Risk, Diversification and the Optimal Number of Export Destinations

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Abstract

We study the implications of introducing demand uncertainty into the canonical Melitz trade model. Uncertainty incentivizes firms to export as a means of reducing their overall risk, a mechanism we refer to as the diversification motive for exporting. A major result is that uncertainty per se can give rise to an export pattern in which there exist firms that serve any particular number of markets. Thus, we show that uncertainty by itself is sufficient to generate realistic export patterns that cannot be generated by the canonical model but that have, nevertheless, been documented in the data. Whether a firm becomes active or not and an active firm's optimal number of export destinations depend on the nature of the shocks the firm faces. In particular, we explore how these decisions are impacted by: (1) riskier global shocks (an identical increase in risk in all destinations); (2) riskier home-country shocks; (3) riskier foreign-country shocks; and (4) the correlation of shocks across foreign markets.

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1 Introduction

It is by now well-established that the number of firms exporting to a given number of destinations declines with the number of destinations.[1] Moreover, recent studies show that firms do not enter export markets according to a common hierarchy.[2] Standard trade models, which posit productivity as the main determinant of export status, cannot easily reconcile these patterns. A common workaround in the literature is to appeal to ad-hoc assumptions of additional firm- or country-level heterogeneity.[3] This, however, only trades one unexplained heterogeneity for another. As we show in this paper, not only does uncertainty obviate the need for additional heterogeneity to explain these patterns, but augmented with uncertainty, an otherwise standard model can also explain other features of the data, such as the negative correlation between home and foreign sales.[4]

Besides having the virtue of providing theoretical rationalizations for certain patterns in the data, introducing uncertainty into trade models seems natural given the ubiquitous nature of uncertainty. All firms face uncertainty, whether about demand for their goods, production costs or a plethora of other economic conditions. Beyond these uncertainties, exporters may face yet more uncertainty due to unpredictable transportation costs or unstable trade policies. Despite this, within the vast literature that examines why and to where firms export, the role of uncertainty has been relegated to relative insignificance.

Perhaps this gap in the literature exists because, it is thought, risk-neutral firms should be indifferent to uncertainty, or perhaps because uncertainty, even when modeled, is washed away by the law of large numbers.[5] But this presumed indifference of firms to risk belies

[1] See Eaton, Kortum and Kramarz (2004).

[2] See Lawless (2009) and Eaton, Kortum and Kramarz (2011).

[3] For instance, Arkolakis (2010) and Eaton, Kortum and Kramarz (2011) explain these patterns by assuming additional heterogeneity in market sizes and entry costs.

[4] See Blum, Claro and Horstmann (2013), Vannoorenberghe (2012) and Ahn and McQuoid (2015). These authors attribute the negative correlation to either convex production costs or capacity constraints.

[5] In Costinot (2009) whether a worker's contract will be enforced is uncertain and in Helpman, Itskhoki and Redding (2010) the match-specific productivity of any particular worker-firm pair is uncertain. Nevertheless, in both cases the uncertainty at the worker-firm level does not translate into uncertainty for the firm because each firm has a continuum of such worker-firm pairs.

a voluminous literature specifically dedicated to studying the impact of risk on firms' decision making, while the assumptions that do away with uncertainty are merely cosmetic modeling conveniences.[6] In this paper, we begin filling this gap in the literature by studying how uncertainty, in an otherwise standard Melitz framework, affects a firm's choice of export destinations. In the process we uncover an as yet unexplored motive for exporting, the diversification motive. In addition, the augmented model provides a wealth of novel predictions that relate the uncertainty faced by the firm to its choice of export destinations.

Granted, demand or cost uncertainty leads to profit uncertainty. But why might this affect the behavior of a risk-neutral firm? In particular, why might the presence of uncertainty influence such a firm's incentive to export? The following simple example will serve to illustrate.

Consider a firm that can serve only its domestic market and is able to produce a good at a marginal cost of four. Due to a demand shock, the good's price is uncertain and is, with equal probability, either zero or six. The firm's realized profit is then either negative four or two per unit produced depending on the realized price. Thus, if the firm learns the price before committing to production it would produce nothing when the price is zero but as much as technologically feasible otherwise. But what if the firm must commit to production before it learns the price realization? In that case, its expected profit would be negative one per unit produced, and as a result, a risk-neutral firm would produce nothing.

Now suppose that the firm, while still required to commit to production before the shock realizations, has the option, in addition to serving its home market, to export to a foreign market that is, prior to the realization of the demand shock, identical to the home market. Would this change the firm's behavior? The answer depends on how correlated the shocks are across the two markets. If shocks are perfectly positively correlated, the

[6] According to a Deloitte (2013) survey of over 300 mostly C-level executives, 81% of companies explicitly focus on managing strategic risk. Hoyt, Moore and Liebenberg (2008) find that the value of firms that invest in enterprise risk management is 17% higher than firms that do not. In the trade context, Hericort and Nedoncelle (2015) find that firms tend to reallocate exports away from destinations with high real-exchange rate volatility.

firm would still choose not to produce as it would still in expectation lose one per unit produced regardless of where it ultimately sells the good. However, this would not be the case if shocks are perfectly negatively correlated, that is, whenever the realized price in the domestic market is zero, it is six in the foreign market, and vice versa. Indeed, the firm's profit would then always be two per unit produced regardless of the particular price realization in each of the destinations, as the firm would always sell whatever it produced in the location with the higher realized price. Not only does access to the additional market prevent the firm from shutting down, the firm would be willing to pay for this access even before learning whether the positive shock will be at home or abroad. All this despite the fact that each market, when considered in isolation, is a loser for the firm.

The underlying intuition is this: when the home and foreign shocks are imperfectly correlated, the two markets can act as mutual shock absorbers. If the home country receives a bad shock while the foreign country receives a good shock, then the foreign destination is able to absorb the output. If the shocks are reversed, then the output otherwise slated for the foreign country can be sold at home to take advantage of the higher domestic price. This illustrates the fact that firms may export not only as a means to access a larger customer base, but also as a means to diversify their sources of demand, which provides an insurance policy against negative shocks. This market-diversification mechanism is what we refer to as the diversification motive for exporting.

This mechanism, of course, extends beyond the barebones two-country setting. Even in our much richer and more realistic many-country general-equilibrium setting, not only does this basic insight survive, but it generates a rich set of novel predictions.

Our model is based on the standard Melitz (2003) framework. In particular, monopolistically competitive firms with heterogeneous productivities decide whether to become active and which export markets, if any, to serve. Due to fixed production and export costs, more productive firms are more likely to be active and to export. Unlike in the Melitz model, firms face destination-specific shocks whose distribution depends on the sector. To capture the fact that firms must make certain irreversible decisions before uncertainty is resolved, in the model firms choose their export destinations and employment prior to learning the realization of shocks.[7] Thus, the optimal number of destinations depends on the *distribution* of shocks rather than on the eventual *realization* of the shocks.

Choosing export markets and committing to employment before learning the realizations exposes the firm to risk. However, the firm can generally alleviate some of this risk through market diversification, thereby raising its expected profit. Specifically, foreign countries can mitigate risk stemming from the home country; the home country can mitigate risk stemming from the foreign-country; and one foreign country can mitigate risk stemming from other foreign countries. We demonstrate how each of these three risk-reducing channels can increase a firm's incentive to export. In this way, we show that market diversification constitutes an independent motive for exporting and can even increase the probability that a firm remains active.

After illustrating the diversification motive for exporting, we go on to show that the counterfactual prediction of a bang-bang solution (if countries are symmetric, firms either do not export or export to all destinations) in the standard Melitz model need not hold. This is because although each destination confers a risk-reducing benefit, the marginal benefit may decrease, leading to an expected profit function that is strictly concave in the number of export destinations. As a consequence, even if the firm finds it profitable to export to one, or even to several, destinations, there will, in general, be some number of export destinations beyond which it is no longer profitable to export. This particular number depends on the firm's productivity. Indeed, we show that if foreign shocks are perfectly correlated, then for any given number of export destinations, there exist firms that optimally serve that number of destinations. This result highlights that uncertainty by itself can explain why firms choose to export to some destinations and not others even when, from the point of view of the firm, the destinations are indistinguishable.

Having established that an exporting firm generally will export to some but not all

[7] Our results would be similar if production would require both capital and labor with firms choosing the level of capital before the realization of shocks and choosing employment after the realization of shocks.

destinations, we next consider the impact of risk on a firm's choices.[8] Although risk has an unambiguously negative impact on a firm's expected profit, the impact on the optimal number of export destinations is less obvious. This is because although increased risk lowers the *level* of a firm's expected profit, it may increase the *marginal* expected profit from exporting to an additional destination since more risk means greater benefit from diversifying through exporting.

We show that in sectors where global shocks (shocks that are perfectly correlated across all destinations) are riskier, firms are less likely to be active and active firms tend to export to fewer destinations. We also show that while firms in sectors with riskier foreign shocks are no less likely to be active, firms in such sectors tend to export to fewer destinations if the shocks are perfectly correlated across foreign markets. Moreover, in sectors where shocks across foreign destinations are less correlated, exporting firms tend to export to more destinations. The logic is that when shocks are less correlated there is a greater scope for reducing risk through market diversification and hence a stronger diversification motive for exporting. Finally, we show that in sectors with riskier home-country shocks, more firms choose to be inactive. However, those firms that do choose to be active are more likely to export and tend to export to more destinations as compared with firms in sectors with less risky home-country shocks.

A growing literature examines the relationship between trade openness and volatility.[9] However, this literature mainly focusses on macroeconomic patterns and takes trade openness as an exogenous variable. As such, while being instructive regarding empirical patterns, this literature sheds little light on the causal relationship between trade and volatility on the microeconomic level. In this paper, we take the view that volatility itself can be an important determinant of international trading activity on the firm level.[10]

Besides contributing to the literature on the determinants of a firm's export status, our

[8] We use the Rothschild and Stiglitz (1970) definition of risk that the distribution S'' is riskier than the distribution S' if S'' can be obtained from S' by a mean-preserving spread.

[9] See, for example, Giovanni and Levchenko (2010).

[10] In a related contribution, Krishna and Levchenko (2013) posit that less developed countries have a comparative advantage in less complex goods, which, in turn, are characterized by higher levels of volatility.

paper is related to a new strand of the trade literature which highlights uncertainty and volatility as important determinants of observed trade patterns. This literature has been inspired by the finding that individual firms' entry into and exit out of export markets are important drivers of observed trade flows.[11]

One common explanation for this finding is that firms experiment by exporting to different markets as a way of learning about foreign demand for their product.[12] Another explanation is that firms serve a market in periods when demand is relatively high, but not in periods when demand is relatively low. Thus, if demand is volatile, so too is a firm's exporting decision.[13] In contrast to our model, in this class of models uncertainty is always resolved before the decision to export is made. As a result, it is the realizations of the shocks that determine whether or not a firm exports rather than the very existence of uncertainty. Put another way, in those models the results would be identical if demand were volatile but certain, that is, if demand fluctuated in a foreseeable way, whereas in our model it is the uncertainty as such that drives the export decision.[14]

2 The Model

There is a finite number $N \geq 2$ of symmetric countries indexed by i. Each country is populated by a continuum of $\mathcal{L}$ homogeneous workers who each supply one unit of labor. There is a set $\mathcal{S}$ of sectors indexed by S. Firms in sector S produce intermediate goods using labor, the only factor of production. Workers, who provide this labor, derive utility by consuming nontradable final goods, each of which is an aggregate of these sector-specific intermediate goods.

[11] See Eaton, Kortum and Kramarz (2004), Bernard, Jensen, Redding and Schott (2007) and Eaton, Eslava, Kugler and Tybout (2008).

[12] See Akhmetova and Mitaritonna (2012), Albornoz, Corcos and Pardo (2012) and Nguyen (2010).

[13] See Vannoorenberghe, Wang and Yu (2014). Blum, Claro and Horstmann (2013) show that in the presence of increasing costs a negative shock at home can incentivize a firm to export.

[14] In this sense our paper is closer in spirit to Juvenal and Santos Monteiro (2013) where the decision to invest in productivity improvements depends on how diversified the firm's export portolio is. However, in that paper, unlike in ours, the decision to export is exogenous.

Workers' utility in country i is

$$U_i = \left[\sum_{S \in \mathcal{S}} Q_i(S)^{\rho} \right]^{1/\rho}, \tag{1}$$

where $Q_i(S)$ is the quantity supplied of the sector-S final good in country i and $\rho \equiv (\sigma - 1)/\sigma$ with $\sigma > 1$. The supply of the sector-S final good in country i is

$$Q_i(S) = \left[\int_{\omega \in \Omega_i(S)} \gamma_i(\omega)^{1/\sigma} q_i(\omega)^{\rho} d\omega \right]^{1/\rho}, \tag{2}$$

where $\Omega_i(S)$ is the endogenous set of sector-S intermediate goods available in country i, $q_i(\omega)$ is the quantity supplied of the sector-S intermediate-good ω in country i, and $\gamma_i(\omega)$ is the realization of a country-specific demand shock to ω.[15]

Given this nested CES structure, workers in country i can equivalently be considered to consume a quantity $Q_i \equiv U_i$ of an aggregate final good whose price is

$$P_i = \left[\sum_{S \in \mathcal{S}} P_i(S)^{1-\sigma} \right]^{1/(1-\sigma)}, \tag{3}$$

where $P_i(S)$ is the price of the sector-S final good in country i. This price is given by

$$P_i(S) = \left[\int_{\omega \in \Omega_i(S)} p_i(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}, \tag{4}$$

where $p_i(\omega)$ is the price of intermediate-good ω in country i.

