Learning by Fund-raising*

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Abstract

From experience, fund-raisers often learn to become more efficient solicitors. This paper incorporates fund-raising technology into the theory of charitable giving. A full characterization of the solicitation strategy that maximizes donations net of fund-raising costs is provided. The strategy identifies a fund-raiser incentive to invest in learning in the form of soliciting some early donors who would give less than their solicitation costs. By defining a notion of "excessive" fund-raising, it is shown that it may worsen with learning. An extension with rising solicitation costs is also considered.

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JEL Classification: H00, H30, H50

1 Introduction

Charitable fund-raising[1] is a highly professional activity. The Association for Professional Fundraisers (AFP) represents 30,000 members, and every year more than 115,000 non-profit organizations consult these professionals, costing 2 billion dollars (Kelly, 1998).[2] It

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[1] Charitable sector is a significant part of the U.S. economy. For instance, in 2008, total donations amounted to \$307 billion. \$229 billion of this total came from individuals, corresponding to 1.61% of GDP (*Giving USA*, 2009). See Andreoni (2006a) and List (2011) for an overview of this sector and the literature.

[2] Fund-raising expenses, however, are not limited to hiring professionals. Andreoni and Payne (2003, 2011) as well as Greenfield (2001) estimate that 5 to 25 percent of donations cover fund-raising expenses, including direct mailing, telemarketing, face-to-face solicitations, and staffing. In fact, given the significance of fund-raising expenses, several watchdog groups have emerged to inform the public about the efficiency of a large set of charities and, in turn, to guide potential donors. For example, BBB Wise Giving Alliance has suggested that fund-raising costs should not represent more than 35% of funds. For its part,Charity Navigator has stipulated that the figure should not exceed 33%.

is strongly believed that fund-raising is learned on the job, raising the demand for those professionals who are more experienced. For instance, a recent survey by Cygnus Applied Research reveals that most succesful fund-raisers are on the job just three to six months before being recruited for another.[3] As the president of Cygnus puts it: "Only one out of three fund-raisers experiences even a day without a job". Professional fund-raisers also place a great value on experience as suggested in this quote from a fund-raiser's webpage: "Fund Development Associates is the regional expert in fund-raising. No one has more direct, hands-on experience. By selecting our firm, you will have a team of professionals with more than one hundred years of combined successful fund-raising experience who have assisted hundreds of charitable organizations achieve their goals".[4]

Both practitioners and researchers agree that one of the most important fund-raising techniques is directly asking people (Andreoni and Payne 2003; Yoruk, 2009; Meer and Rosen 2011). It is believed that people often have good intentions to give, but unless they are solicited, these intentions may not turn into donations. In this paper, we contend that such direct solicitations are also the source of learning for the fund-raiser. Our main objective is to investigate how learning shapes the fund-raising strategy and if it may cause "excessive" fund-raising.

Our formal setup adds an "active" fund-raiser to the "standard" model of giving in which donors consume two goods: a private good as well as a public good.[5] We consider a charity which occasionally runs fund-drives. The fund-raiser's role then consists in informing potential donors individually about the charitable cause, as in Name-Correa and Yildirim (2012). Asking people is costly. The presence of learning economies, however, enables the charity to reduce the marginal cost of fund-raising as the charity solicits more people.

Our first observation is that the fund-raising cost introduces a provision point to the public good, but under an optimal solicitation procedure, a coordination problem among donors does not arise. The charity contacts individuals according to income, starting with the wealthiest. A sufficient condition to solicit one more individual is that she is expected to provide a gift above the marginal cost or become a "net contributor". We show that identifying these net contributors in our model is equivalent to identifying the contributors

[3] The survey includes 1,700 fund-raisers and 8,000 nonprofit chief executives. Results are available at http://www.cygresearch.com/files/AFP_Intl-Conf_Vancouver_April_2_2012-PenelopeBurk.pdf

[4] See http://www.funddevelopmentassociates.com/associates.html

[5] See, e.g., Warr (1983); Roberts (1984); Bergstrom, Blume, and Varian (1986); and Andreoni (1988).

in a model with constant marginal cost except that each donor's wealth is reduced by the variable part of its marginal cost. This important equivalence allows us to utilize the charaterization in Name-Correa and Yildirim (2012) who assume away learning.

Absent learning economies, the charity considers contacting first the richest donor; once this donor is in the "game", the charity becomes more conservative about contacting the second richest donor due to the free-riding incentive, which depends on their income difference. Sequentially applied, this logic implies once the charity identifies a "net free-rider", the solicitations optimally stop.

In the presence of learning, the fund-raiser may, however, solicit a net free-rider, as long as this solicitation enables the fund-raiser to *substantially* move down her learning curve. In this sense, negative net contributions represent the fund-raiser's investment in learning. We provide the exact equilibrium condition determining whether investing in learning is worthy or not. While we assume that the solicitation set is observed by the contacted donors, our characterization is robust to unobservability under reasonable (off-equilibrium) beliefs.

Watchdogs groups evaluate a charity efficiency according to its cost structure. They recommend managing a low fixed cost. For instance, Charity Navigator considers that administrative costs should not represent more than 20% of total costs. My model also applies to a situation in which the presence of a fixed cost generates returns to scale in fund-raising. When a higher setup cost does not totally discourage fund-raising, it increases current donations and encourages the charity to solicit more. Despite these two positive effects, the public good provision diminishes.

I build a benchmark in which the fund-raiser establishes for each donor a minimum gift size and commits to it. We show that this commitment allows the charity to obtain extra-large gifts from the wealthiest donors. With respect to this benchmark I find that the charity conducts excessive fund-raising regardless of the solicitation technology. Moreover, we show that learning is another source of excessive fund-raising. We find, however, that a higher learning rate does not necessarily generate a greater extent of excessive fund-raising.

I extent the model to incorporate a warm-glow motive for giving (Andreoni 1989) and show that my results follow under such added realism. In another extension, we show that when the fund-raiser separates the population in groups and learning is group specific, the charity may favor contacting groups with lower expected income but with more potential for learning. Finally, we show that under decreasing returns to scale it is never optimal to contact a net free-rider.

In addition to the papers mentioned above, our work fits with a small body of theoretical literature on strategic fund-raising as means of: advertising and reducing donors' search costs (Rose Ackerman 1983; and Andreoni and Payne 2003), providing prestige to donors (Glazer and Konrad 1996; Harbaugh 1998; and Romano and Yildirim 2001), signaling the project quality (Vesterlund 2003; and Andreoni 2006b), and organizing lotteries (Morgan 2000). Our work is also related to the models of strategic fund-raising to overcome zero-contribution equilibrium under non-convex production either by securing seed money (Andreoni 1998) or by collecting donations in piece-meals (Marx and Matthews 2000). None of these papers, however, consider endogenous, costly solicitations and learning by fund-raising. Other models consider learning about the project quality by providing the charitable good within a dynamic framework. In these models learning is faster when the cumulative production of the good is larger (Bolton and Harris 1999; and Yildirim 2003).

The closest work to ours is Name-Correa and Yildirim (2012); henceforth, NY (2012). They build a model in which donors do not consider giving unless asked by the fund-raiser. They fully incorporate fund-raising costs to determine the fund-raiser's solicitation strategy. The charity commits to that strategy and sucessfully launches a fund-drive. Our work is similar to theirs; instead of attaching a cost to each donor, though, we explicitly introduce a fund-raising cost structure, which is unrelated to donors' identities. This allows us to model the learning aspect of soliciting as decreasing marginal costs in fund-raising.

Rose-Ackerman (1982) is the first to build a model of costly fund-raising in which donors, as in mine, are unaware of a charity until they receive a solicitation letter. She, however, does not construct donors' responses from an equilibrium play. She was also the first in positing that fund-raising is likely to be conducted in excess. Her argument is that competition among charities triggers high expenses in fund-raising without bringing further benefits to donors. This happens whenever fund-raising diverts funds from one charity that donors value to another they like the same. On the contrary, in our model we build the concept of excessive fund-raising in a non-competitive framework. The term "excessive" comes from the fact that relatively more cost is incurred when contributions are voluntary and those extra resources are wasted,valued neither value by donors nor by the charity.

In addition to the theoretical literature, more extensive empirical and experimental literature exists on charitable giving, to which we will refer below. For recent surveys of the literature, see the reviews by Andreoni (2006a) and List (2011).

The rest of the paper is organized as follows. In Section 2, we set up the model.

In Section 3, we determine the optimal fund-raising strategy. In Section 4 we introduce returns to scale generated by a fixed cost. In Section 5 we consider excessive fund-raising. We present the extensions in Section 6, and conclude in Section 7.

2 Model

Our formal setup extends the standard model of privately provided public goods (e.g., Warr 1983; Roberts 1984; Bergstrom et al. 1986; and Andreoni 1988). Thus, it is useful to briefly review this basic framework before introducing fund-raising costs.

Standard Model. There is a set of individuals, $N = \{1, ..., n\}$, who each allocates his wealth, $w_i > 0$, between a private good consumption, $x_i \geq 0$, and a gift to the public good or charity, $g_i \geq 0$. Units are normalized so that $x_i + g_i = w_i$. At the outset, every person is fully aware of the charitable fund-drive and is in the "contribution game". Letting $G = \sum_{i \in N} g_i$ be the supply of the public good, individual i's preference is represented by the utility function $u(x_i, G)$, which is strictly increasing, strictly quasi-concave, and twice differentiable. Individual i's (Marshallian) demand for the public good, denoted by $f_i(w)$, satisfies the strict normality: $0 < f_i'(w) \leq \theta < 1$ for some parameter θ.[6] Donors simultaneously decide on their gifts and, under strict normality, there is a unique Nash equilibrium, $\{g_1^*, ..., g_n^*\}$.

To isolate any source of zero provision, we will assume that the standard model produces a positive level of the public good in equilibrium, $G^* > 0$. One sufficient condition for this is that $f_i(0) = 0$ for all $i \in N$, which we will maintain throughout. Together with the strict normality, this condition implies that each individual's stand-alone value is positive.

Costly Fund-raising. In the standard model there is no role for strategic fund-raising since all potential donors are already aware of the public good provision.[7] Thus, as with Rose-Ackerman (1982); and Andreoni and Payne (2003), we assume that each person i becomes informed of the fund-drive only if solicited by the fund-raiser.[8] We assume for

[6] The existence of parameter θ facilitates our analysis by ensuring a finite G_i^0 below. It is also commonly assumed in the literature (e.g., Andreoni 1988; Fries, Golding, and Romano 1991).

[7] Alternatively, in the standard model, the fund-raiser would trivially ask everyone for donations since the equilibrium provision of the public good never decrease by adding an individual (e.g., Andreoni and McGuire 1993).

