shrinking of the bands testifies to the fact that agents learn reasonably fast in this model. Some uncertainty, however, survives leaving room for long-lasting real effects.

Before closing this discussion it is worth highlighting how the Central Bank in this setup has perfect information so all the confusion arises in the private sector, while in Mendes (2007)[27] the Central Bank responds to noisy indicators. Studying how different Central Bankers' information sets affect the transmission of policy is certainly an interesting application of this model that I intend to pursue.

The fact that all variables in this model are stationary does not make it particularly suitable to explain technology shocks quantitatively. Some comments, however, are in order.

In particular, as Figure 5 shows, inflation drops immediately to its lowest value which is in line with empirical evidence. The response of output, while being of the same order of magnitude as that in Christiano, Trabandt and walentin (2010)[12], peaks before relative to what appears to happen in the data which is hardly a surprise given the simpler structure of this model.

The dynamics of a response to a technology shock are also easier to describe in that technology shocks are the most likely source of variations in this model and agents learn their technology at the beginning of each period so they are able to immediately respond slashing prices.

The confusion between the aggregate and the idiosyncratic component, which is certainly present because the agents only observe the sum of the two, is not bound to have a major impact on prices because marginal costs would drop no matter the nature of the shock. Certainly an aggregate shock would affect the wages paid out to hired workers and also the demand for goods but at a qualitative level both an idiosyncratic and an aggregate shock would induce agents to lower prices, which is indeed what happens right away.

5.2 Unemployment

Before delving into the discussion of the responses of unemployment to the economic shocks, I would like to stress the role of different timing conventions in the assessment of participation, and hence unemployment, when dispersed information is considered.

In this sense, it is important to notice that in log-linear terms participation can be defined as:

$$n^*_{ht} = \frac{1}{\sigma}\left(w_{ht} - p_{ht} - \hat{\chi}_{ht} - \gamma c_{ht}\right) \tag{35}$$

The difference between the mass of type-h workers willing to work n^*_{ht} and the those actually hired n_{ht} defines the unemployment of the labor force of type h[28]:

$$u_{ht} \equiv n^*_{ht} - n_{ht} \tag{36}$$

Now note that equation (35) has no expected values in it, which means it is an end-of-the-period assessment of participation[29].

If, however, we think that in the real world the participation decision has to be made before the period is over in order for one to have time to get a job, one could think of asking the heads of the families how many of their fellow family members they wish could work at the beginning of the period[30].

If we entertain that possibility, then we should account for the fact that while the wage w_{ht} and consumption are known at the beginning of period as they are being set exactly then, the price of the goods they will get a chance to buy come the end of the period is only known in expectations, the same being true for the preference shock.

[28] Aggregating over h delivers the aggregate result.

[29] By the end of the period or the beginning of the following (which is the same) all those variables are known as one can verify from the discussion above.

[30] It is true that in this setup there is no search because once the wage is posted all members of the family could potentially be asked to work, whether they deem it convenient or not. This, however, does not make the timing of the question as to how many family members one wishes could find a job irrelevant. In this sense, it seems reasonable to ask about what one wishes would happen over a certain time span at the beginning of that same time period.

The definition of participation then becomes:

$$\tilde{n}^{*}_{ht} = \frac{1}{\sigma}\left(w_{ht} - E_{ht}[p_{ht}] - E_{ht}[\hat{\chi}_{ht}] - \gamma c_{ht}\right) \tag{37}$$

In the following discussion I will compare the two definitions, highlighting how this different timing convention might make a difference in the assessment of the results.

Figure 6 illustrates how unemployment and participation respond to a monetary policy shock based on both the ex-post (solid) and and ex-ante (dashed) definitions of participation[31].

First notice that while unemployment per se is implied by the imperfect competition on the job market, its variations in response to a monetary policy shock depend on the sluggish adjustment of prices and wage following a monetary policy shock.

In particular, as expected unemployment shoots up after a contractionary monetary policy shock. On the other hand, one might be surprised to see participation increase.

This is, indeed, a feature of the definition of participation in Gali' (2009). As equations (35) and (37) show, in this context in which there are no search frictions participation essentially depends on two key variables: the real wage and the marginal utility of consumption[32].

When they increase, they make working more convenient in terms of the extra consumption that could deliver.

Following a monetary policy shock consumption falls with output, thus causing an increase in the marginal utility of consumption. This effect dominates the contemporaneous fall in the real wage, which is natural in a setting that is meant to prevent excessive swings in prices. So, overall participation grows.

Gali', Smets and Wouters (2011)[16] and Gali' (2011)[15] consider an endogenous preference shifter to deliver a more realistic process for participation. The calibration I present, on the other hand, does not rely on that because I try to mitigate this problem exploiting the dispersed information

[31] Note that changes are in percent of steady state values. For instance, unemployment is around 8.7 percent in steady state in this setup, so if unemployment goes down by, say, 1 percent it goes down by one percent of the steady state value.

[32] The other element is the preference shock

structure of the model, leaving the study of an endogenous preference shifter to a possible extension. In this sense, I am very conservative in the way I try to address this problem.

In spite of this, I couple of comments can be made. Firstly, notice that the price puzzle effect helps further reduce the real wage which falls more than the nominal wage in the first period, thus lowering the increase in participation. In this respect, it is important to note that participation only explains less than half the increase in unemployment that follows an increase in the aggregate interest rate.

Moreover, if one focuses on the dashed line, which corresponds to the ex-ante definition of participation, one realizes how the timing convention has interesting effects.

In particular, in the context of this model, it makes participation fall below zero two periods after the shock hits and then essentially don't move much at all.

While quantitatively the difference between the two cases may appear rather small, it shows how the underestimation in the fall of the price level can have an important role in the participation decision, with no need for preference shifters. It also seems to suggest that if consumption was also not known at the beginning of the period, there would be even more room to make participation fall after a monetary policy shock[33].

Figure 7 shows the response of unemployment to an increase in aggregate technology.

Hours worked drop following a positive technology shock because agents want to spend some of their increased wealth "buying" more leisure. Secondly, participation is reduced because families expect their consumption to increase thus reducing their marginal utility. The former effect dominates and, as a result, unemployment shoots up on impact but this effect dies out relatively fast and unemployment goes below its steady state level, albeit slightly, after the fifth period. Considering the ex-ante definition of participation and unemployment, the unemployment burst is slightly smaller, but the drop in hours is still dominant.

[33] I leave the possibility of a more thorough analysis of this particular aspect for future research as, changing the timing of the consumption decision is bound to impact the entire model. If, however, the response of expected consumption was smaller, in absolute value, than that of actual consumption, that alone could help dramatically.

5.3 Phillips Curve

Explicitly considering unemployment, allows me to address the classic question of the correlation between inflation and unemployment.

In particular Gali' (2009)[14], finds a negative correlation between unemployment and wage inflation in the sample spanning 1986-2007.