Sectors differ in the distribution of γ_i-shocks that firms face. The nested CES structure with the same elasticity of substitution within and across sectors allows us to hone in on the impact of this difference, while controlling for others, such as differences in sector size and productivity. Straightforward consumer optimization shows that the demand for a firm's intermediate good is $q_i(\omega) = Q_i P_i^{\sigma} \gamma_i(\omega) p_i(\omega)^{-\sigma}$, which is independent of the firm's

[15] For simplicity, we refer to the $\gamma_i(\omega)$'s as demand shocks, but they can also be interpreted as cost shocks.

sector. Thus, the demand for a firm's intermediate good in a country depends only on the realization of its country-specific shock and economy-wide, rather than sector-specific, aggregates.

We will focus on symmetric equilibria. This implies that the wage, w_i, as well as the economy-wide aggregate variables Q_i, P_i, and total expenditure in all sectors, R_i, will be the same in each country. We therefore omit the subscript i in these variables. Furthermore, we normalize the price of the aggregate final good, P, to unity so that market clearing implies that $R = Q$. The subsequent analysis is from the point of view of country 1, the home country, with the understanding that the results are identical for countries $2, 3, \cdots, N$, the foreign countries.

2.1 Firms

In each sector, S, in the home country, there is a unit continuum of expected-profit-maximizing firms whose productivities, φ, are distributed according to the cumulative distribution function, $G(\cdot)$, that strictly increases on $(0, \infty)$. A sector-S firm with productivity φ requires $1/\varphi$ units of production labor to produce one unit of its unique sector-specific intermediate good. Firms are owned equally by all workers, who therefore each receive an equal share of firm profits.

Sector S is characterized by a distribution representing the probability with which firms, independently from one another, draw any particular N-dimensional vector of country-specific demand shocks, $\Gamma \equiv (\gamma_1, \gamma_2, \cdots, \gamma_N)$. Since sectors differ only in this distribution, we use S to denote both the sector and the sector's characterizing distribution. In sector S, for any home-country shock, γ_1, the foreign-country shocks are, with probability $\theta_S \in [0, 1]$, *common*, that is, $\gamma_2 = \gamma_3 = \cdots = \gamma_N$, or, with probability $1 - \theta_S$, *idiosyncratic*, that is, $\gamma_2, \gamma_3, \cdots, \gamma_N$ are independently and identically distributed. Thus, conditional on the home country shock, γ_1, the common foreign shocks are drawn from the distribution $S_c(\gamma \mid \gamma_1)$ when foreign shocks are common, and each country's idiosyncratic shock is independently drawn from the distribution $S_d(\gamma \mid \gamma_1)$ when foreign shocks are idiosyncratic.[16]

Knowing its productivity, φ, every firm in sector S chooses a subset $\mathcal{N} \subseteq \{1, 2, ..., N\}$ of markets to serve. A firm can always ensure zero profit by remaining inactive, which corresponds to choosing $\mathcal{N} = \varnothing$. Alternatively, a firm can become active, that is, choose $\mathcal{N} \neq \varnothing$, by paying a fixed overhead cost consisting of $f_o > 0$ units of labor. Serving the home market requires no additional fixed cost, and therefore, due to the CES demand structure, an active firm will always include the home market in $\mathcal{N}$. Serving foreign markets, however, requires the firm to incur an additional fixed exporting cost of $f_x > 0$ units of labor per market. Incurring this cost allows the firm to sell as much as it wishes at its chosen export destinations, but due to an iceberg cost, the firm must ship $\tau > 1$ units of its good for each unit it sells abroad.

Concurrently with its choice of markets, an active firm also chooses the measure L of labor to hire, where L must be sufficiently large to at least cover the fixed costs, $f_o + nf_x$ of labor, where $n \equiv |\mathcal{N}| - 1$ is the number of export destinations. As a result, a firm with productivity φ that exports to n countries will produce $\varphi\ell$ units of its intermediate good, where $\ell \equiv L - f_o - nf_x$ is the measure of production labor hired by the firm. Only after a sector-S firm has chosen $\mathcal{N}$ and L, it draws the shocks Γ from the distribution S. Aware that the demand for its sector-S intermediate good in country i is $q_i = Q\gamma_i(\omega)p_i(\omega)^{-\sigma}$, the firm then sets prices and associated quantities in its chosen markets.

Summarizing the timing of the model: In the first stage, in each sector firms decide whether or not to become active and which markets to serve. In the second stage, active firms decide how much labor to hire. Finally, in the third stage, the realizations of shocks are revealed and active firms set prices and quantities in their chosen markets.

[16] Formally, for all γ_1, if foreign shocks are common, then $S(\gamma_2, ..., \gamma_N|\gamma_1) = S_c(\gamma_2|\gamma_1)$ and $\gamma_2 = \cdots = \gamma_N$, and if foreign shocks are idiosyncratic, then $S(\gamma_2, ..., \gamma_N|\gamma_1) = \prod_{i=2}^{N} S_d(\gamma_i|\gamma_1)$.

2.2 Firm Optimization

The firm's optimization problem can be solved by iterating backwards over the three stages outlined above. In maximizing its expected profit, an individual firm views the wage and final good expenditure in each country as exogenous, although they are endogenously determined in general equilibrium. Moreover, as there is no aggregate uncertainty, the firm views these variables as constants rather than as random variables.

Since all countries are ex-ante identical, a firm's *expected* profit, which is what matters when it chooses its export destinations, depends on their number rather than on the specific destinations. Therefore, in what follows, we will consider a firm's choice to be of the number of export destinations, n, rather than of the set of export destinations, $\mathcal{N}\setminus\{1\}$.[17] Nevertheless, because the realizations of shocks potentially differ across destinations, a firm's *realized* profit will depend on its particular choice of $\mathcal{N}$.

Third Stage An active firm in sector S with productivity φ determines the prices that maximize its profit for every triplet $(\mathcal{N}, L, \Gamma)$, that is, for every potential choice of export destinations and employment as well as realization of shocks. Consider a particular set of these variables. Since the firm produces $\varphi\ell$ units of its sector-S intermediate good, it will set its prices to

$$p_1 = \left[\frac{QA(\Gamma, \mathcal{N})}{\varphi\ell}\right]^{1/\sigma}$$

and $p_i = \tau p_1$ for $i \in \mathcal{N}\setminus\{1\}$, where

$$A(\Gamma, \mathcal{N}) \equiv \gamma_1 + \tau^{1-\sigma} \sum_{i\in\mathcal{N}\setminus\{1\}} \gamma_i$$

is the sum of the firm's country-specific shocks accounting for the iceberg cost. Given φ and the triplet $(\mathcal{N}, L, \Gamma)$, the firm's revenue is $\varphi^{\rho} Q^{1/\sigma} \ell^{\rho} A(\Gamma, \mathcal{N})^{1/\sigma}$.

Second Stage The active firm in sector S with productivity φ determines the employment that maximizes its expected variable profit for any $\mathcal{N}$. The expected variable profit

[17] The n export destinations are chosen randomly from the $N-1$ foreign countries.

function is (recalling that $n \equiv |\mathcal{N}| - 1$)

$$\varphi^{\rho} Q^{1/\sigma} \ell^{\rho} b(S, n)^{1/\sigma} - w\ell, \tag{5}$$

where

$$b(S, n) \equiv \left[\int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} \, dS(\Gamma) \right]^{\sigma}$$

is the expected generalized sum (of degree $1/\sigma$) of the shocks in the home country and the selected export destinations taking τ into account. We can interpret $b(S, n)$ as the certainty-equivalent generalized sum of the relevant shocks for a sector-S firm that exports to n countries. That is, the expected variable profit is the same as if $b(S, n)$ were the certain generalized sum of the firm's shocks in the home country and the n foreign countries taking the iceberg cost into account.

Differentiating (5) with respect to ℓ shows that for a given n the optimal employment of production workers is given by

$$\ell(\varphi, S, n) = \frac{\varphi^{\sigma-1} Q \rho^{\sigma} b(S, n)}{w^{\sigma}}. \tag{6}$$

Substituting this optimal employment back into the expected variable profit function shows that a sector-S firm's maximized expected variable profit for a given n is

$$\pi(\varphi, S, n) \equiv \varphi^{\sigma-1} k b(S, n),$$

where

$$k \equiv \frac{Q \rho^{\sigma-1}}{\sigma w^{\sigma-1}}$$

is independent of the firm's productivity, the sector, the realization of shocks, and the number of export destinations.

First Stage The firm determines the number of export destinations that together with the optimal employment maximizes its expected profit. Let the expected profit of an active firm in sector S that exports to n destinations be denoted by

$$\Pi(\varphi, S, n) \equiv \pi(\varphi, S, n) - w(f_o + nf_x).$$

Thus, for an active firm the optimal number of export destinations is given by $n(\varphi, S) = \text{argmax}_n \Pi(\varphi, S, n)$. Accordingly, the firm becomes active and exports to $n(\varphi, S)$ countries if $\Pi[\varphi, S, n(\varphi, S)] > 0$, and remains inactive otherwise. Therefore, using eq. (6), the optimal employment for an active firm is given by

$$L(\varphi, S) = \frac{\varphi^{\sigma-1} Q \rho^{\sigma} b[S, n(\varphi, S)]}{w^{\sigma}} + f_o + n(\varphi, S) f_x.$$

Finally, a firm will choose to be active only if its variable profit is sufficient to cover the fixed costs associated with production and exporting. Thus, a firm in sector S will choose to be active only if its productivity is above an endogenous activity cutoff $\varphi_0(S)$, and it will choose to remain inactive otherwise. This activity cutoff must exist because a firm's expected variable profit, conditional on sector, increases in φ, while the fixed overhead cost is the same for all firms.

3 Export Cutoffs

We assume that in every sector there are active firms that do not export.[18] Indeed, this is the most relevant scenario, as one would be hard pressed to find a sector in which all active firms export. This assumption, however, is made for expositional clarity, and, as will become apparent, its violation does not alter the results in any meaningful way.

Conditional on a sector's distribution of shocks, $S(\cdot)$, for any given number of export

[18] If the fixed overhead cost is small relative to the fixed exporting cost, and in particular if $f_o \to 0$, then it can be guaranteed that for any S there exist active firms that do not export. However, this will hold also for many other possible values of the model's parameters.

destinations, n, the gain in expected variable profit from exporting to one additional country, $\pi(\varphi, S, n+1) - \pi(\varphi, S, n)$, is proportional to $\varphi^{\sigma-1}$. At the same time, the fixed cost of exporting to one additional country, f_x, is independent of φ. Therefore, within a sector, the number of export destinations weakly increases in φ. Thus, there exist endogenous sector-specific export cutoffs of productivities, $\varphi_1(S) \leq \varphi_2(S) \leq \cdots \leq \varphi_{N-1}(S)$, such that a firm in sector S exports to exactly n countries if its productivity falls in the interval $\Phi_n(S) \equiv (\varphi_n(S), \varphi_{n+1}(S)]$, where $\varphi_N(S) \equiv \infty$. If $\varphi_n(S) = \varphi_{n+1}(S)$, then $\Phi_n(S)$ is empty so that no firms in sector S export to exactly n countries.

That some active firms do not export is tantamount to $\varphi_0(S) < \varphi_1(S)$, so that $\Phi_0(S)$ is nonempty. Also, since f_x is finite, there will always be very productive firms that prefer to export to all foreign countries. It follows that $\varphi_{N-1}(S) < \infty$ so that $\Phi_{N-1}(S)$ is nonempty. However, as we shall see, some or all of $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ may be empty, with the implication of an empty $\Phi_n(S)$ being that no sector-S firm exports to precisely n destinations.

4 General Equilibrium

General equilibrium in the economy requires that both the labor and goods markets clear in every country. These market-clearing conditions jointly determine w and Q.

The labor demand of home-country firms in sector S is

$$L^D(S) = \sum_{n=0}^{N-1} \int_{\varphi_n(S)}^{\varphi_{n+1}(S)} [\ell(\varphi, S, n) + f_o + n f_x] \, dG(\varphi).$$

Thus, the labor market clearing requires that the labor demand of home-country firms in all sectors equals the labor supply,

$$\sum_{S \in \mathcal{S}} L^D(S) = \mathcal{L}. \tag{7}$$

An active home-country firm in sector S with productivity φ that exports to n countries produces $\varphi\ell(\varphi,S,n)$ units of its intermediate good. If the realization of its shocks is Γ, it sells the fraction $\gamma_1/A(\Gamma,\mathcal{N})$ of its output at home and the fraction $\gamma_i\tau^{-\sigma}/A(\Gamma,\mathcal{N})$ of its output in country $i\in\mathcal{N}\setminus\{1\}$. The remaining production is dissipated by the iceberg cost. Consequently, such a firm sells $q_1(\eta)=\gamma_1\varphi\ell(\varphi,S,n)/A(\Gamma,\mathcal{N})$ of its intermediate good at home, and $q_i(\eta)=\gamma_i\tau^{-\sigma}\varphi\ell(\varphi,S,n)/A(\Gamma,\mathcal{N})$ of its intermediate good in foreign country $i\in\mathcal{N}\setminus\{1\}$, where $\eta\equiv(\varphi,S,n,\Gamma)$.

