Marriage, Fertility and Step-Families: An Equilibrium Analysis[*]
PRELIMINARY!

John Kennes and John Knowles[†]

May 2010

Abstract

We develop a simple competitive equilibrium model of the marriage market with single mothers and step-families. We then explore the role of trends in relative wages and other observed changes, such as women's wages and contraception technology, in accounting for the evolution of the distribution of family types since 1945.

1   Introduction

"But step-families are different. The stepparent has shopped for a spouse, not a child; the child is a cost that comes as part of the deal...step-parenthood is the strongest risk factor for child abuse ever identified".

–Steven Pinker, How the Mind Works, 1997

Consider the cluster of dramatic and well-known changes in family behavior observed in the US, the UK and similar countries over the last 50 years, such as the decline in aggregate fertility, the fall in marriage rates, the rise in unmarried fertility, and the move of married women into paid employment. A useful measure of the importance of this family reshuffling is that, according to the U.S. Census Bureau, 30% of U.S. children are not living with both parents; 26% live with their mother but not the father, and 4% with the father and not their mother.[1]

Because of the size and duration of these changes, it would seem essential for any explanation to consider the interaction between men's and women's optimal marriage decisions. However while the economic literature has developed theoretic tools for the analysis of both marriage and fertility separately, there has been little work on the equilibrium implications of interaction between the two.

This paper explores a particularly simple example of such interaction; the idea that optimal marriage choices by men discourage unmarried fertility but are themselves responses to unmarried fertility. The idea is not new, however while previous models of the marriage market tend to either ignore the presence of children or limit the analysis to one or two periods, the current paper allows for women to accumulate many children with different fathers over many years, and allows men to respond optimally to the distribution of women over children. To model this type of interaction requires a theory in which people have many matching opportunities over the lifecycle,

---

[1]Source: U.S. Census Bureau, Current Population Survey, 2009 Annual Social and Economic Supplement. The US Census for 2000 reports that only 6 percent of children lived in unmarried-partner households. Hence the simple partition of parents into married or single, while less accurate than it has been in the past, is still a reasonable first pass.

where children from previous periods are state variables today and into the future, and where choices governing fertility are taken both inside and outside the match.

The premise of the model deveoped here is that women are better off when fertility begins after marriage. Women are assumed to choose the probability of having a child each period. Because it may take a long time to get married, it may not be optimal for women to exert much effort to prevent child-bearing before marriage. Single men prefer women without children, but competition for wives reduces both the probability of marriage and the husband's share of the surplus, so some may choose to court single mothers . Women without children therefore attract more suitors, marry at a higher rate, and get more surplus within marriage. They face therefore a disincentive for unmarried fertility whose strength is increasing in the gains from marriage.[2]

The expected gains from marriage are themselves increasing in the anticipated fertility of a married couple. Any systemic shock therefore that reduces the optimal fertility of a married couple, relative to that of single women, will shift the equilibrium towards lower marriage rates and higher extra-marital fertility. Examples of such shocks might include the rise of women's wages relative to men's, the advent of more effective contraception, rising transfer payments to single mothers and changes in divorce regulation.

We calibrate our model to US wage, time-use and vital-statistics data for the 1990s, and then ask what happens to marriage rates and fertility in the model when parameters that are re-set to values into to reflect exogenous changes over time. The analysis considers two stylized views of the data; the "1950's", where single women minimized fertility and single mothers did not marry, and the "1990s", where single-mother fertility is prevalent, single mothers do get married, non-mothers marry at a lower rate, and married women choose lower fertility rates. The benchmark version of the model is parametrized so that simulations of the model match averages over marital and fertility behavior for women aged 18-44 from US household data in the 1990s.

The main results are that the model does indeed generate large effects of relative

---

[2]This is consistent with the empirical results of Rosenzweig (1999), which finds that a fall in marital prospects significantly raises the chances that young US women will choose non-marital fertility.

wages on fertility and divorce.

While there is a large literature on the determinants of unmarried parenthood, very few published papers consider the impact on marriage-market equilibria posed by the choice between fertility inside and outside of marriage. Most papers that consider fertility in the context of marriage-market equilibria such as Fernández, Guner, and Knowles (2005), assume fertility within marriage only. Akerlof, Yellen, and Katz (1996) for instance, women prefer not to have children at all (i.e. pregnancy is simply a side-effect of intercourse), while in Weiss and Willis (1997), Aiyagari, Greenwood, and Guner (2000) and Chiappori and Weiss (2006), fertility is exogenous. In Akerlof, Yellen, and Katz (1996) for instance, women prefer not to have children at all, while in Weiss and Willis (1997), Aiyagari, Greenwood, and Guner (2000) and Chiappori and Weiss (2006), fertility is exogenous.

The main exceptions are Neal (2004), which examines the interaction between welfare payments and marriage-market equilibria that differ in unmarried fertility rates, Greenwood, Guner, and Knowles (2000) (GGK hereafter), which shows how marriage-market dynamics and human-capital investment perpetuate the effect of rising welfare payments on unmarried fertility, and Chiappori and Oreffice (2008), which shows how improved contraception technology raises the equilibrium price of wives by reducing the fertility risk of single women.[3] An important feature of these papers is that they model, inter alia, the impact of pre-marital fertility on the household allocations of married couples through the mechanism of marriage-market equilibrium. All of these papers suffer however from an extreme compression of the lifecycle; of the three, only GGK allows for divorce and remarriage, but even there, marriage is only allowed in two periods and divorce only in one.

In the search-and-matching literature, models with repeated matching opportunities are entirely standard, however this is typically achieved by abstracting from choices, such as investment, that permanently change the state of an agent. In the marriage-market model of ?, for instance, based on the job-search framework of Burdett and Wright (1998), agents experience an infinite succession of marriages and

---

[3]The theoretical framework underlying Greenwood, Guner, and Knowles (2000) is developed in Greenwood, Guner, and Knowles (2003). Two closely-related papers that use a similar framework are Caucutt, Guner, and Knowles (2002) and Guner and Knowles (2008).

divorces in response to changes in match quality, which is represented by an iid random variable. The matching literature has also considered the analysis of marriage markets with ex-ante heterogeneous agents, as in Burdett and Coles (1997), where agents sort into marriages on the basis of quality differences which are assumed to be permanent. Recently the literature has begun to consider matching with pre-marital investments, as in Burdett and Coles (2001), but that literature does not consider the margin between investments inside and outside the match.[4]

The analysis here combines concerns explored separately in several other unpublished papers. The closest in spirit is Regalia and Ríos-Rull (1999) , which develops a life-cycle model to analyze trends in marriage, divorce and fertility. They find that reducing the wage gender gap by 19% increases the fraction of women who are single by 59% and the fraction of single mothers by 47%. Their model is significantly richer than that developed here, particularly in terms of human capital investment and wage dispersion, but abstracts from the male side of the marriage market and hence from the marriage-market dynamics explored in the current paper.[5] Greenwood and Guner (2004) argues that technological progress in home goods reduced the economic gains from marriage, making potential matches more unstable, which both reduced marriage rates and increased divorce rates. Greenwood and Guner (2005) models the segregation of young singles into sexually promiscuous and abstinent groups in response to improvements in contraception technology. Knowles (2008) models the impact of abortion, marital instability and contraception technology on fertility, wages and occupational choice, but takes marital status as exogenous.

2 Empirical Background

Empirical evidence for the basic mechanism in the model comes from the lower marriage rates of single mothers, the lower share of the marriage output allocated to

---

[4]A related literature on matching with pre-marital investments, such as Iyigun and Walsh (2007), does not allow for the investment to take place after marriage, and so cannot account for variation over time in the timing of investments like fertility.

[5]A weakness of the current analysis is that it abstracts from wage dispersion within sexes; it is interesting therefore that one of the main conclusions of that paper is that changes in wage dispersion do little to account for the trends in marriage and fertiltiy.

single mothers when they do marry and a host of anecdotes across many cultures illustrating the tension between children and step parents (see Pinker (1999) for a summary)[6].

Beaujouan (2009) finds that remarriage rates in France are significantly lower for single mothers than for child-less women or single men with or without children. She shows however that this asymmetry between men and women is entirely accounted for by the fact that single mothers are much more likely to live with the children than are single males. Figure 1 is taken from her paper. Similarly Browning and Bonke (2006) find that having children from a previous marriage does not reduce the intra-household allocation to Danish husbands in subsequent marriages, but has a strong negative effect on the allocation for wives. Again the explanation appears likely to be co-residence of mothers with their children, although the survey lacks the variables required to test this hypothesis.

Despite such disincentives, the fraction of U.S. births accounted for by unmarried women has risen steadily, from 5% in 1945 to 40% in 2009, according to Ventura (2009). Most of the increase in births to unmarried women since the early 1980's was in births to unmarried cohabiting women; in the years 1980–84, 29 percent of out-of-wedlock births were to cohabiting couples; by 1990–94, this proportion increased to 39 percent. This would seem to suggest the futility of modeling unmarried fertility without a model of cohabitation. The fertility trend however dates back to at least 1945, long before the cohabitation trend. Furthermore, even in 2009, 60% of unmarried births are to women who are not co-habiting. This suggests that relying on cohabitation to explain unmarried fertility is not enough; if anything cohabitation is more likely to be the result of the trend in unmarried fertility, an issue to which we return below in our discussion of future research.

In the argument to follow, a key role is played by the assumption that men are relatively unenthusiastic about children, even their own. A recent empirical paper that strongly supports this view is Kohler, Behrman, and Skytthe (2005), based on a study of the reported happiness of Danish twins. Their results, while agreeing with

---

[6]Pinker: In one study of emotionally healthy middle-class families in the U.S., only half of the stepfathers and a quarter of the stepmothers claimed to have "parental feelings" toward their step-children.

Figure 1: Re-marriage rates for French sample as a function of years since separation. From Beaujouan(2009).
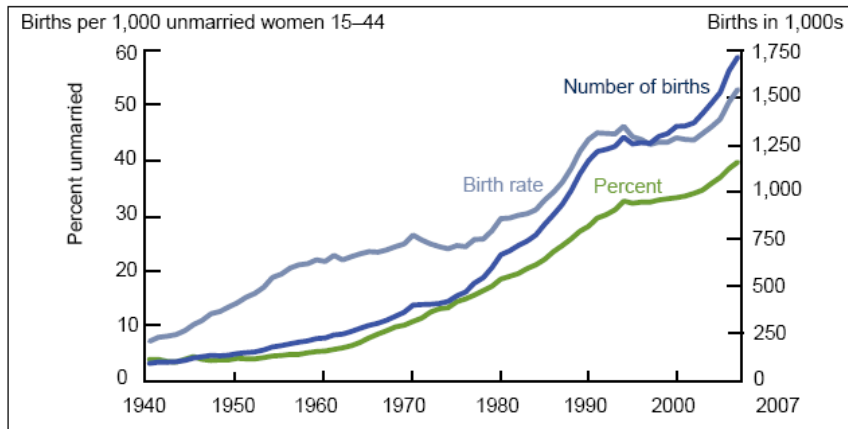
Figure 2: Trends in Non-Marital Fertility, Ventura (2009).

previous research that marriage is associated with greater happiness for both men and women, finds that this happiness bonus is associated for women with the children that marriage produces, while for married men, children strongly reduce happiness.

3   The Model

The population consists of infinitely-lived adults, with a continuum of each sex denoted by $\{M, F\}$ and mass N_M and N_F. Individuals have zero mass. Life is divided into discrete periods. Women are of sex f and may produce up to K children. Adults enjoy a consumption good c, production of which requires only inputs of adult time; a unit of women's time yields w_f units of the good, while a unit of men's time yields w_m.

