Outsourcing versus Vertical Integration: A Dynamic Model of Industry Equilibrium.[1]

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Abstract:

Empirical evidence shows that vertically integrated producers are more productive, bigger and are matched to better suppliers (with high productivity and size). I present a dynamic stochastic model of an industry with heterogeneous firms interacting as buyers and sellers, and market frictions that induce a hold-up problem to the manufacturers to account for these facts. In the model economy, an industrial structure emerges as the result of optimal investment decisions that firms undertake under uncertainty. Firms choose whether to integrate, link to external sellers or buy inputs in the market. This theoretical environment provides a natural framework to answer several questions: Why do supply relations vary across industries and across firms within industries? Why aren't all large firms vertically integrated? How do changes in the properties of uncertainty at firm level determine differences in the vertical structure of an industry? We find that higher uncertainty is associated with higher likelihood of outsourcing; vertically integrated firms are larger and more efficient; otherwise identical downstream firms may differ in their vertical structure, and those that are vertically integrated can end up disintegrated or remain integrated. We also analyze the effects of changes in costs of vertical integration and outsourcing on welfare, aggregate output and productivity.
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1 Introduction

The organization of economic activity has been a field of extensive research in economics. This literature, which goes back to the seminal paper by Coase (1937), has focused on the scope of the market versus the firm. Since then, important contributions on transaction cost economics and contract theory have been emphasizing the role of transaction costs, asset specificity, supply uncertainty, incomplete contracting, market power and regulation on vertical integration.[3] These models, however are silent about firm dynamics. This is in contrast with new evidence, by Hortaçsu and Syverson (2009), which shows that there is a close relationship between the vertical structure of firms and key determinants (size and productivity) of the dynamic behavior of producers. In particular, vertically integrated producers are more productive, bigger and are matched to better suppliers (with high productivity and size). Similarly, there is a large empirical and theoretical literature on firm dynamics studying size distribution of firms, turnover, mobility and productivity, among other issues.[4,5] Given the lack of data, however, this literature has abstracted from the vertical relations firms optimally choose. This is the gap the current model tries to fill.

Introducing endogenous vertical structure decisions (i.e. vertical integration versus outsourcing) into industry equilibrium has implications for key variables of interest, such as size distributions, turnover, etc. For example, vertical integration (we refer to it as VI), in contrast with outsourcing, allows firms to avoid hold-up problems, transactions costs, and cost fluctu-

[3] The literature, at the broadest level, has considered the following perspectives on vertical integration: agency theory articles include Alchian and Demsetz (1972) and Holmstrom (1982); transaction costs theory research includes Williamson (1979); and the references for the property right theory are Grossman and Hart (1986) and Hart and Moore (1990). Gibbons (2005) provides a summary and a comparison of these theories. The most recent surveys include Joskow (2005) and Lafontaine and Slade (2007). Recent theoretical and empirical research on the study of the determinants and effects of vertical integration within and across industries include McLaren (2000), Grossman and Helpman (2002), Antras (2003), Acemoglu et al. (2004) and (2005), Novak and Stern (2007a,b), Ciliberto and Panzar (2009), Legros and Newman (2009) and Gibbons, Holden and Powell (2010).

[4] Empirical research documents stylized facts on entry, exit, growth, and the size distribution of firms: Mansfield (1962); Dunne, Roberts and Samuelson (1988) and (1999a,b); Davis and Haltiwanger (1992); Sutton (1997); Caves (1998); Bartelsman, Scarpetta, and Schivardi (2003); Bartelsman, Haltiwanger and Scarpetta (2004); Axtel (2001); Foster Haltiwanger and Kirzan (2001); Cabral and Mata (2003); Cooper and Haltiwanger (2006); Foster, Haltiwanger and Syverson (2008); Bernard, Redding and Scott (2009); and Hsieh and Klenow (2009); among others.

[5] The theoretical work on industry dynamics tries to provide interpretations of the observed heterogeneity across individual producers: Simon and Bonini (1958); Lucas (1978); Jovanovic (1982), Hopenhayn (1992 a,b); Ericson and Pakes (1995); Pakes and Ericson (1998); Cooley and Quadrini (2001); Melitz (2003); Albuquerque and Hopenhayn (2004); Klette and Kortum (2004); Clementi and Hopenhayn (2006); Luttmer (2007); Asplund and Nocke (2007), Rossi-Hansberg and Wright (2007); Hopenhayn and Vereshchagina (2009); and Chatterjee and Rossi-Hansberg (2011); among others.

ations; and insure specialized input procurement, but also increases managerial costs. Thus, differences in costs and benefits in VI across industries may have an impact on firms' profitability and survival, determining differences in size distribution of firms and average productivity of an industry.

This paper builds a long run dynamic entry and exit equilibrium model of heterogeneous upstream (suppliers) and downstream (manufacturers) firms and market frictions that induce a hold-up problem to the manufacturers. Firms choose whether to integrate, link to external sellers or buy inputs in the market. An industrial structure is the result of optimal investment decisions that firms undertake under uncertainty. In this environment, we seek to understand the determinants of the new stylized facts characterizing the vertical relations of firms. Several questions naturally arise in this environment: Why does the share of vertically integrated firms differ across industries and across firms within industries? How is the vertical structure of firms and industries endogenously determined? What are the implications of firms vertical structure on the size distribution of firms, the firms turnover and the firms value? Why aren't all large firms vertically integrated? How do changes in the stochastic process (i.e. persistence) governing the uncertainty at firm level determine differences in the vertical structure of an industry (i.e., the share of vertically integrated firms)? This paper focus on these questions.

Our results show that, consistent with the facts presented by Hortacsu and Syverson (2009), vertically integrated are larger and more productive. Furthermore, more productive manufacturers tend to integrate with more productive manufacturers. The productivity process of the manufacturers as well as the cost of vertical relations play a key role in the model. We show that when the productivity shocks for manufacturers are less persistent, i.e. there is more uncertainty, the fraction of manufacturers decline. This is consistent with the evidence provided by Kranton and Meinhart (2000). Hence the observed difference in the level of idiosyncratic risk across industry, as documented by Castro, Clementi and MacDonald (2009), are likely to play an important role in vertical relations within industries.

The current paper is related to two literature. First, it introduces vertical relations into industry dynamics models (see Hopenhayn 1992 and Hopenhayn H., and R. Rogerson 1993). Second, it is related to recent papers that study how different organizational forms might emerge as optimal decisions by the firms. In particular, McLaren (2000) and Grossman and Helpman (2002) propose frameworks of incomplete contracting in which final goods manufacturers decide whether to outsource production of intermediate goods or produce them in-house. The key factor determining the organizational structure is the externality effect yielding the thickness of the market for inputs: The more other final goods manufacturers choose to outsource productions of intermediate goods, the more attractive it becomes for one manufacturer to do so as well. These papers, however, consider homogenous producers who decide on their vertical relations within a static environment without any shocks.

1.1 Facts on Vertical Integration

Hortaçsu and Syverson (2009) show that VI status is related to differences in establishment types for the U.S economy.[6] As Table 1 shows, vertically integrated establishments are larger on average. Between 1977 and 1997 vertically integrated plants constitute relatively small fraction, 8 to 9.5 percent, of all establishments of the economy (row 4). Focusing only on multi-unit establishments, vertically integrated plants account for roughly 35 to 40 percent of these multi-unit businesses (row1/row2). Despite their modest share of the overall number of establishments, vertically integrated businesses account for a much larger employment share, 25-30 percent, and roughly half of multi-unit employment (last row).

Table 1: Aggregate Patterns of Vertical Integration, 1977-1997

Non-farm Private Economy

Year	1977	1982	1987	1992	1997
VI Establishments (thousands)	384.3	421.7	546.7	519.8	549.3
Multi-unit establishments (thousands)	1033.7	1167.0	1336.8	1476.6	1605.6
Total establishments (thousands)	4862.2	5049.8	5855.5	6253.2	6831.1
VI establishment share (percent)	7.9	8.4	9.4	8.3	8.0
VI employment (millions)	20.4	21.5	26.9	26.5	28.3
Multi-unit employment (millions)	38.2	42.7	48.3	53.9	60.7
Total employment (millions)	68.1	75.7	87.7	93.6	106.1
VI employment (percent)	29.8	28.4	30.7	28.3	26.7

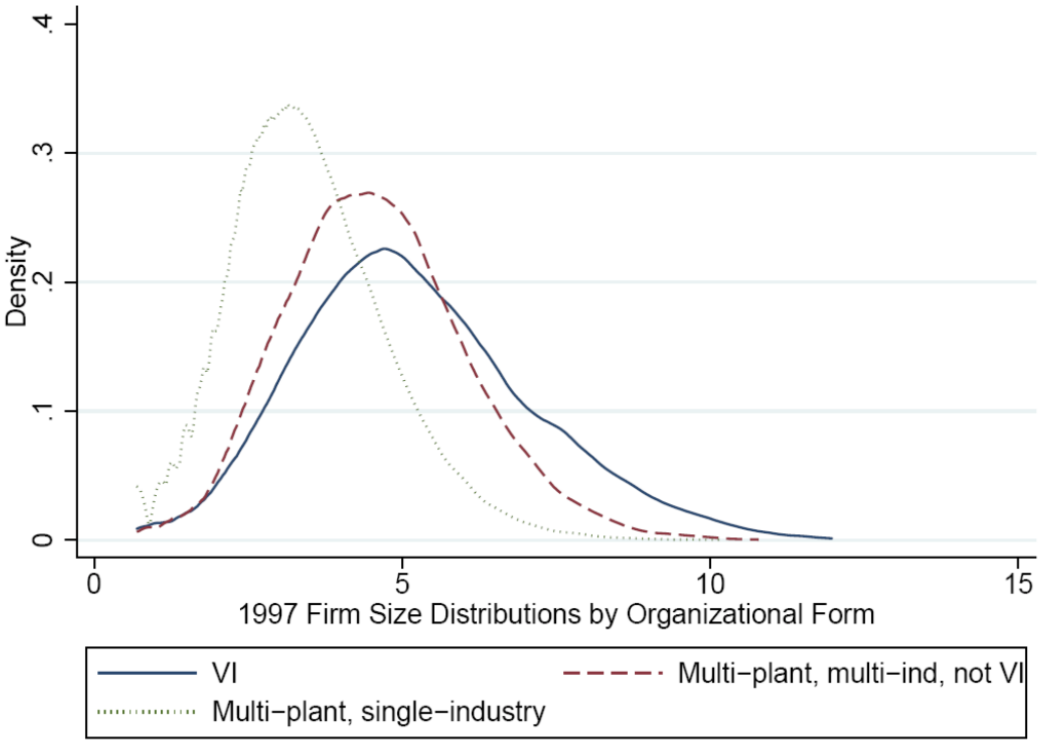
Source: Taken from Hortaçsu and Syverson (2007).

[6] In order to state if a firm is VI first they determine the industry affiliation of every establishment in the Economic Census (EC), using the Input-Output Industry Classification System (IOIC) by the Bureau of Economic Analysis (EC contains SIC codes so they reclassify it into IOIC). Second, they identify in which industry firms operate. Third, they verify whether any substantial links are present between pairs of industries based on volume trade flows using 1987 I-O Tables: a substantial link exists between an industry A and any other industry if A buys at least five percent of its intermediate materials, or any other industry to which A sells at least 5% of its output. Finally, they find all establishments that the firm owns on both ends of a substantial vertical link and classify them as being vertically integrated.

Therefore, vertically integrated establishments are larger on average than single-unit businesses or non-integrated multi-units. Furthermore, the share of plants that are vertically integrated increases with plant's within-industry size percentiles. While smallest plants in an industry are almost never integrated, 7 percent of the median-sized plant are integrated, and 67 percent of plants in the top percentile of their industry size distribution are integrated.

Figure 1 presents the size densities at firm level. It can be seen that central tendencies are clearly different: vertically integrated firms are the largest on average and their distribution is more skewed. Their size dominates, in first order stochastic dominance (FOSD) sense, to the size of not vertically integrated manufacturers. Notice that there is an overlap among these distributions (firms with the same employment levels have different vertical status).

Figure 1: Firm Size Distributions for Multi-Unit Firms, 1997.



Notes: This figure shows density plots of the firm size distributions (measured by logged total employees) for the three types of multi-establishment firms. See text for details.

Source: Taken from Hortaçsu and Syverson (2009).

Hortaçsu and Syverson (2009) also present a conditional analysis where they regress plant's observables types like size, productivity, and factor intensities (all of them related to plant survival) on an indicator for plants' integration status and a set of control variables (including industry by year fixed effects). The results show that, besides being larger, vertically integrated producers display higher productivity levels (they are on average 40 percent more productive than their unintegrated industry cohorts). Moreover, they investigate why plants have these characteristics and conclude that vertically integrated plants are more productive, larger, and more capital intensive primarily because they were either born into integrated structures that way, or because firms with vertically integrated structures that choose to expand through mergers or acquisitions do so by incorporating existing plants that are also of high-type.

Kranton and Minehart (2000) study the relationship between the vertical structure of firms and idiosyncratic uncertainty in demand, putting special emphasis in a special case of vertical relation, networks (an intermediate level of organization between VI and markets). In the last few decades the importance of input procurement by manufacturer-supplier exchange networks has increased a lot.[7] Therefore, Kranton and Minehart (2000) study the conditions under which industries are likely to be organized as networks.

In their model, manufacturers can decide to build a dedicated asset to produce their own inputs, or they can invest in links to external sellers from which they buy specialized inputs or, alternatively, they get inputs from arm-length markets. The results indicate that there is a connection between industrial structure and uncertainty in demand. Networks appear to be more efficient than vertically integrated structures when uncertainty in demand is substantial: higher dispersion of buyer's idiosyncratic demand shocks should be associated with network-like industrial structures and more connected network structures.

Their result is consistent with several case studies. They cite the case of the US automobile industry in 1920, when there was an increase in uncertainty because of competition from the emerging used-car market and new independent manufacturers. After that, the big automakers Ford and GMC moved away from vertical integration to flexible arrangements with independent suppliers (suggesting that disintegration is a response to underlying environmental uncertainty). The same trend occurred in the film industry in the 1940s, when the volatility in demand for

[7] For example, from 1980 to 1990, the major car manufacturers reduced their number of direct input suppliers by more than 50 percent (Noteboom, 1999). This trend is more prominent in Japanese automobile and electronic manufacturing. The number of direct suppliers to Japanese car manufacturers in 1988 was roughly one half of what it was for American or European manufacturers, for similar volumes of production (Lamming, 1993). For electronics and automobiles, Nishiguchi (1994) presents wide-ranging evidence from Japan on how firms rely more and more on a subset of suppliers with whom they maintain close business ties. In the period from 1980 to 1990, Fuji Electric Tokyo bought an additional 7 percent of its inputs from sub-contractors but it has reduced the number of principal subcontractors by 38 percent. On average, electronic assembly contractors have 3.36 regular costumer each of whom placed orders several times per year.

Hollywood movies increased due to the advent of television, and firms moved away from vertical integration to a more flexible system with outsourcing for many aspects of film production.