The probability that a firm that exports to n countries exports to any one particular country is $n/(N-1)$. Using this fact and symmetry across countries yield the supply of the sector-S final good

$$Q\left(S\right)=\left(\sum_{n=0}^{N-1}\left\{\int_{\varphi_n(S)}^{\varphi_{n+1}(S)}\int_{\Gamma\in\mathbf{R}^N}\left[{\gamma_1}^{1/\sigma}q_1(\eta)^\rho+\frac{n}{N-1}\sum_{i=1}^{n}{\gamma_i}^{1/\sigma}q_i(\eta)^\rho\right]dS\left(\Gamma\right)dG\left(\varphi\right)\right\}\right)^{1/\rho}.$$

This, together with equation (1), yields the supply of the aggregate final good, Q. Since workers are the only source of demand for goods, expenditure on the final good must equal their labor income, $w\mathcal{L}$, plus distributed profits,

$$\Psi=\sum_{S\in\mathcal{S}}\left\{\sum_{n=0}^{N-1}\left[\int_{\varphi_n(S)}^{\varphi_{n+1}(S)}\Pi(\varphi,S,n)dG\left(\varphi\right)\right]\right\}.$$

Final good market clearing then requires $Q=w\mathcal{L}+\Psi$.

5 The Diversification Motive for Exporting

Suppose a firm had to commit to the specific quantity it will sell in each destination prior to learning the realization of shocks. In such a case, the firm may as well consider each destination in isolation. However, when, as in our model, a firm only determines its total output rather than how it will eventually distribute this output after it learns the shock realizations, considering each destination in isolation is no longer optimal. Rather, the firm gains by taking advantage of the option of adjusting the quantity it sells in each destination after learning the realization of the shocks. Since nothing prevents the firm from foregoing this option, its expected profit must be at least as large as in the absence of this option.

The above argument demonstrates that firms may benefit from selling to different markets since this reduces overall risk. In other words, firms are incentivized to export as a way of diversifying their sources of uncertain demand, which is why we call this mechanism the diversification motive for exporting. Indeed, in our model overall risk can be reduced in three ways: (1) foreign countries can mitigate risk stemming from the home country; (2) the home country can mitigate risk stemming from foreign countries; and (3) one foreign country can mitigate risk stemming from other foreign countries.

Before moving on to the main results in the paper, it is instructive to consider how each of these above-mentioned ways in which a firm can reduce overall risk affects its incentive to export. Since this incentive to export depends on the difference between the firm's expected profit when it serves only the domestic market and its expected profit when it exports, it will prove useful to define the expected variable profit from exporting to n destinations, $\pi^x(\varphi, S, n)$, as the expected variable profit when serving n foreign markets (in addition to the home market) less the expected variable profit when serving only the domestic market, i.e., $\pi^x(\varphi, S, n) \equiv \pi(\varphi, S, n) - \pi(\varphi, S, 0)$.

5.1 Foreign Countries Mitigate Risk Stemming from the Home Country

To highlight in the starkest way how firms can reduce overall risk by using foreign countries to mitigate risk stemming from the home country, consider first a sector in which there is no uncertainty at all, neither at home nor abroad. This sector will have an export cutoff above which firms export to all destinations and below which firms do not export at all. Next, consider a sector with the same certain foreign level of demand, but with home-country shocks drawn from a nondegenerate distribution, *any* nondegenerate distribution. What then happens to the gain from exporting? The following proposition provides the answer.[19]

Proposition 1 *In sector S', demand is certain everywhere. In sector S'', home-country demand is uncertain while foreign demand is certain and identical to demand in sector S'. Then $\pi^x(\varphi, S', n) < \pi^x(\varphi, S'', n)$ for all $n \geq 1$. Therefore, more firms in sector S'' export than in sector S'.*

Proposition 1 shows that a firm has greater incentive to export when the home country faces uncertainty. This result is all the more striking because it does not depend on how the home-country shocks are distributed, so long as there remain active firms that do not export. In particular, even if *all* the home-country shock realizations are lower than the home-country realization in the certain sector, more firms in the uncertain sector will export. What is going on here?

Since nothing about the foreign countries is changed by the uncertainty in the home country, exporting must be at least as profitable as in the full-certainty scenario. After all, with full certainty a firm can without loss consider each market in isolation. However, in the presence of uncertainty, doing so would be suboptimal for the firm. In particular, if the home country receives a good shock, some output, which would otherwise be shipped abroad, can be sold at home to take advantage of the relatively high home demand. Conversely, if the home country receives a bad shock, output otherwise destined for home can be shipped abroad where demand is relatively high. Since this gain from using exports as a buffer to absorb shocks at home exists only when home shocks are uncertain, the benefit from exporting is greater in the presence of uncertainty at home than in the case of full certainty.

[19]The proofs of all the propositions are in the Appendix.

5.2 The Home Country Mitigates Risk Stemming from Foreign Countries

In the previous subsection we considered a scenario with no uncertainty in the foreign countries, and therefore it was the foreign markets that absorbed risk stemming from the home country, but not vice versa. However, when foreign-shock realizations are uncertain, then the home country is able to absorb some foreign risk. To highlight the role of the home country in mitigating risk stemming from foreign countries as clearly as possible, consider now two sectors in which there is no home-country uncertainty, so that the only uncertainty being the foreign shocks.[20] Moreover, the two sectors are identical in every way except that in one sector the certain home level of demand is greater than in the other. The following proposition shows that the incentive to export is greater in the sector with the greater home demand.

Proposition 2 *Sectors S' and S'' have certain home-country demand and identical distributions of the foreign-country demand shocks. If $\gamma_1' < \gamma_1''$, then $\pi^x(\varphi, S', n) < \pi^x(\varphi, S'', n)$ for all $n \geq 1$. Therefore, more firms in sector S'' export than in sector S'.*

Just like in the scenario considered in Proposition 1, there is no difference in the foreign countries, and yet the higher home-country demand increases the incentive to export. However, in this case it is because a firm with a bigger home-country base is able to absorb more foreign risk so that the scope for mitigating risk stemming from the foreign country is greater. Thus, if a foreign country receives a favorable shock, a bigger home-country demand means that more output can be sent abroad to take advantage of the relatively high foreign demand. If a foreign country receives an unfavorable shock, some output, otherwise destined for sale abroad, can be diverted to the home market. The greater the level of demand at home, the smaller negative impact will this diversion have on the price the firm can charge at home.

[20] In this proposition (and the next) we assume certainty at home, even though uncertainty at home would not change the results.

5.3 One Foreign Country Mitigates Risk Stemming from Other Foreign Countries

The final way a firm can reduce overall risk is by using one foreign country to mitigate risk stemming from other foreign countries. To isolate the impact of this type of risk reduction, we consider sectors with the same certain level of home-country demand and in which the common foreign shocks and each foreign country's idiosyncratic shock are drawn from the same distribution. In such sectors, a higher likelihood of common foreign shocks, θ_S, does not affect the probability of any particular realization in any particular market, but does imply a higher probability that foreign markets receive the same realizations. Thus, a higher θ_S leads to a higher correlation of shock realizations across the foreign markets. The following proposition shows that the correlation of foreign shocks does not affect the incentive to export to one country, but that a lower correlation of foreign shocks provides an incentive to export to more than one foreign country.

Proposition 3 *Sectors S' and S'' have the same certain home-country demand ($\gamma_1' = \gamma_1''$) and $S_c(\gamma_2' \mid \gamma_1') = S_d(\gamma_i' \mid \gamma_1') = S_c(\gamma_2'' \mid \gamma_1'') = S_d(\gamma_i'' \mid \gamma_1'')$ for $i \geq 1$. If $\theta_{S'} > \theta_{S''}$, then*

1. $\pi^x(\varphi, S', 1) = \pi^x(\varphi, S'', 1)$;
2. $\pi^x(\varphi, S', n) < \pi^x(\varphi, S'', n)$ *for all* $n \geq 2$.

Therefore, more firms in sector S'' export to multiple destinations than in sector S'.

Like in both previous propositions, the comparative statics considered here compares scenarios in which each individual foreign country and even the home country are identical. Despite this, θ_S does affect the incentive to export. The reason is that when foreign shocks are idiosyncratic, an export market can alleviate the risk stemming from other export markets. Not so in the case of common foreign shocks, where one foreign market cannot alleviate the risk stemming from other foreign markets. Therefore, for firms that export to at least two countries, when the two types of foreign shocks have the same distribution, the expected profit is always greater when the idiosyncratic shocks are more likely. Since one foreign country can mitigate risk stemming from another foreign country only when firms export to at least two destinations, the particular value of θ_S is of no consequence to firms that do not export or export to only one destination.

5.4 The Optimal Number of Export Destinations

In Propositions 1-3, we considered only how the reduction of risk affects the *level* of the expected profit from exporting to n destinations. This is sufficient for illustrating how uncertainty and the ability to alleviate risk through market diversification affect whether or not firms choose to export (or to export to multiple destinations in the case of Proposition 3). However, in the remainder of this paper our goal is rather more ambitious. We are interested in how uncertainty affects the optimal number of export destinations. To this end it is necessary to examine the impact of uncertainty on the *marginal* expected profit from adding an export destination, that is, the difference in the expected profit when exporting to n rather than $n-1$ destinations.

The analysis in this section has shown that by adding an export destination a firm not only increases its market size, the standard justification for exporting, but also buys an insurance policy against shocks in its other markets. Therefore, for the analysis that follows it will be useful to decompose the marginal expected profit from one additional export destination into the marginal expected direct profit and the marginal expected profit from diversification. The marginal expected direct profit is defined as the portion of the marginal expected profit that the firm would earn if it considered the new destination in isolation, that is, the direct gain to a firm from having a larger customer base. The marginal expected profit from diversification is defined as the additional marginal expected profit the firm earns by taking advantage of its ability to adjust the quantity it sells in the additional destination after learning the realizations of shocks, that is, the total marginal expected profit less the marginal expected direct profit. This profit from diversification is unique to our paper and it is its properties that we will examine.

6 Bang-Bang Exporting?

In what follows, we explore the impact of uncertainty on firms' activity and export cutoffs and ultimately the optimal number of export destinations. In this section we ask: (1) Under what conditions does the model predict the bang-bang solution that firms either do not export or export to all destinations? (2) Under what conditions is uncertainty alone enough to overturn this counterfactual prediction of the standard Melitz model? These two questions are interesting in their own right, but more importantly, their answers shed light on the mechanisms that arise when firms must account for uncertainty when choosing their export destinations.

To explore the implication of uncertainty on firms' exporting policy, we consider three types of shocks: global shocks, where shocks are perfectly correlated across all destinations; non-global common foreign shocks, where only foreign shocks are perfectly correlated; and idiosyncratic foreign shocks, where each foreign destination receives a shock independent of the others. Each type of uncertainty affects a firm's incentive to export in different ways. Therefore, to most clearly highlight the new insights that emerge from the introduction of uncertainty, we begin the analysis by considering each type of shock in isolation.

Why might the bang-bang solution to a firm's exporting decision no longer hold when uncertainty is introduced? In the absence of uncertainty, symmetry across countries implies that the marginal direct profit is equal to the total marginal profit from an additional export destination and equal for every n. As a result, if it is worthwhile to export to one foreign country, it is worthwhile to export to all foreign countries. However, this is no longer true in the presence of uncertainty.

If firms determine how many workers to hire before learning the realization of their country-specific shocks, variable profits in each market are now not the same as if these markets were considered in isolation. This is because the particular realization of shocks affects how the firm chooses to allocate its output across all destinations. Thus, due to uncertainty, variable profits in the different countries are inextricably linked. This linkage across destinations is captured by the marginal expected diversification profit. Since the marginal expected direct profit is equal for all n, the shape of the expected variable profit function depends on the marginal expected diversification profit. In particular, if the marginal expected profit from diversification decreases with the number of export destinations, then the expected variable profit function will be strictly concave, implying that the bang-bang solution no longer holds. We will see in what follows, that for certain types of shocks this is indeed the case.

6.1 Global Shocks

The first type of shock we consider is the simplest, that is, one in which the home- and foreign-country shocks are perfectly correlated. These types of shocks may, for instance, arise in sectors where consumer preferences are highly correlated across countries. Sectors that produce goods whose popularity depends on fads and fashions may fall into this category. In particular, S is a global-shock distribution if there exists a $\nu > 0$ such that for any Γ in the support of S, $\gamma_1 = \nu\gamma_i$ for all $i > 0$. In other words, when shocks are global, it is as if the firm receives the same shock in all countries, with the iceberg cost adjusted to $\tau\nu^{1/(1-\sigma)}$, hence the term "global". We now show that a bang-bang solution still emerges in sectors characterized by global shocks.

