A Model of Recommended Retail Prices

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Abstract

Manufacturers in a variety of industries frequently use list prices, manufacturer suggested retail prices, or similar forms of cheap public price recommendations. Despite their prevalence, it is still not well understood what purpose these non-binding recommendations serve for manufacturers and what effect they have on retailers and consumers. I present a model in which a manufacturer's price recommendation provides information that influences consumers' search behavior. By publicly informing consumers of aggregate market conditions, the manufacturer affects consumers' reservation prices and hence the prices that retailers charge. The manufacturer faces a tradeoff in affecting search: inducing lower reservation prices reduces retailer markups but also inhibits the manufacturer's ability to extract surplus from consumers with a high willingness to pay. I show that the manufacturer can influence consumer search behavior by credibly providing information through cheap talk. Furthermore, I find that a ban on recommendations can be welfare reducing, harming both consumers and the manufacturer. The model also illuminates the difference between price recommendations and explicit price ceilings by demonstrating that the latter induce higher sales but may increase or decrease welfare and consumer surplus.

1 Introduction

Manufacturers routinely use non-binding recommended retail prices in markets ranging from common household goods found at the grocery store to big ticket items such as electronics, appliances, and cars. These recommendations come in a variety of forms (list prices, manufacturer suggested retail prices (MSRPs), sticker prices, etc.) and are made visible to consumers whether they shop at a brick and mortar retailer or online. There is consensus that price recommendations are closely linked to real market outcomes. This relationship has both been shown empirically (e.g. Faber and Janssen (2008)) and also implicitly assumed in the myriad studies that use recommendations as a proxy for transaction prices (e.g. Berry, Levinsohn, and Pakes (1995)). There is also anecdotal evidence that recommendations can directly affect the decisions of market participants. For example, when buying a new car consumers know not to accept a price at or above MSRP and strategic dealers seem to take this into account as they set prices.[1] However, despite the evidence *that* price recommendations effect behavior, our understanding of *how* they do so is quite limited. In part due to the fact that price recommendations are non-binding, the mechanism by which they have an impact and the motives of the manufacturer in making these recommendations are still not well understood.

In practice, most products tend to sell at or below their recommended price, hence a common explanation is that recommendations act as price ceilings. This story is compelling because how a manufacturer benefits from a price ceiling *is* well understood. Retailers with market power impose additional markups which hurt the manufacturer's sales and the manufacturer can resolve this problem with a price ceiling (Mathewson and Winter (1984)). Yet such an explanation of recommendations is incomplete. Since price recommendations are non-binding, at least in name, it is not clear why a manufacturer would make a recommendation instead of just imposing a price ceiling directly. In addition, manufacturers often go to great lengths to publicize their recommendations through advertising or by printing them on product packaging. An explanation of recommendations as explicit price ceilings ignores the potential role played by consumers.

This paper provides an alternative explanation in which price recommendations directly affect consumers' search behavior. Price recommendations provide consumers with information rather than explicitly restraining retailers. Consumers are uncertain about aggregate market conditions and when they engage in costly sequential price discovery, they do not know the distribution of retail prices. Consequently, when a consumer observes a particular price, she does not know if it represents a "good deal" or whether she should keep searching. The manufacturer makes a price recommendation that reveals this information and helps consumers decide between purchasing or continuing to search. Retailers anticipate consumers' reactions to the recommendation and adjust their prices accordingly. By this mechanism, non-binding price recommendations directed at consumers have a real impact on consumer and retailer behavior, and thus on market outcomes.

Price recommendations help consumers avoid the costs of learning about market conditions. But what incentive does the manufacturer have to provide this information? I show that by informing consumers and affecting search, the manufacturer faces a classic price-quantity tradeoff. In the main model consumers have either a high or a low valuation for the manufacturer's product. When

[1] While the majority of vehicles sell for prices strictly below MSRP there have been a few notable exceptions such as the Toyota Prius.

a price recommendation induces consumers to reject high prices, the manufacturer restricts his ability to set a high wholesale price and cannot extract as much surplus from high valuation consumers. At the same time, when consumers reject high prices retailers are forced to reduce their markups and this increases sales to consumers with low valuations. Hence inducing more search trades off serving more low valuation consumers for extracting surplus from high valuation consumers, and which of these two effects is more important for the manufacturer depends on market conditions.

Since the manufacturer has a vested interest in how much search is undertaken, there is an issue of credibility. If the he has incentive to mislead consumers, consumers would rationally ignore recommendations and then no information can be transmitted. Furthermore, assuming this issue away by imposing an exogenous penalty for lying may be unrealistic. For example consider a book cover with one of two possible statements: "best seller" or "MSRP $19.99". Assuming that the first statement must be truthful is reasonable since whether a book is a best seller is easily verified. But the price recommendation is not a falsifiable statement; it is not immediately obvious how a manufacturer can be accused of recommending a "false" price. In a main result, I show that credibility often does not have to be assumed and that the manufacturer can communicate market conditions to consumers using cheap talk.

The cheap talk result stems from the fact that the manufacturer's and consumers' interests can be aligned. In the main model, there is uncertainty about aggregate demand, modeled as the proportion of consumers with a high valuation. The manufacturer and retailers observe aggregate demand but consumers do not. Recalling the manufacturer's tradeoff to inducing search, he prefers less search in states where consumers are predominantly high types and more search in states when consumers are predominantly low types. For their part, consumers expect retailers to charge high prices when aggregate demand is high and low prices when aggregate demand is low and accordingly prefer to search more in the low demand state and search less in the high demand state. Thus, since the two parties agree on whether more or less search is desirable in either state, by the logic in Crawford and Sobel (1982) the manufacturer can credibly convey information to consumers and the cheap talk result goes through.

To address how one should consider price recommendations from an antitrust perspective I present two policy experiments. First, I examine a ban on recommendations and find that in situations where aggregate demand is sufficiently uncertain, the ban can reduce welfare and in particular the surplus of the manufacturer and consumers. I show that in equilibrium only consumers with high valuations engage in search, and the welfare result hinges on this observation. High valuation consumers, by virtue of having learned their type, believe it is more likely that aggregate demand is also high and hence they are on average overly pessimistic about the distribution of prices. As the amount of uncertainty about aggregate demand is increased, high demand consumers become more pessimistic and eventually no search can be supported. This results in a loss of sales to consumers with low demand due to higher downstream prices and a loss of sales to consumers with high demand that may exit the market prior to observing a price they would accept.

Since taking away the manufacturer's ability to make price recommendations can be detrimental to market outcomes, in the second policy experiment I examine the effects endowing the manufacturer with more control. Specifically I explore the often made comparison between recommendations and explicit price ceilings. In a state of high demand, the manufacturer prefers to induce high prices

downstream in order to extract surplus from high demand consumers and as a result has little use for a price ceiling. Thus when aggregate demand is high, market outcomes are unchanged relative the setting with recommendations. In the low demand state however, the manufacturer is concerned with double marginalization and does benefit by setting a binding ceiling that induces lower prices than with price recommendations alone. This improves welfare by creating more sales and funneling consumers towards lower cost retailers.

While there is an extensive theoretical and empirical literature that deals with vertical market relationships, an explicit treatment of the role of price recommendations has been largely lacking. Price recommendations have long been lumped in as an instrument of resale price maintenance with the mechanism behind them left unexplored. Recently though, the question of what impact recommended retail prices can have given that they are *non-binding* has been posed in two papers. Buhler and Gartner (2009) propose a repeated setting where the recommendation guides a retailer about which price to set under threat of future punishment. A different approach is taken by Puppe and Rosenkranz (2006), in which the price recommendation directly enters loss-averse consumers' utility functions as the reference point. My model provides a different mechanism from both of these papers. I depart from Buhler and Gartner by modeling recommendations as messages to consumers and in doing so argue that recommendations are more than vertical restraints. In addition, unlike Puppe and Rosenkranz, I provide an explanation where the role of recommendations is not to change consumer preferences but rather to provide information. That recommendations affect consumers' behavior is not assumed but instead emerges in equilibrium, and the effect that recommendations have is not fixed by the model but rather is determined by market conditions.

Although I focus on price recommendations in vertical markets, there is a literature on the role of non-binding list prices in markets where producers sell directly. An example is residential housing, in which studies such as Horowitz (1992) have empirically demonstrated the relationship between list and transaction prices. Still, the theoretical foundations for why this relationship should exist have been largely lacking. For instance, while Horowitz conjectures that list prices help inform consumers of a seller's reservation price, he does not show that such information can be credibly communicated in an equilibrium. By contrast, theoretical models of list prices in this setting rely on them being more than just information. For example, Gill and Thanassoulis (2010) explicitly characterize the effect of list prices on industry competition, but in doing so assume that list prices act as binding price ceilings. I propose a mechanism by which price communications are truly non-binding, and yet have a real impact by providing consumers with information that affects their search. While a vertical market structure plays an important role in my results, this approach of modeling list prices as cheap talk can potentially help us understand their role in the housing and other related markets.

Lastly, this paper provides a modest methodological contribution to the literature on price search with aggregate uncertainty. Models in this literature either have sequential search but only two firms (Benabou and Gertner (1993)) or a larger number of firms and non-sequential search (Yang and Ye (2008)). The issue is tractability: sequential search with many firms potentially allows for equilibria where consumers follow non-stationary strategies. I provide a model in which search is sequential and there is continuum of potential sellers, yet search strategies are stationary because information is communicated credibly using cheap talk. Hence, I show that in principle sequential search among a large number of sellers can be modeled tractably.

The rest of this paper starts with Section 2 which presents the model. Section 3 characterizes the full information equilibrium, followed by Section 4 which shows the result that cheap signaling can credibly convey information. Sections 5 and 6 present welfare results for the two policy experiments: a ban on cheap communication and an introduction of price ceilings, respectively. Section 7 then concludes.