[8] We envision a charity that occasionally runs fund-drives. In this scenario, it is reasonable to think that donors are unaware of the charitable good provision. However, even if a donor expects a fund-drive to be made, she may procrastinate in giving (O'Donoghue and Rabin 1999) or just wait for the solicitation to save on search costs (Andreoni and Payne 2001).

simplicity that each solicitation reaches the donor with certainty. It costs $c(i) = c + s(i)$ to solicit the ith individual in a sequence. The fixed marginal cost $c > 0$ reflects minimum expenses in telemarketing, face to face solicitations, envelopes procurement, and mailing costs. The variable marginal cost $s(i)$ is non increasing in i, perhaps because of the fund-raiser learning on the job or because of scale economies purchasing inputs at a discount. We assume that this cost structure is known by contacted individuals. Absent the variable cost, our model would reduce to NY (2012) with homogeneous preferences and constant marginal cost.

Let $F \subseteq N$ be the set of donors contacted by the fund-raiser, or the *fund-raiser set*. **I**In the basic model, we assume that the contacted donors know those in the fund-raiser set, though we relax this assumption in Section 3.2. [9]As in the standard setup, let $g_i^*(F)$ be donor i's equilibrium gift engendered by the simultaneous play in F. Then, the total fund-raising cost and the gross donations are defined, respectively, by $C(F) = \sum_{i=1}^{|F|} c(i)$ and $G^*(F) = \sum_{i \in F} g_i^*(F)$, where $C(\varnothing) = 0$ and $g_i^*(\varnothing) = 0$ by convention. The charity chooses F that maximizes the supply of the public good (or net donations):

$$\overline{G}^*(F) = \max \{G^*(F) - C(F), 0\}. \tag{1}$$

Eq. (1) implies that if insufficient funds are received to cover the cost, then no public good is provided, which simply refers to a failed fund-raising in our model.[10] We assume that the charity dislikes fundraising in that when two fund-raiser sets yield the same amount of public good, the charity prefers the one with the lower cost.[11]

Our fund-raising game, then, proceeds as follows. First, the charity decides whether or not to launch a fund-drive. If one is launched, then the charity reaches out to a (optimal) set F^o of potential donors, who all become aware of both the fund-drive and the others solicited. Finally, the contacted donors simultaneously contribute to the public good, leading to equilibrium gifts $\{g_i^*(F^o)\}_{i \in F^o}$ and the public good $\overline{G}^*(F^o)$. Our solution concept is subgame perfect Nash equilibrium in pure strategies.

[9]That donors may know the fund-raiser set prior to giving is not completely unrealistic. For instance, charities organize fund-raising events where donors meet each other

[10]In the case of a failed fund-raising, we assume for simplicity that either the donations are not refunded or they are used for other causes

[11]One justification for this could be that the charity has some concern about its cost/donation rating by the watchdog groups. Formally, if $F' \neq F$ are two fund-raiser sets such that $G' - C' = G - C$ and $C' > C$, then it follows that $C'/G' > C/G$.

3 Optimal fund-raising

In this section we fully characterize the fund-raising equilibrium in terms of the primitives of the model. Before that, we point out that although donors may end up contributing nothing for an arbitrary fund-raiser set, the same cannot happen if the set is optimally chosen.

3.1 Characterization

To characterize the equilibrium contributions, consider first person i's solo decision to cover the entire fund-raising cost, C. Note that person i would receive utility $u_i(w_i, 0)$, if he contributed nothing. Otherwise, he would have to choose $g_i \geq C$ to maximize $u_i(w_i - g_i, g_i - C)$. Let $V_i(w_i - C)$ be i's indirect utility in the latter case, which is increasing in the (net) income. For $C = 0$, clearly $V_i(w_i) > u_i(w_i, 0)$ because $f_i(w_i) > 0$, whereas for $C = w_i$, we have $V_i(0) \leq u_i(w_i, 0)$. Hence, there is a unique cutoff cost, $\widehat{C}_i \in (0, w_i]$ such that when alone, person i would consume some public good if and only if $C < \widehat{C}_i$.[12] The following result shows that although donors together may contribute nothing in some situations, in equilibrium a launched fund-drive is always successful.

Proposition 1 *Fix any arbitrary fund-raiser set, $F \neq \varnothing$, whose fund-raising cost is $C(F)$. If $\max_{i \in F} \widehat{C}_i \leq C(F)$, then there is a zero-contribution equilibrium, generating $\overline{G}^*(F) = 0$. However, in a fund-raising equilibrium, $F^o \neq \varnothing$ if and only if $\overline{G}^*(F^o) > 0$.*

The first part of Proposition 1 says that if no person can bear the cost alone, then the zero-contribution profile becomes an equilibrium. Hence, when fund-raising entails significant costs, a carefully planned strategy of whom to ask for donations seems to be of utmost importance both to control the expenses and to encourage giving.[13]

The second part of the Proposition highlights that in a setting in which the fund-raiser set is observable, an optimizing charity would never start fund-raising if it did not expect that donations would exceed the cost. Together, this proposition means that in our model, the charity can fail to provide the public good despite fund-raising only because it

[12] For the CES utility: $u_i = (x_i^{\rho_i} + (\overline{G})^{\rho_i})^{1/\rho_i}$, with $\rho_i < 1$, it is easily verified that $\widehat{C}_i = [1 - (1/2)^{\frac{1-\rho_i}{\rho_i}}]w_i$ for $\rho_i \in (0, 1)$, and $\widehat{C}_i = w_i$ for $\rho_i \leq 0$ (including the Cobb-Douglas specification at $\rho_i = 0$).

[13] Since the fund-raising cost introduces a threshold to the public good provision, Proposition 1 is a reminiscent of the equilibrium characterization in Andreoni (1998). Unlike his model, however, the provision point in ours will be endogenous to fund-raising strategy as opposed to being a capital requirement.

suboptimally sets the fund-raising strategy.[14] While enlightening, Proposition 1 does not inform us about the charity's solicitation strategy.

In order to do so, note two observations for any fixed fund-raiser set; (1) the incurred cost just depends on the number of solicitations, (2) the higher the income of an individual, the more she gives, as shown in Andreoni (1988). We intuitively observe that the fund-raiser solicits the highest income individual(s). Without loss of generality, index subjects in a descending order of their wealth: $w_1 \geq w_2 \geq ... \geq w_n$.

Observation 1. *For any optimal fund-drive size k, the top k individuals are the ones being solicited.*

From the previous observation we may consider that soliciting individual i costs $c(i)$ to the fund-raiser. In other words, the charity may view fund-raising costs as identity dependent, keeping in mind that soliciting individual $i+1$ implies that individual i is already included in the fund-raiser set. According to NY (2012), when costs are purely identity dependent, the fund-raiser designs a strategy where individual donors are solicited *at the margin* whenever their gifts exceed solicitation costs; such donors are *net contributors*. This marginal strategy leads to an optimal fund-raiser set, F^o, in which every solicited individual becomes a net contributor, even without a cost sharing agreement, since contributions are voluntary, and all of them just take into account the whole fund-raising cost $C(F^o)$.

Once we introduce learning economies, it is possible that the fund-raiser at the margin optimally solicits an individual i,who provides a gift below the marginal cost $c(i)$; in other words, the donor is a net free rider. We illustrate this point with a numerical example, which also motivates our subsequent analysis.

Example 1. *Let $N = \{1, 2, 3\}$ and $u_i = x_i^{1-\alpha}(\overline{G})^{\alpha}$, with $\alpha = 0.3$. Individuals' wealth and solicitation costs are such that $(w_1, w_2, w_3) = (20, 14, 14)$, $c = 1$.*

Consider first no scale economies, i.e., $s(i) = 0$. The following table reports donor equilibrium, and highlights the optimal fund-raiser set.

[14] As noted in the Introduction, charities spend billions of dollars on professional fund-raisers. For instance, the Association of Fundraising Professionals (AFP) represents 30,000 such fund-raisers.

F	$g_1^* - c$	$g_2^* - c$	$g_3^* - c$	$G^* - C$
$\{1\}$	*5.7*			*5.7*
$\{1,2\}$	5.82	-0.177		5.65
$\{1,2,3\}$	5.875	-0.125	-0.125	5.62

Table 1:Donor equilibrium without learning.

Tables 1 reveals that it is optimal to contact only donor 1. Donor 2 and 3 are not included in the set because their contributions never exceed the marginal cost .

Keeping donors' characteristics as above and $c = 1$*, consider* $s(i) = (7, 5, 1)$

F	$g_1^* - c(1)$	$g_2^* - c(2)$	$g_3^* - c(3)$	$G^* - C$
$\{1\}$	3.9			3.9
$\{1,2\}$	3.94	-0.06		3.88
$\{1,2,3\}$	**2.79**	$\mathbf{-1.21}$	**2.79**	**4.37**

Table 2: Donor equilibrium with learning economies

Table 2 shows that it is optimal to contact donors 1,2, and 3. Without donor 3, donor 2, whose gift remains below $c(2)$*, diminishes the charitable good provision. By additionally soliciting individual 3, however, the public good reaches its optimal level. Finally, it is clear that even with three donors, a direct approach to identify the extent of fund-raising is non trivial.*

To develop a simple, intuitive characterization of the fund-raiser set, we interpret sequential costs as taxes on individuals. In this sense, let $\widehat{w}_i = w_i - s(i)$ be individual $i's$ "disposable income". Under this formulation, for a given set F, $i's$ gift is $g_i(F) - s(i)$. We show this in two steps. Consider person i's maximization problem:

$$\max_{x_i, g_i} U(x_i, G - \sum_{j \in F} c(j))$$
$$\text{s.t. } x_i + g_i = w_i$$

As a first step, consider substituting for $\overline{w}_i \equiv w_i - s(i) - c$ and $\overline{g}_i \equiv g_i - s(i) - c$, person i can be deemed as choosing the level of the charitable good:

$$\max_{x_i, \overline{G}} U(x_i, \overline{G})$$
$$\text{s.t. } x_i + \overline{G} = \overline{w}_i + \overline{G}_{-i}$$
$$\overline{G} \geq \overline{G}_{-i}$$

The solution to this maximization yields i's demand function for the charitable good given net contributions by others,$\overline{G}_{-i}$:

$$\overline{G} = \max\{f(\overline{w}_i + \overline{G}_{-i}), \overline{G}_{-i}\}.$$

As a second step, from this whole normalization, we define $\widehat{w}_i \equiv \overline{w}_i - c \equiv w_i - s(i)$ and $\widehat{g}_i \equiv \overline{g}_i - c \equiv g_i - s(i)$. This change of variables allows us to reformulate our original problem with learning economies to a constant return to scale setting with marginal cost c and nominal income distribution$\{\widehat{w}_i\}$.