Summarizing, we want to focus on the following empirical facts documented in Hortaçsu and Syverson (2009) and the main result presented in Kranton and Meinhart (2000):

- *Fact 1)* vertically integrated plants are larger on average and their size distribution is more skewed
- *Fact 2)* When vertically integrating, big and efficient downstream firms choose to acquire upstream production units that are also big and efficient
- *Fact 3)* The fraction of vertically integrated plants increases with the plant's within-industry size percentiles
- *Fact 4)* Besides being larger, vertically integrated plants have higher productivity
- *Fact 5)* When uncertainty in demand is substantial, firms are more likely to invest in links with specific investments (rather than becoming vertically integrated or transact standardized inputs in the market)

2 Environment

2.1 Key features of the model

We develop a long-run dynamic industry equilibrium model with heterogeneous firms interacting as buyers and sellers of inputs. Final good manufacturers face idiosyncratic productivity shocks, denoted by z (as in Hopenhayn 1992a). For production they need one unit of input from suppliers, and with this unit they produce z units of final good. Final good is homogeneous and is sold in a competitive market. Each supplier is characterized by a productivity level ε.

When a manufacturer enters the industry it has to obtain its inputs from the market for inputs. In particular, they pay a price p_u to buy one unit of input. It is assumed that this price is determined by Bertrand competition among suppliers that all produce an homogeneous input. Once p_u is paid, the manufacturer learns the productivity ε, of the supplier. Given the (z, ε) pair, the manufacturer, if it does not exit the industry, has three options: first, it can simply ignore ε and use the *standardized input.* In this case the manufacturer simply produces z units of final good and pays fixed cost of production (C_f). It is assumed that productivity of standardized supplier is *iid* over time. Next period this manufacturer will start the period in exactly the same situation, this is paying C_f, buying one unit of input and learning a new ε.

Second, given (z, ε), the manufacturer can invest (h) to become linked with the particular supplier (we refer to links as L). In this case the manufacturer produces z and pays $C_f - c(z, \varepsilon)$, where $c(z, \varepsilon)$ represents the cost advantage associated with getting a *specialized input* from a particular supplier. A manufacturer pays for a specialized input p_u^L, which is determined by Nash Bargaining. As long as the manufacturer and supplier remain linked, ε remains the same. Next period if z remains the same, the pair continues to be linked. If z changes, however, manufacturer starts next period as a standardized manufacturer (i.e. it has all the same options) with a particular ε at hand. Finally, the manufacturer can pay $h + P_{VI}$ and become vertically integrated with a particular supplier. In this case, it produces z and faces the cost $C_f + C_f^{VI} - c(z, \varepsilon)$. Here C_f^{VI} represents the additional cost of being vertically integrated. Once a manufacturer and a supplier become vertically integrated, they continue to do so until z changes upon which manufacturer can reoptimize, although in order to continue vertically integrated the manufacturer does not need to make any investment.

This environment gives rise to rich industry dynamics as manufacturer enter, exit and decide how to obtain their inputs (i.e. to obtain them from the market, to establish links or get vertically integrated). In this framework, once a manufacturer buys form a supplier it cannot switch partner until next period, thus market frictions induce a hold-up problem (as in Grossman and Hart 1986) to linked manufacturers.[8] Moreover, uncertainty plays a key role. Given that under vertical integration manufacturers face a relatively high cost of governance (as in Grossman and Helpman 2002), reflected by a higher fixed cost of production, vertical integration reduces flexibility when facing a negative shock (compared to links and the use of standardized inputs).

Therefore, there is a clear trade-off between links and vertical integration. On the one hand, a linked manufacturer has lower fixed costs but, faces higher endogenous variable costs (determined by the input price negotiation, as it will be explained later on). On the other hand, becoming vertically integrated requires a bigger investment, and imply higher fixed costs, but lower variable costs to manufacturers. From now on, we use the terms manufacturer and downstream firms, as well as suppliers and upstream firms, interchangeably (notice subscripts

[8] After investments have been made, the supplier can renegotiate the input price, increasing the incentives of the manufacturer to buy standardized inputs or become vertically integrated.

and superscripts d and u, for downstream and upstream firms, respectively).

2.2 Incumbent firm's problem

We assume that there is no aggregate uncertainty. Thus, by a law of large numbers, all aggregate quantities and prices are deterministic over time, although at the firm level, from the point of view of a manufacturer, each firm still faces idiosyncratic uncertainty. We will focus on steady-state stationary equilibrium in which all aggregate variables are constant over time.

2.2.1 Manufacturers

By using one unit of input, a manufacturer produces a quantity z of homogeneous final goods, where z indicates the manufacturer's managerial ability, and sell the production in a competitive market at a price p. Moreover, we assume that z is independent across firms and follows a Markov process $F(z'/z)$ with density function $f(z'/z)$. In addition, we assume that F is strictly decreasing in z and $z \in Z$, where $Z = \{z_1, z_2, ..., z_n\}$ and $z_{i+1} > z_i$ for all i. In other words, the higher is the managerial ability of a manufacturer today, the more likely it will be higher tomorrow [9]

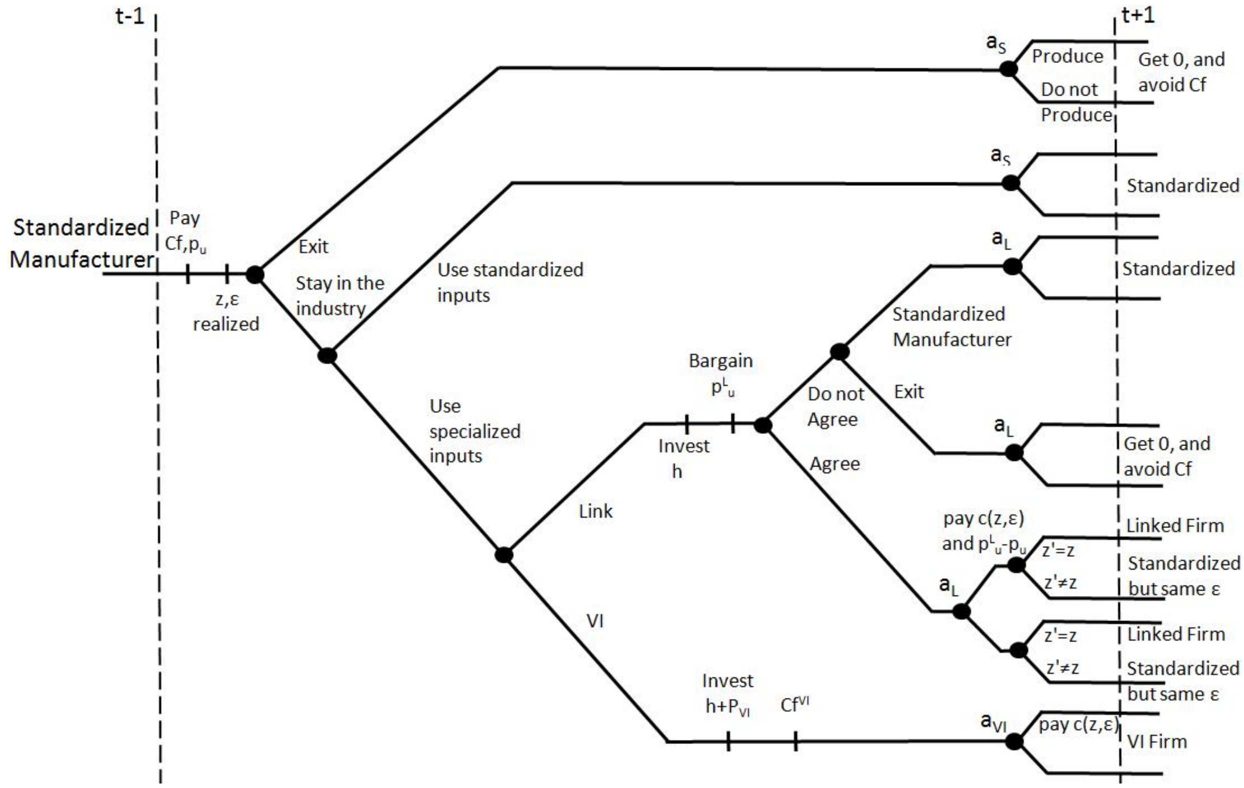
Standardized manufacturer

A standardized manufacturer, at the beginning of every period, before the current productivity shock is realized, has to pay a fixed cost of production, C_f. In addition, it pays an up-front a price, p_u, for the standardized input to a randomly matched standardized supplier. Figure 2

[9] As in Hopenhayn (1992a), this assumption implies that expected discounted profits are an increasing function of firm's current productivity shock.

represents the decisions and timing.

Figure 2: Timing for a standardized manufacturer.



Once C_f and p_u are paid, the idiosyncratic productivity shock, z, is realized and the manufacturer learns the quality of the specialized input the supplier can produce. We assume that the supplier's type, ε, has density function $g^u(\varepsilon)$. As explained before, ε is a match-specific productivity, which can also be interpreted as the managerial ability of the supplier to design and produce a new good and input, and to synchronize production process, together with the matched manufacturer.

Once z and ε are known, the manufacturer decides whether to stay or exit the industry for the next period, and, if it stays in the industry, it must decide whether to use standardized inputs or specialized inputs. In addition, in each situation, it has also to decide whether to produce or not. Thus, if the productivity is very low, in order to avoid paying the fixed costs and the cost of standardized input, the manufacturers may decide to exit the industry for next period. Therefore, as in standard industry dynamics models, there is endogenous exit, hence, in steady state, there is ongoing entry and exit of manufacturers. If the manufacturer stays in the industry, it has to decide whether to use standardized inputs or switch to using specialized inputs. If it uses standardized inputs, next period it will get a new ε.

In order to use specialized inputs the standardized manufacturer has two alternatives, either to become linked with the supplier or become vertically integrated with it (acquire supplier's plant). In both cases, the manufacturer must make specific investments, h (this cost can be thought of as cots in designing a suitable input for the pair z and ε -which is specific to the match- i.e. training costs, costs of providing equipment, know-how, etc.). This investment has two effects, to keep the same supplier's type ε, and to reduce the variable costs to $c(z, \varepsilon)$, where z and ε are complements.[10,11]

If the manufacturer becomes linked with the supplier, we assume that the reduction in variable costs lasts until z changes, and in that case, in order to take advantage of specialized inputs the manufacturer has to invest again h designing a suitable input for the new pair (z, ε). Moreover, once the specific investment is sunk, the price for the specialized input, $p_u^L(z, \varepsilon; p)$, is negotiated (determined by Nash Bargaining Solution, NBS). Hence specific investments are subject to hold-up problem which increases the incentives to buy standardized inputs or become vertically integrated, as explained before.[12] Notice that a linked manufacturer firm has same fixed costs (C_f) and lower variable costs ($p_u^L(z, \varepsilon; p) - c(z, \varepsilon)$) relative to a standardized firm. If the manufacturer decides to become vertically integrated, in addition to the specific investment, h, it has to pay an acquisition price P_{VI} to the supplier (as it will become clear later P_{VI} will correspond to the market value of the supplier). By becoming vertically integrated the manufacturer avoids the hold-up problem.

As in Grossman and Helpman (2002), due to the lack of complete specialization and the extra governance costs associated to managing different plants, we assume that VI increases manufacturer's fixed production costs. This means that a vertically integrated manufacturer

[10] This will be a result of the calibration. We will assume a flexible variable cost function that allows for complementarities between manufacturer and supplier types and, as it will be shown in the following sections, when we calibrate our model so that the industry stationary steady-state equilibrium matches selected characteristics of the U.S. manufacturing sector, complementarities will appear.

[11] The assumptions made on the variable cost function generates a firm behavior which, as it will be shown later on, is in line with new empirical evidence. In particular, Kuglery and Verhoogen (2011), using data from the Colombian manufacturing census, documents that larger plants charge more for their outputs and pay more for their material inputs, and proposes a model of endogenous input and output quality choices by heterogeneous firms to explain the observed patterns.

[12] The hold-up problem is induced by the opportunistic behavior of the supplier. After matching with a given supplier, once the manufacturer has sunk the investment h, there is a bilateral monopoly situation and the supplier seeks to renegotiate the agreement increasing the input price from p_u to p_u^L. Thus the manufacturer is not the full residual claimant of the additional returns the investment generates. Anticipating this, the buyer has an incentive to take the supplier into the firm (becoming VI).

has to pay, in addition to the same fixed cost as the standardized manufacturer, C_f, and the fixed cost of the acquired supplier, C_f^u, a managerial fixed cost, λ (which is assumed to be positive, but it will be a result of the calibration). Furthermore, notice that uncertainty plays a key role: VI increases firm's fixed costs to $C_f + C_f^u + \lambda$ reducing its flexibility when facing a negative shock (when compared to links and market transactions).

When becoming vertically integrated we assume that, in contrast with the link case, the cost advantage $c(z, \varepsilon)$, for different levels of z, is permanent. We assume that, by paying a higher fixed cost of production every period, a vertically integrated firm redesigns the input every time z changes without any additional cost. Therefore, the state variables for a standardized manufacturer is its idiosyncratic productivity, z, the quality of its supplier, ε, and prices p and p_u. Thus, assuming stationarity (distributions, and thus also prices, do not change over time), the value function for the standardized manufacturer firm is:

$$V^S(z,\varepsilon;p,p_u) = \max \left\{ \begin{array}{l} \underbrace{\max_{a_s(z;p,p_u)} p a_s(z;p,p_u) z - p_u - C_f + \beta \max \left\{ \underbrace{0}_{\text{Exit}}, \underbrace{\sum_{z'} \sum_{\varepsilon'} V^S(z',\varepsilon';p,p_u) f(z'|z) g^u(\varepsilon')}_{\text{Standardized (new draw of supplier)}} \right\}}_{\text{Standardized}} \\ \underbrace{\begin{array}{l} \max\limits_{a_L(z,\varepsilon;p), p_u^L(z,\varepsilon;p,p_u)} a_L(z,\varepsilon;p)[pz - p_u^L(z,\varepsilon;p) + c(z,\varepsilon)] - C_f - h \\ +\beta \left[V^L(z,\varepsilon;p,p_u^L) f(z'=z|z) + \sum_{z'} V^S(z',\varepsilon;p,p_u) f(z' \neq z|z) \right], \end{array}}_{\text{Link}} \\ \underbrace{\max_{a_{VI}(z,\varepsilon;p)} a_{VI}(z,\varepsilon;p)[pz + c(z,\varepsilon)] - \underbrace{C_f^{VI}}_{C_f^u + \lambda} - C_f - (P_{VI} + h) + \beta \sum_{z'} V^{VI}(z',\varepsilon;p) f(z'|z),}_{\text{Vertical Integration}} \end{array} \right\}, \tag{1}$$

where $a_s(z;p,p_u)$, $a_L(z;p,p_u)$ and $a_{VI}(z;p)$ are the static production decision rules, and C_f^{VI} is the additional fixed cost of production of a vertically integrated manufacturer, $C_f^{VI} = C_f^u + \lambda$. By standard dynamic programming arguments (e.g., see Stokey and Lucas (1989)), one can show that there is a unique value function satisfying the Bellman equation. The same applies to the following value functions.

