Hierarchy in Microfinance: Embezzlement and the Optimality of Rigid Repayment Schedules and Joint Liability*

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Abstract

We consider the agency problem of a bank staff member managing microfinancing programs, who can abuse his discretion to embezzle borrowers’ repayments. In this context, we study the optimal lending contract. The fact that most borrowers of microfinancing programs are illiterate and live in rural areas where transportation costs are very high make staff’s embezzlement particularly relevant as is documented by Mknely and Kevane (2002). We study the trade-off between the optimal rigid contract and the optimal discretionary contract and how joint liability affects the performance of each contract and the trade-off. Our analysis explains rigid repayment schedules used by the Grameen bank as an optimal response to the bank staff’s agency problem. Joint liability reduces borrowers’ burden of respecting the rigid repayment schedules by providing them with partial insurance. However, the same insurance can be provided by borrowers themselves under individual liability through a side-contract. We also find that the staff’s agency problem creates biases in project selection toward projects of small scale and small risk.

JEL Code: O16, D82, G20

Key words: Microfinance, Group Lending, Joint Liability, Embezzlement, Hierarchy, Contract

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1 Introduction

The remarkable success of microfinance programs in making loans to (and recovering them from) the poor has received world-wide attention and generated a global microfinance movement which has been growing rapidly: according to the report from the Microcredit Summit Campaign, by the end of 2002, 67.6 million clients were served worldwide by over 2,500 microfinance institutions (Armendáriz de Aghion and Morduch, 2005, p.3). The original ideas of microfinance are due to Muhammad Yunus, the founder of the Grameen bank, who started making small loans to groups of poor people in rural area in Bangladesh in the 1970s. Today the Grameen bank is a large financial organization: it disbursed $482.90 million during the 12 months from May 2004 to April 2005 to about 4.5 million borrowers and its loan recovery rate is about 99 percent.1

Most of the existing literature on microfinance2 centers on how group lending, in particular joint liability, affects adverse selection (Ghatak, 1999, 2000, Van Tassel, 1999, Armendáriz de Aghion and Gollier 2000, Laffont and N’Guessan 2000, Laffont, 2003), moral hazard in terms of loan repayment (Besley and Coate, 1995, Armendáriz de Aghion 1999, Rai and Sjöström, 2004) and moral hazard before return realizations such as work incentive (Stiglitz 1990, Varian 1990, Conning 1999, Che 2002, Laffont and Rey, 2003). Despite the variety of the issues that the papers examine, all of them, except Conning (1999)3, consider only borrowers’ incentives and do not study the incentive issues of the staff managing the loans. Furthermore, most papers consider state-contingent repayments without investigating a staff member’s incentive to reveal his4 information about the realized state of nature to the bank. However, corruption is rampant in most organizations in underdeveloped countries and the Grameen bank is a large organization involving several layers of hierarchy.5 Hence, understanding the success of the Grameen bank requires also understanding how its lending contracts successfully solve (or mitigate) agency problems inherent in a hierarchy. Our paper addresses the question of how an organization that

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3 However, Conning does not address embezzlement.
4 We use she for the lender and he for a staff member (or supervisor) or a borrower.
5 At the time of April 2005, the Grameen bank has 4.48 million borrowers and 1,456 branches. It works in 51,687 villages and its total staff members are 13,492. (http://www.grameen-info.org/bank/GBGlance.htm). The hierarchy is composed of head office, zonal office, area office, branch office, center, group, member (Bornstein, 1997 p. 176).
manages so much money in a corrupt society can remain honest.\textsuperscript{6}

Since most borrowers of microfinancing programs in poor countries are illiterate\textsuperscript{7} and means of transportation are primitive, they get informed about the conditions of the loan exclusively through the bank staff member who visits their villages to collect repayments and to monitor their behavior (regarding how the loan is invested and whether they undertake adequate effort). This creates significant scope for the member’s misconduct, and for embezzlement in particular, as documented by Bornstein (1996)\textsuperscript{8} and Mknelly and Kevane (2002). For instance, according to Mknelly and Kevane (2002), room for embezzlement arises because illiterate borrowers cannot maintain their account books.\textsuperscript{9} Furthermore, contract enforcement problems related to limited liability exist not only at the level of borrowers but also at the level of staff.

In this paper, we study the optimal lending and supervisory contracts when the bank staff can embezzle repayments by misrepresenting the realized state when a loan contract specifies state-contingent repayments. To focus on the staff’s incentive problem, we assume away any adverse selection or moral hazard at the level of borrower(s). Therefore, our approach is complementary to the approach taken by the existing literature which focuses on the borrowers’ incentive problems and allows us to explain two main features of the lending contracts used by the Grameen bank, rigid repayment schedules and joint liability, as optimal responses to bank staff’s agency problem. Contrary to the findings of the most papers on group lending that show the optimality of stage-contingent repayments, the lending contracts of the Grameen bank specify a rigid repayment schedule which does not depend on the realization of the state; almost immediately after receiving a loan, a borrower has to start to pay back his weekly repayment which is fixed for the whole period of the lending contract.\textsuperscript{10}

We consider a simple model of hierarchy: there are a lender, a supervisor (a bank staff member) and borrower(s). The supervisor has the task to check the success or failure of the project(s) undertaken by the borrower(s) and to collect the repayment(s). The lender

\textsuperscript{6}An employee of the Grameen bank says “Every other organization in Bangladesh is full of corruption. Here, you can be an honest person and it’s possible to remain so.” (Bornstein 1997, p. 167)

\textsuperscript{7}According to Yunus (1998, p. 24), ”We have worked with ... women who cannot read and write.....”

\textsuperscript{8}Bornstein writes about the embezzlement in the early period of the Grameen bank (pp. 169-174).

\textsuperscript{9}For this reason, staff often take the responsibility of maintaining account books. However, when the authors investigated a staff member’s accounting book, they found ”... the account book had no entry for the group fund. There was no carryover from one cycle to the next, and no entries in the log of deposits and withdrawals” (p. 2028).

\textsuperscript{10}Yunus (1998, p.110) describes the repayment mechanism of the Grameen bank as: (i) one year loan (ii) equal weekly installments (iii) repayment starts one week after the loan etc.
can use either a rigid contract in which the borrower makes the same payment regardless of the realization of the state or a discretionary contract in which the repayment depends on the realization of the state. If the lender uses the latter, the supervisor has a discretion in that when a project succeeded, he can report that the project failed and embezzle the difference between the payment upon success and the payment upon failure. The lender can use incentive pays and/or audit to induce the supervisor to behave well. We assume that, when a cheating supervisor is discovered, the lender can recover the original payment made by the borrower and punish the supervisor by not paying him his salary but cannot impose any further fine because of limited liability.

As is usual in the literature on group lending\textsuperscript{11}, the lender is assumed to maximize the borrower(s)’ payoff subject to the break-even constraint. We assume that a borrower’s marginal cost of paying back one unit of money is higher when the project fails than when it succeeds. Therefore, in the benchmark of a honest supervisor, it is optimal for the bank to provide insurance to the borrower by recovering all financing cost through a repayment upon success.

In section 3, we consider the case of a single borrower and study the optimal lending contract and the optimal supervisory contract. Individual lending programs are often used by microfinance programs\textsuperscript{12} and our single borrower case clarifies the trade-off between the optimal rigid contract and the optimal discretionary contract. We find first the optimal supervisory contract given a discretionary lending contract. It turns out that it is optimal to use either incentive pays only or audit only. We prove that the optimal lending contract is either a rigid contract or a contract with a maximum discretion in which the borrower pays only when his project succeeds. In the former case, the supervisor has no discretion and neither bonus nor audit is necessary while using only audit is optimal in the latter case. The optimal contract is rigid when the audit cost is larger than the borrower’s burden of respecting the fixed repayment schedule when his project fails. In other words, the optimal contract is rigid when the audit cost is larger than the gain from providing insurance for the borrower.

In section 4, we consider the case of two borrowers and derive the optimal lending contract and the optimal supervisory contract both with and without joint liability. Note first that in our model joint liability does not affect the net present value (NPV) of the project when the supervisor is honest and hence there is no particular reason to

\textsuperscript{11}For instances, see Armendáriz de Aghion (1999), Ghatak (1999, 2000), Rai and Sjöström (2004).

\textsuperscript{12}According to the Microbanking Bulletin (2002), 73 over 147 best microfinance programs surveyed make loans to individuals.
adopt it in the absence of the supervisor’s agency problem. We find that joint liability strictly increases the NPV both under the optimal rigid contract and under the optimal discretionary contract. Joint liability can thus be regarded as an optimal response to the embezzlement problem. For instance, if the auditing cost is high such that a rigid contract is optimal, then joint liability reduces the borrowers’ burden of respecting a rigid repayment schedule by providing them with partial insurance: although the total repayment does not depend on the realized state in a rigid contract, some partial insurance can be provided in the state in which only one borrower’s project succeeds by increasing his repayment and reducing the other’s repayment. However, we show that if the borrowers can sign a side-contract for mutual insurance, the outcome of the optimal rigid contract under joint liability can be achieved also under individual liability.

Agency problems in hierarchies have been a major theme of research in economics. For instance, Williamson (1967) and Calvo and Wellisz (1978) study when and how loss of control in a hierarchy limits the size of firm. Our paper is more closely related to the literature on collusion between a supervisor and the agent(s) in the mechanism design theory (Tirole 1986, Laffont and Tirole 1991, Kofman and Lawarrée 1993 and Faure-Grimaud, Laffont and Martimort 2003). In the literature, they derive the optimal collusion-proof contract when the supervisor can manipulate the information he reports to the principal about the agent’s type in exchange for a bribe. We do not study the collusion between a supervisor and a borrower, but we focus on the supervisor’s incentive to manipulate the information he reports to the lender regarding the realized return of the project(s).\textsuperscript{13}

Although our paper is related to the literature on costly state verification (Townsend 1979, Diamond 1984, Gale and Hellwig 1985) in that the lender can conduct an audit to verify the realized return, to our knowledge, it is the first paper that considers a hierarchy in the framework of costly state verification and tackles the supervisor’s incentive to embezzle borrowers’ repayments. Another difference is that while the previous literature assumes that the borrower cannot repay the loan for some states and derive the optimality of a state-contingent repayment contract such as a debt contract, we assume that the borrower is able to repay the loan even when the project fails, which is consistent with the 99% of repayment rate in the Grameen bank, and focus on the trade-off between a rigid repayment schedule and a state-contingent one.

In the group lending literature, Conning (1999) is the only one, to our knowledge, who considers a hierarchy; he studies both the borrower’s incentive to choose effort and

\textsuperscript{13}Note that in section 4.4, we consider side-contracting between borrowers.
the staff’s incentive to monitor the former’s effort choice. However, he does not study
the staff’s incentive to embezzle repayments since he assumes that the realized return is
common knowledge as most papers on group lending do. Furthermore, he considers only
one instrument (i.e. incentive pay) to affect the staff’s incentive while we consider two
instruments (i.e. incentive pay and audit). In Armendáriz de Aghion (1999), Armendáriz
de Aghion and Gollier (2000), Rai and Sjöström (2004), the realized return of a borrower’s
project is his private information. However, none of them consider a hierarchy. For
instance, Rai and Sjöström (2004) study the efficient lending contracts that induce the
borrowers to truthfully report their information about the realized returns and show, for
instance, that in the absence of the collusion between the borrowers, the outcome of the
efficient contract (which is a debt contract) can be implemented through a cross-reporting
mechanism when they share information about the returns.\textsuperscript{14} By contrast, we consider the
efficient lending contracts that induce the supervisor to truthfully report his information
about the realized returns and derive the condition under which a state-independent
contract is optimal. Our results suggest that giving discretion to staff to enable them to
use more local information for borrowers’ benefit, as is suggested by cross-reporting, can
involve the cost of increasing the scope for their misconduct and thereby can backfire by
reducing the borrowers’ payoffs.

Section 2 presents the model for the case of a single borrower and Section 3 analyzes
this case. Section 4 analyzes the case of two borrowers. Section 5 discuss our results and
section 6 concludes. All the proofs that are not presented in the main texts are gathered
in Appendix.

\section{Basic model}

The model has three agents: one lender, one supervisor and one borrower. The borrower
borrows one unit of money from the lender and invests it in a project. The lender is
risk neutral and designs the contracts. She is assumed to maximize the borrower’s payoff
as long as her own break-even constraint is satisfied. More precisely, to break even, she
needs to be paid back an amount $\rho > 1$, the opportunity cost of the loan, plus the wage
bill paid to the supervisor and the cost of audit.

\textsuperscript{14}Laffont and Rey (2003) also show the optimality of cross-reporting in the absence of collusion in
terms of inducing effort.

Let $Y$ denote the revenue generated by the borrower’s investment: with probability
$p \in (0, 1)$, the project succeeds and generates a revenue of $Y = Y_S > 0$ units of money;
with probability $1 - p$ it fails and generates $Y = Y_F \equiv 0$ units of money. A lending contract, represented by $\{r_S, r_F\}$, specifies a repayment contingent on the state: the borrower should pay $r_S$ when $Y = Y_S$ and $r_F$ when $Y = Y_F$. In order to focus on the moral hazard of the supervisor, we assume that the borrower always pays $r_i$ back in state $i = S, F$.\footnote{In reality, dynamic incentives, not modeled in this paper, can induce borrowers to pay back the loan since they highly value the opportunity to borrow larger and larger amount of money in the future.} Without loss of generality\footnote{We can obtain our results even though $r_i$ can be negative but considering this case makes the proofs much longer. Even though the constraint $r_F \geq 0$ binds in our analysis, one cannot strictly improve the borrowers’ payoff by considering $r_F < 0$ as is explained in remark 1 in section 3.}, we require $r_i$ to be non-negative.