The theoretical moments implied by this calibration of my model imply a negative correlation of both quarterly wage inflation and four-quarter centered wage inflation[34]. In particular the former takes on value $-.11$ while the latter $-.04$. While these numbers are smaller than those found by Gali' for the 1986-2007 sample, it is important to note that because they are theoretical moments there is no uncertainty about their values so one can confidently say that the correlation is indeed negative.

Interestingly, the tradeoff appears to be stronger when only monetary policy shocks (the shock that can be considered a policy instrument in this model) are allowed to hit the economy. Quarterly inflation correlation to unemployment drops all the way to $-.43$, while considering the four-quarter centered measure of inflation I get a correlation coefficient of $-.46$. These numbers are important not only for their policy implications but also because they provide an insightful snapshot of the role of information dispersion. In fact, while steady-state unemployment in this model is due to the fact that wages are essentially to high because of workers' wage setting power, frictions determine the co-movement.

A higher degree of information dispersion and less curvature in the disutility of working[35] seem likely candidates to make the correlation coefficients quantitatively closer to those observed in the data.

[34] This is the one Gali' (2009)[14] studies.

[35] The current parametrization would imply a Frisch elasticity of .5 which appears to be at the lower end of micro estimates. Increasing the Frisch elasticity (i.e. reducing the curvature in the disutility of working) is bound to help in making wages more sluggish and hours (or number of workers hired) more responsive, other things the same.

6 Conclusions

While maintaing that prices can only be re-set at random intervals is a strong assumption, it seems natural to think that different agents, interacting with the rest of the economy, gather different bits of information so that each of them has a different perception of the state of economy.

In light of this, I consider a very simple New-Keynesian-like model with wage-setting workers and price-setting firms, in which I substitute an information friction for multi-period Calvo price and wage setting. Relative to Lorenzoni (2009)[23] and Mendes (2007)[27], I explicitly model the labor market doing away with the assumption of self production[36].

I show how this simple model features long-lasting real effects of monetary policy, driven by the agents being confused between aggregate and idiosyncratic shocks as well as between technology and monetary policy shocks. The initial confusion between technology and monetary policy shocks can also generate the price puzzle, i.e. inflation temporarily going into positive territory after an increase in the Taylor-rule interest rate.

Moreover, results are not only qualitatively in line with common economic wisdom, but responses of inflation and output to a monetary policy shock are also, to a first approximation, quantitatively consistent with VAR evidence by Christiano, Trabandt and Walentin (2010)[12].

The introduction of an imperfectly competitive labor market lends itself to the study of unemployment in this setting. I follow Gali' (2009)[14], reinterpreting hours worked as the number of workers hired and find that my setup produces a negative correlation between wage inflation and the unemployment, or a Phillips Curve. Furthermore, imperfect information helps, under the circumstances discussed above, to generate a more realistic response of participation to a monetary policy shock.

Finally, from a more technical standpoint, I propose a fairly simple fix to the unruly wealth distribution that generally plagues this class of models with heterogeneous agents and incomplete

[36] Please refer to the discussion of the related literature to see in greater detail how my model fits into the picture of dispersed information models.

markets. I study how making the discount factor endogenous, in the spirit of Schmitt-Grohe' and Uribe(2003)[29] and introducing a small tax on savings is enough to generate a stationary wealth distribution which is crucial if one is to take the log-linearization around the non-stochastic steady state seriously.

I find this technical contribution to be important in making this entire class of model a potential workhorse for macro modeling and policy analysis.

In this perspective, the present model is just s first step in that direction. For instance a search-based labor market in the spirit of Venkateswaran (2011)[35] could be introduced in this context[37] to analyze the effects of a search friction on unemployment and monetary policy transmission.

An in-depth analysis of the impact of different information sets of the Central Bank on its policy are also worth investigating[38], as they could potentially provide a novel explanation of different regimes in the price puzzle, as documented by Castelnuovo and Surico (2009)[10], as well as some insight into whether it would be optimal for a Central Bank to try and get more precise information on the state of the economy. This would also be a step into a more thorough consideration of optimal monetary policy in the context of a model which is more quantitative than Lorenzoni (2009)[22] and at the same time considers information dispersion explicitly (as compared to Svensson and Woodford (2003)[32]) and does not assume that the private sector has full information, as in Boivin and Giannoni (2008)[7] among others[39].

Finally, introducing an investment decision in this context is an obviously interesting exercise which would build on a seminal paper in the field of dispersed information, namely Townsend (1983)[33].

[37] Venkateswaran (2011) [35] does not address monetary policy in his dispersed information model with search.

[38] Melosi (2011)[26] and Walsh (2010)[36] are two important references in this sense, which I would build on in light of the different environment my model provides.

[39] Such analysis would also complement that of Paciello and Wiederholt (2011)[28] who, in the context of rational inattention, primarily focus on the information set of agents.

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A Solution

A.1 Non-stochastic Symmetric Steady State

I report the steady state values for the main variables. I normalize the price level to one and express most of them as functions of one another to keep things compact. However, every variable can be expressed as a function of N_s which is in turn a function of underlying parameters.

$$P_s = 1 \tag{38}$$

$$N_s = \left(\frac{(1-\alpha)(\eta-1)}{\eta}\right)^{\frac{1}{\alpha(-\gamma)+\alpha+\gamma+\sigma}} \tag{39}$$

$$W_s = \frac{\eta N_s^{\sigma}\left(N_s^{1-\alpha}\right)^{\gamma}}{\eta-1} \tag{40}$$

$$Y_s = N_s^{1-\alpha} \tag{41}$$

$$C_s = Y_s \tag{42}$$

$$\Lambda_s = W_s^{\eta} N_s \tag{43}$$

$$\Xi_s = P_s^{\epsilon} Y_s \tag{44}$$

$$\lambda_s = \frac{C_s^{-\gamma}}{P_s} \tag{45}$$

$$T_s = -\tau P_s Y_s \tag{46}$$

A.2 State Space

I define the individual, i.e. island specific, state as:

$$z_{ht} \equiv \begin{bmatrix} \zeta_{ht}^{P} \\ \zeta_{ht}^{W} \\ \zeta_{ht}^{\Lambda} \\ \zeta_{ht}^{\Xi} \\ \varepsilon_{ht}^{R} \\ \varepsilon_{ht}^{\chi} \\ \varepsilon_{ht}^{T} \\ \varepsilon_{ht}^{A} \\ b_{h,t-1} \\ a_t \\ y_t \\ p_t \\ w_t \\ r_t \\ v_t \end{bmatrix} \tag{47}$$

$$\mathbf{Z}_{ht} \equiv \begin{bmatrix} z_{ht} \\ z_{h,t-1} \\ \cdots \\ z_{h,t-T} \end{bmatrix} \tag{48}$$

Notice that I stack the variables so that the island-specific variables are listed first and the economy-wide variables come in after them.
The state equation is then:

$$\begin{aligned} \mathbf{Z}_{ht} &= A\mathbf{Z}_{h,t-1} + B\mathbf{W}\underline{u}_{ht} \qquad (49) \\ & \qquad (50) \end{aligned}$$