Proposition 4 *If sector S is exposed to global shocks, then $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ are empty. Therefore, in sector S firms either do not export or export to all foreign countries.*

From the point of view of the firm, choosing its export destinations and employment is equivalent to choosing its total output. Given its total output and having learned its realization of shocks, the firm maximizes its profit by optimally dividing this output between its chosen markets. This optimal division is achieved by equalizing marginal revenue across markets. However, when shocks are global, marginal revenues across markets differ by the same proportionality factor for every realization of Γ. Therefore, the output sold in the home country as well as in each export destination is independent of the particular realization of the shocks. In other words, a firm cannot gain by adjusting its allocation of output across destinations for different realizations of the shocks, which implies that the expected profit from diversification in this case is zero. As a consequence, when the firm chooses the number of export destinations, it, in effect, does so by considering each destination in isolation. Since the foreign countries are symmetric, it follows that if it is worthwhile to export to one foreign country, it is worthwhile to export to all foreign countries.

The mathematical manifestation of this intuition is apparent in the certainty-equivalent generalized sum of the shocks since it fully captures the effect of uncertainty. When shocks are global, this certainty equivalent, $b(S,n) = (1+\tau^{1-\sigma}n\nu)b(S,0)$, is linear in n. It is this linearity in n that leads firms either not to export or to export to all foreign countries. Furthermore, the linearity makes it natural to interpret $b(S,0)$ as a certainty-equivalent shock in the home country and $\nu b(S,0)$ as a certainty-equivalent shock in all the foreign countries. Indeed, all firm exporting and output decisions would be identical to those in a model with no uncertainty, where $b\,(S,0)$ and $\nu b(S,0)$ are the certain home and foreign levels of demand, respectively.

6.2 Non-Global Shocks

If shocks are global, there is no possibility of reducing risk through market diversification so that the expected variable profit function is linear with respect to the number of export destinations. However, when shocks are not global this need not be the case. Indeed, the expected profit from diversification will be positive as firms take advantage of the imperfect correlation of shocks across destinations by selling more where shocks are favorable and less where shocks are unfavorable. Through its effect on the expected profit from diversification, the nature of the shock distribution will then determine the (nonlinear) shape of the expected variable profit function, and ultimately the number of export destinations that a firm will choose.

6.2.1 Common Foreign Shocks

We now consider sectors that are exposed to shocks that are not global, but in which all foreign shocks are common, that is, $\theta_S = 1$. This type of uncertainty may arise in sectors with firm- or product-specific uncertainty regarding transportation costs, or alternatively, in sectors with firm- or product-specific home biases of uncertain magnitude. Besides their plausibility, common foreign shocks are of particular interest because they provide a relatively simple case where the bang-bang exporting solution does not hold. In particular, in Proposition 5 we show that in sectors where all foreign shocks are common, any particular number of export destinations $n = 0, 1, \cdots, N-1$ is optimal for some firms, with more productive firms tending to export to more countries.

Proposition 5 *If shocks in sector S are not completely global and $\theta_S = 1$, then $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ are nonempty. Therefore, in sector S there are positive masses of firms exporting to $n = 0, 1, \cdots, N-1$ foreign countries.*

The essence of the proof is to demonstrate that when foreign shocks are common, for a given productivity, the expected variable profit function is strictly concave in the number of export destinations. Due to this strict concavity, it is no longer the case that if a firm finds it worthwhile to export to one destination, it then finds it worthwhile to export to all destinations. In particular, for any sector S with common foreign shocks and every $n = 1, 2, ..., N-1$ there exists a nondegenerate interval of productivities such that for firms in this interval the marginal expected variable profit from exporting to the n-th foreign country exceeds the fixed cost of exporting, but not so for the marginal expected variable profit from exporting to the $n+1$-th country. These firms export to precisely n destinations. Therefore, there exist firms that export to any particular number of export destinations.

The strict concavity in the case of common foreign shocks is a result of the decreasing marginal expected profit from diversification. To understand this decrease, consider first the impact of exporting on the risk emanating from the home-country shock. If the firm serves only the home market, it is fully exposed to the risk stemming from this shock. Adding one export destination makes it possible for the firm to alleviate some of the negative impact of this risk. Adding more export destinations further facilitates the risk reduction and thereby alleviates the impact of the risk stemming from the home country even more. However, the benefit from this reduction in risk diminishes in the number of export destinations. This is because the greater the firm's market diversification, the smaller is the remaining risk and hence the gain from further diversification.

Consider next the impact of exporting on the risk emanating from foreign-country shocks. If the firm does not export, it is shielded from these shocks. Exporting introduces the risk stemming from the foreign-country shocks, which, since the shocks in the export destinations are identical, can only be alleviated by the home country. This risk-reduction benefit too decreases with the number of export destinations since the home country can mitigate less of the added risk the more foreign risk it is already absorbing. As a result, each additional export destination increases the risk the firm faces from foreign-country shocks at an increasing rate. The upshot is that the risk-reducing benefit from diversification diminishes with the number of export destinations because of both the decreasing gain from a further reduction in the impact of the home-country risk and the decrease in the ability of the home country to mitigate additional foreign risk.

6.2.2 Idiosyncratic Foreign Shocks

We next consider sectors where the only type of foreign shocks is idiosyncratic, that is, $\theta_S = 0$. These types of shocks could arise because, for cultural, environmental or other reasons, consumers in different countries may have different tastes, which firms cannot easily predict. Again, whether or not a firm's optimal export policy takes the form of a bang-bang solution depends on the shape of the expected variable profit function, which, in turn, depends on whether the marginal expected profit from diversification increases or decreases with the number of export destinations. Interestingly, as Proposition 6 shows, when all foreign shocks are idiosyncratic, firms may have a bang-bang solution in some sectors but not in others.

Proposition 6 *If $\theta_S = 0$ and the idiosyncratic foreign shocks in sector S are not too risky, then:*

1. *If there is no home-country risk, then $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ are empty. Therefore, in sector S firms either do not export or export to all foreign countries.*
2. *If there is home country risk, then $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ are nonempty. Therefore, in sector S there are positive masses of firms exporting to $n = 0, 1, \cdots, N-1$ foreign countries.*

With idiosyncratic foreign shocks, not only is there a possibility for the home country to mitigate risk stemming from foreign countries, but there is also the possibility, unavailable in the case of common foreign shocks, for one foreign country to mitigate risk stemming from other foreign countries. It is precisely this latter possibility that leads to the ambiguity in the case of idiosyncratic shocks. And why might the benefit from this type of risk reduction either increase or decrease the marginal expected profit from diversification? This is because each additional export destination can mitigate some of the risk stemming from the existing foreign destinations, but at the same time brings with it its own idiosyncratic risk. Therefore, the net change in the impact of foreign risk from adding one more export destination is ambiguous. Consequently, the scope for reducing risk stemming from the foreign destinations, and by extension, the marginal expected profit from diversification, may either increase or decrease with the number of export destinations.[21]

[21] With riskier idiosyncratic shocks it is straightforward to construct numerical examples in which some sectors have a bang-bang solution and others do not.

7 Comparative Statics of Shock Riskiness

Armed with a deeper understanding of the mechanisms involved in determining how different shock types affect a firm's activity and export cutoffs, we are now ready to consider distributions that contain at the same time all the types of shocks considered above. In particular, our main goal is to explore the effects of changes in the riskiness of these shocks, where an increase in the riskiness of a shock is equivalent to a mean-preserving spread of its distribution (Rothschild and Stiglitz, 1970). We confine our analysis to the empirically relevant cases of sectors in which there are some firms that export to some but not all destinations, that is, sectors where the bang-bang solution does not hold.[22]

A firm's decisions depend on how risk affects the firm's marginal expected profit rather than the level of its expected profit. Thus, while the firm will certainly be less profitable when it faces greater risk, the risk may, nevertheless, induce the firm to export to more destinations. With this in mind, we now examine how riskiness affects: (1) the probability that a firm chooses to be active; (2) the probability that a firm chooses to export; and (3) the number of countries to which an active firm chooses to export. In the absence of uncertainty, (2) and (3) are indistinguishable since if a firm decides to export at all, it will export to all destinations. Not so in the presence of uncertainty. Indeed, one of the main insights of our framework is that uncertainty can explain why firms export to some but not all destinations.

[22]Since Proposition 5 shows that for $\theta_S = 1$ there exist parameter values for which the solution is not bang-bang, by continuity, this must also be the case for $\theta_S < 1$. Of course, there are reasons other than uncertainty that firms may export to some but not all destinations. For instance, this will be the case if the fixed exporting cost increases with the number of destinations, or alternatively, if the fixed exporting cost differs across countries. Since our results hinge on the effect of uncertainty on a firm's variable profit function, our comparative-statics results will carry through in more general settings where the bang-bang solution does not hold for reasons other than uncertainty.

7.1 Global Risk

We begin by considering the impact of the riskiness of global shocks on a firm's decisions. Any distribution S can be equivalently considered as a distribution whose realizations are the elementwise product of the realization of a global distribution, S^g, and an independent non-global (or, rather, not necessarily global) distribution, S^{ng}.[23] Distinguishing between these two components of the shock distribution is useful because it allows us to focus on how firm decisions are influenced by the shocks that affect worldwide demand, while not limiting the applicability to those sectors where firms face only global shocks.

Proposition 7 *If S' and S'' have identical non-global components, but S'' has a riskier global component than S', then $\varphi_n(S') < \varphi_n(S'')$ for all $n \geq 0$.*

Proposition 7 shows that firms in a sector characterized by distributions with riskier global components are less likely to be active. Moreover, fewer firms in the riskier sector will export, and those that do export will export to fewer destinations. The reason is that a riskier global-shock distribution reduces the expected variable profit from the sale of any given output in any destination, domestic or foreign, in the same proportion. This proportionality implies that the marginal expected profit from serving any particular destination also decreases with the riskiness of the global shock. As a result, the riskier the distribution of global shocks, the greater a firm's productivity must be to justify incurring the fixed overhead cost to become active, and, if active, to justify incurring the fixed cost of exporting to one additional destination.[24] Nevertheless, the effect on the proportion of exporters among active firms as well as the average number of destinations per exporter is ambiguous as the activity and all export cutoffs shift in the same direction.

[23] That is, there exists a $\nu > 0$ such that any realization of S can be written as $\Gamma = (\gamma^g\gamma_1^{ng}, \nu\gamma^g\gamma_2^{ng}, ..., \nu\gamma^g\gamma_N^{ng})$, where the realization of S^g is $\Gamma^g = (\gamma^g, \nu\gamma^g, ..., \nu\gamma^g)$ and the realization of S^{ng} is independently drawn and equal to $\Gamma^{ng} = (\gamma_1^{ng}, \gamma_2^{ng}, ..., \gamma_N^{ng})$. Such a decomposition is always possible because the global component can be, for example, a vector of ones with certainty. Moreover, this decomposition is not unique, so that Proposition 7 holds for any possible decomposition into S^g and S^{ng}.

[24] Since the certainty-equivalent sum of shocks is proportional to $\left[\int_{\mathbf{R}^N} \gamma^{g^{1/\sigma}} dS^g(\Gamma^g)\right]^\sigma$, the changes in the activity and export cutoffs are proportional to the change in $\left[\int_{\mathbf{R}^N} \gamma^{g^{1/\sigma}} dS^g(\Gamma^g)\right]^{-1/\rho}$.

7.2 Foreign Risk

We next turn our attention to foreign-country shocks by considering two sectors which differ only in that the foreign shocks are riskier in one sector than in the other. If firms with productivity φ in either sector do not export, then their behavior is identical in both sectors because they are unaffected by the riskiness of the foreign shocks regardless of whether it is in the common or idiosyncratic component. In particular, the activity cutoffs are the same and, therefore, the mass of active firms is identical in both sectors. The riskiness of the shocks do, however, affect the export cutoffs.

7.2.1 Common Foreign Shocks

We start by considering the case of two sectors that have the same distribution of home-country shocks while the distribution of the common foreign shocks is riskier in one sector than the other. We focus on scenarios in which the additional foreign risk is conditioned on the home-country shock. This allows us to isolate the impact of foreign riskiness by abstracting from any potential correlation of the additional foreign risk with the home-country shock. The scenario we consider is rather general and, in particular, includes the special case that the additional foreign risk is independent of the home-country shock, that is, the two sectors are identical except that the riskier sector has an additional (additive) zero-mean common foreign shock whose realization is independent of the home-country shock realization. As we will show, in addition to the activity cutoffs being the same, the sector with the riskier common foreign shocks will have higher export cutoffs.

Proposition 8 *If S'' and S' are identical except that, conditional on at least some home-country shocks, S'' has riskier common foreign shocks than S', then:*

1. $\varphi_0(S') = \varphi_0(S'')$;

2. $\varphi_n(S') < \varphi_n(S'')$ *for all* $n \geq 1$.

The proposition states that in a sector with riskier common foreign shocks, firms need to be more productive in order to export to any given number of destinations as compared with firms in a less risky sector. Since the activity cutoff is identical in the two sectors, as discussed above, it follows that the proportion of exporting firms among active firms will be smaller in the riskier sector. Moreover, active firms will on average serve fewer markets in the riskier sector.