The borrower has no collateral and therefore, in the case of failure, his only way to generate cash to pay $r_F$ back consists in reducing his consumption. This is costly in the following sense: generating $r$ units of money by reducing consumption costs $\psi(r)$ to the borrower, with $\psi(0) = 0$, $\psi'(r) > 1$ and $\psi''(r) \geq 0$ for any $r > 0$; hence, $\psi(r) > r$ for $r > 0$.\footnote{For instance, Bornstein (1997) documents the story of a woman who purchased a cow with the money borrowed from the Grameen bank. Since the cow stopped lactating midway through the year, she had to cut down on the family’s eating to pay back the weekly installments (p.149).} Alternatively, $\psi(r)$ can be interpreted as the cost of borrowing money from local money lenders.\footnote{Yunus (1998) describes that the credit market for the poor, before the launch of the Grameen bank, was taken over by local money lenders charging usurious interest rates.} The borrower’s expected payoff upon accepting the contract is given by:\footnote{Notice that (1) assumes $r_S \leq Y_S$, which is necessary for the borrower to obtain a positive payoff.}

$$p(Y_S - r_S) - (1 - p)\psi(r_F).$$

We remark that given a state $i = S, F$, the borrower’s marginal utility of one unit of money is 1 if $Y_i - r_i \geq 0$ while it is larger than 1 if $Y_i - r_i < 0$. This decreasing marginal utility of money makes the borrower risk averse. We say that the borrower is fully insured if $r_F = 0$ (and $r_S \leq Y_S$); then, he makes a payment to the lender only in state $S$ and the marginal utility from additional money is equal to one regardless of the realized state.

The supervisor has the task to check and report the state (whether $Y = Y_S$ or $Y = Y_F$) and to collect the borrower’s repayment $r_S$ or $r_F$. A supervisory contract specifies a wage for the supervisor contingent on the state he reports: the wage is $w_S$ when $Y = Y_S$ and $w_F$ when $Y = Y_F$. Since the means of transportation are primitive, the supervisor must visit the borrower to get the repayment and we assume that as long as he visits the borrower, he can costlessly verify the state. Furthermore, we assume that the supervisor is protected by limited liability and therefore his wage cannot be lower than a certain $w \geq 0$ (i.e., $\min\{w_S, w_F\} \geq w$), which is the supervisor’s reservation utility or the minimum
wage. Limited liability can arise for instance from the supervisor’s having freedom to quit.\footnote{For instance, Conning (1999) assumes limited liability with $w = 0$.} Although most of our characterization of the optimal supervisory contract given a lending contract is done for $w \geq 0$, we consider $w = 0$ for the characterization of the optimal lending contract for tractability reasons.

We focus on the moral hazard of the supervisor, who can misrepresent the state to the lender. For instance, when $\Delta \equiv r_S - r_F > 0$, by reporting $Y = Y_F$ when $Y = Y_S$ the supervisor can embezzle $\Delta$. Since embezzlement still requires visiting the borrower, we assume that the cost of visiting the borrower is zero for simplicity. The lender can audit the actual payment made by the borrower at the cost of $k(>0)$. When cheating is discovered, the lender can recover $\Delta$ and refuse to pay any wage to the supervisor. However, since the supervisor is protected by limited liability, we assume that the lender cannot impose any further fine. If she can impose a large fine, it is easy to show that she can eliminate the moral hazard at almost zero cost: for instance, by conducting an audit with a small probability $\varepsilon (>0)$, she can solve the moral hazard at the cost of $\varepsilon k$. Hence, we focus on the case of zero-fine. A supervisory contract is represented by $\{q_i, w_i\}$ with $i = S, F$, where $q_i$ represents the probability of audit when the supervisor reports $i$. For simplicity, the supervisor is assumed to be risk-neutral.

A grand-contract is composed of a lending contract and a supervisory contract $\{r_i, w_i, q_i\}$. We assume that the lender makes a take-it-or-leave-it offer to both the supervisor and the borrower. When the supervisor is honest, $\{r_S = (\rho + w)/p, r_F = q_F = q_S = 0, w_S = w_F = w\}$ is the optimal grand-contract. In particular, it provides the borrower with full insurance since no repayment is required when the project fails (i.e. $r_F = 0$).

For expositional facility, we define two kinds of lending contracts, rigid and discretionary, and two kinds of supervisory contracts, with a carrot and with a stick.

**Definition:** A lending contract is said to be rigid if $\Delta = 0$. A lending contract is said to be discretionary if $\Delta \neq 0$. Given a total cost $C$ of financing the project, a contract with $r_S = C/p$ and $r_F = 0$ is called a contract with maximum discretion and full insurance.

**Definition:** A supervisory contract with $(w_S - w_F)(r_S - r_F) > 0$ and $q^F = q^S = 0$ is called a supervisory contract with a carrot. A supervisory contract with $w_S = w_F$ and $q^F > 0$ and/or $q^S > 0$ is called a supervisory contract with a stick.

In a rigid lending contract, the supervisor has no discretion since the borrower’s payment does not depend on the state of nature; hence, the lender expects to receive a fixed
amount of money from the supervisor. By contrast, the supervisor has some discretion when $\Delta \neq 0$; the amount of discretion is given by $|\Delta|$. We show later on that we need to consider only lending contracts with $\Delta \geq 0$. Given a total cost $C$ of financing the project, a contract with $r_S = C/p$ and $r_F = 0$ gives the maximum discretion to the supervisor while providing full insurance for the borrower. When $\Delta > 0$, there are two different ways to induce the supervisor not to embezzle the repayment. If the lender does not use audit ($q_S = q_S = 0$), she must award an incentive pay: $w_S$ must be (sufficiently) larger than $w_F$. If no incentive pay is used, audit must be used frequently enough. In the former case the lender uses a carrot (i.e. the incentive pay), while, in the latter case, she uses a stick in that if an embezzlement is detected, the supervisor loses his wage and the stolen repayment is recovered by the lender.

In section 3 in which we consider the case of one borrower, we first study the optimal supervisory contract given $\{r_S, r_F\}$ and then derive the optimal lending contract by comparing the optimal rigid contract with the optimal discretionary contract. Since we are mainly interested in this comparative static, we make the following assumption:

**A1**: $Y_S$ is large enough such that the net present value (NPV) of the project (i.e. the borrower’s expected utility) is positive at the equilibrium. More precisely, when $w = 0$, A1 is equivalent to $pY_S > \rho + (1 - p) \min \{\psi(\rho) - \rho, k\}$.

For instance, when the supervisor is honest, the project’s NPV is positive if and only if $pY_S - \rho - w > 0$. A1 means that $pY_S$ is much larger than $\rho + w$ since the NPV is positive even in the presence of the supervisor’s moral hazard. This assumption allows us to focus on the comparison between the optimal rigid contract and the optimal discretionary contract without being worried about whether or not the NPV is positive; as a consequence, we neglect the borrower’s participation constraint.

### 3 Single borrower case

In this section, we derive the optimal grand-contract in the case of a single borrower. From the revelation principle, there is no loss of generality in restricting our attention to direct revelation mechanisms\(^{21}\) that induce the supervisor to report the true state to the lender. Therefore, a grand-contract should satisfy the following incentive constraints to

\(^{21}\)According to Laffont and Martimort (2002, section 3.6), the revelation principle holds when the principal can commit to audit mechanisms as in our case.
induce the supervisor to report truthfully the state of the world:

\[
\begin{align*}
(IC_{SF}) & \quad w_S \geq (1 - q_F)(r_S - r_F + w_F); \\
(IC_{FS}) & \quad w_F \geq (1 - q_S)(r_F - r_S + w_S).
\end{align*}
\]

A grand-contract must also satisfy the lender’s break-even constraint given by:

\[
(BE) \quad pr_S + (1 - p)r_F \geq \rho + pw_S + (1 - p)w_F + [pq_S + (1 - p)q_F] k, \tag{2}
\]

where the right hand side represents the total financing cost that the lender needs to recover. The lender’s optimization problem, denoted by \((L^S)^{22}\), is defined as follows:

\[
\max_{\{w_i,q_i,r_i\}_{i=SF}} p(Y_S - r_S) - (1 - p)\psi(r_F) \quad \text{subject to} \quad \begin{align*}
(BE), & \quad (IC_{SF}), & \quad (IC_{FS}), & \quad w_S \geq w \quad \text{and} \quad w_F \geq w.
\end{align*} \tag{3}
\]

The following useful lemma (i) proves an important property of the borrower’s utility function which is repeatedly used in our paper and (ii) shows that we can focus on contracts with \(r_S \geq r_F\).

**Lemma 1** When there is a single borrower,

(i) Suppose that the lending contracts \(\{r_S,r_F\}\) and \(\{r'_S,r'_F\}\) have the same expected payment and \(r_F > r'_F \geq 0\). Then the borrower prefers \(\{r'_S,r'_F\}\) to \(\{r_S,r_F\}\).

(ii) The optimal lending contract is such that \(\Delta \geq 0\).

**Proof.** (i) If \(pr'_S + (1 - p)r'_F = pr_S + (1 - p)r_F \equiv r^e\), then \(p(Y_S - r^e) - (1 - p)\psi(r^e_F) = pY_S - r^e + (1 - p)(r'_F - \psi(r'_F))\). This function of \(r'_F\) is equal to (1) at \(r'_F = r_F\) and is decreasing with respect to \(r'_F\).

(ii) Suppose that a grand contract \(G = \{r_S,r_F,w_S,w_F,q_S,q_F\}\) satisfies (3) and is such that \(r_S < r_F\). We now find \(G' = \{r'_S,r'_F,w'_S,w'_F,q'_S,q'_F\}\) which satisfies (3) and increases the borrower’s payoff with respect to \(G\). Precisely, let \(r'_S = r'_F = pr_S + (1 - p)r_F\) and \(w'_S = w'_F = w, q'_S = q'_F = 0\). Then, (a) \(G'\) satisfies \((IC_{SF})\) and \((IC_{FS})\); (b) the financing cost with \(G'\) is equal to \(\rho + w\), the minimum feasible value; therefore, it cannot be larger than the cost under \(G\); (c) the borrower’s expected payment (the left hand side of \((BE))\)

\[\]
with $G'$ is the same as with $G$. Hence, $G'$ satisfies (3) and the borrower’s expected utility is higher with $G'$ by lemma 1(i) since $r_F < r'_F$. ■

The intuition for lemma 1(i) is that although the expected payment is the same, the borrower pays less in state $F$ with $\{r'_S, r'_F\}$ than with $\{r_S, r_F\}$; thus, he bears a smaller cost of reducing consumption with $\{r'_S, r'_F\}$. Lemma 1(ii) relies on lemma 1(i) to show that we can restrict our attention to lending contracts with $\Delta \geq 0$. Precisely, it proves that if $r_S < r_F$, then increasing the payment in state $S$ and decreasing the payment in state $F$ without modifying the expected payment has the effect of relaxing the incentive constraints (because the room for embezzlement is reduced) and increasing the borrower’s expected utility.

When $\Delta \geq 0$ holds, the supervisor has no incentive to misrepresent the state from $F$ to $S$ in the absence of any incentive pay or audit. Accordingly, we consider the relaxed problem in which $(IC_{FS})$ is neglected; we verify ex post that $(IC_{FS})$ is satisfied in the solution of the relaxed problem. We observe that in the relaxed problem it is optimal to set $q_S = 0$ in order to minimize the financing cost. For notational simplicity, in the rest of this section we let $q \equiv q_F$.

We now solve $(LS)$ in two steps. First, given a lending contract $\{r_S, r_F\}$, we find the optimal supervisory contract $\{w_S, w_F, q\}$ that minimizes the cost of financing the project, $\rho + pw_S + (1-p)w_F + (1-p)qk$, subject to $(IC_{SF})$, $w_S \geq w$ and $w_F \geq w$; it is important to notice that $\{r_S, r_F\}$ affects $(IC_{SF})$ only through $\Delta \geq 0$. Second, we maximize the borrower’s expected utility with respect to $(r_S, r_F)$ subject to $(BE)$.

### 3.1 The optimal supervisory contract given $\Delta$

In this subsection we find the optimal supervisory contract given a lending contract (i.e. given $\Delta \geq 0$) by solving the following problem, denoted by $(SS)^24$: \[
\min_{w_S, w_F, q} \rho + pw_S + (1-p)w_F + (1-p)qk
\]
subject to \[(IC_{SF}), \quad w_S \geq w \quad \text{and} \quad w_F \geq w.\]

A first step is given by a simple lemma:

\[23\] This methodology is similar to the one followed in a standard model of moral hazard in which (i) given an effort level to implement, one finds the incentive scheme which minimizes the cost of implementing that effort and (ii) one finds the optimal effort (Laffont and Martimort, 2002).

\[24\] The large $S$ means a "supervisory contract".
Lemma 2 When there is a single borrower, the optimal supervisory contract for a given \( \Delta \geq 0 \) is such that

(i) \( w_F = \underline{w} \);

(ii) if \( \Delta = 0 \), then \( w_S = \underline{w} \) and \( q = 0 \);

(iii) if \( \Delta > 0 \), then \( (IC_{SF}) \) binds.

Proof. (i) Reducing \( w_F \) improves the objective function and relaxes \( (IC_{SF}) \). Therefore \( w_F \) is equal to \( \underline{w} \), the smallest feasible value.

(ii) Since if the supervisor has no discretion \( (\Delta = 0) \), there is no need to make incentive pay or audit.

(iii) Suppose that \( \Delta > 0 \) and \( (IC_{SF}) \) is slack. Then it is optimal to set \( w_S = \underline{w} \) and \( q = 0 \), but a contradiction arises because this contract violates \( (IC_{SF}) \) since \( \Delta > 0 \).

