Skill Premium, College Enrollment and Education Signals

Job Market Paper

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Abstract

This paper asks if "higher education as a signal" helps explain the comovements between college enrollment rate and skill premium for younger workers in the US from the 1970s. In my model a continuum of agents, heterogeneous in talent and initial wealth, make schooling and working decisions: work now or take up college first? When college is very expensive only the wealthy can afford it, hence the lack of a college degree does not signal much as far as talent is concerned. When college becomes more affordable the degree is a better signal of talent. If talent is valuable, per se, on the work place, the college premium should increase. The model has closed-form solutions. When calibrated, it provides a robust estimate of the signaling effect, which accounts for around 17% of the growth in skill premium.

Keywords: College Premium, education signalling, college enrollment

JEL codes: J31, D82

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1 Introduction

The rise in the college wage premium - defined as the differential between the wages of college and high school graduates - is a well-documented fact. As Card and Lemieux (2001) have shown, the wage premium has evolved differently for different age groups: younger workers account for most of the growth of the premium. In line with the cohort-based perspective, this paper looks at college premium for workers age 23-26 and asks: how much of this evolution can be reasonably explained by the idea that higher education is (also) a signal of talent? The answer is motivated by the observation that the college premium and college enrollment rates have closely tracked each other during the past four decades (Figure 1). The story I submit is the following: as college education becomes more accessible, the lack of a college degree becomes an increasingly clear signal of poor talent; if talent, per se, is useful in the working place but unobservable, the college degree will be rewarded by an increasing premium relative to the high school diploma. The paper provides both a signaling model with closed-form solutions and a robust estimate of the signaling effect for the US economy from 1972 to 2005. Within a broadly defined class of models, the signaling mechanism accounts for about 17% of the growth in college premium.

[Figure 1 about here.]

How a college degree should be interpreted depends on the nature of the hurdles one must overcome to reach it. Imagine a world where agents differ in talent and wealth and higher talent can complete college with higher probability. If college is very costly and most people are financially constrained, possession of a college degree is a weak signal of personal talent and a strong signal of family's wealth. At the opposite extreme, assume college were costless: then the talented people would all be attending it and the less talented ones would not. In this case a college degree would be a very strong signal of personal talent. As we move from one extreme to the other, and college becomes affordable for a larger share of the population, not having a college degree, i.e. having only a high school degree, becomes a strong signal of low talent. This brings about a deterioration in the wage of high school graduates relative to that of college graduates. In other words: if college enrollment increased because it became more affordable, we should have observed also an increase in the college wage premium. In a stationary environment, without technological progress and capital accumulation, the increase is due to a decrease in the wage of high school graduates. In a more general environment, it is due to the fact that the wage of high school graduates grows less than the average and a lot less than that of college graduates, as we observe to be the case in the US data. Figure 2 plots the HP-filtered log weekly wage of college graduates and that of high school graduates over time together with their estimated trends. The wage of college graduates appears to be constant until 1993, after which it grows at 1.7% a year. On the other hand, the wage of high school graduates deteriorates from 1970 to 1993 at an annual rate of 1.1% and afterwards rises half as quickly as that of college graduates, at 0.8% per year.

[Figure 2 about here.]

The model developed below formalizes the intuition for the case of a stationary economy. When I take the model to data, I interpret the increase in the wage to college graduates as due partly to capital accumulation and TFP growth, and partly, as an improvement in college's talent discrimination technology, summarized by the probability of graduating from college given talent. If people of all talents choose to go to college, the discrimination technology boils down to the average college completion rate. If the technology improves - i.e. it becomes easier to complete college if talented and harder if not - the average completion rate increases. Thus an improvement in the discrimination technology leads to a rising wage to college graduates and an increasing college completion rate, as in the data (see Figure 6).

Because the empirical relevance of my theory requires to be plausible the assumption that college has become progressively more affordable because financial constraints were relaxed, I should discuss here the relevant evidence. Baumol and Blackman (1995) and Archibald and Feldman (2008) are two of the very few papers that address the change of college affordability over time directly. Both papers recognize the rise of college price as a cost disease phenomenon. Although the share of income spent on college education has gone up, the relative price of other goods, which have experienced rapid productivity growth, fell so much that given income, one could actually afford more college education and more other goods. Archibald and Feldman (2008) argue that the difference between income and college expense is a better measure of affordability than share of income spent on education. They show that during 1990-2007 the median income left over after paying for college expense increased for both public and private institutions, with larger gains in public institutions. Here I re-construct this measure to cover the period from 1975/76 to 2008/09. Figure 3 plots the time series of the difference between the HP-filtered median household income and the net college price, together with the net college price as a share of median household income. The net college price is obtained by substracting average total aids per full-time-equivalent (FTE) student from average tuition, fees, room and board (TFRB). The total aids include grant aids, federal loans, educaiton tax benefits and federal work-study. The result is broadly consistent with the aforementioned findings. The residual income shows an upward sloping trend, indicating an increase in college affordability, even though the share of income keeps rising too.

[Figure 3 about here.]

Micro data tells a similar story. The National Postsecondary Student Aid Study (NPSAS) contains student-level information on financial aid provided by the federal government, the states, postsecondary institutions, employers, and private agencies, along with demographic and enrollment data My sample consists of all students who are dependent and enrolled in a bachelor's degree program in NPSAS 87, 90, 93, 96, 00, 04 and 08. Tables 1 shows the difference between the mean of parents' income and tuition and fees net grants and federal loans, by household income quintile and type of institution. The growth in the residual income is more apparent for 4-year public institutions than for 4-year non-for-profit private institutions. Notably, the increasing trend holds across all income groups for public colleges. In so far as the marginally constrained student is more likely to attend a public school, the evidence is supportive. The case of selective private colleges is examined by Hill, Winston and Boyd (2005). In their sample of 28 highly selective COFHE[1] colleges and universities,

[1]Consortium on the Financing of Higher Education. All private institutions.

the real net price of attending those institutions as share of income fell for all income groups and the most dramatic decline was at the lowest quintile income group. The above analysis assumes a constant family size. If one takes into account that the number of own children under 18 per family has decreased from 1.28 in 1971 to 0.84 in 2009[2], the residual income after paying for children's college expenses should increase even more.

[Tables 1 about here.]

Some more evidence is available from the literature on the effect of aid on college enrollment. Federal grants and loans have increased dramatically on a per FTE student basis (Figure 4). The two key questions are how sensitive enrollment is to college price and how effectively the grants and loans programs are in promoting college access. The empirical evidence is mixed (for a review, cf. Kane, 2006). Most of the estimations that exploit cross-sectional variability reach an estimate that $1,000 reduction of college tuition increased the enrollment rate by three to five percentage points. See for example Kane (1994), Dynarski (2003), and Winter (2009). However, those studies that look at the enrollment of high- and low-income students before and after Pell Grant was launched in 1973 do not find relative increase in attendance in the low-income group, but those models are typically not well identified (Kane, 1995 and Leslie and Brinkman, 1983). Long (2007) finds a positive effect of loans on enrollment for those families who had just become eligible and the effect was concentrated in full-time enrollment. By and large, the evidence seems to favor a positive effect from grants and loans programs.

[Figure 4 about here.]

In the line of research that focuses on differential enrollment behaviors across racial/ethnic groups, Cameron and Heckman (2001), Carneiro and Heckman (2002 and 2003) argue that long-run factors that determine the preparedness for college are more important than short-term cash constraints in making schooling decision. Their point can be translated into a high

[2] U.S. Census Bureau, Families and Living Arrangements 2009, Table FM-3.

correlation between family income and ability in my model. As long as the correlation is not 1, in which case ability is observable and there is no role for college as a signal, the signaling mechanism in this paper still works, though the college premium would be smaller. In fact, recognizing the positive correlation between family income and talent helps my argument in the sense that the true marginal student, who can benefit from college and is barely financially constrained, is likely from the middle income group instead of the lowest one. I have shown in the preceding paragraphs that the college indeed has become more affordable to the middle income families for both types of institutions. While my model does not aim to provide a theory of enrollment decision per se, the only, realistic, assumption that I need is that college enrollment rates have risen over the years because a bigger and bigger fraction of the population can go to college when they choose to.

2 A Brief Literature Review

I briefly review the related literature. The evolution of the aggregate skill premium is described, among others, by Autor, Katz and Kearney (2008). Katz and Murphy (1992) provide a supply and demand framework to account for the dynamics of wages. Autor, Katz and Krueger (1998) rely on skill-biased technological change to rationalize the demand for skilled labor outpacing the supply. While their model involves assumptions on the unobservable quality of labor, Krusell, Ohanian, Rios-Rull and Violante (2000) show that the capital-skill complementarity can account for almost all of the growth in aggregate skill premium without any change in the trend of the unobservable.

While all of the papers above look at wage differentials by education attainment across all age groups, Card and DiNardo (2002) point out that the skill premium does not grow at the same rate across age groups. Further, Card and Lemieux (2001) estimate a production model with imperfect substitution between workers from different age groups and attribute the rising college premium for younger workers to the slowdown in the rate of growth of educational attainment starting with the 1950 cohorts. My paper shares with their work this cohort-based perspective. Guvenen and Kuruscu (2009) calibrate a overlapping generations model of human capital accumulation with skill-biased technical change and heterogeneous agents differing in the ability to accumulate human capital. Their model generate behaviors of the overall wage inequality and college premium for young workers that are consistent with the data. This paper differs from all of the above papers in that I abstract away the technological progress in the production process that change the labor demand. Instead, I focus on the implication of the signaling effect of education in an environment in which the suppliers of labor are less and less financially constrained in their schooling decision.

While the application of signaling theory to the college wage premium is relatively new, the idea of education-as-a-signal is obviously not: it dates back to Spence (1973). Hendel, Shapiro and Willen (2005) argue that decreasing interest rates on borrowing or decreasing tuition has the unintended consequence of widening the wage gap for similar reasons to the ones in this paper. They develop a model with imperfect capital markets and look at a separating equilibrium with two types, in which only the high ability type can benefit from college. The presence of the wedge between the borrowing and lending rates of interest enriches the dynamics of the skill premium and college attendance and allow them to discuss policies such as college loans. In contrast, this paper looks at a pooling equilibrium where all agents having continuously distributed abilities can benefit from college as long as they can afford it, while shutting down credit markets completely. The change in affordability, which depends on the availability of financial aids and loans, is governed by the speed with which the budget constraints are relaxed, a parameter which is calibrated to match the observed enrollment rates. The convenience of a pooling equilibrium is technical. The equilibrium dynamics has a closed form which facilitates the calibration. However, it is plausible to me that a high school graduate believes that he can benefit from college given the option of dropping out. Bedard (2001) lends some support to this by showing that high school dropout rates are higher in areas with greater university access. When more high school graduates have access to college, being a high school graduate without college enrollment is not worthy of the effort to complete the high school. While both my model and Hendel et al. (2005) predict no variation over time in the wage offer to college graduates, Balart (2010) specifies conditions on the wealth distribution under which more access to higher education decreases earnings for all education groups within the framework of Hendel et. al.

This paper also contributes to the literature which quantifies the relative importance of college education as a process of human capital enhancement and as a signaling device. Riley (2001) summarizes a large body of empirical research that tests the educational screening hypothesis against the human capital accumulation hypothesis, reaching mixed conclusions. I refer the reader to the references therein. Recognizing both roles of college education in generating college premium, Fang (2006) estimates a structural static model of endogenous education choices and wage determination and finds that productivity enhancement accounts for at least two-thirds of the college wage premium. On the other hand, Taber (2001) develops a dynamic programming selection model and finds evidence that the change in college premium in the 1980s was more plausibly driven by increasing demand for unobservable abilities than for skills acquired at school. While Taber (2001) suggests that the educational signal was likely to play a big role, he does not model the education signaling explicitly. He assumes the within-cohort ability differential between college graduates and high school graduates to be constant over time, eliminating the cohort effect on the evolution of college premium. It is precisely this cohort effect that is the focus of this paper. More specifically, to borrow from Taber's terminology, the change of college premium has three potential sources: the change in the payoff to skills acquired in college, the change in the payoff to unobservable ability, the change in the ability differential conditioning on education outcome. Fang (2006) suggests that the first source is important, because in a static setting the college premium is determined mostly by the payoff to skills learned in school. Taber's (2001) argument is that the second source seems to play a larger role than the first, ignoring the third possibility. In contrast, my paper argues, roughly, that regardless of the relative importance of the first two roles, the third source, the cohort effect, accounts for around 17% of the growth in college premium.

The rest of the paper is organized as follows. Section 2 presents the theory, while Section 3 simulates the model and provides a measurement of the effect of signals on the growth of skill premium. Section 4 concludes. All proofs are in the Appendix.

3 Model

3.1 A Static Model: the Working of the Education Signal

A static model may help the reader's intuition. Assume personal talent is private information that is nevertheless useful in production. Firms can base their wage offer only on the observable signal, which consists of having attained, or not, a college degree. Everyone is born with a high school diploma.