Let F_i be the set of the top i individuals. The next Lemma shows that individual $i's$ incentive to provide a donation above the marginal cost, c, in F_i can be represented by a cost cutoff.

Lemma 1 *Let $\overline{\phi}(G) \equiv \phi(G) - G$, where $\phi = f^{-1}$, and donor i's cost cutoff be given by*

$$\overline{c}_i = \widehat{w}_i - \overline{\phi}(\textstyle\sum_{j=1}^{i}(\widehat{w}_j - \widehat{w}_i)). \tag{2}$$

Individual i is a net contributor in F_i iff $c < \overline{c}_i$

By strict normality $\overline{\phi}'(.) > 0$. Therefore, $i\ s$ cutoff cost decreases in others' disposable incomes and increases in i s own.

Observation 2. *Absent the sequential component, we obtain:* $\overline{c}_1 \geq \overline{c}_2 \geq .. \geq \overline{c}_n$ and $F^0 = \{i \in N \mid c < \overline{c}_i(\widehat{w}_i)\}$ (NY, 2012)

Note first that under no sequential cost, $\widehat{w}_i = w_i$. Hence, for any subeconomy F_i, individuals are ranked according to their net gifts $g_i^*(F_i) - c$, since $w_1 - c \geq w_2 - c.. \geq w_n - c$. It is clear that $\overline{c}_i$ is less than w_i, except for the first individual, and it diminishes in i. Intuitively, once the richest donor is solicited, the second individual is less likely to cover the marginal cost c as a consequence of the free rider problem. In general, as the charity keeps fund-raising, free riding becomes more and more severe and it is less likely that an additional person will be solicited. Once a net free rider is identified, fund-raising must stop. Otherwise, given that individuals are ranked according to their net gifts' sizes, additional solicitations would bring only negative net donations. This would hurt the public good provision, as shown in Lemma 1 in NY (2012). Re-consider Example 1 above, when $s(i) = 0$. From eq. (2), it is easily verified that $\overline{c}_1 = 20$, $\overline{c}_2 = 0$, and $\overline{c}_3 = 0$,which implies that $F^o = \{1\}$.

The free-rider problem is still present when $s(i) > 0$. However, on the upside, fund-raising allows the charity to reach learning economies, thus partially counteracting free riding.

To be more precise, let $a_{\widehat{w}}(i,k)$ be the average disposable income from individuals i to k where $i \leq k$. By convention, $a_{\widehat{w}}(i,i) = w_i - s(i)$. By applying the next proposition iteratively we obtain a full characterization of the fund-raiser's strategy.

Proposition 2 *Suppose either (1)* $i = 1$ *or (2)* $i > 1$ *and individuals* 1 *to* $i-1$ *are solicited by the fund-raiser. Then,* i *is solicited iff there is an individual* $k \geq i$ *such that* $c < \bar{c}_i(a_{\widehat{w}}(i,k))$*. Moreover if* $k > i$ *is the closest individual to* i *satisfying the previous inequality, then donors from* $i+1$ *up to* k *must also be solicited.*

Proposition 2 says that to contact an *additional* individual i, it is sufficient that she pays for her marginal cost at the margin, i.e., if the economy were F_i. It does not matter whether or not she becomes a net contributor in F^o.[15]

Even though the free-rider problem is more pronounced the more a charity fund-raises, it is also true that more fund-raising generates more experience for the charity. Thus, Proposition 3 also says that despite individual i being a net free rider at the margin, she is solicited as long as subsequent cost decreases turn out to be *substantial.*

This proposition contrasts with the equilibrium characterization in NY(2012), where every individual in F^o is a net contributor. In this sense, the presence of net free riders in F^o can be thought of a charity's investment in acquiring experience.

Re-consider Example 1 above, under learning economies. From eq. (2), it folows that $\bar{c}_1 = 13, \bar{c}_2 = -0.33 < c < \bar{c}_3 = 22.33$. Moreover, $\bar{c}_3(a_{\widehat{w}}(2,3)) = 6.33 > c$. Thus, according to Proposition 2, $F^o = \{1,2,3\}$.

Finally, the fund-raiser considers the set resulting from iteratively applying proposition 2 as a *candidate* equilibrium strategy. This set will be optimal if, given the total fund-raising cost, $\sum_{j=1}^{|F^o|} c(j)$, incurred, each individual decides to contribute rather than consume only the private good; i.e., if, in equilibrium, her net cost, $\sum_{j=1}^{|F^o|} c(j) - G^*_{-i}$, is strictly less than her cutoff, $\widehat{C}_i$. The next condition guarantees that this happens for every individual included in the set.

[15] A donor i may be a net contributor in the subeconomy F_i but not in F^o because the sequence of disposable incomes is not necesarily monotonically decreasing. Therefore, after soliciting individual i, another subject providing a higher net gift may also be contacted, thus driving i's contribution below $c(i)$ as a consequence of the free rider problem.

Assumption S. Let $k \in N$ be the largest index such that $c < \bar{c}_i$. Then it follows that (i) $\sum_{i=1}^{k}(w_i - c(i)) > 0$, and (ii) for $i \leq k$: $f(w_i - \widehat{C}_i) \leq \Phi_k^{-1}(\sum_{i=1}^{k}(w_i - c(i))$, where $\Phi_k(\overline{G}) \equiv \sum_{j=1}^{k}(\phi(\overline{G}) - \overline{G}) + \overline{G}$.

We define drastic learning as a sequence of variable costs $\{s(i)\}$ generating a monotonically increasing sequence of disposable incomes. The next corollary shows the fund-raiser's response to drastic learning.

Corollary 1 *Under drastic learning all potential donors are solicited.*

This corollary says that in some cases the learning curve may be steep enough such that each additional solicitation would bring the greatest net gift among already requested individuals. Thus, the fund-raiser faces strong incentives to fund-raise more. Indeed, she ends up soliciting all potential donors to fully take advantage of cost savings.

3.2 Unobservability of the Fund-raiser Set

Our assumption regarding the observability of the the fund-raiser set is reasonable for small fund–raising campaigns. For others, it is not feasible for donors to keep track of the charity's solicitations, but hold beliefs about them.

Given the optimal fund-raiser set F^0, one natural belief system is as follows: a solicited donor who is also in F^0 believes that the charity sticks to the solicitation strategy F^0, whereas a solicited donor outside F^o believes that every richer individual is also solicited while lower income individuals are not.[16] Each donor assumes that others act according to the stated beliefs. We show in the next proposition that when donors share these beliefs, the fund-raiser's equilibrium strategy is the same whether or not it is observable.

Proposition 3 *Suppose the fund-raiser set is unobservable to donors. Let k be the highest index individual in F^o. Suppose gains from learning are exhausted,i.e., $c \geq \bar{c}_j$ for every $j > k$. Then, under the beliefs described above, F^o is sustained as a perfect Bayesian equilibrium.*

It is plausible in big fund-raising campaigns that total initial donations do not cover total initial costs. Despite that, we observe that fund-drives are launched and charitable

[16] This belief system is grounded in a learning by fund-raising setting. To gain experience in the field, fund-raising may be carried out by few persons. Thus donors may not perceive deviations from F^o as uncorrelated or isolated mistakes.

goods are provided from net donations because initial donors expect the charity to continue fund-rasing up to individual k to take advantage of learning economies. Thus, they know that eventually total donations exceed total costs.

Under the belief system described above, the fund-raiser does not necessarily have a commitment problem to its target strategy. Problems may arise if people hold a different belief system. For example, consider the following beliefs: if a donor in F^o is contacted, he learns about the fund-drive and believes that the rest of F^o will also be contacted, whereas if a donor outside F^o is contacted, he attributes this to a mistake and believes that he is the only one contacted besides F^o.[17] To illustrate the tension between charity and donors, consider a case in which the fund-raiser solicits individuals in F^o but there is still potential for learning. Notice that any solicited individual $i > k$ would take others' contributions as: $G^*(F^0) - C(F^0) - c(|F^o| + 1)$. But, in fact, as the charity keeps fund-raising more and more, subsequent cost decreases are obtained without being noticed by additional donors. Consequently, the free rider problem is curbed to some extent, thus undermining the charity's credibility to F^o. As a result, more than optimal fund-raising may be conducted at expense of the charitable good provision. This is consistent with the anecdotal evidence that schools often announce a target level of funds to be raised as well as the length of the fund-drive. [18]

4 Effects of a fixed cost on optimal fund-raising

Fixed costs, also called overhead costs–expenses such as rent, utilities, technology, accounting costs, legal costs, and marketing costs–are an important component of a charity's cost structure. Donors and foundations are aware of the potential detrimental impact of these costs on the charitable good provision. Indeed, watchdog groups rank charities' efficiency based on the administrative cost to total cost ratio. For instance, Charity Navigator suggests that for an acceptable charity this ratio ranges from 15% to 20%. Moreover, a study conducted by the center of philanthropy at Indiana University shows that of the 710 foundations that responded to the survey, 69% responded that their donations were intended to support charity's overhead expenses.

[17] These beliefs are similar to "passive" beliefs often used in bilateral contracting in which one party privately contracts with several others (e.g., Cremer and Riordan 1987; McAfee and Schwartz 1994).

[18] For example, Duke University recently announced a new five-year fundraising campaign to raise $3.25 billion for academic programs, medical education and health research, and its endowment.

To isolate the effect of a fixed cost on optimal fund-raising, we consider the following particular cost structure: a fixed cost s and a constant marginal cost c. This is captured in my model by making $s(1) = s > 0$ and $s(i) = 0$ for every $i > 1$. Let $F^o(s)$ be the fund-raiser set when the fixed cost amounts to s.

Proposition 4 *Consider two fixed cost levels, s and s' such that $s < s'$ and $F^o(s)$ as well as $F^o(s')$ are non-empty. Then,*

(a) *Fund-raising increases in the setup cost ,i.e, $F^o(s) \subseteq F^o(s')$*

(b) *Individual gross donations augments in the setup cost, i.e, $g_i(F^o(s)) < g_i(F^o(s'))$ for every $i \in F^o(s')$, but*

(c) *The public good amount falls in the setup cost, i.e., $\overline{G}^*(F^o(s)) > \overline{G}^*(F^o(s'))$*

The intuition behind this Proposition is simple. From (2) it is clear that for individuals $i > 1$ cutoff costs rise in the fixed cost. Thus, given a higher fixed cost, the charity solicits more because it anticipates that individuals are more willing to give in order to *partially* recover the cost increase. Despite the rise in total gross donations generated by current and additional solicited donors, the level of the public good falls since individuals collectively do not make up for the totality of the rise in the cost. Thus, the two positive effects of the setup cost increase, more fund-raising and more gross donations, are neutralized by the negative effect of a rising cost burden on the supplied public good.