In what follows we consider the case of \( \Delta > 0 \). Since \( (IC_{SF}) \) binds, we find \( w_S = (1 - q)(\Delta + \underline{w}) \); hence \( w_S \geq \underline{w} \) requires \( q \leq \overline{q}(\Delta) \equiv \Delta/\left(\Delta + \underline{w}\right) \leq 1 \). The objective function after replacing \( w_S \) with \( (1 - q)(\Delta + \underline{w}) \) is

\[
\rho + p(1-q)(\Delta + \underline{w}) + (1-p)\underline{w} + (1-p)qk
\]

We need to minimize this function with respect to \( q \), for \( q \in [0, \overline{q}(\Delta)] \). Since the function is linear in \( q \), the optimum is easily found as follows. Let \( \overline{\Delta} \equiv (1-p)k/p - \underline{w} \). Then, \( q = 0 \) is optimal if \( \Delta \leq \overline{\Delta} \) while \( q = \overline{q}(\Delta) \) is optimal if \( \Delta > \overline{\Delta} \). The following proposition summarizes these results by characterizing the optimal supervisory contract as a function of \( \Delta \geq 0 \) and gives the associated financing cost.\(^{25}\)

Proposition 1 When there is a single borrower, given a lending contract with \( \Delta \geq 0 \), the optimal supervisory contract is characterized as follows. There exists \( \overline{\Delta} \equiv (1-p)k/p - \underline{w} \) such that

(i) \( (Carrot \text{ regime}) \) For \( 0 \leq \Delta \leq \max \{0, \overline{\Delta}\} \), \( q = 0 \) and \( w_S = \underline{w} + \Delta \), \( w_F = \underline{w} \); the financing cost is \( \rho + p\Delta + \underline{w} \).

(ii) \( (Stick \text{ regime}) \) For \( \Delta \geq \max \{0, \overline{\Delta}\} \), \( q = \overline{q}(\Delta) \equiv \Delta/(\Delta + \underline{w}) \) and \( w_S = w_F = \underline{w} \); the financing cost is \( \rho + \underline{w} + (1-p)\overline{q}(\Delta)k \).

We now provide an intuitive explanation of the optimal supervisory contract by focusing on the case in which \( \overline{\Delta} > 0 \). Note first that because the total financing cost in

\(^{25}\) Notice that, in both contracts mentioned by proposition 1, \( (IC_{FS}) \) is satisfied even though it has been neglected in the optimization process. Hence, it is correct to state that proposition 1 characterizes the optimal supervisory contract.
the objective function and \((IC_{SF})\) are linear with respect to \(q\), the optimal \(q\) is either zero (i.e. no audit) or \(\bar{q}(\Delta)\), which is the minimal \(q\) that satisfies \((IC_{SF})\) at the minimum wage \(w_S = \bar{w}\), given that \(w_F = \bar{w}\). With \(q = 0\), the lender must give a carrot \((w_S - w_F)\) equal to the amount of discretion \((\Delta)\) to eliminate the supervisor’s incentive to embezzle; when \(q = \bar{q}(\Delta)\), instead, the contract induces truthtelling by using only stick since \(w_S = w_F = \bar{w}\). In order to see which method between the carrot and the stick performs better, we compare the extra cost generated by the supervisor’s moral hazard with respect to the financing cost in the absence of the moral hazard. Without the moral hazard, the financing cost is simply \(\rho + \bar{w}\). The extra cost under the carrot contract is \(p\Delta\), while the extra cost under the stick contract is \((1 - p)\bar{q}(\Delta)k = (1 - p)\Delta k/(\Delta + \bar{w})\). The two are the same when \(\Delta = \bar{\Delta}\), but the former increases faster than the latter as \(\Delta \geq \bar{\Delta}\) increases. Hence, using only an incentive pay is optimal for \(\Delta \leq \bar{\Delta}\) while using only audit is optimal for \(\Delta \geq \bar{\Delta}\). In other terms, when \(\bar{\Delta} > 0\), it is optimal to use the carrot contract for small discretion but the stick contract is better for large discretion. For expositional facility, we will say that a supervisory contract belongs to the stick regime when \(\Delta \geq \max\{0, \bar{\Delta}\}\) and to the carrot regime when \(0 \leq \Delta \leq \max\{0, \bar{\Delta}\}\). Note however that in the special case of \(\Delta = 0\), lemma 2(ii) says that the lender needs neither carrot nor stick.

3.2 The optimal lending contract

We now find the optimal lending contract by maximizing the borrower’s payoff subject to the lender’s break-even constraint \((BE)\). Given a lending contract \(\{r_S, r_F\}\), proposition 1 identifies the minimum financing cost that determines the right hand side of \((BE)\). Furthermore, since any increase in \(r_S\) or \(r_F\) reduces the borrower’s payoff, \((BE)\) must bind in the optimum. We first find the optimal lending contract conditional on the carrot or the stick regime and then derive the optimal contract.

Consider first the carrot regime, which occurs if \(0 \leq \Delta \leq \max\{0, \bar{\Delta}\}\). Then, writing \((BE)\) with equality yields

\[
r_F = \rho + \bar{w}.
\]

Since \(r_F\) is equal to \(\rho + \bar{w}\) regardless of the value of \(\Delta\), the objective of \((G^S)\) is given by

\[
p(Y_S - \rho - \bar{w} - \Delta) - (1 - p)\psi(\rho + \bar{w}).
\] (4)

Then it is obvious that in the carrot regime a rigid contract (i.e. \(\Delta = 0\)) is optimal. The intuition for this result is straightforward if we notice that \(w_S = w_F + \Delta\) in the carrot regime; hence, \(\Delta > 0\) increases the financing cost (with respect to \(\Delta = 0\)) without affecting \(r_F\). Thus, choosing \(\Delta = 0\) is optimal and this implies \(r_S = r_F = \rho + \bar{w}\).
Consider now the stick regime, which occurs if \( \Delta \geq \max \{0, \overline{\Delta}\} \). Now the binding (BE) is given by

\[
r_F = \rho + w + (1 - p) \frac{\Delta}{\Delta + w} k - p\Delta.
\]  

(5)

We define \( r_F^{\text{stick}}(\Delta) \) as the right hand side of (5), but we notice that not all \( \Delta \) satisfying \( \Delta \geq \max \{0, \overline{\Delta}\} \) is feasible since \( r_F \) cannot be negative; therefore, the condition \( r_F^{\text{stick}}(\Delta) \geq 0 \) needs to be satisfied. It turns out that \( r_F^{\text{stick}} \) is decreasing in \( \Delta \) because

\[
\frac{dr_F^{\text{stick}}}{d\Delta} = (1 - p) \frac{\Delta}{(\Delta + w)^2} k - p < 0 \quad \text{for any } \Delta \geq \max \{0, \overline{\Delta}\}.
\]

Hence, there exists a unique \( \Delta > \max \{0, \overline{\Delta}\} \) such that \( r_F^{\text{stick}}(\Delta) = 0 \); we use \( \Delta^{\text{max}} \) to denote this value of \( \Delta \). Therefore, the lender chooses \( \Delta \) in \([\max \{0, \overline{\Delta}\}, \Delta^{\text{max}}]\) to maximize the following objective of \((G^S)\), given by:

\[
p \left[ Y_S - r_F^{\text{stick}}(\Delta) - \Delta \right] - (1 - p) \psi(r_F^{\text{stick}}(\Delta)).
\]  

(6)

In determining the optimal \( \Delta \), we notice from (5) that an increase in \( \Delta \) has two opposing effects on \( r_F^{\text{stick}} \). On the one hand, \( r_F^{\text{stick}} \) is reduced because of the term \(-p\Delta\) which shifts the borrower’s payment from state \( F \) to state \( S \) without changing his expected payment; this increases the payoff of the borrower by lemma 1(i). On the other hand, increasing \( \Delta \) raises the auditing cost because, for a given \( q < 1 \), it gives the supervisor a higher incentive to embezzle and a larger \( q \) is needed to satisfy \((IC_{SF})\) with \( w_S = w \). This increases \( r_F^{\text{stick}} \) through the term \((1 - p) \frac{\Delta}{\Delta + w} k \) and reduces the borrower’s payoff. This trade-off makes it difficult to find the exact optimal value of \( \Delta \) in the stick regime, as we see from the derivative of (6).\(^{26}\)

In order to eliminate the ambiguity, we introduce the simplifying assumption of \( w = 0 \). In this case we have \( \overline{\Delta} > 0 \) and \( \overline{q}(\Delta) = 1 \) for any \( \Delta \geq \overline{\Delta} \). Therefore, an increase of \( \Delta \) in \([\overline{\Delta}, \Delta^{\text{max}}]\) does not increase the auditing cost but only reduces \( r_F^{\text{stick}} \) and increases \( r_S \) without modifying the borrower’s expected payment. Hence, the optimal \( \Delta \) under the stick regime is \( \Delta^{\text{max}} \), which is equal to \( [\rho + (1 - p)k] / p \) when \( w = 0 \). This result implies that the optimal lending contract is either a rigid contract or a contract with a maximum discretion and stick as the following proposition states:

**Proposition 2** When there is a single borrower, under A1 and \( w = 0 \),

\[^{26}\text{The derivative is } -p \left[ \frac{dr_F^{\text{stick}}}{d\Delta} + 1 \right] - (1 - p) \psi'(r_F^{\text{stick}}(\Delta)) \frac{dr_F^{\text{stick}}}{d\Delta}.
\]

\[^{27}\text{Formally, when } w = 0 \text{ the derivative of (6) is } p(1 - p)(\psi'(r_F^{\text{stick}}(\Delta)) - 1) > 0 \text{ for any } \Delta \in [\overline{\Delta}, \Delta^{\text{max}}].
\]
(i) The optimal grand contract is one of the two following:

a. a rigid lending contract \( \{ r_S = r_F = \rho \} \) with \( \{ w_S = w_F = 0, q = 0 \} \)

b. a lending contract with a maximum discretion \( \{ r_S = [\rho + (1 - p)k] / p, r_F = 0 \} \) with \( \{ w_S = w_F = 0, q = 1 \} \).

(ii) The optimal lending contract is rigid if and only if \( k \geq \psi(\rho) - \rho \) holds.

Proposition 2 says that the optimal lending contract specifies either no discretion or the maximum discretion. In the former case, the lender needs neither carrot nor stick because there is no money to embezzle. Thus, financing costs are low but \( r_F = \rho > 0 \) implies that the borrower is not fully insured since his cost of paying \( \rho \) when the project fails is \( \psi(\rho) (> \rho) \). The contract with maximum discretion has \( r_S > r_F = 0 \) and provides full insurance. However, the possibility of embezzlement requires the lender to conduct audit frequently enough such that \((IC_{SF})\) is satisfied at the minimum wage. When \( w = 0 \), in particular, the audit should occur whenever the supervisor reports that the project failed and this increases the borrower’s expected payment to \( \rho + (1 - p)k \). From our discussion, it is obvious that the optimal lending contract is rigid if the audit cost is large with respect to the borrower’s cost of respecting the rigid repayment schedule when the project fails. The borrower’s payoff is \( pY_S - \rho - (1 - p) \min \{ \psi(\rho) - \rho, k \} \), which is positive by A1.

Remark 1 Even if \( r_F \) can be negative, it does not affect the optimal contracts characterized in proposition 2. First, obviously, it does not affect the optimal rigid contract. Second, in the case of the optimal discretion contract, \( r_F < 0 \) does not improve insurance with respect to \( r_F = 0 \) and does not reduce the audit cost which should occur whenever the project fails.

4 Group lending

Suppose now that there are two borrowers living in the same village. Each borrower finances a project with one unit of money borrowed from the lender and a supervisor monitors both of them. Each project has the same probability of success \( p \in (0,1) \) and the success of one project is independent from that of the other. We assume that both borrowers and the supervisor observe whether each project succeeds or fails. We distinguish two cases depending on whether there is joint liability or individual liability between the two borrowers.

Under individual liability, the lender signs an individual lending contract with each borrower which takes the same form \( \{ r_S, r_F \} \) as before. However, the supervisory con-
tract is different since the supervisor now monitors two borrowers. Let $w_n$ represent the supervisor's wage and $q_n$ the probability of conducting an audit when he reports that $n$ number of projects succeeded, with $n \in \{0, 1, 2\}$. Therefore, a supervisory contract is given by $\{w_n, q_n\}$ for $n = 0, 1, 2$; the minimum wage constraint requires $w_n \geq \underline{w}$ for each $n$. Since both borrowers live in the same village, the cost of audit does not depend on whether the lender audits the payment of one borrower or those of both borrowers$^{28}$ and is equal to $k > 0$.

Introducing joint liability affects the form of the lending contract but not the form of the supervisory contract. The lender signs a lending contract with the group of the two borrowers which takes the form $\{r_{SS}, r_{SF}, r_{FS}, r_{FF}\}$ under joint liability$^{29}$ where, for instance, $r_{SF}$ represents the payment that a borrower whose project succeeded has to make when the other’s project failed. Without loss of generality,$^{30}$ we assume that $r_{ij}$ cannot be negative for $i, j = S, F$.

A lending contract with joint liability is called a group lending contract. Note first that the set of lending contracts under individual liability can be seen as the (strict) subset of the group lending contracts that satisfy $r_{SS} = r_{SF}$ and $r_{FS} = r_{FF}$. Observe also that if the supervisor is honest, the first-best outcome can be achieved under individual liability by choosing $p r_S = \rho + \underline{w}$ and $r_F = 0$. Therefore, we have two observations:

**Observation 1**: The set of lending contracts under individual liability is a strict subset of the set of group lending contracts.

**Observation 2**: If the supervisor is honest, the first-best outcome can be achieved without joint liability.

We define a rigid or a discretionary group lending contract:

**Definition**: A group lending contract is rigid if $2r_{SS} = r_{SF} + r_{FS} = 2r_{FF}$; otherwise, it is discretionary.

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$^{28}$The main cost of audit is the cost of visiting the village and the marginal cost of visiting one more borrower in the same village is negligible.

$^{29}$According to Yunus (1998), joint liability implies “In practice, when a member has difficulty repaying a loan, the other members of the group work out a solution that assures repayment to the bank (p.109).” In the literature, joint liability has often been formalized as in our paper.