Note that matrices A and B which describe the evolution of the state are the same for every island. Where:

$$\underline{u}_{ht} \equiv \begin{bmatrix} u_{ht}^{\zeta^P} \\ u_{ht}^{\zeta^W} \\ u_{ht}^{\zeta^\Lambda} \\ u_{ht}^{\zeta^\Xi} \\ u_{ht}^{\varepsilon^R} \\ u_{ht}^{\varepsilon^\chi} \\ u_{ht}^{\varepsilon^T} \\ u_{ht}^{\varepsilon^A} \\ u_t^A \\ u_t^R \\ u_t^v \end{bmatrix} \qquad (51)$$

Note that rows 9, 11, 12, 13 in matrices A and B are to determined in equilibrium. Rows 1 through 8, 10 and 15 are purely exogenous, while row 14 is a bit of hybrid in that it is a known linear function of endog variables.
The observation equation includes 10 signals and takes on the following structure[40]:

$$\underline{\mathbf{s}}_{\mathbf{ht}} = HZ_{h,t-1} + V\mathbf{W}\underline{u}_{ht} \qquad (52)$$

The matrix structure is the same for every island.
I introduce $\mathbf{W}$ to make my setup consistent with Hansen and Sargent (2008)[18] p. 106 who assume that the innovations have unitary variance so now I have:

$$\begin{aligned} Var(\underline{u}_{ht}) &= I_{11} \qquad (53) \\ Var(\text{"}economic\ shocks\text{"}) &= \mathbf{W}\mathbf{W}' \qquad (54) \end{aligned}$$

Matrix $\mathbf{W}$ has the standard deviations of the shock on the diagonal and zeros off.

A.3 Guesses

For consumption I posit:

$$c_{ht} = \phi_1 b_{h,t-1} + \phi_2 a_{ht} + \underline{\phi} E_{ht}\mathbf{Z}_{ht} \qquad (55)$$

[40] Here I follow Hansen and Sargent (2008)[18] setup because it allows for the same shocks to show up both in the state and in the observation equations.

After log-linearizing and substituting the guesses into the Euler equation I get the following:

$$\begin{aligned} c_{ht} &= \varrho_1 E_{ht} r_t + \varrho_2 E_{ht} \varepsilon^R_{ht} + \varrho_3 E_{ht} p_{t+1} + \varrho_4 E_{ht} a_{t+1} + \varrho_5 \underline{\phi} \mathbf{E}_{ht} Z_{h,t+1} + \varrho_6 E_{ht} \varepsilon^A_{h,t+1} + \varrho_7 E_{ht} \zeta^P_{h,t+1} + \varrho_8 E_{ht} \varepsilon^T_{ht} \\ &+ \varrho_{11} E_{ht} \zeta^W_{ht} + \varrho_{12} E_{ht} \zeta^\Xi_{ht} + \varrho_{13} E_{ht} \zeta^P_{ht} + \varrho_{14} E_{ht} p_t + \varrho_{15} \underline{\phi} E_{ht} \mathbf{Z}_{ht} + \varrho_{16} b_{h,t-1} + \varrho_{17} a_{ht} + \varrho_{18} E_{ht} \varepsilon^\chi_{ht} \\ &+ \varrho_{19} E_{ht} \zeta^\Lambda_{ht} + \varrho_{20} E_{ht} w_t + \varrho_{21} E_{ht} y_t + \varrho_{22} E_{ht} a_t \end{aligned} \tag{56}$$

Where ϱ_i are known functions of underlying parameters. Noting that by the autoregressive structure of the problem $E_{ht}\mathbf{Z}_{h,t+1} = AE_{ht}\mathbf{Z}_{ht}$, the guess is verified. In fact, all the variables in expectations enter the current or future state vector while only two variables are not in expectations, savings and the technology shock. Also, because matrix A impacts the expectations of $\mathbf{Z}_{h,t+1}$ it is clear that $\underline{\phi}$ will depend on it.

Similarly for wages I posit:

$$w_{ht} = \theta_1 b_{h,t-1} + \theta_2 a_{ht} + \underline{\theta}(A) E_{ht} \mathbf{Z}_{ht} \tag{57}$$

And from the log-linearized model and some algebra I get:

$$w_{ht} = \varrho_{31} p_t + \varrho_{32} w_t + \varrho_{33} a_t + \varrho_{34} y_t + \varrho_{35} \zeta^P_{ht} + \varrho_{36} \zeta^\Lambda_{ht} + \varrho_{37} \underline{\phi} \mathbf{Z}_{ht} + \varrho_{38} a_{ht} + \varrho_{39} b_{h,t-1} + \varrho_{40} \varepsilon^\chi_{ht} \tag{58}$$

Which shows that the guess is verified. Please note that $\underline{\theta}$ is also a function of A because it depends on $\underline{\phi}$ which in turn depends on A. In other words, it depends on A because wage-setting depends on the consumption.

Finally, for prices I have posited:

$$\check{p}_{ht} = -\frac{a_{ht}}{1+\alpha(\epsilon-1)} + \underline{\pi} E_{ht} \mathbf{Z}_{ht} \tag{59}$$

And get:

$$\check{p}_{ht} = \varrho_{51} a_{ht} + \varrho_{52} w_t + \varrho_{53} \zeta^W_{ht} + \varrho_{54} p_t + \varrho_{55} y_t + \varrho_{56} \zeta^\Xi_{ht} \tag{60}$$

Which verifies the guess and shows that $\underline{\pi}$ does not depend on A.

A.4 Expectation Approximation

Given the Kalman filter setup described above the expectations take on the following process:

$$\mathbf{Z}_{ht|ht} = \mathbb{A}(L) \underline{u}_{ht} \tag{61}$$

$$\mathbb{A}(L) \equiv \left[I - (A - CH)L\right]^{-1} \left[CH(I - AL)^{-1} B\mathbf{W}L + CV\mathbf{W}\right] \tag{62}$$

Although, this is the correct process for expectations, I need expectations as a simple linear function of the state for the updating procedure.
To this end I consider the projection of the correct process for expectations onto a subset of the elements of the state vector chosen to mimic to some extent the information set and to avoid

collinearity.
In particular I define matrix $\mathbf{R}$ as a matrix of zeros and ones, such that:

$$\mathbf{RZ}_{ht} = \begin{bmatrix} \zeta^P_{h,t-1} \\ \zeta^W_{h,t-1} \\ \zeta^\Lambda_{h,t-1} \\ \zeta^\Xi_{h,t-1} \\ \varepsilon^R_{h,t-1} \\ \varepsilon^\chi_{h,t-1} \\ \varepsilon^T_{h,t-1} \\ \varepsilon^A_{ht} \\ b_{h,t-1} \\ a_t \\ y_{t-1} \\ p_{t-1} \\ w_{t-1} \\ r_{t-1} \\ v_{t-1} \end{bmatrix} \tag{63}$$