Proposition 8 follows because a riskier common foreign shock leads to a decrease in the marginal expected profit from exporting for any n. On the one hand, the riskier common foreign shocks reduce the marginal expected direct profit from exporting. On the other hand, the higher risk abroad enhances the scope for the home country to mitigate risk stemming from the marginal foreign country, thereby increasing the marginal expected profit from diversification. However, the increased benefit from mitigating this foreign risk, i.e., the increase in the marginal expected profit from diversification, cannot possibly be greater than the loss from introducing this risk in the first place, i.e., the decrease in the marginal expected direct profit.[25]

7.2.2 Idiosyncratic Foreign Shocks

The situation with idiosyncratic foreign shocks is more complicated. Unlike the case of common foreign shocks, riskier idiosyncratic foreign shocks (again, conditional on the home-country shock) increase the ability of one foreign country to mitigate risk stemming from other foreign countries. As a result, it could be the case that riskier idiosyncratic foreign shocks increase the marginal expected profit from diversification by more than the decrease in the marginal expected direct profit for some n's. This situation, impossible in the case of riskier common foreign shocks, would lead to an increase in the total marginal expected profit for these n's. Proposition 9 will make clear that this is not only a theoretical

[25] The condition in Proposition 8 (and 9) that the additional riskiness in S'' is conditional on the home-country shock is necessary in order to rule out a scenario in which the additional risk is negatively correlated with the home-country shocks. Indeed, if this were the case, then the additional risk may make exporting more attractive as it could potentially reduce the firm's overall risk.

possibility, but that riskier idiosyncratic foreign shocks must lead to an increase in the total marginal expected profit for some n's and to a decrease for others if the number of countries is large enough.

Proposition 9 *If S'' and S' are identical except that, conditional on at least some home-country shocks, S'' has riskier idiosyncratic foreign shocks than S', then:*

1. $\varphi_0(S') = \varphi_0(S'')$;
2. $\varphi_1(S') < \varphi_1(S'')$;
3. *If N is large enough, there exists an n such that $\varphi_n(S') > \varphi_n(S'')$.*

The first part of the proposition follows, as mentioned above, from the fact that non-exporting firms are unaffected by foreign shocks. The second part follows from the second part of Proposition 8 because there is no difference between a common and idiosyncratic foreign shock when a firm exports to only one destination. Finally, the third part of the proposition follows from the fact that as n tends to infinity all idiosyncratic foreign risk is fully dissipated. As a result, as n grows large, the expected profit in the two sectors converge. Since for $n = 1$ the level of expected profit is lower in the sector with the riskier idiosyncratic foreign shocks (as is evident from the first two parts of the proposition), it must be that for some $n > 1$ the marginal expected profit from an additional destination is greater in the riskier sector. Thus, if N is large enough, then firms in the riskier sector will export to more destinations than firms in the less risky sector for some range of productivities. The reason is that firms in the riskier sector face more risk in each destination and therefore may find it worthwhile to add more destinations to help mitigate the higher risk stemming from the previous export destinations despite the fact that the additional destinations are riskier as well.

The upshot of the proposition is that just as with common foreign shocks, a sector with riskier idiosyncratic foreign shocks will have a smaller proportion of active firms exporting.

However, unlike the case with common foreign shocks, the active firms in the riskier sector may actually serve more markets, on average, than firms in the less risky sector.

7.3 Correlation Between Foreign Shocks

We now examine how the impact of common and idiosyncratic foreign shocks differ by considering the effect of the likelihood that foreign shocks are idiosyncratic, θ_S, on firm behavior. To focus on the impact of θ_S, we restrict attention to sectors in which the common foreign shocks and each foreign country's idiosyncratic shock are drawn from the same distribution, that is, $S_c = S_d$.[26] An increase in θ_S then decreases the correlation of shock realizations across foreign markets without changing the likelihood of any particular outcome in any given destination. The following proposition shows that the correlation of foreign shocks does not affect a firm's probability of being active or of exporting, but does affect the number of export destinations.

Proposition 10 *Sectors S' and S'' have the same home-country uncertainty and $S_c(\gamma_2' \mid \gamma_1') = S_d(\gamma_i' \mid \gamma_1') = S_c(\gamma_2'' \mid \gamma_1'') = S_d(\gamma_i'' \mid \gamma_1'')$ for $i \geq 1$ and all γ_1' and γ_1''. If $\theta_{S'} > \theta_{S''}$, then*

1. $\varphi_n(S') = \varphi_n(S'')$; *for* $n = 0, 1$;
2. $\varphi_n(S') > \varphi_n(S'')$ *for all* $n \geq 2$.

The first part of the proposition states that both the activity cutoff and the cutoff for beginning to export are not influenced by θ_S. This is because foreign shocks do not affect non-exporters, and common and idiosyncratic foreign shocks are identical from the perspective of a firm that exports to only one country. Thus, θ_S affects neither the mass

[26] Of course, without restrictions on the relationship between the distributions of the common and idiosyncratic foreign shocks it is not possible to say very much. After all, if the distribution of the common foreign shocks were particularly favorable (unfavorable) compared to the distribution of the idiosyncratic foreign shocks, then an increase in θ_S would clearly make exporting more (less) likely.

of active firms in a sector nor the proportion of active firms that export. Nevertheless, the second part of the proposition implies that the likelihood that the foreign shocks are common does affect the optimal number of export destinations. In particular, the higher is θ_S, the higher are the export cutoffs for $n \geq 2$, and therefore the fewer export markets will each exporting firm serve, on average.

This proposition is a generalization of Proposition 3 not only because it allows for more general shock distributions, but also because it shows that when idiosyncratic shocks are more likely, then not only is the *level* of the expected profit from exporting higher for any $n \geq 2$, but so too is the *marginal* expected profit from an additional destination. The logic too is similar. Adding an export destination beyond the first allows the firm to mitigate existing foreign risk so long as the shock in the additional destination is not perfectly correlated with the shocks in the other foreign destinations. The lower the correlation, viz. the lower θ_S, the greater is the ability of the additional foreign destination to mitigate existing foreign risk, and consequently, the greater is the incentive to add another export destination.

7.4 Home-Country Risk

We now turn to the impact of the distribution of home-country shocks on a firm's behavior. We will show that, while riskier home-country shocks have an unambiguously adverse effect on a firm's decision to be active, the impact on the number of export destinations may depend on the elasticity of substitution between goods. As in the case of foreign shocks, our goal is to isolate the impact of riskiness at home from any confounding effect of the potential correlation between this additional home-country risk and the foreign shocks. Therefore, here, analogously to Propositions 8 and 9, we consider cases in which the home-country shock is riskier conditional of the foreign shock. This, of course, includes the special case that the additional home-country risk is independent of the foreign shocks.

Proposition 11 *If S'' and S' are identical except that, conditional on at least some foreign* *shocks, S'' has riskier home-country shocks than S', then:*

1. $\varphi_0(S') < \varphi_0(S'')$;
2. *If σ is sufficiently small, then $\varphi_n(S') > \varphi_n(S'')$ for all $n \geq 1$.*

Riskier home-country shocks will always raise the activity cutoff because increased riskiness makes the home country less profitable. In addition, if the elasticity of substitution, σ, is not too large, then riskier home-country shocks will increase the incentive to export. The logic underlying this result, which we elucidate in the following paragraphs, is that a home-country risk has two opposing effects on the incentive to export, the relative strengths of which depend on σ.

Consider a non-exporting firm in a sector with just two equally likely outcomes for the home-country shock, high (γ^*) and low (γ_*). How would such a firm respond if the high outcome doubled to $2\gamma^*$? Clearly the firm would increase its output to take advantage of the higher potential realization. However, because it must still account for the possibility of receiving the low shock, the firm will less than double its output. The end result is that the gap between the firm's actual output and the output it would have chosen had it known its shock realization with certainty is now larger for either of the two potential realizations. Such a firm now has more to gain from exporting. Indeed, if it receives the high shock the firm can take advantage of the relatively high demand by selling domestically some of the output which would otherwise be destined for the foreign markets. If instead it receives the low shock, the firm can export more of its output rather than being forced to sell the excess at home where it would fetch a lower price.

The intuition just outlined shows why increasing a relatively high home-country demand realization increases the incentive to export. Of course, the reverse is also true: lowering a relatively high realization decreases the incentive to export. It is now becoming clear why increased riskiness of home-country shocks has an ambiguous effect on the incentive to export. Since increased riskiness consists of increasing one outcome while decreasing another by an equal amount, it exerts two opposing forces on the incentive to export.

But why do the relative strengths of these opposing forces depend on σ? For concreteness, consider a sector just as the one described above. However, now, instead of doubling the high shock, suppose that the high shock is split into two equally likely shocks, so that the firm receives the shock $\gamma^* + \varepsilon$ or $\gamma^* - \varepsilon$ each with probability $1/4$ (in addition to γ_* with a probability of $1/2$). This represents increased riskiness of the home-country shock, and as just described exerts two opposing pressures on the incentives to export. On the one hand, increasing the realization from γ^* to $\gamma^* + \varepsilon$ increases the incentive to export. On the other hand, decreasing the realization from γ^* to $\gamma^* - \varepsilon$ decreases the incentive to export.

Since a higher σ implies that the expected profit function is more concave with respect to the shocks, the greater is σ, the greater impact does the decrease of the high shock to $\gamma^* - \varepsilon$ have relative to the increase of the high shock to $\gamma^* + \varepsilon$ on the incentive to export. As a result, a greater σ gives more weight to the force favoring less exporting relative to the force favoring more exporting. Thus, when σ is large enough, the former may dominate.[27]

8 Conclusion

Our goal in this paper has been to study the implications of uncertainty in the canonical Melitz (2003) trade model. We introduce uncertainty as a country-specific demand shock, the realization of which firms learn only after choosing their export destinations and making an irreversible employment decision. We show that due to the irreversibility of the employment decision, firms may choose to export as a means of diversifying their sources of demand. In other words, exporting provides the firm with an insurance policy.

After illustrating the diversification motive for exporting, we go on to show that the gain from market diversification may decrease with the number of export destinations.

[27] Suppose instead that the increased riskiness of the home-country shock is obtained by splitting the low shock, γ_*, into $\gamma_* + \varepsilon$ and $\gamma_* - \varepsilon$. The increase to $\gamma_* + \varepsilon$ decreases the incentive to export, while the decrease to $\gamma_* - \varepsilon$ increases the incentive to export. In this case, just as in the case considered in the main text, the greater is σ, the greater the increase in the incentive to export caused by the riskier environment.

Consequently, a firm's marginal expected variable profit may decline with the number of export destinations. This is significant because while a firm might find it worthwhile to pay the fixed cost to export to some destinations, it may, at some point, reach a number of destinations for which it no longer finds it worthwhile to incur this per-country fixed cost. Indeed, we prove that if foreign shocks are common, then there exist firms that export to any particular number of export destinations, with more productive firms serving more markets. Importantly, this result is obtained even though countries are symmetric – they do not differ in size, demand structure, cost of exporting or in any other way. Thus, without adding any additional heterogeneity beyond that present in the original Melitz model, our model is able to explain why similar firms do not enter export markets according to a common hierarchy.

Nevertheless, the logic underlying our results is not tied to the symmetric-country assumption. Our comparative-static results regarding the riskiness of shocks would remain unchanged even if country sizes or exporting costs were heterogeneous. Indeed, since the method of our proofs was to show the impact of risk on the expected marginal profit from exporting, the proofs would hardly require adjustment when additional layers of heterogeneity are introduced. Additional heterogeneity would not change the results that in sectors with riskier global shocks fewer firms will tend to be active and that active firms will tend to export to fewer destinations. Nor would additional heterogeneity overturn our results relating to the riskiness of foreign shocks. It would still be the case that firms in sectors with riskier foreign shocks are just as likely to be active and will tend to export to fewer destinations if it is the common component of the foreign shock that is riskier. Likewise, it would still be the case that a higher correlation of the foreign shocks tends to reduce the number of countries to which an active firm exports.

Of course, leaving the symmetric-country framework does raise new and interesting questions since a firm would then no longer be content to choose its export destinations randomly among the foreign countries. It would be of interest to determine how different types of risk would interact with country size and other country characteristics to determine a firm's choice of specific export destinations. Similarly, if some foreign shocks were more correlated with home-country shocks or more correlated with shocks in some foreign destinations and less with shocks in others, how would this affect the choice of export destinations? These are certainly fruitful directions for future research.

As this is the first paper to explore the diversification motive for exporting, our goal has been to develop its theoretical implications. Although our analysis has concentrated on the effect of different types of risk on the optimal number of export destinations, our model also yields a wealth of related predictions concerning the impact of riskiness on firms' revenues and profits as well as on the correlation of sales in different countries. All of these predictions provide ample testable implications which can help aid future research in verifying whether the diversification motive for exporting is empirically important.