More fund-raising, even when optimally conducted, may in some cases indicate that the charity is actually less productive. This observation contrasts with our intuitive understanding of public good provision in a costless economy where, fixing individuals' characteristics, a larger set of contributors signals a greater supply of the public good.

5 Excessive Fund-raising

Do charities spend too much in fund-raising? Does the cost structure matter in providing an answer for the previous question? To answer these questions, we first consider a benchmark setting in which the fund-raiser fixes for each donor i a minimum gift size t_i. She publicly announces these and refuses donations below the respective thresholds.[19] In some sense

[19] This is actually a case of multilateral "contracting" under positive externalities as in Segal (1999). It also resembles Andreoni 1988, in which the threshold for public good provision is determined by the production technology. In our setting, donors face individual thresholds endogenously determined by the fund-raiser.

the charity is exerting some individual *pressure* on each donor, even though giving is still voluntary. Thus, the free rider problem is still present.

For a fixed set F, the fund-raiser maximization problem is:

$$\begin{aligned} \max_{\{t_i\}_{i=1}^{|F|}} \sum t_i \qquad (3) \\ \text{s.t. } U(w_i - t_i, T) \quad \geq \quad U(w_i, \max\{T_{-i} - C(F), 0\}) \text{ for every } i \in F^o \end{aligned}$$

where $T = \sum_{j=1}^{|F|} t_j$. [20][21]

The next observation shows that in the benchmark the fund-raiser also starts soliciting from the richest individual.

Observation 3. *Individual i does not provide a gift above t_i^*. Moreover $t_{i+1}^* > 0$ implies $t_i^* > t_{i+1}^*$.*

Observation 3 says that threshold gifts leave each individual indifferent to contributing the "suggested" amount or not giving at all. As in the case with purely voluntary contributions and homogeneous preferences, the richer is the individual, the higher is the threshold gift imposed on her.

Given the commitment to minimum gift sizes in the benchmark, the following observation is very intuitive:

Observation 4. *The voluntary provision of the public good is below that in the benchmark.*

Noteworthy, the fund-raiser can feasibly set a minimum gift size to individual i corresponding to her voluntary contribution under $F^o, g_i^*(F^o)$. In other words, the equilibrium

[20] The most acute form of commitment or pressure would add a target level of the charitable good such that if total donations are below that target, neither provision takes place nor refund is made. In this extreme case, the fund-raiser extracts from each individual, g_i^0 where it solves $U(w_i - g_i^0, g_i^0) = U(w_i, 0)$. The critical public good level would be $\sum_{j=1}^n g_i^0$.

[21] Let $\widehat{T}(w_i + T_{-i} - C, T_{-i} - C,)$ represent the level of the public good that makes individual i indifferent between making up for that level given others' contributions T_{-i}, and not contributing at all. Thus, $t_i = \widehat{T}(w_i + T_{-i} - C, T_{-i} - C,) - (T_{-i} - C)$. To establish some comparisons with regard to the pure altruism case, consider $C = 0$.There are two effects of T_{-i}, on $\widehat{T}$: The first effect operates through aggregate income $w_i + T_{-i}$ as in the voluntary case: $\widehat{T}_1 > 0$. The second effect is negative and operates by increasing $i's$ outside utility. Thus, $\widehat{T}_2 < 0$. Moreover, if the latter effect is stronger than the former, then $\widehat{T}_1 + \widehat{T}_2 < 0$, which implies $\frac{dt_i}{dT_{-i}} < -1$. In this case, the substitution effects would be much stronger than in the pure voluntary case, as we see under Cobb-Douglas preferences with $\alpha < \frac{1}{2}$.

voluntary contribution profile $\{g_i^*(F^o)\}_{i\in F^o}$, is a feasible solution to (3) when $F = F^o$. We then show that the fund-raiser may profitably deviate from that solution. To see this, suppose the charity exclusively "pressures" individual 1. By quasiconcavity of the utility function, the fund-raiser is able to extract from him a larger gift than voluntarily provided. In response, other individuals lower their contributions. Overall, the public good amount increases above the level supplied under voluntary contributions, by the strict normality assumption.

We observe that on the benchmark the charity aims to supply the greatest feasible level of the public good, even at the expense of donors' aggregate welfare. In this sense, the resulting outcome is not efficient in a Samuelsian sense. However, a lack of commitment to minimum donations might lead the charity to conduct excessive fund-raising, in the sense that more solicitation expenses would have to be incurred to optimally supply a relatively low charitable good provision. To be more precise, let F^* be the fund-raiser set on the benchmark.

Definition 1 *We say that a charity conducts excessive fund-raising whenever she solicits a larger number of donors with respect to the benchmark, i.e.,* $F^* \subset F^o$.

The next proposition states that excessive fund-raising occurs when individuals have a low preference for the public good. For instance, when individuals have Cobb-Douglass preferences and the demand for the public good is αw, this is the case for $\alpha < \frac{1}{2}$. Indeed, this case is the most relevant from an empirical perspective. Some works, as Zieschang (1985), have estimated α to be 0.0342.

To understand the driving force causing excessive fund-raising, we first focus in the benchmark when fund-raising is costless, and make the following assumption that is satisfied under a low preference for the public good.

Assumption: *In a costless economy* $\frac{dt_i}{dT_{-i}} < -1$.

A low preference for the public good generates a strong substitution effect among feasible requested donations. Thus, when the fund-raiser increases for one individual the minimum donation size by one dollar, the potential gift size for everyone else drops by more than one dollar. As a result, optimal fund-raising entails receiving an extra-larger donation from the richest individual and nothing from the rest. Now, once cost is introduced, it may be the case that the fund-raiser solicits more individuals to partially recover the initial cost,

$c(1)$. This means that each solicited individual $i > 1$ provides a positive net donation and also that $\sum_{i>1}(g_i(F^*) - c(i)) < c(1)$. Indeed, it is shown in the appendix that if more than two individuals are solicited, all of them are pivotal, in the sense that each individual contribution is critical to the public good provision. Since gifts are smaller when charity lacks commitment, it is then intuitive that relatively more fund-raising is conducted to recover the initial cost, as shown in Proposition 5.

Proposition 5 *Suppose more than two individuals are solicited under F^o. Then, there is excessive fund-raising.*

Rose-Ackerman (1982) was the first to introduce the concept of excessive fund-raising in a competitive charitable market under costly fund-raising. She has in mind a benchmark in which charities act coordinately to maximize aggregate net donations. She points out that competition for donations triggers a relatively high level of fund-raising, without increasing aggregate gross donations. Rather, competition causes a switch of gifts among charities equally valued by donors. Thus, ultimately less public good is provided.

In contrast, I have a single charity in my setup, and the more fund-raising conducted, the greater the level of gross donations collected. The main source of excessive fund-raising in my model, then, is the lack of commitment to gift sizes.

Is learning an additional source of excessive fund-raising? Does excessive fund-raising worsen with a faster learning process?

To answer these questions we build on the following sequential cost function:

$$s(i) = \max\{s - \delta(i-1), 0\} \tag{4}$$

where δ represents the learning rate. The next proposition shows how excessive fundraising changes when we move from constant returns to scale in fund-raising to learning by fund-raising.

Proposition 6 *Consider two scenarios: constant returns to scale, $\delta_{nl} = 0$, and learning by fund-raising, $\delta_l \in (0, s)$. Excessive fund-raising is higher under learning, $\delta_l > 0$.*

Proposition 6 says that excessive fund-raising worsens with learning. This result can be explained in terms of the effect of learning on optimal fund-raising in both cases, when charity commits to minimum gift size and when this is not feasible. On one hand, the charity fund-raises more to take advantage of cost decreases when there is no commitment. On the other hand, recall that if more than two individuals are solicited in the benchmark, it is just out of a cost recovery motive; in other words, all individuals are pivotal. Then, fund-raising shrinks with learning because for any subeconomy F_i, $i > 1$, total cost diminishes. Both effects push excessive fund-raising to a higher extent.

Following this logic, it seems intuitive that any increase in the rate of learning widens excessive fund-raising. Surprisingly, this statement is not necesarily correct.

Proposition 7 *Excessive fund-raising is (potentially) non-monotonic in δ.*

Proposition 7 shows that excessive fund-raising is affected by the rate of learning in a complex way. The underlying force driving the previous result is that in the purely voluntary contribution case, the propensity to fund-raise an individual $i > 2$, reflected in her cutoff cost, is non-monotonic in the rate of learning, reaching an interior optimum. Now, to understand the source of this non-monotonicity, note first from (4) that (i) there is some threshold rate for individual $i > 2$, δ_i^*, such that $s(i)$ decreases in δ for $\delta < \delta_i^*$ and remains constant for $\delta \geq \delta_i^*$. (ii) $\delta_1^* > \delta_2^*.. > \delta_n^*$. From these two points we see that each disposable income difference between individual i and the other lower index individuals decreases for $\delta \leq \delta_i^*$, and it increases for $\delta > \delta_i^*$. Because i's cutoff cost is decreasing in the sum of these differences, (see 2) the marginal propensity to fund-raise individual i increases for $\delta \leq \delta_i^*$ This result can be interpreted as coming from a relative cost-saving effect that makes it more likely that individual i becomes a net contributor. The opposite happens when $\delta > \delta_i^*$

Even though a slower learning process may actually bring more excessive fund-raising, it may surprisingly permit the fund-raiser to accumulate more experience as well, as formalized in the next Lemma.

Lemma 2 *Consider two rates of learning:δ_h and δ_l such that $\delta_h > \delta_l > 0$. Then $c(|F^o(\delta_h)|) \leq c(|F^o(\delta_l)|)$ is not always the case.*

A slower learning process on one hand makes fund-raising a given number of individuals more costly, but on the other hand, it may encourage the charity to solicit more people, thus fostering learning. If the difference between learning rates is low enough, the latter effect may outweigh the former one. Consequently, a charity learning more slowly may end up accumulating more fund-raising experience reflected in a lower marginal cost. This may be important for a charity periodically running fund-drives because learning spillovers would also be intertemporal in this case.

In summary, a slower learning process may have negative consequences in a static sense because of excessive fund-raising. This same learning process may generate positive dynamic consequences because of deeper learning.