2 The Model

On the demand side there is a continuum of consumers with measure one. Each consumer demands a single unit of a good. Consumers draw their valuation for the good from a distribution where with probability φ they draw the high value of 1 and with probability $1-\varphi$ they draw the low value $v < 1$. The parameter φ is uncertain and has two equally likely realizations φ_L and φ_H. Consumers learn only their own valuation and not the realization of φ.

The supply side consists of a monopolist manufacturer and a continuum of retailers with measure one. The manufacturer has zero production costs and is restricted to setting a uniform linear wholesale price w for all retailers. Each retailer has constant marginal costs made up of two components: the common wholesale price w and an idiosyncratic cost c. Retailers independently draw c from a continuous and differentiable distribution $F(\cdot)$ with support on $[0,1]$.

The game proceeds as follows. First, nature selects φ and the realization is observed by the manufacturer and retailers but not by the consumers. The manufacturer then sets wholesale price w and signal $\sigma \in \{\sigma_L, \sigma_H\}$[2]. Next, retailers, having observed φ, w, σ and their own cost c simultaneously set prices. These prices are then fixed for the rest of the game. Next, each consumer learns her own valuation and observes the manufacturer's signal σ and a price p from a randomly selected retailer. The consumer can either purchase the good at price p and exit the market, exit the market without purchasing, or search. If she chooses to search, the consumer pays a search cost s, observes another price at a randomly selected retailer,[3] and at this point has the option to purchase the good at any of the prices she has seen so far, exit without purchasing, or search again. This process continues until every consumer has exited. There is no time discounting.

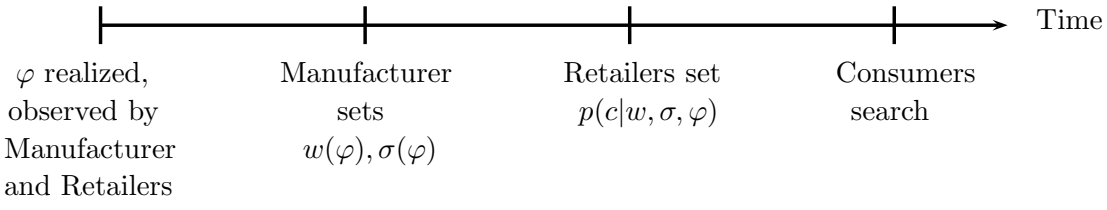

Figure 1: **Model Timing**

[2]This restriction on the signal space is without loss of generality given that there are two states of nature.
[3]Specifically, I assume that when consumers search they draw every retailer with equal chance.

Strategies are $w(\varphi)$, $\sigma(\varphi)$ for the manufacturer and $p(c|w,\sigma,\varphi)$ for retailers. Define $\vec{\mathbb{P}}$ as the set of all possible price histories, so that

$$\vec{\mathbb{P}} \equiv \{\vec{p} = \{p_i\}_{i=1,\ldots,n} \;\mid\; n \in \mathbb{I},\; p_i \in \mathbb{R}^+\}$$

Conditional on her valuation $\theta \in \{v, 1\}$, a consumer has strategy $A(\sigma, \vec{p}|\theta)$ and beliefs $\mu(\sigma, \vec{p}|\theta)$ where

$$A:\ \{\sigma_L, \sigma_H\} \times \vec{\mathbb{P}} \times \{v, 1\} \to \{\text{exit, purchase, search}\}$$
$$\mu:\ \{\sigma_L, \sigma_H\} \times \vec{\mathbb{P}} \times \{v, 1\} \to [0, 1]$$

I use the Perfect Bayesian Equilibrium solution concept, where all strategies are mutual best responses and beliefs are formed using Bayes rule whenever possible.

2.1 Model Discussion

I introduce downstream retailer heterogeneity to induce price dispersion as in Reinganum (1979). On the consumer side, I depart from Reinganum's setting by having heterogenous consumers with unit demand.[4] The choice of unit demand is motivated in part by the fact that many of the goods that come with price recommendations (cars, electronics, books, etc.) are purchased one at a time as opposed to in continuous quantities. Unit demand also improves tractability while still delivering important qualitative features like downstream price dispersion and search.

The key to ensuring that the manufacturer's signals have content is to endow the manufacturer with information that consumers do not have. I have chosen aggregate demand as the source of uncertainty to reflect the fact that in many markets consumers expect prices to depend on how popular a product may be, and that sellers are more aware of this information than consumers through marketing research or other such means. In principle uncertainty can come from other sources, for instance manufacturer costs. I the conclusion, I will argue that in this scenario there is good reason to believe that the manufacturer can still inform consumers with cheap talk.

As a final note, I will find that in equilibrium some retailers do not makes sales due to their high costs and the number of these retailers matters for real market outcomes. I restrict the support of retailer costs to $[0, 1]$ in order to include only retailers that can have positive gains from trade.

In the ensuing analysis, I look for an equilibrium in which price recommendations reveal the state via cheap talk. I solve for this equilibrium in two steps. In Section 3, I solve an auxiliary model in which there is complete information about aggregate demand φ. I then use this solution in Section 4 to help characterize an equilibrium of the main model where the manufacturer's signal perfectly reveals the state to consumers.

3 Equilibrium in a Full Information Setting

In this section I solve the model above but as if φ is common knowledge. I first solve for the downstream equilibrium between consumers and retailers conditional on wholesale price w and then use this solution to characterize the manufacturer's optimal choice of w.

[4] Reinganum's model has homogeneous consumers with continuous demand. In either case, the key to generating price dispersion is that every retailer has different costs and faces an elastic demand function.

3.1 Downstream Equilibrium

I show that any downstream equilibrium must be of the following form.

- low valuation consumers never search and either buy immediately or quit,
- high valuation consumers use a threshold search strategy $\bar{p}$, and
- retailers makes sales at one of two prices: either v or $\bar{p}$.

To start, in equilibrium consumers know the distribution of prices from which they sample and by a standard result in McCall (1970) they optimally follow stationary threshold strategies. Every consumer has search cost s and one implication of this is that given an equilibrium distribution of prices, if the lowest price charged is $\underline{p}$, no consumer will reject any price $p \in [\underline{p}, \underline{p} + s)$ unless it is bigger than her valuation.

Next, in any equilibrium the lowest price charged, $\underline{p}$, cannot be smaller than v. Toward a contradiction, imagine an equilibrium where $\underline{p} < v$. In such an equilibrium no consumer would reject a price $p \in [\underline{p}\ ,\ \min\{\underline{p} + s, v\})$. This implies that the retailer charging $\underline{p}$ is not maximizing profits: he can increase his price slightly and not lose any sales. Hence, no equilibrium can be supported where a price below v is charged.[5]

Given this fact, low valuation consumers will not collect surplus in any equilibrium and consequently never find it optimal to search after their first price observation (which is free). Thus in any equilibrium low valuation consumers either purchase at the first price if it is v, otherwise they exit.[6] High valuation consumers on the other hand may have incentive to search. Let $\bar{p}$ be their threshold and since no price is ever charged below v, it must be that $\bar{p} \geq v + s$.

Now consider the prices set by retailers. Retailers potentially face three kinds of consumers: new low valuation consumers, new high valuation consumers, and high valuation consumers that have already seen some prices but continued to search. Hence retailers face a step demand function: they can set a price $p \leq v$ and serve all the consumers that visit them or they can set a price $v < p \leq \bar{p}$ and serve only the high types. A price $p > \bar{p}$ would result in no sales. The retailer's demand function is thus

$$q(p) = \begin{cases} 1 + \varphi\kappa & \text{if } p \leq v \\ \varphi + \varphi\kappa & \text{if } v < p \leq \bar{p} \\ 0 & \text{if } p > \bar{p} \end{cases}$$

and is illustrated in Figure 2, where $\varphi\kappa$ is the number of searchers that visit any particular retailer.

[5] A similar argument can be found in Diamond (1971).

[6] Low valuation consumers are actually indifferent between buying at price v or exiting. However, for a technical reason no equilibrium can be supported when low valuation consumers reject v with positive probability. Given v is the low types' threshold, if they reject v with probability $\varepsilon > 0$, there always exists a $\delta > 0$ small enough where a retailer can charge $v - \delta$, gain the ε in sales, and make higher profits. A retailer's best response is to choose the highest price from the set of prices strictly smaller than v, but since this is an open set no such price exists. So by a closure problem an equilibrium cannot be supported for any $\varepsilon > 0$. By the same logic, high valuation consumers must accept with probability 1 at their threshold as well.

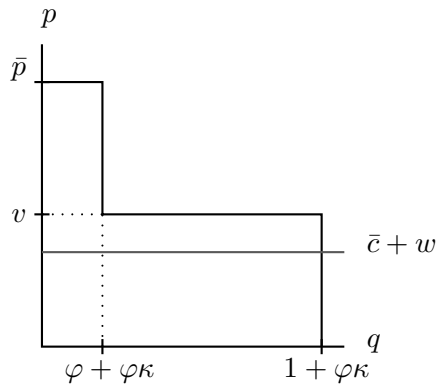

Figure 2: Retailer demand

Retailers choose price p to maximize their profit, given by

$$\pi(p|c) = (p - c - w)q(p)$$

Any retailer with a cost $c > \bar{p} - w$ is priced out of the market. For the purpose of exposition, I assume that all priced out retailers charge a price $p = \infty$.[7] Retailers that can afford to participate then charge either v or $\bar{p}$ depending on their cost. Define $\bar{c}$ as the cost at which a retailer is indifferent between these two prices:

$$\pi(v|\bar{c}) = \pi(\bar{p}|\bar{c}) \qquad \Leftrightarrow \qquad \bar{c} = v - w - \frac{\varphi}{1-\varphi}(\bar{p} - v)(1 + \kappa) \tag{1}$$

Whenever the $\bar{c}$ that solves equation (1) is negative, it must be that $\pi(v|c) < \pi(\bar{p}|c)$ for every $c \geq 0$ and no retailer charges $p = v$. Figure 3 illustrates an equilibrium distribution of prices when $\bar{c} > 0$.