$^{30}$As in the case of the single borrower; we can obtain our results even though $r_{ij}$ can be negative but considering this possibility makes the proofs much longer. Even though the constraint $r_{FF} \geq 0$ binds in our analysis, one cannot strictly improve the borrowers’ payoff by considering $r_{FF} < 0$ for the reasons similar to those explained in remark 1 in section 3.
In this section, we make the following assumption:

\[ A1': \text{When } w = 0, \text{ we assume (i) } pY_S > \rho + (1 - p)^2 \min \{\psi(\rho) - \rho, \frac{1}{2}k\} \text{ and (ii) } pY_S \geq \frac{2p + (1 - p)^2k}{2 - p}. \]

A1'(i) is similar to A1 and implies that the NPV of the project is positive at equilibrium. A1'(ii) simplifies our analysis of the optimal lending contract under joint liability in that \( r_{FS} = 0 \) becomes optimal: A1'(ii) is introduced since the characterization of the optimal supervisory and lending contracts under joint liability is somewhat technically demanding. Note that both A1'(i) and A1'(ii) are satisfied if \( pY_S \) is large enough.

Consider the case of joint liability. As we have argued in section 3, by the revelation principle there is no loss of generality in restricting our attention to direct revelation mechanisms that induce the supervisor to report the true state. Therefore, the following incentive constraints should be satisfied:

\[(IC_{n\hat{n}}) \quad w_n \geq (1 - q_n)[R(n) - R(\hat{n}) + w_{\hat{n}}] \quad \text{for } (n, \hat{n}) \in \{0, 1, 2\}^2, \quad (7)\]

where \( R(2) \equiv 2r_{SS}, \ R(1) \equiv r_{SF} + r_{FS}, \ R(0) \equiv 2r_{FF}. \) The lender’s break-even constraint is now

\[
(BE) \quad 2p^2r_{SS} + 2p(1 - p)(r_{SF} + r_{FS}) + 2(1 - p)^2r_{FF} \\
\geq \quad 2\rho + p^2(w_2 + kq_2) + 2p(1 - p)(w_1 + kq_1) + (1 - p)^2(w_0 + kq_0)
\]

and the lender’s program under joint liability, denoted by \( (L^J) \), is defined as follows:

\[
\max_{r_{SS},r_{SF},r_{FS},r_{FF},w_n,q_n \text{ for } n=0,1,2} p^2(2Y_S - 2r_{SS}) + 2p(1 - p)(Y_S - r_{SF} - \psi(r_{FS})) - (1 - p)^22\psi(r_{FF})
\]

subject to

\[(BE), \ (7), \ r_{ij} \text{ for } i, j = S, F, \ w_n \geq w \text{ for } n = 0, 1, 2.\]

Note that in the objective function in \( (L^J) \) we assume \( r_{SF} \leq Y_S \) and \( r_{SS} \leq Y_S \), which in the proof of Proposition 4 we verify to be satisfied under A1'.

Let \( \Delta_1 \equiv r_{SF} + r_{FS} - 2r_{FF} \) and \( \Delta_2 \equiv 2r_{SS} - r_{SF} - r_{FS}. \) For the case of one borrower, lemma 1(ii) shows that the optimal lending contract is such that \( \Delta \geq 0. \) For the case of two borrowers, an analogous result holds, as stated by lemma 3 below: the optimal lending contract satisfies \( \Delta_1 \geq 0 \) and \( \Delta_2 \geq 0. \)
Lemma 3 When there are two borrowers, under joint liability, \( w = 0 \) and \( A_1 \), the optimal lending contract is such that \( \Delta_1 \geq 0 \) and \( \Delta_2 \geq 0 \).

When \( \Delta_1 \geq 0 \) and \( \Delta_2 \geq 0 \), it is easy to see that the supervisor has no incentive to report a state \( \hat{n} \) larger than the true state \( n \) in the absence of any incentive pay or audit. Therefore, we consider a relaxed problem in which the upward incentive constraints \((IC_{02}), (IC_{01}), (IC_{12})\) are neglected.\(^{31}\) In this relaxed problem, it is optimal to set \( q_2 = 0 \) because \( q_2 \) appears only in the right hand side of (BE).

The program with individual liability, denoted by \( (L^i) \), is defined as \( (L^j) \) except that \( r_S \) replaces \( r_{SS} \) and \( r_{SF} \) and \( r_F \) replaces \( r_{FF} \) and \( r_{FS} \). Notice that under individual liability the proof of lemma 1(ii) applies and therefore we consider only lending contracts with \( r_S - r_F \geq 0 \).

As in the previous section, we perform our analysis in two steps: we first find the optimal supervisory contract given a lending contract and then find the optimal lending contract.

4.1 The optimal supervisory contract

In this subsection, we find the optimal supervisory contract, given a lending contract \( \{r_{SS}, r_{SF}, r_{FS}, r_{FF}\} \), by minimizing the right hand side of (BE) with respect to \( (w_2, w_1, w_0, q_1, q_0) \), subject to \((IC_{21}), (IC_{20}) \) and \((IC_{10}) \). We use \( (S^j) \) to denote this program. Since a lending contract affects the incentive constraints only through \( \Delta_1 \) and \( \Delta_2 \), \( (S^j) \) can be stated as follows:

\[
\min_{w_2, w_1, w_0, q_1, q_0} 2\rho + p^2 w_2 + 2p(1-p)(w_1 + kq_1) + (1-p)^2(w_0 + kq_0) \tag{8}
\]

subject to

\[
\begin{align*}
(IC_{21}) & \quad w_2 \geq (1-q_1)(\Delta_2 + w_1); \\
(IC_{20}) & \quad w_2 \geq (1-q_0)(\Delta_2 + \Delta_1 + w_0); \\
(IC_{10}) & \quad w_1 \geq (1-q_0)(\Delta_1 + w_0); \\
(LL) & \quad w_n \geq w \quad \text{for} \quad n = 0, 1, 2
\end{align*}
\tag{9}
\]

We use \( C^j(\Delta_1, \Delta_2) \) to denote the value of the objective function in (8) at the optimal supervisory contract.

Solving \( (S^j) \) is not straightforward because the incentive constraints are not linear in the instrumental variables (although they are linear in each single variable). This leads

\(^{31}\) As in section 3, we verify ex post that these constraints are satisfied in the solution to the relaxed problem.
to minimizing a function which is neither concave nor convex, over a non-convex feasible set. We first prove some useful properties of \((S^f)\):

**Lemma 4** When there are two borrowers, under joint liability, the optimal supervisory contract given \((\Delta_1, \Delta_2) \geq (0, 0)\) is such that:

(i) \(w_0 = w\);

(ii) if \((IC_{20})\) binds, then also \((IC_{21})\) binds;

(iii) at least two incentive constraints in (9) bind.

Lemma 4(ii)-(iii) implies that there exist three regimes for the binding incentive constraints. Precisely, it is possible that the binding constraints are \((IC_{21})\) and \((IC_{20})\), or \((IC_{10})\) and \((IC_{21})\), or all the three constraints in (9). The next lemma shows that in each regime, the optimal supervisory contract belongs to a set with at most three elements, which in turn implies that five different supervisory contracts can be optimal for different parameter values. For expository convenience, we denote them as \(\alpha, \beta, \gamma, \delta, \eta\) and define them below; \(C_i\) represents the value of the objective function in (8) under contract \(i = \alpha, \beta, \gamma, \delta, \eta\).

- \(\alpha : (q_0, q_1) = (0, 0), \ (w_0, w_1, w_2) = (w, w + \Delta_1, w + \Delta_1 + \Delta_2), \ C_\alpha = 2p + w + p(2 - p)\Delta_1 + p^2\Delta_2;\)

- \(\beta : (q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + w}, 0), \ (w_0, w_1, w_2) = (w, w, w + \Delta_2), \ C_\beta = 2p + w + (1 - p)^2k\frac{\Delta_1}{\Delta_1 + w} + p^2\Delta_2;\)

- \(\gamma : (q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + w}, \frac{\Delta_2}{\Delta_2 + w}(\Delta_3 + w)), \ (w_0, w_1, w_2) = (w, w, w + w\frac{\Delta_2}{\Delta_3 + w}), \ C_\gamma = 2p + w + p^2w\frac{\Delta_2}{\Delta_1 + w} + 2p(1 - p)k\frac{\Delta_2}{(\Delta_2 + w)(\Delta_1 + w)} + (1 - p)^2k\frac{\Delta_2}{\Delta_1 + w};\)

- \(\delta : q_0 = q_0^* = 1 + \frac{\Delta_2}{\Delta_1 + w} - \frac{1}{\Delta_1 + w}\sqrt{\frac{2p(1 - p)k\Delta_2(\Delta_2 + \Delta_1 + w)}{p(p\Delta_2 + (2 - p)(\Delta_1 + w)) - (1 - p)^2k^2}}, \ q_1 = f(q_0^*) \) with \(f(q_0) \equiv q_0\frac{\Delta_2}{\Delta_2 + (1 - q_0)(\Delta_1 + w)}, \ (w, w, w_2) = (w, w + (1 - q_0^*)\Delta_1 + w), \ w + (1 - q_0^*)\Delta_2 + w)\), \(C_\delta = 2p + (1 - q_0^*)[p^2\Delta_2 + (2 - p)p(\Delta_1 + w)] + 2p(1 - p)kf(q_0^*) + (1 - p)^2kq_2^* + w;\)

- \(\eta : (q_0, q_1) = (\frac{\Delta_2 + \Delta_1}{\Delta_2 + \Delta_1 + w}, \frac{\Delta_2}{\Delta_2 + w}), \ (w, w, w_2) = (w, w, w), \ C_\eta = 2p + w + 2p(1 - p)k\frac{\Delta_2}{\Delta_2 + \Delta_1 + w} + (1 - p)^2k\frac{\Delta_2}{\Delta_2 + \Delta_1 + w}.\)

We have:

\(^{32}\text{This contract is defined if and only if } 2p(1 - p)\frac{\Delta_2}{\Delta_2 + \Delta_1 + w} k < p(2p + 2p(\Delta_2 + (2 - p)(\Delta_1 + w)) - (1 - p)^2k < 2p(1 - p)\frac{\Delta_2}{\Delta_2 + \Delta_1 + w}; \text{ these conditions are equivalent to } q_0^* \in (0, \frac{\Delta_2}{\Delta_1 + w}).\)
Lemma 5  When there are two borrowers, under joint liability,
(i) Suppose that $\Delta_1 > 0$ and $\Delta_2 > 0$.
   a. If only $(IC_{21})$ and $(IC_{20})$ bind in the optimum, then $\eta$ is the optimal supervisory contract.
   b. If only $(IC_{10})$ and $(IC_{21})$ bind in the optimum, then $\beta$ is the optimal supervisory contract.
   c. If all the constraints in (9) bind in the optimum, then the optimal supervisory contract belongs to $\{\alpha, \delta, \gamma\}$.
(ii) If $\Delta_1 = 0$ and/or $\Delta_2 = 0$, then the optimal supervisory contract belongs to $\{\alpha, \beta, \eta\}$.

For each of the five contracts mentioned above, characterizing the set of $(\Delta_1, \Delta_2)$ in which a given contract is better than the others is a very complicated task when $w > 0$. For this reason, we consider $w = 0$ from now on; this allows us to compare the cost of different contracts in a simple way and, furthermore, reduces the set of possible optimal contracts to four because of the following observation:

Observation 3: When $w = 0$, contract $\eta$ is equivalent to contract $\gamma$.

A straightforward consequence of lemma 5 and observation 3 is the following proposition.

Proposition 3  When there are two borrowers, under joint liability and $w = 0$, the optimal supervisory contract is the lowest cost contract among $\alpha, \beta, \gamma, \delta$. Hence, $C_J(\Delta_1, \Delta_2) \equiv \min \{C_\alpha, C_\beta, C_\gamma, C_\delta\}$.

Hence, we need to deal with only contracts $\alpha, \beta, \gamma, \delta$. Contract $\alpha$ specifies no audit but intensively uses carrots since $w_1 - w_0 = \Delta_1$ and $w_2 - w_1 = \Delta_2$. Contract $\gamma$, by contrast, gives no incentive pay but intensively uses sticks since the lender conducts audit with probability one whenever the supervisor reports that at least one project failed. Contract $\beta$ uses either carrot or stick depending on the supervisor’s report in the following sense. When the supervisor reports $n = 0$, the lender audits with probability one ($q_0 = 1$) while the supervisor receives a carrot $w_2 - w_1 = \Delta_2$ when he reports $n = 2$. Finally, contract $\delta$ mixes carrots and sticks not over different states of nature as contract $\beta$, but within the same state of nature; since $\Delta_1 > w_1 - w_0 > 0$ and $\Delta_2 > w_2 - w_1 > 0$, there is a positive probability of audit both if the supervisor reports $n = 1$ and if he reports $n = 0$.  

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4.2 The optimal lending contract under joint liability

In this subsection we find the optimal lending contract under joint liability by maximizing the borrowers’ payoff subject to (BE) of which the right hand side is replaced with $C^J(\Delta_1, \Delta_2)$, the financing cost under the optimal supervisory contract given $(\Delta_1, \Delta_2)$. By using $r_{SF} = \Delta_1 + 2r_{FF} - r_{FS}$ and $2r_{SS} = \Delta_2 + r_{SF} + r_{FS} = \Delta_2 + \Delta_1 + 2r_{FF}$, we can now write the lender’s program, denoted by $(L')$, as follows:

$$\max_{\Delta_1 \geq 0, \Delta_2 \geq 0, r_{FS}, r_{FF}} \quad 2p Y_S - p(2-p)(2r_{FF} + \Delta_1) - p^2 \Delta_2 - 2p(1-p)(\psi(r_{FS}) - r_{FS}) - (1-p)^2 2\psi(r_{FF})$$

subject to

$$(BE) \quad 2r_{FF} + p(2-p)\Delta_1 + p^2 \Delta_2 = C^J(\Delta_1, \Delta_2);$$

and

$$r_{FS} \geq 0, r_{FF} \geq 0.$$

We have written $(BE)$ with equality since it binds in the optimum in the case of two borrowers as well.