6 Extensions

In this section we provide three extensions. In the first one we introduce warm-glow giving. In the second one we consider the case in which population is divided among professional groups and learning is group specific. The second one addresses a setting in which there are decreasing returns to scale in fund-raising.

6.1 Warm-Glow Giving

In this section we consider warm-glow as an additional motive for giving and show how fund-raising incentives are affected by it. As in NY(2012), we assume that an individual gets warm-glow from her *net* contribution. Thus, let $u = u(x_i, \overline{G}, g_i - c(i))$ be person i's utility function, which is increasing and strictly quasi-concave. Person i's demand for the public good in a Nash equilibrium can be written as: $\overline{G}^* = \widehat{f}(\overline{w}_i + G^*_{-i} - C(F_{-i}), G^*_{-i} - C(F_{-i}))$, where $\overline{w}_i = w_i - c(i)$ and $F_{-i} = F \setminus \{i\}$. Partial derivatives satisfy $0 < \widehat{f}_{i1}, < 1$ and $\widehat{f}_{i2} \geq 0$ by normality of goods. If, in addition, $0 < \widehat{f}_1 + \widehat{f}_2 \leq \theta < 1$, then a unique Nash equilibrium obtains. Note that for $\widehat{f}_2 = 0$, the warm-glow model reduces to the standard model.

To obtain a closed form solution that facilitates our comparative statics analysis, we consider the following utility for all i:

$$U_i(x_i, G, g_i) = (1-\beta)\ln x_i + \beta \ln(\gamma G + (1-\gamma) g_i)$$

where $\beta \in (0,1)$, $\gamma = \frac{\alpha-\beta}{\alpha(1-\beta)}$ and $\alpha \in (\beta, 1)$. Under this specification warm glow is a substitute for altruism. The demand for the public good in this case is $\overline{G}^* = \beta w_i + \frac{\beta}{\alpha} G_{-i}$. Ignoring the costly aspect of fund-raising, note that when $\alpha = 1$, $\overline{G}^* = \beta(w_i + G_{-i})$. Thus, individuals give out of a pure public good motive. On the other hand, when $\alpha = \beta$, then $\overline{G}^* = \beta w_i + G_{-i}$ and $g^*_i = \beta w_i$. Hence, individuals give motivated by pure warm-glow.[22] Thus, the lower α is, the stronger is the warm-glow motive.

[22] The parameter α represents the altruism coefficient as introduced in Andreoni (1989). It is a measure of the relative strength of the public good motive for giving.

It can be shown that Proposition 2 holds under this utility specification, and individual i s cutoff cost is given by

$$\bar{c}_i = \widehat{w}_i - \frac{\alpha - \beta}{\beta} \sum_{j=1}^{i} (\widehat{w}_j - \widehat{w}_i). \tag{5}$$

It is intuitive that the more warm-glow people experience, the more incentived is the fund-raiser to solicit more, since the free rider problem is less severe. Indeed, Eq.(5) implies that $\bar{c}_i$ increases as α decreases. Thus, fund-raisers learn more on the job when the warm-glow motive is strong. Moreover, if a net free-rider is identified, the fund-raiser is more likely to solicit her as a learning investment.

6.2 Group specific learning

Suppose the fund-raiser divides the set of potential donors into $m \geq 1$ groups, depending on their professional activities. She believes that each member of group i independently draws his income from a discrete distribution, $\widetilde{w}_i$, with mean $E[\widetilde{w}_i]$. We assume that the charity learns by fund-raising within a group, but this experience does not translate into cost decreases in soliciting members of other groups.Thus, let $s_i(j)$ be the sequential cost of fund-raising the jth individual in group i. The fund-raiser's strategy is to choose the number of donors to be contacted from each group. To focus the analysis on the fund-raiser side, we continue to assume that donors have no uncertainty about the income profile in the population. Moreover, to simplify the analysis, we consider identical homothetic preferences so that $f(w) = \alpha w$ for some $\alpha \in (0, 1)$. Without loss of generality we rank groups according to their average disposable incomes: $E[\widetilde{w}_1] - a_{s_1} \geq ... \geq E[\widetilde{w}_m] - a_{s_m}$. The fund-raiser's equilibrium strategy is stated in Proposition 9.

Proposition 8 *Let group i s cutoff be given by*

$$\bar{c}_i = E[\widetilde{w}_i] - a_{s_i} - \frac{1-\alpha}{\alpha} \textstyle\sum_{j=1}^{i} n_j \left[(E[\widetilde{w}_j] - E[\widetilde{w}_i]) + (a_{s_i} - a_{s_j}) \right]$$

Then $\bar{c}_1 \geq \bar{c}_2 \geq ... \geq \bar{c}_n$. Moreover, every member of group i is solicited iff $c < \bar{c}_i$

The fund-raiser optimally treats each group member as having mean disposable income $E[\widetilde{w}_i] - a_{s_i}$.It is intuitive, then, that the fund-raiser either contacts no members of group i iff $c \geq \bar{c}_i$ or solicits all of them iff $c < \bar{c}_i$. Thus, group i's cutoff cost is interpreted as the average propensity of its members to pay for c. Note that an increase in the extent of learning economies within a group i, either due to the presence of more members or to a higher speed of learning, augments the group's mean disposable income. Thus, group i is more likely to be solicited and any other group less so.

We say that groups i and j merge if the fund-raiser knows n_i and n_j but is not able to distinguish among members of these groups. (NY, 2012)

Consider the case in which the technology for fund-raising any given group is $s(j)$ and two groups merge. We assume full learning spillovers within the merged group. That is, the fund-raising cost function for this group is still $s(j)$. One may think that since the merger brings more potential for learning, the public good provision increases. But this is not always the case. After a merger, available information becomes coarser in the sense that the fund-raiser does not distinguish individuals in the merged group. This effect potentially hurts the public good provision as shown in the next Lemma:

Lemma 3 *Suppose groups i is solicited and group j is not and they merge. If the merged group ij is not solicited, i.e.,*

$$E[\widetilde{w}_{ij}] - a_{s_{ij}} - \frac{1-\alpha}{\alpha} \sum_{\{k \neq i,j : E[\widetilde{w}_k] > E[\widetilde{w}_{ij}]\}} n_k \left[(E[\widetilde{w}_k] - E[\widetilde{w}_{ik}]) + (a_{s_k} - a_{s_{ij}}) \right] \leq c,$$

where $E[\widetilde{w}_{ij}]$ and $a_{s_{ij}}$ are respectively the mean income and average cost of the merged group ij, then the public good provision after the merger diminishes

Lemma 3 makes explicit the tradeoff generated after a merger. On one hand, learning increases, i.e., $a_{s_{ij}} < a_{s_i}$, which makes $\bar{c}_{ij}$ increase with respect to $\bar{c}_i$. On the other hand, coarser information hurts fund-raising in the sense that $E[\widetilde{w}_{ij}] < E[\widetilde{w}_i]$. This effect makes $\bar{c}_{ij}$ fall below $\bar{c}_i$. If the latter effect is stronger, the merged group ij is not fund-raised. Thus, members of group i, who were optimally fund-raised before the merger, are no longer identified by the fund-raiser, learning spillovers do not justify the inclusion of members of the merger group in the solicitation set. As a result, the public good provision declines.

Even though we consider homogeneous preferences in this work, this result trivially extends to a setting in which donors may have different taste toward the public good, α_i.

Previous works, such as Andreoni (2013), examine the effect of diversity on the public good provision from the donor's side. The results of this section suggest that a better understanding of this matter must include as well the fund-raiser's response given potential learning spillovers.

6.3 Decreasing returns to scale

In this section we consider a charity constrained by physical and human resources. We envision fund-raising as an increasingly costly process. The next proposition formalizes the fund-raiser's solicitation strategy in this setting.

Proposition 9 *Suppose $s(1) \leq s(2) \leq s(3).. \leq s(n)$. Let $\overline{\phi}(G) \equiv \phi(G) - G$, and donor i's cost cutoff be given by*

$$\overline{c}_i(\widehat{w}_i) = \widehat{w}_i - \overline{\Phi}(\textstyle\sum_{j=1}^{i-1}(\widehat{w}_j - \widehat{w}_i) \qquad \text{(cutoff costs)}$$

Then, $\overline{c}_1 \geq \overline{c}_2 \geq ... \geq \overline{c}_n$, and $F^o = \{i \in N | c < \overline{c}_i\}$.

In general, as costs increase, the charity becomes more conservative soliciting an additional subject, because of both the free-rider problem and the increase in marginal cost. Furthermore, absent a learning motive, once a net free rider is identified, the solicitation process stops. Consequently, as in NY (2012), every solicited individual is a net contributor in F^o. Indeed, it is intuitive that as the charity experiences more rapid diseconomies of scale, the fund-raiser set shrinks as well as the public good provision. Moreover, the degree of excessive fund-raising tends to diminish.

7 Conclusion

In this paper we extend the literature on charitable fund-raising by bringing to the center of the analysis the role of solicitation technology in optimal fund-raising, which is characterized in terms of donors' preference and incomes as well as solicitation costs. We also define excessive fund-raising in a single charity environment with respect to a setting in which the fund-raiser commits to minimum gift sizes.

We specially consider a charity which becomes a more efficient solicitor through time. This fact is not innocuous in terms of optimal fund-raising and excessive fund-raising. On the contrary, on one hand, it determines an investment in learning incentive. For instance, some charities may launch a fund-drive even when initial donations are not sufficient to cover initial costs. However, it is common knowledge that the charity fund-raises more to achieve cost reductions, which ensures the charitable good provision. On the other hand, excessive fund-raising worsens when we move from a constant return to scale technology to a setting with learning by fund-raising. Moreover, excessive fund-raising is non-monotonic in the rate of learning.

From a policy perspective, the introduction of government grants, either direct or matching ones, could reinforce or counteract the advantages of learning. Our results also suggest that some sort of commitment to alleviate the free-rider problem is more valuable in environments where the fund-raiser learns on the job.

As an extension we work a setting in which the population is divided among professional groups and the fund-raiser believes that each member of a given group independently draws his income from a discrete distribution. Moreover, learning takes place exclusively within each group. We find that moving to a less diverse population by merging groups may hurt public good provision, despite full learning spillovers within merged groups. Thus, diversity may actually be beneficial for the fund-raiser.

For future work, it may be interesting to consider how experience generates wage premiums in the market for fund-raisers. On one hand, a more experienced fund-raiser is highly demanded since she rises the public good provision, but on the other hand, a higher wage increases fund-raising costs, thus dampening the charitable good.

It would also be interesting to address in a formal model the divergence of objectives between a charity and a fund-raiser. The charity's objective is to maximize current level of the public good. However, given a high demand for experienced fund-raisers, they may over-solicit to learn more. This sort of reasoning may justify why fund-raisers are paid a fixed wage regardless of the level of funds they raise.