The population has size one, half is endowed with high talent, $\overline{\theta}$, and half with low talent $\underline{\theta}$. Let the distribution of wealth in the population be $F(\Omega)$. College education has a fixed cost of Q. Assume that all those with wealth $\Omega > Q$ go to college, hence, the fraction of people who goes to college is $F(Q)$. Assume there is randomness in successfully completing college. The probability of a high (low) talent person to complete college is $\overline{p}$ ($\underline{p}$), with $\overline{p} > \underline{p}$. The wage offer is simply the expected talent conditional on the signal received.

With some algebra, we have the wage offer to college graduates $\overline{W}$ and to high school graduates $\underline{W}$,

$$
\begin{aligned}
\overline{W} &= \frac{\overline{p}}{\overline{p}+\underline{p}}\overline{\theta} + \frac{\underline{p}}{\overline{p}+\underline{p}}\underline{\theta}, \\
\underline{W} &= \frac{1-\overline{p}[1-F(Q)]}{2-(\overline{p}+\underline{p})[1-F(Q)]}\overline{\theta} + \frac{1-\underline{p}[1-F(Q)]}{2-(\overline{p}+\underline{p})[1-F(Q)]}\underline{\theta}.
\end{aligned}
$$

While $\overline{W}$ is a constant, $\underline{W}$ depends on the fraction of people that can afford to go to college. Write $x = 1-F(Q)$, we have $\underline{W}'(x) < 0$, implying that the wage differential increases together with college attendance. Next we embed this simple mechanism in a dynamic model of production.

3.2 Embedding the Signals in a Dynastic Model

This is a continuous time discrete-choice problem. Each agent is indexed by the pair (θ, k_0), where θ denotes talent, distributed in $[0, \overline{\theta}]$ according to a cumulative distribution function $G(\theta)$, and k_0 is the initial endowment of capital from a distribution $F(k_0)$ over $[0, \overline{k}_0]$. The distributions $G(\cdot)$ and $F(\cdot)$ are independent. Each agent is endowed with 1 unit of labor. In each instant, an agent faces a discrete choice of whether going to college or not. There are two implicit assumptions in this formulation. One, the offspring of the high (low) type remains high (low); since our main concern is not about social mobility, this assumption seems innocuous. Two, firms cannot, through repeated interaction with an agent from the same dynasty, infer her type. Agents save a constant fraction of their income in each instant. Saving must be positive, i.e. agents cannot borrow against future income. We will relax this assumption later. College education requires a fixed cost $Q > 0$. The rest is the same as in the static model, with $p(\theta)$, a monotone increasing function, representing the probability of completing college for type θ.

3.2.1 The Agents' Problem

At each instant of time, an agent (θ, k_0) decides whether to go to college or directly to the labor market. If he decides to go to college, he pays the fixed cost Q, after which one of the two possible states of nature is realized: he either completes college or not. After finding a job, he works, consumes and saves a fraction σ of his income. Agents are risk neutral and maximize the discounted sum of future consumption taking the rental rate of capital $R(t)$ and the wages $\overline{W}(t)$, $\underline{W}(t)$ as given: $U(c(t)) = \int_0^\infty c(t)e^{-rt}dt$.

Since there is no disutility from labor, all agents supply 1 unit of labor inelastically. There is no capital depreciation. For ease of exposition, the time argument is suppressed when it does not cause confusion.

Lemma 1 *If it is optimal for an agent with talent θ to go to college at t, then it is optimal for any agent who has talent greater than θ to go to college at t as long as his current capital holding $k \geq Q$.*

Intuitively, for an agent with talent θ attending college is convenient if $p(\theta)(\overline{W} - \underline{W}) - RQ$ is positive. Because $p(\theta)$ is increasing, this implies the result.

3.2.2 Production

In each period the representative firm rents capital from the households and hires workers. I will look at two different classes of production functions. The first class, call it $P1$, is

$$Y(K, L_H, L_C) = [\lambda L_H^\rho E(\theta|HSG) + \upsilon K^\rho + (1 - \lambda - \upsilon) L_C^\rho E(\theta|CG)]^{1/\rho}, \ \rho \leq 1, \qquad \text{(P1)}$$

where L_H is the number of high school graduates and L_C is the number of college graduates. Here high school graduates and college graduates are perceived as different inputs, i.e. they are assigned different jobs. The productivity of each group is its average talent, by Law of Large Numbers. Implicitly, college education here is productive in the sense that successfully completing college equips the college graduates with a particular set of skills that allow them to undertake a particular task. The elasticity of substitution between two types of labor is the same as their elasticity with capital. In contrast, the second class of production functions only employs aggregate labor and capital as its inputs, that is, skilled and unskilled labor are perfect substitutes:

$$Y(K, L) = A[\alpha K^\rho + \beta(L \cdot E(\theta))^\rho]^{1/\rho}. \qquad \text{(P2)}$$

In both cases, markets are competitive and the high school (or college) graduates will be paid by their marginal product conditional on the signal. Later, in the calibration section, I will explore the different quantitative implications of the two production functions. The total stock of capital is $K(t) = \int_0^{\overline{k_0}} k(t) dF(k_0)$ and the total labor supply $L(t) = 1, \forall t$. Following the tradition, skilled labor (or, unskilled) and college graduates (or, high school graduates) are used interchangeably.

3.2.3 Equilibrium

Definition 1 Equilibrium without credit markets

An equilibrium without credit markets of this economy is a list $(c(t), k(t), sh(t))$ for each agent (θ, k_0) and a list of prices $(R(t), \overline{W}(t), \underline{W}(t))$ given initial capital distribution $F(\cdot)$ and distribution of talent $G(\cdot)$, the exogenous positive saving rate σ and the production technology, so that

(i) Agents optimally make schooling decision $sh(K(t))$, given $R(t)$, $\overline{W}(t)$, $\underline{W}(t)$;

(ii) Firm maximizes period profit;

(iii) Factor Markets clear.

To provide an analytically convenient environment, we will look at a special class of the equilibrium defined above, the pooling equilibria in which all agents optimally go to college as soon as they can afford it. More discussion on equilibrium selection can be found at the end of this section. Before proving the existence of the pooling equilibria, I will prove the monotonicity of the wage differential in enrollment under the proposed strategy profile, which will be useful in the construction of the equilibrium later. Let x be the fraction of agents who go to school and we have $x = 1-F(Q)$. The theoretical results here are presented mainly for $P1$. An analogous characterization of equilibria with $P2$ can be obtained from the author upon request.

Lemma 2 *For $P1$, under the strategy profile that all types of agents go to college as soon as their current capital holdings $k \geq Q$, for high ρ and low Q, $\ln(\overline{W}/\underline{W})$ is increasing in the fraction, x, of agents going to college.*

To facilitate interpretation, the wage differential has the form of $\frac{\overline{W}}{\underline{W}} = \frac{1-\lambda-\upsilon}{\lambda}(\frac{L_C}{L_H})^{\rho-1}\frac{E[\theta|CG]}{E[\theta|HSG]}$. An increase in the attendance will unambiguously lead to a higher ratio of expected talents, $\frac{E[\theta|CG]}{E[\theta|HSG]}$, by exactly the same logic as in the static model. Imagine $\rho = 1$, then the wage differential will unambiguously go up. However, for $\rho < 1$, the general equilibrium effect kicks in. Since college graduates become more abundant, its marginal productivity decreases relative to that of high school graduates, and this mitigates the effects of the signals. For every Q, I can find a $\widehat{\rho} \leq 1$, such that for all $\rho \geq \widehat{\rho}$, this monotonicity property of the wage gap holds. In general, the monotonicity of wage differential rely also on small Q and high ρ. Consider a separating equilibrium, in which higher types opt for school and lower types don't. Suppose that college is very expensive, hence few people can afford it. Then a college degree is more correlated with wealth than with talent and the signal it contains is weak. The marginal productivity of skilled labor is high, hence skilled labor would be receiving a high payment, if identifiable. But holding a college degree is not such a clear signal of talent, as only the rich can afford it. If college enrollment increases while its cost is constant the signal's quality does not improve as the high cost of attending college implies we are scrapping the "bottom of the barrel" among wealthy people. More generally, this is true also when the cost of attending college decreases as long as it is high and the distribution of wealth is not concentrated at high values of wealth. The marginal productivity of skilled labor decreases, though, relative to that of unskilled labor and, as a result, we may have a range in which increasing college attendance brings about a decrease of the wage premium.

Proposition 1 *Under some assumptions, for Q sufficiently small, there exists a pooling equilibrium where all types of agents choose to go to college as soon as $k \geq Q$.*

To guarantee that the net benefit of college attendance, $p(\theta)(\overline{W}(t) - \underline{W}(t)) - R(t)Q$ is positive for all t, Q cannot be too high. A sufficient upper bound, $\widehat{Q}$, is the solution (which exists) to $p(0)(\overline{W}(0) - \underline{W}(0)) = \upsilon(K(0) - \widehat{Q})^{\rho-1}\widehat{Q}$.

Remark 1 The bound of admissible Q, $\widehat{Q}$, is (i) increasing in x_0; (ii) increasing in $p(0)$; necessarily $p(0) > 0$; (iii) increasing in $K(0)$.

The above proposition has nice implications about the trends of college enrollment rate and of skill premium.

Corollary 1 *There is a cut-off level of the initial wealth for a given $t, k_0(t)$, so that for all agents whose endowment $k_0 \geq k_0(t)$, they will choose college education at t. That is, the college enrollment rate is increasing over time.*

Observe that all agents who haven't attended college accumulate capital in exactly the same fashion: $\dot{k^i} = \sigma[R(t)k^i + \underline{W}(t)]$. Therefore, $k_0(t)$ satisfies $k_0(t) = Q - \int_0^t \dot{k}(s)ds$,where the evolution of $k(s)$ follows $\dot{k} = \sigma[R(s)k(s) + \underline{W}(s)]$, $0 \leq s \leq t$. Obviously, $k_0(t)$ is decreasing over time along the equilibrium path.

Corollary 2 *The wage gap is widening over time along the equilibrium path.*

The equilibrium path is completely characterized in terms of the aggregate capital, $K(t)$, and the cut-off wealth level, $k_0(t)$:

$$\begin{cases} \dot{K}(t) = \sigma Y(K(t) - x(t)Q, 1 - x(t)\int pdG, x(t)\int pdG) \\ \dot{k_0}(t) = -\sigma[R(t)Q + \underline{W}(t)] \end{cases} \tag{1}$$

$$s.t.\ k_0(t) \geq 0$$

$$\text{with } K(0) = \int_0^{\overline{k_0}} k_0 dF(k_0) \text{ and } k_0(0) = Q;$$

where $Y(K, L_H, L_C)$ is given by (P1), $R(t)$ given by (A1), $\underline{W}(t)$ given by (A2) and $x(t) = 1 - F(k_0(t))$,

I will use this dynamic system to simulate the model in Section 4.

For $P2$, $\frac{\overline{W}}{\underline{W}} = \frac{E(\theta|CG)}{E(\theta|HSG)}$. I can establish the existence of the pooling equilibrium under even weaker assumptions.

Lemma 2' *For $P2$, under the strategy profile that all types of agents go to college as soon as their current capital holdings $k \geq Q$, $\ln(\overline{W}/\underline{W})$ is increasing in the fraction, x, of agents going to college.*

Proposition 1' *For $P2$, under the assumption that $Q < K(0)$, for Q sufficiently small, there exists a pooling equilibrium where all types of agents choose to go to college as soon as $k \geq Q$.*

The two corollaries continue to hold and the dynamic system that characterizes the equilibrium path remains valid with modified production technology and prices.

In general, there may exist separating equilibria in the sense that only a fraction of agents who can benefit from college self-select to attend college. In this case, Lemma 1 continues to hold, so the college-goers are those whose talent is above some threshold and who are not financially constrained. I discuss conditions for the existence of a separating equilibrium in the Appendix. The equilibrium evolution of the enrollment rates, the cut-off values of talent or the college premium is not necessarily monotonic. Furthermore, the actual enrollment rates and college completion rates imply that under mild conditions, the college premium is increasing in the cut-off value of talent. This means for a given enrollment rate, the lower the cut-off the smaller the wage gap. In other words, if we interpret the rising college premium as attracting less talented high school graduates to go to college, the decreasing minimum talent level tends to dampen the college premium. Intuitively, as we move to the extreme case of a pooling equilibrium, the effect of changes in the budget constraint on the college premium is smallest. Since talent is unobservable, the data is silent on the equilibirum selection. I restrict my attention to the pooling equilibrium for the following reasons: (1) the solution is closed-form and has nice properties; (2) the signaling effect brought by relaxing budget constraints in a separating equilibrium is likely to be even greater than that in a pooling equilibrium; (3) if we think empirically the talent cut-off in the separating equilibrium is decreasing, then the pooling equilibrium can be seen as a limiting case; (4) since our starting point is high school graduates, it is reasonable to assume that someone who can successfully complete the high school curriculum is prepared for college.