The first term of this value function corresponds to the case where the manufacturer remains using standardized inputs in current production and, if it does not exit, for the next period as it is presented in the continuation value. The second term corresponds to the situation in which the manufacturer uses specialized inputs by linking with the supplier. Therefore, the manufacturer decides whether to produce or not and negotiates the input price, $p_u^L(z, \varepsilon; p)$, with the supplier. As long as z remains the same for the next period, the pair continues to be linked (as it can be seen in the first term in the continuation value). If z changes, however, manufacturer starts next period as a standardized manufacturer with the same previous ε at hand (look at the second term in the continuation value). The third term represents the value of becoming vertically integrated with the supplier. Thus, the manufacturer decides whether to produce or not and it will start the next period as a vertically integrated firm, that is, with the same cost advantage as a firm that continues linked, but with higher fixed costs of production.

Moreover, notice that by becoming a linked firm, the manufacturer faces lower fixed costs (just C_f) and higher variable costs ($p_u^L(z, \varepsilon; p) - c(z, \varepsilon)$) relative to becoming a vertically integrated firm. Besides, by becoming vertically integrated, the manufacturer faces higher fixed costs ($C_f + C_f^{VI}$) and lower variable costs (it does not pay p_u and receives the cost advantage $c(z, \varepsilon)$) relative to a standardized manufacturer; and has higher fixed costs ($C_f + C_f^{VI}$) and lower variable costs (doesn't pay $p_u^L(z, \varepsilon; p)$) relative to a linked firm. Thus, there is a clear trade-off of linking versus becoming vertically integrated. We will discuss later on how the properties of the stochastic process (i.e. persistence) governing the uncertainty at firm level also plays a role in these trade-offs, and thus determine differences in the vertical structure of firms across industries.

Linked manufacturer

At the beginning of every period, a manufacturer linked with a supplier of type ε pays a fixed cost of production C_f , and productivity z is realized. If the new productivity shock z is equal to the previous shock, then the link continues and firms trade inputs at the same negotiated input price $p_u^L(z, \varepsilon; p)$ from the previous period and production takes place. Otherwise, if the realization of the new shock z is different from the previous one, the link is broken and the manufacturer has to decide again whether to invest in a link or not. Moreover, if the link is broken, it becomes again a standardized manufacturer, hence it has the same continuation options as a standardized firm, with the only difference that it is matched with the same supplier as in the previous period.

The value function of a linked manufacturer when z has not changed is given by

$$\begin{aligned} V^L(z,\varepsilon;p,p_u^L) = \quad & pz - p_u^L(z,\varepsilon;p) + c(z,\epsilon) - C_f \\ & \beta\left\{V^L(z,\varepsilon;p,p_u^L)f(z'=z|z) + \sum_{z'} V^S(z',\varepsilon;p,p_u)f(z'\neq z|z)\right\}, \end{aligned} \tag{2}$$

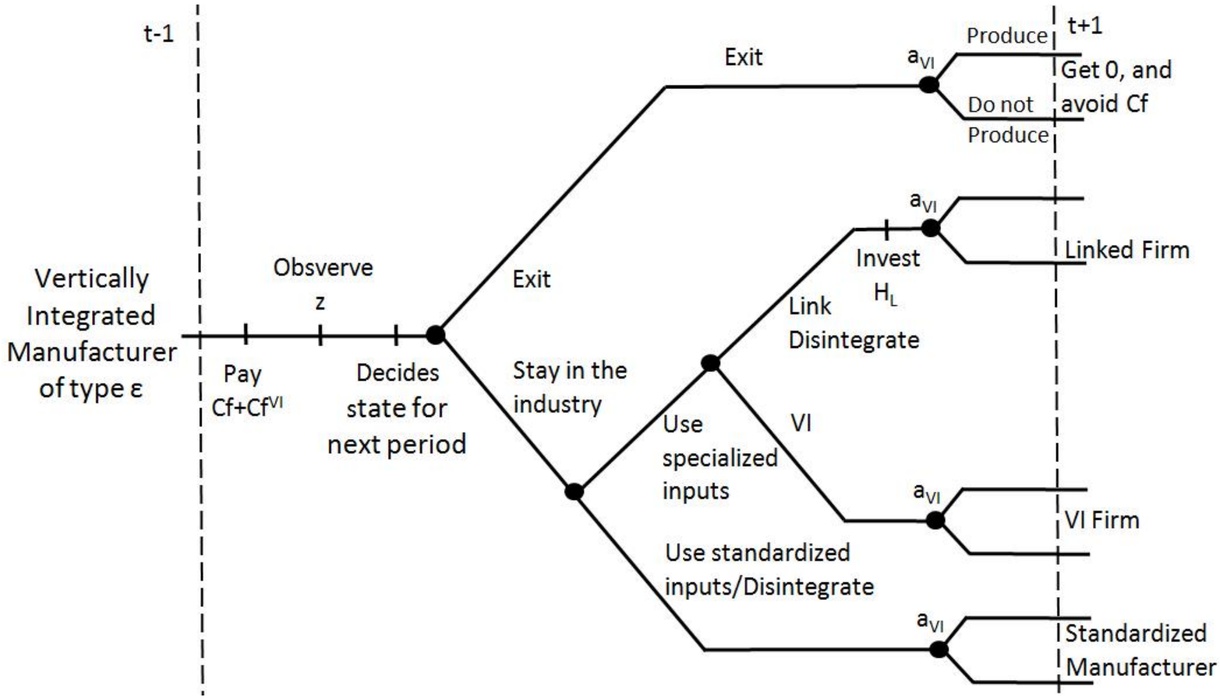
which, after some simple operations, becomes

$$V^L(z,\varepsilon;p,p_u^L) = \quad \tfrac{pz-p_u^L(z,\varepsilon;p)+c(z,\varepsilon)-C_f}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)} \sum_{z'} V^S(z',\varepsilon;p,p_u)f(z'\neq z|z). \tag{3}$$

Vertically integrated manufacturer

A manufacturer that is vertically integrated with a supplier of type ε pays fixed costs of production C_f and C_f^{VI}, productivity z is realized (while ε remains the same). Therefore, it decides current production, $a_{VI} \in \{0,1\}$, and the state for the next period (Figure 3). It has the same continuation options as for the standardized firm (invest in L, get a new supplier, exit the industry), but in order to continue vertically integrated with same supplier it has to make no additional investment. In the case of investing in a link (disintegrate but remain matched with same supplier) the manufacturer produces today as a vertically integrated firm and, since next period on, it has to pay a negotiated input price $p_u^L(z,\varepsilon;p)$.

Figure 3: Timing for a vertically integrated manufacturer.



According to the previous timing, the value function for a vertically integrated manufacturer looks like in Equation 4. A manufacturer with productivity z that enters the current period being vertically integrated with a supplier of type ε , after paying the fixed costs, has to decide whether to produce or not so as to maximize the per period profit. Next, it has to decides in which state it enters the next period, this is either continue vertically integrated (with the second continuation value), without making any additional investment, or disintegrate. In case it decides to disintegrate, it still has the option to continue producing with the same supplier by becoming linked with it after investing h (with the first continuation value). Finally, it can also become a standardized manufacturer starting the next period with a new ε, as it is indicated by the third continuation value, or exit the industry.

$$
\begin{aligned}
V^{VI}(z,\varepsilon;p) = \quad & \max_{a_{VI}(z,\varepsilon;p),x'_{VI}(z,\varepsilon;p)} a_{VI}(z,\varepsilon;p)[pz + c(z,\varepsilon)] - C_f^{VI} - C_f \\
& -hI^{VI}(x'_{VI}(z,\varepsilon;p) = L) \\
& +\beta \left\{ \underbrace{\left[V^L(z,\varepsilon;p,p_u^L) f(z' = z|z) + \sum_{z'} V^S(z',\varepsilon;p,p_u) f(z' \neq z|z) \right]}_{\text{Link}} \right. \\
& \underbrace{+I^{VI}(x'_{VI}(z,\varepsilon;p) = VI) a_{VI}(z,\varepsilon;p) \sum_{z'} V^{VI}(z',\varepsilon;p) f(z'|z)}_{\text{Vertical Integration}} \qquad (4) \\
& \underbrace{+I^{VI}(x'_{VI}(z,\varepsilon;p) = S) \sum_{z'} \sum_{\varepsilon'} V^S(z',\varepsilon',p,p_u) f(z'|z) g^u(\varepsilon')}_{\text{Standardized (new draw)}} \\
& \left. \underbrace{+I^{VI}(x'_{VI}(z,\varepsilon;p) = Exit) 0}_{\text{Exit}} \right\},
\end{aligned}
$$

where $x'_{VI}(\cdot) \in \{VI, L, S, Exit\}$ is the decision rule, that is the state chosen for the next period, and $I^{VI}(\cdot)$ are indicator functions given $x'_{VI}(\cdot)$.

2.2.2 Suppliers

Standardized supplier

Standardized suppliers produce one unit of an homogeneous input and compete in prices. They have zero marginal cost and pay a fixed cost, C_f^u, every period. Once they match with a manufacturer, the quality ε of the specialized input they are able to produce is realized. In case they remain as standardized input supplier the quality of the match, ε, is i.i.d. over time and across suppliers. The value function of a standardized supplier is

$$
\begin{aligned}
W^S(z,\varepsilon;p,p_u) = \;& p_u - C_f^u \\
& +\beta \Bigg\{ \underbrace{I^S(x_S'(\cdot)=L)\left[W^L(z,\varepsilon;p,p_u^L)f(z'=z|z)+\sum_{z'}W^S(z,\varepsilon;p,p_u)f(z'\neq z|z)\right]}_{\text{manufacturer decides to become linked}} \\
& +\underbrace{I^S(x_S'(z,\varepsilon;p,p_u)=VI)P_{VI}(z,\varepsilon)}_{\text{manufacturer decides to become VI}}+ \\
& +\underbrace{\big(1-I^S(x_S'(\cdot)=VI)-I^S(x_S'(\cdot)=L)\big)\sum_{\varepsilon'}\sum_{z'}W^S(z',\varepsilon';p,p_u)J^d(z')g^u(\varepsilon')}_{\text{manufacturer decides to be use standardized inputs}} \Bigg\},
\end{aligned} \tag{5}
$$

where $x_S'(\cdot) \in \{VI, L, S, Exit\}$ is the decision rule of the standardized manufacturer that is matched with this supplier, and $I^S(\cdot)$ are the corresponding indicator functions given $x_S'(\cdot)$. The function $J^d(z')$ is an equilibrium object that represents the density of manufacturers, for each particular productivity level z, that will be looking for a standardized supplier in the next period. For each value of z the density $J^d(z')$ is determined by the process of entry, exit, investment in links and vertical integration.

Specialized supplier

A specialized (linked) supplier produce one unit of the input using the same technology as a standardized supplier. It offers an input of heterogeneous quality which is permanent over time (as explained before, conditional on producing with the same manufacturer every period). In addition it negotiates the input price in a bilateral monopoly situation with the manufacturer, due to the market frictions (once the manufacturer is matched with a supplier it cannot switch partner until next period).

The value function of a linked supplier is

$$
W^L(z,\varepsilon;p,p_u^L) = p_u^L(z,\varepsilon;p) - C_f^u + \beta\left\{W^L(z,\varepsilon;p,p_u^L)f(z'=z|z)+\sum_{z'}W^S(z',\varepsilon;p,p_u)f(z'\neq z|z)\right\}, \tag{6}
$$

which, after some simple operations, becomes

$$W^L(z,\varepsilon;p,p_u) = \frac{p_u^L(z,\varepsilon;p) - C_f^u}{1-\beta f(z'=z|z)} + \frac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)} \sum_{z'} W^S(z',\varepsilon;p,p_u) f(z' \neq z|z). \quad (7)$$

We assume that, if a linked manufacturer breaks the link with a supplier, then the supplier returns to the standardized inputs market, gets matched with another standardized manufacturer, and gets a new draw of ε from $g^u(\varepsilon)$. In addition, if a supplier becomes vertically integrated it gets P_{VI} and disappears. Furthermore, if the manufacturer disintegrates, then the supplier appears again as a standardized supplier.

2.2.3 Equilibrium prices

Price for the specialized input and acquisition price

Given all the previous value functions, we can now define the prices for the specialized inputs and the acquisition price for a supplier firm that a manufacturer pays when vertically integrating. The first one is defined, according to Nash Bargaining, as follows:

$$\begin{aligned} p_u^L(z,\varepsilon;p) = \quad & \arg\max_{p_u^L} \Bigg[\tfrac{pz - p_u^L(\cdot) + c(\cdot) - C_f}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)} \sum_{z'} V^S(z',\varepsilon;p,p_u) f(z' \neq z|z) \\ & - \Bigg(\underbrace{pz - p_u - C_f + \beta \max \Bigg\{ \underbrace{0}_{\text{Exit}}, \underbrace{\sum_{z'} \sum_{\varepsilon'} V^S(z',\varepsilon';p,p_u) f(z'|z) g^u(\varepsilon')}_{\text{Standardized Manufacturer}} \Bigg\}}_{\text{Manufacturer outside option}} \Bigg) \Bigg]^{\theta} \\ & \Bigg[\tfrac{p_u^L(\cdot) - C_f^u}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)} \sum_{z'} W^S(z',\varepsilon;p,p_u) f(z' \neq z|z) \\ & - \Bigg(\underbrace{p_u - C_f^u + \beta \underbrace{\sum_{\varepsilon'} \sum_{z'} W^S(z',\varepsilon';p,p_u) J^d(z') g^u(\varepsilon')}_{\text{Standardized Supplier}}}_{\text{Supplier outside option}} \Bigg) \Bigg]^{1-\theta}, \end{aligned} \quad (8)$$

where θ is the bargaining power of the manufacturer. Thus solving for the bargained specialized input price we get:

$$\begin{aligned} p_u^L(z,\varepsilon;p) = \quad & (1-\beta f(z'=z|z))(1-\theta)\Bigg[\tfrac{pz+c(\cdot)-C_f}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)}\sum_{z'} V^S(\cdot) f(z'\neq z|z) \\ & -\left(pz - p_u - C_f + \beta \max\left\{0, \sum_{z'}\sum_{\varepsilon'} V^S(z',\varepsilon';p,p_u) f(z'|z) g^u(\varepsilon')\right\}\right)\Bigg] \\ & -\theta\Bigg[\tfrac{-C_f^u}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)}\sum_{z'} W^S(z',\varepsilon;p,p_u) f(z'\neq z|z) \\ & -\left(p_u - C_f^u + \beta\sum_{\varepsilon'}\sum_{z'} W^S(z',\varepsilon';p,p_u) J^d(z') g^u(\varepsilon')\right)\Bigg]. \end{aligned} \tag{9}$$

Thus, the specialized input price depends only on the value functions of the standardized manufacturer and supplier. Moreover, I assume that a standardized manufacturer which optimally chooses to become vertically integrated makes a take-it-of-leave-it offer to the supplier and pays to him a price P_{VI} that is the present discounted value of being a standardized supplier. This is, we assume that the market value of the supplier is $P_{VI} = \beta E_{z',\varepsilon'} W^S(z',\varepsilon';p,p_u)$.

2.2.4 Free Entry Condition

There is free entry of manufacturers who are ex-ante identical. We assume that manufacturer firms that enter the industry make no specific investment. This means that entrants cannot enter the industry being vertically integrated or linked firms, they just enter as standardized manufacturers.