Appendix

Proof of Proposition 1: Foreign Countries Mitigate Risk Stemming from the Home Country

Since $\pi(\varphi, S, n) = \varphi^{\sigma-1} k b(S, n)$, we need to show that $b(S', n) - b(S', 0) < b(S'', n) - b(S'', 0)$ for all $n \geq 1$. Note that $A(\Gamma, \mathcal{N}) = \gamma_1 + \tau^{1-\sigma} n \gamma_2$ and consider n as a continuous variable. For any nondegenerate distribution of γ_1'' we have that

$$\begin{aligned} \frac{db(S'', n)}{dn} &= \tau^{1-\sigma}\gamma_2'' \left[\int_{\mathbf{R}^N} (\gamma_1'' + \tau^{1-\sigma} n \gamma_2'')^{1/\sigma} dS''(\Gamma) \right]^{\sigma-1} \int_{\mathbf{R}^N} (\gamma_1'' + \tau^{1-\sigma} n \gamma_2'')^{(1-\sigma)/\sigma} dS''(\Gamma) \\ &> \tau^{1-\sigma}\gamma_2'' \left[\int_{\mathbf{R}^N} (\gamma_1'' + \tau^{1-\sigma} n \gamma_2'')^{1/\sigma} dS''(\Gamma) \right]^{\sigma-1} \left[\int_{\mathbf{R}^N} (\gamma_1'' + \tau^{1-\sigma} n \gamma_2'')^{1/\sigma} dS''(\Gamma) \right]^{1-\sigma} \\ &= \tau^{1-\sigma}\gamma_2'' \end{aligned}$$

where the inequality follows from Jensen's inequality. Since $\gamma_1' = \gamma_1''$ and for any certain γ_1' we have that $db(S', n)/dn = \tau^{1-\sigma}\gamma_2'$, it follows that $b(S', n) - b(S', 0) < b(S'', n) - b(S'', 0)$ for all $n \geq 1$, which proves the proposition. ■

Proof of Proposition 2: The Home Country Mitigates Risk Stemming from Foreign Countries Country

We need to show that if γ_1 is certain and γ_2 is uncertain, then $b(S, n) - b(S, 0)$ increases in γ_1. Since $b(S, 0) = \gamma_1$, this involves showing that $b(S, n)$ increases more than the increase in γ_1 for $n \geq 1$. For any certain γ_1 and uncertain γ_2 we have that for $n \geq 1$

$$\begin{aligned} \frac{db(S, n)}{d\gamma_1} &= \left[\int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} dS(\Gamma) \right]^{\sigma-1} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} dS(\Gamma) \\ &> \left[\int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} dS(\Gamma) \right]^{\sigma-1} \left[\int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} dS(\Gamma) \right]^{1-\sigma} \\ &= 1, \end{aligned}$$

where the inequality follows from Jensen's inequality. Hence, since $db(S, 0)/d\gamma_1 = 1$, $b(S', n) - b(S', 0) < b(S'', n) - b(S'', 0)$ for $n \geq 1$, which proves the proposition. ■

Proof of Proposition 3: One Foreign Country Mitigates Risk Stemming from Other Foreign Countries

Part 1 Since $b(S',0) = b(S'',0)$ and $b(S',1) = b(S'',1)$, it follows that $\pi(\varphi, S', 1) - \pi(\varphi, S', 0) = \pi(\varphi, S'', 1) - \pi(\varphi, S'', 0)$.

Part 2 Since $b(S',0) = b(S'',0)$, we need to show that $b(S',n) < b(S'',n)$ for $n \geq 2$, which is equivalent to showing that $b(S,n)$ decreases with θ_S for $n \geq 2$. To do so, let $a_n \equiv \gamma_1 + \tau^{1-\sigma} n \gamma_2$ and $\tilde{a}_n \equiv \gamma_1 + \tau^{1-\sigma} \sum_{i \in \mathcal{N} \setminus \{1\}} \gamma_i$. Thus, a_n ($\tilde{a}_n$) is the sum of the certain home-country demand and the common (idiosyncratic) foreign demand shocks for n export destinations, net of the iceberg cost. Then

$$b(S,n) = \left[\theta_S \int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma) + (1 - \theta_S) \int_{\mathbf{R}^N} \tilde{a}_n^{1/\sigma} dS_d(\Gamma) \right]^{\sigma}.$$

The derivative with respect to θ_S has the same sign as

$$\int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N} \tilde{a}_n^{1/\sigma} dS_d(\Gamma). \tag{8}$$

Since the idiosyncratic foreign shocks are independently and identically distributed according to the same distribution as the common shock, it follows that a_n and $\tilde{a}_n$ have the same mean, with a_n being riskier than $\tilde{a}_n$ for $n \geq 2$. The power $1/\sigma$ is a strictly concave function, so (8) is negative for $n \geq 2$. Hence $b(S',n) < b(S'',n)$ for $n \geq 2$ which proves part 2 of the proposition. ■

Proof of Proposition 4: Global Shocks

Shocks being global implies that

$$\begin{aligned} A(\Gamma, \mathcal{N}) &= \gamma_1 + \tau^{1-\sigma} n \nu \gamma_1 \\ \Rightarrow \qquad b(S,n) &= (1 + \tau^{1-\sigma} n \nu)\, b(S,0). \end{aligned}$$

Thus,

$$\pi(\varphi, S, n+1) - \pi(\varphi, S, n) = \varphi^{\sigma-1} k \tau^{1-\sigma} \nu b(S, 0)$$

is independent of n. It follows that $\varphi_1(S) = \varphi_2(S) = \cdots = \varphi_{N-1}(S)$, implying that $\Phi_1(S) = \Phi_2(S) = \cdots = \Phi_{N-2}(S) = \varnothing$. Hence, an active firm in sector S will not export if $\varphi \leq \varphi_1(S)$ and will export to all foreign countries if $\varphi > \varphi_1(S)$. ■

Proof of Proposition 5: Common Foreign Shocks

We will first show that the series $\{\pi(\varphi, S, n)\}_{n=1}^{N-1}$, or, equivalently, the series $\{b(S, n)\}_{n=1}^{N-1}$, is strictly concave in n. Now, $\theta_S = 1$ implies that $A(\Gamma, \mathcal{N}) = \gamma_1 + \tau^{1-\sigma} n \gamma_2$. Considering n as a continuous variable, we have that

$$\begin{aligned}
\frac{\partial^2 b(S,n)}{\partial n^2} &= (\sigma-1) b(S,n)^{(\sigma-2)/\sigma} \left[\frac{\partial b(S,n)^{1/\sigma}}{\partial n}\right]^2 + b(S,n)^{(\sigma-1)/\sigma} \frac{\partial b^2(S,n)^{1/\sigma}}{\partial n^2} \\
&= b(S,n)^{(\sigma-2)/\sigma} \left\{ (\sigma-1) \left[\frac{\partial b(S,n)^{1/\sigma}}{\partial n}\right]^2 + b(S,n)^{1/\sigma} \frac{\partial b^2(S,n)^{1/\sigma}}{\partial n^2} \right\}.
\end{aligned}$$

As $b(S, n) > 0$, it suffices to show that

$$(\sigma-1) \left[\frac{\partial b(S,n)^{1/\sigma}}{\partial n}\right]^2 + b(S,n)^{1/\sigma} \frac{\partial b^2(S,n)^{1/\sigma}}{\partial n^2} \tag{9}$$

is negative. Since

$$\begin{aligned}
\frac{\partial b(S,n)^{1/\sigma}}{\partial n} &= \frac{\tau^{1-\sigma}}{\sigma} \int_{\mathbf{R}^N} \gamma_2 A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} dS(\Gamma), \\
\frac{\partial b^2(S,n)^{1/\sigma}}{\partial n^2} &= \frac{(1-\sigma)\tau^{2-2\sigma}}{\sigma^2} \int_{\mathbf{R}^N} \gamma_2^2 A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} dS(\Gamma),
\end{aligned}$$

(9) becomes

$$\begin{aligned}
&(\sigma-1) \left[\frac{\tau^{1-\sigma}}{\sigma} \int_{\mathbf{R}^N} \gamma_2 A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} dS(\Gamma)\right]^2 \\
&+ \frac{(1-\sigma)\tau^{2-2\sigma}}{\sigma^2} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} dS(\Gamma) \int_{\mathbf{R}^N} \gamma_2^2 A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} dS(\Gamma),
\end{aligned}$$

which has the same sign as

$$\left[\int_{\mathbf{R}^N} \gamma_2 A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} dS(\Gamma)\right] \left[\int_{\mathbf{R}^N} \gamma_2 A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} dS(\Gamma)\right]$$
$$- \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{1/\sigma} dS(\Gamma) \int_{\mathbf{R}^N} \gamma_2^2 A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} dS(\Gamma).$$

Using primes for realizations in the second integral in each term, this can be written as

$$\begin{aligned}
& \int_{\mathbf{R}^N} \int_{\mathbf{R}^N} \left[\gamma_2 A(\Gamma, \mathcal{N})^{(1-\sigma)/\sigma} \gamma_2' A(\Gamma', \mathcal{N})^{(1-\sigma)/\sigma} - A(\Gamma, \mathcal{N})^{1/\sigma} \gamma_2'^2 A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma}\right] dS(\Gamma) dS(\Gamma') \\
= & \int_{\mathbf{R}^N} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma} \left[\gamma_2 A(\Gamma, \mathcal{N}) \gamma_2' A(\Gamma', \mathcal{N}) - A(\Gamma, \mathcal{N})^2 \gamma_2'^2\right] dS(\Gamma) dS(\Gamma'),
\end{aligned}$$

or as

$$\begin{aligned}
& \tfrac{1}{2}\left\{\int_{\mathbf{R}^N} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma} \left[\gamma_2 A(\Gamma, \mathcal{N}) \gamma_2' A(\Gamma', \mathcal{N}) - A(\Gamma, \mathcal{N})^2 \gamma_2'^2\right] dS(\Gamma) dS(\Gamma')\right. \\
& \left. + \int_{\mathbf{R}^N} \int_{\mathbf{R}^N} A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma} A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} \left[\gamma_2' A(\Gamma', \mathcal{N}) \gamma_2 A(\Gamma, \mathcal{N}) - \gamma_2^2 A(\Gamma, \mathcal{N})^2\right] dS(\Gamma) dS(\Gamma')\right\},
\end{aligned}$$

which follows from the fact that the expressions in the two double integrals are identical except for the switching of the primes. Since each pair of realizations (Γ, Γ') in the first double integral has the same density as the pair of realizations (Γ', Γ) in the second double integral, the last expression becomes

$$\begin{aligned}
& \tfrac{1}{2} \int_{\mathbf{R}^N} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma} \\
& \left[2\gamma_2 A(\Gamma, \mathcal{N}) \gamma_2' A(\Gamma', \mathcal{N}) - A(\Gamma, \mathcal{N})^2 \gamma_2'^2 - \gamma_2^2 A(\Gamma', \mathcal{N})^2\right] dS(\Gamma) dS(\Gamma') \\
= & -\tfrac{1}{2} \int_{\mathbf{R}^N} \int_{\mathbf{R}^N} A(\Gamma, \mathcal{N})^{(1-2\sigma)/\sigma} A(\Gamma', \mathcal{N})^{(1-2\sigma)/\sigma} \left[A(\Gamma, \mathcal{N}) \gamma_2' - \gamma_2 A(\Gamma', \mathcal{N})\right]^2 dS(\Gamma) dS(\Gamma').
\end{aligned} \tag{10}$$

As shocks are not global, $A(\Gamma, \mathcal{N})\gamma_2' - \gamma_2 A(\Gamma', \mathcal{N}) = \gamma_1 \gamma_2' - \gamma_2 \gamma_1'$ is sometimes different from zero. It follows that (10) and hence (9) is negative. Thus, the series $\{\pi(\varphi, S, n)\}_{n=1}^{N-1}$

is strictly concave in n. The series is also increasing in n and it therefore follows that $\varphi_1(S) < \varphi_2(S) < \cdots < \varphi_{N-1}(S)$ and hence that each of $\Phi_1, \Phi_2, \cdots, \Phi_{N-2}$ is nonempty. Consequently, there are active firms that export to $0, 1, \cdots, N-1$ foreign countries. ■

Proof of Proposition 6: Idiosyncratic Foreign Shocks

Part 1 We will first show that the series $\{\pi(\varphi, S, n)\}_{n=1}^{N-1}$ is strictly convex in n. This requires showing that

$$\begin{aligned} b(S,n+1) - b(S,n) &< b(S,n+2) - b(S,n+1) \\ \Leftrightarrow 2b(S,n+1) - b(S,n) - b(S,n+2) &< 0 \end{aligned} \tag{11}$$

for all $n \geq 0$.

There exists a $\zeta \geq 0$ such that γ_i for $i \geq 2$ can be written as $\gamma_i = \overline{\gamma} + \zeta\epsilon_i$, where $\overline{\gamma} \equiv \int_{\mathbf{R}^N} \gamma_i dS(\Gamma)$ for $i \geq 2$ is the mean of each of the idiosyncratic foreign shocks and $\epsilon_2, \epsilon_3, \cdots, \epsilon_N$ are independently and identically distributed random variables with zero mean. Define $a(\Gamma, n, \xi) \equiv \gamma_1 + \tau^{1-\sigma} n\overline{\gamma} + \tau^{1-\sigma}\xi \sum_{i=2}^{n+1} \epsilon_i$ so that $A(\Gamma, \mathcal{N}) = a(\Gamma, n, \zeta)$ and $\xi \geq 0$ is a scale parameter for risk such that the idiosyncratic foreign risk decreases as ξ decreases and the risk vanishes for $\xi = 0$.