Appendix

Proof of Proposition 1. Fix an arbitrary fund-raiser set, $F \neq \varnothing$, whose total fund-raising cost is $C(F) > 0$. Suppose $C(F) > \max_{i \in F} \widehat{C}_i$. Then, individual i's best response to $G^*_{-j}(F) = 0$ is $g^*_i(F) = 0$. Thus, the zero-contribution profile is an equilibrium, resulting in $\overline{G}^*(F) = 0$.

Consider the optimal fund-raiser set, F^o. Clearly, $\overline{G}^*(F^o) > 0$ implies that some agents have been contacted, and thus $F^o \neq \varnothing$. Conversely, suppose that in equilibrium, $F^o \neq \varnothing$, but $\overline{G}^*(F^o) = 0$. Then, since $C(F^o) > 0$, the charity has a strict incentive to choose $F = \varnothing$ and incur no cost. Hence, $\overline{G}^*(F^o) > 0$. ■

Proof of Observation 1. By closely following Andreoni (1988), equilibrium contributions in any subgame F can be characterized as

$$g^*_i(F) - c = (w_i - c) - w^*(F) \tag{6}$$

where $w^*(F) = \phi(\overline{G}^*(F)) - \overline{G}^*(F))$ and $\phi \equiv f^{-1}$. Moreover, $g_i(F^o) - c > 0$ for every $i \in F^o$.Otherwise, the fund-raiser may save on costs, which contradicts the optimality of F^o. Suppose the optimal strategy is to fund-raise k individuals. Then $C(F^o) = \sum_{j=1}^{k} c_i$. Note that if $k = 1$ the result trivially follows since $f'(.) > 0$ and $w_1 - c_1 > w_j - c_j$ for any $j > 1$. Next, consider the case in which the first i individuals are included in the set, where $i < k$. Denote $\overline{G}^*(F_{+l}) = \overline{G}^*(F \cup \{l\})$ Note that by including any individual $l < i$ such that $g_l^* > 0$, by (6), it follows that every individual $j \leq i$ is also a contributor. Moreover, $\overline{G}^*(F_{+l})$ solves

$$
\begin{aligned}
& ((i+1)(\phi(\overline{G}^*(F_{+l}) - \overline{G}^*(F_{+l})) + \overline{G}^*(F_{+l})) - \\
& \textstyle\sum_{j=1}^{i}(w_i - c_i) + \sum_{j=1}^{i+1} s(j) \\
= \; & w_l - c_l
\end{aligned}
$$

Thus, $\overline{G}^*(F_{+(i+1)}) \geq \overline{G}^*(F_{+l})$ for any $l \geq i+1$, since $w_{i+1} - c_{i+1} \geq w_l - c_l$ and $\phi' > 0$. ■

Definition. *Let $G_i^0(c)$ be the "drop-out" level of the public good for person i under net income $w - c$, which uniquely solves $f(w_i - c + G_i^0) = G_i^0$. By convention $G_i^0(c) = 0$ whenever $w_i - c \leq 0$.*

Lemma A1. *If $\overline{G}^*(F_i) > 0$ for some F_i, then $g_i^*(F_i) - c(i) > 0$ if and only if $G_i^0(c(i)) > \overline{G}^*(F_i)$*

Proof. Following closely Lemma 1 in NY(2012), note that $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) = w_i - g_i^*(F)$, or equivalently $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) = (w_i - c(i)) - (g_i^*(F_i) - c(i))$ if $g_i^*(F_i) > 0$; and $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) \geq w_i$ if $g_i^*(F_i) = 0$. Since $\phi_i(G_i^0(c(i))) - G_i^0(c(i)) = w_i - c(i)$, and $\phi_i' > 1$, the Lemma follows. ■

Definition. Let $\Phi_i(\overline{G}) \equiv \sum_{j=1}^{i}(\phi(\overline{G}) - \overline{G}) + \overline{G}$, where $\phi \equiv f^{-1}$ and $\Phi_i'(\overline{G}) > 0$. Define $\Delta_i(c(i)) \equiv \Phi_i(G_i^0(c(i))) - \sum_{j=1}^{i}(w_j - c(j))$

Corollary 2 *If $\overline{G}^*(F_i) > 0$ for some F_i, then $g_i^*(F_i) - c(i) > 0$ if and only if $\Delta_i(c(i)) > 0$.*

Proof. This follows from Lemma A.1. and Andreoni and McGuire (1993). ■

Proof of Lemma 1. This proof follows closely NY(2012). Define $\bar{c}_i$ the value of c making $\Delta_i(c(i)) = 0$. Simplifying terms, $\bar{c}_i$ solves:

$$
i[\phi(G_i^0(c(i))) - G_i^0(c(i))] + G_i^0(c(i)) - \textstyle\sum_{j=1}^{i}(w_j - s(j)) + ic = 0.
$$

Since $\phi(G_i^0(c(i))) - G_i^0(c(i)) = w_i - s(i) - c$, from the equation above, we have

$$G_i^0(\overline{c}_i + s(i)) = \sum_{j=1}^{i} \left[(w_j - w_i) + (s(i) - s(j)) \right].$$

In addition, given that $\overline{\phi}(G) \equiv \phi(G) - G$, we also have $\overline{\phi}(G_i^0(\overline{c}_i + s(i))) = w_i - s(i) - \overline{c}_i = \overline{\phi}\left(\sum_{j=1}^{i} \left[(w_j - w_i) + (s(i) - s(j)) \right]\right)$, which reduces to

$$\overline{c}_i = w_i - s(i) - \overline{\phi}(\textstyle\sum_{j=1}^{i} \left[(w_j - w_i) + (s(i) - s(j)) \right]).$$

Let $\widehat{w}_i = w_i - s(i)$. Then $\overline{c}_i = \widehat{w}_i - \overline{\phi}(\sum_{j=1}^{i} (\widehat{w}_j - \widehat{w}_i))$. Finally, notice that $\Delta_i > 0$ iff $c < \overline{c}_i$, then, by the previous corollary, the proposition follows. ■

Proof of Proposition 2. By noting that if $\overline{G}^*(F_i) > 0$ then it satisfies $\Phi_i(\overline{G}^*(F_i)) = \sum_{j=1}^{i} (w_j - c(j))$ and by Lemma A1, it follows that $\Delta_i(w_i - c(i)) > 0$ iff $g_i^*(F_i) - c(i) > 0$. Consider first the case where $i = 1$. Take the lowest index indvidual $k \geq i$ s.t. (i) $\Delta_k(w_k - c(k)) > 0$ and (ii) $\sum_{j=1}^{k} (w_j - c(j))$. Clearly $\overline{G}^*(F_k) > G_k^0(c(k)) > 0 = \overline{G}^*(\{\emptyset\})$. Now consider $i > 1$ and individuals $1, 2, .., i-1$ are solicited. Take the lowest index indvidual $k \geq i$ s.t. $\Delta_k(w_k - c(k)) > 0$ Notice that $g_k^*(F^k) > 0$. Thus,$\overline{G}^*(F^k) > 0$. Note also that $\Phi_{i-1}(\overline{G}^*(F^k)) = \sum_{j=1}^{i-1} (w_j - c(j)) + \sum_{j=i}^{k} \left[g_j^*(F^k) - c(j) \right]$.Thus, $\overline{G}^*(F^k) > \overline{G}^*(F^{i-1})$ iff $\sum_{j=i}^{k} \left[g_j^*(F^k) - c(j) \right] > 0$. Let $w_j' - c'(j) = w_j - c(j)$ for $j < i$ and $w_j' - c'(j) = a_{w-c}(ik)$ for $i \leq j \leq k$. This implies $\Delta_i(w_i' - c'(i)) = \Delta_{i+1}(w_{i+1}' - c'(i+1)) = .. = \Delta_k(w_k' - c'(k))$. Thus $g_j^*(w_j', F^k) - c'(j) > 0$ for every $j = i, i+1, .., k$.iff $\Delta_i(a_{w-c}(ik)) > 0$. Note also that

$$\begin{aligned}
\textstyle\sum_{j=i}^{k} \left[g_j^*(w_j', F^k) - c'(j) \right] &= \sum_{j=i}^{k} a_{w-c}(ik) + \sum_{j=1}^{i-1} (w_i - c(i)) \\
&\quad - \textstyle\sum_{j=1}^{i-1} \left[g_j^*(w_j', F^k) - c'(j) \right] \\
&= \sum_{j=1}^{k} (w_i - c(i)) - \textstyle\sum_{j=1}^{i-1} \left[g_j^*(w_j, F^k) - c(j) \right] \\
&= \textstyle\sum_{j=i}^{k} \left[g_j^*(w_j, F^k) - c(j) \right]
\end{aligned}$$

The first equality above is valid since individuals $i, i+1, .., k$ are *gross* contributors, under both income distributions. Thus, we obtain the result

$$\overline{G}^*(F^k) > \overline{G}^*(F^{i-1}) iff \textstyle\sum_{j=i}^{k} \left[g_j^*(F^k) - c(j) \right] > 0 \text{ iff } \Delta_i(a_{w-c}(ik)) > 0.$$

Consider the case in which $k > i$. Since there is no $i \leq l < k$ such that $\Delta_l(a_{w-c}(ik)) > 0$, then it is optimal to include $i, i+1, .., k$ in F^o. Thus, by Lemma 1 the proposition follows. ■

Proof of Proposition 3. (i) follows by noticing that cost cutoff for individuals $i > 1$ are increasing in the setup cost and the fund-drive is launched for both fixed cost levels under consideration. To prove (ii) by Let $\Phi_{F^o(s)}(\overline{G}) \equiv \sum_{i \in F^o(s)}(\phi(\overline{G}) - \overline{G}) + \overline{G}$ and k be the number of solicitations under $F^o(s')$. Then, by using equilibrium conditions

$$\begin{aligned} \Phi_{F^o(s')}(\overline{G}^*) &= \textstyle\sum_{i \in F^o(s')}(w_i - c) - s \\ &> \textstyle\sum_{F^o(s')}(w_i - c) - s' = \Phi_{F^o(s')}(\overline{G}^*(F^o(s'))) \end{aligned}$$