They must pay a sunk downstream entry cost, $C_e^d \geq 0$, the fixed cost of production, $C_f \geq 0$ and the buy one unit of the standardized input paying p_u After that, they draw z from $g^d(z)$ and then match randomly with a supplier according to $g^u(\varepsilon)$. For entrants that survive, for next period, their productivity shocks, z, evolve according to $F(z'/z)$. Thus, the value of the expected future discounted profits of a new downstream firm is

$$V_e^d(p,p_u) = \sum_{\varepsilon}\sum_{z} V^S(z,\varepsilon;p,p_u) g^d(z) g^u(\varepsilon). \tag{10}$$

In the input industry there is also free entry. Entrants are ex-ante homogeneous producers and enter the input industry as standardized suppliers. They first have to pay a sunk upstream entry cost, $C_e^u \geq 0$, and fixed cost, $C_f^u \geq 0$. After doing so, they earn p_u and matches randomly with a manufacturer, according to $J^d(z)$, and their type ε is revealed according to $g^u(\varepsilon)$. Thus, the value at entry for an upstream firm is

$$W_e^u(p_u, p) = \sum_{\varepsilon} \sum_{z} W^S(z, \varepsilon; p, p_u) J^d(z) g^u(\varepsilon). \tag{11}$$

2.3 Characterization of Equilibrium

Before defining the stationary equilibrium in this model we first make some additional assumptions and state new definitions. As mentioned before, the analysis in this section considers the existence of complementarity between manufacturer and supplier's types but, assumption that will be confirmed in the calibration. Thus, let us assume that the variable cost function $c(z, \varepsilon)$ satisfies increasing differences.[13] In other words, manufacturers of different types can produce more efficiently with a supplier of high ε-type, but the cost advantage is greater for producers of high z-types. Therefore, assuming a given functional form for $c(z, \varepsilon)$ we can plot $c(z, \varepsilon) + Cf$ (solid grey curves in Figure 4) which is weakly decreasing in ε, together with the revenue function, pz, (solid black curve in Figure 4) for a standardized manufacturer. The distance between the latter and $p_u + Cf$ is the per period profit of a standardized manufacturer.

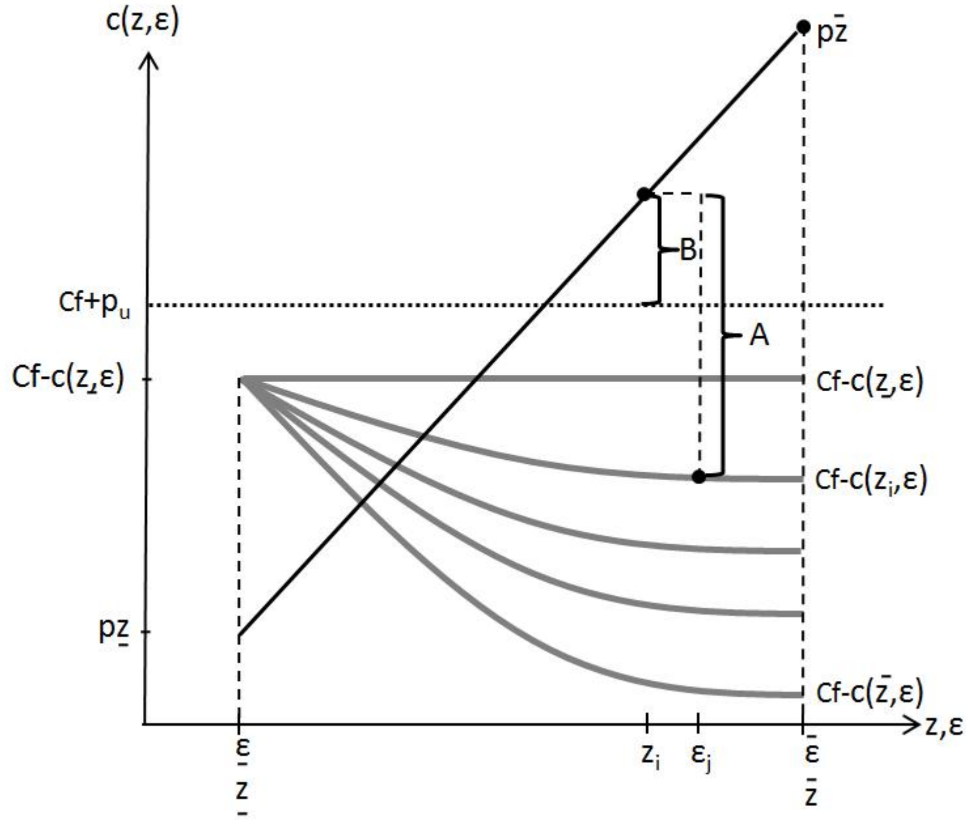
The upper curve $c(z, \varepsilon) + Cf$ (the straight line) in in Figure 4 represents the case of the least efficient manufacturer (denoted by $\underline{z}$) . We can see that when it is matched with the most efficient supplier (denoted by $\overline{\varepsilon}$) it does not improve in costs. In contrast, when the most efficient manufacturer (denoted by $\overline{z}$) is matched with the most efficient supplier (denoted by $\overline{\varepsilon}$) there is

[13] A manufacturer of type z that is matched with a supplier of type ε has cost advantage $c(z, \varepsilon)$ that satisfies the following property:

$$c(z_i, \varepsilon_j) - c(z_i, \varepsilon_{j-1}) > c(z_{i-1}, \varepsilon_j) - c(z_{i-1}, \varepsilon_{j-1}) \;\forall i, j = 1, .., n$$

a big decline in total costs (lower curve).

Figure 4: Costs, revenues and profits of a neither VI nor linked firm.



Furthermore, for a standardized manufacturer of type z_i matched with a supplier of type ε_j, the static gain from using specialized inputs (net of costs corresponding to the cases of VI or link) is the difference between the distances A and B. Clearly, as it can be seen in the picture, the static gain from using specialized inputs is increasing in z and ε. We will use this property of the profit functions, together with the characteristics assumed on F, to state that the value of investing in the use of specialized inputs is increasing in z and ε.

To gain more intuition about how the model works let's show which vertical structure a manufacturer chooses for next period given the current productivity. In the following proposition we focus on a standardized manufacturer firm, but the same reasoning should be followed for the case of a vertically integrated firm and a linked one:

Proposition 1 There exist two sets of pairs (z, ε) that define sets of thresholds $S^* \subset Z \times E$ and $\widetilde{S} \subset Z \times E$ for a a standardized manufacturer firm such that:

i) If $(z,\varepsilon) \notin UCS(S^*)$, the firm exits the industry (where $UCS(S^*)$ is upper contour set of S^*)

ii) If $(z,\varepsilon) \in UCS(\widetilde{S})$, the firm stays in the industry and decides to be vertically integrated or set up links

iii) If $(z,\varepsilon) \notin UCS(\widetilde{S})$ and $(z,\varepsilon) \in UCS(S^*)$, the firm stays in the industry being a standardized manufacturer firm.

Proof Let's first define z^* as the minimum productivity level at which a standardized manufacturer, before observing the current supplier type ε it is matched with, decides to stay in the industry and get a new draw of supplier for next period. Let's compare the two continuations values in the first term of equation (1). Given that F is decreasing in z and $c(z,\varepsilon)$ is increasing in z, the continuation value of getting a new draw of ε, $E_{z',\varepsilon'}\left[V^S(z',\varepsilon';p,p_u)/z\right]$, is monotone increasing in z. Therefore, as $V^S(\cdot)$ is continuous in z, by the intermediate value theorem there exists a thresholds z^* and it is singled valued, and defined as in Hopenhayn (1992):

$$z^* = \inf\left\{z \in Z : \sum_{z'}\sum_{\varepsilon'} V^S(z',\varepsilon';p,p_u) f(z'|z) g^u(\varepsilon') \geq 0\right\}.$$

Now let's focus on finding $\widetilde{S}$ and postpone S^* for a moment. For that we are looking for the set of minimum productivity levels for z and ε at which the current and expected continuation value of using specialized goods (becoming vertically integrated or linked) is greater than or equal to being a standardized manufacturer. We know that for pairs of (z,ε) formed by low values of z and ε, given the assumptions on costs and sunk specific investment, the firm does not decide to become vertically integrated or set up links. Furthermore, in order to have available the continuation values corresponding to VI or L the firm has to invest $h + P_{VI}$ or h, respectively. This means that the corresponding expected future discounted profits plus present revenues must be high enough to recover the costs $h + P_{VI}$ or h. But, given that the continuation values of becoming vertically integrated or linked are monotone increasing in z and ε, and as $V^{VI}(\cdot)$ and $V^L(\cdot)$ are continuous in z and ε, for each value of z, by the intermediate value theorem, there exists a level for ε (a threshold), which is singled valued, at which the standardized manufacturer decides to become vertically integrated or linked. This reasoning allows us to define a correspondence $\widetilde{S}(z)$ that maps values of z into values for ε, $\widetilde{S}(z) : Z \to \widetilde{E}$, where $\widetilde{E}$ is a subset of E. Thus $\widetilde{S}(z)$ is formally defined as

$$\widetilde{S}(z) \equiv \left\{ \varepsilon \in \widetilde{E} : \text{given } z, \left(\begin{array}{c} \max \left\langle \underbrace{V^L(z,\varepsilon;p,p_u^L)}_{\text{Value of becoming Linked today}} - h, \underbrace{V^{VI}(z,\varepsilon;p)}_{\text{Value of becoming VI today}} - h + P_{VI} \right\rangle \\ \geq pa_s(z;p,p_u)z - p_u - C_f + \beta \max \left\{ \underbrace{0}_{\text{Exit}}, \underbrace{\sum_{z'} \sum_{\varepsilon'} V^S(z',\varepsilon';p,p_u) f(z'|z) g^u(\varepsilon')}_{\text{Standardized (new draw of supplier)}} \right\} \end{array} \right) \right.$$

Then, the set $\widetilde{S}$ is defined as

$$\widetilde{S} \equiv \left\{ (z,\vartheta) \in Z \times \widetilde{E} : \begin{array}{l} i)\ \vartheta \equiv \inf \left(\widetilde{S}(z) \right),\ \text{and} \\ ii)\ \widetilde{S}(z) \neq \varnothing \end{array} \right\}.$$

Thus, the definition for the $UCS(\widetilde{S})$ is as follows

$$UCS(\widetilde{S}) \equiv \left\{ \left(z, \widetilde{S}(z) \right) \in Z \times \widetilde{E} : \left\{ \begin{array}{l} i)\ \widetilde{S}(z) \text{ defined as before, and} \\ ii)\ \widetilde{S}(z) \neq \varnothing \end{array} \right. \right\}.$$

In words, all the values $(z,\varepsilon) \in UCS(\widetilde{S})$ define a subset of $Z \times E$ at which the downstream firm decides to be vertically integrated or have links with the supplier for the following period. This means that the productivity of the match is high enough so that the manufacturer wants to keep the same supplier and it covers all the corresponding investment costs.

Now we can use the previous defined objects (thresholds z^* and $\widetilde{S}$) to define S^*.

$$S^* \equiv \left\{ (z,\varepsilon) \in Z \times E : \left\{ \begin{array}{ll} z = z^*,\ \forall \varepsilon & \text{if } z^* \cap \widetilde{S}_1 = \varnothing \\ (z,\varepsilon) \in (z^*,\widehat{\varepsilon}) \cup (\widetilde{z},\widetilde{\varepsilon}) & \text{if } z^* \cap \widetilde{S}_1 \neq \varnothing \end{array} \right. \right\}, \tag{12}$$

where $\widehat{\varepsilon} = \{\varepsilon : \varepsilon = \inf(\widetilde{S}(z^*))\}$; $\widetilde{z} = \{z : z \leq z^*\}$, $\widetilde{\varepsilon} = \{\varepsilon : \varepsilon \in \widetilde{S}(\widetilde{z})\}$ and $\widetilde{S}_1$ is the first element of all the pairs defined by $\widetilde{S}_1$.■

Basically the next proposition states that if a standardized manufacturer firm with a given productivity pair (z,ε) decides to become vertically integrated or linked, then any firm with higher efficiency levels (z,ε) will also become vertically integrated or linked.

Proposition 2 Given $(\widetilde{z},\widetilde{\varepsilon}) \in \widetilde{S}$, $\forall z \geq \widetilde{z}$, $\varepsilon \geq \widetilde{\varepsilon} : UCS(z,\varepsilon) \subseteq UCS(\widetilde{z},\widetilde{\varepsilon})$.

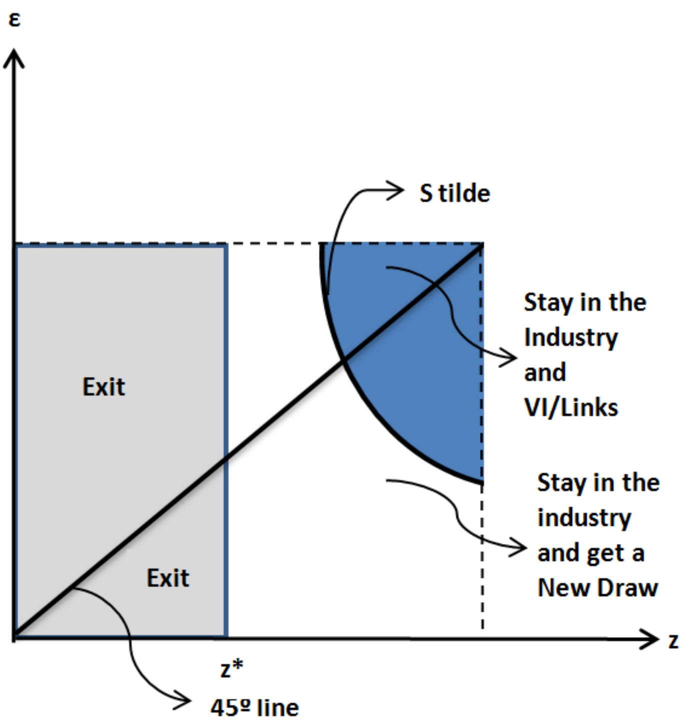
Proof Take $(z_n, \widetilde{S}(z_n)) \in \widetilde{S}$. As the maximum continuation value for the firm is $E_{z'}[V^{VI}(\cdot)]$ or $E_{z'}[V^L(\cdot)]$, and given that $c(z,\varepsilon)$ is increasing in ε, both of these continuation values are increasing in ε. Then $\forall\varepsilon > \widetilde{S}(z_n)$ we have that $(z_n,\varepsilon) \in UCS(z_n,\widetilde{S}(z_n))$. Furthermore, $UCS(z_n,\varepsilon) \subset UCS(z_n,\widetilde{S}(z_n))$. By the same argument, if $\exists(z_{n-j},\widetilde{S}(z_{n-j})) \in \widetilde{S} \Rightarrow \forall\varepsilon > \widetilde{S}(z_{n-j})$ we have that $(z_{n-j},\varepsilon) \in UCS(z_{n-j},\widetilde{S}(z_{n-j}))$ and $UCS(z_{n-j},\varepsilon) \subset UCS(z_{n-j},\widetilde{S}(z_{n-j}))$ $\forall j = 1,..,n-1$. Moreover, as $E_{z'}[V^{VI}(\cdot)]$ and $E_{z'}[V^{L}(\cdot)]$ are also increasing in z, given $(z_i,\varepsilon_h) \in UCS(\widetilde{S}) \Rightarrow \forall(z,\varepsilon)$ s.t. $z \geq z_i, \varepsilon \geq \varepsilon_h$ we have that $UCS(z,\varepsilon) \subseteq UCS(z_i,\varepsilon_h)$ for $i,h = 1,..,n$.$\blacksquare$

Intuitively the previous two propositions are a characterization of the decision rules for a standardized manufacturer. They state that, under the assumptions made on costs, these decision rules look like presented in Figure 5. In the horizontal axis we have the productivity of the manufacturer and in the vertical axis the productivity of the supplier. The figure shows the regions of (z,ϵ) under which a standardized manufacturer decides to exit the industry, to become vertically integrated, to set up link or to continue standardized for next period.