3.2.4 A Theoretical Bound of the Effect of the Signals

The next question is *how much* this story can account for the growth in the skill premium. This is of course an empirical question, but here I will derive a theoretical bound of the force of the signals. A widely held opinion is that compositional change in the labor force has little effect on the distribution of wage. This exercise addresses this concern theoretically and hopefully sheds some light on the kind of environment in which the force of signals tends to be strong.

Following Krusell et al. (2000), the growth rate in skill premium can be decomposed into two effects for the model with $P1$, the relative quantity effect and the relative efficiency effect,

$$g_{\ln \frac{\overline{W}}{\underline{W}}} \simeq (1-\rho)(g_{h_u} - g_{h_s}) + \rho(g_{\psi_s} - g_{\psi_u}),$$

where $g_x = \frac{dx/dt}{x}$; $h_s = x \int pdG$; $h_u = 1 - x \int pdG$; $\psi_s = E[\theta|CG]$ and $\psi_u = E[\theta|HSG]$.

The change in the distribution of signals leads to a change in the average talent given a signal, which amounts to a change in the efficiency of skilled labor relative to that of unskilled labor. To maximize the effect of the signals, we must choose the underlying parameters to maximize the relative efficiency effect $g_{\psi_s} - g_{\psi_u}$:

$$\sup_{\substack{G_t(\cdot) \\ p_t(\cdot)}} \frac{\int_0^{\bar{\theta}} \theta p(\theta) dG - \int_0^{\bar{\theta}} p(\theta) dG \int_0^{\bar{\theta}} \theta dG}{(1 - x(t) \int_0^{\bar{\theta}} p(\theta) dG)(\int_0^{\bar{\theta}} \theta dG - x(t) \int_0^{\bar{\theta}} \theta p(\theta) dG)} \dot{x}(t).$$

Remark 2 (1) $x(t)$ and $\dot{x}(t)$ are conveniently taken as given at each t. Though they are endogenous variables, I calibrate the enrollment rates to replicate those in data. So we may well take it as exogenous here.

(2) We allow $G_t(\cdot)$ and $p_t(\cdot)$ to be time-dependent. This maximizes the possible explanatory power of the signals and makes per period problem exactly the same. From now on, we will suppress the time subscript t.

Proposition 2 *The effect of signals is bounded by the negative growth rate of the fraction of people that don't attend college (if finite)*:

$$g_{\psi_s} - g_{\psi_u} \leq \frac{\dot{x}}{1-x} = -g_{1-x}.$$

This result suggests that the signals work most effectively when the education can perfectly sort out the highest talents. Consider the following example in which there are only two talents, 1 or 0.

Example 1 *There is a fraction of ε (close to 0) of people with talent of 1 and the remaining are of talent 0. As a result, $E(\theta) = \varepsilon$. Suppose people with high talent can pass the exam almost surely, while people with low talent have the probability of success decreasing overtime in the following fashion:*

$$p_t(0) = \frac{1}{1 + x(t)}.$$

Note that at each instant of time the probability of success is still weakly increasing in the talents. The exam costs nothing. Then, one can verify that

$$\frac{E(\theta | \textit{with degree})}{E(\theta | \textit{without degree})} \to \frac{1}{1-x}, \textit{as } \varepsilon \to 0.$$

$$g_{\psi_s} - g_{\psi_u} = \frac{d}{dt} \ln \frac{E(\theta | \textit{with degree})}{E(\theta | \textit{without degree})} \to -g_{1-x}, \textit{as } \varepsilon \to 0.$$

Note that in this example, the sorting mechanism becomes more and more efficient overtime, which also contributes to the growth of skill premium. This example shows that the suggested bound can be achieved in the limit. However, in the setting where the probabilities of success are constant overtime, we would expect in general slower growth in skill premium. The bottom line is that in an economy in which the distribution of degrees is highly upward skewed, the education signal has a bigger force.

Now we do a simple counterfactual calculation. Take the college enrollment rates from 1969 to 2005 and compute g_{1-x}[3]. Then, I take the skill premium in 1969, and let it grow at the maximum theoretical bound $-g_{1-x}$, whereby I get the fictitious wage gap in the dashed line contrasted with the real data, as is illustrated in Figure 5.

[Figure 5 about here.]

The signals, theoretically, have the potential to generate all of the growth in skill premium. But as will be clear in Section 4, our hands are tied significantly by the specification and parameterization of the model.

3.3 Optimality

In the current environment, there are two potential sources of inefficiency: the information problem represented by the private information of talents and the problem of missing credit market. We will investigate the consequences of these two problems one by one. In both cases, the objective of the social planner is to maximize period total output.

[3]Since in the proof of the above proposition, $\dot{x}$ is assumed to be positive. I simply replace any negative growth in the data with zero.

3.3.1 Benchmark One: Complete Information

Assume a social planner observes the individual talents. For $P1$, the social planner simply chooses θ^* so that all agents with talent above θ^* are educated at a cost Q:

$$\begin{aligned}\Gamma(\theta^*) &= \max_{\theta^*}\{\lambda(1-\int_{\theta^*}^{\overline{\theta}} pdG)^{\rho-1}(\int_0^{\overline{\theta}} \theta dG-\int_{\theta^*}^{\overline{\theta}} \theta pdG)+\upsilon[K-(1-G(\theta^*))Q]^{\rho} \\ &\quad +(1-\lambda-\upsilon)(\int_{\theta^*}^{\overline{\theta}} pdG)^{\rho-1}\int_{\theta^*}^{\overline{\theta}} \theta pdG\}^{1/\rho} \\ &\quad s.t.\ \ 0\leq\theta^*\leq\overline{\theta}.\end{aligned}$$

This is not a concave problem and the solution is messy. Let $\rho = 1$ for tractability.

Proposition 3 *Consider* $\rho = 1$ with $P1$. *In cases in which* $2\lambda \geq 1-\upsilon$ *holds or both* $2\lambda < 1-\upsilon$ *and* $(1-2\lambda-\upsilon)\overline{\theta}p(\overline{\theta}) < \upsilon Q$ *hold, it is optimal not to provide education at all. If* $2\lambda < 1-\upsilon$ and $(1-2\lambda-\upsilon)\overline{\theta}p(\overline{\theta}) \geq \upsilon Q$*, the optimal cut-off in talent* θ^* *is given by* $(1-2\lambda-\upsilon)\theta^* p(\theta^*) = \upsilon Q$.

In cases where production relies more on unskilled labor than on skilled labor, or in cases where the opportunity cost of investing in education is high, it may be optimal not to provide education at all. But with incomplete information, there may still exist pooling equilibria defined in Section 3.2.3. The individual incentive to self-signal the talent causes both misallocation of factors and a waste of resources. More generally, in all those pooling equilibria, after some finite length of time, the economy will always over-invest in education, even though it may never reach the optimal amount of skilled labor even in the limit.

With $P2$, the degrees are irrelevant since talents are perfectly substitutable and the social planner simply uses all available resources.

Proposition 3' *For* $P2$*, the social planner employs all labor and capital and the period output is* $A[\alpha K^{\rho} + \beta(E(\theta))^{\rho}]^{1/\rho}$.

In the case with $P2$, there is no need to invest in education if education serves purely as a signal.

3.3.2 Benchmark Two: Relaxing Borrowing Constraints

In this section, agents of the same generation are allowed to borrow from each other. Let $b(t)$ be the amount of debt (or credit) that the agent acquires before he receives his income, which has to be paid back at the end of that period.

Definition 2 Equilibrium with within-generation credit markets

An equilibrium of this economy is a list $(c(t), k(t), sh(t), b(t), R(t), \overline{W}(t), \underline{W}(t))$ for each agent (θ, k_0), given initial capital distribution $F(\cdot)$ and distribution of talent $G(\cdot)$ and the exogenous positive saving rate σ and the production technology, so that

(i) Agents optimally choose $sh(K(t))$ and $b(t)$, given $R(t)$, $\overline{W}(t)$, $\underline{W}(t)$;

(ii) Firm maximizes period profit;

(iii) Factor markets clear;

(iv) Credit markets clear: $\int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\overline{k_0}} b(t; \theta, k_0) dG(\theta) dF(k_0) = 0$.

Notice that *Lemma 1* still holds. It is easy to construct an equilibrium in which all agents go to college from day 1.

Proposition 4 *Under Assumptions 1- 3 and* $P1$*, for* Q *sufficiently small, there exists an equilibrium in which all agents go to college from day 1.*

In this equilibrium, the college attendance rate is always 1 and the wage gap remains constant

$$\frac{\overline{W}}{\underline{W}} = \frac{1-\lambda-\upsilon}{\lambda}\left(\frac{\int pdG}{1-\int pdG}\right)^{\rho-1}\frac{\int \theta pdG}{\int pdG}\frac{1-\int pdG}{\int \theta dG - \int \theta pdG}.$$

Furthermore, for all economies that have an equilibrium with borrowing constraints as is defined in *Definition 1*, there is also an equilibrium with with-in generation credit markets as is defined in *Definition 2,* in which there is full attendance. The equilibrium with within generation credit markets is easier to support: it exists for even higher cost of education. Now the evolution of the aggregate capital is described by

$$\dot{K}(t) = \sigma[\upsilon(K(t)-Q)^{\rho} + \Pi]^{\frac{1}{\rho}}\text{[4]}.$$

[4] $\Pi = \lambda(1-\int pdG)^{\rho-1}(\int \theta dG - \int \theta pdG) + (1-\lambda-\upsilon)(\int pdG)^{\rho-1}\int \theta pdG.$

For the same set of parameters, the equilibrium with within generation credit markets has more skilled labor, less unskilled labor and less capital. Hence, only in an economy where skilled labor is very productive, the relaxation of borrowing constraint may bring about more output. More generally, from a social planner's point of view, relaxing borrowing constraint does not necessarily lead to a Pareto improvement with transfers, since this allows for more competition through unproductive signals. The equilibrium without credit markets converges to the benchmark equilibrium in the limit.

4 Calibration

4.1 Data

The relevant data series are the log wage gap between college graduates and high school graduates, the college enrollment rate and the college completion rate.

Skill premium. To be consistent with the theoretic prediction that cohorts born more recently when the signaling effect of a degree is stronger face higher premium than what earlier cohorts face, the calculation of college premium should be cohort-based. I computed the wage series using the CPS March data from 1969 to 2005 by age groups and focus on the age group 23-6. The construction process is essentially the same as Autor, Katz and Kearney (2008).

College enrollment rate. The college enrollment rate is available from 1960 to 2006 from the American College Testing Program on NCES website. The enrollment rate is obtained by dividing the total number of college enrollment in a given year by the total number of high school completers, who graduated from high school and completed GED within the preceding 12 months.

College completion rate. Take the number of bachelor's degrees conferred by degree-granting institutions each year and divide it by the total college enrollment four years before. The degree data are available by year from 1970 to 2006 from NCES. The model counterpart is $\int_0^{\overline{\theta}} p(\theta)dG(\theta)$, the average passing rate of college-goers. I plot the series of college completion rates in Figure 6.

[Figure 6 about here.]

Initial income distribution in 1972. I take the wage/salary income distribution of the fulltime-fullyear-employed 40-50 years old in 1972 from CPS March. These people were likely to have children around 20-year-old in the same year. CPS sampling weights are used.

Cost of college. The cost of college in the model is the tuition, fees, room and board (TFBR) net grants and aids. The TFBR is available from 1976 to 2005 from College Board and the Grants and Aids are available from 1986 to 2006 on selected years. After interpolating the missing observations linearly, the real net cost is almost constant from 1986 to 2006, averaged at 5467 in 2006 dollars.

4.2 Calibration Strategy

The data is structured as follows. The model year refers to the year for which the skill premium is calculated. Within the same period in the model, the enrollment rate six years and college completion rate two years before the model year are used. This is to accommodate the fact that the skill premium is calculated for the age group 23-26. Since the annual degree data starts in 1970 and the skill premium series ends in 2005, the first period in the model is 1972, while the last is 2005.

In order to introduce more variability to the model, I allow the average talent given a college degree to grow and transform the formulae of wage gap to make use of the data of college completion rate. More specifically, let the average talent given a college degree follow a linear trend

$$h_t \equiv E[\theta|CG] = \frac{\int_0^{\bar{\theta}} \theta p_t(\theta) dG}{\int_0^{\bar{\theta}} p_t(\theta) dG} = h_0 + \gamma t.$$

The model is silent about the change in h_t, since the signaling effect from increasing enrollment works through a deteriorating wage offer to unskilled labor. In reality, there are reasons to believe that the average talent of a college graduate grows over time: better screening mechanism in college admission, or better college financing to the talented, or improving human capital accumulation through college, among others. Permitting h_t to grow over time in this reduced form of course increases the overall fit of our model to data, but we will see that the magnitude of the signaling effect modeled in this paper does *not* hinge much on the growth rate of h_t.