From (10) and (11) we easily discover one important effect of joint liability:

**Lemma 6** When there are two borrowers, under A1’ and joint liability, it is optimal to choose $r_{FS} = 0$ and therefore $r_{SF} = 2r_{FF} + \Delta_1$.

**Proof.** Let $r_{FF}$ and $\Delta_1$ be given; hence, also $r_{SF} + r_{FS} = 2r_{FF} + \Delta_1$ is given. We prove that under A1’ it is optimal to set $r_{FS} = 0$ and therefore $r_{SF} = 2r_{FF} + \Delta_1$. Note that $r_{FS}$ does not affect (11). In writing the objective function in (10), we implicitly assumed that $r_{SF} = 2r_{FF} + \Delta_1 - r_{FS}$ is not larger than $Y_S$. As long as this condition is satisfied, maximizing (10) with respect to $r_{FS}$ tells us that any $r_{FS} > 0$ is dominated by $r_{FS} = 0$ since $\psi'(r) > 1$ for any $r > 0$. With $r_{FS} = 0$, the condition $r_{SF} \leq Y_S$ turns out to be satisfied in the optimal lending contract given that A1’(ii) holds (see Proposition 4 and its proof).33

**Lemma 6** says that when one borrower is successful and the other is not, it is optimal that the unsuccessful borrower pays nothing ($r_{FS} = 0$) and the lender’s revenue comes only

33 Furthermore, if we consider $r_{FS} < 0$, then the objective function in (10) becomes $2p Y_S - p(2-p)(2r_{FF} + \Delta_1) - p^2 \Delta_2 - (1-p)^2 2\psi(r_{FF})$ and does not depend on $r_{FS}$ (intuitively, the marginal utility of money for an unsuccessful borrower in state $FS$ is 1 when $r_{FS} < 0$ and therefore the precise value of $r_{FS}(< 0)$ is irrelevant if $r_{SF} + r_{FS}$ is given). However, since $r_{SF} = 2r_{FF} + \Delta_1 - r_{FS}$, a negative $r_{FS}$ increases $r_{SF}$ with respect to $r_{FS} = 0$ and makes it more difficult to satisfy $r_{SF} \leq Y_S$. 20
from the successful borrower \((r_{SF} = \Delta_1 + 2r_{FF})\). The reason is that letting the unsuccessful borrower pay a positive amount requires him to reduce his consumption, which is more costly for borrowers than simply using the money generated by the successful project. Therefore, joint liability provides borrowers with insurance in the states of the world where only one project is successful. Clearly, this approach is viable only if the successful borrower has enough money to pay \(\Delta_1 + 2r_{FF}\); we show later on that \(\Delta_1 + 2r_{FF} \leq Y_S\) holds in the optimal contract under A1’.

Given lemma 6, (10) boils down to

\[
2pY_S - p(2 - p)(2r_{FF} + \Delta_1) - p^2\Delta_2 - (1 - p)^22\psi(r_{FF}).
\]

We need to maximize (12) subject to (11) with respect to \((\Delta_1, \Delta_2) \in \mathbb{R}_+^2\). For this purpose, it is useful to define \(S_i\) as the subset of \(\mathbb{R}_+^2\) in which the supervisory contract \(i\) is optimal: \(S_i = \{(\Delta_1, \Delta_2) \in \mathbb{R}_+^2 : C^J(\Delta_1, \Delta_2) = C_i\}\). For each \(i = \alpha, \beta, \gamma, \delta\), we maximize (12) with respect to \((\Delta_1, \Delta_2) \in S_i\) subject to (11) in which the right hand side is replaced by \(C_i\). By letting \(M_i\) denote the maximum of (12) over \(S_i\), we can find the optimal lending contract by comparing the four values \(M_\alpha, M_\beta, M_\gamma, M_\delta\).

**Lemma 7** When there are two borrowers, under A1’, \(\underline{w} = 0\) and joint liability, in the optimal lending contract the supervisory contract \(\delta\) is not the unique optimal supervisory contract. Therefore, we can look for the optimal supervisory contract among \(\alpha, \beta, \gamma\) without loss of generality.

In the proof of lemma 7 we show that when we maximize (12) subject to (11) with respect to \((\Delta_1, \Delta_2) \in S_\delta\), the optimal \((\Delta_1, \Delta_2)\) lies on the boundary of \(S_\delta\) and therefore there is a contract \(i \in \{\alpha, \beta, \gamma\}\) which satisfies \(C_i = C_\delta\). Therefore, we can look for the optimal supervisory contract among \(\alpha, \beta, \gamma\) without loss of generality; this fact considerably simplifies the analysis because \((q_0, q_1) \in \{0, 1\}^2\) in \(\alpha, \beta, \gamma\) while \((q_0, q_1) \in (0, 1)^2\) in \(\delta\). As a consequence of lemma 7, in the right hand side of (11) we replace \(C^J(\Delta_1, \Delta_2)\) with \(c^J(\Delta_1, \Delta_2) \equiv \min\{C_\alpha, C_\beta, C_\gamma\}\) and obtain

\[
2r_{FF} + p(2 - p)\Delta_1 + p^2\Delta_2 = c^J(\Delta_1, \Delta_2).
\]

With some abuse of notation, we use \(S_i\) to denote the set of \((\Delta_1, \Delta_2) \in \mathbb{R}_+^2\) such that
Figure 1: The optimal supervisory contract for given $(\Delta_1, \Delta_2)$ in $\mathbb{R}_+^2$ when $\omega = 0$

c^I(\Delta_1, \Delta_2) = C_i$. Precisely, $c^I(\Delta_1, \Delta_2)$ is defined as follows:
\[
c^I(\Delta_1, \Delta_2) = \begin{cases} 
  C_\alpha & \text{if } (\Delta_1, \Delta_2) \in S_\alpha \equiv \\{ (\Delta_1, \Delta_2) \in \mathbb{R}_+^2 : \Delta_1 \leq \frac{(1-p)^2 k}{p(2-p)} \land \Delta_2 \leq \frac{p(2-p)\Delta_1}{p^2 k} \land \Delta_1 \leq \frac{(1-p)^2 k}{p^2 k} \} \\
  C_\beta & \text{if } (\Delta_1, \Delta_2) \in S_\beta \equiv \\{ (\Delta_1, \Delta_2) \in \mathbb{R}_+^2 : \Delta_1 \geq \frac{(1-p)^2 k}{p(2-p)} \land \Delta_2 \leq \frac{2(1-p)k}{p} \} \\
  C_\gamma & \text{if } (\Delta_1, \Delta_2) \in S_\gamma \equiv \\{ (\Delta_1, \Delta_2) \in \mathbb{R}_+^2 : \Delta_2 \geq \frac{2(1-p)k}{p} \land \Delta_1 \geq \frac{(1-p)^2 k}{p^2 k} \} 
\end{cases}
\] (14)

and the sets $S_\alpha$, $S_\beta$ and $S_\gamma$ are represented graphically in figure 1.

We observe, however, that not all $(\Delta_1, \Delta_2)$ in $\mathbb{R}_+^2$ are feasible because the condition $r_{FF} \geq 0$ must be satisfied. In particular, if $(\Delta_1, \Delta_2) \in S_\beta$ then (13) reduces to
\[
r_{FF} = \rho + \frac{(1-p)^2 k}{2} - \frac{p(2-p)}{2} \Delta_1
\] (15)

and $r_{FF} \geq 0$ is equivalent to $\Delta_1 \leq \Delta_1^{\max} \equiv \frac{2p/(1-p)k}{p(2-p)}$ (see the vertical dotted line in figure 1). If instead $(\Delta_1, \Delta_2) \in S_\gamma$, then (13) is
\[
r_{FF} = \rho + \frac{(1-p^2)k - p(2-p)\Delta_1 - p^2 \Delta_2}{2}
\] (16)
and \( r_{FF} \geq 0 \) is equivalent to (see the diagonal dotted line in figure 1)

\[
2\rho + (1 - p^2)k \geq p(2 - p)\Delta_1 + p^2\Delta_2.
\]

(17)

Finally, any point in \( S_\alpha \) is feasible because (13) implies \( r_{FF} = \rho \) when \( c'(\Delta_1, \Delta_2) = C_\alpha \). Therefore, we define the feasible set \( S' \) for \((\Delta_1, \Delta_2)\) as \( S' = S'_\alpha \cup S'_{\beta} \cup S'_\gamma \), where \( S'_\alpha = S_\alpha \), \( S'_{\beta} \) is the subset of \( S_\beta \) in which \( \Delta_1 \leq \Delta_1^{max} \) and \( S'_\gamma \) is the subset of \( S_\gamma \) in which (17) holds. Since \( S' \) is a compact subset of \( \mathbb{R}^2 \), a solution to \((L')\) exists. Next proposition describes the optimal \((\Delta_1, \Delta_2)\) and the optimal grand contract.

**Proposition 4** When there are two borrowers, under \( A1' \), \( w = 0 \) and joint liability,

(i) the optimal grand contract is one of the two following:

a. A rigid lending contract \( \{r_{SS} = r_{FF} = r_{FS} = \rho, r_{FS} = 0\} \) with \( \{q_0 = q_1 = q_2 = 0, w_0 = w_2 = w_2 = 0\} \) (the supervisory contract is \( \alpha \) with \( \Delta_1 = \Delta_2 = 0 \)).

b. A discretionary contract \( \{r_{FF} = r_{FS} = 0, 2r_{SS} = r_{SF} = \frac{2\rho + (1 - p^2)k}{p(2 - p)}\} \) with \( \{q_0 = 1, q_1 = q_2 = 0, w_0 = w_2 = w_2 = 0\} \) (the supervisory contract is \( \beta \) with \( \Delta_1 = \frac{2\rho + (1 - p^2)k}{p(2 - p)}, \Delta_2 = 0 \)).

(ii) The rigid contract is optimal if and only if \( k \geq 2(\psi(\rho) - \rho) \).

In the proof of proposition 4, we derive the optimal \((\Delta_1, \Delta_2)\) for each \( i = \alpha, \beta, \gamma \) given that \((\Delta_1, \Delta_2) \in S'_i \). The supervisory contract \( \alpha \) uses only incentive pay and \( w_1 - w_0 \) and \( w_2 - w_1 \) are equal to \( \Delta_1 \) and \( \Delta_2 \), respectively. As a consequence, when \((\Delta_1, \Delta_2) \in S'_\alpha \), the only effect of \( \Delta_1 > 0 \) and/or \( \Delta_2 > 0 \) is to increase the financing cost with respect to \( \Delta_1 = \Delta_2 = 0 \) and therefore \( \Delta_1 = \Delta_2 = 0 \) is optimal; this yields the rigid contract described in proposition 4(i)a. The supervisory contract \( \beta \) combines an incentive pay \((w_2 - w_1 = \Delta_2)\) and the audit \((q_0 = 1)\). For the same reason that makes \( \Delta_1 = \Delta_2 = 0 \) optimal when \((\Delta_1, \Delta_2) \in S'_\alpha \), \( \Delta_2 = 0 \) is optimal in \( S'_{\beta} \): an increase in \( \Delta_2 \) only induces an increase in financing cost. By contrast, regarding \( \Delta_1 \), it is optimal to choose the maximal \( \Delta_1 \) satisfying the feasibility constraint (i.e. \( \Delta_1 \leq \Delta_1^{max} \)) since (i) the auditing cost does not depend on \( \Delta_1 \); (ii) an increase in \( \Delta_1 \) reduces \( r_{FF} \) and increases both \( r_{SF} \) and \( r_{SS} \) (by the same amount) without changing the borrowers’ expected payment. The logic underlying lemma 1(i) implies that the value of \( \Delta_1 \) which makes \( r_{FF} = 0 \) is optimal. This gives the discretionary contract described in proposition 4(i)b. Finally, the best contract when \((\Delta_1, \Delta_2) \in S'_\gamma \) is such that \( r_{FF} = 0 \) holds as in the best contract in \( S'_{\beta} \) (the familiar logic behind lemma 1(i) applies) but it has a higher auditing cost because \((q_0, q_1) = (1, 1)\) with \( \gamma \) while \((q_0, q_1) = (1, 0)\) with \( \beta \). This occurs since \( \Delta_2 > 0 \), which holds for all contracts in \( S'_{\gamma} \), requires \( q_1 = 1 \) in order to avoid embezzlement. Thus, the best contract when \((\Delta_1, \Delta_2) \in S'_\gamma \) is dominated by the best contract when \((\Delta_1, \Delta_2) \in S'_{\beta} \). Finally, the
rigid contract is optimal if and only if \( k \geq 2(\psi(\rho) - \rho) \) (i.e. when the audit cost is large enough with respect to the gain from providing full insurance for the borrowers).

4.3 Individual liability

In the case of individual liability, a lending contract is such that \( r_{SS} = r_{SF} = r_S \) and \( r_{FS} = r_{FF} = r_F \), which implies \( \Delta_1 = \Delta_2 \equiv \Delta \). In order to have an idea of the effects of individual liability, notice first that in the case of joint liability, given \( r_{FF} \geq 0 \) and \( \Delta_1 \geq 0 \), it is possible to set \( r_{FS} = 0 \) and \( r_{SF} = 2r_{FF} + \Delta_1 \) (if \( 2r_{FF} + \Delta_1 \leq Y_S \)) and thereby to provide some insurance to an unsuccessful borrower when the other borrower has success (lemma 6). Under individual liability, by contrast, \( r_{FS} = 0 \) implies \( r_{FF} = 0 \) and therefore, for instance, the optimal rigid contract under joint liability described by proposition 4(i)a is not feasible. As another example, notice that the optimal discretionary contract under joint liability described in Proposition 4(i)b is not feasible under individual liability because it specifies \( \Delta_1 > 0 = \Delta_2 \).