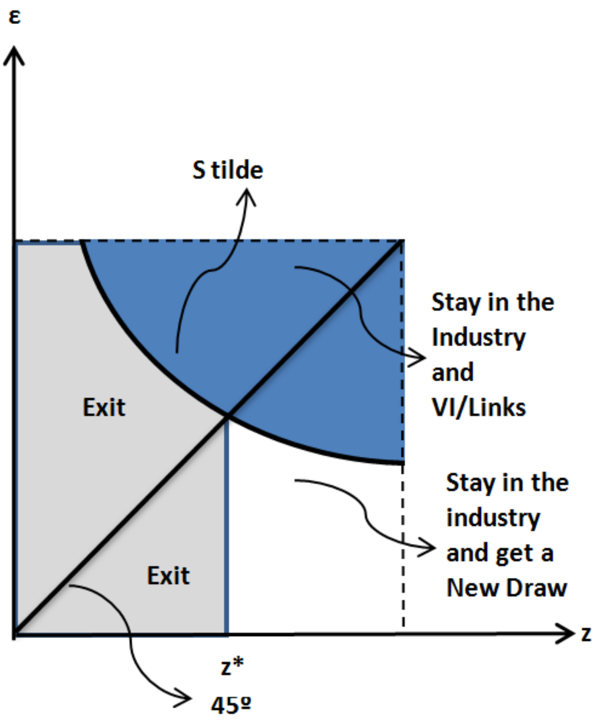
In panel A we have the case in which $z^* \cap \widetilde{S} = \varnothing$ in our expression (12), and thus there is only one relevant threshold (z^*) that manufacturers consider to exit the industry. This is, a manufacturer with a productivity shock bellow z^* decides to exit the industry independently to which supplier's type it is matched with. If its productivity level z is above that threshold, the firm decides to remain active in the industry, and if it is matched with an efficient supplier it decides to become vertically integrated or linked.

In panel B we have the case in which $z^* \cap \widetilde{S} \neq \varnothing$ in our expression (12), and thus there is a set of relevant thresholds (S^*) that manufacturers consider to exit the industry. Furthermore, in contrast with Panel A, a manufacturer with a productivity shock bellow z^* can survive if it is matched with an efficient supplier. The equilibrium shape of the set of relevant thresholds will depend on the parametrization of the model. We will focus on that in the calibration section.

Figure 5: Decision rule for a (z, ε)-standardized manufacturer for next period.



Panel A



Panel B

2.4 Stationary Equilibrium

Because there is a continuum of firms that are subject to idiosyncratic shocks, there is a cross sectional distribution of firms over the states (z, ε) and over different vertical structures. We call Φ^S the stationary distribution of downstream standardized firms, and Φ^{VI},Φ^L,Ξ^S and Ξ^L the stationary distribution of vertically integrated manufacturers, linked manufacturers, standardized suppliers and specialized suppliers, respectively. Let′s define $D(p)$ as the aggregate demand, that is continuous and strictly decreasing. Then, the stationary equilibrium is standard:

A stationary equilibrium in this model is a list of value functions for manufacturers and suppliers $(V^S(z, \varepsilon; p, p_u)$, $V^L(z, \varepsilon; p, p_u)$, $V^{VI}(z, \varepsilon; p)$, $W^S(z, \varepsilon; p, p_u)$, $W^L(z, \varepsilon; p, p_u)$, $V_e^d(p)$, $W_e^u(p_u))$, policy functions $(a_S(z, \varepsilon; p, p_u)$, $x'_S(z, \varepsilon; p, p_u)$, $a_L(z, \varepsilon; p, p_u)$, $a_{VI}(z, \varepsilon; p)$, $x'_{VI}(z, \varepsilon; p))$, prices p

and p_u and price functions $p_u^L(z,\varepsilon;p)$ and $P_{VI}(z,\varepsilon)$, invariant measures for downstream standardized firms Φ^S, vertically integrated firms Φ^{VI} and linked firms Φ^L and invariant measures for upstream standardized firms Ξ^S and upstream linked firms Ξ^L, an invariant density $J^d(z)$, a mass of downstream and upstream entrants m^d and m^u, and sets of thresholds S^* and $\widetilde{S}$, given the aggregate demand function for final goods $D^d(p)$ such that:

i) Input prices $p_u^L(z,\varepsilon;p)$ and acquisition prices $p_{VI}(z,\varepsilon)$ are given by NBS

ii) Given p, p_u, $p_u^L(z,\varepsilon;p,p_u)$ and $P_{VI}(z,\varepsilon)$, policy functions $a_S(z,\varepsilon;p,p_u)$, $a_{VI}(z,\varepsilon;p)$ and $a_L(z,\varepsilon;p,p_u)$ solve the static input decisions

iii) Given p, p_u, $p_u^L(z,\varepsilon;p,p_u)$ and $P_{VI}(z,\varepsilon)$, policy functions $x'_S(z,\varepsilon;p,p_u)$ and $x'_{VI}(z,\varepsilon;p)$ solve the dynamic decisions of firms

iv) Free entry conditions are satisfied for manufacturers

$$C_e^d = V_e^d(p;p_u) = \sum_{\varepsilon}\sum_{z} V^S(z,\varepsilon;p,p_u)g^d(z)g^u(\varepsilon), \tag{13}$$

and for suppliers

$$C_e^u = W_e^u(p_u;p) = \sum_{\varepsilon}\sum_{z} W^S(z,\varepsilon;p,p_u)J^d(z)g^u(\varepsilon). \tag{14}$$

v) Market clearing conditions are satisfied in the market for final goods $D^d(p) = S^d(p)$ and in the market for standardized inputs $D^u(p_u) = S^u(p_u)$ where

$$\begin{aligned} S^d(p) = \ & \sum_z\sum_\varepsilon z\Phi^S(z,\varepsilon) + \sum_z\sum_\varepsilon z a_{VI}(z,\varepsilon;p)\Phi^{VI}(z,\varepsilon) \\ & + \sum_z\sum_\varepsilon z a_L(z,\varepsilon;p,p_u^L)\Phi^L(z,\varepsilon). \end{aligned} \tag{15}$$

vi) Laws of motion of states are consistent with individual decisions (stationary measures Φ^S,Φ^{VI},Φ^L,Ξ^S and Ξ^L are fixed points). As mentioned before the heterogeneity of a market firm is described by $\Phi^S(B)$ measure on $(S,\mathcal{B})$, where $S = Z\times E$ and B_s = all possible subsets of S, and $B\epsilon\mathcal{B}_s$. Then we have the following fixed point of the form

$\Phi^S = T(\Phi^S, m^d)$:

$$
\begin{aligned}
\Phi^S(B) = \; & \underbrace{\sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \underbrace{\Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right)}_{\text{Element of the Markov chain}} \quad \underbrace{I^S(x'_S(z,\varepsilon;p,p_u)=S)}_{\text{Indicator function from policy functions}} \quad \Phi^S(z,\varepsilon)}_{\text{Incumbent who survive}} \\
& + \underbrace{\sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{VI}(x'_{VI}(z,\varepsilon;p)=S)\Phi^{VI}(z,\varepsilon)}_{\text{Vertically Integrated Incumbent who survive}} \\
& + \underbrace{\sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{L}(x'_{L}(z,\varepsilon;p,p_u^L)=S)\Phi^{L}(z,\varepsilon)}_{\text{Linked Incumbent who survive}} \; \forall B\epsilon\mathcal{B}_s. \\
& \underbrace{\sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \underbrace{\Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right)}_{\text{Element of the Markov chain}} \quad \underbrace{I^S(x'_S(z,\varepsilon;p,p_u)=S)}_{\text{Indicator function from policy functions}} \quad m^d g^d(z) g^u(\varepsilon))}_{\text{Entrants}}
\end{aligned} \tag{16}
$$

In a similar way, the heterogeneity of incumbent downstream firms that are vertically integrated and linked is described by $\Phi^{VI}(B)$ measure on $(S, \mathcal{B})$ and $\Phi^L(B)$ measure on $(S, \mathcal{B})$. Then we have $\Phi^{VI} = T^{VI}(\Phi^{VI})$ and $\Phi^L = T^L(\Phi^L)$:

$$
\begin{aligned}
\Phi^{VI}(B) = \; & \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^S(x'_S(z,\varepsilon;p,p_u)=VI)\Phi^{ND}(z,\varepsilon) \\
& + \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{VI}(x'_{VI}(z,\varepsilon;p)=VI)\Phi^{VI}(z,\varepsilon) \\
& + \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{L}(x'_{L}(z,\varepsilon;p,p_u^L)=VI)\Phi^{L}(z,\varepsilon) \; \forall B\epsilon\mathcal{B}_s.
\end{aligned} \tag{17}
$$

And finally, we have the following fixed point for the measures of linked firms

$$
\begin{aligned}
\Phi^{L}(B) = \; & \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^S(x'_S(z,\varepsilon;p,p_u)=L)\Phi^{ND}(z,\varepsilon) \\
& + \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{VI}(x'_{VI}(z,\varepsilon;p)=L)\Phi^{VI}(z,\varepsilon) \\
& + \sum_{z}^{s_n}\sum_{\varepsilon}^{s_n} \Pr\left(\left(z',\varepsilon'\right)\epsilon B|z,\varepsilon\right) I^{L}(x'_{L}(z,\varepsilon;p,p_u^L)=L)\Phi^{L}(z,\varepsilon) \; \forall B\epsilon\mathcal{B}_s.
\end{aligned} \tag{18}
$$

vii) The mass of suppliers, m^u, equal the mass of standardized and linked manufacturers

$$m^u = \sum_{z}\sum_{\varepsilon} \Phi^S(z,\varepsilon) + \sum_{z}\sum_{\varepsilon} \Phi^L(z,\varepsilon)$$

In the appendix, it is explained the algorithm used to compute the equilibrium.

3 Quantitative Analysis

3.1 Calibration-Preliminary Results

To solve the model numerically, we need to specify functional forms for the demand and firms technology and assign parameter values. Basically, we calibrate our model so that the industry stationary equilibrium matches selected characteristics of the U.S. manufacturing sector taken from the U.S. Census Bureau and from Hortaçsu and Syverson (2007 and 2009). Table 2 summarizes the values for the parameters set a priori.

Table 2: Parameters set a priori

Parameters	Definition	Value	
θ	Bargaining power of the buyer	0.5	assumed
β	Discount factor	0.96	assumed
B	Inverse of demand elasticity	1.164	Nicholson (1989)
ρ	Autoregressive parameter	0.93	Hopenhayn and Rogerson (1993)

Manufacturers and suppliers are assumed to have the same bargaining power, $\theta = 1/2$. In addition, we set a discount factor value $\beta = 0.96$ consistent with a 4% interest rate. We assume a constant elasticity of demand, $p = \frac{A}{Y^B}$, where Y is the aggregate production, A is a scaling factor and B is the inverse demand elasticity which we take equal to 1.164.[14] The parameter A is normalized to one (it has no impact on the relevant endogenous variables).[15]

We assume that shocks z has lognormal distribution and follows an $AR(1)$ process,

$$ln\ z_t = \delta + \rho \ln\ z_{t-1} + \mu_t, \quad \text{with } \mu_t \sim N(0, \sigma_\mu^2),$$

where μ_t is the *iid* shock, and the parameter ρ is a measure of persistence of the idiosyncratic productivity process. Changes in the persistence of the shocks will have an impact on how a firm decides its vertical structure given the properties of the costs. Therefore, if persistence is very high, then, loosely speaking, an efficient firm expects that high shocks today will be around for a long time. Conversely, if shocks are not very persistent, then the manufacturer will take into account the possibility of incurring high losses (due to high fixed costs) or not recovering the

[14] We take the average of the elasticity values published in Nicholson (1989): Food 0.21, Medical Services 0.20, Automobiles 1.20, Housing (Rental) 0.18, Housing (Owner-Occupied) 1.2, Gasoline 0.54, Electricity 1.14, Giving to Charity 1.29, Beer 1.13, Marijuana 1.5.

[15] Sensitivity analysis with respect to A, B, θ and other parameters was performed and it is presented later on.

irreversible investment ($h+P_{VI}$), because there is a strong possibility that they will be incurred relatively soon.

A 25-points grid was assumed for both discretized shocks z and ε, where we assume $Z = E$ to simplify.[16] The transition matrix for z was obtained by Tauchen's method which approximates the previous $AR(1)$ process for the idiosyncratic shocks. The estimation of its persistence parameter ρ was taken from Hopenhayn and Rogerson (1993), assuming that firms in both models are hit by the same stochastic idiosyncratic productivity process[17]. We took the invariant distribution of the Markov chain matrix for z as the initial distribution $g^d(z)$ and as $g^u(\varepsilon)$.

With respect to the function $c(z, \varepsilon)$ we assume a function as follows

$$c(z_i, \varepsilon_j) = T_1 \left(\frac{z_i - z_1}{z_n - z_1} \right)^{\alpha} \left(\frac{\varepsilon_j - \varepsilon_1}{\varepsilon_n - \varepsilon_1} \right)^{1-\alpha} + T_2 I_{\{VI,Link\}},$$

which is increasing in z_i and ε_j, with $\alpha \in [0, 1]$. The parameter T_1 is the maximum gain from searching a supplier, for the most efficient manufacturer (being z_n and matched with an ε_n supplier reduces the nonsunk cost T_1); and T_2 is the gain from investment (by investing $h+P_{VI}$ in becoming vertically integrated, or h in becoming linked, the manufacturer reduce the nonsunk cost in this amount T_2, independently on the type of the supplier it is matched with). The parameter α indicates how important is the manufacturer's type in the effect of the cost reducing investment. Notice that $c(z, \varepsilon)$ is flexible, in the sense that it allows for the absence of increasing differences.

Table 3 presents the value for the calibrated parameters with the corresponding moments the model tries to match. Figure 6 shows the shape of the function $c(z_i, \epsilon_j)$ for the parameter values presented above.