The data counterpart of college completion rate π_t is $\int p(\theta)dG$, whereby the models of wage differential are transformed into

$$\frac{\overline{W}}{\underline{W}_t} = \frac{1-\lambda-\upsilon}{\lambda}\left(\frac{x_t\pi_t}{1-x_t\pi_t}\right)^{\rho-1}\frac{(h_0+\gamma t)(1-x_t\pi_t)}{\int\theta dG - x_t\pi_t(h_0+\gamma t)}; \tag{P1}$$

$$\frac{\overline{W}}{\underline{W}_t} = \frac{(h_0+\gamma t)(1-x_t\pi_t)}{\int\theta dG - x_t\pi_t(h_0+\gamma t)}; \tag{P2}$$

where x_t is the college enrollment rate. From the last section, the college completion rates rose sharply during the period 1985 to 1995. What does this imply? Assume $\gamma = 0$. It is easy to show that with $P2$, the wage gap is increasing in college completion rate as long as $h_0 \geq \int\theta dG$. With $P1$, the wage gap is increasing only if $h_0 > \int\theta dG$ and ρ is sufficiently close to 1. In the calibrated models, it is true for both productions that the growth in completion rates helps generating some portion of the college premium. This may be interpreted as a change of the talent distribution over time, or changes in the college screening technology.

Now we are ready to discuss the measurement of the signaling effect. To facilitate discussion, I restrict my attention to $P2$. Recall from Section 3.2.4 that in the model, the growth rate of skill premium has two components, the relative quantity effect and the relative efficiency effect. $P2$ only has the relative efficiency effect: there is no general equilibrium effect of changes in skilled/unskilled labor composition on college premium. In other words, when I vary the enrollment rate, the variation in the skill premium reflects solely the relative efficiency effect, which is exactly the signaling effect that I'm interested in. Hence, the signaling effect can be measured by a counterfactual simulation, in which I fix the enrollment rate constant at the initial level and simulate the wage gap. In the absence of college completion rate data, the wage gap is constant if $\gamma = 0$. However, in the transformed model with the completion rate data, there is some growth in the wage gap even if $\gamma = 0$. The signaling effect is then the residual contribution to the growth in college premium on top of the prediction of the counterfactual model. I calculate the compound annual growth rate (CAGR) of the wage gap predicted by a model holding enrollment fixed and compare it with the CAGR of the wage gap predicted by a calibrated model with endogenous enrollment rates.

The measure of the signaling effect is the percentage of growth rate that is contributed by varying enrollment rates:

$$1 - \frac{CAGR(\ln \frac{\overline{W}}{\underline{W}}|\text{holding enrollment fixed}, \gamma)}{CAGR(\ln \frac{\overline{W}}{\underline{W}}|\gamma)}. \tag{2}$$

Essentially, for any γ, I can compute the measure of the signaling effect within a model parametrized by γ (call it model γ) in the above way. As I vary γ, the overall fit of the model varies and can be measured likewise by

$$\frac{CAGR(\ln \frac{\overline{W}}{\underline{W}}|\gamma)}{CAGR(\ln \frac{\overline{W}}{\underline{W}}|\text{data})}.$$

This is the percentage of growth explained by model γ with respect to data. Multiplying the above two measures, I come to a measure of the overall signaling effect of model γ. We will see that this measure is remarkably stable across different values of γ.

To tackle the difficult problem brought by the unobservables, I ask the following two questions: one, what is the contribution of signals given that the unobservable behave in the most favorable way to me; two, what is the effect of signals as I limit the contribution of the unobservables. To answer them, I follow three steps.

In the first step, I jointly estimate some key parameters in a non-linear-least-square model of wage gap. More specifically, for $P1$, I normalize $h_0 = 1$, take $\upsilon = \lambda = \frac{1}{3}$, and jointly estimate γ, $E\theta$ and ρ; for $P2$, I take $\alpha = \frac{1}{3}, \beta = \frac{2}{3}$, normalize $h_0 = 1$, and estimate γ and $E\theta$. But, all I take from this stage is the value of γ. I interpret this value as representing the most favorable term I can get from the unobservables. Details of the estimation are available upon request.

In the second step, I calibrate the model in the standard fashion, taking γ from the first step as given. In particular, the saving rate σ is pinned down by minimizing the distance between the model enrollment rates and the data.

In the third step, I calibrate models which correspond to different values of γ, ranging from 0 to the first step estimate. I look at the measurement of the signaling effect and find it to be quite constant across different γ.

4.3 Calibration Results

4.3.1 P1

The first stage estimation for P1 yields $\gamma = 0.712\%$. To gain a sense of the magnitude of γ, the avarage talent of college graduates grows to 1.23 times the original level within 33 periods. It turns out that with the first stage estimate of γ, the model over-predicts the growth in college premium. Hence, in the calibration, I pick the γ that matches the model prediction of college premium in the last period with that in the data.

In the second stage, I calibrate the model as follows.

$Model$	$Value$	$Source$
M^5	45000	Decision rule
γ	0.5%	Match last period model college premium with data
h_0	1	Normalization
x_0	0.5006	College enrollment rate in 1966[6]
Q	5467	Real TFRB net aids averaged over 1986 and 2006
$F(\cdot)$	$--$	Income distribution in 1972 times $F^{-1}(1-x_0)$
K_0	20816	Mean of $F(\cdot)$
υ	0.3	Average capital share of national income in NIPA
ρ	0.98	Monotonicity of skill premium in enrollment rate
λ	0.3283	To match the initial college premium in 1972
σ	$2.82e-7$	To match model enrollment rate with the data

M requires some explanation. M scales the productivity of talent to a scale comparable to that of capital, so that in each period the decision rule $p(0)(\overline{W} - \underline{W}) - RQ > 0$ holds. The value of ρ implies strong substitutability among the three inputs. Krusell et al.(2000) estimate the elasticity of substitution between unskilled labor and equipment to be 1.67 and that between skilled labor and equipment to be 0.67, which suggests some substitutability

[5] $h_t = M(h_0 + \gamma t)$.To guarantee the existence of the pooling equilibrium, I need $p(0)(\overline{W} - \underline{W}) - RQ > 0$. A sufficient condition is that $\pi(\overline{W} - \underline{W}) - RQ > 0$. The scale of h_t guarantees that.

[6] The enrollment rate in 1966 is 0.5011. The difference results from a kernel density estimation of the income distribution.

between unskilled labor and the combo of skilled labor and capital. In this model, ρ must be high enough to guarantee the monotonicity of the wage gap in enrollment rate. Both the growth rate γ and the trend in college completion rates contribute to the growth of the college premium. When holding the college enrollment rate fixed at the initial condition, the model still predicts around 85% of the growth. To be more specific, the CAGR of college premium in the model is 3.46%, while in the counterfactual with constant enrollment, it is 2.98%. This suggests that the signaling effect contributes around 14% in the growth of college premium (Panel 1).

[Panel 1 about here.]

4.3.2 P2

In the model with $P2$, the same parameter values apply unless noted below.

$Model$	$Value$	$Source$
M	100	Decision rule
γ	0.3435%	1st stage estimation
$E\theta$	0.9058	To match the initial college premium in 1972
Q	5467	Real TFRB net aids averaged over 1986 and 2006
α	1/3	NIPA
β	2/3	NIPA
ρ	-1	Empirical estimate, see Antras (2004)
σ	$4.5239e-005$	To match the model enrollment rate with the data

Now the CAGR of the model college premium is 3.36%, while in the counterfactual model it is 2.76%. Therefore, the signals contribute about 18% in the growth of college premium (Panel 2). Note that the model, by itself, is not an elaborate model about the evolution of the college enrollment, therefore it fails to catch the swing in the college enrollment rates. However, even if I feed the actual enrollment rate into the model, the prediction of college premium doesn't change much (Panel 3). The counterfactual prediction accentuates the trough and peak for obvious reasons. But the model is able to replicate the long-run trends subject to the limited source of variability.

[Panels 2 and 3 about here.]

4.3.3 Measuring the Signaling Effect

Now I restrict my attention to $P2$. I recalibrate the model for 30 equally spaced values of γ ranging from 0 to 0.3435%.

As is expected, the explanatory power of the model increases as I increase γ (see Figure 7). However, Figure 8 shows that the signaling effect of model γ is actually decreasing in γ. Hence, in terms of the overall effect of signaling, the estimate stays fairly constant within the range of 16-18.5% (see Figure 9).

The merit of this exercise is that we can be reasonably confident in saying that around 17% of the growth rate in college premium comes from the signaling mechanism modeled here. This estimate allows rooms for many other potential explanations to be at play at the same time, be it demographic change or skill-biased technological change or capital-skill complementarity, since it is conceptually equivalent to a γ less than the first step estimate. In general, with productions that allow decreasing return to scale in the skills, the increasing trend of enrollment rates changes the relative supply of skilled labor, which will tend to dampen the signaling effect. Hence the measure as defined in (2) tends to underestimate the effect of signaling, since it is a product not only of the signaling effect but also the general equilibrium effect of increased supply of skilled labor.

[Figures 7, 8 and 9 about here.]

5 Conclusion

Though the idea of education as a job market signal is well known, its application to the evolution of wage distribution hasn't been well articulated in theory. This paper is such an attempt. I have developed a model with agents heterogeneous in initial wealth and talent, who make schooling decisions. The growth in the college enrollment rate due to increased accessibility to college makes a high school diploma a clearer signal of low talent. If talent is useful in production, the college degree will be rewarded a higher premium relative to the high school diploma. This brings about a growing wage gap between college graduates and high school graduates. The model is calibrated, with two specifications of production technologies. The effect of signals on the college premium is estimated to be around 17% for models that can potentially allow for other explanations of rising college premium. Simplistic as it seems, the theory has a big potential to explain a wider range of phenomena. I close the paper with directions for future research.

One immediate extension is to extend the two dimensional choice variable to the multi-dimensional choice of getting bachelor's, master's or doctor's degree. Eckstein and Nagypal (2004) argues that the most important group contributing to the increase in college wage premium is workers with a postgraduate degree. This is consistent with my theory. The increase in the number of Bachelor's degrees issued will demand even higher degrees to effectively signal one's talent, which leads to the growing graduate school premium. It is conceivable that with a continuum of choice of levels of education, that varies from community colleges to the Ph.D. programs in top universities, the distribution of the education premium to each will fan out over time as the signals work their way through the distribution.

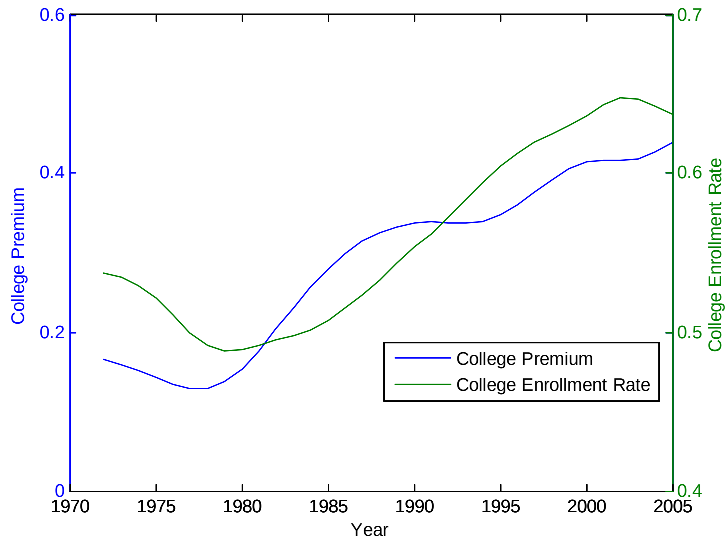
The framework can also be easily adapted to explaining the increasingly high premium of attending elite colleges. By casual observation, the best schools are becoming more and more accessible to the high talented students, thanks to more effective admission processes and more generous financial aid. As a result, the degree of elite schools must have become more correlated with talent than before. To estimate the fancy college premium and observe its evolution over time would be an interesting empirical question.

Another direction of research is to model the supply side of the college education. The key to the growing enrollment rate is the relaxation of household budget constraint over time through capital accumulation. But in reality there may be other ways that achieve the same effect. One example is the relaxation of the borrowing constraints, as is studied in Hendel, Shapiro and Willen (2001). Incorporating a sector of college will be a first step toward a general equilibrium approach. Colleges maximize some objective function by choosing costly admission processes. They can either admit students without much screening or undertake costly selection procedure. Colleges can be endowed with reputation such that in equilibrium some reputedly good colleges choose to be more selective, but will be compensated by higher prices they charge the students. Students in turn will be compensated by the top college premium. The story is more relevant if we can document the growing tuitions of top-notch schools and the growing returns to elite education.