The idea is that all the components of the state, except the aggregate and idiosyncratic technology components, are not known until the end of the period. So I select this subset of variables to approximate expectations.
So matrix Θ will simply be the projection matrix (transposed) of the true process for expectations onto the space spanned by the variables in the vector I just described times $\mathbf{R}$. It can be efficiently computed evaluating numerically the the stochastic density functions of these processes.
Notice that this procedure is different relative to the usual notion of truncation in that I am not computing expectations using only a limited number of lags but rather I am computing the true process for expectation and only then I approximate it.
Also, in a separate paper, currently in progress, I compare this projection method with the one used in Lorenzoni (2009)[23] in a controlled environment (i.e. one in which there is no need for updating and the all the relevant stochastic processes are known in detail). Preliminary results seem to suggest that the method I propose here does better at low frequencies while Lorenzoni's is superior at high frequencies. It also appears to be the case that the share of the variance of the true process for expectations explained by my method is higher, especially when highly autocorrelated variables are considered. For these reasons I opt for this approximation scheme.
Another benefit of my approach is that it does not require the assumption that variables prior to a certain number of periods in the past take on value zero. This results in the ability to "safely" reduce the number of lags included in the state vector (T). The results in this paper are obtained for $T = 2$.
Finally, note that this approximation method, although obviously close along certain dimensions, differs in a crucial way from the classic reference of Krusell and Smith (1998)[21]. In a world with linear decision-making rules (guesses) and Gaussian innovations, the distribution is not the object of the approximation. Rather, the approximation should be thought of as reducing a very complicated filter describing expectations to a simple linear function of the state which allows an updating procedure similar to that in Lorenzoni (2009)[23].

A.5 Updating

The evolution of bond holdings gets updated with each successive computation of Θ because of the expectational term, while for the other endogenous variables the updating depends on the guesses. In particular, aggregating them and using the approximation for expectations[41]:

$$
\begin{aligned}
c_t &= \phi_2 a_t + \underline{\phi}\overline{\Theta}\mathbf{Z}_{ht} &(64)\\
w_t &= \theta_2 a_t + \underline{\theta}\overline{\Theta}\mathbf{Z}_{ht} &(65)\\
p_t &= \pi_2 a_t + \underline{\pi}\overline{\Theta}\mathbf{Z}_{ht} &(66)
\end{aligned}
$$

Expressing these same equations using indicator vectors I obtain:

$$
\begin{aligned}
\mathbf{e}'_{11}\mathbf{Z}_{ht} &= \left(\phi_2\mathbf{e}'_{10} + \underline{\phi}\overline{\Theta}\right)\mathbf{Z}_{ht} &(67)\\
\mathbf{e}'_{13}\mathbf{Z}_{ht} &= \left(\theta_2\mathbf{e}'_{10} + \underline{\theta}\overline{\Theta}\right)\mathbf{Z}_{ht} &(68)\\
\mathbf{e}'_{12}\mathbf{Z}_{ht} &= \left(\pi_2\mathbf{e}'_{10} + \underline{\pi}\overline{\Theta}\right)\mathbf{Z}_{ht} &(69)
\end{aligned}
$$

These equations implicitly define the updating procedure
Theoretically one could think of solving the equations above as a function of A, letting the computer search for the values of A that make the RHS as close to the LHS as possible. However it seems way more efficient to implement an updating scheme rather than letting the computer wander around searching for the fixed point.
So I implement an updating procedure similar to Lorenzoni (2009) which I show in detail only for the first equation:

$$
\begin{aligned}
\mathbf{e}'_{11}\mathbf{Z}_{ht} &= \left(\phi_2\mathbf{e}'_{10} + \underline{\phi}\overline{\Theta}\right)\mathbf{Z}_{ht} &(70)\\
\mathbf{e}'_{11}\left(A\mathbf{Z}_{h,t-1} + B\underline{u}_{ht}\right) &= \left(\phi_2\mathbf{e}'_{10} + \underline{\phi}\overline{\Theta}\right)\left(A\mathbf{Z}_{h,t-1} + B\underline{u}_{ht}\right) &(71)
\end{aligned}
$$

Then I consider the restrictions stemming from the fact that the above equations have to hold for every value of the state and the innovation process:

$$
\begin{aligned}
\mathbf{e}'_{11}A^{updated} &= \left(\phi_2\mathbf{e}'_{10} + \underline{\phi}\overline{\Theta}\right)A &(72)\\
\mathbf{e}'_{11}B^{updated} &= \left(\phi_2\mathbf{e}'_{10} + \underline{\phi}\overline{\Theta}\right)B &(73)
\end{aligned}
$$

Finally I allow for some "smoothness" in the updating procedure[42]:

$$
\begin{aligned}
A_{new} &= qA^{updated} + (1-q)A \qquad 0 < q \leq 1 &(74)\\
B_{new} &= qB^{updated} + (1-q)B &(75)
\end{aligned}
$$

When the square distance between A and A_{new} and B and B_{new} is small enough[43] convergence is achieved.

[41] The matrix $\overline{\Theta}$ accounts for the fact that aggregating all the idiosyncratic components become zero.

[42] The results I present in this paper where obtained for a value of $q = .1$. It is important to bear in mind that while this slows down the updating it does not prevent the updating procedure to wander potentially very far from the initial conditions.

[43] I minimize the largest square difference among all the elements of matrices A and B.

B Calibration

Economic Parameters							
β_0	.99	α	.3	γ	2	σ	2
η	6	ϵ	11	ψ_1	-.0011	τ	.1
τ_B	.0012	ι_π	1.5	ι_y	0		
Aggregate Shocks Variances and Autocorrelations							
$\sigma^2_{u^a}$	$1.96e-04$	$\sigma^2_{u^r}$	$3.61e-06$	$\sigma^2_{u^v}$	$2.81e-05$		
ρ	.86	ρ_r	.7	ρ_v	0		
Idiosyncratic Components Variances and Autocorrelations							
$\sigma^2_{u^{\varepsilon^A}}$	$4.84e-04$	$\sigma^2_{u^{\zeta^P}}$	$1e-04$	$\sigma^2_{u^{\zeta^W}}$	$4e-04$	$\sigma^2_{u^{\zeta^\Lambda}}$	$1e-04$
ρ_{ε^A}	.86	ρ_{ζ^P}	.9	ρ_{ζ^W}	.99	ρ_{ζ^Λ}	.99
$\sigma^2_{u^{\zeta^\Xi}}$	$5.07e-05$	$\sigma^2_{u^{\varepsilon^R}}$	$9e-06$	$\sigma^2_{u^{\varepsilon^\chi}}$	$4e-06$	$\sigma^2_{u^{\varepsilon^T}}$	$4e-06$
$\rho_{\zeta^X i}$	.9	ρ_{ε^R}	. 7	ρ_{ε^χ}	.7	ρ_{ε^T}	.7