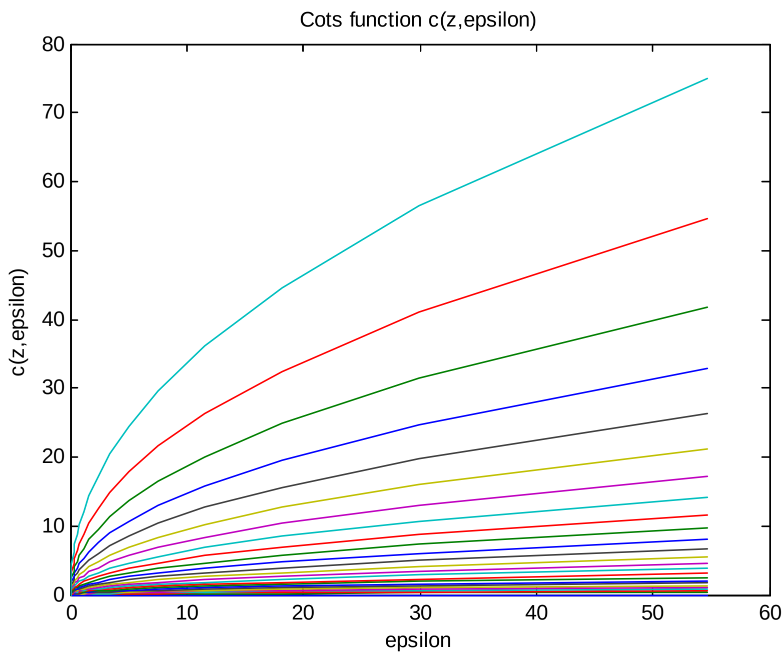
[16] The number of grid points was selected so as to have a smooth enough behavior of firms' decisions.

[17] One can also assume that, under a Leontieff production function, employment follows the same stochastic process as revenues.

Table 3: Calibrated Parameters and moments to fit.

	Definition		Target	Definition
δ	Autoregressive intercept	0	revenue distrib. of firms	
σ^2_μ	Standard deviation of μ.	0.15		
T_1	Gain from searching for high ε	75		
T_2	Cost reduction	0		
C_f	Fixed cost	0.40	10%	Annual exit rate (Bartelsman, Haltiwanger and Scarpetta 2000)
λ	Extra managerial fixed cost of a vertically integrated firm	3.15	$8\% - 9\%$	%VI firms (Hortaçsu and Syverson 2009)
h	Investment cost of L	1.3	25%	% L firms (Uzzi 1996)
α	Relative weight of z in cost reduction	0.47	7%	times the median-sized manufacturing plant is VI (Hortaçsu and Syverson, 2009)
C^d_e	Sunk cost of entry	3.01	$V^e(1)$	Entry value given $p = 1$

Figure 6: Cost function $c(z, \epsilon)$



The value of the intercept, δ, and the variance of the error term, σ^2_μ, of the AR(1) stochastic process for z, as well as T_1 and T_2 are chosen so as to fit the size (revenue) distribution of firms of the US manufacturing sector. Revenue values in the model are expressed in millions of dollars. In particular, we use the U.S. Census Bureau tabulated data prepared by the Small Business Administration (SBA) for year 2002.

Table 4 that indicates a mean revenues for all firms of 11,434 millions of dollars. In addition, the share of firms in the first interval of revenues (0-0.99) of 51.45%, and the shares of firms with revenues between (1-4.99), (5-9.99) and (10-49.99) are 22.7%, 5.7% and 7.5%, respectively. Finally, the share of the biggest firms that have revenues above 50 millions is 12.6%. Hence we choose δ, σ_μ^2, T_2 and T_2 in order to minimize the Euclidean distance between the data and model densities of firms in each scale interval so as to generate a revenue distribution that is in line with Table 4.

Table 4: Size (revenue) distribution of firms

	Receipt Size of Manufacturing Establishments (in millions of dollars)					
	Total	0-0.99	1-4.99	5-9.99	10-49.99	50+
Establishments	344,341	177,099	78,026	19,774	25,893	43,549
		51.4%	22.7%	5.7%	7.5%	12.6%
Receipts ($000)	3,937,164,576	56,607,235	173,543,614	122,826,132	361,399,818	3,222,847,777
		1.4%	4.4%	3.1%	9.2%	81.9%
Mean	11,434					

Source : Based on Census Bureau 2002 tabulated data prepared by the SBA.

The fixed cost C_f is selected to fit an exit rate of 10% (taken from Bartelsman, Scarpetta and Shivardi, 2003) given a normalized final good price $p = 1$; and the level for the sunk entry cost C_e^d was selected so as to satisfy the free entry condition of manufacturers. In addition, the value for fixed cost, C_f^u, as well as the entry cost, C_e^u, of suppliers were assumed to be equal to the fixed cost C_f and entry cost of manufacturers, respectively.

The extra managerial cost for a vertically integrated manufacturer, λ, and the investment cost, h, were chosen to match a share of 8 to 9 % of vertically integrated firms and a share of linked firms 25%, respectively.[18] And finally, the value for the relative weight of z in cost

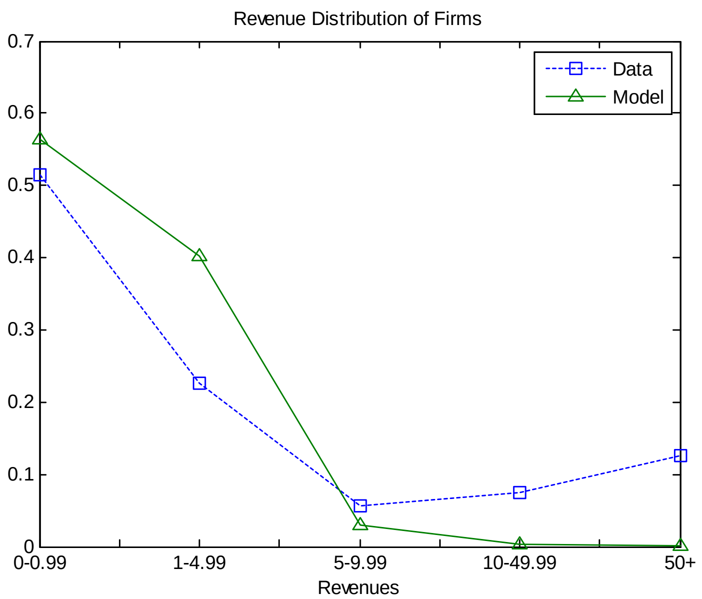
[18] Uzzi (1996) studies the Women's Dress industry where manufacturers and contractors are linked by long-term ongoing relationships. He finds that about 25 percent of the manufacturers have networks composed of 5 or fewer exchange partners; 30 percent have exchanges with 5 to 12 partners, while about 40 percent maintain business ties with more than 20 contractors. We take a value of 25% for our calibration given that in our model each manufacturer is supplied with just one supplier. Notice that the exercise we will perform in the following section is to decrease the persistence of the z shocks and look at what happen with the number and share of VI and L firms. And the value of the H's parameters determines the sensitivity of the decision rules to the persistence of z.

complementarity, α, was chosen so as to fit the percentage of median sized manufacturing plants that are vertically integrated.[19] Table 5 shows the calibration results. It can be seen that the annual exit rate, the share of vertically integrated and linked firms, and the percentage of vertically integrated plants in the median-sized plants are well fitted, while the fit of the size distribution of firms can be improved (Figure 7).

Table 5: Data moments and model moments.

	Model	Data
Share of firms by size		
(revenues in millions of U.S. dollars)		
0-0.99	56.4%	51.4%
1-4.99	40.2%	22.7%
5-9.99	3.1%	5.7%
10-49.99	0.4%	7.5%
50+	0.0%	12.6%
Annual exit rate	8.6%	10%
Share of Linked firms	25.7%	25%
Share of vertically integrated firms	8.4%	8%-9%
Share of vertically integrated median-sized firms	5.2%	7%

Figure 7. Size distribution of firms.



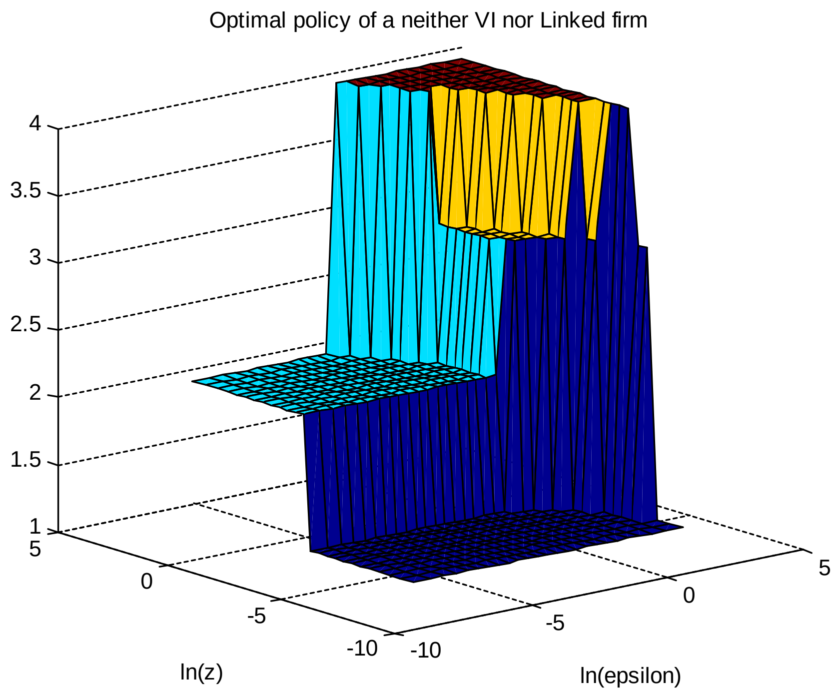
[19]The share of VI plants, as well as the percent of the median-sized plants that are integrated, were taken from Hortaçsu and Syverson (2009), as exposed in the introduction.

3.2 Benchmark Economy

3.2.1 Equilibrium decision rules, revenue distribution of firms and vertical relations.

Figure 8 shows the policy functions of a standardized firm. The associated values of the decision rule are as follows. The number 1 represents exit the industry, 2 stay in the industry and get a new draw of supplier (continue being standardized), 3 stay in the industry and set up a link, and 4 stay in the industry and become vertically integrated.

Figure 8. Policy function of a standardized firm.



In figure 9 we expose the same policy function as in figure 8 but in the (z, ε) plane. Thus, it shows the same results derived from the theoretical section 2.3, and, in particular, the issues exposed in figure 5. The areas plotted in Figure 9 correspond to the characterization of the decision rules made in Propositions 1 and 2.

The numbers inside each cell of Figure 9 are the numbers (and corresponding decisions) that indicate the height of the surface plotted in Figure 8. Cells containing the same number define the vertical status for different firms. Besides, the least efficient firms decide to exit the industry. As it was explained in section 2, firms with pairs of productivity levels below the set of thresholds S^* exit the industry (area indicated by cells containing number 1). Manufacturer firms that are efficient but matched with inefficient suppliers decide to continue active and get a new draw for next period (area indicated by cells containing number 2).

The most efficient manufacturer firms (the ones with highest levels of z) decide to become vertically integrated when they are matched with efficient suppliers. There are some manufacturers with intermediate productivity levels, which have drawn an efficient supplier, and decide to keep the same supplier by setting up a link (number 3-area). The increasing differences in cost function generates the correlation of types for high productivity levels.

Figure 9. Policy function of a standardized firm.

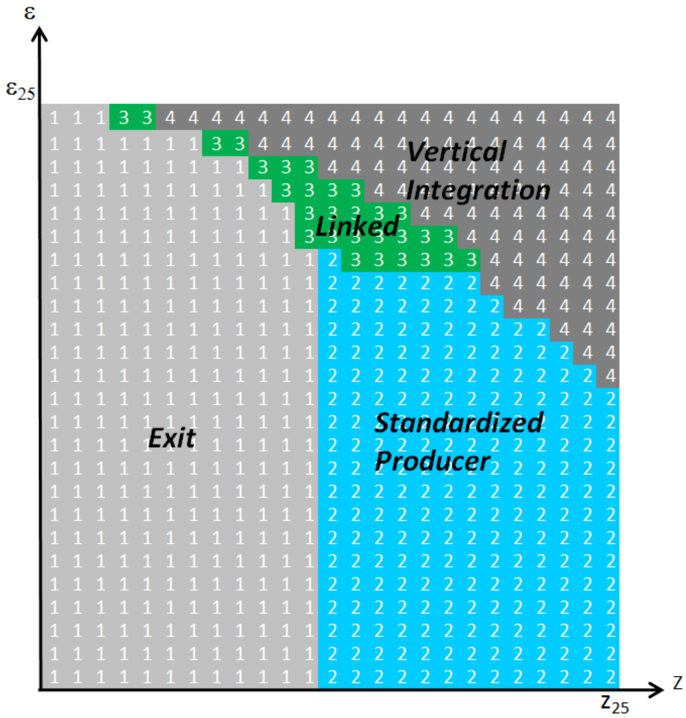
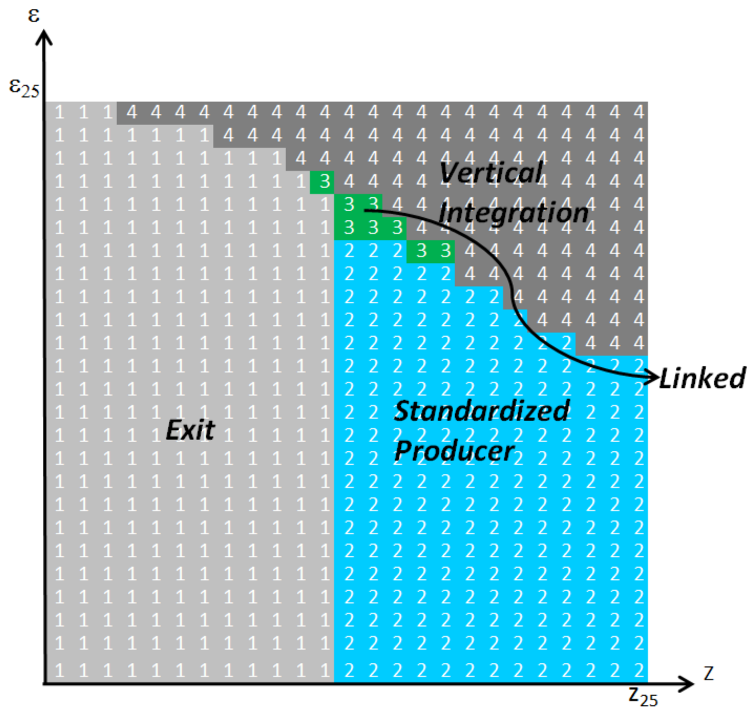


Figure 10 shows the decision rules of a vertically integrated firm and has the same interpretations as before. A particularly interesting point here is that the model generates vertical disintegration of plants. Moreover, identical manufacturers may differ in their vertical structure, and those that are vertically integrated can end up disintegrated or remain integrated. For example, taking a firm with high z-productivity and an intermediate upper level for ε, start decreasing the level for z and keep ε fixed (given that ε does not evolve over time). Then if its z-productivity decreases enough over time, this manufacturer will decides to disintegrate and become linked, outsourcing the input production. Furthermore, if it productivity continues to decrease, it may decide to change supplier or exit the industry.

Figure 10. Policy function of a VI firm.