There are three types of households; single males, single females, and married couples. Married adults live together as husband and wife with all the children ever born to the female spouse. Each period, married couples become permanently sterile with probability δ, exit the marriage market, and remain married to each other forever. Non-sterile couples experience random shocks q to the quality of the marriage; each gets utility flow q from remaining married. Singles also become permanently sterile with probability δ each period.

Let k be the number of kids in a married-couple household, and $k_m \leq k$ be the number of the husband's biological (own) kids. We use the indirect utility functions $\widetilde{u}_{SM}$, $\widetilde{u}_{SF}(k)$ and $\widetilde{u}_M(k, k_m)$, for, respectively, single males, single women and married-couples, to represent the maximized utility flow each period from consumption and children. The critical assumption is that children generate more utility within a marriage than without:

$$\widetilde{u}_{SF}(k+1) - \widetilde{u}_{SF}(k) < \widetilde{u}_M(k+1, k_m-1) - \widetilde{u}_M(k, k_m)$$

Utility within married couples is perfectly transferable. This means that utility of the couples can be traded off on a one-for-one basis. Assuming full-commitment,therefore, all allocations of the surplus can be efficiently achieved by maximizing the equally-weighted sum of the welfare of husband and wife.

The details of the within-period decision-making are not critical for the theory at this point, but what we have in mind here is the idea that parents get less utility from

step children than from their own children, so that an additional child within a marriage raises the father's utility more than a pre-existing child would. We also follow AGG and many other papers in assuming that children outside the household do not enter the parent's utility function. For a more explicit treatment of the household structure, the reader is referred to the calibration section below.

Only non-sterile women with fewer than K kids are fecund. Suppose that for each marital status m, the support Π of the fertility probability choice set is bounded below by π_k^{mL} and above by π_k^{mH}.[7] We assume that for each marital status, there is a "normal" fertility rate $\widehat{\pi}_k^i$ and that the cost of choosing a different rate is increasing in the distance from $\widehat{\pi}_k^m$, as given by the function $\Theta\left(\pi^{mF}|\widehat{\pi}_k^{mF}\right)$. Note that the notation allows the normal fertility, as well as the bounds, to also depend on the number of children a woman has had already, but rules out dependence on the fertility history of the husband.

3.1 Frictional assignment

Transitions between household types occur through marriage and divorce. The marriage "market" consists of all single males and females; i.e. new entrants and those who were single or became divorced last period. The number of single female marriages with k children active in the marriage market is denoted by $S_F(k)$. We also assume that while entry into the marriage market is cost-less for women, there is an entry cost $\gamma > 0$ that single men must pay.

The timing of events in each period is that first singles make marriage decisions, then all married couples (including those who married in the current period) learn the current match-quality shock, and decide whether to divorce. Then fertility decisions take place, and then fertility, utility and, finally, sterility are realized. At the time of marriage therefore the match quality in the first period of the marriage is not known.

Each period there is random matching within $K+1$ marriage markets. Single men are identical ex ante and can choose which market to enter each period, but can only enter one market per period. Within each market k, the women all have k children already. Suppose there are $S_M(k)$ men who enter marriage markets of type k. Let

---

[7]In earlier versions, to keep the analysis simple, we supposed that fertility choice is costless. This meant that the optimal choice was a corner solution: $\pi_k^{i*} \in \left\{\pi_k^{iL}, \pi_k^{iH}\right\}$.

9

$\phi_k = S_M(k)/S_F(k)$ denote the tightness of market k.

At the beginning of the period, single men (suitors) are randomly assigned to a woman within the market they have chosen. Each single woman therefore starts the period with k children and an integer number of suitors. Thus the probability that a female gets z males in her local marriage market is given by

$$\omega_z = \frac{\phi^z e^{-\phi}}{z!} \tag{1}$$

Each suitor makes a proposal that consists of an allocation of the *ex ante* surplus between the two spouses. The allocation mechanism in the market is a second-price auction. When a woman receives more than one proposal, she is allocated the entire surplus of the marriage; otherwise the surplus is allocated to her husband. The probability that the husband receives the surplus is therefore $\omega_0(\phi_k)$, the probability that he was the only entrant.

In order to allow for both large gains from marriage and low probability of marriage, we assume there is a friction in the marriage market; with probability p_z a woman cannot marry, regardless of her state or the number of suitors. This can be seen as the reduced form of a matching friction induced by some transient heterogeneity.

Should the marriage occur, the couple then learns the current value of q. Since utility is transferable, decisions within the marriage such as fertility, maximize the expected surplus, contingent on (k, k_m) and the current value of q.

3.2 Expected payoffs

The aggregate state of the stationary economy is given by the market-tightness vector ϕ, where $\phi = \{\phi_0, ., .., \phi_K\}$.

It is convenient to divide the period into the stage before and the stage after marital events. Let the values on entering the period, for men and women, respectively, be denoted $V_{SM}^E(k, \phi)$ and $V_{SF}^E(k, \phi)$. Let $Y^E(k, k_m|\phi, q_{-1})$, denote the expected value, on entering the period, of a marriage consisting of a woman with k kids of her own of which k_m are fathered with her current husband, where q_{-1} denotes the previous-period's realization of q. Let the initial value of q be $\bar{q}$, so that we can write the value of a new marriage as $Y^0(k, \phi) \equiv Y^E(k, 0|\phi, \bar{q})$.

Let $Y^R(k, k_m|\phi, q)$ be the value of the marriage after q is realized but before fertility realizations are made. Let the ex ante value of a new marriage be given by $Y^E(k, k_m|\phi, q_{-1})$. The alternative to any given marriage is to remain single for the period. Let $V_{SM}^R(\phi)$ and $V_{SF}^R(k, \phi)$ denote the continuation values as singles for men and women, respectively, at the close of the marriage market. The ex ante surplus from a new marriage in market k is

$$S^0(k, \phi) = Y^0(k, \phi) - \left(V_{SM}^R(\phi) + V_{SF}^R(k, \phi)\right)$$

Suppose that the surplus from a new marriage is declining monotonically in k [8]. As meetings are assumed to generate marriage whenever the expected surplus is positive, we can define a threshold k_ϕ such that a marriage market k operates if and only if $k \leq k_\phi$. A market that violates this condition is said to be "inactive". We assume that single men are allocated only to those markets with $k \leq k_\phi$ which we call "active markets".

3.2.1 Divorce Decisions

The marriage is assumed to end whenever the surplus plus the divorce cost C_D is negative. Define the threshold function $q^*(k, k_M)$ such that the *ex post* marriage surplus plus the divorce cost equals zero at $q = q^*(k)$. That is

$$q^*(k, k_M) = -\left(Y^R(k, k_m|\phi, q) - V_{SF}^R(k, \phi) - V_{SM}^R(\phi)\right) + C_D \tag{2}$$

. For marriages with positive systematic gains, $q^*(k)$ will be negative.

So now we can write the divorce probability arising from the optimal divorce decision rule as:

$$\pi_{k, k_m}^D(q_{-1}) = F\left(q^*(k, k_M), q_{-1}\right) \tag{3}$$

.

---

[8] With concave utility, a woman's gains from the higher income associated with marriage will increase with the number of children she already has. Since marriage rates in the data are declining in the number of previous children (for evidence from France, see Beaujouan (2010)), the surplus must be declining. To ensure this is the case in the model we allow the utility of husbands to be declining in the number of previous children the wife brings to the marriage.

3.2.2 Married Couples

Letting the married-couple's optimal divorce and fertility probabilities be π^D_{k,k_m} and π^{MF}_{k,k_m}, respectively, we have

$$Y^E(k, k_m|\phi, q_{-1}) = \pi^D_{k,k_m}(q_{-1})\left[V^R_{SM} + V^R_{SF}(k, \phi) - C_D\right] \tag{4}$$
$$+ \int_{q^*(k,k_M)} Y^R(k, k_m|\phi, q)\, f(q, q_{-1})\, dq$$

where

$$Y^R(k, k_m|\phi, q) = q + \pi^{MF}_{k,k_m}\left[u_M(k+1, k_m+1) + \widetilde{\beta}Y^E(k+1, k_m+1|\phi, q)\right] \tag{5}$$
$$+ \left(1 - \pi^{MF}_{k,k_m}\right)\left[u_M(k, k_m) + \widetilde{\beta}Y^E(k, k_m|\phi, q)\right] - \Theta\left(\pi^{MF}_{k,k_m}|\widehat{\pi}^M_k\right)$$

3.2.3 Singles

The *ex ante* net value of a man's prospects in marriage market k is given by

$$p_z\omega_0(\phi_k)\left(Y^E(k, k_m|\phi, q_{-1}) - V^R_{SF}(k, \phi) - V^R_{SM}\right) = p_z\omega_0(\phi_k)\, S(k, 0|\phi) \tag{6}$$

Recalling the definition of the value functions, we can write, respectively, the *ex ante* net value of entering marriage market k and the continuation values for single men as:

$$V^E_{SM}(k) = V^R_{SM} + p_z\omega_0(\phi_k)\, S(k, 0|\phi) - \gamma \tag{7}$$
$$V^R_{SM} = \max_k\left\{u_{SM} + \beta V^E_{SM}(k)\right\} \tag{8}$$

. Similarly for single women with k children, the *ex ante* net value of entering the marriage market is:

$$V^E_{SF}(k) = V^R_{SF}(k, \phi) + p_z\left[1 - \omega_0(\phi_k) - \omega_1(\phi_k)\right] S(k, 0|\phi) \tag{9}$$

. If π^{SF}_k is the optimal fertility probability, the continuation values for single women are:

$$V^R_{SF}(k, \phi) = \left(1 - \pi^{SF}_k\right)\left[u_{SF}(k) + \beta V^E_{SF}(k)\right] + \pi^{SF}_k\left[u_{SF}(k+1) + \beta V^E_{SF}(k+1)\right] - \Theta\left(\pi^{SF}_k|\widehat{\pi}^{SF}_k\right) \tag{10}$$

3.3 Fertility Decisions

The main issue here is that having a child changes the state of the marriage and the outside option of the single female. It may also increase the utility of the father after a divorce; for now we assume that away. The net benefit of having a child therefore depends on forecast of the divorce probability, which may depend on the current value of q, if this helps to predict future values. In this case q becomes a state variable which makes the analysis somewhat more complicated.

3.3.1 Single women

Single women with less than K kids choose fertility $\pi_k^{SF} \in \Pi$ to solve:

$$V_{SF}^R(k, \phi) = \max_{\pi_k^{SF}} \left\{ \left(1 - \pi_k^{SF}\right) \left[u_{SF}(k) + \beta V_{SF}^E(k, \phi) \right] \right. \tag{11}$$
$$\left. + \pi_k^{SF} \left[u_{SF}(k+1) + \beta V_{SF}^E(k+1, \phi) \right] - \Theta\left(\pi_k^{SF} | \widehat{\pi}_k^S \right) \right\}$$

The first-order condition for this is:

$$\Theta'\left(\pi_k^{SF} | \widehat{\pi}_k^S \right) = \Delta u_{SF}(k) + \beta(1 - \delta) \Delta V_{SF}^E(k, \phi)$$

where

$$\Delta u_{SF}(k) \equiv u_{SF}(k+1) - u_{SF}(k)$$
$$\Delta V_{SF}^E(k, \phi) \equiv V_{SF}^E(k+1, \phi) - V_{SF}^E(k, \phi)$$

.