Finally, one can conceive a full dynamic model, in which agents optimize over consumption and saving. Intuitively, this will help us more. Since the skill premium is growing over time, for subjective discount rate that is not too high, later cohorts will optimally choose to save more, which will allow their children to go to even fancier colleges or allow them to pursue postgraduate degrees, that will further enlarge the associated higher education premium. Combining a full dynamic model with a multiple or even continuum choice of levels of education would certainly make an elaborate model, though possibly analytically intractable. One would want to pay the extra cost of computation for more precise quantitative and policy-oriented analysis. After all, the parsimonious model we have here lays out the essential economic intuition just as well.

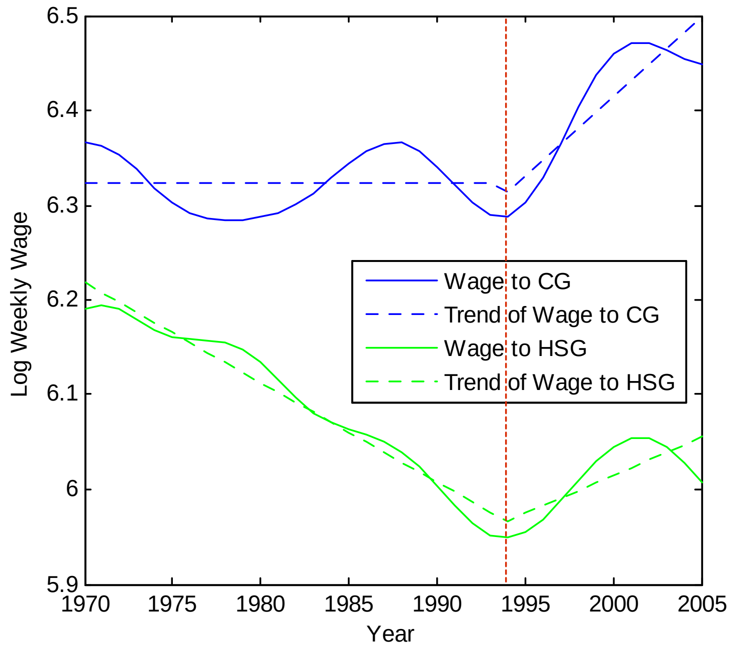
Figures and Tables

Figure 1: College Enrollment Rate and College Premium: HP-filtered, U.S. 1972-2005[7]



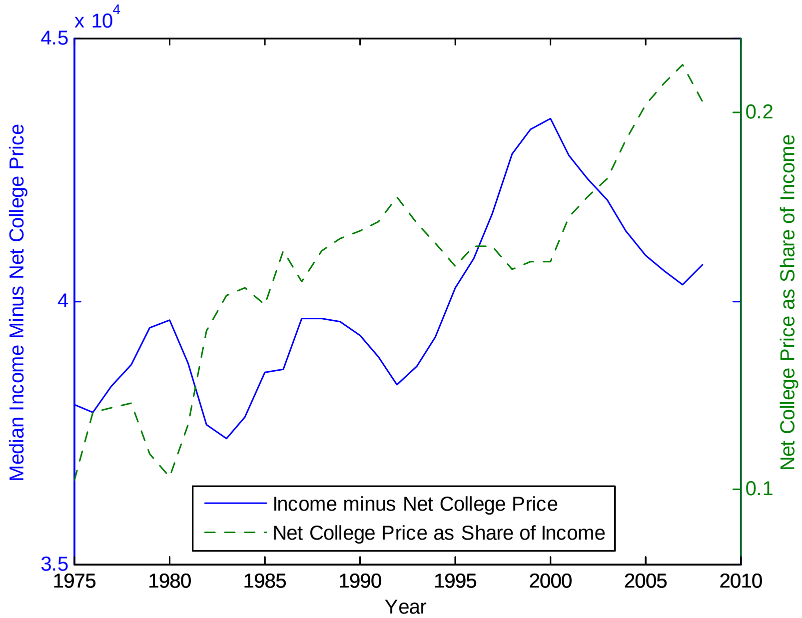
[7]The college premium is the log weekly wage difference of a college graduate and a high school graduate for the age group 23-6, constructed from March CPS. Data are filtered by the Hodrick-Prescott Filter to remove the cycle.

Figure 2: HP-Filtered Log Weekly Wage to College Graduates and High School Graduates[8]



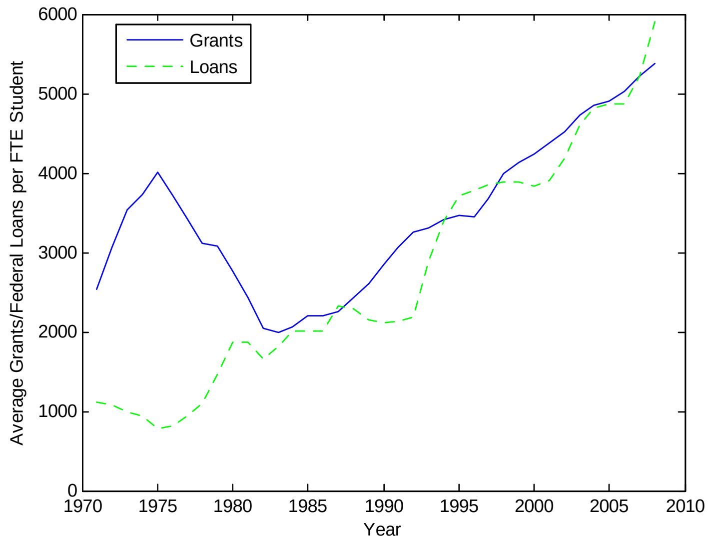
[8]Fitting a linear trend to the HP-filtered log weekly wage series yields: no trend with an average of 6.32 in $\log W^{CG}$ until 1993 while $\log W^{HSG} = 26.96 - 0.011 * Year$; from 1994 to 2005, $\log W^{CG} = -27.2323 + 0.0168 * Year$ and $\log W^{HSG} = -10.10 + 0.0081 * Year$; . All coefficients significant at 1%. The smoothness paramater in the HP-filter is 6.25.

Figure 3: The Difference of HP-filtered Median Household Income and Net College Price versus Net College Price as Share of Median Household Income (in 2008 Dollars) 1975/76-2007/08[9]



[9]Data source: Trends in Student Aid 2009, Table 3; Trends in College Pricing 2009, Figure 5; U.S. Census Bureau, CPS, Annual Social and Economic Supplements, Tables H-6, H-8.

Figure 4: Average Grants and Federal Loans Per Full-Time-Equivalent Student (in 2008 Dollar) 1970/71-2008/09[10]



[10]Source: *Trends in Student Aid 2009*, Table 3.

Figure 5 Real and Fictitious Wage Gap

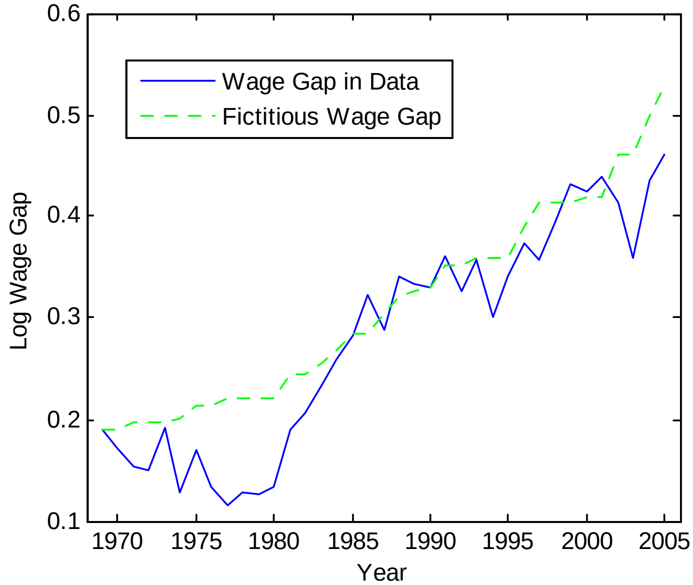
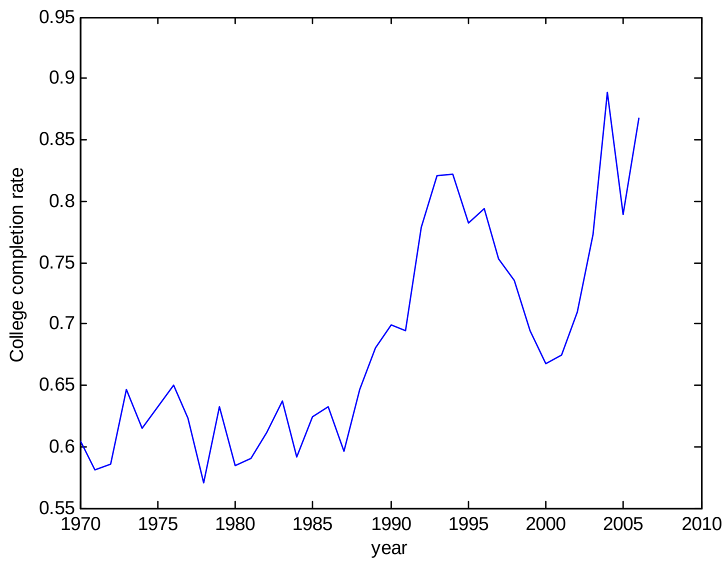
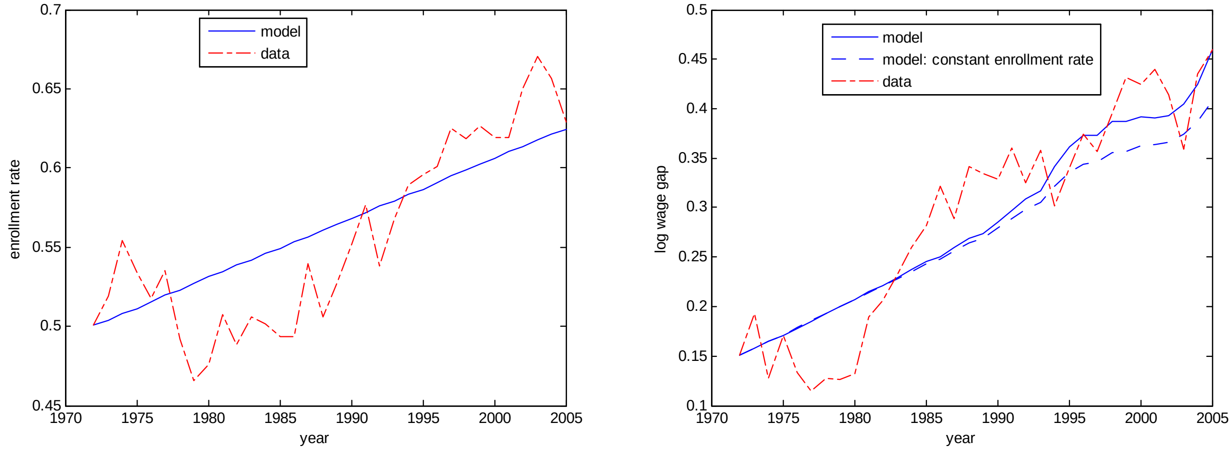


Figure 6 College Completion Rates[11]

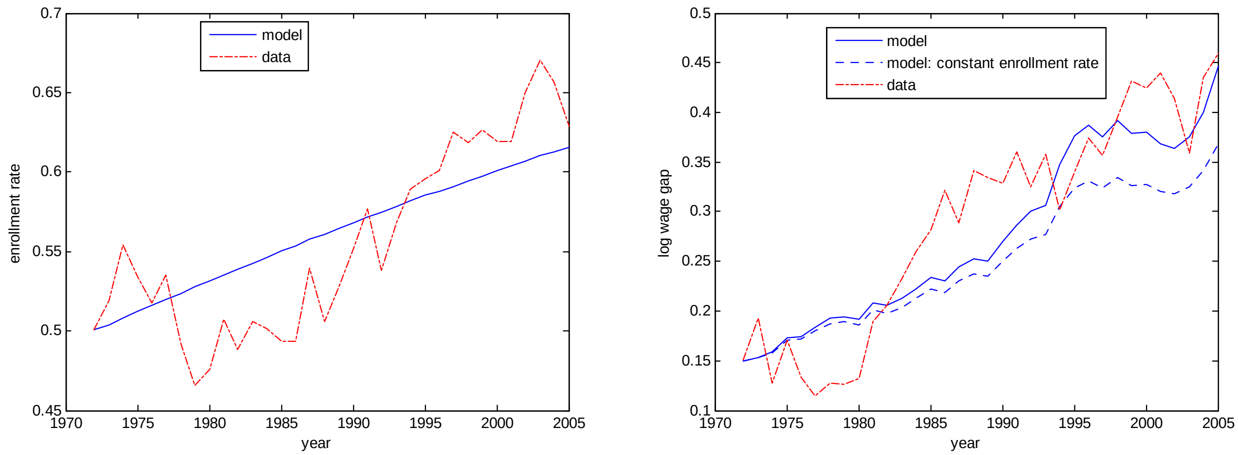


[11]Source: NCES

Panel 1: Model prediction of college premium for $P1 : h_0 = 1, \rho = 0.98, \gamma = 0.5\%$