Please note that reported values are variances and variances of the innovations not of processes themselves, e.g. the variance of the aggregate interest rate innovation implies a one-standard deviation shock of 19bp which is the size of the shock used for impulse responses.
The calibration of the idiosyncratic and aggregate process for technology follow Mendes (2007) who in turn cites the work of Kahn and Thomas (2007)[20]. The volatility of the shock to the demand for goods is also borrowed from Mendes (2007) who relies on estimates by Busato(2004)[8].
The other coefficients governing the idiosyncratic coefficients are calibrated following a simple idea. Labor market variances and autocorrelations are assumed to be larger than corresponding values for goods market statistics because it seems obvious that the types of labor hired by the average firm tend to be "less representative" than the goods bought by the average family and because the types of labor needed by a business tend to change more slowly than the bundle of goods purchased by consumers.
The processes for the idiosyncratic components of transfers and the interest rate and the preference shocks are simply set at what seems a suitably small level while preventing full revelation.
In this respect the interest rate process deserves some further comments because of its key role. In particular it should be noted that, while the variance of the innovation to the idiosyncratic component is larger than that on the aggregate component, when one considers the overall variance of the island-level interest rate in this economy he will find that only 19 percent of its variance is explained by idiosyncratic components. The difference is explained by inflation entering the Taylor rule so that the variance of the aggregate components is much larger than it might seem simply looking at the innovation.

C Impulse Responses

While I have discussed the most interesting features of the model in the main body of the paper[44], for the sake of completeness I report the impulse responses of aggregate variables to the aggregate shocks in Figures 8, 9 and 10.

[44] A project I am working on with Alessia Paccagnini, tries to exploit data revisions to identify noise shocks in a VAR setting. Results from that paper will provide a benchmark to which I could compare the responses to noise shocks.

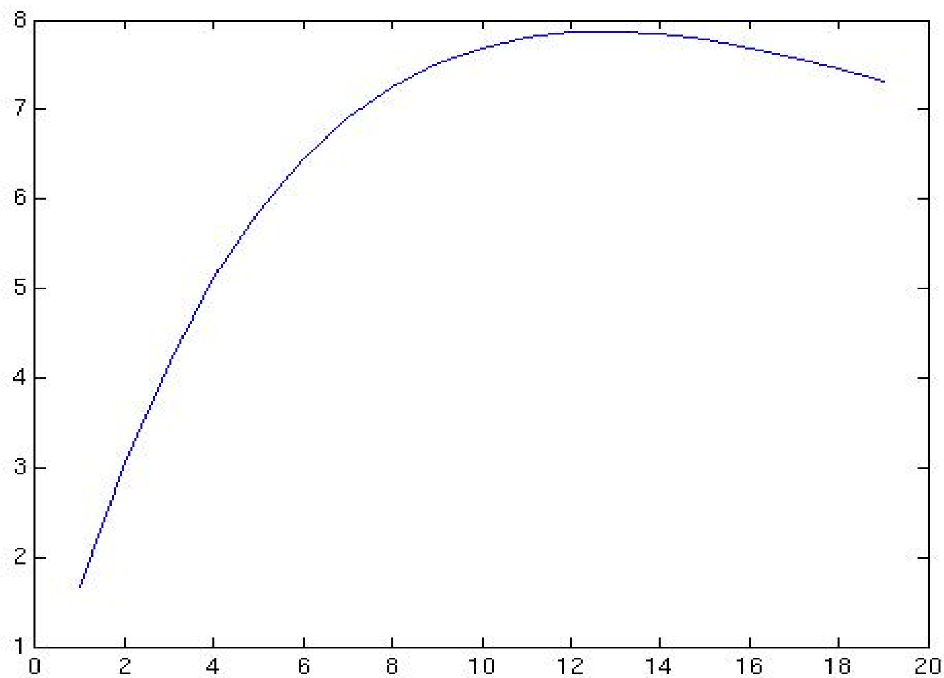

Figure 1: Response of savings to a 2.2 percent increase in the idiosyncratic technology component. The units on the vertical axis are percent of the steady-state consumption.

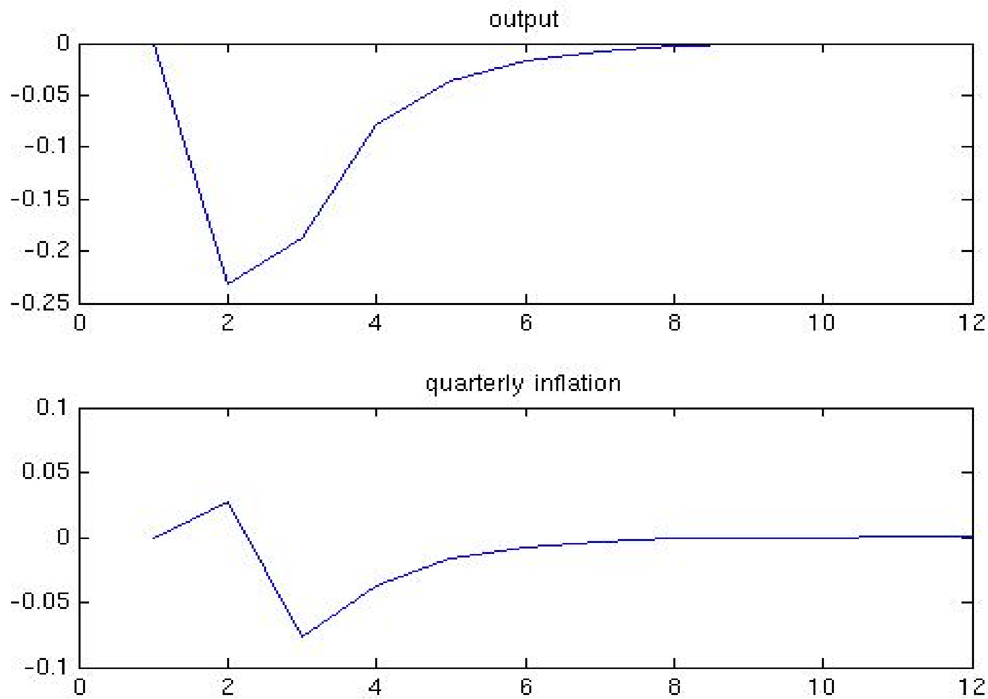

Figure 2: Responses of output and inflation to a 19bp increase in the Taylor-rule interest rate