The optimal supervisory contract under individual liability is a special case of the optimal supervisory one under joint liability characterized in proposition 3, where we replace \( \Delta_1 \) and \( \Delta_2 \) with \( \Delta \). Let \( C^I(\Delta) \equiv C^J(\Delta, \Delta) \) denote the minimized financing cost under individual liability given \( \Delta \). Therefore, we can now write the lender’s program under individual liability, denoted by \( (L') \), as follows:

\[
\max_{\Delta \geq 0, r_F \geq 0} \quad 2pY_S - 2p(r_F + \Delta) - 2(1 - p)\psi(r_F) \\
(BE) \quad 2r_F + 2p\Delta = C^I(\Delta) 
\]

where the right hand side of \( (BE) \) represents the financing cost derived from the optimal supervisory contract.

As in the case of joint liability, comparing all five supervisory contracts which can be optimal for different values of \( \Delta \) is very cumbersome. Hence, we study the case in which \( \underline{w} = 0 \) and hence \( \eta = \gamma \). Also in this case a useful lemma holds:

**Lemma 8** When there are two borrowers, under A1’, \( \underline{w} = 0 \) and individual liability, contract \( \delta \) is never an optimal supervisory contract.

Lemma 8 says that when \( \underline{w} = 0 \), as under joint liability, the supervisory contract \( \delta \) can be neglected, although for a different reason; when \( \delta \) is defined, it induces a higher financing cost than \( \beta \). Hence, we replace \( C^I(\Delta) \) with \( c^I(\Delta) \equiv c^J(\Delta, \Delta) \) where
Figure 2: The optimal supervisory contract for given $\Delta(=\Delta_1=\Delta_2)$ in $\mathbb{R}_+$ when $w=0$

c^f(\Delta_1, \Delta_2) \equiv \min \{C_\alpha, C_\beta, C_\gamma\}$ has been introduced in subsection 4.2. We make a weak assumption 
$$(1-p)(3-p)k \leq 2\rho$$
and use (14) to find

c^f(\Delta) = \begin{cases} 
2\rho + 2p\Delta & \text{if } \Delta \leq \frac{(1-p)^2}{p(2-p)}k \\
2\rho + (1-p)^2k + p^2\Delta & \text{if } \frac{(1-p)^2}{p(2-p)}k \leq \Delta \leq \frac{2(1-p)}{p}k \\
2\rho + (1-p^2)k & \text{if } \frac{2(1-p)}{p}k \leq \Delta 
\end{cases}

Figure 2 describes the optimal supervisory contract for $\Delta \geq 0$ in this case.

After replacing the right hand side in (19) with $c^f(\Delta)$, we obtain

$$r_F = \frac{c^f(\Delta)}{2} - p\Delta = \begin{cases} 
\rho & \text{if } \Delta \leq \frac{(1-p)^2}{p(2-p)}k \\
\rho + \frac{(1-p)^2}{2}k + \frac{p^2}{2}\Delta - p\Delta & \text{if } \frac{(1-p)^2}{p(2-p)}k \leq \Delta \leq \frac{2(1-p)}{p}k \\
\rho + \frac{1-p^2}{2}k - p\Delta & \text{if } \frac{2(1-p)}{p}k \leq \Delta 
\end{cases}$$

(20)

From (20) we see that the condition $r_F \geq 0$ is equivalent to $\Delta \leq \frac{2\rho + (1-p^2)k}{2p}$.

We plug (20) into (18) and observe that the borrowers’ payoff (i) decreases with $\Delta$ for $\Delta \in [0, \frac{(1-p)^2}{p(2-p)}k]$ because $r_F = \rho$ does not depend on $\Delta$ (an increase in $\Delta$ only increases the financing cost); (ii) increases with $\Delta$ for $\Delta \in [\frac{2(1-p)}{p}k, \frac{2\rho + (1-p^2)k}{2p}]$ because $\Delta$ does not

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34The inequality means that the audit cost is relatively smaller than the cost of capital. For instance, if $p = 1/2$, the inequality is equivalent to $\frac{5}{8}k \leq \rho$. 25
affect the financing cost and reduces $r_F$ without changing the expected payment. For
$\Delta \in \left[\frac{(1-p)^2}{p(2-p)}, \frac{2(1-p)^2}{2p}\right]$, increasing $\Delta$ has the positive effect of reducing $r_F$ (and increasing $r_S$) by $-p\Delta$ but also has the negative effect of increasing the financing cost by $\frac{p^2}{2}\Delta$ because of the carrot $w_2 - w_1 = \Delta$ which is awarded to the supervisor by contract $\beta$
(under joint liability, the increase in financing cost is avoided by setting $\Delta_2 = 0$ while $\Delta_1$ is maximized). This trade-off makes it hard to determine the optimal $\Delta$ in $\left[\frac{(1-p)^2}{p(2-p)}, \frac{2(1-p)^2}{2p}\right]$ in an unambiguous way. Therefore, we assume that $\psi$ is linear: $\psi(r) = \theta r$ with $\theta > 1$.

This implies that the borrowers’ payoff is monotone (increasing or decreasing) with respect to $\Delta$ for $\Delta \in \left(\frac{(1-p)^2}{p(2-p)}, \frac{2(1-p)^2}{2p}\right)$ and by using (i)-(ii) above we infer that the optimal $\Delta$ is either equal to 0 or to $\frac{2p+(1-p)^2k}{2p}$. The next proposition characterizes the optimal contract under individual liability.

**Proposition 5** When there are two borrowers under individual liability, suppose that $A1'$ is satisfied, $\omega = 0$, $(1-p)(3-p)k \leq 2\rho$ and $\psi(r) = \theta r$ with $\theta > 1$. Then

(i) The optimal grand contract is one of the two following:

a. A rigid lending contract $\{r_S = r_F = \rho\}$ with $\{q_0 = q_1 = 0, \ w_0 = w_1 = w_2 = 0\}$ (the supervisory contract is $\alpha$ with $\Delta = 0$).

b. A discretionary contract $\{r_S = \frac{2p+(1-p)^2k}{2p}, r_F = 0\}$ with $\{q_0 = q_1 = 1, \ w_0 = w_1 = w_2 = 0\}$ (the supervisory contract is $\gamma$ with $\Delta = \frac{2p+(1-p)^2k}{2p}$).

(ii) The rigid contract is optimal if and only if $(1+p)k \geq 2(\psi(\rho) - \rho)$.

The optimal contract under individual liability characterized above is identical to the one in the single-borrower case characterized in proposition 2 except that under the optimal discretionary contract in proposition 5(i)b, when both borrowers fail, the lender needs to incur the audit cost only once instead of twice. This is why the condition for the optimal contract to be rigid is stronger in proposition 5 than in proposition 2.

**Remark 2** When $(1-p)(3-p)k > 2\rho$ holds, then the condition $r_F \geq 0$ is equivalent to

$\Delta \leq \frac{2p+(1-p)^2k}{p(2-p)} \Rightarrow$ and we have

$$r_F = \frac{c'(\Delta)}{2} - p\Delta = \begin{cases} \rho & \text{if } \Delta \leq \frac{(1-p)^2k}{p(2-p)} \\ \frac{(1-p)^2k}{2} + \frac{p^2}{2}\Delta - p\Delta & \text{if } \frac{(1-p)^2k}{p(2-p)} \leq \Delta \leq \frac{2p+(1-p)^2k}{p(2-p)} \end{cases}$$

The optimal grand contract is either the rigid one in the above proposition or a discretionary one $\{r_S = \frac{2p+(1-p)^2k}{p(2-p)}, r_F = 0\}$ with $\{q_0 = 1, q_1 = 0, \ w_0 = w_1 = w_2 = 0\}$ (the supervisory contract is $\beta$ with $\Delta = \frac{2p+(1-p)^2k}{p(2-p)}$). The rigid contract is optimal if and only if $pp + (1-p)^2k \geq (1-p)(2-p)(\psi(\rho) - \rho)$.

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4.4 Comparison: joint liability versus individual liability

The following table summarizes the total NPV when there are two borrowers under the optimal rigid or discretionary contract\(^{35}\) depending on the form of the liability.

<table>
<thead>
<tr>
<th>Liability</th>
<th>The optimal rigid contract</th>
<th>The optimal discretionary contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint</td>
<td>( V - (1 - p)^22(\psi(\rho) - \rho) )</td>
<td>( V - (1 - p)^2k )</td>
</tr>
<tr>
<td>Individual</td>
<td>( V - [(1 - p)^2 + p(1 - p)] 2(\psi(\rho) - \rho) )</td>
<td>( V - [(1 - p)^2 + 2p(1 - p)] k )</td>
</tr>
</tbody>
</table>

where \( V \equiv 2pY_S - 2\rho \) represents the NPV when the supervisor is honest.

Conditional on that the lender uses the optimal rigid contract (or the optimal discretionary contract), the change from individual liability to joint liability increases the NPV. In the case of the optimal rigid contract, joint liability provides a partial insurance in that when only one project succeeds, the borrower whose project failed does not pay anything (i.e. \( r_{FS} = 0 \)) while he has to pay \( \rho \) under individual liability. This increase in the NPV from the partial insurance is equal to \( 2p(1 - p)(\psi(\rho) - \rho) \). In the case of the optimal discretionary contract, the introduction of joint liability does not affect any insurance provision since full insurance is provided regardless of the type of liability. However, it reduces the cost of audit: by making the borrower’s total repayment when both projects succeed equal to the payment when only one succeeds (i.e. \( \Delta_2 = 0 \)), the lender needs to conduct an audit to avoid embezzlement only when both projects fail. By contrast, under individual liability, the lender should audit whenever at least one project fails since \( \Delta > 0 \) leaves room for embezzlement both when \( n = 2 \) and when \( n = 1 \). The reduction in the audit cost is equal to \( 2p(1 - p)k \).

We also find that joint liability makes a discretionary contract more likely to be optimal since if a discretionary contract is optimal under individual liability (i.e. \( (1 + p)k < 2(\psi(\rho) - \rho) \) holds from proposition 5), then a discretionary contract is optimal under joint liability (i.e. \( k < 2(\psi(\rho) - \rho) \) holds from proposition 4).

Summarizing, we have

**Proposition 6** When there are two borrowers, suppose \( A1', w = 0, (1 - p)(3 - p)k \leq 2\rho \) and \( \psi(r) = \theta r \) with \( \theta > 1 \).

(i) When the supervisor is honest, joint liability does not affect the financing cost.

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\(^{35}\)We abuse a little bit of terminology by using the term ”the optimal discretionary contract” in the following sense. Although the contract is the optimal one among all the discretionary contracts if it is better than the optimal rigid contract, it may not be optimal among discretionary contracts if it is worse than the optimal rigid contract.
(ii) When the supervisor can misbehave, it is strictly optimal to introduce joint liability since it either reduces the financing cost or provides borrowers more insurance.

(iii) A discretionary contract is more likely optimal under joint liability than under individual liability.

4.5 Joint liability versus mutual insurance

Up to now, we have not considered the possibility that two borrowers can sign a side-contract between themselves. Actually, Laffont and N’Guessan (2000), Laffont (2003), Laffont and Rey (2003) and Rai and Sjöström (2004) consider side-contracting. In our model, if the lender uses individual liability, then the borrowers might have an interest to sign a side-contract to provide mutual insurance. More precisely, consider the following timing in which after accepting the lending contract and before the realization of the state of nature, the borrowers sign a binding side-contract which specifies a state-contingent side-payment from one borrower to the other. Since the lending contract and the agents are symmetric, a side-contract does not need to specify any side-payment when both projects succeed or both fail.\(^\text{36}\) Hence, a side-contract only specifies a monetary transfer \(x\) that a borrower whose project succeeds makes to a borrower whose project fails such that the latter uses \(x\) to make his repayment to the lender. Note first that given a grand-contract, side-contracting has no impact on the supervisor’s incentive since it does not affect the borrowers’ total payment schedule.

Consider first joint liability. Then, we can show in two steps that the optimal grand-contract without side-contracting that we characterized in proposition 4 is still the optimal contract even though the borrowers can sign a side-contract. First, we can prove that there is no loss of generality in restricting attention to the lending contracts which induce no side-contracting (i.e. \(x = 0\)). This is what is called the collusion-proofness principle\(^\text{37}\) in the literature on the mechanism design under collusion and this principle holds in our context. Suppose that a grand-contract \(G^J = \{r_{SS}, r_{SF}, r_{FS}, r_{FF}, w_{n}, q_{n}\}\) induces the borrowers to sign a side-contract specifying a side transfer \(x^* \neq 0\) as their optimal response. Then, consider another grand-contract \(G^{J^*}\) which is identical to \(G^J\) except \(r'_{SF} = r_{SF} + x^*\) and \(r'_{FS} = r_{FS} - x^*\). If the lender proposes \(G^{J^*}\), it must be optimal for the borrowers to sign a side-contract specifying zero side-transfer (i.e. it is optimal for them

---

\(^{36}\)Actually, if \(\psi\) is strictly convex then a side payment in state \(FF\) from a borrower to the other borrower reduces the borrowers’ sum of payoffs.

\(^{37}\)Tirole (1986) is the first who introduced the concept. For instance, Laffont and Martimort (2000) and Jeon and Menicuci (2005) use this principle to design the optimal mechanism under collusion.
not to sign any side-contract). Otherwise, \(x^*\) cannot be an optimal response to \(G^J\) and we have a contradiction. The collusion-proofness principle implies that side-contracting simply adds additional constraints to the lender’s optimization problem; hence, the lender cannot achieve a strictly better outcome with side-contracting than without it. Second, it is easy to see that when the lender uses the optimal grand-contract without side-contracting in proposition 4, it is optimal for the borrowers not to sign any side-contract since the grand-contract provides full insurance \((r_{FS} = 0)\) at the state in which only one project succeeds. Therefore, we can conclude that the optimal grand-contract without side-contracting in proposition 4 is optimal even when borrowers can sign a side-contract.