Panel 2: Model prediction of college premium for P2: $\gamma = 0.3435\%$



Panel 3: Prediction of college premium using endogenous enrollment rates vs. data

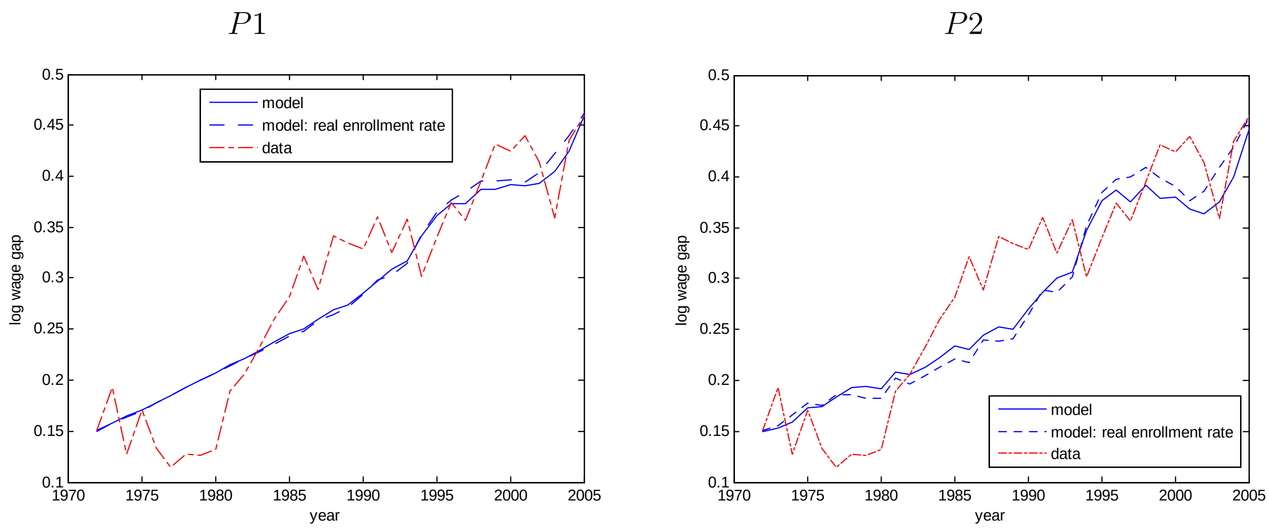


Figure 7: % of CAGR in College Premium Explained by Model γ

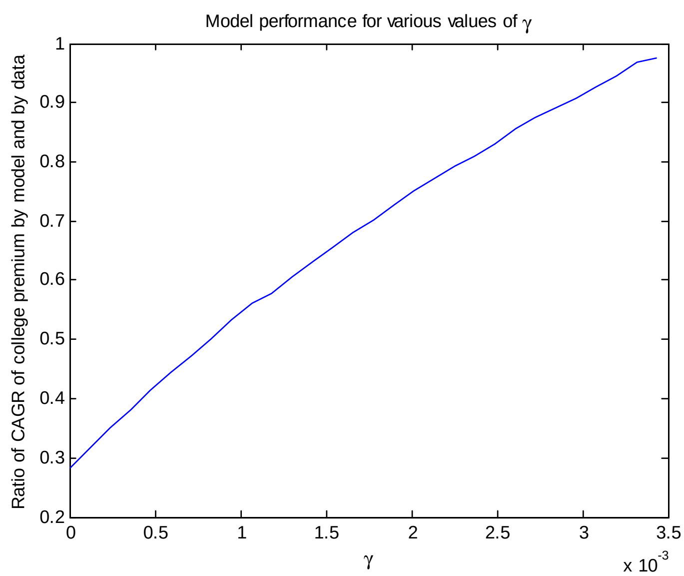


Figure 8: % of CAGR in College Premium in Model γ Explained by Signaling

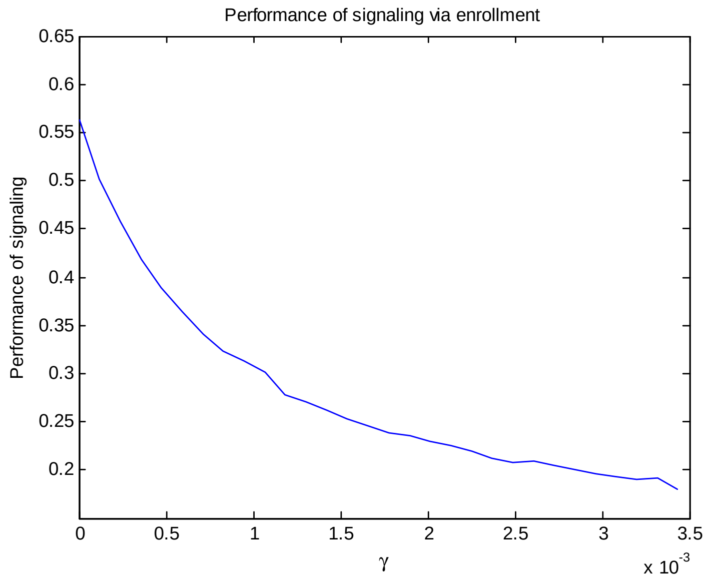


Figure 9: % of CAGR in College Premium Explained by Signaling for Model γ

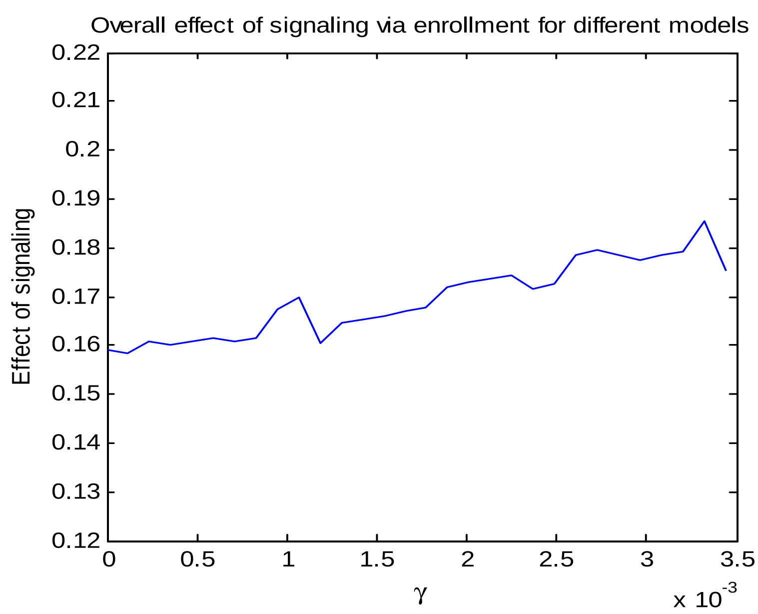


Table 1

Difference between Mean Parents' Income and Tuition and Fees net Grants and Federal Loans, by Income Groups and Types of Intitution, selected years (in 2008 dollars)

Year	Lowest 5th	Second 5th	Third 5th	Fourth 5th	Highest 5th
Public 4-year Institutions					
1985	15,110.83	36,668.34	55,474.54	79,295.33	145,075.60
1988	13,207.92	37,307.48	58,180.59	82,020.43	151,674.68
1991	16,125.12	36,804.96	55,521.14	80,999.30	156,513.43
1994	16,562.50	36,218.12	56,319.46	81,763.71	155,444.26
1998	18,708.25	39,304.02	60,786.51	88,908.78	160,174.07
2002	18,574.82	39,332.83	60,830.58	89,701.64	161,131.78
2006	18,505.38	39,554.77	61,375.99	92,128.54	168,814.00
Private Not-for-profit 4-year Institutions					
1985	13,440.63	35,098.99	53,755.32	75,810.07	168,132.03
1988	10,037.89	33,389.80	53,464.12	76,961.45	167,005.24
1991	13,456.60	33,829.92	50,992.03	76,435.25	153,830.67
1994	13,436.82	32,813.51	52,238.00	76,540.80	157,520.44
1998	15,863.34	35,309.56	56,792.18	82,824.42	156,307.34
2002	13,544.82	34,069.10	54,987.60	84,082.46	156,285.44
2006	13,430.70	34,850.36	54,063.27	83,832.11	167,342.44

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6 Appendix

6.1 Theoretical Derivation

Lemma 1 Proof The value function is $v^i(k(t)) = \max\{v^{ci}(k(t)), v^{nci}(k(t))\}$, where

$$\begin{aligned} rv^c(k(t)) &= p(\theta)\{(1-\sigma)[R(t)(k(t)-Q)+\overline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)(k(t)-Q)+\overline{W}(t)]\} \\ &\quad + [1-p(\theta)]\{(1-\sigma)[R(t)(k(t)-Q)+\underline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)(k(t)-Q)+\underline{W}(t)]\}. \\ s.t. k(t) &\geq Q \\ rv^{nc}(k(t)) &= (1-\sigma)[R(t)k(t)+\underline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)k(t)+\underline{W}(t)]. \end{aligned}$$

Given $\overline{W}, \underline{W}, R$, since it is optimal for(k_0, θ) to go to college,

$$\begin{aligned} \Delta(k,\theta) \equiv v^c(k) - v^{nc}(k) &= (1-\sigma+\sigma\frac{dv}{dk})[p(\theta)(\overline{W}-\underline{W}) - RQ] > 0 \\ &\Rightarrow \Delta(k,\theta') = (1-\sigma+\sigma\frac{dv(k;\theta',k_0)}{dk})[p(\theta')(\overline{W}-\underline{W}) - RQ] \\ &> 0. \forall \theta' > \theta \end{aligned}$$

Hence, independent of the state variable k, (k_0, θ') would always prefers college *as long as* going to college is feasible, i.e. $k \geq Q$. Q.E.D.

.

Assumption 1 $\rho > 1 - \frac{(\int_0^{\overline{\theta}} \theta p dG - \int_0^{\overline{\theta}} p dG \int_0^{\overline{\theta}} \theta dG)(1-F(Q))}{\int_0^{\overline{\theta}} \theta dG - (1-F(Q)) \int_0^{\overline{\theta}} \theta p dG}$.

Lemma 2 Proof Under the specified strategy profile, the output and factor prices are

$$R(t) = \upsilon \Upsilon (K(t) - x(t)Q)^{\rho-1}. \tag{A1}$$

$$\begin{aligned} \overline{W}(t) &= (1-\lambda-\upsilon)\Upsilon(x(t)\int_0^{\overline{\theta}} pdG)^{\rho-1} \frac{\int_0^{\overline{\theta}} \theta p dG}{\int_0^{\overline{\theta}} p dG}. \qquad \text{(A2)} \\ \underline{W}(t) &= \lambda \Upsilon (1 - x(t)\int_0^{\overline{\theta}} pdG)^{\rho-1} \frac{\int_0^{\overline{\theta}} \theta dG - x(t)\int_0^{\overline{\theta}} \theta p dG}{1 - x(t)\int_0^{\overline{\theta}} p dG}, \end{aligned}$$

where $\Upsilon = \{\lambda(1-x\int_0^{\overline{\theta}} pdG)^{\rho}\frac{\int_0^{\overline{\theta}}\theta dG - x\int_0^{\overline{\theta}}\theta pdG}{1-x\int_0^{\overline{\theta}} pdG} + \upsilon(K(t)-xQ)^{\rho} + (1-\lambda-\upsilon)(x\int_0^{\overline{\theta}} pdG)^{\rho}\frac{\int_0^{\overline{\theta}}\theta pdG}{\int_0^{\overline{\theta}} pdG}\}^{\frac{1}{\rho}-1}$.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{\overline{W}}{\underline{W}}\right) &= \frac{1}{1-x\int pdG}\left(\frac{\rho-1}{x} + \frac{\int\theta pdG - \int pdG\int\theta dG}{\int\theta dG - x\int\theta pdG}\right) \\
&\geq \frac{1}{1-x\int pdG}\left(\frac{\rho-1}{x_0} + \frac{\int\theta pdG - \int pdG\int\theta dG}{\int\theta dG - x_0\int\theta pdG}\right) \\
&\geq 0, \text{ by } \textit{Assumption 1} \text{ and } x_0 = 1-F(Q).
\end{aligned}$$

This implies that $\ln(\frac{\overline{W}}{\underline{W}})$ is increasing in x. Note that $\forall Q$, *Assumption 1* is not empty. Q.E.D.

Assumption 2 $\frac{1-\lambda-\upsilon}{\lambda} \geq (\frac{1-x_0\int pdG}{x_0\int pdG})^{\rho-2}\frac{\int\theta dG - x_0\int\theta pdG}{x_0\int\theta pdG}$.