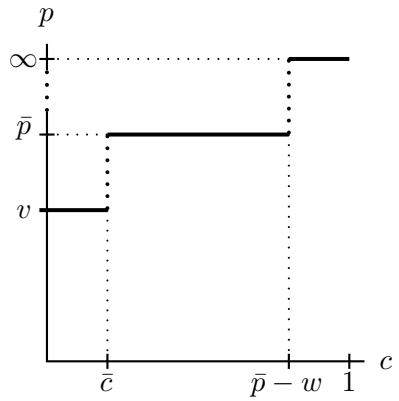

Figure 3: Downstream prices

Back to consumers' strategies, in equilibrium high types' search threshold $\bar{p}$ must be rational conditional on the distribution of prices. Let $V(p)$ be the high type's value function given that p is the lowest price she has seen. By definition,

[7] In any equilibrium in the full information setting where the highest accepted price is $\bar{p}$, the set of prices set by priced out retailers does not affect any equilibrium outcomes.

$$V(p) = \max\{0,\ 1-p,\ V^s(p)\} \tag{2}$$

where the consumer's options are to exit, accept p, or continue to search and receive continuation value $V^s(p) \equiv E[V(p')|p] - s$. Given the distribution of equilibrium prices as described by $\bar{c}$, in equilibrium the high type's threshold must satisfy

$$\begin{aligned} 1-\bar{p} &= \max\Big\{0, V^s(\bar{p})\Big\} \\ &= \max\Big\{0,\ F(\bar{c})(1-v) + (1-F(\bar{c}))(1-\bar{p}) - s\Big\} \end{aligned} \tag{3}$$

The left hand side is the value to accepting $\bar{p}$, the right hand side gives the maximum of either the value of exiting or the continuation value of searching. When continuing to search, a consumer will pay cost s and observe price v with probability $F(\bar{c})$ or a price at least as high as $\bar{p}$ with probability $1-F(\bar{c})$. If v is observed, the consumer will accept it and obtain payoff $1-v$. If a price of $\bar{p}$ or higher is observed the continuation payoff to the consumer is $1-\bar{p}$ since she is indifferent to accepting $\bar{p}$ at that point. When $\bar{p} < 1$ solves the equation above, the expression simplifies to

$$(\bar{p} - v)F(\bar{c}) = s \tag{4}$$

This gives a natural interpretation for the threshold, with the left hand side being the expected benefit from another observation and then right hand side being the expected cost.

To fully describe a high type consumer's strategy, one must specify whether she exits or continues to search conditional when rejecting a price above $\bar{p}$. Which of these two options is optimal depends on the distribution of prices. Given she follows the threshold strategy $\bar{p}$, a consumer's continuation value to searching is

$$V^s(p \geq \bar{p}) = F(\bar{c})(1-v) + \Big(F(\bar{p}-w) - F(\bar{c})\Big)(1-\bar{p}) + \Big(1 - F(\bar{p}-w)\Big)\max\Big\{0, V^s(p \geq \bar{p})\Big\} - s \tag{5}$$

That is, a consumer can either observe and accept a price of v or $\bar{p}$ or observe a higher price and have the option to search or exit.[8] Let α be the probability with which a consumer searches conditional on rejecting. When $\bar{p} < 1$ solves equation (3), the continuation value to searching must be strictly positive, hence $\alpha = 1$. When $\bar{p} = 1$, that is when high type consumers accept any price at or below their valuation, the continuation value to searching is either exactly zero or strictly negative (see equation (2)). In this case equation (5) simplifies to

$$V^s(p \geq \bar{p} = 1) = F(\bar{c})(1-v) - s$$

If $F(\bar{c})(1-v) - s < 0$ exiting is strictly preferred to searching so $\alpha = 0$. If instead $F(\bar{c})(1-v) - s = 0$, then any $\alpha \in [0,1]$ is optimal for the consumer. To summarize, α must satisfy

$$\alpha = \begin{cases} 1 & \text{if } \bar{p} < 1 \\ 0 & \text{if } \bar{p} = 1,\ F(\bar{c})(1-v) - s < 0 \end{cases} \tag{6}$$

[8] Note that implicit in equation (5) is the fact $V^s(p \geq \bar{p})$ is constant in p. This is a standard result, stemming from the fact that having access to a price one will never accept in the future provides the same continuation value regardless of what that price actually is.

Lastly, the equilibrium must specify the number of searchers $\varphi \cdot \kappa$ received by each retailer. The probability that a high type consumer rejects her first price but eventually purchases is given by

$$\Pr(\text{search and buy}) = F(\bar{p} - w) \sum_{i=1}^{\infty} \Big(\alpha(1 - F(\bar{p} - w))\Big)^i = F(\bar{p} - w)\frac{\alpha(1 - F(\bar{p} - w)}{1 - \alpha(1 - F(\bar{p} - w)}$$

Since consumers are equally likely to visit every retailer on any draw, in equilibrium all participating retailers serve the same number of searchers. Hence,

$$\kappa = \frac{\Pr(\text{search and buy})}{F(\bar{p} - w)} = \frac{\alpha(1 - F(\bar{p} - w)}{1 - \alpha(1 - F(\bar{p} - w)} \tag{7}$$

Note than when $\alpha = 1$ and high types continue to search until they buy, this expression reduces to $\kappa = (1 - F(\bar{p} - w))/F(\bar{p} - w)$, that is the ratio of the number of retailers that create searchers and the number that receive searchers. When $\alpha = 0$, $\kappa = 0$ since high types either purchase at the first retailer or exit. The retailer's threshold can now be directly expressed as a function of high types strategy $\bar{p}$ and α and is given by

$$\bar{c} = v - w - \frac{\varphi}{1 - \varphi} \frac{\bar{p} - v}{1 - \alpha(1 - F(\bar{p} - w))} \tag{8}$$

With consumer behavior described by equations (3) and (6) and retailers' prices described by equation (8), the following proposition summarizes the structure of the downstream equilibrium.

Proposition 1 *Under full information, any downstream equilibrium is characterized by thresholds $\bar{p}$ and $\bar{c}$, and probability α that satisfy equations (3), (6), and (8) with strategies as follows:*

- *low valuation consumers accept any $p \leq v$, else they exit*
- *high valuation consumers accept any $p \leq \bar{p}$, else they search with probability α and exit with probability $1 - \alpha$*
- *retailers set prices according to*

$$p(c) = \begin{cases} v & c \in [0, \bar{c}] \\ \bar{p} & c \in (\bar{c}, \bar{p} - w] \\ \infty & c \in (\bar{p} - w, 1] \end{cases}$$

Furthermore, a downstream equilibrium always exists.

That any equilibrium must be characterized in this way follows from the line of argument presented in preceding text. Existence can be proven by showing that equations (3) and (8) must intersect and the formal proof can be found in Appendix A.

Lastly, note that while the equilibrium must be of the form described in Proposition 1, there may still be multiple equilibria. This is due to the complementarity between $\bar{c}$ and $\bar{p}$. A higher search threshold can lead to fewer retailers charging v which can then justify the increased search threshold. For clarity of exposition I restrict attention to the downstream equilibrium with the lowest prices.

3.2 Upstream Decision

Recall that while retailers observe and react to the wholesale price w, consumers only observe retail prices. Hence, the manufacturer knows that while changing the wholesale price affects the prices set by retailers (as summarized by $\bar{c}$), it would not affect the search threshold $\bar{p}$ or search probability α of the high valuation consumers. The demand function faced by a manufacturer is given by

$$\begin{aligned} Q(w) = & \underbrace{(1-\varphi)F(\bar{c}(w))}_{\text{sales to low types}} + \underbrace{\varphi F(\bar{p}-w)(1+\kappa(w))}_{\text{sales to high types}} \\ = & (1-\varphi)F(\bar{c}(w)) + \varphi\frac{F(\bar{p}-w)}{1-\alpha(1-F(\bar{p}-w))} \end{aligned} \tag{9}$$

The first term is the total sales to low types. From equation (8),

$$\frac{\partial \bar{c}}{\partial w} = -1 - \frac{\varphi}{1-\varphi}(\bar{p}-v)\frac{\alpha f(\bar{p}-w)}{1-\alpha(1-F(\bar{p}-w))} < 0$$

A higher wholesale price has two effects on retailer profits: it reduces the markup and increases the number of searchers. Both of these effects make charging the price $\bar{p}$ relatively more attractive than v. I call the effect on $\bar{c}$ the double marginalization effect. In the vertical markets literature, double marginalization refers to the fact that when retailers have market power they impose additional markups and sell a lower quantity. In my setting, retailers enjoy some market power due to the search friction. By switching from a price v to a price $\bar{p}$, retailers impose a higher markup and lose sales to low valuation consumers.

The second term in the demand function is the total sales to high types. Here the probability α that high types continue to search plays an important role. When $\alpha = 1$, the manufacturer's demand reduces to

$$Q(w|\alpha = 1) = (1-\varphi)F(\bar{c}(w)) + \varphi \cdot \mathbb{I}(w < \bar{p}) \tag{10}$$

Recall that consumers do not observe the wholesale price. Rather, their only information comes from the prices they observe when searching. Given an equilibrium in which $\alpha = 1$, high type consumers never exit and continue to search until they see a price below their threshold $\bar{p}$. A manufacturer can take advantage of this behavior by increasing his wholesale price to a point at which very few retailers remain in the market. In this situation high type consumers would eventually purchase, but only after searching for a long time. Consumers cannot, from the length of their search, infer that the manufacturer has increased his wholesale price, instead they interpret a long sequence of very high prices as bad luck.