3.3.2 Married

Married couples choose fertility to maximize the joint returns to the family. The married couple with $k < K$ kids chooses π^f to solve

$$Y^R(k, k_m | \phi, q) = q + \max_{\pi_{k,k_m}^{MF} \in \Pi} \left\{ \pi_{k,k_m}^{MF} \left[u_M(k+1, k_m+1) + \beta Y^E(k+1, k_m+1 | \phi, q) \right] \right. \tag{12}$$
$$\left. + \left(1 - \pi_{k,k_m}^{MF}\right) \left[u_M(k, k_m) + \beta Y^E(k, k_m | \phi, q) \right] - \Theta\left(\pi_{k,k_m}^{MF} | \widehat{\pi}_k^M \right) \right\}$$

The first-order condition for this is:

$$\Theta'\left(\pi_{k,k_m}^{MF} | \widehat{\pi}_k^M \right) = \Delta u_M(k, k_m) + \beta \left[\Delta Y^E(k, k_m | \phi, q) \right]$$

where

$$\Delta u_M (k, k_m) = u_M (k+1, k_m+1) - u_M (k, k_m)$$

$$\Delta Y^E (k, k_m|\phi, q) \equiv Y^E (k+1, k_m+1|\phi, q) - Y^E (k, k_m|\phi, q)$$

3.3.3 Market-Clearing: Determination of ϕ_k

In much of the directed-search literature it is assumed that there is an excess supply of potential entrants, so the markets satisfy a free-entry condition, i.e. that men are indifferent between entering a given market and not participating at all. This means the expected gains $V_{SM}^E (k) - V_{SM}^R$ equal the entry cost γ.

The alternative is to assume that all (fecund) singles are active in the marriage market; this allows for expected gains to exceed the entry cost. In this case there are two equilibrium conditions: expected gains are equalized across all active markets, and demand for single men equals the supply of single men.

The current model has two types of equilibria; one where the free-entry condition binds and single men are in excess supply, and one where the supply and demand of single men are equalized.

Let $\mathcal{M}$ be the set of active marriage markets of type k. The free-entry condition is that for all active marriage markets of type k, the value of entering is at least as great as the value of staying out:

$$V_{SM}^E (k, \phi) - \gamma \geq V_{SM}^R (\phi) \quad \forall k \in \mathcal{M} \tag{13}$$

Suppose that we know the value of V_{SM}^R. Since this is a sufficient statistic for conditions in the other markets, we can write the surplus as $S \left(k, 0 | \phi_k, V_{SM}^R \right)$. By definition,

$$V_{SM}^E (k) = V_{SM}^R + p_z \omega_0 (\phi_k) S \left(k, 0 | \phi_k, V_{SM}^R \right)$$

The equilibrium therefore requires, when the free-entry condition binds, that

$$V_{SM}^R + p_z \omega_0 (\phi_k) S (k, 0|\phi) - \gamma = V_{SM}^R$$

or

$$\omega_0 (\phi_k) = \gamma / (p_z S (k, 0|\phi)) \tag{14}$$

14

. [9] There is however no guarantee that the free-entry condition is solved by a positive ϕ_k in every market. In those markets where the solution would require $\phi_k < 0$, the non-negativity constraint binds, so in equilibrium these markets do not operate because men prefer to enter another market or to remain single.

Since men are indifferent in this equilibrium, between entering the marriage market and staying out, the value of staying out equals the autarky value:

$$V_{SM}^R = \frac{u_{SM}}{\beta (1 - \delta)}$$

In principle, the above relationship pins down ϕ_k, given the value V_{SM}^R. The random assignment rule implies that $\omega_0 (\phi_k) = e^{-\phi_k}$, so free entry implies $\phi_k = -\log (\omega_0) = \log (\gamma / (z S (k, 0|\phi)))$.

. Now suppose that single men strictly prefer entry into active marriage markets. Another way to think of this is that there is excess demand for husbands; the supply constraint binds. This constraint is

$$\sum_k S_M (k) \leq S_M \qquad (15)$$

using the definition of market tightness: $S_M^D \left(V_{SM}^R \right)$

$$\phi_k = \frac{S_M (k)}{S_F (k)}$$

the binding of this supply constraint can be written as:

$$\sum_{k \in \mathcal{M}} \phi_k S_F (k) = \sum_{k \in \mathcal{M}} S_M (k) = S_M$$

. Since the market-clearing implies that ϕ_k is decreasing in V_{SM}^R, then it is easy to solve for equilibrium by increasing V_{SM}^R from the autarky level until this constraint holds with equality

3.4 Equilibrium

We summarize the model with a formal definition of the stationary equilibrium of the directed-search marriage market.

---

[9]So if there are no entry costs, then $\omega_0 (\phi_k) = 0$. Since this would require $\phi_k = \infty$, that means that the supply constraint would bind.

Definition 1 *A stationary equilibrium of the directed-search marriage market with a maximum kids K consists of the following objects: a list of decision rules for fertility $\left\{\pi_k^{SF}, \{\pi_{k,k_m}^{MF}\}_{k_m=0}^{k}\right\}_{k=0}^{K-1}$, and divorce $\left\{\{\{q^*(k,k_m)\}\}_{k_m=0}^{k}\right\}_{k=0}^{K-1}$, a list of ex-ante value functions $\left\{V_{SF}^E(k), V_{SM}^E(k)\right\}_{k=0}^{K}$ for singles and $\left\{\ \{Y^E(k,k_m,q)\}_{k_m=0}^{k}\right\}_{k=0}^{K}$ for married, a list of ex-post value functions $\left\{V_{SF}^R(k), V_{SM}^R(k)\right\}_{k=0}^{K}$ for singles and $\left\{\ \{Y^R(k,k_m,q)\}_{k_m=0}^{k}\right\}_{k=}$ for married, a list of distributions $\left\{S_F(k), \{F(k,k_m)\}_{k_m=0}^{k}\right\}_{k=0}^{K}$, a rule $\{\phi_k\}_{k=0}^{K}$ for assigning men to markets , a rule $\{R_F(z), R_M(z)\}$ for assigning the surplus as a function of the number of suitors z, and a law of motion $\left\{T_S(k), \{T_M(k,k_m,q)\}_{k_m=0}^{k}\right\}_{k=0}^{K}$ for the distributions. This list must satisfy the following conditions:*

1. *Optimality:*

   (a) *The fertility decision rules are solutions to the individual optimization problems (12) and (11), given the value functions.*

   (b) *the divorce thresholds $\left\{\{\{q^*(k,k_m)\}\}_{k_m=0}^{k}\right\}_{k=0}^{K-1}$ set the marriage surplus to zero.*

   (c) *For each k, the value functions solve the the system of ex ante asset equations (??),(7), (4) and ex post equations (10),(8), and (??), (4) ,given the laws of motion for the state variables k,k_m,q and marital status and the surplus allocation rules .*

2. *Market-clearing: the market tightness vector $\{\phi_k\}_{k=0}^{K}$ satisfies these conditions:*

   (a) *Feasibility: the supply constraint (15) is satisfied.*

   (b) *Indifference: $V_{SF}^E(k) - \gamma = V_{SM}^R$ over all markets where $\phi_k > 0$*

   (c) *Free entry: if the supply constraint (15) does not bind, then $V_{SM}^R = V_{SM}^A$,the value of autarky for single males.*

   (d) *Allocation: the surplus allocation rules $\{R_M(k), R_F(k)\}$ are consistent with the outcomes of a second-price auction within each market k.*

3. *Aggregation: The laws of motion of the distributions satisfy*

(a) *Consistency with individual decisions: Equations (??),(??), (??), (??) and (??) are satisfied*

(b) *Stationarity: The distributions are the fixed points of their laws of motion.*

.

4 Solving the Model

4.1 Asset Equations

The main benefit of the model's structure is recursivity: since the decision rules depend on the distributions only through the value of V_{SM}^R we can solve the asset equations contingent on a conjecture of V_{SM}^R and iterate on this conjecture until convergence. Furthermore, using backwards induction from $k = K$, we can solve each level of k separately.

Suppose the shock q has a n_q-point support and that marriage market k is active. Lets assume that we know the value functions for $k + 1$, the fertility and divorce decisions for $\{(k, q_i)\}_{i=1}^{n_q}$ and the *ex post* value V_{SM}^R of being a single male. A very convenient feature of the model is that these assumptions allow us to write the asset equations relevant to the marriage market for women with k children as the following linear system:

$$
\begin{bmatrix}
V_{SF}^E (k) \\
Y^E (k, 0, q_1) \\
... \\
Y^E (k, 0, q_{n_q})
\end{bmatrix}
= A
\begin{bmatrix}
V_{SF}^E (k) \\
Y^E (k, 0, q_1) \\
... \\
Y^E (k, 0, q_{n_q})
\end{bmatrix}
+
\begin{bmatrix}
d_0 \\
d_1 \\
... \\
d_{n_q}
\end{bmatrix}
$$

The elements of A are derived in the appendix. Although the coefficients get quite complicated, the computation itself is very straightforward. Since the system is based on conjectures about the fertility and divorce decisions, these have to be tested and the step repeated with new conjectures if the current conjectures for women with k children are not verified. The important point is that, given the system has already been solved for $k + 1$ and higher, this iterative step is carried out only at level k, rather then the entire system of asset equations.

4.2 Distributions

Supposing that we have solved the asset equations based on a conjectured value of V_{SM}^R. Using the marriage and fertility decision rules, we can compute the steady-state distributions of the household types. The strategy is to first solve for the stationary distribution of households with zero kids, then use the results to solve for $k = 1$, then $k = 2$, and so on up to $k = K$.

Recall that the fertility rule is π_k^{SF} for single women and $\pi_{k,k_m}^{MF}(q)$ for married, and that the flow rate of people into and out of the population is δ. We assume that newly married have the same distribution of q as ongoing couples who had realization q_1 last period.

Let the probability that a woman of type k marries be $\mu_k = 1 - e^{-\phi_k}$. The probability that a woman who is single today is single next period is $(1 - \mu_k) + \mu_k \pi_k^D$, while for a married woman, the probability is simply π_k^D. Note that as divorce takes place before fertility decisions, all women flowing into the singles state next period must have chosen the single fertility rate this period, even though some flow in from (a short-lived) marriage.

Let the next-period mass of the singles and married at each state be given by $S_F'(k)$ and $H'(k, k_m, q)$, respectively. It is easy to show that, when q follows a Markov chain with n_q values, we can write the law of motion of the distribution of singles and marriages without husband's children as a collection of linear systems that can be solved sequentially for the stationary distributions:

$$
\begin{bmatrix} S_F'(k) \\ H'(k, 0, q_1) \\ \dots \\ H'(k, 0, q_{n_q}) \end{bmatrix} = B \begin{bmatrix} S_F(k) \\ H(k, 0, q_1) \\ \dots \\ H(k, 0, q_{n_q}) \end{bmatrix} + \begin{bmatrix} d_1^k \\ 0 \\ \dots \\ 0 \end{bmatrix}
$$

The elements of B are derived in the appendix. This linear system is easily solved for the stationary values $S_F^*(k)$ and $H^*(k, 0, q)$. For any $k > 0$, it is easy to solve for the stationary distributions of married couples with $k_m > 0$, once the masses $S_F^*(k-1)$ and $\{H^*(k-1, 0, q_i)\}_{i=1}^{n_q}$ are known.

5   Calibration

The purpose of calibrating the model is to assess the ability of the theory to account for current patterns of average fertility, marriage rates and the distribution of family types, and to assess the contributions of various types of shocks to the differences in these patterns between the 1990s and the 1950s. We can also use the calibrated model to measure how important are marriage-market prospects for single fertility; the difficulty of doing this econometrically is succinctly described in Rosenzweig (1999). Finally, we expect divergences between the model results and the empirical statistics to help us identify new research questions that the model raises but does not resolve.