The inequality comes from $s < s'$. By strict normality, $\Phi'_{F^o(s)}(.) > 0$.Thus, $\overline{G}^* > \overline{G}^*(F^o(s')$. Note that $\overline{G}^* \leq \overline{G}^*(F^o(s))$, by a revealed preference argument. Thus, $\overline{G}^*(F^o(s)) > \overline{G}^*(F^o(s'))$.Finally, note that in equilibrium $g_i^*(F^o(s')) = w_i + \overline{G}^*(F^o(s')) - \phi_i(F^o(s'))$. Since $\overline{G}^*(F^o(s)) > \overline{G}^*(F^o(s'))$ and $\phi'_i > 1$, it follows that $g_i^*(F^o(s')) > g_i^*(F^o(s))$ for every $i \in F^o(s')$. Thus, (iii) follows. ■

Proof of Proposition 4. Let $\mathcal{F}_i$ be donor i's belief about the fund-raiser set when he is contacted. Then, as stated in the text, $\mathcal{F}_i = F^o$ if $i \in F^o$, and $\mathcal{F}_i = \{1, 2, 3, .., i\}$ if $i \notin F^o$. We will show that given the beliefs $\{\mathcal{F}_i\}_{i=1}^n$, contacting $j \notin F^o$ is not a profitable deviation for the fund-raiser. Let k be the lowest index individual being solicited under F^o.Thus, individual $k+1$ provides the biggest gift among the individuals not in F^o . Let g_{k+1}^o be $k+1$'s contribution under the stated belief system. We first show closely following NY(2102) that $g_{k+1}^o \leq c(k+1)$. Suppose not. Upon being contacted, person $k+1$ would expect others' gross contributions to be $G^*(F^o)$, resulting in

$$\phi_{k+1}(\overline{G}^*(F^o) + g_{k+1}^o - c(k+1)) - (\overline{G}^*(F^o) + g_{k+1}^o - c_{k+1}) = w_{k+1} - g_{k+1}^o. \tag{7}$$

On the other hand, if the individuals in F^o knew about the presence of $k+1$ before contributing, then,

$$\phi_{k+1}(\overline{G}^*(F_{k+1})) - \overline{G}^*(F_{k+1}) \geq w_{k+1} - g_{k+1}^*(F_{k+1}). \tag{8}$$

Suppose $g_{k+1}^o > c(k+1)$.It would directly imply $g_{k+1}^o > g_{k+1}^*(F_{k+1})$. Therefore, $w_{k+1} - g_{k+1}^*(F_{k+1}) > w_{k+1} - g_{k+1}^o$. Then, since $\phi'_{k+1} > 1$, eq.(7) and (8) reveal that $\overline{G}^*(F_{k+1}) > \overline{G}^*(F^o) + g_{k+1}^o - c(k+1)$. This contradicts $\overline{G}^*(F_{k+1}) \leq \overline{G}^*(F^o)$. Thus, $g_{k+1}^o \leq c(k+1)$.

Denote $F_k^{+\{k+1\}}$ be the two stage game where in stage 1, individuals in F_k contributes simultaneously believing that no person outside F is contacted and in stage 2, given total contributions in stage 1, individual k+1 decides on her gift.

Consider two cases:

(1) $\overline{G}^*(F_{k+1}) \geq \overline{G}^*(F^o) + g^o_{k+1}(F^o) - c(k+1)$.By strict normality, this in turn implies that

$$\overline{G}^*(F^o)+g^o_{k+1}(F^o)-c(k+1)+g^o_{k+2}(F^{o+\{k+1\}})-c(k+2) \leq \overline{G}^*(F_{k+1})+g^o_{k+2}(F_{k+1})-c(k+2) \tag{9}$$

Moreover, since $g^*_{k+2}(F_{k+2}) \leq c(k+2)$, then by exactly following the same argumentation as above, it implies $g^o_{k+2}(F_{k+1}) \leq c(k+2)$. Consequently

$$\overline{G}^*(F_{k+1}) + g^o_{k+2}(F_{k+1}) - c(k+2) \leq \overline{G}^*(F_{k+1}) \leq \overline{G}^*(F^o) \tag{10}$$

Combining (9) and (10) and since $C(F_{k+1}) > C(F^o)$, we obtain that the fund-raiser is better off sticking to F^o.

(2)$\overline{G}^*(F_{k+1}) < \overline{G}^*(F^o) + g^o_{k+1}(F^o) - c(k+1)$.

Again, $g^*_{k+2}(F_{k+2}) \leq c(k+2)$ implies $g^o_{k+2}(F_{k+1}) - c(k+2) \leq 0$. Moreover, by strict normality $g^o_{k+2}(F^{o+k+1}) < g^o_{k+2}(F_{k+1})$. Thus, $g^o_{k+2}(F^{o+k+1}) < c(k+2)$.By recalling that $g^o_{k+1}(F^o) - c(k+1) \leq 0$ we get $\overline{G}^*(F^o) + g^o_{k+1}(F^o) - c(k+1) + g^o_{k+2}(F^{o+k+1}) - c(k+2) < \overline{G}^*(F^o)$. Thus, the fund-raiser is better off sticking to F^o. By inductively applying this argument, the result follows ■

Definition. *Let $t_i(T_{-i})$ be the value of t_i satisfying $U(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) - U(w_i, T_{-i}) = 0$*

Denote $G^0_i(0)$ simply as G^0_i.

Lemma A2. *$t_i(T_{-i})$ may be defined as $\widehat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i}$, where $\widehat{T}$ satisfies:*

1 $\widehat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i} > 0$ for every $T_{-i} \in \left[0, G^0_i\right)$

2 $\widehat{T}(w_i + G^0_i, G^0_i, w_i) = G^0_i$

3 $\widehat{T}_1 > 0, \widehat{T}_2 < 0, \widehat{T}_3 < 0$

4 $\widehat{T}_1 + \widehat{T}_3 > 0$

5 $\widehat{T}(w_i + T_{-i}, T_{-i}, w_i) > f(w_i + T_{-i})$ for every $T_{-i} \in \left[0, G^0_i\right)$

Proof. By quasiconcavity of the utility function $t_i(T_{-i}) > g_i(T_{-i})$. Therefore $U_1 > U_2$. Moreover, by quasiconcavity of the utility function

$U_{ii} < 0$ and $U_{12} > 0$. Together, it implies $\frac{\partial t_i}{dT_{-i}} = \frac{U_2(w_i, T_{-i}) - U_2(w_i - t_i, T_{-i} + t_i)}{U_2(w_i - t_i, T_{-i} + t_i) - U_1(w_i - t_i, T_{-i} + t_i)} < 0$. By noting that $x_i + T = w_i + T_{-i}$,then the first term of $\widehat{T}$ captures the direct positive effect of T_{-i} on T, as in the classic public good model. The second term of $\widehat{T}$ captures the negative effect of T_{-i} on T through a higher outside option. The third term captures the negative effect of w_i on T through a higher outside option. Thus , (3) follows. (4) follows by quasiconcavity of the utility function.

Let t_i^s satisfies $U(w_i - t_i^s, t_i^s) = U(w_i, 0)$. By quasiconcavity of the utility function, $t_i^s > 0$. Moreover $t_i^s = \widehat{T}(w_i, 0) > f(w_i) = g_i^s$. Note that by definition of G_i^0, $U(w_i, G_i^0) > U(w_i - g_i, G_i^0 + g_i)$ for any $g_i > 0$. Thus, (1) and (2) follows. Part (5) follows from definition of $\widehat{T}$ and quasiconcavity of the utility function. ■

For the following lemmas and propositions we omit the third argument of $\widehat{T}$,knowing that $\widehat{T}$ is increasing in w_i, by Lemma A2-3.

Lemma A3. *In the costless case, there exists a solution to the taxation problem unique up to total taxation T^*. Moreover, the solution is unique up to individual taxation if $0 < \widehat{T}_1 + \widehat{T}_2 < 1$*

Proof. Existence follows directly from Brower's fixed point . Uniqueness when $0 < \widehat{T}_1 + \widehat{T}_2 < 1$ and $\widehat{T}(w_i, 0) < G_i^0$ follows from a standard contraction mapping argument, the same used in the voluntary contribution literature (Cornes and Sandler 1998). However, there is an additional point to be stressed here. Since $\widehat{T}(w_i, 0)$ may be greater than G_i^0 in this framework, it may happen that the optimal solution entails taxing just one individual. Thus, if individuals are ex-ante identical, the fund-raiser is indifferent with regard to whom to tax. This generates multiplicity. ■

Lemma A4. *Suppose that under costless fund-raising, $\frac{dt_i}{dT_{-i}} < -1$, then, facing a total cost C, more individual(s) are contacted if $G_2^0 > t_1(0) - C$. If that is the case, then, they partially recover the cost,i.e., $\sum_{i>1} t_i^* < C$. Moreover,$\frac{dt_1^*}{dT_{-1}^*} > 0$. If more than two individuals are solicited, then $\frac{dt_i^*}{dT_{-i}^*} > 0$ for every $i > 1$ s.t. $t_i^* > 0$*

Proof. Since in a costless economy $t_1^* > G_1^0$ then, it also follows that $t_1^* > G_1^0$ for $i > 1$. Thus, once we introduce a total cost C, it must be the case that $\sum_{i>1} t_i^* < C$. Otherwise, more charitable good could be obtained by increasing individual 1's gift size, since $\frac{dt_i}{dT_{-i}} < -1$ in a costless economy. Note that $\sum_{i>1} t_i^* < C$ implies $t_1^{*\prime} > 0$.

Then, if the optimal set is $\{1,2\}$, it must be the case that $t_2^* < 0$.Moreover, note that if the fund-raiser receive gifts from more than two individuals, for $2 \leq i < j$ such that $t_i^*, t_j^* > 0$ it must be the case that $t_i^{*\prime}, t_j^{*\prime} > 0$. That is, all individuals are pivotal. Suppose not, let $t_i^{*\prime} < 0, i.e., T_{-i} - C > 0$ for some $i > 1$, then by reducing $j's$ gift size by one unit, the fund-raiser is enable to rise individual $i's$ threshold gift by more than one unit, which constitutes a profitable deviation. Contradiction. ■

Proof Observation 3. Consider first the case in which $\frac{dt_i}{dT_{-i}} > -1$ for $C = 0$,i.e., $0 < \widehat{T}_1 + \widehat{T}_2 < 1$. We show that $t_i^* > t_{i+1}^*$. By way of contradiction suppose $t_{i+1}^* > t_i^*$. Note that there exists an inverse function $\widehat{\phi}(\overline{T}^*, w_i)$ such that $\widehat{\phi}_1 > 0$ and $\widehat{\phi}_2 < 0$ such that

$t_i^* = \overline{T}^* - \widehat{\phi}(\overline{T}^*, w_i) < t_{i+1}^* = \overline{T}^* - \widehat{\phi}(\overline{T}^*, w_{i+1})$. Since $\widehat{\phi}_2 < 0$, it implies $w_i < w_{i+1}$.