Figure 4 illustrates a downstream demand function $Q(w|\alpha = 1, \bar{p})$, assuming that when $w = 0$ some retailers choose to sell to low types. The figure is meant only to show manufacturer incentives *conditional* on the strategy $(\alpha = 1,\ \bar{p})$ of high type consumers and does not necessarily represent an equilibrium.

The downward sloping part of the demand curve corresponds to wholesale prices that induce sales to both high and low type consumers. From equation (10), as the wholesale price w increases, sales

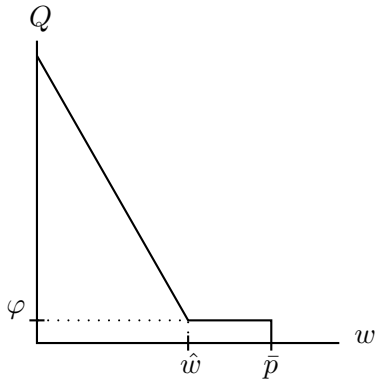

Figure 4: Downstream demand when $\alpha = 1$

to low types fall because $\frac{\partial \bar{c}}{\partial w} < 0$, and sales to high types remain unchanged. The demand curve kinks at the wholesale price $\hat{w}$ where $\bar{c}(\hat{w}) = 0$.[9] For higher wholesale prices, all high types continue to be served and thus the demand curve is flat. Once $w > \bar{p}$, no retailers can afford to serve the high types and the manufacturer induces no sales.

Next consider the manufacturer's decision when high type consumers do not always search. Then $\alpha < 1$ and demand is given by

$$Q(w|\alpha < 1) = (1 - \varphi)F(\bar{c}(w)) + \varphi \frac{F(\bar{p} - w)}{1 - \alpha(1 - F(\bar{p} - w))} \tag{11}$$

In this case not all high type consumers are served. Every time consumers reject a price, with chance $1 - \alpha$ they will exit the market. For example, when $\alpha = 0$ the manufacturer is only able to sell to high types with probability $F(\bar{p} - w)$, since any consumer that visits a priced out retailer will exit without purchasing. Figure 5 shows an example of such a demand function, $Q(w|\alpha = 0, \bar{p} = 1)$, with again a uniform distribution of retailer costs F. Again, the same caveat as in Figure 4 applies in that the consumer behavior is taken as given and not shown to be an equilibrium behavior.

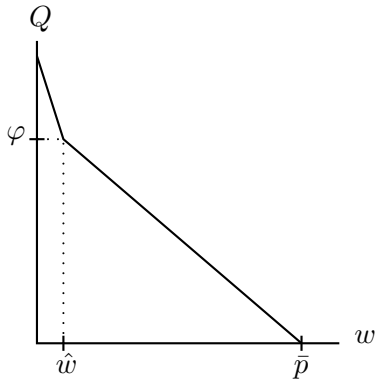

Figure 5: Downstream demand when $\alpha = 1$

There is still a kink in the demand curve at the wholesale price $\hat{w}$ where $\bar{c}(\hat{w}) = 0$ and low types are

[9] By equation (1) for any $\bar{p}$ there exists a w at which $\bar{c} = 0$.

no longer included. However, beyond this point increasing the wholesale price continues to result in a loss of sales. Since I assume that $\alpha = 0$, it must be that $\bar{p} = 1$ hence this is the highest wholesale price at which the manufacturer can make sales.

While the figures are meant only as illustrations of the problem faced by the manufacturer, Figure 4 suggests that for these parameter values it will be optimal for the manufacturer to set a wholesale price lower than the kink. In other words he will find it optimal to include low valuation consumers. Figure 5, however, depicts a scenario where it will be optimal for the manufacturer to charge a wholesale price higher than the kink, that is to serve only the high types. Next, I show that in fact Figure 4 and Figure 5 depict an equilibrium, each for a different set of the parameters of the model.

Claim 2 *When φ is small enough, the optimal wholesale price chosen by the manufacturer induces sales to low types.*

Proof of Claim I first show that when the proportion of high valuation consumers φ is small enough it is feasible for the manufacturer to induce sales to low types. I then show that it is optimal for him to do so.

By equation (8)

$$\begin{aligned}\bar{c} &= v - w - \frac{\varphi}{1-\varphi}\ \frac{\bar{p}-v}{1-\alpha(1-F(\bar{p}-w))} \\ &\geq v - w - \frac{\varphi}{1-\varphi}\ \frac{1-v}{F(v-w)}\end{aligned}$$

By inspection, the above expression shows that when φ is sufficiently small $\bar{c}(w = 0) > 0$. In fact, as for sufficiently small φ there will a range of wholesale prices that will induce search to low types. Equation (9) then shows that that as φ is decreased, the quantity sold by setting a wholesale price that serves only high types goes to zero while the quantity that can be sold with a wholesale price that includes low types is bounded strictly above zero. Hence, it is also optimal for the manufacturer to choose a wholesale price that serves low types. ■

In fact, I prove a stronger statement about the low state. Specifically, in addition to the state φ being low enough, given that search cost s is also small enough, the manufacturer will optimally induce a downstream equilibrium in which $\alpha = 1$.

Lemma 3 *When φ and search cost s are small enough, high type consumers search in equilibrium with probability $\alpha = 1$.*

Proof of Lemma Fix a high type consumer's strategy $\bar{p} < 1$ and $\alpha = 1$. By Claim 2 there exists a φ low enough so that the manufacturer chooses a w to induce sales to low types, and let $\bar{c}$ be the ensuing retailer threshold induced by w. In order for this to be an equilibrium, it must be that $\bar{p} < 1$ is a best response to the induced price distribution. By inspection of equation (3), there exists for any $\bar{c}$ and $\bar{p} > v$, there exists an s small enough to make the equation hold. Hence $\bar{p} < 1$ can be supported in equilibrium and as a result $\alpha = 1$ is supported. ■

I have shown that when φ is small, the manufacturer sets an equilibrium wholesale price that induces sales to low types. Furthermore, when the search cost s is small, consumers will search in

equilibrium. Next I argue that when φ is high enough the manufacturer will set a wholesale price that excludes low types.

Lemma 4 *When φ is large enough, for any search threshold $\bar{p} > v$ the manufacturer charges a wholesale price w that excludes low types.*

Proof of Lemma By equation (1)

$$\bar{c} = v - w - \frac{\varphi}{1-\varphi}(\bar{p} - v)(1+\kappa) \leq v - \frac{\varphi}{1-\varphi} \cdot s$$

The second inequality follows from the fact that $\bar{p} - v \geq s$ and $1 + \kappa \geq 1$. The above expression is negative for φ large enough, hence the manufacturer will not have the option of inducing sales to low types. ■

Lastly, I argue that if there is an equilibrium in which the manufacturer only serves high types, it must be that there is no search and $\alpha = 0$. The reason is that if only high types are included, and high types follow a threshold strategy, then only a price of 1 can be supported downstream. If the only price charged is 1 then search is never worthwhile.

The results of this section are summarized in the following proposition.

Proposition 5 *The full information equilibrium is characterized by Proposition 1 and wholesale price $w = \arg\max\ w \cdot Q(w)$, as given by equation (9). Furthermore, when the proportion of high types φ and search cost s are sufficiently small, $\alpha = 1$ and high types search until they purchase. When φ is sufficiently large no search is induced so $\alpha = 0$ and every consumer either buys from the first retailer or quits.*

4 Cheap Signaling

I look for an equilibrium in which consumers learn the aggregate demand φ immediately upon observing the recommendation σ at the first retailer that they visit. In such an equilibrium, consumers will act as if they are fully informed, hence I can use the preceding full information analysis to characterize the equilibrium outcomes. But for this to be an equilibrium, when given the opportunity the manufacturer cannot have incentive to mislead consumers. That is, given that the manufacturer can induce consumers to search as if it were the high state or as if it were the low state, I must check whether he will choose to signal truthfully in both states. It is important to note that the manufacturer sets the signal and wholesale price simultaneously, so for credibility I must show that he will not have incentive to send the wrong signal for *any* wholesale price he might secretly (for the consumers) adjust to. I will not find that in general cheap signaling is credible, instead my aim is to show that cheap signaling is possible and to highlight the conditions under which cheap signaling can arise.

First, since I have switched back to a game of incomplete information I must now specify consumers' beliefs. A high type consumer's belief that it is a state of high demand, $\mu(\sigma, \vec{p})$, depends on the manufacturer's signal σ and the set of prices she has observed thus far $\vec{p}$. In an equilibrium where signals reveal the state, beliefs must be consistent with equilibrium play so that $\mu(\sigma_H, \vec{p}) = 1$ whenever every $p \in \vec{p}$ is charged by some retailer in the high state and $\mu(\sigma_L, \vec{p}) = 0$ whenever

every $p \in \vec{p}$ is charged by some retailer in in the low state.[10] It is important to note that the consumer's strategy is not necessarily a stationary threshold strategy. For instance, when there is a price $p \in \vec{p}$ so that p is never charged in the high state, then a consumer's belief $\mu(\sigma_H, \vec{p})$ is not restricted to be 1 since the consumer finds herself in an off the equilibrium path information set. This means that in principle, for two different histories of prices $\vec{p}$ and $\vec{\rho}$, if $\mu(\sigma_H, \vec{p}) > \mu(\sigma_H, \vec{\rho})$ then the consumer may accept some price p after having observed the history $\vec{p}$ but reject that same price p after having observed the history $\vec{\rho}$. To address this, I restrict attention to equilibria where a consumer's belief $\mu(\sigma_H, \cdot\,) = 1$ and $\mu(\sigma_L, \cdot\,) = 0$. This implies that when a consumer sees a price and signal combination that are inconsistent, she trusts the manufacturer's signal.