Assumption 2 guarantees $\overline{W}(t) > \underline{W}(t)$.

Assumption 3 $Q < K(0)$.

Proposition 1 Proof The key is to verify that in the suggested equilibrium, all agents optimally make the schooling decision.

By *Lemma 1*, it is sufficient to look at the agent with the lowest talent and make sure he prefers to go to college. Suppose the college attendance is growing over time.

$$\begin{aligned}
& p(0)[\overline{W}(t) - \underline{W}(t)] - R(t)Q \\
= \; & \Upsilon\{p(0)[(1-\lambda-\upsilon)(x\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG} \\
& -\lambda(1-x\int pdG)^{\rho-1}\frac{\int\theta dG - x\int\theta pdG}{1-x\int pdG}] - \upsilon(K(t)-xQ)^{\rho-1}Q\},
\end{aligned}$$

where Υ, as is defined in *Lemma 2*, is positive. *Assumption 2* and *Lemma 2* implies $(1-\lambda-\upsilon)(x\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG} - \lambda(1-x\int pdG)^{\rho-1}\frac{\int\theta dG - x\int\theta pdG}{1-x\int pdG}$ is increasing in x. Now

$$\begin{aligned}
& p(0)[\overline{W}(t) - \underline{W}(t)] - R(t)Q \geq \Upsilon\{p(0)[(1-\lambda-\upsilon)(x_0\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG} \\
& -\lambda(1-x_0\int pdG)^{\rho-1}\frac{\int\theta dG - x_0\int\theta pdG}{1-x_0\int pdG}] - \upsilon(K(0)-Q)^{\rho-1}Q\} \geq 0. \\
\Rightarrow \; & p(0)[(1-\lambda-\upsilon)(x_0\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG} - \lambda(1-x_0\int pdG)^{\rho-1}\frac{\int\theta dG - x_0\int\theta pdG}{1-x_0\int pdG}] \\
\geq \; & \upsilon(K(0)-Q)^{\rho-1}Q \equiv \Psi(Q).
\end{aligned}$$

By *Assumption 3,* $\frac{d\Psi(Q)}{dQ} = [K(0)-Q]^{\rho-2}[K(0)-\rho Q] > 0$,with $\Psi(0)=0$; $\lim_{Q\to K(0)}\Psi(Q) = +\infty$. By *Assumption 2,*

$$p(0)[(1-\lambda-\upsilon)(x_0\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG}-\lambda(1-x_0\int pdG)^{\rho-1}\frac{\int\theta dG - x_0\int\theta pdG}{1-x_0\int pdG}] > 0,$$

then there exists a $\widehat{Q}$ such that

$$\Psi(\widehat{Q}) = p(0)[(1-\lambda-\upsilon)(x_0\int pdG)^{\rho-1}\frac{\int\theta pdG}{\int pdG}-\lambda(1-x_0\int pdG)^{\rho-1}\frac{\int\theta dG - x_0\int\theta pdG}{1-x_0\int pdG}].$$

For all $Q \leq \widehat{Q}$, $p(0)[\overline{W}(t)-\underline{W}(t)] - R(t)Q > 0, \forall t$. So by *Lemma 1,* for Q sufficiently small, all agents want to go to college as soon as they can afford it. Lastly, for all those who are constrained, $\dot{k^i} = \sigma[R(t)k^i + \underline{W}(t)] > 0$. This implies that indeed in the equilibrium there will be an increasing fraction of people who can afford education. Q.E.D.

Corollary 1 Proof An agent starts to go to college at time t that satisfies $k_0^i(t)+\int_0^t \dot{k^i}(s)ds = Q$, where the evolution of k^i follows $\dot{k^i} = \sigma[R(t)k^i(t) + \underline{W}(t)]$. At time t the faction of agents that goes to college is $1-F(k_0^i(t))$, which is increasing in t, since $k_0^i(t)$ is decreasing in t. Q.E.D.

Proposition 2 Proof I proceed in three steps.

Step 1: Transformation. Let $\widehat{p}(\theta) = \overline{\theta}p(\theta)g(\theta)$, which necessarily satisfies $\widehat{p}(\theta) \geq 0, 0 \leq \int_0^{\overline{\theta}}\widehat{p}(\theta)d\theta \leq \overline{\theta}$. Let $\int_0^{\overline{\theta}}\theta dG \equiv a$. This problem is equivalent to a two-step maximization. Given a,

$$\sup_{\widehat{p}(\theta)} \dot{x}\overline{\theta}\frac{\int_0^{\overline{\theta}}\theta\widehat{p}d\theta - a\int_0^{\overline{\theta}}\widehat{p}d\theta}{(\overline{\theta}-x\int_0^{\overline{\theta}}\widehat{p}d\theta)(\overline{\theta}a - x\int_0^{\overline{\theta}}\theta\widehat{p}d\theta)}$$

$$s.t.\widehat{p}(\theta) \geq 0, 0 \leq \int_0^{\overline{\theta}}\widehat{p}(\theta)d\theta \leq \overline{\theta}, 0 \leq \int_0^{\overline{\theta}}\theta\widehat{p}d\theta \leq a\overline{\theta}$$

Then, maximize over all possible a.

Step 2: Change of variables. Let $y(\theta) = \int_0^{\theta}\widehat{p}(v)dv$. Integration by part gives $\int_0^{\overline{\theta}}\theta\widehat{p}d\theta = \int_0^{\overline{\theta}}\theta y'(\theta)d\theta = \overline{\theta}y(\overline{\theta}) - \int_0^{\overline{\theta}}y(\theta)d\theta$. The problem can be rewritten as

$$\sup_{\substack{y(\overline{\theta}),\\ \int_0^{\overline{\theta}}y(\theta)d\theta}} \dot{x}\overline{\theta}\frac{(\overline{\theta}-a)y(\overline{\theta}) - \int_0^{\overline{\theta}}y(\theta)d\theta}{(\overline{\theta}-xy(\overline{\theta}))[x\int_0^{\overline{\theta}}y(\theta)d\theta + \overline{\theta}(a-xy(\overline{\theta})]}$$

$$s.t.\begin{Bmatrix} 0 \leq y(\overline{\theta}) \leq \overline{\theta}; y\ '(\theta) \geq 0; \\ \max\{0, \overline{\theta}(y(\overline{\theta})-a)\} \leq \int_0^{\overline{\theta}}y(\theta)d\theta \leq (\overline{\theta}-a)y(\overline{\theta}). \end{Bmatrix}$$

Step 3: Maximization. Firstly, $y(\overline{\theta})$ and $\int_0^{\overline{\theta}} y(\theta)d\theta$ can take values independently. Secondly, the objective is increasing in $y(\overline{\theta})$, but decreasing in $\int_0^{\overline{\theta}} y(\theta)d\theta$. But bigger $y(\overline{\theta})$ will increase the lowest level that $\int_0^{\overline{\theta}} y(\theta)d\theta$ can take.

If $y(\overline{\theta}) \leq a$, then the optimal values are $y(\overline{\theta}) = a$ and $\int_0^{\overline{\theta}} y(\theta)d\theta = 0$.

If $y(\overline{\theta}) \geq a$. Then at the optimum, no matter what value $y(\overline{\theta})$ takes, $\int_0^{\overline{\theta}} y(\theta)d\theta = \overline{\theta}(y(\overline{\theta}) - a)$. Substituting this relation into the objective function $\sup_{y(\overline{\theta})} \dot{x}\frac{\overline{\theta}-y(\overline{\theta})}{(\overline{\theta}-xy(\overline{\theta}))(1-x)}$. It is decreasing in $y(\overline{\theta})$. Hence, at the optimum, $y(\overline{\theta}) = a$ and $\int_0^{\overline{\theta}} y(\theta)d\theta = 0$.

In both cases, the maximum of the objective function is $\sup(g_{\psi_s} - g_{\psi_u}) = \dot{x}\frac{\overline{\theta}-a}{(\overline{\theta}-xa)(1-x)}$.

Now maximize with respect to a, $\sup(g_{\psi_s} - g_{\psi_u}) = \frac{\dot{x}}{1-x} = -g_{1-x}$,as $a \to 0$.Q.E.D.

Proposition 3 Proof Differentiate the objective function with respect to θ^* gives $g(\theta^*)[(2\lambda + \upsilon - 1)\theta^* p(\theta^*) + \upsilon Q]$. If $2\lambda \geq 1-\upsilon$, maximum is obtained at $\theta^* = \overline{\theta}$. Suppose $2\lambda < 1-\upsilon$. If $(1-2\lambda-\upsilon)\overline{\theta}p(\overline{\theta}) < \upsilon Q$, maximum is obtained at $\theta^* = \overline{\theta}$. Otherwise, first order necessary condition requires for $\theta^* \in [0, \overline{\theta}], (1-2\lambda-\upsilon)\theta^* p(\theta^*) = \upsilon Q$. SOC at θ^* gives $[\lambda - (1-\lambda-\upsilon)][p(\theta^*) + \theta^* p'(\theta^*)] < 0$. Hence $\Gamma(\theta^*)$ is a local maximum. It is easily shown that $\Gamma(\theta^*) > \Gamma(\overline{\theta})$ and $\Gamma(\theta^*) > \Gamma(\overline{\theta})$. Hence, θ^* achieves the global maximum. Q.E.D.

Proposition 4 Proof The agents' problem: the value function is $v^i(k(t)) = \max\{v^{ci}(k(t)), v^{nci}(k(t))\}$, where,

$$rv^c(k(t)) = p(\theta)\{(1-\sigma)[R(t)(k(t)-Q) + \overline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)(k(t)-Q) + \overline{W}(t)]\}$$
$$+[1-p(\theta)]\{(1-\sigma)[R(t)(k(t)-Q) + \underline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)(k(t)-Q) + \underline{W}(t)]\}$$
$$s.t. k(t) + b(t) \geq Q.$$
$$rv^{nc}(k(t)) = (1-\sigma)[R(t)k(t) + \underline{W}(t)] + \frac{dv^i}{dk}\sigma[R(t)k(t) + \underline{W}(t)].$$

By the same logic as in *Proposition 1*,$\forall t, v^{ci}(k(t)) - v^{nci}(k(t)) = p(0)[\overline{W}(t) - \underline{W}(t)] - R(t)Q > 0$. Now the factor prices in the proposed equilibrium are

$$\begin{aligned}
\overline{W}(t) &= (1-\lambda-\upsilon)\widehat{\Upsilon}(\int pdG)^{\rho-2}\int \theta pdG. \\
\underline{W}(t) &= \lambda\widehat{\Upsilon}(1-\int pdG)^{\rho-2}(\int \theta dG - \int \theta pdG). \\
R(t) &= \upsilon\widehat{\Upsilon}(K(t)-Q)^{\rho-1},
\end{aligned}$$

where $\Upsilon = \{\lambda(1-\int pdG)^{\rho-1}(\int \theta dG - \int \theta pdG) + \upsilon(K-Q)^{\rho} + (1-\lambda-\upsilon)(\int pdG)^{\rho-1}\int \theta pdG\}^{\frac{1}{\rho}-1}$.

$$\begin{aligned}
& p(0)[\overline{W}(t) - \underline{W}(t)] - R(t)Q \\
= \; & \widehat{\Upsilon}\{p(0)[(1-\lambda-\upsilon)(\int pdG)^{\rho-2}\int \theta pdG - \lambda(1-\int pdG)^{\rho-2}(\int \theta dG - \int \theta pdG)] \\
& -\upsilon(K(t)-Q)^{\rho-1}Q\},
\end{aligned}$$

By *Assumptions 1-3*,

$$\begin{aligned}
& (1-\lambda-\upsilon)(\int pdG)^{\rho-2}\int \theta pdG - \lambda(1-\int pdG)^{\rho-2}(\int \theta dG - \int \theta pdG) \\
> \; & (1-\lambda-\upsilon)(x_0\int pdG)^{\rho-1}\frac{\int \theta pdG}{\int pdG} - \lambda(1-x_0\int pdG)^{\rho-1}\frac{\int \theta dG - x_0\int \theta pdG}{1-x_0\int pdG} > 0.
\end{aligned}$$

$\exists\, \widehat{Q}^*$ such that

$$\begin{aligned}
& p(0)[(1-\lambda-\upsilon)(\int pdG)^{\rho-2}\int \theta pdG - \lambda(1-\int pdG)^{\rho-2}(\int \theta dG - \int \theta pdG) \\
= \; & \upsilon(K(0)-\widehat{Q}^*)^{\rho-1}.
\end{aligned}$$

It is readily seen that $\widehat{Q}^* > \widehat{Q}$. $\forall Q < \widehat{Q}^*$, $p(0)[\overline{W}(t) - \underline{W}(t)] - R(t)Q \geq 0$. Hence, everyone attends college at all times, while the wage gap remains constant. Q.E.D.