5.1   Simulation

As is standard in the macro literature, model statistics are produced by simulation of the model's stationary steady state at a given set of parameter values.   The population size is set to $N = 1000$ for each sex, and the simulation follows each woman for 27 years, to correspond to the 18-44 age group that is usually considered by statistical reports on fertility. The initial conditions at entry are marital status and number of children. These are set to match the average for 18 year olds in 1996-2007, according to the March CPS; 10% of women are married, and x% have children. For each woman, the realizations of the stochastic processes governing marriage, fertility, divorce are given by 27 iid draws of a uniform random variable of dimension 5. The aggregate statistics are then computed by pooling the observations over the entire population, over all ages.

While age is not a state variable in the model, it is possible of course to track age in the simulation, and so the lifecycle profiles over this age interval can be traced and compared to the analogous profiles in the data. This is discussed below in the section on further research; for the current work, the simulation targets will be aggregates over the age intervals.

5.2   Within-household structure

We take a period to be one year and normalize the time endowment to 1 unit of time per person per year. Each household takes $\theta_H < 1$ units of time per period to

maintain. Adults consume goods, which in turn require time to produce. A unit of women's time produces w_f units of consumption good, a unit of men's time w_m. Raising children requires mother's time; the cost $\Theta_i(k)$ is increasing in the number of kids, but bounded below $1 - \theta_H$. The function is indexed by household type, as the time requirements may differ if a husband is present. In addition to the required time, there is a discretionary component, which we call investment, $I \in \{0, 1\}$. Choosing $I = 1$ costs θ_I units per child of the mother's time. Investment is not divisible among the children; each child in a household gets the same amount.

Utility is linear in leisure; husbands do not contribute their time to home production, child care, or investment because they have higher productivity $w_m > w_f$. The utility function is concave in consumption and concave in own children, with parameters that vary only by sex i, not by marital status. Letting the number of step-children be k_o and disutility per step child be χ_i, we can write this as:

$$u_i(c, k, I, k_o) = \begin{cases} \log c + \alpha_i \log(k(1 + \gamma_i)I) - \chi_i k_o & k > 0 \\ \log c + \alpha_{0i} - \chi_i k_o & k = 0 \end{cases} \quad (16)$$

where c is given by the budget constraint:

Single men produce $c_{SM} = w_m(1 - \theta_H)$ and single women with k children produce

$$c_{SF}(k) = w_f(1 - \theta_H - k(\kappa_{SF} + I_{SF}(k)\theta_I - \tau_k))$$

.Husbands produce $w_m(1 - k\kappa_M)$ and wives $w_f(1 - \theta_H - k(\kappa_{MF} + I_M(k)\theta_I))$, so that the budget constraint of a married couple is

$$c_{MF} + c_{MM} = w_m(1 - k\kappa_M) + w_f(1 - \theta_H - k(\kappa_{MF} + I_M(k)\theta_I))$$

.

The flow utility received by each household member therefore varies according to the number of children (k, k_m) and the marital status. We assume the utility function is linear with parameters that vary only by sex i:

$$u_i(c, k, I) = c + (\alpha_i + I\gamma_i)k \quad (17)$$

For single males, flow utility is

$$u_{SM} = w_m(1 - \theta_H)$$

.For single woman households, the state is k , so the total utility flow is:

$$u_{SF}(k) = c_{SF}(k) + \left(\alpha_f + I_{sf}(k)\gamma_f\right)k \tag{18}$$

$$= w_f(1 - \theta_H) + \left(\alpha_f + I_{sf}(k)\left(\gamma_f - \theta_I w_f\right) - \kappa_{sf} + \tau\right)k \tag{19}$$

.For married households the state is (k, k_m) . The consumption of each spouse (c_{MF}, c_{MM}) is determined in equilibrium. The total utility flow is

$$y_M(k, k_m) = u_{MF}(c_{MF}, k, I_M) + u_{MM}(c_m, k, I_M) \tag{20}$$

$$= q + c_{MF} + \left(\alpha_f + I_M(k, k_m)\gamma_f\right)k + c_{MM} + (\alpha_m + I_M(k, k_m)\gamma_m)k_m \tag{21}$$

where q is exogenous marriage quality.

Two important features of these assumptions are that married men will tend to spend more time than singles producing the consumption good, and that married mothers will tend to spend less time in production than single mothers. It is easy to see that when there are no step children, married couples gain more than single females from making the investment; the single female gets $\gamma_f k$, while the married household gets $(\gamma_m + \gamma_f)k$. Whenever $w_f/\theta_I \in (\gamma_f, \gamma_m + \gamma_f)$ then single females will not make the investment and married couples will. Since married women are assumed to do all the housework θ_H, this means that married mothers will necessarily work fewer hours producing the consumption good than single mothers. Singles will also work fewer hours than married men, as the singles have to the housework themselves.

5.3 Fertility Probability and Effort

For non-sterile women, the probability that a child will arrive next period is assumed to be a declining function of contraceptive effort, which is modeled as a utility cost $\Theta\left(\pi_i^F\right)$ to the household. Therefore those who prefer to have a child will exert zero effort. Let the fertility probability at zero effort, for a woman of marital status i be $\hat{\pi}_i$. For fertility-cost parameter $\zeta > 0$, the effort-probability frontier is given by:

$$\Theta\left(\pi_i^F\right) = \sqrt{\frac{\zeta}{\max\left(1e - 5, \pi_i^F\right)}} - \sqrt{\frac{\zeta}{\hat{\pi}}}$$

5.4 Parameters set *a priori*

Most of the parameters can be set independently of the marriage-market equilibrium. This part of the calibration relies mainly on statistics from government publications and other papers. The probability δ of exiting the reproductive state, as in Regalia and Ríos-Rull (1999), is set so as to replicate the average number of years a woman spends in the reproductive state. We compute this by summing the fraction of women who are fecund at each age between 16 and 44, as estimated by Trussell and Wilson (1985).[10] This results in a total of 20.45 fecund years per woman, so we set $\delta = 0.0489$.

Wages are set to the medians for each sex from the 1995 CPS for the age group 25-45. For men the median hourly wage is $10; so we set $w_m = 10$. For women, the median hourly wage is $8.17, so we set $w_f = 8.31$.[11] Wages at younger ages would not be informative about the cost of time, as younger people are likely to be in school or provisional jobs. We set $\beta = 0.96$, the standard value in the macroeconomics literature; in models with savings, this value ensures that the risk-free interest matches the US long-run average of 0.04. We set the exogenous part of the divorce probability to $\pi_d = 0.01$ to match the average divorce rate for US women aged 20-44 in 1960, as reported in Carter and Glick (1970).

The time-allocation parameters are set to match the time remaining after home production and child care in the model to weekly working hours according to the 1990 US Census , for people aged 25-45 years, by sex, marital status, and in the case of women, number of children. This also requires us to set the investment parameters so that single women do not invest but married women do, at least when the children are all from the husband. The match is not exact but rather is given by the parameters which yield the minimum Euclidean distance between the empirical targets and the model outputs. The comparison between the model and hours data is shown in Table X.

These fixed-parameter values are shown in Table Y.

---

[10] The numbers we use are based on the interpolated series reported in Sommer (2008).

[11] Might be better simply to take numbers from Blau and Khan for FTFY workers.

5.5   Free Parameters

The model has 9 free parameters that are set so that the model's stationary distribution matches an equal number of statistical targets. In addition, a number of other free parameters are normalized to a fixed value, as these are not identified by the targets. For instance, variation in p_z,which is the probability that marriage is permitted in a local market, is roughly equivalent to adjusting other parameters that affect marriage probabilities via preferences, such as the utility level of singles relative to married. can , and the preference parameters. We set these parameters so as to ensure that the calibrated model met the conditions that are required for the model to match the marital and fertility patterns of the 1990s. The value of the low-quality love shock q_L is set to $-q_H$.These values are show in Table Z.

6   Results

The results presented here are entirely preliminary!

The analysis in this section is divided into two parts: a numerical example with $K = 1$, and the benchmark model with $K = 5$. The purpose of the numerical example is to illustrate how the model responds as candidates for the exogenous shocks are allowed to vary across an interval that includes the parameter values for both the 1950s and the 1990s. The purpose of the benchmark model is to provide a tighter fit to data that can be used to assess the proposed explanations of the marital transformation between these two periods.

6.1   Comparative Statics: The K=1 Model

The K=1 parametrization is the result of setting the free parameters in Table 3(a) so that moments from the steady state distribution of the model approximate the empirical moments in Table 3(b). The main restriction of course is that women can have at most one child, so this imposes a significant handicap on the model in terms of fitting data. However the benefit is that the comparative statics are relatively easy to understand.

Fig. 3 below shows the response of the $K = 1$ model to a rise in the women's wage
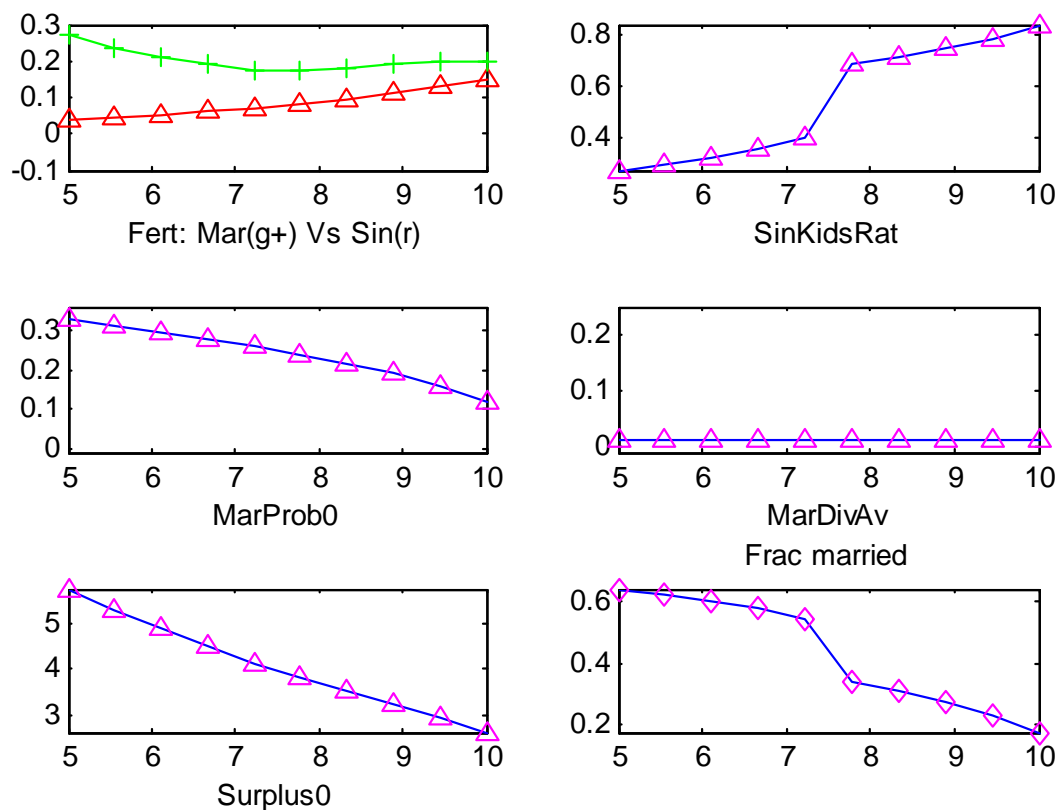
Figure 3: The response of the benchmark model to a rise in the female wage for 5 to 10 per year.