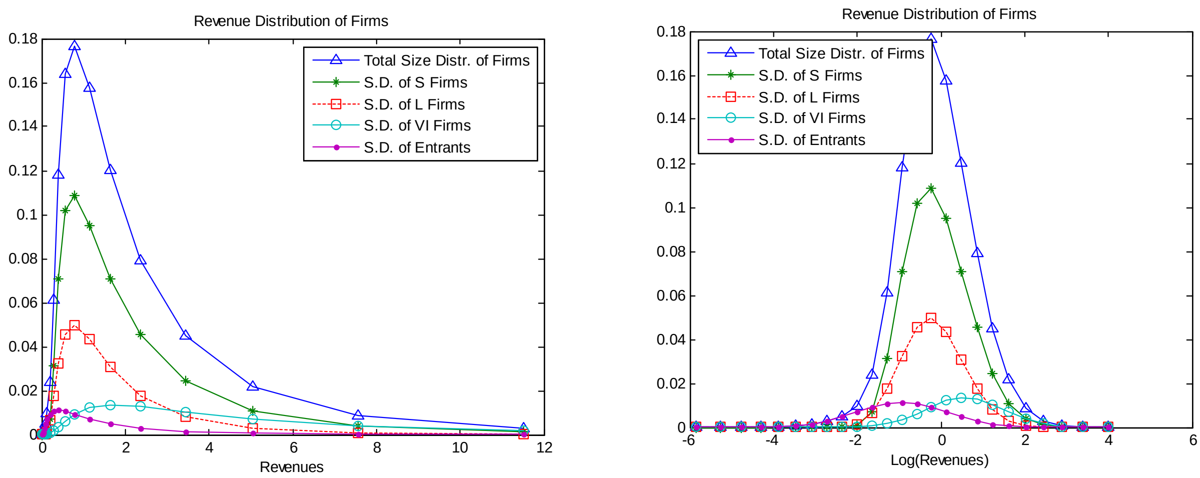
To summarize, we can see that our model induces the following behavior of firms. Vertically integrated manufacturer firms are larger and more efficient on average. Big and efficient standardized manufacturers that seek to expand though vertical integration choose suppliers that are also large and efficient as found in Hortaçsu and Syverson (2009).

In equilibrium the model generates some big manufacturers that are not vertically integrated, in line with the fact exposed in Figure 1. In Figure 11, panel A presents the equilibrium size (revenue) distribution of manufacturing plants[20]. The line with triangles represents the total size distribution of firms, while the other lines represent, for each size, the proportion of each type of firm (S, VI, L and Entrants) to the total share of firms for each particular size (this is, the area below each line adds up to the share of each category in the total number of plants).

[20] Figure 11 excludes the highest values for z so as to present a better exposition of the distributions at the lowest productivity levels. Figure 12 presents the whole range of the log of z.

Panel B shows the same picture in logarithmic scale.

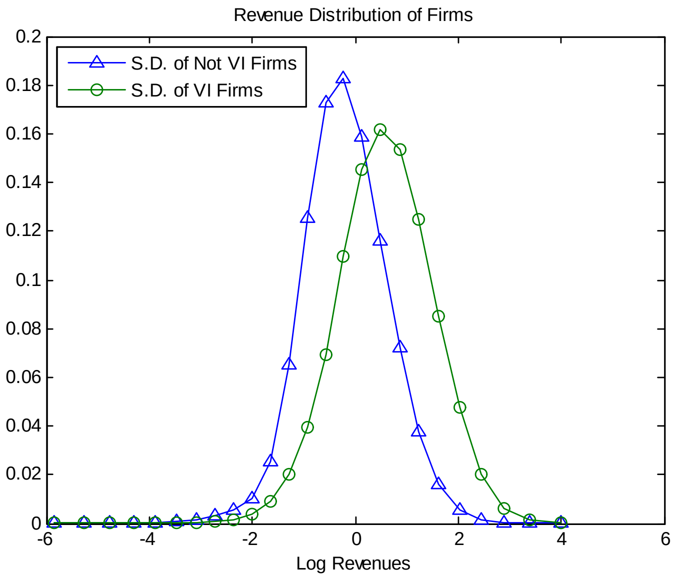
Figure 11. Size distribution of firms.



A-Revenue distribution of firms. B-Revenue dist. of firms (log. scale).

Notice that there is an overlap between these distributions: downstream firms with the same high z-productivity levels differ in their vertical structure in the steady state. The explanation for this, according to our model, is that some efficient manufacturers decide not to become vertically integrated and instead get a new draw while still looking for a more efficient supplier. The previous two graphs show that the fraction of vertically integrated plants increases with the plant size. In addition, it can also be seen that vertically integrated firms dominates (in first order stochastic dominance sense) to the size distribution of not vertically integrated firms. This last fact is exposed better in Figure 12 which presents just the size distribution of vertically integrated and not vertically integrated manufacturing plants (now each line is the share of plants as a proportion of all plants in a particular vertical structure -the total area below each line adds up to one-).

Figure 12. Size distribution of vertically integrated and not vertically integrated manufacturers.



3.3 How does the model work?

The model economy presented above, gives rise to rich industry dynamics as manufacturer enter, exit and decide how to obtain their inputs. In this environment an industrial structure emerges as the result of optimal investment decisions that firms undertake under uncertainty. Differences across industries that affect firms' incentives to use the VI or L margins determine firm level TFP dynamics and have an impact on profitability, survival, size distribution of firms and average productivity of an industry. In the following sections we use the model to addresses the questions on why supply relations vary across industries and across firms within industries, and how these relations affect size distribution of firms, turnover, mobility, welfare, aggregate output and productivity.

3.3.1 Bargaining power and vertical structure

In this section we analyze the effect of changes in the bargaining power of the manufacturer (Table 6). When the bargaining power of the manufacturer increases, downstream firms face a less severe hold-up problem. The average specialized input price, $p_u^L(\cdot)$, decreases from 1.39 to 1.17, which leads the manufacturers to become linked instead of vertically integrated. As it can be seen in the table, the share of vertically integrated firms decreases and the share of linked ones increases (the mass of vertically integrated and linked firms reacts in the same direction). The slight decline in the final good price from 1 to 0.98, that yields an increase in consumer surplus, together with a reduction in the average specialized input price, which generates an increase in producer surplus, yields a higher aggregate welfare. Furthermore, as the total investment increases, TFP increases.

Table 6: Changes in bargaining power

	θ		
	0.5	0.6	0.7
Price	1.00	0.99	0.98
Exit Rate	0.09	0.08	0.08
Agg. Output	100.0	100.0	100.4
TFP	100.0	102.1	103.3
Welfare	100.0	100.2	101.0
Consumer surplus	100.0	100.0	100.9
Producer surplus	100.0	100.9	105.7
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.084	0.072	0.071
$\frac{VI}{L}$	0.328	0.238	0.204

3.3.2 Costs of VI and L and vertical structure

Let's now focus on the specific investment cost, h. An increase in h generates a decline in the value at entry of manufacturers, and this leads to a higher final good price, lower output (thus lower consumer surplus), and higher exit rate (Table 7). As the cost of becoming linked is higher, relative to becoming vertically integrated, the ratio VI to L rises.

Despite the increase in the exit rate, there is a decline in TFP. The lower TFP level is caused by a decrease in the TFP of suppliers. Given that small and medium sized manufacturers use links more intensively relative to VI, the increase in h has a big impact on this group of firms. In addition, as small and medium sized manufacturers are more selective in the ε they choose to invest in, this leads to lower RTFP of suppliers (from 1.8 to 1.6). In line with this reasoning, it can be seen that some firms that invested in L, now do not invest at all, and some other ones invest in VI, as shown by the increase in the percentage of median-sized firms that invest in VI from 0.052 to 0.064. As a result, even though there is higher selection, TFP decreases, producer surplus decreases and total welfare decreases.

Table 7: Changes in specific investment and VI fixed costs.

	h			λ		
	1.3	1.4	1.5	3.15	3.25	3.35
Price	1.00	1.01	1.03	1.00	1.00	1.00
Exit rate	0.086	0.091	0.091	0.086	0.090	0.09
Agg. Output	100.0	98.9	97.9	100.0	100.0	100.0
TFP	100.0	99.1	97.6	100.0	101.4	101.4
Welfare	100.0	97.5	95.1	100.0	100.0	100.0
Consumer surplus	100.0	97.7	95.4	100.0	100.0	100.0
Producer surplus	100.0	91.9	84.2	100.0	99.9	99.6
Share of Vertically Integrated Firms						
$\frac{VI}{Total\ Firms}$	0.084	0.090	0.096	0.084	0.062	0.060
$\frac{VI}{L}$	0.328	0.427	0.589	0.328	0.251	0.242

When the additional managerial fixed cost of a vertically integrated manufacturer (λ) increases, the share of vertically integrated firms, as well as the ratio of vertically integrated to linked firms, decreases (Table 7). Furthermore, the increase in the fixed cost of a vertically integrated firm does not seem to have an effect on the value at entry of manufacturers, because the possibility to become a big vertically integrated firm is strongly discounted upon entry. Therefore, the equilibrium price remain the same as before (so does the consumer surplus), but the exit rate increases. In addition, the TFP increases a bit while producer surplus slightly decreases. Thus there is no effect on total welfare.

3.3.3 Complementarity and vertical structure

When T_1 increases, it increases the complementarity between manufacturer and supplier's type making the effects of cost reducing investment more important, thus the mass of firms that become vertically integrated and linked increases (Table 8). The exit rate decreases and it is cheaper to invest and thus to survive. The larger proportion of inefficient firms offsets the original decline in costs, thus the TFP decreases. Finally, total welfare increases.

Table 8: **Changes in complementarity.**

	T_1			T_2			α		
	70	75	80	-0.5	0	0.5	0.45	0.47	0.49
Price	1.02	1.00	0.97	1.06	1.00	0.838	1.00	1.00	0.99
Exit rate	0.090	0.086	0.086	0.093	0.086	0.072	0.090	0.086	0.088
Agg. Output	100.0	102.1	104.4	100.0	105.3	122.7	100.0	100.5	101.1
TFP	100.0	97.9	95.8	100.0	101.0	103.6	100.0	100.3	98.6
Welfare	100.0	104.8	109.9	100.0	112.6	158.1	100.0	101.3	102.4
Consumer surplus	100.0	104.8	110.0	100.0	112.2	157.1	100.0	101.2	102.4
Producer surplus	100.0	104.8	106.0	100.0	128.8	208.0	100.0	105.6	101.0
Share of VI									
$\frac{VI}{Total\ Firms}$	0.058	0.084	0.103	0.047	0.084	0.107	0.077	0.084	0.104
$\frac{VI}{L}$	0.235	0.328	0.431	0.294	0.328	0.233	0.331	0.328	0.480

The increase in T_2 generates an increase in the value at entry, which makes the equilibrium final good price and exit rate lower. When T_1 increases, every manufacturer increases VI and L with less efficient suppliers. In contrast, when T_2 increases it is the least efficient active manufacturers that were in the margin of setting up links and becoming vertically integrated the ones that start playing an important role in the total investment. As explained above, in figure 5, these group of manufacturers are more selective with respect to the supplier they choose to become vertically integrated or linked. They have to find a very efficient supplier in order to do so. Thus, an increase in T_2 generates an increase in TFP, in contrast with what happens when T_1 increases.[21]

The parameter α indicates how important is the manufacturer's type, z, relative to supplier's type, ε, in the effect of the cost reducing investment. If α increases it is less important than before, in terms of reductions in variable cost, how efficient is the supplier. Thus, when α increases it makes manufacturers less selective on the type of supplier they choose to invest in VI and L. As a result TFP decreases. In addition, the share of vertically integrated to linked manufacturers increases. Moreover, as it is easier to become more productive when linking or becoming vertically integrated (it depends less on how efficient is the supplier), the value at entry increases and the equilibrium price decreases. The decline in final good price leads to an increase in total production and consumer surplus. Finally, total welfare increases.

[21] The RTFP of suppliers increases from 1.6 to 2.2.

3.3.4 Discount factor and vertical structure

With respect to a change in the discount factor, as firms value more the future they have more incentives to invest, thus the total investment in VI and L increases (the measure of vertically integrated and linked firms rise), and the share of firms using specialized inputs increases (Table 9). As the value at entry increases, the equilibrium final good price decreases and consumer surplus increases. Given that there is less selection, in equilibrium there are more inefficient firms active in the industry, and TFP decreases. Furthermore, as the decrease in TFP does not seems to have a big impact on aggregate profitability, the total producer surplus increases and, as a result, total welfare increases.

Table 9: Changes in discount factor.

	β		
	0.95	0.96	0.97
Price	1.08	1.00	0.913
Exit Rate	0.090	0.086	0.083
Agg. Output	100.0	107.5	116.3
TFP	100.0	95.5	90.9
Welfare	100.0	117.0	138.7
Consumer surplus	100.0	117.3	139.6
Producer surplus	100.0	103.0	104.4
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.078	0.084	0.091
$\frac{VI}{L}$	0.342	0.328	0.341

3.3.5 Fixed entry and production costs and vertical structure

When manufacturer's fixed cost of production is higher, the equilibrium price increases and consumer surplus decreases (Table 10). The exit rate increases, which generates an increase in TFP. An increase in the fixed cost of suppliers has similar effects. In both cases total welfare decreases.

The effect of changes in entry costs of manufacturers and suppliers is as follows (Table 10 and 11). When C_e^d increases, the equilibrium price increases and production, as well as consumer surplus, decreases. The increase in price generates more investments in VI, in particular by small firms (the percentage of median sized manufacturing plants that are vertically integrated increases). There is also a relative increase in the share of big firms. This explains the rise in TFP. Although there is an increase in TFP, producer surplus decreases due to the decline of entry and the total mass of firms.

Table 10: Changes in fixed costs and entry costs.

	C_f			C_f^u			C_e^d		
	0.35	0.40	0.45	0.35	0.40	0.45	2.5	3.0	3.5
Price	0.96	1.00	1.04	0.96	1.00	1.04	0.93	1.00	1.07
Exit rate	0.08	0.08	0.09	0.08	0.08	0.09	0.08	0.08	0.09
Agg. Output	100.0	96.77	93.28	100.0	96.77	93.28	100.0	94.06	88.40
TFP	100.0	102.13	102.41	100.0	95.04	89.45	100.0	105.32	108.18
Welfare	100.0	93.19	85.87	100.0	93.19	85.85	100.0	87.70	76.52
Consumer surplus	100.0	92.99	85.70	100.0	92.99	85.70	100.0	87.34	76.09
Producer surplus	100.0	102.81	93.71	100.0	102.81	92.74	100.0	106.46	98.93
Share of VI									
$\frac{VI}{Total\ Firms}$	0.087	0.084	0.086	0.087	0.084	0.085	0.084	0.084	0.093
$\frac{VI}{L}$	0.219	0.257	0.220	0.219	0.257	0.217	0.223	0.257	0.210

A rise in the entry cost of suppliers induces an increase in the standardized input price (from 0.42 to 0.46). Thus, there is an increase the exit rate of manufacturers and in the final good price which yields a decline in consumer surplus. The increase in the standardized input price induces an increase in VI and a decline in L.

Table 11: Changes in entry costs.