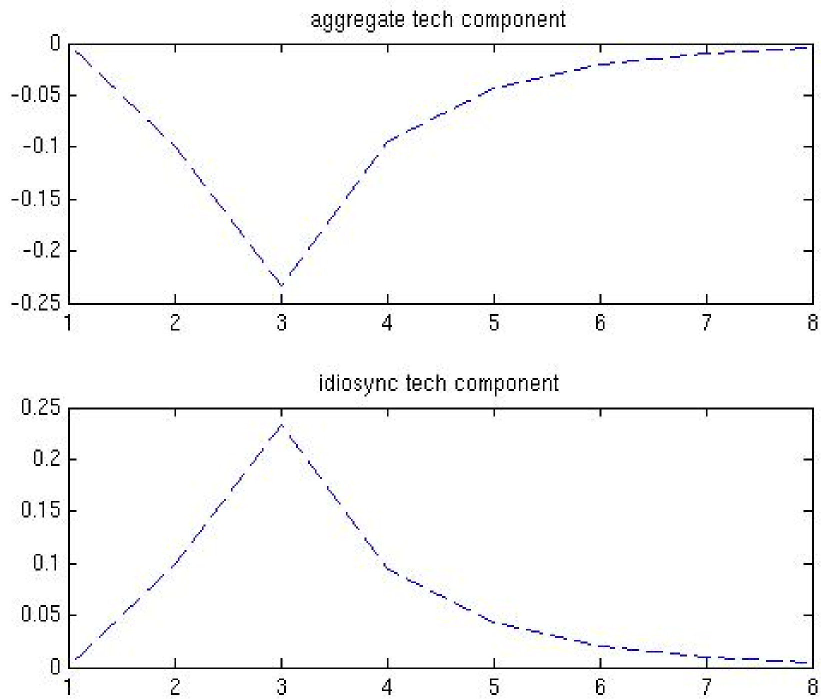

Figure 3: Evolution of the approximated expectations of the aggregate (top) and idiosyncratic (bottom) components of the technology process.

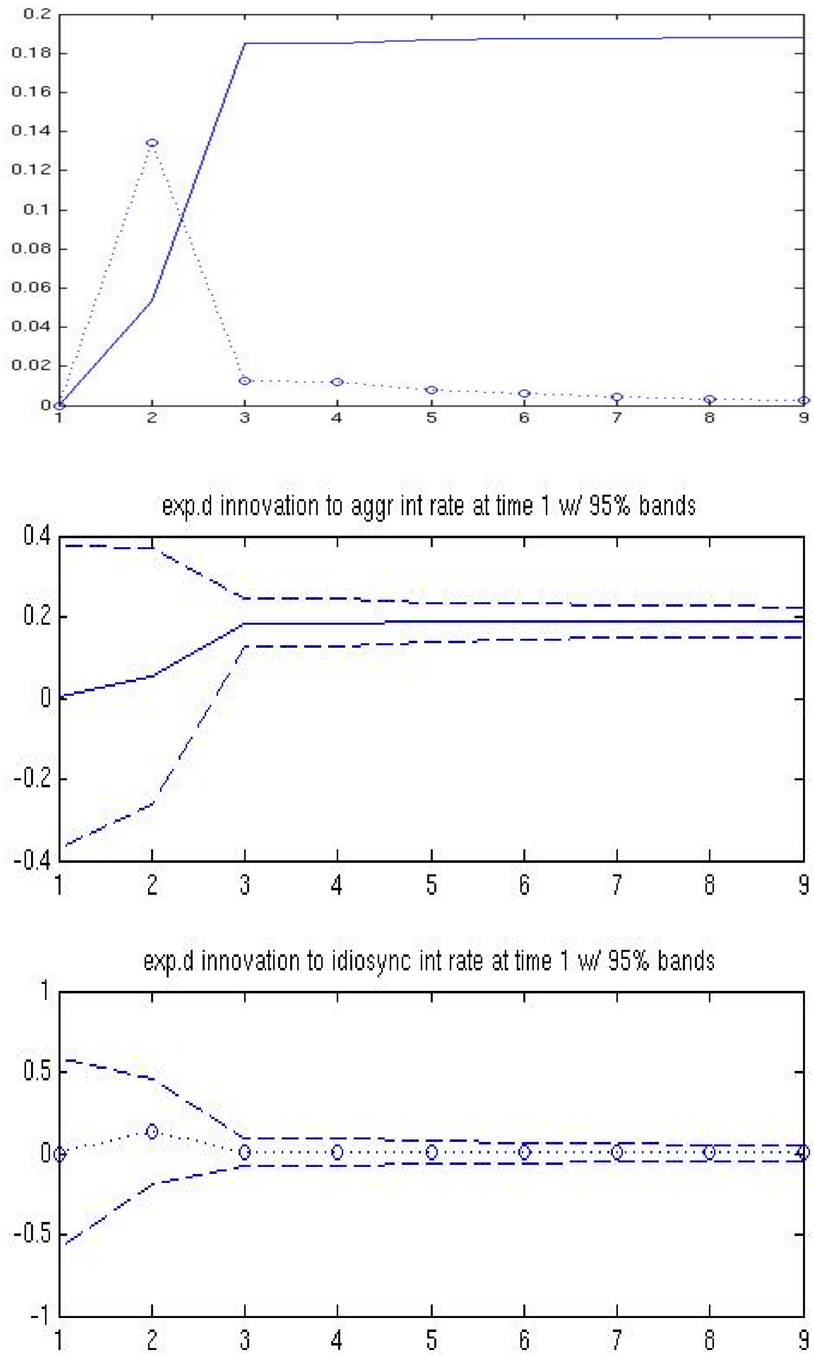

Figure 4: $E_{ht}[u_1^R]$ (solid) vs $E_{ht}[u_1^{\varepsilon^R}]$ (dotted) in the top pane, same with 95 percent bands in the lower panes

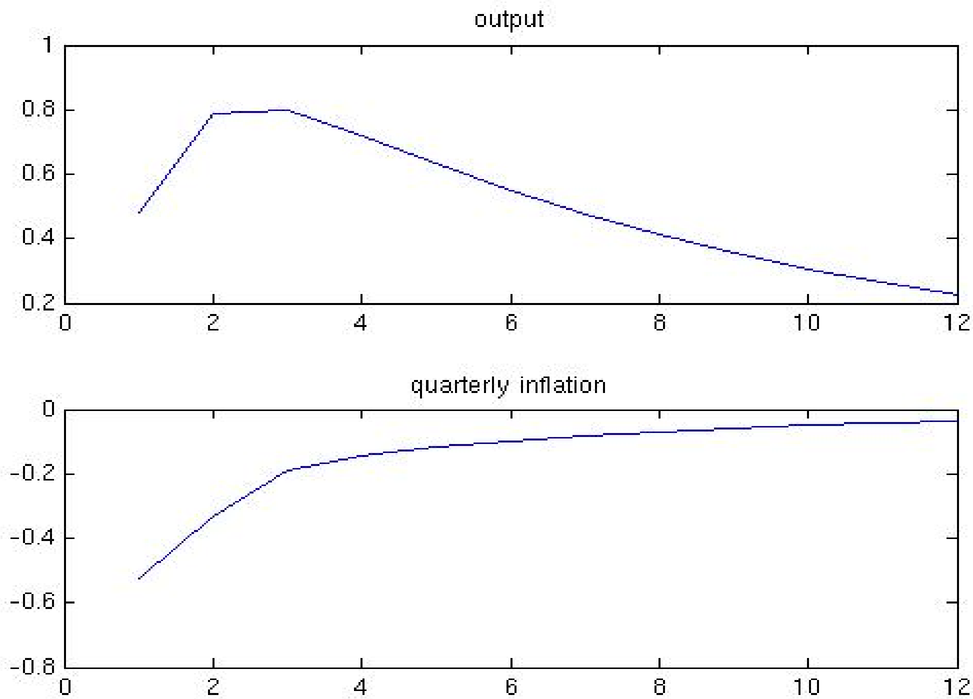

Figure 5: Responses of output and inflation to a positive one-stadard-deviation technological shock