Consider

$$2\left[\int_{\mathbf{R}^N} a(\Gamma, n+1, \xi)^{1/\sigma} dS(\Gamma)\right]^\sigma - \left[\int_{\mathbf{R}^N} a(\Gamma, n, \xi)^{1/\sigma} dS(\Gamma)\right]^\sigma - \left[\int_{\mathbf{R}^N} a(\Gamma, n+2, \xi)^{1/\sigma} dS(\Gamma)\right]^\sigma, \tag{12}$$

which equals the left-hand side of (11) when $\xi = \zeta$. Since (12) equals zero at $\xi = 0$, to show that the left-hand side of (11) is negative when the idiosyncratic foreign shocks are not too risky, i.e., when ζ is sufficiently close to zero, it is sufficient to show both that the first derivative of (12) with respect to ξ equals zero at $\xi = 0$, and that the second derivative of (12) with respect to ξ is negative at $\xi = 0$. Now, the derivative of (12) with respect to

ξ is

$$\begin{aligned}
&2\tau^{1-\sigma}\left[\int_{\mathbf{R}^N} a(\Gamma,n+1,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+2}\epsilon_i a(\Gamma,n+1,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma)\\
&-\tau^{1-\sigma}\left[\int_{\mathbf{R}^N} a(\Gamma,n,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+1}\epsilon_i a(\Gamma,n,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma)\\
&-\tau^{1-\sigma}\left[\int_{\mathbf{R}^N} a(\Gamma,n+2,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+3}\epsilon_i a(\Gamma,n+2,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma),
\end{aligned}$$

which, since $\xi=0\Rightarrow a(\Gamma,n)=\gamma_1+\tau^{1-\sigma}n\overline{\gamma}$ and the ϵ_i's have zero mean, for $\xi=0$ equals

$$2\tau^{1-\sigma}\int_{\mathbf{R}^N}\left(\sum_{i=1}^{n+2}\epsilon_i\right)dS(\Gamma)-\tau^{1-\sigma}\int_{\mathbf{R}^N}\left(\sum_{i=1}^{n+1}\epsilon_i\right)dS(\Gamma)-\tau^{1-\sigma}\int_{\mathbf{R}^N}\left(\sum_{i=1}^{n+3}\epsilon_i\right)dS(\Gamma)=0.$$

The second derivative of (12) with respect to ξ has the same sign as

$$\begin{aligned}
&2\left[\int_{\mathbf{R}^N} a(\Gamma,n+1,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-2}\left\{\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+2}\epsilon_i a(\Gamma,n+1,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma)\right\}^2\\
&-2\left[\int_{\mathbf{R}^N} a(\Gamma,n+1,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\left(\sum_{i=1}^{n+2}\epsilon_i\right)^2 a(\Gamma,n+1,\xi)^{(1-2\sigma)/\sigma}\right]dS(\Gamma)\\
&-\left[\int_{\mathbf{R}^N} a(\Gamma,n,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-2}\left\{\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+1}\epsilon_i a(\Gamma,n,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma)\right\}^2\\
&+\left[\int_{\mathbf{R}^N} a(\Gamma,n,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\left(\sum_{i=1}^{n+1}\epsilon_i\right)^2 a(\Gamma,n,\xi)^{(1-2\sigma)/\sigma}\right]dS(\Gamma)\\
&-\left[\int_{\mathbf{R}^N} a(\Gamma,n+2,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-2}\left\{\int_{\mathbf{R}^N}\left[\sum_{i=1}^{n+3}\epsilon_i a(\Gamma,n+2,\xi)^{(1-\sigma)/\sigma}\right]dS(\Gamma)\right\}^2\\
&+\left[\int_{\mathbf{R}^N} a(\Gamma,n+2,\xi)^{1/\sigma}dS(\Gamma)\right]^{\sigma-1}\int_{\mathbf{R}^N}\left[\left(\sum_{i=1}^{n+3}\epsilon_i\right)^2 a(\Gamma,n+2,\xi)^{(1-2\sigma)/\sigma}\right]dS(\Gamma),
\end{aligned}$$

which for $\xi = 0$ equals

$$-\frac{2}{\gamma_1 + \tau^{1-\sigma}(n+1)\overline{\gamma}} \int_{\mathbf{R}^N} \left(\sum_{i=1}^{n+2} \epsilon_i \right)^2 dS(\Gamma) + \frac{1}{\gamma_1 + \tau^{1-\sigma} n \overline{\gamma}} \int_{\mathbf{R}^N} \left(\sum_{i=1}^{n+1} \epsilon_i \right)^2 dS(\Gamma)$$
$$+\frac{1}{\gamma_1 + \tau^{1-\sigma}(n+2)\overline{\gamma}} \int_{\mathbf{R}^N} \left(\sum_{i=1}^{n+3} \epsilon_i \right)^2 dS(\Gamma).$$

Since the idiosyncratic foreign shocks are independently and identically distributed, this equals

$$\begin{aligned} & -\frac{2(n+1)\mathrm{var}(\epsilon_2)}{\gamma_1 + \tau^{1-\sigma}(n+1)\overline{\gamma}} + \frac{n\mathrm{var}(\epsilon_2)}{\gamma_1 + \tau^{1-\sigma} n \overline{\gamma}} + \frac{(n+2)\mathrm{var}(\epsilon_2)}{\gamma_1 + \tau^{1-\sigma}(n+2)\overline{\gamma}} \\ = \; & -\frac{2\tau^{1-\sigma}\gamma_1 \overline{\gamma} \mathrm{var}(\epsilon_2)}{[\gamma_1 + (n+1)\tau^{1-\sigma}\overline{\gamma}]\,(\gamma_1 + n\tau^{1-\sigma}\overline{\gamma})\,[\gamma_1 + (2+n)\tau^{1-\sigma}\overline{\gamma}]}, \end{aligned}$$

which is negative. Therefore, if ζ is sufficiently small, $\{b(S,n)\}_{n=0}^{N-1}$, and hence $\{\pi(\varphi, S, n)\}_{n=0}^{N-1}$, is a strictly convex series in n. It follows that $\varphi_1(S) = \varphi_2(S) = \cdots = \varphi_{N-1}(S)$ so that each of $\Phi_1(S), \Phi_2(S), \cdots, \Phi_{N-2}(S)$ is empty. Consequently, if idiosyncratic risk is sufficiently small then an active firm will not export if $\varphi \leq \varphi_1(S)$ and will export to all foreign countries if $\varphi > \varphi_1(S)$.

Part 2 This follows from the proof of Proposition 5 by noting that with home-country shocks and no (common or idiosyncratic) foreign shocks, the series $\{\pi(\varphi, S, n)\}_{n=1}^{N-1}$ is strictly concave in n (because Proposition 5 does not rule out certain foreign demand). By continuity, the series will remain strictly concave for a sufficiently low level of riskiness of the idiosyncratic shocks. ■

Proof of Proposition 7: Riskier Global Shocks

In view of the fact that (see footnote 23)

$$b(S,n) = \left[\int_{\mathbf{R}^N} (\gamma^g)^{1/\sigma} dS^g(\Gamma^g) \right]^\sigma \left[\int_{\mathbf{R}^N} \left(\gamma_1^{ng} + \tau^{1-\sigma} \nu \sum_{i \in \mathcal{N} \setminus \{1\}} \gamma_i^{ng} \right)^{1/\sigma} dS^{ng}(\Gamma^{ng}) \right]^\sigma,$$

we only need to show that riskier global shocks lead to a decrease in $\int_{\mathbf{R}^N} (\gamma^g)^{1/\sigma} dS^g(\Gamma^g)$. However, this follows from the power $1/\sigma$ being a strictly concave function.

$\blacksquare$

Proof of Proposition 8: Riskier Common Foreign Shocks

Part 1 This follows from the fact that $\pi(\varphi, S, 0)$ is unaffected by the riskiness of the common foreign shocks.

Part 2 We first show that riskier common foreign shocks are associated with a lower $b(S,n) - b(S,n-1)$ for $n \geq 1$, or equivalently, that the effect of riskier common foreign shocks on $b(S,n)$ decreases with n. Note that we can write $b(S,n) = [\theta_S \kappa_{cn} + (1-\theta_S)\kappa_{dn}]^{\sigma}$, where

$$\begin{aligned}\kappa_{cn} &\equiv \int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma), \\ \kappa_{dn} &\equiv \int_{\mathbf{R}^N} \tilde{a}^{1/\sigma} dS_d(\Gamma).\end{aligned}$$

Any increase in the riskiness of the common foreign shocks can be obtained by adding a series of symmetric binomial gambles to different realizations of the foreign shocks.[28] Therefore, let $\hat{\Gamma} = (\hat{\gamma}_1, \hat{\gamma}_2, \cdots, \hat{\gamma}_N)$ indicate a particular realization of Γ drawn from S_c which is exposed to the symmetric binomial gamble $\pm\delta$, $\delta > 0$, added to each of $\hat{\gamma}_2 = \hat{\gamma}_3 = \cdots = \hat{\gamma}_N$. Having added the gamble, κ_{cn} becomes

$$\int_{\mathbf{R}^N} \ddot{a}_n dS_c(\Gamma), \tag{13}$$

where $\ddot{a}_n \equiv a_n^{1/\sigma}$ for $\Gamma \in \mathbf{R}^N \backslash \hat{\Gamma}$, $\ddot{a}_n \equiv \frac{1}{2}\left(\hat{a}_{n+}^{1/\sigma} + \hat{a}_{n-}^{1/\sigma}\right)$ for $\Gamma = \hat{\Gamma}$, $\hat{a}_{n+} \equiv \hat{\gamma}_1 + \tau^{1-\sigma} n(\hat{\gamma}_2 + \delta)$ and $\hat{a}_{n-} \equiv \hat{\gamma}_1 + \tau^{1-\sigma} n(\hat{\gamma}_2 - \delta)$.

[28] See Rothchild and Stiglitz (1970).

Differentiating $b(S,n)$ with respect to δ yields

$$\tfrac{1}{2}\theta_S\tau^{1-\sigma}nb(S,n)^{(\sigma-1)/\sigma}\left[\hat{a}_{n+}^{(1-\sigma)/\sigma}-\hat{a}_{n-}^{(1-\sigma)/\sigma}\right]dS_c(\hat{\Gamma}),$$

which we want to show decreases in n. Since $b(S,n)^{(\sigma-1)/\sigma}$ increases in n and $\hat{a}_{n+}^{(1-\sigma)/\sigma}-\hat{a}_{n-}^{(1-\sigma)/\sigma}<0$, it is sufficient to show that

$$n\left[\hat{a}_{n+}^{(1-\sigma)/\sigma}-\hat{a}_{n-}^{(1-\sigma)/\sigma}\right] \tag{14}$$

decreases in n. Differentiating (14) with respect to n yields

$$\hat{a}_{n+}^{(1-\sigma)/\sigma}-\hat{a}_{n-}^{(1-\sigma)/\sigma}-\rho\tau^{1-\sigma}n\left[(\hat{\gamma}_2+\delta)\hat{a}_{n+}^{(1-2\sigma)/\sigma}-(\hat{\gamma}_2-\delta)\hat{a}_{n-}^{(1-2\sigma)/\sigma}\right]. \tag{15}$$

If $(\hat{\gamma}_2+\delta)\hat{a}_{n+}^{(1-2\sigma)/\sigma}\geq(\hat{\gamma}_2-\delta)\hat{a}_{n-}^{(1-2\sigma)/\sigma}$, then (15) is negative. To show that (15) is also negative if $(\hat{\gamma}_2+\delta)\hat{a}_{n+}^{(1-2\sigma)/\sigma}<(\hat{\gamma}_2-\delta)\hat{a}_{n-}^{(1-2\sigma)/\sigma}$, note that (15) is less than

$$\begin{aligned}&\hat{a}_{n+}^{(1-\sigma)/\sigma}-\hat{a}_{n-}^{(1-\sigma)/\sigma}-\tau^{1-\sigma}n\left[(\hat{\gamma}_2+\delta)\hat{a}_{n+}^{(1-2\sigma)/\sigma}-(\hat{\gamma}_2-\delta)\hat{a}_{n-}^{(1-2\sigma)/\sigma}\right]\\ =\;&\hat{\gamma}_1\left[\hat{a}_{n+}^{(1-2\sigma)/\sigma}-\hat{a}_{n-}^{(1-2\sigma)/\sigma}\right],\end{aligned}$$

which is negative since $\delta>0\Rightarrow\hat{a}_{n+}^{(1-2\sigma)/\sigma}<\hat{a}_{n-}^{(1-2\sigma)/\sigma}$. Accordingly, (15) is negative so that (14) decreases with n. It follows that riskier common foreign shocks are associated with a lower $b(S,n)-b(S,n-1)$, and hence a lower $\pi(\varphi,S,n)-\pi(\varphi,S,n-1)$. Part 2 of the proposition follows. ■

Proof of Proposition 9: Riskier Idiosyncratic Foreign Shocks

Part 1 This follows from $\pi(\varphi,S,0)$ being unaffected by the riskiness of the idiosyncratic foreign shocks.