Consider now individual liability. First, we note that the outcomes that the lender can achieve under individual liability are a subset of the outcomes achievable under joint liability regardless of whether or not the borrowers can sign a side-contract. Therefore, the lender cannot achieve under individual liability an outcome superior to the best she can achieve under joint liability. Second, we can show that if side-contracting is possible, under individual liability the lender can achieve the outcome of the optimal rigid contract under joint liability (see proposition 4(i)a) by offering \(G^{I*} = \{r_S = r_F = \rho, w_n = q_n = 0\}\). Under \(G^{I*}\), it is optimal for the borrowers to sign a side-contract specifying \(x^* = \rho\) because it maximizes the borrowers’ expected payoffs. Therefore, \(G^{I*}\) induces the outcome of the optimal rigid contract under joint liability. Last, when there is individual liability, side-contracting does not allow the lender to achieve the outcome of the optimal discretionary contract under joint liability in proposition 4(i)b. The latter contract under joint liability specifies a total repayment schedule such that \(2r_{SS} = r_{SF} + r_{FS} > 2r_{FF} = 0\). This kind of repayment schedule cannot be achieved under individual liability because \(r_{SS} = r_{SF}\) and \(r_{FS} = r_{FF}\) must hold and side-contracting has no impact on the total repayment schedule. Summarizing, we have:

**Proposition 7** When there are two borrowers;

(i) Under joint liability, the optimal grand-contract is the same regardless of whether or not the borrowers can sign a side-contract and the possibility of side-contracting has no impact on their payoffs.

(ii) When the borrowers can sign a side-contract,

a. the borrowers’ payoffs cannot be higher under individual liability than under joint liability.

b. If a rigid contract is optimal under joint liability, the maximal payoffs under joint liability can be achieved under individual liability by a grand-contact which induces the borrowers to sign a suitable side-contract.
c. If a discretionary contract is optimal under joint liability, the borrowers’ payoffs under joint liability are strictly higher than under individual liability.

Proposition 7(ii)b implies that conditional on that a rigid contract is optimal, our model does not necessarily predict that we should observe the use of joint liability. Actually, the Grameen bank seems to recently have discontinued its previous practice of joint liability.38

5 Discussions

Consider the case of a single borrower who can choose between two investment projects of different size, a small one and a large one. The small project is the one described in section 3. In the large project the borrower invests \(a(> 1)\) units of money and the investment generates the revenue \(aY_S\) with probability \(p \in (0, 1)\), the revenue zero with probability \(1 - p\). The cost of capital is \(a\rho\) and, in order to use our result in proposition 2, we assume \(w = 0\). When the supervisor is honest, choosing the large project is optimal if and only if \(pY_S - \rho\) is positive because there are constant returns to scale. In other words, as long as the small project has a positive NPV, it is optimal to choose the large one. However, when the supervisor can be dishonest, it can be optimal to choose the small project even though \(pY_S - \rho > 0\). Conditional on choosing the large project, proposition 2 proves that a rigid contract is optimal if and only if \(k \geq \psi(a\rho) - a\rho\). Since this inequality implies \(k > \psi(\rho) - \rho\), a rigid contract is optimal in the small project if it is optimal in the large project. When a rigid contract is optimal regardless of the size of the project, the large one is chosen if and only if \((a - 1)p(Y_S - \rho) > (1 - p)[\psi(a\rho) - \psi(\rho)]\) and this inequality may not hold even though \(pY_S > \rho\). Therefore, the staff’s incentive problem creates a bias in project selection toward projects of small scale.

Suppose now that the borrower can choose between two investment projects of different probability of success: each of them requires one unit of money to invest and has the same positive NPV but a different probability of success. A (relatively) safe project produces a return of \(Y_S\) with probability \(p\) and zero return with probability \(1 - p\). A risky project produces a return of \(Y_0 > Y_S\) with probability \(p'\) and zero return with probability \(1 - p'\). We assume \(p > p'\) and \(pY_S = p'Y_0\). Therefore, when the supervisor is honest, the borrower is indifferent between the two projects. When the supervisor can misbehave and \(w = 0\), from

38See http://www.grameen-info.org/bank/GBGlance.htm. We asked them when and why they eliminated joint liability but got no response.
proposition 2, the borrower’s expected payoff conditional on choosing the safer project is \( pY^S - \rho - (1 - p) \min \{ \psi(\rho) - \rho, k \} \), which is strictly larger than the expected payoff conditional on choosing the riskier project: \( pY^R - \rho - (1 - p') \min \{ \psi(\rho) - \rho, k \} \). Therefore, the staff’s incentive problem creates a bias in project selection toward safe projects.

In this paper, we considered a three-tier hierarchy for simplicity. However, when we add additional layers into the hierarchy, a rigid lending contract is more likely to be optimal than a discretionary contract. Actually, the hierarchy of the Grameen bank is composed of head office, zonal office, area office, branch office, center, group, member. As long as the total payment that each staff member at the bottom of the hierarchy should collect does not depend on the realized returns of his borrowers, no staff member at a higher level of hierarchy has any discretion. Therefore, when a rigid contract is used, adding layers of hierarchy to our setting involves no extra cost except the minimum wages paid to the staff. By contrast, when a discretionary lending contract is used, even though the staff at the bottom are induced to behave well through audit or incentive pays, the staff at a higher level have also discretion and the bank has to incur additional costs to induce them to behave well.

In spite of the benefit of protecting borrowers from staff’s potential misconduct, a rigid lending contract has the cost of providing poor insurance to them. In the Grameen bank, this problem is mitigated to some extent by the Group and Emergency funds. The Group fund is a collective saving from which villagers could borrow when they need a short-term loan. The Emergency fund is a reserve into which borrowers have to contribute a fee at the end of the year and has evolved into a form of life insurance. Even though making each borrower’s payment responsive to his individual shock can be very costly because of the staff’s potential misbehavior, making it responsive to a publicly observable shock affecting a whole region such as natural catastrophes can be managed in a centralized way with little agency cost. Actually, the Grameen bank provides Disaster funds to areas affected by natural catastrophes.

Staff’s career concern is an important factor affecting their behavior. We can incorporate it into our model in the following way. Suppose that the supervisor derives a rent \( U > 0 \) from having a job in the Grameen bank. Then the incentive constraint \((IC_{SF})\) in section 3 is written as follows:

\[
w_S + U \geq (1 - q_F)(r_S - r_F + w_F + U),
\]

which is equivalent to

\[
w_S \geq (1 - q_F)(r_S - r_F + w_F) - q_F U.
\]
As $U$ increases, the probability of audit necessary to satisfy $(IC_{SF})$ decreases. Therefore, a positive rent makes it easier to induce a staff member to behave well. In general, $U$ would depend on his expectation about how long his organization would survive and what kind of careers he could make in the organization. As long as a microfinancing organization continues to grow and to maintain its good reputation, its staff would attach a high rent to their jobs and this makes it easier to induce their good behavior.

In our model, the only discretion that a staff member can have is in terms of making a report about the realized returns. However, in reality, there are other sources of discretion. In particular, a staff member can exercise his discretion when deciding whether or not a villager is eligible for a loan or how much loan a member can obtain, etc. This might induce staff's misconduct. Therefore, even though fixed repayment schedules are used, some monitoring of the staff’s actions should be made. In fact, according to Yunus, his bank could maintain excellence ”only if its monitoring system can reach out to all the remote and dark corners of the system and keep them clean”. (Bornstein, 1997, p. 171)

6 Conclusion

Our paper tried to answer the following challenging question: how could one make a large organization managing a lot of money, such as the Grameen bank, honest while corruption is a norm in the country? We addressed the question by focusing on the staff’s incentive to embezzle borrowers’ repayments and found the rigid repayment schedules which minimize (or eliminate) the staff’s discretion as an optimal response. We also found that joint liability reduces borrowers’ burden of respecting the rigid repayment schedules by providing them with partial insurance. However, the same insurance can be provided by borrowers themselves under individual liability since they have an incentive to sign a side-contract for mutual insurance. We also found that the staff’s incentive problem creates biases in project selection toward projects of small scale and small risk.

Recently, Yunus (2002) announced in ”Grameen Bank II” some changes in the payment mechanism. The changes are introduced to reduce the tension between a borrower and his staff member which arises under a rigid repayment schedule when the former has difficulty making his repayment due to bad shocks. Grameen Bank II introduces more flexibility into the system: borrowers can reschedule repayments and both the loan duration and the size of weekly installment can be varied. Our paper explains borrowers’ burden of respecting a rigid repayment schedule and suggests that increasing staff’s discretion may even harm borrowers by increasing the scope for staff’s misconduct.
Appendix

Proof of lemma 3

Since proving lemma 3 is not as easy as proving lemma 1(ii) and the proof of lemma 3 is very long and repeats what we do in section 4 to find the optimal grand-contract, we here prove only that if $2r_{FF} > \max \{r_{FS} + r_{FS}, 2r_{SS}\}$, then it is possible to increase the borrowers’ payoff (this proof is similar to the proof of lemma 1(ii)). Suppose that $G = \{r_{SS}, r_{SF}, r_{FS}, r_{FF}, w_2, w_1, w_0, q_2, q_1, q_0\}$ satisfies all the constraints in $(G')$ and is such that $2r_{FF} > \max \{r_{FS} + r_{FS}, 2r_{SS}\}$. Then, a grand contract with $r'_{SS} = r'_{SF} + r'_{FS} = r'_{FF} = p^2 r_{SS} + 2p(1-p)(r_{SF} + r_{FS}) + (1-p)^2 r_{FF}$ and $w_2' = w_1' = w_0' = 0, q_2' = q_1' = q_0' = 0$ satisfies all incentive constraints (with equality) and (BE) and increases the borrower’s payoff by lemma1(i).

The proof that the other cases in which $\Delta_1 < 0$ or $\Delta_2 < 0$ are not optimal is very similar to the derivation of the optimal contract for the case of $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$; it is omitted for the sake of brevity. 39

Proof of lemma 4

(i) It suffices to observe that the lower is $w_0$, the lower is the objective function and the more relaxed are (IC$_{20}$) and (IC$_{10}$).

(ii) We argue by contradiction. If (IC$_{20}$) binds and (IC$_{21}$) is slack, then $w_2 = (1 - q_0)\Delta_2 + (1 - q_0)(\Delta_1 + w)$ and $q_1 = 0$ (otherwise it is profitable and feasible to reduce $q_0$). As a consequence, (IC$_{21}$) slack requires $(1 - q_0)\Delta_2 + (1 - q_0)(\Delta_1 + w) > \Delta_2 + w_1$, or $(1 - q_0)(\Delta_1 + w) > q_0\Delta_2 + w_1$ but $\Delta_2 \geq 0$ and (IC$_{10}$) imply that this inequality is violated.

(iii) Lemma 4(ii) rules out the case in which only (IC$_{20}$) binds. If (IC$_{21}$) binds and (IC$_{10}$) and (IC$_{20}$) are slack, then $q_0 = 0$ and $w_1 = w$ (otherwise it is profitable and feasible to reduce $q_0$ and $w_1$); but then (IC$_{10}$) slack reduces to $w > \Delta_1 + w$, which fails to hold since $\Delta_1 \geq 0$. If (IC$_{10}$) binds and (IC$_{21}$) and (IC$_{20}$) are slack, then $q_1 = 0$ and $w_2 = w$; but then (IC$_{21}$) slack reduces to $w > \Delta_2 + w_1$, which does not hold since $\Delta_2 \geq 0$ and $w_1 \geq w$.

Proof of lemma 5

(i)a. From (IC$_{21}$) and (IC$_{20}$) binding and (IC$_{10}$) slack we obtain

$$w_1 = w, \quad w_2 = (1 - q_0)(\Delta_2 + \Delta_1 + w), \quad q_1 = 1 - \frac{(1 - q_0)(\Delta_2 + \Delta_1 + w)}{\Delta_2 + w} \quad (21)$$

39 The complete proof can be obtained from the authors upon request.
Hence, \( w_2 \geq w \) and \( q_1 \geq 0 \) are equivalent to \( q_0 \in [\frac{\Delta_1}{\Delta_2 + \Delta_1 + w}, \frac{\Delta_2 + \Delta_1}{\Delta_2 + \Delta_1 + w}] \) and (IC\(_{10}\)) slack reduces to \( q_0 > \frac{\Delta_1}{\Delta_2 + \Delta_1 + w} \). By plugging (21) into the objective function we find a linear function of \( q_0 \) which is minimized either at \( q_0 = \frac{\Delta_1}{\Delta_2 + \Delta_1 + w} \) or at \( q_0 = \frac{\Delta_2 + \Delta_1}{\Delta_2 + \Delta_1 + w} \). In the second case, which implies \( q_1 = \frac{\Delta_2}{\Delta_2 + \Delta_1 + w} \) and \( w_2 = \frac{w}{\Delta_2 + \Delta_1 + w} \), (IC\(_{10}\)) is slack and contract \( \eta \) is obtained. In the first case, instead, (IC\(_{10}\)) is not slack.

(i)b. From (IC\(_{10}\)) and (IC\(_{21}\)) binding we find

\[
W_1 = (1-q_0)(\Delta_1 + w) \quad \text{and} \quad W_2 = (1-q_1)(\Delta_2 + (1-q_0)(\Delta_1 + w))
\]  

(22)

Hence, \( W_1 \geq w \) is equivalent to \( q_0 \leq \frac{\Delta_1}{\Delta_2 + \Delta_1 + w} \) and \( W_2 \geq w \) is equivalent to

\[
(1-q_1)(\Delta_2 + (1-q_0)(\Delta_1 + w)) \geq w
\]  

(23)

From (22) follows that (IC\(_{20}\)) reduces to \( q_1 \leq f(q_0) \), where \( f(q_0) \equiv q_0 \frac{\Delta_2 + \Delta_1 + w - q_0(\Delta_1 + w)}{\Delta_2 + \Delta_1 + w} \) is an increasing and convex function such that \( f(0) = 0 \) and \( f(\frac{\Delta_1}{\Delta_2 + \Delta_1 + w}) = \frac{\Delta_2}{\Delta_2 + \Delta_1 + w} \).