Given this restriction on beliefs, consumers act as if they are fully informed about the state. Retailers anticipate consumers' behavior as a function of recommendation σ, and since retailers expect consumer to still follow threshold strategies, they still set retail prices according to equation (8). I now consider the manufacturer's incentives for revealing truthfully.

Let $(\bar{p}_L, \alpha_L)$ and $(\bar{p}_H, \alpha_H)$ be the search strategies employed by consumers in the full information equilibrium in the low and high demand state, respectively. I focus on parameters such that under full information, search is induced in the low demand state ($\alpha_L = 1$, $\bar{p}_L < 1$) but not in the high demand state ($\alpha_H = 0$, $\bar{p}_H = 1$). By Lemmas 3 and 4, this requires that φ_L and s are sufficiently small and φ_H is sufficiently large. To prove the existence of a cheap talk equilibrium, I verify that conditional on being in the high demand state, the manufacturer is better off inducing $(\bar{p}_H, \alpha_H)$ than $(\bar{p}_L, \alpha_L)$ and vice versa in the low demand state. In doing this, it is important to remember that retailers are aware of the true state. While in equilibrium consumers' beliefs and hence their search behavior change in response to the manufacturer's signal, retailers respond only in anticipation of the consumers' actions. The retailers' threshold cost is given by $\bar{c}(w, \varphi, \bar{p}(\sigma), \alpha(\sigma))$ and depends on the wholesale price w, the true state φ, and the anticipated consumer behavior $\bar{p}(\sigma)$ and $\alpha(\sigma)$.

First, suppose that aggregate demand is high. By sending signal σ_H and revealing the high state, the manufacturer earns a profit

$$\begin{aligned}\Pi^*(\sigma_H|\varphi_H) &= \max_w \; w \cdot Q(w, \sigma_H|\varphi_H) \\ &= \max_w \; w\Big((1-\varphi_H)F(\bar{c}(w, \varphi_H, 1, 0)) + \varphi_H \cdot F(1-w)\Big)\end{aligned}$$

The downstream demand is given by equation (11), and by assumption $\alpha_H = 0$ and $\bar{p}_H = 1$. That is, by signaling the high demand state the manufacturer induces consumers to either accept their first observed price or exit. If the manufacturer were instead to send the false signal σ_L, he would induce $\bar{p}_L < 1$ and $\alpha_L = 1$ and obtain a profit

$$\begin{aligned}\Pi^*(\sigma_L|\varphi_H) &= \max_w \; w \cdot Q(w, \sigma_L|\varphi_H) \\ &= \max_w \; w\Big((1-\varphi_H)F(\bar{c}(w, \varphi_H, \bar{p}_L, 1)) + \varphi_H \cdot \mathbb{I}(w \leq \bar{p}_L)\Big)\end{aligned}$$

The demand function here follows from equation (10), with high type consumers searching with probability 1 until they find a price below $\bar{p}_L$. When choosing between the two recommendations σ_L

[10] Formally, given an equilibrium price distribution $G(p)$, any price p where $\lim_{\varepsilon \to 0} \frac{G(p+\varepsilon) - G(p-\varepsilon)}{2\varepsilon} > 0$ is on the equilibrium path, which I refer to as being "charged by some retailer".

and σ_H, the manufacturer chooses between inducing two different downstream demand functions. Figure 6 depicts the demand faced by a manufacturer, for either signal, when demand is high and retailers' costs are distributed uniformly.

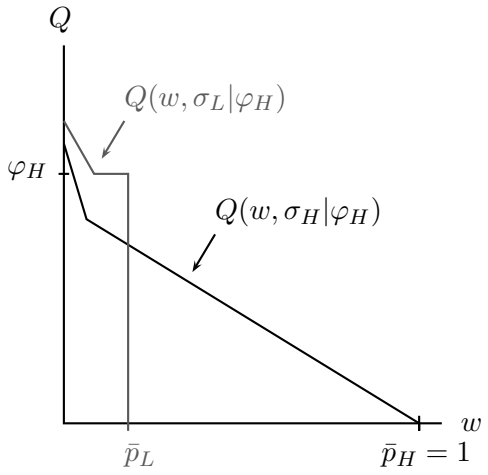

Figure 6: Signaling in state φ_H

When the manufacturer signals the high state with σ_H, his demand is the same as described in Figure 5. Recall that when wholesale price w is small, increasing it results in a loss of sales to low types through the effect on threshold $\bar{c}$ and a loss of sales to high types who exit upon visiting a priced out retailer. The kink occurs at the wholesale price at which retailers stop serving low types. If the manufacturer reveals the low state with σ_L, for every wholesale price he alters his ability to sell both to low and high types. The effect sending the false recommendation σ_L on sales to low types is ambiguous.[11] But by assumption, and as depicted in Figure 6, the state φ_H is high enough where the manufacturer would choose a wholesale price to serve only high types regardless of search threshold $\bar{p}$.

The effect of signaling σ_L on sales to high types is also ambiguous. By inducing $\alpha_L = 1$ (as opposed to $\alpha_H = 0$), the manufacturer can now sell to high types who otherwise would have exited if their first draw was from a priced out retailer. This means that for any $w \leq \bar{p}_L$, the manufacturer sells to all high types, as demonstrated by the flat portion of $Q(w, \sigma_L|\varphi_H)$. However, for any $w > \bar{p}_L$, the manufacturer cannot make any sales. From Figure 6, it is clear that as $\bar{p}_L$ becomes smaller, the maximized profit from inducing state σ_L also shrinks which makes the manufacturer prefer to reveal the high state truthfully. The task is to show that there are market parameters for which $\bar{p}_L$ is small while the demand function $Q(w, \sigma_H|\varphi_H)$ is unchanged.

[11] In principle, whether the manufacturer can sell more or less to low types by sending signal σ_L is ambiguous. To see this, using equation (8) consider at any wholesale price the change in $\bar{c}$ from sending signal σ_L instead of σ_H:

$$\bar{c}(w, \varphi_H, \bar{p}_L, 1) - \bar{c}(w, \varphi_H, 1, 0) = \frac{\varphi_H}{1 - \varphi_H} \left(1 - v - \frac{\bar{p}_L - v}{F(\bar{p}_L - w)}\right) \tag{12}$$

The sign of this expression is ambiguous: lowering $\bar{p}$ from $\bar{p}_H = 1$ to $\bar{p}_L$ means a retailer serving only high types collects a lower markup but also serves more searchers.

Lemma 6 *Given φ_H is high enough so that no low valuation consumers are served under full information, if s, φ_L, and v are sufficiently small then the manufacturer prefers to reveal the high demand state truthfully.*

Sketch of Proof Using the fact that $\alpha_L = 1$ and $\bar{p}_L < 1$, re-arrange equation (3) to get

$$\bar{p}_L = v + \frac{s}{F(\bar{c}_L)} \tag{13}$$

Note that $\bar{c}_L$ is an equilibrium object that depends in part on the values v and s. I show in Appendix B that when s, v, and φ_L are appropriately small, the full information $\bar{p}_L$ is arbitrarily close to 0, which makes $\Pi^*(\sigma_L, \varphi_H)$ also arbitrarily close to zero. To get some intuition for this, consider the extreme case with $s = 0$. Here, all consumers will use threshold $\bar{p} = v$, and thus as v is reduced toward zero, so is the equilibrium $\bar{p}$. At the same time, fixing those values of v and s, for any φ_H the profit to truth telling $\Pi(\sigma_H, \varphi_H)$ is bounded from below by $\max_w \ w \cdot \varphi_H F(1-w) > 0$. Hence inducing $\bar{p}_L$ will reduce profits. ■

Next consider the low demand state φ_L. Recall the assumption that φ_L is small enough so that the manufacturer finds it optimal to induce sales to low types for any $\bar{p} > v + s$. Hence, for the analysis I restrict attention to wholesale prices $w < v$. The manufacturer's profit from inducing signal σ_L is

$$\Pi^*(\sigma_L|\varphi_L) = \max_w \ w\Big((1-\varphi_L)F(\bar{c}(w, \varphi_L, \bar{p}_L, 1)) + \varphi_L \mathbb{I}(w \leq \bar{p}_L)\Big)$$

and the profit from inducing σ_H is

$$\Pi^*(\sigma_H|\varphi_L) = \max_w \ w\Big((1-\varphi_L)F(\bar{c}(w, \varphi_L, 1, 0)) + \varphi_L F(1-w)\Big)$$

Define the profit maximizing wholesale price $\tilde{w}$ conditional on sending false signal σ_H:

$$\tilde{w} \equiv \arg\max_w \ \Pi^*(\sigma_H, \varphi_L)$$

Note that by assumption, φ_L is low enough so that $\tilde{w} < v$ and the manufacturer optimally induces sales to low types. To show that the manufacturer prefers to send the truthful signal σ_L, it is sufficient to show that $Q(\tilde{w}, \sigma_L|\varphi_L) \geq Q(\tilde{w}, \sigma_H|\varphi_L)$.

Consider first the sales to high types. Since $\tilde{w} < v \leq \bar{p}_L$, signaling σ_L and inducing $\alpha = 1$ means the manufacturer will sell to all high types while signaling σ_H and inducing $\alpha = 0$ means high types do not search and some will be lost. Thus at wholesale price $\tilde{w}$, sales to high types are higher.

Sales to low types depend on how retailer threshold $\bar{c}$ responds to a change in $\bar{p}$ and α. Adapting expression (12) to the low state obtains

$$\bar{c}(\tilde{w}, \varphi_L, \bar{p}_L, 1) - \bar{c}(\tilde{w}, \varphi_L, 1, 0) = \frac{\varphi_L}{1-\varphi_L}\left(1 - v - \frac{\bar{p}_L - v}{F(\bar{p}_L - \tilde{w})}\right)$$

By the proof in Appendix B, the expression $\frac{\bar{p}_L - v}{F(\bar{p}_L - \tilde{w})}$ can be made arbitrarily small by choosing appropriately small values for φ_L and s. Hence, for small enough φ_L and s, it must be that

$\bar{c}(\tilde{w}, \varphi_L, \bar{p}_L, 1) > \bar{c}(\tilde{w}, \varphi_L, 1, 0)$ and thus sending the signal σ_L allows the manufacturer to serve more low types at wholesale price $\tilde{w}$. Since sending the truthful signal σ_L in the low demand state increases sales at $\tilde{w}$, the optimal wholesale price conditional on lying, it must be then truthful revelation is a best response.