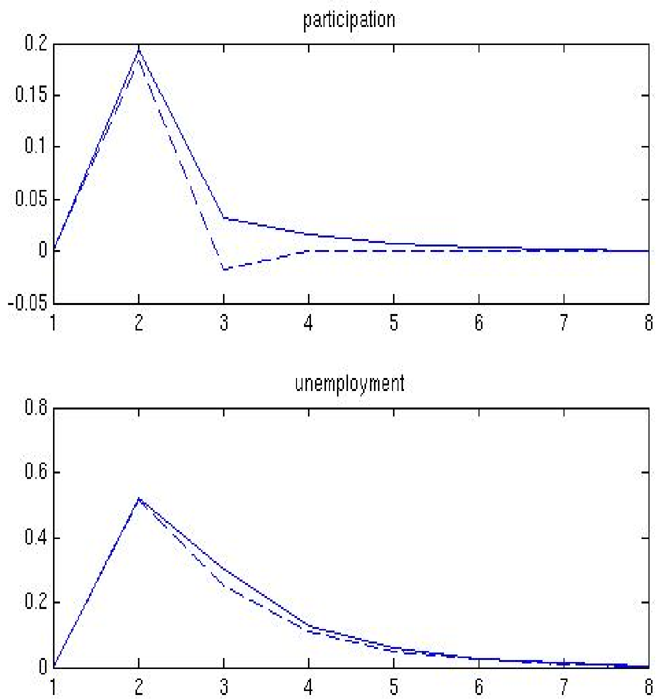

Figure 6: Responses of participation and unemployment to a MP shock for both the ex-post (solid) and ex-ante (dashed) definitions of participation.

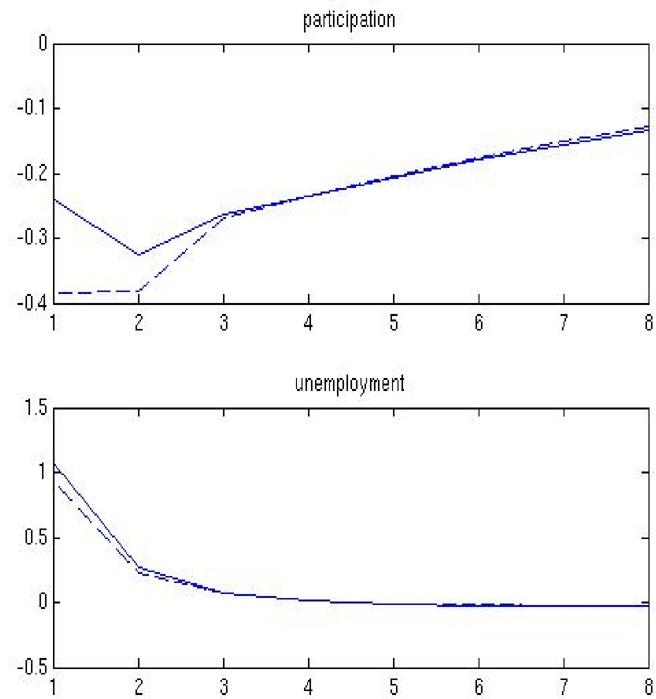

Figure 7: Responses of participation and unemployment to a tech shock for both the ex-post (solid) and ex-ante (dashed) definitions of participation.

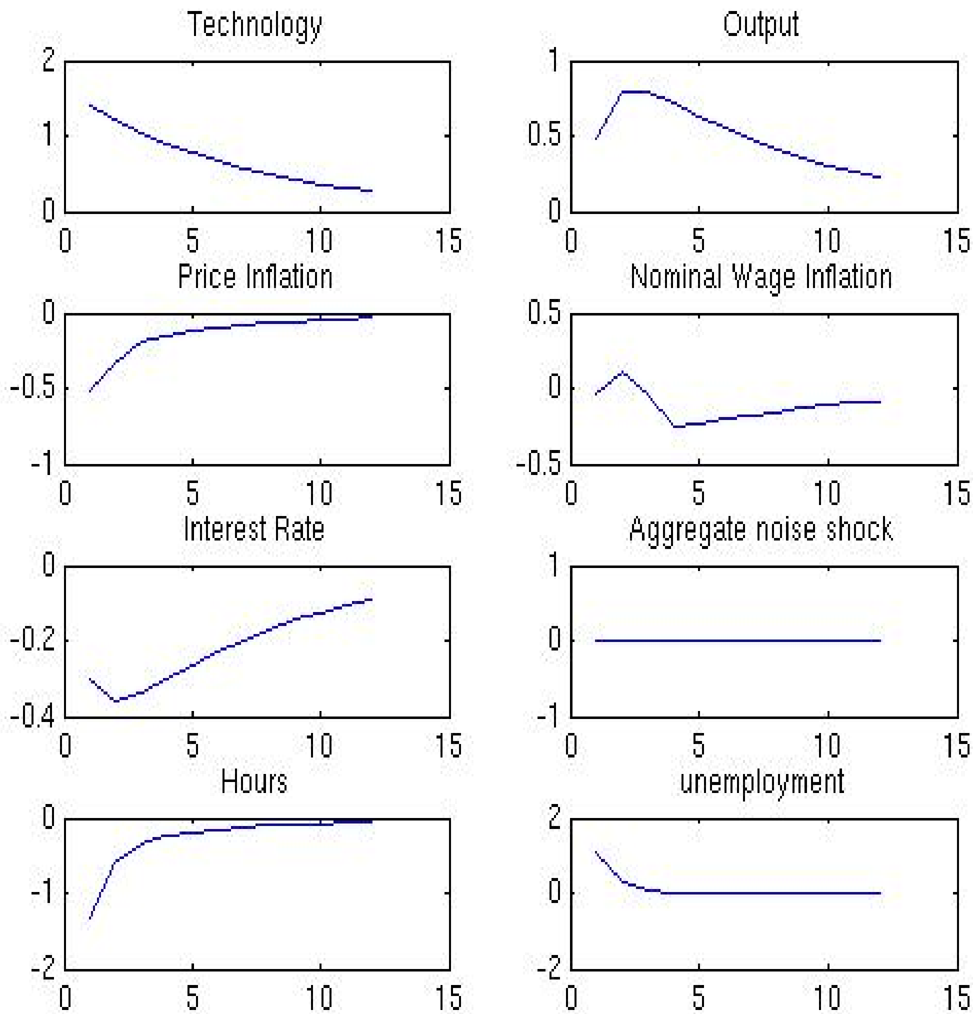

Figure 8: Responses of the main aggregate variables to an aggregate one-standard-deviation technology shock