Part 2 This follows from part 2 of Proposition 8 by setting $S_c(\gamma_2\mid\gamma_1)=S_d(\gamma_i\mid\gamma_1)$ for $i\geq1$. In other words, common and idiosyncratic foreign shocks are identical when firms export to only one country.

Part 3 Since $\pi(\varphi, S', 1) > \pi(\varphi, S'', 1)$ (from part 2) and $\lim_{n\to\infty} \pi(\varphi, S', n) = \lim_{n\to\infty} \pi(\varphi, S'', n)$, there exists an n^* such that $\pi(\varphi, S', n^*) - \pi(\varphi, S', n^* - 1) < \pi(\varphi, S'', n^*) - \pi(\varphi, S'', n^* - 1)$. This implies that $\varphi_{n^*}(S') > \varphi_{n^*}(S'')$. ■

Proof of Proposition 10: Correlation Between Foreign Shocks

Part 1 Since $b(S', n) = b(S'', n)$ for $n = 0, 1$, it follows that $\varphi_n(S') = \varphi_n(S'')$ for $n = 0, 1$.

Part 2 Since $b(S, 1) - b(S, 0)$ is independent of θ_S (from part 1), we need to show that $b(S, n+1) - b(S, n)$ strictly decreases with θ_S for all $1 \leq n \leq N - 2$. This requires showing that

$$\left[\theta_S \int_{\mathbf{R}^N} a_{n+1}^{1/\sigma} dS_c(\Gamma) + (1 - \theta_S) \int_{\mathbf{R}^N} \tilde{a}_{n+1}^{1/\sigma} dS_d(\Gamma)\right]^\sigma - \left[\theta_S \int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma) + (1 - \theta_S) \int_{\mathbf{R}^N} \tilde{a}_n^{1/\sigma} dS_d(\Gamma)\right]^\sigma \quad (16)$$

decreases with θ_S for all $1 \leq n \leq N - 2$, where a_n and $\tilde{a}_n$ were defined in the proof of Proposition 3.

The derivative of (16) with respect to θ_S has the same sign as

$$\left[\theta_S \int_{\mathbf{R}^N} a_{n+1}^{1/\sigma} dS_c(\Gamma) + (1 - \theta_S) \int_{\mathbf{R}^N} \tilde{a}_{n+1}^{1/\sigma} dS_d(\Gamma)\right]^{\sigma-1} \left[\int_{\mathbf{R}^N} a_{n+1}^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N} \tilde{a}_{n+1}^{1/\sigma} dS_d(\Gamma)\right]$$
$$- \left[\theta_S \int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma) + (1 - \theta_S) \int_{\mathbf{R}^N} \tilde{a}_n^{1/\sigma} dS_d(\Gamma)\right]^{\sigma-1} \left[\int_{\mathbf{R}^N} a_n^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N} \tilde{a}_n^{1/\sigma} dS_d(\Gamma)\right]. \quad (17)$$

Since $a_{n+1} > a_n$ and $\tilde{a}_{n+1} > \tilde{a}_n$, the positive term in the left brackets on the first line of (17) exceeds the positive term in the left brackets on the second line of (17). It follows from the proof of Proposition 3 that the terms in the right brackets on the first and second lines of (17) are negative. To show that (16) decreases in θ_S for $1 \leq n \leq N - 2$ it therefore suffices to demonstrate that the term in the right brackets on the first line of (17) is less than the term in the right brackets on the second line of (17).

To show this, let

$$a_{n\mu} \equiv \frac{\sum_{i\in\mathcal{N}\setminus\{1\}}\gamma_i + \mu\gamma_j}{n+\mu},$$

where $\mu \in [0,1]$ and $j \notin \mathcal{N}$. Such j exists because $n \leq N-2$. Note that $a_{n\mu}$ has the same mean as γ_2 but is less risky for $\mu > 0$. Further, the riskiness of $a_{n\mu}$ decreases with μ. Consider

$$\begin{aligned}
&\int_{\mathbf{R}^N}\left[\gamma_1 + \tau^{1-\sigma}(n+\mu)\gamma_2\right]^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N}\left[\gamma_1 + \tau^{1-\sigma}\left(\sum_{i=1}^{n}\gamma_{i+1} + \mu\gamma_j\right)\right]^{1/\sigma} dS_d(\Gamma)\\
= \;& (n+\mu)^{1/\sigma}\left[\int_{\mathbf{R}^N}\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}\gamma_2\right)^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N}\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}a_{n\mu}\right)^{1/\sigma} dS_d(\Gamma)\right],
\end{aligned} \tag{18}$$

which for $\mu = 1$ would equal the term in the right brackets on the first line of (17) and for $\mu = 0$ would equal the term in the right brackets on the second line of (17).

The derivative of (18) with respect to μ has the same sign as

$$\begin{aligned}
&(n+\mu)^{1/\sigma-1}\left[\int_{\mathbf{R}^N}\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}\gamma_2\right)^{1/\sigma} dS_c(\Gamma) - \int_{\mathbf{R}^N}\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}a_{n,\mu}\right)^{1/\sigma} dS_d(\Gamma)\right]\\
&-(n+\mu)^{1/\sigma-2}\left[\int_{\mathbf{R}^N}\gamma_1\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}\gamma_2\right)^{1/\sigma-1} dS_c(\Gamma) - \int_{\mathbf{R}^N}\gamma_1\left(\frac{\gamma_1}{n+\mu} + \tau^{1-\sigma}a_{n\mu}\right)^{1/\sigma-1} dS_d(\Gamma)\right]\\
&-(n+\mu)^{1/\sigma}S_d^{\#},
\end{aligned} \tag{19}$$

where $S_d^{\#}$ is the effect on $\int_{\mathbf{R}^N}[\gamma_1/(n+\mu) + \tau^{1-\sigma}a_{n\mu}]\, dS_d(\Gamma)$ of the decrease in the riskiness of $a_{n\mu}$ caused by an increase in μ.

Concerning the first and second line in (19): Since $\gamma_1/(n+\mu) + \tau^{1-\sigma}\gamma_2$ has the same mean but is riskier than $\gamma_1/(n+\mu) + \tau^{1-\sigma}a_{n\mu}$ for $\mu > 0$ and the power $1/\sigma$ is a strictly concave function while the power $1/\sigma - 1$ is a strictly convex function, both the first and the second line in (19) are negative for $n \geq 1$.

Concerning the third line in (19): If $n \geq 1$, an increase in μ reduces $1/(n+\mu)$ which is the weight in $a_{n\mu}$ of the realization of each of the independently and identically distributed random variables $\gamma_i \in \mathcal{N}\backslash\{1\}$; instead, the increase in μ increases $\mu/(n+\mu)$ which is the weight in $a_{n\mu}$ of the realization of the independently and identically distributed random variable γ_j. Since the weight of each of $\gamma_i \in \mathcal{N}\backslash\{1\}$ exceeds the weight of γ_j for $\mu < 1$, an increase in μ decreases the riskiness of $a_{n\mu}$. The power $1/\sigma$ is a strictly concave function, so the decrease in the riskiness of $a_{n\mu}$ leads to an increase in $\int_{\mathbf{R}^N} [\gamma_1/(n+\mu) + \tau^{1-\sigma} a_{n\mu}]^{1/\sigma} \, dS_d(\Gamma)$ which implies that $S_d^{\#}$ is positive. Hence, also the third line in (19) is negative.

It follows, then, that (19) is negative so that the term in the right brackets on the first line of (17) is less than the term in the right brackets on the second line of (17). Consequently, the derivative of (16) with respect to θ_S is negative. Thus, $b(S, n+1) - b(S, n)$ strictly decreases with θ_S, and we conclude that $\varphi_n(S') > \varphi_n(S'')$ for all $n \geq 2$. ■

Proof of Proposition 11: Riskier Home-Country Shocks

Part 1 Since $b(S, 0) = \left[\int_{\mathbf{R}^N} \gamma_1^{1/\sigma} dS(\Gamma)\right]^{\sigma}$ and $1/\sigma$ is a strictly concave function, it follows that $b(S, 0)$ decreases with home-country riskiness. Consequently, $\pi(\varphi, S', 0) > \pi(\varphi, S'', 0)$ and hence $\varphi_0(S') < \varphi_0(S'')$.

Part 2 We want to show that $b(S, n+1) - b(S, n)$ increases with home-country risk. For this, it is sufficient to show that the effect of an increase in home-country risk on $b(S, n)$ increases with n. (The effect of home-country risk is negative, so we show that the absolute value of the negative effect decreases with n.) To do so, we now let $\hat{\Gamma}$ indicate a particular realization of Γ which is exposed to the symmetric binomial gamble $\pm\delta$, $\delta > 0$, added to $\hat{\gamma}_1$.

Suppose first that the gamble is added when the foreign shocks are common. Then κ_{cn} equals (13) where now $\hat{a}_{n+} \equiv \hat{\gamma}_1 + \delta + \tau^{1-\sigma} n \hat{\gamma}_2$ and $\hat{a}_{n-} \equiv \hat{\gamma}_1 - \delta + \tau^{1-\sigma} n \hat{\gamma}_2$, while κ_{dn} is unchanged. Differentiating $b(S, n)$ with respect to δ yields

$$\tfrac{1}{2}\theta_S b(S, n)^{\rho} \left(\hat{a}_{n+}^{-\rho} - \hat{a}_{n-}^{-\rho}\right) dS_c(\Gamma), \tag{20}$$

which is negative. We want to show that for a small σ, the absolute value of (20) decreases with the number of export destinations, i.e., that

$$\frac{\hat{\tilde{a}}_{(n+1)+}^{-\rho} - \hat{\tilde{a}}_{(n+1)-}^{-\rho}}{\hat{a}_{n+}^{-\rho} - \hat{a}_{n-}^{-\rho}} < \frac{b(S, n+1)^{-\rho}}{b(S,n)^{-\rho}}$$

for all n. Applying l'Hospital's rule to the left-hand side of this inequality, for $\delta \to 0$ this becomes

$$\left[\frac{\hat{\gamma}_1 + \tau^{1-\sigma}(n+1)\hat{\gamma}_i}{\hat{\gamma}_1 + \tau^{1-\sigma} n \hat{\gamma}_i}\right]^{-\rho-1} < \left[\frac{\theta_S \kappa_{c(n+1)} + (1-\theta_S)\kappa_{d(n+1)}}{\theta_S \kappa_{cn} + (1-\theta_S)\kappa_{dn}}\right]^{1-\sigma}$$

$$\Leftrightarrow \left[\frac{\hat{\gamma}_1 + \tau^{1-\sigma} n \hat{\gamma}_i}{\hat{\gamma}_1 + \tau^{1-\sigma}(n+1)\hat{\gamma}_i}\right]^{2\sigma-1} < \left[\frac{\theta_S \kappa_{cn} + (1-\theta_S)\kappa_{dn}}{\theta_S \kappa_{c(n+1)} + (1-\theta_S)\kappa_{d(n+1)}}\right]^{\sigma(\sigma-1)}$$

for all n. As $\sigma \to 1$ the right-hand side approaches 1, while the left-hand side approaches a number strictly less than 1. Hence, if the home-country shock becomes riskier when the foreign shocks are common, then the absolute value of the negative effect of an increase in the home-country risk on $b(S,n)$ decreases with n.

Suppose next that the gamble is added when the foreign shocks are idiosyncratic. Then κ_{cn} is unchanged while $\kappa_{dn} = \int_{\mathbf{R}^N} \ddot{\tilde{a}}_n dS_d(\Gamma)$, where $\ddot{\tilde{a}}_n \equiv \tilde{a}_n^{1/\sigma}$ for $\Gamma \in \mathbf{R}^N \backslash \hat{\Gamma}$, $\ddot{\tilde{a}}_n \equiv \frac{1}{2}\left(\hat{\tilde{a}}_{n+}^{1/\sigma} + \hat{\tilde{a}}_{n-}^{1/\sigma}\right)$ for $\Gamma = \hat{\Gamma}$, where $\hat{\tilde{a}}_{n+} \equiv \hat{\gamma}_1 + \delta + \tau^{1-\sigma}\sum_{i\in\mathcal{N}\backslash\{1\}} \hat{\gamma}_i$ and $\hat{\tilde{a}}_{n-} \equiv \hat{\gamma}_1 - \delta + \tau^{1-\sigma}\sum_{i\in\mathcal{N}\backslash\{1\}} \hat{\gamma}_i$. Differentiating $b(S,n)$ with respect to δ yields

$$\tfrac{1}{2}\theta_S b(S,n)^{\rho}\left(\hat{\tilde{a}}_{n+}^{-\rho} - \hat{\tilde{a}}_{n-}^{-\rho}\right) dS_d(\Gamma), \tag{21}$$

which is negative. We want to show that for a small σ, the absolute value of (21) decreases with the number of export destinations. The proof is almost identical to the case of common shocks considered above. We conclude that for a sufficiently small σ, home-country risk increases $b(S, n+1) - b(S,n)$, and hence $\varphi_n(S') > \varphi_n(S'')$ for all $n \geq 1$. ■

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