	C_e^u		
	2.5	3.0	3.5
Price	0.97	1.00	1.025
Exit Rate	0.08	0.08	0.09
Agg. Output	100.0	97.85	95.80
TFP	100.0	101.48	101.30
Welfare	100.0	95.41	90.95
Consumer surplus	100.0	95.30	90.92
Producer surplus	100.0	100.20	92.36
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.084	0.084	0.093
$\frac{VI}{L}$	0.257	0.257	0.210

3.4 Idiosyncratic productivity shocks and vertical structure

In our theoretical framework we have three different types of manufacturer firms. First, a standardized manufacturer, which has no variable costs advantage relative to vertically integrated and linked firms. It is not subject to Hold-up and has lower fixed costs relative with a vertically integrated firm. Thus it performs better when facing negative shocks.

Second, a linked firm. It uses specialized inputs and is subject to Hold-up problem. It performs better than a vertically integrated manufacturer firm when negative shocks are realized (avoid higher fixed costs and bound losses).

And third, a vertically integrated firm which has the lowest variable costs. It is not subject to Hold-up. In addition, it pays higher fixed costs and requires higher investment costs ($h + P_{VI}$, which in equilibrium is much higher than h), then perform worst with negative shocks.

In this section we want to address the following question: what is the implication of making the evolution of the manufacturers productivity shocks less persistent? In table 12 we present the comparative statics results. It shows the effect of decreasing the persistence of shocks, ρ, on the vertical relation of the industry. By comparing the first column with the other ones, it can be seen that the share of vertically integrated manufacturers to linked ones decreases, as well as the share of vertically integrated manufacturers, while the mass of vertically integrated firms decreases and the measure of linked ones increases. Moreover, the share of firms that invest in using specialized inputs, $(VI + L)/Total\ Firms$, increases.

Because of cost reducing investment through VI are less attractive when there is a decline in the persistence, manufacturers value at entry decreases. Hence the equilibrium price increases. As a result, the equilibrium output decreases and consumer surplus is lower. In addition, the increase in final good price generates a lower exit rate. Despite the lower selection, there is an increase in producer surplus and total factor productivity (TFP) due to the fact that efficient manufacturers that invest in the use of specialized inputs become more selective about suppliers' type. In other words, in order to invest in VI or L, manufacturers wait more until they get matched with a better supplier. Thus suppliers' productivity increases.[22,23] Finally, as the decline in consumer surplus is bigger than the increase in producer surplus, the total welfare

[22] See the appendix for definitions of total factor productivity (TFP) and revenue TFP (RTFP).

[23] Revenue TFP of suppliers increases significantly, from 1.8 to 2.1, while the RTFP of manufacturers does not change.

decreases.[24]

Table 12: Changes in persistence and variance of shocks.

	ρ			σ^2_μ		
	0.93	0.92	0.91	0.13	0.15	0.17
Price	1.00	1.08	1.11	0.90	1.0	0.99
Exit rate	0.09	0.06	0.06	0.05	0.086	0.092
Agg. Output	100.0	93.6	91.4	100.0	91.3	92.3
TFP	100.0	104.2	108.4	100.0	98.7	93.8
Welfare	100.0	86.8	82.9	100.0	81.2	82.5
Consumer surplus	100.0	86.5	81.9	100.0	81.9	83.9
Producer surplus	100.0	112.6	132.8	100.0	60.0	39.7
Share vertically integrated Firms						
$\frac{VI}{Total\ Firms}$	0.084	0.062	0.059	0.130	0.084	0.041
$\frac{VI}{L}$	0.328	0.206	0.152	0.308	0.328	0.385

If ρ is high, firms anticipate that high shocks today will be around for long time. Thus, by becoming vertically integrated, they strongly discount the realization of a low shock (while paying high fixed costs). Therefore, many firms decide to become vertically integrated.

In contrast, if ρ is low, there is higher mobility across productivity states and the expected duration of being in a high idiosyncratic efficiency level is lower. There is a higher possibility of having a low shock relatively soon, incurring high losses (due to high fixed production costs) or not recovering the investment cost $(h + P_{VI})$. As a result, manufacturers become more flexible, which is reflected by a lower VI to L ratio and a decrease in the share of vertically integrated firms.

To summarize, as found in Kranton and Minehart (2000), our result indicates that the properties of the idiosyncratic risk at firm level plays an important role in determining the vertical structure of firms. The choice of manufacturers between VI and link is nontrivial.

[24] Total welfare is the sum of consumer and producer surplus, which is calculated as follows:

$$\begin{aligned} Welfare &= \frac{A{Y^*}^{1+B}}{1+B} - pY^* + \sum_z \sum_\varepsilon \Big[\pi_d^S(z,\varepsilon)\Phi^S(z,\varepsilon) + \pi_d^L(z,\varepsilon)\Phi^L(z,\varepsilon) \\ &\quad + \pi_d^{VI}(z,\varepsilon)\Phi^{VI}(z,\varepsilon) + \pi_u^S(z,\varepsilon)\Phi^S(z,\varepsilon) + \pi_u^L(z,\varepsilon)\Phi^L(z,\varepsilon)\Big] \end{aligned}$$

It follows from the trade-off between loosing flexibility against negative shocks and sharing a fraction of profits with the supplier.

As the variance σ_μ^2 increases, given that the per period profit is concave in z, the value at entry is lower. Hence the equilibrium price increases and consumer surplus shows a large decline. A higher dispersion in productivity shocks implies that there are entrants with efficiency levels within a wider range of values. The most inefficient ones exit while the most efficient ones survive (each one of which contributes more to total production than before). Thus, there are two forces that diminishes the total number of firms. First, the higher equilibrium prices generates a decline in demand, and therefore there is less space for production units in the market. And second, there are bigger production units that satisfy the lower quantity demanded. What is interesting here is that, even though there is a reallocation of resources from small to medium and big firms (looking at the size distribution of firms, there is an increase in the share of big firms and a decline in the share of small ones), which increases the RTFP of manufacturers, the big decline in total investment (the share, as well as the mass, of firms that become vertically integrated and linked decreases) generates lower supplier's RTFP. As a result, the total RTFP (and TFP) decreases. In line with this, producer surplus is lower, hence total welfare decreases.

4 Conclusion

This paper proposes a dynamic entry and exit model of an industry with vertical structure decisions and specific investments. In the model, the industrial vertical structure is the result of optimal investment decisions that firms make under uncertainty. The model does well in replicating new facts on vertical structures documented in Hortaçsu and Syverson (2009) and Kranton and Minehart (2000). Our results indicate that differences in vertical structures across industries, and across firms within industries, are the result of differences in the properties of the stochastic process governing the uncertainty at firm level, in specific investment costs, in bargaining power of manufacturers and suppliers, and in complementarity of manufacturers and suppliers productivity.

5 Appendix

5.1 Solution Method

The algorithm to compute the equilibrium is as follows:

1) Given initial guesses for the price of the final good, p^0, and for the standardized input price, p_u^0, compute the price for the specialized input, $p_u^{L0}(z,\varepsilon)$, by NBS over current profits, that is, taking

$$p_u^{L0} = p_u - (1-\theta)c(z,\varepsilon),$$

as the solution of expression (8), and take H_{VI} as $\frac{\overbrace{pu - C_f^u}^{p_{VI}^0}}{1-\beta} + h$. Take these prices as the initial guesses for p_u^{L0} and p_{VI}^0

2) Take an initial guess for the density of productivity of manufacturers looking for a standardized suppliers $J_0^d(z)$,

3) Obtain policy functions $a_S(\cdot)$, $x'_S(\cdot)$, $a_L(\cdot)$, $a_{VI}(\cdot), x'_{VI}(\cdot)$ and value functions ,$V^S(\cdot)$, $V^{VI}(\cdot)$, $V^L(\cdot)$, $W^S(\cdot)$ and $W^L(\cdot)$ (equations $1, 3, 4, 5,$ and 7).

4) Compute the price for the specialized input, $p_u^L(z,\varepsilon)$ by NBS taking into account the continuation values (equation 8) and $P_{VI}(z,\varepsilon) = \beta E_{z',\varepsilon'} W^S(z',\varepsilon'; p, p_u)$.

5) Compare $p_u^L(z,\epsilon)$ and $P_{VI}(z,\varepsilon)$ with previous guesses $p_u^{L0}(z,\epsilon)$ and $P_{VI}^0(z,\varepsilon)$.

   i) If they are close⇒guess a new specialized input price, taking:

$$\begin{aligned} p_u^{L0}(z,\epsilon) &= p_u^{L0}(z,\varepsilon) + \Lambda(p_u^L(z,\varepsilon) - p_u^{L0}(z,\varepsilon)), \text{ and} \\ P_{VI}^0(z,\varepsilon) &= P_{VI}^0(z,\varepsilon) + \Lambda(P_{VI}(z,\varepsilon) - P_{VI}^0(z,\varepsilon)), \end{aligned}$$

   where Λ is a convergence tolerance parameter, and repeat from point (3).

   ii) If they are close⇒compute for each price $p_u^L(z,\varepsilon)$ and $p_{VI}(z,\varepsilon)$ the gains from trade for manufacturers and suppliers that trade inputs:

      * If for some (z,ε) gains from trade are negative⇒use an indicator so that under these prices the manufacturer decides not to negotiate, and repeat from point (3) using these new prices.

* If for every (z, ε) gains from trade are positive$\Rightarrow$stop and go to next point.

6) Use the computed decision rules and the transition matrix to compute the invariant density of productivity of manufacturers looking for a standardized suppliers $J^d(z)$, and compare it with $J_0^d(z)$:

   i) If they are not close$\Rightarrow$guess a new one ($J_0^d(z) = J^d(z)$) and repeat from point (2) until they get close.

   ii) If they are close$\Rightarrow$stop and go to next point .

7) Compute $V_e^d(p_u, p)$ and $W_e^u(p_u, p)$ and given the entry costs C_e^d and C_e^u verify if free entry conditions (equations 10 and 11) hold:

   i) If they do not hold:

      * If $V_u^e(p_u) < C_e^u$ and/or $W_e^u(p_u) < C_e^u \Rightarrow$ guess a new higher prices, p and p_u by bisection and repeat from point (1).
      * If $V_u^e(p_u) > C_e^u$ and/or $W_e^u(p_u) > C_e^u \Rightarrow$ guess a new lower prices, p and p_u by bisection and repeat from point (1).

   ii) If $V_s^e(p_u) \approx C_e^u$ and $W_e^u(p_u) \approx C_e^u$ $\Rightarrow$stop and go to next point.

8) Use the computed decision rules and the transition matrix to compute the fixed points of the distribution of manufacturer firm sizes when the mass of firms is one ($m^d = 1$). Thus, we have the fixed points $\widehat{\Phi}^S, \widehat{\Phi}^{VI}$ and $\widehat{\Phi}^L$.

9) Use the linear homogeneity of the $T's$ operators (defined in point vi of the stationary equilibrium definition, in equations 16, 17 and 18) in m^d to obtain the equilibrium value for m^d that satisfies the market clearing condition for the final good: $D^d(p) = S^d(p, m^d)$.

5.2 Physical and revenue TFP

In this section I describe how the physical and revenue total factor productivity is calculated. We denote physical and revenue total factor productivity as TFP and RTFP, respectively. The expression for the revenue TFP is as follows:

$$
\begin{aligned}
RTFP = \quad & \sum_z \sum_\varepsilon a_S(z,\varepsilon) \tfrac{pz}{p_u + C_f} \widetilde{\Phi}^S(z,\varepsilon) + \sum_z \sum_\varepsilon a_L(z,\varepsilon) \tfrac{pz + c(z,\varepsilon)}{p_u^L(z,\varepsilon) + C_f} \widetilde{\Phi}^L(z,\varepsilon) \\
& + \sum_z \sum_\varepsilon a_{VI}(z,\varepsilon) \tfrac{pz + c(z,\varepsilon)}{C_f + C_f^{VI}} \widetilde{\Phi}^{VI}(z,\varepsilon) + \sum_z \sum_\varepsilon \tfrac{p_u}{C_f^u} \widetilde{\Xi}^S(z,\varepsilon) \\
& + \sum_z \sum_\varepsilon \tfrac{p_u^L(z,\varepsilon)}{C_f^u} \widetilde{\Xi}^L(z,\varepsilon),
\end{aligned}
$$

where the first term represents the weighted average (the weight is the share of standardized manufacturers in each state, $\widetilde{\Phi}^S(z,\varepsilon)$) of the ratio of standardized manufacturer's revenues, pz, to their total production cost, $p_u + C_f$.

The second and third terms are the weighted average of the ratio of linked and vertically integrated manufacturer's revenues to their corresponding total production costs. In these cases $\widetilde{\Phi}^L(z,\varepsilon)$ and $\widetilde{\Phi}^{VI}(z,\varepsilon)$ are the share of linked and vertically integrated manufacturers, respectively. In contrast with the first term, in the numerator it appears the variable cost advantage of specific investments, $c(z,\varepsilon)$. The other difference is in the denominator, where it appears as cost of the linked firms the bargained input price $p_u^L(z,\varepsilon)$; and for vertically integrated firms the additional fixed cost C_f^{VI}.

The last two terms correspond to the RTFP of suppliers. There, $\widetilde{\Xi}^S(z,\varepsilon)$ and $\widetilde{\Xi}^L(z,\varepsilon)$ are the share of standardized and linked suppliers, respectively. The fourth term is the RTFP of a standardized supplier, which is the ratio of revenue, p_u, to total cost, C_f^u. For specialized suppliers, the RTFP is similar, but their revenue is $p_u^L(z,\varepsilon)$.

The expression for TFP is as follows

$$
\begin{aligned}
TFP = \quad & \sum_z \sum_\varepsilon a_S(z,\varepsilon) \tfrac{z}{1 + \frac{C_f}{p}} \widetilde{\Phi}^S(z,\varepsilon) + \sum_z \sum_\varepsilon a_L(z,\varepsilon) \tfrac{z + \frac{c(z,\varepsilon)}{p}}{1 + \frac{C_f}{p}} \widetilde{\Phi}^L(z,\varepsilon) \\
& + \sum_z \sum_\varepsilon a_{VI}(z,\varepsilon) \tfrac{z + \frac{c(z,\varepsilon)}{p}}{\frac{C_f + C_f^{VI}}{p}} \widetilde{\Phi}^{VI}(z,\varepsilon) + \sum_z \sum_\varepsilon \tfrac{1}{\frac{C_f^u}{p}} \widetilde{\Xi}^S(z,\varepsilon) \\
& + \sum_z \sum_\varepsilon \tfrac{1}{\frac{C_f^u}{p}} \widetilde{\Xi}^L(z,\varepsilon),
\end{aligned}
$$

in which the difference with the definition for RTFP is the following. For manufacturers, every term reflects the ratio of units produced by each firm to the units of all inputs they use in production. Standardized and linked manufacturers use one unit of input to produce and $\frac{C_f}{p}$ fixed units of physical resources to produce z and $z + \frac{c(z,\varepsilon)}{p}$ units of final goods, respectively.

Every vertically integrated firm produces $z+\frac{c(z,\varepsilon)}{p}$ units of final goods and uses $\frac{C_f+C_f^{VI}}{p}$ fixed units of physical resources to produce. The logic is the same for suppliers.

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