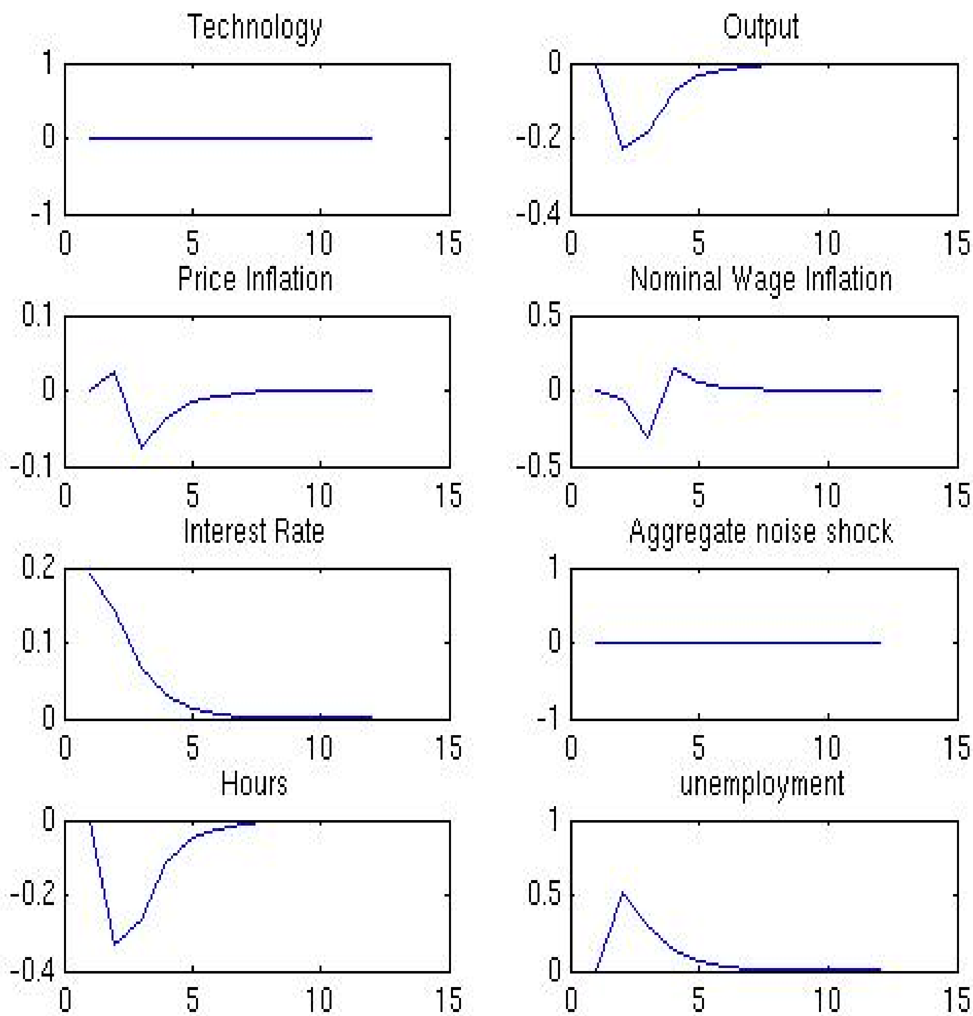

Figure 9: Responses of the main aggregate variables to an aggregate one-standard-deviation monetary policy shock

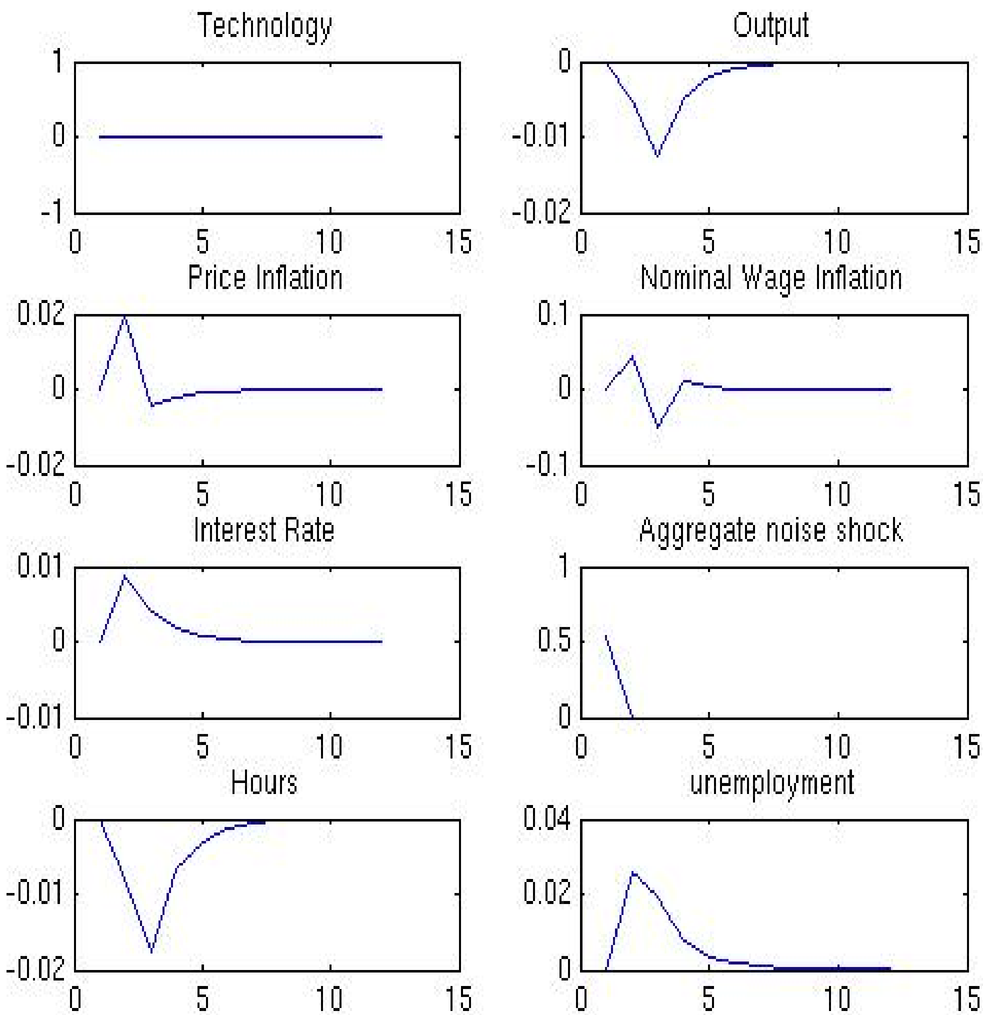

Figure 10: Responses of the main aggregate variables to an aggregate one-standard-deviation noise shock