from 5 to 10, holding constant all the other parameters of the model. We see in the first panel that this leads to contrary trends in fertility; that of single women rises, while that of married declines. The result, shown in the panel to the right, is that the share of single mothers in fertility rises throughout, from about 30% to 80%. As in the US data, this is partly due to the rise in unmarried fertility relative to married, but also, as shown in the second row, to the decline in marriage rates as female wages rise, which is reflected in a decline in the fraction of women married as shown in the last panel, and is driven by the decline of the marital surplus as single women's utility rises with the wage. It is not clear why the divorce rate("MarDivAv") is not trending at the same time.
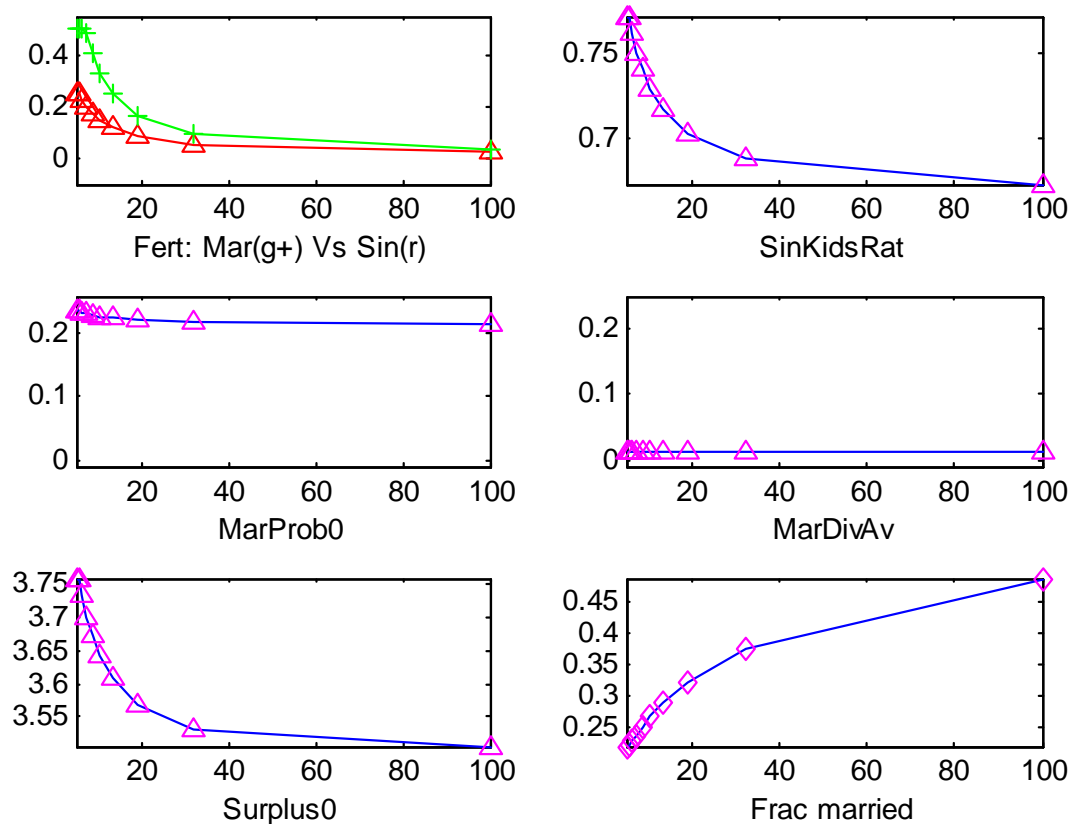
Figure 4: The response of the benchmark model to a rise in the effectiveness of contraception.

To explore the impact of other historical changes, we next consider a rise in the effectiveness of birth control, as illustrated in Figure 4. Effectiveness is defined here as $1/\xi$, the parameter in the effort-probability frontier. The effects on married and single fertility are very similar, though this time they do converge as effectiveness increases. The fertility share of single moms (SinKidsRat) falls, contrary to the data, and the fraction married increases.

Now suppose that it is lump-sum transfers to single mothers that increase over time. Figure 5 shows fertility increases for both married and single women, at roughly the same rate. The fertility share of single moms (SinKidsRat) increases but by much less than in the wage case; the range of variation is 0.08 compared to 0.45 in the wage
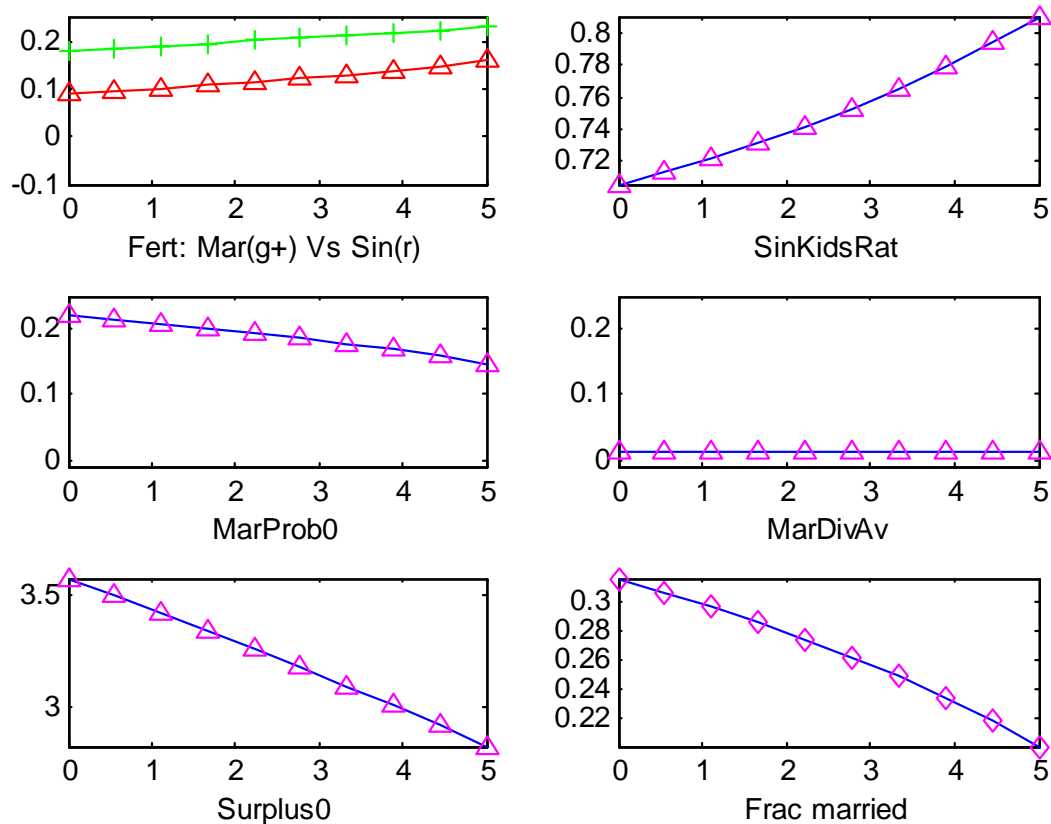
Figure 5: The response of the benchmark model to a risein lump-sum transfers to single mothers, from 0 to 5 per year.

experiment.

Overall the results indicate that the mechanism that motivated the paper, the marriage-market disincentive for extra-marital fertility, does indeed respond to wage trends as hypothesized. This was not obviously true because the mechanism requires both that marriage be rewarding for women and that the effect of premarital fertility undo these rewards. The key is that men in the model must care for marriage only as a route to consumption of goods and q; the parameters in Table 1(b) imply that they do not care for children, particularly those fathered by other men. Women like marriage, for consumption and for the opportunity it provides for additional investment in children, benefits that weaken as women's wages rise, driving down

married fertility and raising consumption for single mothers.

6.2 Computational Measurements: the K=5 Model

The main role for the K=5 experiment is to prepare the ground for a quantitative assessment of the effects of women's wage, relative to men's, as the K=1 experiment showed that the other candidates were unlikely to provide consistent explanations of the rise in the fertility share of single moms. The first column of Table 4(a) shows the statistical targets from the data, while the second shows the corresponding result for the benchmark model. The fit is not terribly good at this point, mainly because there has been little time to calibrate. We expect the fit to improve dramatically as we learn how to work with the model.

The main result so far is the effect of imposing on the model a wage of 6.1, corresponding to women's wage averaging 61% of men's as was the case for FTFY workers in US data for the early 1970s (see Blau and Kahn (1997)). The effects, shown in the (1950s) column of Table 4(a), are dramatic. The fraction of women who are divorced falls more 50%, from 8.3% to 3.9%, while the fraction of births to unmarried women falls from 34% to 26%. This is due to a rise in the married birth rate, from 8 % pa to 8.7%, and a decline in mean birth rate of singles, from 4.3% to 3.9%, and a rise in the median lifespan of a marriage, from 5.5 years to 7.6, as shown in part(b) of the table. Marriage rates however do not rise, remaining stable around 5.5% pa, which is unexpected and contrary to the data; further work is needed to ascertain the causes of this result.

Part (b) of Table 4 shows other (non-targetted) aggregate statistics for these parameterizations. The fraction of children living with their fathers rises in the model from 58% to 66% when the women's wage falls, and the fraction of marriages lasting at least 5 years rises from 80% to 90%.

The parameter values required to obtain these results are shown in Table 4(c). Once again the salient feature is the men's lack of enthusiasm for children, even their own. Being childless raises a man's utility by 0.4 and utility by -.2 per log of children after the first. The aversion per step child is 0.78 which is large relative to the other utility numbers. The model also requires strong persistence of the marriage quality; a high shock today is followed next period by a high shock with probability 97%.,

while for a low shock, the probability of a high shock next period falls to 57%.

Overall the results indicate large responses in the historically-correct directions for most variables of immediate interest, but it is not yet clear whether the model can explain much or any of the shift in marriage rates.

7 Conclusions

Our quantitative results are not meant to be definitive but rather should be taken as illustrations of the usefulness of our approach. The contribution of the current paper is to allow the theory of family structure to account for marriage-market dynamics associated with repeated opportunities to remarry and to have children; to get there we abstracted from important features explored in related papers, such as aging, human-capital investment in children or the impact of means-tested government transfers. There are also important features of marriage, such as the margin between cohabitation and marriage, that are ignored by both the current paper and the bulk of the related literature[12]. However it is easy to see that the approach used here can be extended to deal with these and other features of marriage and fertility.

---

[12] As a first pass, this neglect is not entirely unjustified, as cohabitation for many appears to be a form of extended courtship rather than a substitute for marriage. Spain and Bianchi (1996, p. 49) state that the majority of marriages formed since 1985 began as cohabitation. Overall, they say, cohabitation accounts for 6% of US households.

References

AIYAGARI, S. R., J. GREENWOOD, AND N. GUNER (2000): "On the State of the Union," *Journal of Political Economy*, 108(2), 213–244.

AKERLOF, G. A., J. L. YELLEN, AND M. L. KATZ (1996): "An Analysis of Out-of-Wedlock Childbearing in the United States," *The Quarterly Journal of Economics*, 111(2), 277–317.

BEAUJOUAN, E. (2009): "Se remettre en couple: contrastes hommes-femmes.(Repartnering: contrasts between men and women)," in *Entre famille et travail, des arrangements de couples aux pratiques des employeurs*, ed. by R.-L. Arnaud. Ined-la Découverte, Paris.

BLAU, F. D., AND L. M. KAHN (1997): "Swimming Upstream: Trends in the Gender-Wage Differential in the 1980s," *Journal of Labor Economics*, 15(1, pt. 12), 1–42.

BROWNING, M., AND J. BONKE (2006): "Allocation within the household: direct survey evidence," Economics Series Working Papers 286, University of Oxford, Department of Economics.