Lemma 7 *Given φ_H is high enough so that no low valuation consumers are served under full information, if search cost s and low state aggregate demand φ_L are small enough then the manufacturer prefers to reveal the low demand state truthfully.*

This lemma follows directly from the preceding argument. All that remains to check is that there exist parameter values that satisfy the conditions for truth telling in the low state and the high state. By inspection of the two preceding lemmas, as long as the low state is characterized by a small enough φ_L and v, the high state is characterized by a large enough φ_H, and the search cost s is sufficiently small, signaling can be credible in both states.

Proposition 8 *The manufacturer can credibly communicate via cheap talk when search cost s is sufficiently small, φ_H is sufficiently high and v and φ_L are sufficiently low.*

The intuition behind why the manufacturer could credibly signal comes from understanding the impact of increasing the search threshold $\bar{p}$ on the manufacturer's demand function $Q(w)$. Increasing $\bar{p}$ tends to worsen double-marginalization and reduces sales at low wholesale prices designed to serve low types. However, a higher $\bar{p}$ also means that a higher price is available at which high types can be served, and if the manufacturer intends to set a high w sales will increase there.

Having established that it is possible that price recommendations can act as cheap signals that inform consumers about aggregate demand, I now consider the implications of a policy that would prohibit the manufacturer from doing so.

5 The Effects of a Ban on Recommendations

The ability to make price recommendations endows the manufacturer with some indirect control of downstream prices. This practice can be considered a form of vertical restraint and a natural question from an antitrust perspective is what would happen if such signaling were banned.

Without recommendations the model becomes substantially more difficult to solve. The root cause for this is that now consumers' search strategies are non-stationary; every observed price can potentially change a consumer's beliefs about whether it is the high or low demand state. Retailers in turn would face consumers that hold heterogeneous beliefs and possibly different search thresholds. This paper will not characterize the set of equilibria with no signaling, thus I am unable to make a general statement about the welfare impact of banning signaling. However, I do find a set of parameters that both guarantee credible signaling and provide a tractable solution the no-signaling equilibrium. For these parameters it is possible to outline the effects of a ban on price recommendations.

In solving for the no-signaling equilibrium, the key is the belief of a consumer that has learned her type but not yet observed any prices. If a consumer learns that she is a high type, her belief becomes

$$\mu_0 = \frac{\varphi_H}{\varphi_H + \varphi_L}$$

with associated likelihood ratio $\lambda_0 = \frac{\varphi_H}{\varphi_L}$. Consequently, every high valuation consumer is overly pessimistic about aggregate demand (in the sense that high demand implies high prices) and every low valuation consumer is overly optimistic. Since it is only the high valuation consumers that search, consumers may search less on average than they ought to.

I focus on a setting where λ_0 is a large number. Specifically, I focus on the situation where in the high state φ_H is high enough so that the manufacturer will find it optimal to sell only to high types and in the low state φ_L is low enough so that the manufacturer finds it optimal to sell to all types.

Recall that $\vec{p} = \{p_1, ..., p_n\}$ represents a sequence of observed prices and let $\mu(\vec{p})$ be the high valuation consumer's belief conditional on having observed $\vec{p}$. For tractability as well as to be consistent with the assumption made in the model with recommendations, I consider only equilibria in which price observations off the equilibrium path do not alter beliefs. Formally, restrict attention to equilibria with

$$\mu\big(\{p_i\}_{i=1,...,n}\big) = \mu\big(\{p_j\}_{j\in E}\big) \text{ where } E = \{i \mid p_i \text{ is charged in equilibrium } \} \tag{14}$$

In addition, I look only for equilibria in which retailers that do not make sales convey no information beyond the fact that they are priced out. Specifically, if a retailer has costs $w + c$ and in equilibrium demand at any $p \geq w + c$ is zero, then the retailer charges $p = \infty$.

I define a "no search equilibrium" to be an equilibrium of the following form. In the low demand state retailers charge either v or 1, in the high demand state retailers charge only 1. Consumers do not search after their first price draw and either accept or exit. Beliefs off the equilibrium path at all prices other than v or 1 are $\mu = \frac{\varphi_H}{\varphi_H + \varphi_L}$ by the assumption above.

Lemma 9 *When signaling is banned, the no search equilibrium is supported whenever φ_H is sufficiently large and φ_L is sufficiently small.*

The proof can be found in Appendix C.

Recall that to support a cheap talk equilibrium, it must be that φ_L, v, and s are sufficiently small and φ_H is sufficiently large. Given that the requirement to support the no search equilibrium is also that φ_H is sufficiently large and φ_L is sufficiently small, there exist parameter values where both of these equilibria are supported. I focus my analysis on this set of parameters.

Let the equilibrium in the signaling scenario be described by

$$\Big((w_L^s, \bar{c}_L^s, \bar{p}_L^s, \alpha_L^s),\ (w_H^s, \bar{c}_H^s, \bar{p}_H^s, \alpha_H^s)\Big)$$

Recall that in the cheap talk equilibrium, in the low state search is induced and thus $\bar{p}_L^s < 1$ and $\alpha_L^s = 1$, and some low types are served so that $\bar{c}_L^s > 0$. In the high demand state sales are made only to high types and there is no search, thus $\bar{c}_H^s \leq 0$, $\bar{p}_H^s = 1$, and $\alpha_H^s = 0$. In the no search equilibrium when signaling is banned, consumers do not search and either purchase immediately or exit and manufacturer and retailer strategies are described by

$$(w_L^{ns}, \bar{c}_L^{ns}, w_H^{ns}, \bar{c}_H^{ns})$$

In this equilibrium sales are made to low types only in the low state, thus $\bar{c}_L^{ns} > 0$ and $\bar{c}_H^{ns} \leq 0$.

In the high demand state, both in the signaling and no signaling cases outcomes are identical. Downstream, consumers do not search and φ_H is high enough that for any w, retailers that make sales all charge a price of 1. This implies that the manufacturer faces the same downstream demand function in both situations, and sets a wholesale price that solves

$$w_H^s = w_H^{ns} = \arg\max_w \; w \cdot \varphi_H F(1-w)$$

In the low demand state several differences emerge. First under no signaling fewer low type consumers are served as retailers can now charge 1 instead of $\bar{p}_L^s$ when serving high types, resulting in $\bar{c}_L^{ns} < \bar{c}_L^s$. Also, fewer high valuation consumers are served when signaling is banned. With signaling every high valuation consumer purchases in the low state but with the ban, high valuation consumers purchase with probability $F(\bar{p}_L^{ns} - w_L^{ns}) < 1$. In addition, with no signaling those consumers that purchase do so from retailers with costs that are higher on average, since retailers with costs $c \in [\bar{p}_L^s - w_L^s, 1 - w_L^{ns}]$ can now make sales.[12] Both reduced sales to high and low types and the increase in the average retailer costs diminish welfare. On the other hand, a ban on signaling does allow high valuation consumers that quit to save on search costs. For a high type consumer, the expected total cost of searching in a setting with signaling is given by

$$\sum_{i=1,\ldots,\infty} \Big((1 - F(\bar{p}_L^s - w_L^s) \cdot s\Big)^i F(\bar{p}_L^s - w_L^s) \;=\; F(\bar{p}_L^s - w_L^s) \frac{(1 - F(\bar{p}_L^s - w_L^s)s}{1 - (1 - F(\bar{p}_L^s - w_L^s)s}$$

As expected total search cost becomes arbitrarily small with a shrinking s. At the same time, conditional on consumers not searching both the strategies of the retailers and the manufacturer do not vary with s. Thus, as s falls the savings on search costs associated with a ban on signaling fall while the welfare losses due to fewer sales and higher average costs remain unchanged. For small enough s, the losses must dominate.

Lemma 10 *For parameter values that support a cheap talk equilibrium and an equilibrium with no search without signaling, when search cost s is sufficiently small welfare is higher in the cheap talk equilibrium than in the no search equilibrium.*

The proof is omitted but follows along the lines of the preceding arguments.

Another way to examine the ban on recommendations is to consider its effect on the individual parties. The manufacturer is made unambiguously worse off. If the high state ensues the manufacturer is indifferent, however in the low state the outcome is as if the manufacturer successfully lies about the state to consumers, which by the cheap talk is not in the manufacturer's interest. Low valuation consumers are equally well off as they receive zero surplus under either scenario. High valuation consumers are worse off with the ban; they receive zero surplus in the low state whereas with signaling they expect positive surplus. The effect on retailers as a group is

[12] This statement assumes that $1 - w_L^{ns} > \bar{p}_L^s - w_L^s$. Note that

$$1 - w_L^{ns} - (\bar{p}_L^s - w_L^s) = (1 - \bar{p}_L^s) - (w_L^{ns} - w_L^s)$$

The second term on the right hand side is no larger than v since in the low state both wholesale prices will be set between 0 and v, and the first term can be made arbitrarily close to $1 - v$ by the argument in Appendix B. Hence when v is sufficiently small $1 - w_L^{ns} - (\bar{p}_L^s - w_L^s) > 0$.

ambiguous. Retailers that are priced out in the setting with recommendations can now make sales since $\bar{p}$ has increased to 1. But retailers with low costs make fewer sales, both due to the fact that some consumers are now served by higher cost retailers and because no search is induced.