Now consider $\frac{dt_i}{dT_{-i}} < -1$ when $C = 0$,i.e., $\widehat{T}_1 + \widehat{T}_2 < 0$. Clearly individual 1 is solicited since she is the one providing the greatest stand-alone value. On the other hand, if the optimal solution entails 2 solicitations, then individual 2 is the other one being solicited since $\widehat{T}(w_2 + t, t) > \widehat{T}(w_i + t, t)$ for any $i > 2$ and $t > 0$. If 3 or more solicitations are made $t_i^{*\prime} > 0$ for every i s.t. $t_i^* > 0$. Thus, $\widehat{T}_1 + \widehat{T}_2 = \widehat{T}_1 > 0$ at $\overline{T}^*$ and the inverse function argument given above also applies here. Finally, if some individual provides a gift $y_i^* > t_i^*$, then by quasiconcavity of the utility function there exists $t_i' > y_i^*$ such that $U(x_i - t_i', t_i' + T_{-i}^* - C) = U(w_i, \max\{T_{-i}^* - C, 0\})$, which contradicts the optimality of t_i^*. ■

Proof Observation 4. Note that $\{g_i^*\}_{i=1}^{|F^i|}$ is a feasible solution. Also note that $U(w_1 - g_1^*, g_1^* + G_{-1}^* - C) > U(w_1, G_{-1}^* - C)$ Therefore, since $U()$ is quasiconcave in their arguments it follows that $g_1^* < t_1(G_{-1}^*)$. Hence,by fixing $t_1(G_{-1}^*)$ we have $G^* < t_1(G_{-1}^*) + G_{-1}(t_1)$., where the inequality comes from strict normality. Note also that the RHS of the previous inequality is still a feasible solution of the taxation problem since $G_{-1}(t_1) < G_{-1}^*$.Thus,$U(w_1 - t_1, t_1 + G_{-1}(t_1)) > U(w_1, G_{-1}(t_1))$ Hence, we found a deviation from G^* where more public good is generated. Thus, $G^* < T^*$. ■

Proof of Proposition 5. If $F^* \subseteq \{1,2\}$ then trivially, there is excessive fund-raising. Consider then $|F^*| \geq 3$. We know from Lemma A4, that in this case every individual is *pivotal*,*i.e.*, $\frac{dt_i}{dT_{-i}} > 0$. Thus, $g_i^*(F^*) < t_i^*(F^*)$ for every $i \in F^*$.Thus if $\overline{G}^*(F^*) > 0$, individuals are also pivotal under voluntary contributions. Therefore, $F^* \subseteq F^o$. On the other hand, if $\overline{G}^*(F^*) = 0$, then $F^* \subset F^o$. Thus, there is excessive fund-raising. ■

Lemma A5. *(Voluntary contributions) For every* $i > 2$:$\overline{c}_i(\delta = 0) < \overline{c}_i(\delta = s)$. *Moreover,for any:* $\delta'' > \delta'$ *(i)*$\overline{c}_i(\delta') < \overline{c}_i(\delta'')$ *for* $0 \leq \delta', \delta'' < \frac{s}{i-1}$ *and (ii)* $\overline{c}_i(\delta') \geq \overline{c}_i(\delta'')$ *for*

$\frac{s}{i-1} < \delta', \delta'' \leq s$

Proof. Note that

$$\begin{aligned} \overline{c}_i(s) &= w_i - \Phi(\textstyle\sum_{j=1}^{i-1} \left[(w_j - w_i) - s\right] > \overline{c}_i(0) \\ &= w_i - s - \Phi(\textstyle\sum_{j=1}^{i-1} \left[(w_j - w_i)\right], \end{aligned}$$

establishing the first part of the Lemma. Moreover,

$$\overline{c}_i(\delta) = w_i - s + \delta(i-1) - \Phi(\textstyle\sum_{j=1}^{i-1} \left[(w_j - w_i) - \delta(i-1)\dfrac{i}{2}\right]$$

for $0 \leq \delta < \frac{s}{i-1}$. Thus, $\frac{\partial \overline{c}_i(\delta)}{d\delta} = i - 1 + \Phi'(.)(i-1)\frac{i}{2} > 0$. On the other hand for $\frac{s}{i-1} \leq \delta \leq s$, note that $\overline{c}(\delta) = w_i - \Phi(\sum_{j=1}^{i-1} \left[(w_j - w_i) + (\max\{(i-1)\delta, s\} - s\right]$. Thus, $\frac{\partial \overline{c}_i(\delta)}{d\delta} = -\Phi'(.)\left[\frac{(k-1)k}{2}\right] < 0$ where k is the highest index individual with $(k-1)\delta < s$. ■

Lemma A6. *For any δ_1, δ_2 such that $\delta_1 < \delta_2$ it must follow that $F^*(\delta_2) \subseteq F^*(\delta_1)$*

Proof. Consider first the benchmark with constant marginal cost. Note that if $t_1(0) - c - s \geq G_2^0$, then, it follows that the optimal fund-raising strategy in the benchmark consists in soliciting exclusively individual 1. Moreover, this strategy is *fixed* for any learning rate. On the other hand, consider $|F^*| \geq 3$. By Lemma A4 we know that if an individual $i \geq 3$ is solicited, then she must be pivotal.

Note that an increase in δ lowers $C(F^*)$. Therefore, if individual $i > |F^*|$ was not necessary to cover $C(F^*)$ before the $\delta-$increase, it would not be contacted once δ increases. Thus, for any δ_1, δ_2 such that $\delta_1 < \delta_2$ it must follow that $F^*(\delta_2) \subseteq F^*(\delta_1)$ ■

Proof of Proposition 6. Consider the voluntary case.By Lemma $A4$, :$\overline{c}_i(0) < \overline{c}_i(\delta_l)$ for any $0 < \delta_l \leq s$.Thus, $F^o(0) \subseteq F^o(\delta_l)$. By this result and Lemma A6, the proposition folllows. ■

Proof of Proposition 7. We first show that fund-raising in the pure voluntary contribution case is potentially non-monotonic in δ, consider a case in which $|N| > 2$. Let i be the lowest index in the set. Fix $w_1, w_2, ., w_{i-1}$ such that, $(i) w_j \geq w_{j+1} + s$ for every $j < i, i.e.$, cutoff costs are monotonically decreasing, (ii) $c < \overline{c}_{i-1}(0)$,i.e., every $j < i$ is a net contributor for any $0 \leq \delta \leq s$, Note that there exists $\overline{w}_i > 0$ such that $\overline{c}_i(\overline{w}_i\ , s) = c$, or, equivalently $\overline{w}_i - s - \overline{\phi}(\sum_{j=1}^{i-1}(w_j - \overline{w}_i) - s) = 0$.This follows from $\overline{c}_i(w_{i-1}\ , s) > c$, $\frac{\partial \overline{c}_i(w\ ,s)}{dw} = 1 + \overline{\phi}'(i-1)$. On the other hand, let $\underline{w}_i > 0$ solves $\overline{c}(\underline{w}_i, \frac{s}{(i-1)}) = c$.That is,

$\underline{w}_i - s - \overline{\phi}(\sum_{j=1}^{i-1}\left[(w_j - \underline{w}_i) - \frac{i}{2}s\right]) > 0$.Since $\overline{c}_i(.)$ is increasing in i,it follows that $\overline{w}_i > \underline{w}_i$. Pick any $\overline{w}_i \geq w_i > \underline{w}_i$. Then:

$$\overline{c}_i(0) < \overline{c}_i(s) \leq s < \overline{c}_i(\frac{s}{i-1}) \tag{11}$$

Thus, given that every $j < i$ is in F^0 for any δ,by (ii) and from (11) it follows that $F^o(0) = F^o(s) \subset F^o(\frac{s}{i-1})$.

By this result and Lemma A6, it follows then that excessive fund-raising is potentially non- monotonic in δ. ■

Proof of Lemma 2. A particular example works. Consider $N = \{1, 2, 3\}$. Suppose

$w_1 > w_2 + c$ and $w_2 > w_3$ $+c$.Let $\overline{w}_3$ solves $\overline{c}(\overline{w}_3\ , c - c_l) = c$. That is,

$$\overline{w}_3 = \overline{\phi}(\sum_{j=1}^{2}\left[(w_j - w_3) - (c - c_l)\right]).$$

Let $\underline{w}_3$ solves $\overline{c}(\underline{w}_3, \frac{c-c_l}{2}) = c$. That is,

$$\underline{w}_3 = \overline{\phi}(\sum_{j=1}^{2}\left[(w_j - \underline{w}_i) - \frac{3}{2}(c - c_l)\right])$$

Check that w_1 and w_2 are big enough such that $\underline{w}_3 > 0$. Pick any $\overline{w}_3 \geq w_3 > \underline{w}_3$. Then, $\overline{c}_3(w_3, 0) < \overline{c}_3(w_3, c - c_l) \leq c < \overline{c}_3(w_3, \frac{c-c_l}{2})$. Now, let δ^* solves $\overline{c}_3(w_3, \delta^*) = c$. Let $\delta_h = \delta^* + \epsilon, \delta_l = \delta^*$. So, $F^o(\delta = \delta_h) = \{1, 2\}$ and $F^o(\delta = \delta_l) = \{1, 2, 3\}$. Then, $c(2, \delta_h) - c(3, \delta_l) = \delta_h - 2\epsilon > 0$ since $\delta_h > \frac{c-c_l}{2}$. ■

Proof of Proposition 9. In NY(2012) it is proven that without learning, either all members of a given group are solicited or neither of them are. Once we introduce learning, this result is reinforced in the sense that being j a member of group i, then $E[\widetilde{w}_i] - s(j) < E[\widetilde{w}_i] - s(j + 1)$.

Thus, the cutoff cost of individual $j + 1$ is higher than the cutoff cost of individual j.By following the corollary of proposition 2, it, then, also follows that either all members of a given group are solicited or neither of them are. Therefore, we can redistribute income among members of group i such that each of them is allocated with mean income $E[\widetilde{w}_i] - a_{s_i}$. As in the proof of proposition 2, such a redistribution is neutral. Thus, the result follows by applying proposition A1 in NY (2012). ■

Proof of Lemma 3. Notice that if group ij is not solicited, then no additional learning is brought is generated by the fund-raiser strategy. On the other hand, since group

i was solicited before the merger, a revealed preference argument shows that a strictly lower public good provision is expected after the merger. ■

Proof of Proposition 10. Notice that under decreasing returns to scale, the cost function is non-decreasing. Therefore, $\{\widehat{w}_i\}$ is a non-increasing sequence. From (2) the result follows. ■

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