BURDETT, K., AND M. G. COLES (1997): "Marriage and Class," *The Quarterly Journal of Economics*, 112(1), 141–68.

——— (2001): "Transplants and Implants: The Economics of Self-Improvement," *International Economic Review*, 42(3), 597–616.

BURDETT, K., AND R. WRIGHT (1998): "Two-Sided Search with Nontransferable Utility," *Review of Economic Dynamics*, 1(1), 220–245.

CARTER, H., AND P. C. GLICK (1970): *Marriage and divorce: a social and economic study [by] Hugh Carter and Paul C. Glick*. Harvard University Press, Cambridge, Mass.,.

CAUCUTT, E. M., N. GUNER, AND J. KNOWLES (2002): "Why Do Women Wait? Matching, Wage Inequality, and the Incentives to Fertility Delay," *Review of Economic Dynamics*, 5(4), 815–855.

CHIAPPORI, P.-A., AND S. OREFFICE (2008): "Birth Control and Female Empowerment: An Equilibrium Analysis," *Journal of Political Economy*, 116, 113–140.

CHIAPPORI, P.-A., AND Y. WEISS (2006): "Divorce, Remarriage, and Welfare: A General Equilibrium Approach," *Journal of the European Economic Association*, 4(2-3), 415–426.

FERNÁNDEZ, R., N. GUNER, AND J. KNOWLES (2005): "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," *The Quarterly Journal of Economics*, 120(1), 273–344.

GREENWOOD, J., AND N. GUNER (2004): "Marriage and Divorce Since World War II: Analyzing the Role of Technological Progress on the Formation of Households," Discussion paper, NBER.

———— (2005): "Social Change," Economie d'Avant Garde Research Reports 9, Economie d'Avant Garde.

GREENWOOD, J., N. GUNER, AND J. KNOWLES (2000): "Women on Welfare," *American Economic Review*, 90(Papers and Proceedings).

———— (2003): "More on Marriage, Fertility and the Distribution of Income," *International Economic Review*, 44(3), 827–862.

GUNER, N., AND J. KNOWLES (2008): "Why is the Rate of Single Parenthood Lower in Canada than in the U.S.?," *Canadian Journal Economics*, Forthcoming.

IYIGUN, M., AND R. P. WALSH (2007): "Building the Family Nest: Premarital Investments, Marriage Markets, and Spousal Allocations," *Review of Economic Studies*, 74(2), 507–535.

KNOWLES, J. (2008): "High-Powered Jobs: Can Contraception Technology Explain Trends in Women's Occupational Choice?," Discussion paper, University of Southampton.

KOHLER, H.-P., J. BEHRMAN, AND A. SKYTTHE (2005): "Partner + children = Happiness? The effects of partnerships and fertility on happiness," *Population and Development Review*, 31(3), 407–445.

NEAL, D. (2004): "The Relationship between Marriage Market Prospects and Never-Married Motherhood," *The Journal of Human Resources*, 39(4), 938–957.

PINKER, S. (1999): *How the Mind Works*. W. W. Norton and Company.

REGALIA, F., AND J.-V. RÍOS-RULL (1999): "What Accounts for the Increase in Single Households and the Stability in Fertility?," Barcelona, Spain: Universitat Pompeu Fabra, Department of Economics, Working Paper No. 1999.

ROSENZWEIG, M. R. (1999): "Welfare, Marital Prospects, and Nonmarital Childbearing," *The Journal of Political Economy*, 107(6), S3–S32.

SOMMER, K. (2008): "Fertility Choice in a Lifecycle Model with Idiosyncratic Uninsurable Earnings Uncertainty," Georgetown University, Mimeo.

TRUSSELL, J., AND C. WILSON (1985): "Sterility in a population with natural fertility," *Population Studies*, 39(2), 269–286.

VENTURA, S. J. (2009): "Changing Patterns of Nonmarital Childbearing in the United States," *NCHS Data Brief*, (18).

WEISS, Y., AND R. J. WILLIS (1997): "Match Quality, New Information, and Marital Dissolution," *Journal of Labor Economics*, 15(1), S293–329.

A   Solving the Marriage Market Asset Equations

Let the seed value of q be q_1, so that the probability of the first shock in a marriage being q_i is $f(q_i, q_1)$. This is not an innocent assumption; since q_1 is the highest value, persistence implies strangers are optimistic here about their chances in love. If people *require* $q = q_1$ to marry, then they will divorce whenever $q = q_0$. So for the 1950s equilibrium, we need that people would have married even if $q = q_0$.

We now proceed to work out the coefficients of the system.

A.1   Single Female

Let the probability that a single female obtains the marriage surplus be

$$p_S(\phi_k) \equiv p_z \left[1 - \omega_0(\phi_k) - \omega_1(\phi_k)\right]$$

The *ex ante* value of being a single female with k kids is:

$$
\begin{aligned}
V_{SF}^E(k) &= V_{SF}^R(k,\phi) + p_S(\phi_k) S(k,0|\phi) \\
& \quad V_{SF}^R(k,\phi) + p_S(\phi_k)\left[Y^E(k,k_m|\phi,q_0) - V_{SF}^R(k,\phi) - V_{SM}^R\right] \\
V_{SF}^E(k) &= V_{SF}^R(k,\phi)\left[1 - p_S(\phi_k)\right] + p_S(\phi_k)\left[Y^E(k,k_m|\phi,q_1) - V_{SM}^R\right]
\end{aligned}
$$

, where $S(k,0|\phi) = Y^E(k,k_m|\phi,q_0) - V_{SF}^R(k,\phi) - V_{SM}^R$.

Using the expression for $V_{SF}^R(k)$:

$$
\begin{aligned}
V_{SF}^R(k) &= \left(1 - \pi_k^{SF}\right) u_{SF}(k) + \pi_k^{SF} u_{SF}(k+1) - \Theta\left(\pi_k^{SF}\right) \\
& \quad + \beta(1-\delta)\left[\left(1 - \pi_k^{SF}\right) V_{SF}^E(k) + \pi_k^{SF} V_{SF}^E(k+1)\right] \tag{22}
\end{aligned}
$$

we can write

$$
V_{SF}^R(k) = \tilde{d}_1 + \beta(1-\delta)\left(1 - \pi_k^{SF}\right) V_{SF}^E(k)
$$

, where

$$
\begin{aligned}
\tilde{d}_1 &= \left(1 - \pi_k^{SF}\right) u_{SF}(k) + \pi_k^{SF} u_{SF}(k+1) \\
& \quad - \Theta\left(\pi_k^{SF}\right) + \beta(1-\delta)\pi_k^{SF} V_{SF}^E(k+1)
\end{aligned}
$$

Plugging this back into the definition of $V_{SF}^E(k)$, we get,

$$
\begin{aligned}
V_{SF}^E(k) &= \left[\tilde{d}_1 + \beta(1-\delta)\left(1 - \pi_k^{SF}\right) V_{SF}^E(k)\right]\left[1 - p_S(\phi_k)\right] + p_S(\phi_k)\left[Y^E(k,k_m|\phi,q_1) - V_{SM}^R\right] \\
&= a_{11} V_{SF}^E(k) + a_{13} Y^E(k,k_m|\phi,q_1) + d_1
\end{aligned}
$$

where

$$
\begin{aligned}
a_{11} &= \beta(1-\delta)\left(1 - \pi_k^{SF}\right)\left[1 - p_S(\phi_k)\right] \\
a_{13} &= p_S(\phi_k) \\
d_1 &= \left[1 - p_S(\phi_k)\right]\tilde{d}_1 - p_S(\phi_k) V_{SM}^R
\end{aligned}
$$

we have written this as a linear function of terms in k which are to be determined, and terms in $k+1$, which are already known.

DEBUG:

If no prospect of marriage, this simplifies to:

$$
\begin{aligned}
V_{SF}^E(k) &= \left[\tilde{d}_1 + \beta(1-\delta)\left(1 - \pi_k^{SF}\right) V_{SF}^E(k)\right] \\
&= \tilde{d}_1 / \left[1 - \beta(1-\delta)\left(1 - \pi_k^{SF}\right)\right]
\end{aligned}
$$

where

A.2   New Marriages

Let the divorce probability of a marriage with new realization q be $\pi^D_{k,k_m}(q)$. the probability that the spouses, should they stay together, are not sterile next period is $(1-\delta)$.

The value of a new marriage where the bride already has k children is:

$$Y^E(k,0,q_1) = f(q_0,q_1)\left[Y^R(k,0|\phi,q_0)\left(1-\pi^D_{k,0}(q_0)\right)+\pi^D_{k,0}(q_0)\left[V^R_{SF}(k,\phi)+V^R_{SM}-C_D\right]\right]$$
$$+f(q_1,q_1)\left[Y^R(k,0|\phi,q_1)\left(1-\pi^D_{k,0}(q_1)\right)+\pi^D_{k,0}(q_1)\left[V^R_{SF}(k,\phi)+V^R_{SM}-C_D\right]\right]$$

. I find it slightly more convenient to rewrite this as below:

$$Y^E(k,0,q_1) = \left[f(q_0,q_1)\pi^D_{k,0}(q_0)+f(q_1,q_1)\pi^D_{k,0}(q_1)\right]\left[V^R_{SF}(k,\phi)+V^R_{SM}-C_D\right]$$
$$+f(q_0,q_1)\left(1-\pi^D_{k,0}(q_0)\right)Y^R(k,0|\phi,q_0)+f(q_1,q_1)\left(1-\pi^D_{k,0}(q_1)\right)Y^R(k,0|\phi,q_1)$$

If a marriage breaks up, under our assumptions about timing, it is before the fertility decision, so assuming the marriage survives the divorce stage, the value of the marriage, before the fertility realization is known, is

$$Y^R(k,0|\phi,q_i) = EU^M(k,q_i)+\beta(1-\delta)\left[\left(1-\pi^{MF}_{k,0}(q_i)\right)Y^E(k,0,q_i)+\pi^{MF}_{k,0}(q_i)Y^E(k+1,1,q_i)\right]$$

, where

$$EU^M(k,q_i) = q_i+\left(1-\pi^{MF}_{k,0}(q_i)\right)u_M(k,0)+\pi^{MF}_{k,0}(q_i)u_M(k+1,1)-\Theta^{MF}\left(\pi^{MF}_{k,0}(q_i)\right)$$

Now using

$$V^R_{SF}(k) = \tilde{d}_1+\beta(1-\delta)\left(1-\pi^{SF}_k\right)V^E_{SF}(k)$$

Ultimately we can write this as:

$$Y^E(k,0,q_i) = a_{2+i,1}V^E_{SF}(k)+a_{2+i,2}Y^E(k,0,q_0)+a_{2+i,3}Y^E(k,0,q_1)+d_{2+i}$$

, where

$$a_{2+i,1} = \left[f(q_0,q_i)\pi^D_{k,k_m}(q_0)+f(q_1,q_i)\pi^D_{k,k_m}(q_1)\right]\beta(1-\delta)\left(1-\pi^{SF}_k\right)$$
$$a_{2+i,2} = f(q_0,q_i)\left(1-\pi^D_{k,0}(q_0)\right)\beta(1-\delta)\left(1-\pi^{MF}_{k,0}(q_0)\right)$$
$$a_{2+i,3} = f(q_1,q_i)\left(1-\pi^D_{k,k_m}(q_1)\right)\beta(1-\delta)\left(1-\pi^{MF}_{k,0}(q_1)\right)$$

33

and the intercept terms are:

$$
\begin{aligned}
d_{2+i} &= f\left(q_0, q_i\right)\left(1 - \pi^D_{k,0}\left(q_0\right)\right)\left[EU^M\left(k, q_0\right) + \beta\left(1 - \delta\right)\pi^{MF}_{k,0}\left(q_0\right)Y^E\left(k+1, 1, q_0\right)\right] \\
&\quad + f\left(q_1, q_i\right)\left(1 - \pi^D_{k,0}\left(q_1\right)\right)\left[EU^M\left(k, q_1\right) + \beta\left(1 - \delta\right)\pi^{MF}_{k,0}\left(q_1\right)Y^E\left(k+1, 1, q_1\right)\right] \\
&\quad + \left[f\left(q_0, q_i\right)\pi^D_{k,k_m}\left(q_0\right) + f\left(q_1, q_i\right)\pi^D_{k,k_m}\left(q_1\right)\right]\left[\tilde{d}_1 + V^R_{SM} - C_D\right]
\end{aligned}
$$

.