6 A Comparison of Recommendations to Price Ceilings

Most products sell at or below their recommended price, and consequently recommendations have often been treated as binding price ceilings in the literature (Gill and Thanassoulis (2010)). While I present an alternative explanation in which recommendations communicate information, the equilibrium is still observationally equivalent to a situation in which the manufacturer sets a price ceiling $\bar{p}_L$ and $\bar{p}_H$ in each state. At the same time, in my model the manufacturer can only induce $\bar{p}_L$ or $\bar{p}_H$ and not any other ceiling. In this section, I explore the implications of endowing the manufacturer with ability to set a price ceiling at any price and examine how market outcomes differ from those where only non-binding recommendations are available.

To make an apples to apples comparison, assume price ceilings are commonly observed and reveal the state φ. This way the informational content of either price ceilings or recommendations is the same. In addition, consider only parameter values that support an informative cheap talk equilibrium with recommendations. Namely, restrict attention to parameters where in state φ_H only high valuation consumers are served and in state φ_L all consumers are served when cheap talk is used.

In the high demand state φ_H, with cheap recommendations there is no search in the equilibrium and only a price of 1 is charged downstream. Consider the effect of a price ceiling p^c. Since no low valuation consumers are served price ceilings do nothing to reduce double marginalization.[13] The manufacturer's profit is given by

$$\Pi(w, p^c) = \max_w \ \varphi_H \cdot w \cdot \frac{F(p^c - w)}{1 - \alpha(1 - F(p^c - w))}$$

The price ceiling has two effects. First the number of priced in retailers is $F(p^c - w)$, hence reducing p^c reduces this number. But also, publicly changing the price ceiling changes consumers' returns to searching. In forcing participating retailers to charge a price $p^c < 1$, the manufacturer improves the returns to searching for consumers whose initial observation is at a priced out retailer. In this setting, the continuation value to searching conditional on visiting a retailer with $p = \infty$ is

$$V^s(\infty|p^c) = F(p^c - w)(1 - p^c) + (1 - F(p^c - w)) \max\{0, V^s(\infty|p^c)\} - s$$

For small values of s, keeping fixed the wholesale price w there will exist some $p^c < 1$ so that the continuation value to searching $V^s(\infty|p^c) > 0$. Hence, setting this p^c would in principle allow the manufacturer to serve all high types by inducing them to continue to search. However, the manufacturer faces a commitment problem here. Because the manufacturer's wholesale price is unobserved to consumers, whenever high valuation consumers follow the strategy of searching until finding a price p^c, the manufacturer should set a w close to p^c and induce very few retailers to make sales. If

[13] Technically the manufacturer can induce sales to low type consumers by setting a ceiling close enough to v to induce some retailers to charge v. Because the price ceiling p^c is also a ceiling on the manufacturer's wholesale price, I claim without explicitly proving it that for large enough φ_H and low enough v, the manufacturer would not optimally impose a ceiling that induces sales to low types.

the manufacturer does this, consumers that search have a very small chance to find an acceptable price on the next draw, thus the continuation value to searching is negative and consumers should exit. Because of this commitment issue, the manufacturer is unable to entice consumers to search in the high state with a price ceiling. He thus optimally sets the price ceiling $p^c \geq 1$ and induces the same downstream outcome that he would in the high state with cheap signaling.

In the low demand state φ_L, when using recommendations the manufacturer optimally sets w to make positive sales to low valuation consumers and this induces search threshold $\bar{p}_L$. Setting a price ceiling $p^c \geq \bar{p}_L$ would be irrelevant; the state would be revealed and the signaling equilibrium would ensue. When setting a binding $p^c \in (v, \bar{p}_L]$ the downstream demand function in this case is given by

$$Q(w, p^c) = (1 - \varphi_L)F(\bar{c}(w, p^c)) + \varphi_L \mathbb{I}(w < p^c) \tag{15}$$

In the first term, the threshold retailer cost $\bar{c}(w, p^c)$ decreases in p^c by the same logic that $\bar{c}(w, \bar{p})$ decreases in $\bar{p}$ in the cheap talk case. This is the double marginalization effect: by reducing the price ceiling the manufacturer induces retailers to switch to serving low type consumers. The second term is also analogous to the case with recommendations, showing that when high valuation consumers search they will eventually end up buying as long as a positive measure of retailers can afford to serve them. An implication of equation (15) is that

$$\frac{dQ(w, p^c > v)}{dp^c} < 0,$$

hence, the optimal binding price ceiling is at v or below. Next observe that p^c will not be set below v in any equilibrium. The only reason for the manufacturer to do this would be to try to induce low types to search, but by the same logic as in the high state no such equilibrium can be supported. If low types are induced to search then the manufacturer would set w close to p^c and make search not worthwhile. This implies that the manufacturer will optimally set a binding price ceiling $p^c = v$.

Because the lowest price charged is v, low valuation consumers are equally well off under either the recommendations or the price ceiling regime. The effect on high valuation consumers of switching from recommendations to price ceilings is unclear. While they pay a lower price and save $\bar{p}_L - v$ upon purchasing, they must also on average search longer to obtain this price. Even for small s, the relative size of the two effects is unclear since as s decreases, both the total expected search cost and the difference $\bar{p}_L - v$ also decrease.

The manufacturer is better off with the price ceiling since imposing the outcome when using only recommendations is an option still available to him. Because he loses fewer consumers to double marginalization, the manufacturer will also charge a higher wholesale price w. Whether retailers are better off depends on their costs. Retailers that used to make positive sales but now are priced out are worse off. The effect on all other retailers is ambiguous. Retailers that used to charge $\bar{p}_L$ and now are forced to charge v collect a smaller markup but receive more searchers. Retailers that charge v under either regime receive more searchers under price ceilings, however their markup is also diminished due to the higher w.

On net the manufacturer induces more downstream sales with price ceilings, which is consistent with the effect of resale price maintenance in a full information setting. But because of search,

the manufacturer induces high valuation consumers to expend higher costs to find a lower price. Because higher sales come at the expense of higher search costs, the net effect is ambiguous.

7 Conclusion

This paper addresses the question of how price recommendations can impact a market for consumer goods. I posit that such communication is an attempt by manufacturers to inform consumers about their returns to searching. When aggregate demand is low, the manufacturer induces more search and in doing so reduces retailer markups and increases sales to consumers with low valuations. When aggregate demand is high, the manufacturer induces less search which allows him to charge a higher wholesale price thereby extracting surplus from consumers with high valuations. I show that when search costs are low and there aggregate demand is sufficiently uncertain, the manufacturer is able to credibly communicate with consumers via cheap talk.

This model draws a sharp distinction between price recommendations and resale price maintenance. Recommendations are not just an indirect method to manage double marginalization. Though the effect of recommendations is observationally similar to that of price ceilings, the mechanism is quite different — in my model the manufacturer influences outcomes by informing consumers rather than imposing constraints on retailers. And while providing information sometimes leads to reduced double marginalization, at other times it leads retailers to actually set higher prices.

While I introduce uncertainty about aggregate demand in the main model, other forms of aggregate uncertainty could potentially make cheap communication possible. For example, consider uncertainty about the manufacturer's costs. A manufacturer facing high costs would be more interested in extracting surplus from high valuation consumers, while a manufacturer facing low costs would be relatively more interested in increasing total sales. At the same time, consumers should expect higher prices in the high cost state and lower prices in the low cost state. I conjecture that when the two cost states are sufficiently different, the manufacturer can credibly communicate his costs via cheap talk.

In practice there is variation in the way actual prices relate to price recommendations. For instance, books often sell for exactly their jacket price while cars tend to sell for strictly less than MSRP. Even within the car market, how far below MSRP a car sells varies by the popularity of that vehicle, and in fact some cars sell above MSRP. Imposing that recommendations are price ceilings or some other exogenously determined restraints precludes the analysis from explaining such variation. Since in my model a recommendation is just a cheap message, I allow for the consumers' interpretation of this message to vary with market conditions and thus can accommodate the varying relationship between prices and recommendations.

The goal of this paper was to highlight a mechanism by which price recommendations serve a purely informational role and still influence market outcomes. Towards this end, the model I develop is quite stylized and thus has limitations for direct use in assessing policy. Nonetheless, the message that seems to emerge is that ceteris paribus price recommendations help consumers, allowing them to make more informed decisions and saving search costs. Hence, an antitrust policy discussion of the merits of price recommendations should keep these informational benefits in mind.

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Appendix A: Proof of Existence of the Downstream Equilibrium Under Full Information

I show that a downstream equilibrium that is characterized in Proposition 1 must exist. I argue that equations (8) and (3) are continuous and that they must intersect.

Recall equation (3) which describes the high valuation consumer's threshold:

$$1 - \bar{p} = \max\{0,\ F(\bar{c})(1 - v) + (1 - F(\bar{c}))(1 - \bar{p}) - s\}$$

and let $\bar{p}^{(3)}(\bar{c})$ be the implicit function implied by this equation. By inspection this function is continuous. First note that $\bar{p}^{(3)}(\bar{c}) > v$ for all $\bar{c}$. This can be seen by contradiction. Rewrite equation (3) as

$$1 - \bar{p} = \max\{0,\ 1 - \bar{p} + F(\bar{c})(\bar{p} - v) - s\}$$

If $\bar{p} < v$ then the left hand side must be larger than the right hand side, hence a contradiction. Next note that because of the max operator, $\bar{p}^{(3)}(\bar{c}) \leq 1$ for all $\bar{c}$.

Now recall equation (8)

$$\bar{c} = v - w - \frac{\varphi}{1 - \varphi}\ \frac{\bar{p} - v}{1 - \alpha(1 - F(\bar{p} - w))}$$

Recall also that $\bar{p} < 1$ implies $\alpha = 1$, else if $\bar{p} = 1$ any $\alpha \in [0, 1]$ can be used to support an equilibrium. In this sense, α helps the existence argument in that for $\bar{p} = 1$, there are many values of $\bar{c}$ that can satisfy equation (1) given a choice of α.