6.2 Separating Equilibrium

I sketch here the proof of the existence of a separating equilibrium for $P2$. This exercise can be repeated for $P1$.

Assumption 4 $p(0) = 0$.

Assumption 5 $g(\overline{\theta}) = 0$.

Assumption 6 $\beta p(E(\theta))[\frac{\int_{E(\theta)}^{\overline{\theta}} \theta pdG}{\int_{E(\theta)}^{\overline{\theta}} pdG} - \frac{\int \theta dG - (1-F(Q))\int_{E(\theta)}^{\overline{\theta}} \theta pdG}{1-(1-F(Q))\int_{E(\theta)}^{\overline{\theta}} pdG}] > \alpha[K_0 - (1-G(E(\theta))Q]^{\rho-1}Q$.

Assumption 7 $1 > 2\int_{E(\theta)}^{\overline{\theta}} pdG$.

First, at time t, fix $x_t = 1 - F(k_{0t})$ and K_t. By Lemma 1, the cut-off level of talent $\widehat{\theta}_t$ satisfies $p(\widehat{\theta}_t)(\overline{W}_t - \underline{W}_t) = R_t Q$, or

$$\beta p(\widehat{\theta}_t)[E_t(\theta|CG) - E_t(\theta|HSG)] = \alpha[K_t - (1 - G(\widehat{\theta}_t))x_t Q]^{\rho-1}Q. \tag{3}$$

The LHS is further equal to $\beta p(\widehat{\theta}_t)[\frac{\int_{\widehat{\theta}_t}^{\overline{\theta}} \theta pdG}{\int_{\widehat{\theta}_t}^{\overline{\theta}} pdG} - \frac{\int \theta dG - x_t \int_{\widehat{\theta}_t}^{\overline{\theta}} \theta pdG}{1 - x_t \int_{\widehat{\theta}_t}^{\overline{\theta}} pdG}]$. One can show that RHS is decreasing in $\widehat{\theta}_t$ while LHS is increasing in $\widehat{\theta}_t$ if

(a) $\widehat{\theta}_t < \int \theta dG$; and (b) $1 > 2x_t \int_{\widehat{\theta}_t}^{\overline{\theta}} pdG$. We will restrict the solution $\widehat{\theta}_t$ to $[0, E(\theta)]$ to guarantee (1). (2) is ensured by (1) and Assumption 7. Note that we can rewrite (2) as $2[x_t(1-G(\widehat{\theta}_t))]\frac{\int_{\widehat{\theta}_t}^{\overline{\theta}} pdG}{1-G(\widehat{\theta}_t)} = 2\cdot$enrollment rate$_t\cdot$college completion rate$_t$. One can verify using the U.S. data from 1972 to 2005 that the above inequality is always satisfied. Under Assumptions 4 and 5, in order for (3) to have a solution in $[0, E(\theta)]$, one requires Assumption 6. Hence, for all values of x_t and K_t, there exists a cut-off point $\widehat{\theta}_t \in [0, E(\theta)]$, such that all agents with $\theta \geq \widehat{\theta}_t$ choose to go to college as long as they can afford it.

Second, the dynamic system that characterizes the equilbrium path is

$$\begin{aligned} \dot{K}_t &= \sigma A[\alpha(K_t - (1 - F(k_{0t}))(1 - G(\widehat{\theta}_t))Q)^\rho + \beta E(\theta)^\rho]^{1/\rho}; \\ \dot{k}_{0t} &= -\sigma[R_t Q + \underline{W}_t]; \end{aligned}$$

where $R_t = \Upsilon\alpha[K_t - (1 - F(k_{0t}))(1 - G(\widehat{\theta}_t))Q]^{\rho-1}$, $\underline{W}_t = \Upsilon\beta[E(\theta)]^{\rho-1}\frac{\int \theta dG - (1-F(k_{0t}))\int_{\widehat{\theta}_t}^{\overline{\theta}} \theta pdG}{1-(1-F(k_{0t}))\int_{\widehat{\theta}_t}^{\overline{\theta}} pdG}$ and $\Upsilon = A[\alpha(K_t - (1 - F(k_{0t}))(1 - G(\widehat{\theta}_t))Q)^\rho + \beta E(\theta)^\rho]^{1/\rho-1}$. The cut-off of talent satisfies

$$\beta p(\widehat{\theta}_t)[\frac{\int_{\widehat{\theta}_t}^{\overline{\theta}} \theta pdG}{\int_{\widehat{\theta}_t}^{\overline{\theta}} pdG} - \frac{\int \theta dG - (1 - F(k_{0t}))\int_{\widehat{\theta}_t}^{\overline{\theta}} \theta pdG}{1 - (1 - F(k_{0t}))\int_{\widehat{\theta}_t}^{\overline{\theta}} pdG}] = \alpha[K_t - (1 - G(\widehat{\theta}_t))(1 - F(k_{0t}))Q]^{\rho-1}Q.$$

The initial conditions are $K_0 = \int_0^{\overline{k}_0} k_0 dF, k_{00} = Q$.

Under Assumptions 4-7, the solution to the above dynamic system exists. However, the equilibrium paths of the cut-off point of talent, the enrollment rates and the college premium are not necessarily monotone.

6.3 Calibration

6.3.1 Data

Skill premium. The raw data are taken from the CPS March from 1969 to 2005. Only fullyear fulltime workers that have positive wage and schooling are considered. They are grouped by ages. The relevant age group here is those age 23 to 26. The log deflated weekly wage, which is the income from wage and salary divided by weeks worked, is then regressed on dummies of education, geographic region and race, by sexes. The education is the highest education attainment reported, high school dropouts, high school graduates, some college, college graduates or above. The geographical region is grouped in four, Northeast region, Midwest region, South region and West region. For more definitions on the data precession, please refer to Autor, Katz and Kearney (2008). For each sex, the log wage gap is the difference between the prediction for a white college graduate (but with no graduate degree) who lives in the average geographic region and that for a high school graduate counterpart. The log wage gap is the mean of the log wage gaps of the two sexes, weighted by their hours worked. CPS weights are used. I have explored variations of this basic set-up, including the log 10-year-income gap, the log wage gap between a 23 year old college graduate and a 19 year old high school graduate, among others. The results don't differ much.

Initial income distribution in 1972. Annual income from wage and salary are converted into 2006 dollars by CPI index. CPS weights are used. To match the initial enrollment rate, which is 0.5006, I find the 50th percentile in the empirical income distribution and normalize it to be equal to Q. That is,

$$F(Q/\xi) = 1 - 0.5006.$$

Further multiply all income in the sample by ξ and this gives the $F(\cdot)$ in the model. ξ can be thought of as the share of income that goes to educational expenses.

Cost of college. The real cost of college, computed using the data published in *Trends in College Pricing 2006* and *Trends in Student Aid 2007,* do not show an obvious trend from 1986 to 2006. I take Q to be the average over all these years, which is 5467.

6.4 Proofs for Model with $P2$

Lemma 2' Proof Let $a = E(\theta)$. The gross output is

$$Y = A\{\alpha(K - xQ)^{\rho} + \beta[L_H E(\theta|HSG) + L_C E(\theta|CG)]^{\rho}\}^{1/\rho}.$$

$$\begin{aligned}\overline{W} &= \Lambda\beta a^{\rho-1}E(\theta|CG);\\ \underline{W} &= \Lambda\beta a^{\rho-1}E(\theta|HSG);\\ R &= \Lambda\alpha(K-xQ)^{\rho-1},\end{aligned}$$

where $\Lambda = A\{\alpha(K-xQ)^{\rho}+\beta[L_H E(\theta|HSG)+L_C E(\theta|CG)]^{\rho}\}^{1/\rho-1}$.

$$\begin{aligned}\ln\frac{\overline{W}}{\underline{W}} &= \frac{E(\theta|CG)}{E(\theta|HSG)}\\ &= \frac{\int\theta pdG}{\int pdG}\frac{1-x\int pdG}{\int\theta dG - x\int\theta pdG}\end{aligned}$$

increasing in x. Q.E.D.

Proposition 1' Proof

$$\begin{aligned}& p(0)(\overline{W}-\underline{W})-RQ\\ =\ & \Lambda\{p(0)\beta a^{\rho-1}[E(\theta|CG)-E(\theta|HSG)]-\alpha(K-xQ)^{\rho-1}Q\}\\ \geq\ & \Lambda\{p(0)\beta a^{\rho-1}[E(\theta|CG)-E(\theta|HSG)]-\alpha(K(0)-Q)^{\rho-1}Q\}.\end{aligned}$$

By the same token, there exists $\widetilde{Q}$, s.t.

$$p(0)\beta a^{\rho-1}[E(\theta|CG)-E(\theta|HSG)] = \alpha(K(0)-\widetilde{Q})^{\rho-1}\widetilde{Q}.$$

For all $Q\leq\widetilde{Q}$,

$$p(0)(\overline{W}-\underline{W})-RQ\geq 0, \forall t.$$

Moreover, when this is the case, there will be indeed an increasing number of agents going to college. Q.E.D.

6.5 1st Stage Estimation Results

P1 The non-linear model for wage gap is

$$\ln(\frac{\overline{W}}{\underline{W}})_t = \ln\frac{1-\lambda-\upsilon}{\lambda}+(\rho-1)\ln(\frac{x_t\pi_t}{1-x_t\pi_t})+\ln\frac{(h_0+\gamma t)(1-x_t\pi_t)}{\int\theta dG - x_t\pi_t(h_0+\gamma t)},$$

which is transform into a statistical model with series of $\ln(\frac{\overline{W}}{\underline{W}})_t$, x_t and π_t:

$$\begin{aligned}y_t &= \ln(\frac{\overline{W}}{\underline{W}})_t+\ln\frac{x_t\pi_t}{1-x_t\pi_t} = \ln\frac{1-\lambda-\upsilon}{\lambda}+\rho\ln\frac{x_t\pi_t}{1-x_t\pi_t}+\ln\frac{(1-x_t\pi_t)(h_0+\gamma t)}{\int\theta dG - x_t\pi_t(h_0+\gamma t)}+\varepsilon_t\\ &= b_0\ln\frac{x_t\pi_t}{1-x_t\pi_t}+\ln\frac{(1-x_t\pi_t)(1+b_2 t)}{b_1-x_t\pi_t(1+b_2 t)}+\varepsilon_t.\end{aligned}$$

Normalize $h_0 = 1$. Take $\upsilon = \lambda = 1/3$, and jointly estimate ρ, $\int \theta dG$ and γ. Note that $\int \theta dG$ and γ are relative to h_0 as a result of normalization.

Source	SS	df	MS	Number of obs=	34	
Model	7.4987078	3	2.49956927	R-squared=	0.9925	
Residual	.05678031	31	.001831623	AdjR-squared=	0.9918	
				RootMSE=	.0427975	
Total	7.55548811	34	.001831623	Res.dev. =	−120.9397	
yt	Coef.	Std.Err.	t	P>t	[95% Conf.Interval]	
$/b0$	.859356	.0352956	24.35	0.000	.7873702	.9313418
$/b1$	.9985415	.0269426	37.06	0.000	.9435917	1.053491
$/b2$	.0071175	.0010561	6.74	0.000	.0049637	.0092714

Parameter b0 taken as constant term in model & ANOVA table

This implies

$$\begin{aligned} \rho &= 0.869356; \\ \int \theta dG &= 0.9986415; \\ \gamma &= 0.71175\%. \end{aligned}$$

P2 The non-linear model for the wage gap is

$$\ln \frac{\overline{W}}{\underline{W}_t} = \ln \frac{(h_0 + \gamma t)(1 - x_t \pi_t)}{\int \theta dG - x_t \pi_t (h_0 + \gamma t)};$$

which is transformed into

$$\begin{aligned} y_t &= \ln(\frac{\overline{W}}{\underline{W}})_t - \ln(1 - x_t \pi_t) = \ln \frac{h_0 + \gamma t}{\int \theta dG - x_t \pi_t (h_0 + \gamma t)} \\ &= \ln \frac{1 + b_0 t}{b_1 - x_t \pi_t (1 + b_0 t)}. \end{aligned}$$

I normalize $h_0 = 1$, and jointly estimate γ and $\int \theta dG$.

Source	SS	df	MS	Number of obs=	33	
Model	22.5626658	2	11.2813329	R-squared=	0.9966	
Residual	.07801796	31	.002516708	AdjR-squared=	0.9963	
				RootMSE=	.0501668	
Total	22.6406837	33	.686081325	Res.dev. =	-105.9117	
yt	Coef.	Std.Err.	t	$P > t$	[95%Conf.Interval]	
$/b0$	.0034347	.0005565	6.17	0.000	.0022996	.0045697
$/b1$	.9032312	.0103055	87.65	0.000	.8822131	.9242494

Parameter b0 taken as constant term in model & ANOVA table

This implies

$$\begin{aligned} \gamma &= 0.3435\%, \\ \int \theta dG &= 0.9032. \end{aligned}$$