A.3 The rest of the value functions

Now that we have computed the value system for single women and newly-weds, it remains to compute the values of single men $V^E_{SM}(k)$ and the values of marriages with husband's children present, $(k_m > 0)$. These are straight-forward. First, for the single men,

$$
V^E_{SM}(k) = V^R_{SM} + \omega_0\left(\phi_k\right)S\left(k, 0 | \phi\right) - \gamma
$$

where $S\left(k, 0 | \phi\right) = Y^E\left(k, k_m | \phi, q_0\right) - V^R_{SF}\left(k, \phi\right) - V^R_{SM}$ and $V^R_{SF}(k) = \tilde{d}_1 + \beta\left(1 - \delta\right)\left(1 - \pi^{SF}_k\right)V^E_{SF}($
So, given V^R_{SM} and ϕ_k, then $V^E_{SM}(k)$ is known.

Later we can use this to compute $V^R_{SM} = \max_k\left\{u_{SM} + \beta V^E_{SM}(k)\right\}$ and verify that we have guessed correctly or update.

The value of an ongoing marriage where the bride already has k children of which $k_m \in \{1, ..., k\}$ are the husband's is:

$$
\begin{aligned}
Y^E\left(k, k_m, q_i\right) &= \left[f\left(q_0, q_i\right)\pi^D_{k,k_m}\left(q_0\right) + f\left(q_1, q_i\right)\pi^D_{k,k_m}\left(q_1\right)\right]\left[V^R_{SF}\left(k, \phi\right) + V^R_{SM} - C_D\right] \\
&\quad + f\left(q_0, q_i\right)\left(1 - \pi^D_{k,k_m}\left(q_0\right)\right)Y^R\left(k, k_m | \phi, q_0\right) + f\left(q_1, q_i\right)\left(1 - \pi^D_{k,k_m}\left(q_1\right)\right)Y^R\left(k, k_m | \phi\right.
\end{aligned}
$$

the unknowns here are the values $Y^R\left(k, k_m | \phi, q_0\right)$.

$$
Y^R\left(k, k_m | \phi, q_i\right) = EU^M\left(k, q_i\right) + \widetilde{\beta}\left(1 - \pi^{MF}_{k,k_m}\left(q_i\right)\right)Y^E\left(k, k_m, q_i\right) + \widetilde{\beta}\pi^{MF}_{k,k_m}\left(q_i\right)Y^E\left(k+1, k_m+1, q_i\right)
$$

where $\widetilde{\beta} \equiv \left(1 - \delta\right)^2\beta$

Substituting we get

34

$$Y^E(k, k_m, q_i) = \left[f(q_0, q_i) \pi^D_{k,k_m}(q_0) + f(q_1, q_i) \pi^D_{k,k_m}(q_1) \right] \left[V^R_{SF}(k, \phi) + V^R_{SM} - C_D \right]$$
$$+ f(q_0, q_i) \left(1 - \pi^D_{k,k_m}(q_0) \right) \left[EU^M(k, q_0) + \widetilde{\beta} \left(1 - \pi^{MF}_{k,k_m}(q_0) \right) Y^E(k, k_m, q_0) \right]$$
$$+ \widetilde{\beta} f(q_0, q_i) \left(1 - \pi^D_{k,k_m}(q_0) \right) \pi^{MF}_{k,k_m}(q_0) Y^E(k+1, k_m+1, q_0)$$
$$+ f(q_1, q_i) \left(1 - \pi^D_{k,k_m}(q_1) \right) \left[EU^M(k, q_1) + \widetilde{\beta} \left(1 - \pi^{MF}_{k,k_m}(q_1) \right) Y^E(k, k_m, q_1) \right]$$
$$+ \widetilde{\beta} f(q_1, q_i) \left(1 - \pi^D_{k,k_m}(q_1) \right) \pi^{MF}_{k,k_m}(q_1) Y^E(k+1, k_m+1, q_1)$$

Suppose that we know the entire system for $k + 1$, now the only unknowns are $Y^E(k, k_m, q_0)$ and $Y^E(k, k_m, q_1)$.

$$\begin{bmatrix} Y^E(k, k_m, q_0) \\ Y^E(k, k_m, q_1) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} Y^E(k, k_m, q_0) \\ Y^E(k, k_m, q_1) \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

, where

$$b_{ij} = f(q_j, q_i) \left(1 - \pi^D_{k,k_m}(q_j) \right) \widetilde{\beta} \left(1 - \pi^{MF}_{k,k_m}(q_j) \right)$$

and

$$d_i = \left[f(q_0, q_i) \pi^D_{k,k_m}(q_0) + f(q_1, q_i) \pi^D_{k,k_m}(q_1) \right] \left[V^R_{SF}(k, \phi) + V^R_{SM} - C_D \right]$$
$$+ f(q_0, q_i) \left(1 - \pi^D_{k,k_m}(q_0) \right) \left[EU^M(k, q_0) + \widetilde{\beta} \pi^{MF}_{k,k_m}(q_0) Y^E(k+1, k_m+1, q_0) \right]$$
$$+ f(q_1, q_i) \left(1 - \pi^D_{k,k_m}(q_1) \right) \left[EU^M(k, q_1) + \widetilde{\beta} \pi^{MF}_{k,k_m}(q_1) Y^E(k+1, k_m+1, q_1) \right]$$

At k=K, of course the fertility terms disappear, so we are left with

$$b_{ij} = f(q_i, q_j) \left(1 - \pi^D_{k,k_m}(q_i) \right) \widetilde{\beta}$$

$$d_i = \left[f(q_0, q_i) \pi^D_{k,k_m}(q_0) + f(q_1, q_i) \pi^D_{k,k_m}(q_1) \right] \left[V^R_{SF}(k, \phi) + V^R_{SM} - C_D \right]$$
$$+ f(q_0, q_i) \left(1 - \pi^D_{k,k_m}(q_0) \right) U^M(k, q_0)$$
$$+ f(q_1, q_i) \left(1 - \pi^D_{k,k_m}(q_1) \right) U^M(k, q_1)$$

A.4   Distributions

Suppose we impose a discrete distribution on q over N_q values. Also, let the probability a single woman with k children marries be

$$\mu_k \equiv p_z \left[1 - \omega_0(\phi_k) \right]$$

A.4.1 case 1: $k = 0$

The laws of motion for $k = 0$ are:

$$S'_F(0) = \delta + \left(1 - \pi_0^{SF}\right)(1 - \delta)\left[(1 - \mu_0)S_F(0) + \int \left[\pi_{0,0}^D(q')H(0,0,q)f(dq',q) + \mu_0 S_F(0)\pi_{0,0}^D(q')\right.\right.$$

$$H'(0,0,q') = \left(1 - \pi_{0,0}^D\right)(1 - \delta)\int \left(1 - \pi_{0,0}^{MF}(q')\right)[H(0,0,q)f(q',q) + \mu_0 S_F(0)f(q',q_1)\ dq']$$

A.4.2 Child-less single women

We can then write the flow into singles as composed of the following elements:

1. Those were single last period and did not marry, plus new entrants: This has
   mass $\delta + \left(1 - \pi_0^{SF}\right)(1 - \delta)(1 - \mu_0)S_F(0)$

2. Those who were single last period and married, then divorced:

$$\left(1 - \pi_0^{SF}\right)(1 - \delta)\mu_0 S_F(0)\sum_{q'}\pi_{0,0}^D(q')f(q',q_1)$$

3. Those who were married last period and divorced:

$$\left(1 - \pi_0^{SF}\right)(1 - \delta)\sum_q\sum_{q'}\pi_{0,0}^D(q')f(q',q)H(0,0,q)$$

We can therefore write the law of motion of the child-less single mass as

$$S'_F(0) = \delta + a_{11}S_F(0) + \sum_q a_{1,q+1}H(0,0,q)$$

$$a_{11} = \left(1 - \pi_0^{SF}\right)(1 - \delta)\left[(1 - \mu_0) + \mu_0\sum_{q'}\pi_{0,0}^D(q')f(q',q_1)\right]$$

$$a_{1,q+1} = \left(1 - \pi_0^{SF}\right)(1 - \delta)\sum_{q'}\pi_{0,0}^D(q')f(q',q)$$

A.4.3 child-less marriages

The mass of child-less marriages with quality q' consists of:

1. Those who were single last period and married this period

$$\left(1 - \pi_{00}^{MF}(q')\right)(1 - \delta)\mu_0 S_F(0)f(q',q_1)$$

2. Those who were married last period and had no children and did not divorce

$$(1 - \delta) \left(1 - \pi_{0,0}^D (q')\right) \left(1 - \pi_{0,0}^{MF} (q_j)\right) \sum_q f(q', q) H(0, 0, q)$$

We can therefore write the law of motion of the mass of child-less marriages as

$$H(0, 0, q_j) = a_{j+1,1} S_F(0) + \sum_{i=1}^{N_q} a_{j+1,i+1} f(q_j, q_i) H(0, 0, q_i)$$

where

$$\begin{aligned} a_{j+1,1} &= (1 - \delta) \mu_0 \left(1 - \pi_{0,0}^D (q_j)\right) \left(1 - \pi_{00}^{MF} (q_j)\right) f(q_j, q_1) \\ a_{j+1,i+1} &= (1 - \delta) \left(1 - \pi_{0,0}^D (q_j)\right) \left(1 - \pi_{0,0}^{MF} (q_j)\right) f(q_j, q_i) \end{aligned}$$

A.4.4   case 2: $k > 0, k_m > 0$

For each $k > 0$ we can also construct a similar linear system, with flows in from the population with $k - 1$ kids and flows out to the system with $k + 1$ kids. There is also a flow in to $S'_F(k)$ from $H(k - 1, k_m, q)$ and $H(k, k_m, q)$. However the flow into $H(k, 0, \cdot)$ can only be from singles, which simplifies the system.