Let equation (8) implicitly define the function $\bar{p}^{(8)}(\bar{c})$. Continuity once again is obvious here. First note that $\bar{p}^{(8)}(v - w) = v$. Also, note that there exists a $\bar{c}$ low enough (and possibly negative) so that $\bar{p}^{(8)}(\bar{c}) = 1$.

Hence, $\bar{p}^{(8)}(v - w) < \bar{p}^{(3)}(v - w)$ and $\bar{p}^{(8)}(\bar{c}) \geq \bar{p}^{(3)}(\bar{c})$ for some small enough $\bar{c}$. Given that $\bar{p}^{(8)}(\bar{c})$ and $\bar{p}^{(3)}(\bar{c})$ are continuous functions, they must then intersect. ■

Appendix B: Proof of Arbitrarily Low Threshold $\bar{p}$ in the Full Information Equilibrium

This section proves that for any small $\bar{p} > 0$, there exist small enough φ, s, and v so that $\bar{p}$ is supported in the full information equilibrium. I will show this by first noting that a manufacturer's profits in any equilibrium are bounded away from zero. Then, I will show that as s and φ are reduced toward zero, either $\bar{p}$ approaches v or the manufacturer's profit approaches zero, which would contradict the first statement. Lastly, since $\bar{p}$ can be made arbitrarily close to v, when v is chosen to be small, $\bar{p}$ will be small as well.

First, recall that by Lemma 3, the manufacturer chooses to set a wholesale price w to induce sales to low types, i.e. he induces $\bar{c} > 0$. It will be useful for this argument to show that as $\varphi \to 0$, the manufacturer's profit is uniformly bounded strictly above 0.

Claim 11 *For any $\delta > 0$, there exists a low enough $\hat{\varphi}$ so that for any $\varphi < \hat{\varphi}$, the manufacturer's equilibrium profit exceeds $v/2 \cdot F(v/2) - \delta$.*

Proof of Claim: The manufacturer's equilibrium profit, given w is the optimally charged wholesale price, is given by

$$\begin{aligned}
\Pi(w,\varphi) &= w\Big((1-\varphi)F(\bar{c}(w,\varphi)) + \varphi \cdot \mathbb{I}(w \leq \bar{p})\Big) \\
&\geq w\left((1-\varphi)F\left(v - w - \frac{\varphi}{1-\varphi}\frac{\bar{p}-v}{F(\bar{p}-w)}\right)\right) \qquad \text{(by equation (8))} \\
&= w\left(F(v-w) - \underbrace{\left(F\left(v - w - \frac{\varphi}{1-\varphi}\frac{\bar{p}-v}{F(\bar{p}-w)}\right) - F(v-w)\right)}_{A} - \underbrace{\varphi F\left(v - w - \frac{\varphi}{1-\varphi}\frac{\bar{p}-v}{F(\bar{p}-w)}\right)}_{B}\right)
\end{aligned}$$

Consider a manufacturer that charges $w = v/2$. At this wholesale price, $\bar{p} - w \geq v + s - w \geq v/2 + s > 0$, hence both terms A and B go to zero as φ goes to zero. For φ low enough, $v/2 \cdot (A + B) < \delta$. Hence, $\Pi(w,\varphi) \geq \Pi(v/2,\varphi) \geq v/2 \cdot F(v/2) - \delta$. ■

Next, I prove by contradiction that as s and φ decrease, $\bar{p} - v$ approaches zero. Suppose toward a contradiction that there exists an $\varepsilon > 0$ such that there is some $\hat{s}$ with the property that for any $s < \hat{s}$, in the full information equilibrium $\bar{p} - v > \varepsilon$. By equation (13), this implies that $\frac{s}{F(\bar{c})} > \varepsilon$ for any $s < \hat{s}$. Recall equation (8):

$$\bar{c} = v - w - \frac{\varphi}{1-\varphi}\frac{\bar{p}-v}{F(\bar{p}-w)}$$

For small enough φ in any equilibrium $w < v$. By the hypothesis above, this implies that $\bar{p} - w \geq \bar{p} - v \geq \varepsilon$ for all $s < \hat{s}$. Then as s and φ both shrink toward zero, the right hand side of the equation above must approach $v - w$. At the same time, since by hypothesis $\frac{s}{F(\bar{c})} > \varepsilon$ for all small s, it must be that $\bar{c}$ is converging toward zero. Hence, it must be that as s and φ shrink toward zero, the equilibrium w approaches v. However, this implies that the manufacturer's equilibrium profit approaches zero which contradicts the claim above since the manufacturer's profit has an absolute lower bound strictly above zero.

Hence I have shown that as search cost s and low state demand φ_L diminish toward zero, the equilibrium search threshold $\bar{p}$ approaches the low types' valuation v. Thus, there always exists a full information equilibrium where the search threshold is arbitrarily close to zero given parameters φ, s, and v are all chosen to be sufficiently small. ■

Appendix C: Proof of Existence of a No Search Equilibrium When Recommendations are Banned

When recommendations are banned, I show that an equilibrium with no search can be supported when initial belief λ_0 is sufficiently high.

Proof In the proposed equilibrium, retailers face the step demand function

$$q(p) = \begin{cases} 1 & \text{if } p \in [0, v] \\ 1-\varphi & \text{if } p \in (v, 1] \\ 0 & \text{if } p \in (1, \infty) \end{cases}$$

Retailers use threshold strategy $\bar{c}$ given by equation (1)

$$\bar{c}(w, \varphi) = v - w - \frac{\varphi}{1-\varphi}(1-v)$$

Given an equilibrium where consumers do not search, the manufacturer solves

$$\max_w \; w \cdot Q(w, \varphi) = w \cdot \Big((1-\varphi) F(\bar{c}(w, \varphi)) + \varphi F(1-w) \Big)$$

in each state φ, with $\bar{c}(w, \varphi)$ given above and decreasing in w. Let $w(\varphi)$ be the solution to the above optimization and note that $0 < w(\varphi) < 1$.

Claim 12 *When φ_H is large enough $\bar{c}(\varphi_H) \leq 0$ and when φ_L is small enough $\bar{c}(\varphi_L) > 0$ in equilibrium.*

Proof of Claim This does not follow immediately because in equation (1) wholesale prices are endogenous. That $\bar{c}(\varphi_H) \leq 0$ for a large enough φ_H does follow directly. From the manufacturer's optimization it is clear that when φ_L is small enough, setting a w that induces $\bar{c} > 0$ is optimal and by (1) also feasible for the manufacturer. Hence, when φ_L is small enough $\bar{c}_L > 0$. ■

On the supply side I have shown that when consumers follow the strategy of no search then only a price of 1 is charged in the high state and prices v and 1 are charged in the low state. Next, I must show that no searching is a best response for consumers. For notational clarity, define

$$\begin{aligned} w_L &\equiv w(\varphi_L), \qquad w_H \equiv w(\varphi_H) \\ \bar{c}_L &\equiv \bar{c}(w_L, \varphi_L), \quad \bar{c}_H \equiv \bar{c}(w_H, \varphi_H) \end{aligned}$$

Low valuation consumers expect no prices strictly below v in either state and will either purchase on their first price draw or exit. High valuation consumers will assign a likelihood to the high state conditional on the price they see according to

$$\lambda(p) = \begin{cases} 0 & \text{if } p = v \\ \lambda_0 \, \frac{F(1-w_H)}{F(1-w_L) - F(\bar{c}_L)} & \text{if } p = 1 \\ \lambda_0 \, \frac{1-F(1-w_H)}{1-F(1-w_L)} & \text{if } p = \infty \\ \lambda_0 & \text{for all other } p \end{cases} \tag{16}$$

Likelihoods at equilibrium prices are computed as the product of the prior likelihood λ_0 and the ratio of the probabilities of seeing the price in either state. Prices off the equilibrium path are by assumption ignored by consumers when forming beliefs. Note that likelihood λ translates into belief $\mu = \frac{\lambda}{\lambda+1}$.

A high type consumer whose lowest observed price is p and who holds belief μ has a value function recursively defined by

$$V(p, \mu) = \max \Big\{ 0, \; 1-p, \; E[V(p', \mu') | p, \mu] - s \Big\} \tag{17}$$

Claim 13 *There exists a high enough belief $\bar{\mu}$ so that whenever $\mu > \bar{\mu}$, the continuation value to searching $E[V(p', \mu')|p, \mu] - s < \max\{0, 1-p\}$ $\forall$ p.*

Proof of Claim I provide an upper bound for the continuation value to searching:

$$E[V(p', \mu')|p, \mu] - s \leq \mu(\max\{1-p, 0\}) + (1-\mu)(1-v) - s$$

If the state is high, the consumer will not see a price below 1 and the highest payoff she can obtain is to accept p if it is less than 1 else exit. If the state is low, the highest payoff the consumer can get is if she observes and accepts price v. For large enough μ, it must then be that $E[V(p', \mu')|p, \mu] - s < \max\{0, 1-p\}$. ■

To restate the claim, for any price p once consumers are convinced enough the state is high they will not search and either purchase or exit.

Claim 14 *There exists a $\bar{\lambda}$ so that for any $\lambda_0 > \bar{\lambda}$, $\mu(p) > \bar{\mu}$ $\forall$ p.*

Proof of Claim The proof follows from equation (16).[14] I have thus shown that for large enough λ_0 no price will induce search and this concludes the proof. ■

[14] Because w_L, w_H, and $\bar{c}_L$ are endogenous objects, one must confirm that as λ_0 grows these remain strictly bounded away from extreme values. Specifically, for the result to hold it is sufficient to show that w_H never approaches 0 or 1 as φ_H grows and φ_L shrinks. The manufacturer's profit function clear shows this to be the case.