Once the system at $k - 1$ is known, it is easy to compute the steady-state distribution for $H(k, k_m, q)$ with $k_m > 0$. This is particularly easy for $k_m > 1$ because the only inflow is from $H(k - 1, k_m - 1, q)$, whereas for $k_m = 1$, we must also allow for inflows from single women with $k - 1$:

$$\begin{aligned} H(k, k_m, q_i) &= \widetilde{a}_{k,k_m} (q_i) + (1 - \delta) \left(1 - \pi_{k,k_m}^D (q_i)\right) \left(1 - \pi_{k,k_m}^{MF} (q_i)\right) \sum_{j=1}^{N_q} H(k, k_m, q_j) f(q_i, q_j) \\ &\quad + (1 - \delta) \left(1 - \pi_{k-1,k_m-1}^D (q_i)\right) \pi_{k-1,k_m-1}^{MF} (q_i) \sum_{j=1}^{N_q} H(k - 1, k_m - 1, q_j) f(q_i, q_j) \end{aligned}$$

, where

$$\widetilde{a}_{k,k_m} (q_i) = \begin{cases} (1 - \delta) \mu_{k-1} S_F(k - 1) \left(1 - \pi_{k-1,k_m-1}^D (q_i)\right) \pi_{k-1,k_m-1}^{MF} (q_i) f(q_i, q_1) & k_m = 1 \\ 0 & k_m > 1 \end{cases}$$

So for the $N_q = 2$ system, we can write this as

$$\begin{bmatrix} H(k, k_m, q_0) \\ H(k, k_m, q_1) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} H(k, k_m, q_0) \\ H(k, k_m, q_1) \end{bmatrix} + \begin{bmatrix} g_0 \\ g_1 \end{bmatrix}$$

where

$$h_{11} = (1-\delta)\left(1 - \pi^D_{k,k_m}(q_0)\right)\left(1 - \pi^{MF}_{k,k_m}(q_0)\right) f(q_0, q_0)$$

$$h_{12} = (1-\delta)\left(1 - \pi^D_{k,k_m}(q_0)\right)\left(1 - \pi^{MF}_{k,k_m}(q_0)\right) f(q_0, q_1)$$

$$h_{21} = (1-\delta)\left(1 - \pi^D_{k,k_m}(q_1)\right)\left(1 - \pi^{MF}_{k,k_m}(q_1)\right) f(q_1, q_0)$$

$$h_{11} = (1-\delta)\left(1 - \pi^D_{k,k_m}(q_1)\right)\left(1 - \pi^{MF}_{k,k_m}(q_1)\right) f(q_1, q_1)$$

and

$$g_i = \widetilde{a}_{k,k_m}(q_i) + (1-\delta)\left(1 - \pi^D_{k-1,k_m-1}(q_i)\right)\pi^{MF}_{k-1,k_m-1}(q_i)$$

$$\times \sum_{j=1}^{N_q} f(q_i, q_j) H(k-1, k_m-1, q_j)$$

A.4.5   case 3: $k > 0, k_m = 0$

Singles   Sticking to the discrete case, for $k > 0$, the flows into $S'_F(k)$ are:

1. From singles at $k-1$ who didn't marry and then had a baby

$$(1-\delta)\pi^{SF}_{k-1}\left(1 - \mu_{k-1}\right) S_F(k-1)$$

2. From singles at $k-1$ who did marry, then divorced and then had a baby

$$(1-\delta)\pi^{SF}_{k-1}\left[\sum_{i=1}^{N_q} \pi^D_{k-1,0}(q_i) f(q_i, q_1)\right]\mu_{k-1}S_F(k-1)$$

3. singles at k who didn't marry and didn't have a baby

$$(1-\delta)\left(1 - \pi^{SF}_k\right)\left(1 - \mu_k\right) S_F(k)$$

4. singles at k who did marry and didn't have a baby

$$(1-\delta)\left(1 - \pi^{SF}_k\right)\left[\sum_{i=1}^{N_q} \pi^D_{k,0}(q_i) f(q_i, q_1)\right]\mu_k S_F(k)$$

5. from married at $k-1$ who divorced and then had a baby

$$\pi^{SF}_{k-1}(1-\delta)\sum_{i=1}^{N_q}\sum_{j=1}^{N_q} \pi^D_{k-1,0}(q_j) f(q_j, q_i) H(k-1, 0, q_i)$$

$$+\pi^{SF}_{k-1}(1-\delta)\sum_{k_m=1}^{k-1}\sum_{j=1}^{N_q}\sum_{i=1}^{N_q} \pi^D_{k-1,k_m}(q_j) f(q_j, q_i) H(k-1, k_m, q_i)$$

6. from married at k who divorced and did not have a baby

$$\left(1 - \pi_k^{SF}\right)\left(1 - \delta\right)\sum_{j=1}^{N_q}\sum_{i=1}^{N_q}\pi_{k,0}^{D}\left(q_j\right)f\left(q_j, q_i\right)H\left(k, 0, q_i\right)$$

$$+\left(1 - \pi_k^{SF}\right)\left(1 - \delta\right)\sum_{k_m=1}^{k}\sum_{j=1}^{N_q}\sum_{i=1}^{N_q}\pi_{k,k_m}^{D}\left(q_j\right)f\left(q_j, q_i\right)H\left(k, k_m, q_i\right)$$

The law of motion for single women is:

$$S_F'\left(k\right) = a_{11}S_F\left(k\right) + \sum_{i=1}^{N_q}a_{1i}H\left(k, 0, q_i\right) + d_1$$

where

$$a_{11} = \left(1 - \delta\right)\left(1 - \pi_k^{SF}\right)\left(\left(1 - \mu_k\right) + \left[\sum_{i=1}^{N_q}\pi_{k,0}^{D}\left(q_i\right)f\left(q_i, q_1\right)\right]\mu_k\right)$$

$$a_{1i+1} = \left(1 - \pi_k^{SF}\right)\left(1 - \delta\right)\sum_{j=1}^{N_q}\pi_{k,0}^{D}\left(q_j\right)f\left(q_j, q_i\right)$$

and

$$d_1 = \left(\left(1 - \delta\right)\pi_{k-1}^{SF}\left(1 - \mu_{k-1}\right) + \left(1 - \delta\right)\pi_{k-1}^{SF}\left[\sum_{i=1}^{N_q}\pi_{k-1,0}^{D}\left(q_i\right)f\left(q_i, q_1\right)\right]\mu_{k-1}\right)S_F\left(k - 1\right)$$

$$+\pi_{k-1}^{SF}\left(1 - \delta\right)\sum_{k_m=0}^{k-1}\sum_{j=1}^{N_q}\sum_{i=1}^{N_q}\pi_{k-1,k_m}^{D}\left(q_j\right)f\left(q_j, q_i\right)H\left(k - 1, k_m, q_i\right)$$

$$+\left(1 - \pi_k^{SF}\right)\left(1 - \delta\right)\sum_{k_m=1}^{k}\sum_{j=1}^{N_q}\sum_{i=1}^{N_q}\pi_{k,k_m}^{D}\left(q_j\right)f\left(q_j, q_i\right)H\left(k, k_m, q_i\right)$$

Married For married women in households with no kids from the husband, the flows into $H'\left(k, 0, q_i\right)$ are:

1. From married with same number of kids

$$\left(1 - \delta\right)\left(1 - \pi_{k,0}^{D}\right)\left(1 - \pi_{k,0}^{MF}\left(q_i\right)\right)\sum_{j=1}^{N_q}H\left(k, 0, q_j\right)f\left(q_i, q_j\right)$$

2. From singles with same number of kids

$$\left(1 - \delta\right)\left(1 - \pi_{k,0}^{D}\right)\left(1 - \pi_{k,0}^{MF}\left(q_i\right)\right)\mu_k S_F\left(k\right)f\left(q_j, q_1\right)$$

which sums to:

$$H'(k, 0, q_i) = (1 - \delta)\left(1 - \pi_{k,0}^D\right)\left(1 - \pi_{k,0}^{MF}(q_i)\right)\left[\sum_{j=1}^{N_q} H(k, 0, q_j) f(q_i, q_j) + \mu_k S_F(k) f(q_j, q_1)\right]$$

. Notice that there cannot be a flow in from $k - 1$ to $H'(k, 0, q_i)$; singles must have gotten married to flow in, but married at $k - 1$ who have a child become type $k, 1$, not $k, 0$.. When we discretize the system and set $N_q = 2$ we get a 3x3 system:

$$\begin{bmatrix} S_F'(k) \\ H'(k, 0, q_1) \\ H'(k, 0, q_2) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{23} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} S_F'(k) \\ H'(k, 0, q_1) \\ H'(k, 0, q_2) \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

where

$$
\begin{aligned}
a_{11} &= \left(1 - \pi_k^{SF}\right)(1 - \delta)\left[(1 - \mu_k) + \mu_k\left[\pi_{k,0}^D(q_0) f(q_0, q_1) + \pi_{k,0}^D(q_1) f(q_1, q_1)\right]\right] \\
a_{12} &= \left(1 - \pi_k^{SF}\right)(1 - \delta)\left[\pi_{k,0}^D(q_0) f(q_0, q_0) + \pi_{k,0}^D(q_1) f(q_1, q_0)\right] \\
a_{13} &= \left(1 - \pi_k^{SF}\right)(1 - \delta)\left[\pi_{k,0}^D(q_0) f(q_0, q_1) + \pi_{k,0}^D(q_1) f(q_1, q_1)\right] \\
a_{21} &= \left(1 - \pi_{k,0}^D(q_0)\right)\left(1 - \pi_{k,0}^{MF}(q_0)\right)\mu_k f(q_0, q_1) \\
a_{22} &= \left(1 - \pi_{k,0}^D(q_0)\right)\left(1 - \pi_{k,0}^{MF}(q_0)\right) f(q_0, q_0) \\
a_{23} &= \left(1 - \pi_{k,0}^D(q_0)\right)\left(1 - \pi_{k,0}^{MF}(q_0)\right) f(q_0, q_1) \\
a_{3,1} &= \left(1 - \pi_{k,0}^D(q_1)\right)\left(1 - \pi_{k,0}^{MF}(q_1)\right)\mu_k f(q_1, q_1) \\
a_{32} &= \left(1 - \pi_{k,0}^D(q_1)\right)\left(1 - \pi_{k,0}^{MF}(q_1)\right) f(q_1, q_0) \\
a_{33} &= \left(1 - \pi_{k,0}^D(q_1)\right)\left(1 - \pi_{k,0}^{MF}(q_1)\right) f(q_1, q_1) \\
a_{j+1,1+i} &= \left(1 - \pi_{k,0}^D(q_j)\right)\left(1 - \pi_{k,0}^{MF}(q_j)\right) f(q_j, q_i)
\end{aligned}
$$

$$
\begin{aligned}
d_1 &= \left(1 - \pi_k^{SF}\right)(1 - \delta)\sum_{k_m=1}^{k}\sum_{j=1}^{N_q}\sum_{i=1}^{N_q} \pi_{k,0}^D(q_j) f(q_j, q_i) H(k, k_m, q_i) \\
d_2 &= d_3 = 0
\end{aligned}
$$

more generally,

$$a_{11} = \left(1 - \pi_k^{SF}\right)(1 - \delta)\left[(1 - \mu_k) + \mu_k \sum_{j=1}^{N_q} \pi_{k,0}^D(q_j) f(q_j, q_1)\right]$$

$$a_{12} = \left(1 - \pi_k^{SF}\right)(1 - \delta) \sum_{i=1}^{N_q} \pi_{k,0}^D(q_i) f(q_i, q_0)$$

$$a_{13} = \left(1 - \pi_k^{SF}\right)(1 - \delta) \sum_{i=1}^{N_q} \pi_{k,0}^D(q_i) f(q_i, q_1)$$

$$a_{j+1,1} = \left(1 - \pi_{k,0}^D(q_j)\right)\left(1 - \pi_{k,0}^{MF}(q_j)\right) \mu_k f(q_j, q_1)$$

$$a_{j+1,1+i} = \left(1 - \pi_{k,0}^D(q_j)\right)\left(1 - \pi_{k,0}^{MF}(q_j)\right) f(q_j, q_i)$$

and

$$d_1 = \left(1 - \pi_k^{SF}\right)(1 - \delta) \sum_{k_m=1}^{k} \sum_{j=1}^{N_q} \sum_{i=1}^{N_q} \pi_{k,0}^D(q_j) f(q_j, q_i) H(k, k_m, q_i)$$

$$d_2 = d_3 = 0$$