The feasible set \( F \) for \((q_0, q_1)\) is therefore \( F = \{(q_0, q_1) \in \mathbb{R}^2_+ : q_0 \leq \frac{\Delta_1}{\Delta_2 + \Delta_1 + w} \text{ and } q_1 \leq f(q_0)\} \) since (IC\(_{20}\)) is equivalent to \( w_2 \geq (1-q_0)\Delta_2 + w_1 \) and in any point of \( F \) (i) (IC\(_{20}\)) holds; (ii) \( w_2 \geq (1-q_0)\Delta_2 + w_1 \geq w \), thus (23) is satisfied. From (22) and (8) we infer the objective function to minimize with respect to \( (q_0, q_1) \in F \):

\[
Q(q_0, q_1) = 2p + p^2(1-q_1)(\Delta_2 + (1-q_0)(\Delta_1 + w)) + 2p(1-p)((1-q_0)(\Delta_1 + w) + kf(q_0)) + (1-p)^2 k q_0
\]

We now prove the statement in lemma 5(i)b. First notice that no point in the interior of \( F \) minimizes \( Q \) since the Hessian matrix of \( Q \) is indefinite for any \((q_0, q_1)\):

\[
\frac{\partial^2 Q}{\partial q_0^2} = \frac{\partial^2 Q}{\partial q_1^2} = 0 \quad \frac{\partial^2 Q}{\partial q_0 \partial q_1} = p^2(\Delta_1 + w) > 0
\]

This violates the second order condition for a minimum, which requires the Hessian matrix to be positive semi-definite.

We can neglect any point \( (\bar{q}_0, 0) \) on the boundary of \( F \) such that \( 0 < \bar{q}_0 < \frac{\Delta_1}{\Delta_1 + w} \) because \( Q \) is linear in \( q_0 \) and this implies \( \min\{Q(0, 0), Q(\frac{\Delta_1}{\Delta_1 + w}, 0)\} \leq Q(\bar{q}_0, 0) \). Likewise, we can neglect any point \( (\frac{\Delta_1}{\Delta_1 + w}, \bar{q}_1) \) on the boundary of \( F \) such that \( 0 < \bar{q}_1 < f(\frac{\Delta_1}{\Delta_1 + w}) \) because \( Q \) is linear in \( q_1 \). As a consequence, only \( (\frac{\Delta_1}{\Delta_1 + w}, 0) \) may be a minimum point for \( Q \) in the subset of \( F \) in which (IC\(_{20}\)) is slack and at \( (q_0, q_1) = (\frac{\Delta_1}{\Delta_1 + w}, 0) \) we find contract \( \beta \).

(i)c. When all the three constraints bind we have \( q_1 = f(q_0) \) and \( Q(q_0, q_1) \) is equal to

\[
Q(q_0) = Q(q_0, f(q_0)) = 2p + p^2(1-f(q_0))(\Delta_2 + (1-q_0)(\Delta_1 + w)) + 2p(1-p)((1-q_0)(\Delta_1 + w) + kf(q_0)) + (1-p)^2 k q_0
\]

\[
= 2p + 2p(1-p)kf(q_0) + (1-p)^2 k q_0 + p(1-q_0)(p \Delta_2 + (2-p)(\Delta_1 + w))
\]
Since \( f \) is convex, also \( \bar{Q} \) is so. Hence, \( \bar{Q} \) is minimized at \( q_0 = 0 \) if \( \bar{Q}'(0) \geq 0 \) (contract \( \alpha \)), at \( q_0 = \frac{\Delta_1}{\Delta_1 + w} \) if \( \bar{Q}'(\frac{\Delta_1}{\Delta_1 + w}) \leq 0 \) (contract \( \gamma \)), at \( q^*_0 \in (0, \frac{\Delta_1}{\Delta_1 + w}) \) if \( \bar{Q}'(0) < 0 < \bar{Q}'(\frac{\Delta_1}{\Delta_1 + w}) \) (contract \( \beta \)).

(ii) Suppose that \( \Delta_2 > 0 = \Delta_1 \). Then \( w_1 = w \) since (IC\( _{10} \)) is \( w_1 \geq (1 - q_0)w \). Lemma 4(ii)-(iii) implies that (IC\( _{21} \)) binds and therefore \( w_2 = (1 - q_1)(\Delta_2 + w) \). (IC\( _{20} \)) reduces to \( q_0 \geq q_1 \) and then \( q_0 = q_1 \) because \( q_0 > q_1 \) implies that (IC\( _{10} \)) is slack and violates lemma 4(iii). Thus, \( q_1 = q_0 \leq \frac{\Delta_2}{\Delta_2 + w} \) in order to satisfy \( w_2 \geq w \). The objective function is then \( 2p + p^2(1 - q_0)(\Delta_2 + w) + 2p(1 - p)(w + kq_0) + (1 - p)^2(w + kq_0) \), which is linear with respect to \( q_0 \in [0, \frac{\Delta_2}{\Delta_2 + w}] \). Thus the optimal \( q_0 \) is either 0 or \( \frac{\Delta_2}{\Delta_2 + w} \) and contract \( \alpha \) is obtained for \( q_0 = 0 \), contract \( \eta \) for \( q_0 = \frac{\Delta_2}{\Delta_2 + w} \). Suppose that \( \Delta_2 = \Delta_1 = 0 \). Then it is possible to satisfy all incentive constraints with \( w_2 = w_1 = w \) and \( q_1 = q_0 = 0 \), which minimizes the financing cost (as if the supervisor were honest). Then contract \( \alpha \) is obtained.

Suppose that \( \Delta_2 = 0 < \Delta_1 \). Then, (IC\( _{21} \)) is redundant because it is easy to see that (IC\( _{20} \)) and (IC\( _{10} \)) jointly imply that (IC\( _{21} \)) is satisfied. Hence, \( q_1 = 0 \). Notice that (IC\( _{20} \)) and (IC\( _{10} \)) reduce to \( w_1 \geq (1 - q_0)(\Delta_1 + w) \) and \( w_2 \geq (1 - q_0)(\Delta_1 + w) \), respectively, hence either they both bind or are slack. They are both slack if and only if \( q_0 > \frac{\Delta_1}{\Delta_1 + w} \) and in this case \( w_1 = w_2 = w \). However, this is suboptimal with respect to \( q_0 = \frac{\Delta_1}{\Delta_1 + w} \). Thus, (IC\( _{20} \)) and (IC\( _{10} \)) both bind and the objective function is linear. Hence, it is optimal to set \( q_1 = 0 \) and we find \( \alpha \) or \( \beta \).

**Proof of Lemma 7**

In the proof, we show that when we maximize (12) subject to (11) with respect to \( (\Delta_1, \Delta_2) \in S_6 \), the optimal \( (\Delta_1, \Delta_2) \) lies on the boundary of \( S_6 \) and therefore there is a contract \( i \in \{ \alpha, \beta, \gamma \} \) which satisfies \( C_i = C_6 \).

Let \( \bar{S}_6^i \) be the set of points \( (\Delta_1, \Delta_2) \in \mathbb{R}^2_+ \) such that \( \delta \) is the unique optimal supervisory contract; then \( r_{FF} = \frac{1}{2}C_\delta - \frac{p(2 - p)}{2}\Delta_1 - \frac{p^2}{2}\Delta_2 \geq 0 \) by (11). Notice that \( \bar{S}_6^i \) is an open set. We start by proving that all point of \( \bar{S}_6^i \) belong to the interior of \( S' \). First, \( \bar{S}_6^i \) includes no \( (\Delta_1, \Delta_2) \) such that \( \Delta_1 = 0 \) and/or \( \Delta_2 = 0 \) because of lemma 5(ii). Furthermore, suppose by contradiction that \( (\Delta_1, \Delta_2) \in \bar{S}_6^i \) with \( \Delta_1 > 0, \Delta_2 > 0 \) and \( (\Delta_1, \Delta_2) \) belongs to the boundary of \( S' \) or does not belong to \( S' \). Then, \( r_{FF} < 0 \) because \( C_\delta < \min \{ C_\beta, C_\gamma \} \) implies \( r_{FF} = \frac{1}{2}C_\delta - \frac{p(2 - p)}{2}\Delta_1 - \frac{p^2}{2}\Delta_2 \leq \frac{1}{2} \min \{ C_\beta, C_\gamma \} - \frac{p(2 - p)}{2}\Delta_1 - \frac{p^2}{2}\Delta_2 \leq 0 \). In particular, the

\[ \bar{Q}'(x) = \frac{2p(1 - p)k\Delta_2 \delta + \Delta_1 w + \Delta_2 w}{\Delta_1 + (1 - \delta)(\Delta_1 + \Delta_2 + w)} + (1 - p)^2k - p(\rho\Delta_2 + (2 - p)(\Delta_1 + w)), \]

the condition \( \bar{Q}'(0) < 0 < \bar{Q}'(\frac{\Delta_1}{\Delta_1 + w}) \) is equivalent to the condition mentioned in footnote 32.
last inequality holds because if \((\Delta_1, \Delta_2)\) belongs to the boundary of \(S'\) or does not belong to \(S'\), then \(r_{FF} = \frac{1}{2} \min\{C_\beta, C_\gamma\} - \frac{p(2-p)}{2} \Delta_1 - \frac{p^2}{2} \Delta_2 \leq 0\) by definition of \(S'\).

After we established that \(\bar{S}'_k\) is a subset of the interior of \(S'\), consider the problem of maximizing (12) subject to \(2r_{FF} + p(2-p)\Delta_1 + p^2\Delta_2 = C_\delta\) with respect to \((\Delta_1, \Delta_2) \in \bar{S}'_k\).

The Lagrangian function is

\[
L(\Delta_1, \Delta_2, r_{FF}) = 2pY_S - p(2-p)(2r_{FF} + \Delta_1) - p^2\Delta_2 - (1-p)^22\psi(r_{FF}) + \lambda [2r_{FF} + p(2-p)\Delta_1 + p^2\Delta_2 - C_\delta]
\]

If there exists a solution to this program in \(\bar{S}'_k\), it is necessary that \(\frac{\partial L}{\partial \Delta_1}\) and \(\frac{\partial L}{\partial \Delta_2}\) both vanish in the solution because \(\bar{S}'_k\) is open:

\[
\frac{\partial L}{\partial \Delta_1} = (-1 + \lambda) p(2-p) - \lambda p(2-p)(1-q_0^*) - 2\lambda p(1-p)k \frac{\partial f(q_0^*)}{\partial \Delta_1} = 0
\]

\[
\frac{\partial L}{\partial \Delta_2} = (-1 + \lambda) p^2 - \lambda p^2(1-q_0^*) - 2\lambda p(1-p)k \frac{\partial f(q_0^*)}{\partial \Delta_2} = 0
\]

These conditions are equivalent to

\[
2\lambda(1-p)k \frac{\partial f(q_0^*)}{\partial \Delta_1} = (2-p)(\lambda q_0^* - 1)
\]

\[
2\lambda(1-p)k \frac{\partial f(q_0^*)}{\partial \Delta_2} = p(\lambda q_0^* - 1)
\]

Since \(\frac{\partial f(q_0^*)}{\partial \Delta_1} = \frac{-q_0^*(1-q_0^*)\Delta_2}{\Delta_2(1-q_0^*)\Delta_1} < 0\), if \(\lambda \leq 0\) then the left hand side of (24) is zero or positive but the right hand side is negative. Hence, \(\lambda > 0\) and (24) implies \(\lambda q_0^* < 1\); however, (25) implies \(\lambda q_0^* > 1\) because \(\frac{\partial f(q_0^*)}{\partial \Delta_2} = \frac{q_0^*(1-q_0^*)\Delta_1}{\Delta_2(1-q_0^*)\Delta_1} > 0\). Therefore, (24)-(25) cannot hold simultaneously and there is no \((\Delta_1, \Delta_2)\) which maximizes (12) subject to \(2r_{FF} + p(2-p)\Delta_1 + p^2\Delta_2 = C_\delta\). Hence, the optimal \((\Delta_1, \Delta_2)\) in \(S'\) is not such that \(\delta\) is the unique optimal supervisory contract and therefore we can neglect \(\delta\) and consider only \(\alpha, \beta, \gamma\).

**Proof of Proposition 4**

The proof consists of four claims.

**Claim 1** The optimal \((\Delta_1, \Delta_2)\) in \(S'_{\alpha}\) is equal to \((0,0)\).

Suppose that \((\Delta_1, \Delta_2)\) belongs to \(S'_{\alpha}\), which implies \(c'(\Delta_1, \Delta_2) = C_\alpha\). Then (13) is equivalent to \(r_{FF} = \rho\) and the borrowers’ payoff (12) is

\[
B'_{\alpha}(\Delta_1, \Delta_2) = 2pY_S - p(2-p)(2\rho + \Delta_1) - p^2\Delta_2 - (1-p)^22\psi(\rho)
\]
Since $B^J_\beta$ is decreasing both in $\Delta_1$ and $\Delta_2$, it is optimal to set $\Delta_1 = \Delta_2 = 0$. Notice that $B^J_\alpha(0, 0) = 2pY_S - 2\rho - 2(1 - p)^2(\psi(\rho) - \rho)$.

**Claim 2** The optimal $(\Delta_1, \Delta_2)$ in $S'_\beta$ is equal to $(\Delta_1^{\text{max}}, 0)$.

Suppose that $(\Delta_1, \Delta_2) \in S'_\beta$, which implies $c^J(\Delta_1, \Delta_2) = 2\rho + (1 - p)^2k + p^2\Delta_2$. Then (13) implies (15) and plugging into (12) we find

$$B^J_\beta(\Delta_1, \Delta_2) = 2pY_S - p(2 - p)\left(2\rho + (1 - p)^2(k + \Delta_1)\right) - p^2\Delta_2 - 2(1 - p)^2\psi\left(\rho + \frac{(1 - p)^2k - p(2 - p)\Delta_1}{2}\right)$$

The optimal $\Delta_2$ is 0 because $B^J_\beta$ is decreasing in $\Delta_2$. Furthermore,

$$\frac{\partial B^J_\beta}{\partial \Delta_1} = p(2 - p)(1 - p)^2\left(-1 + \psi'(\rho + \frac{(1 - p)^2k - p(2 - p)\Delta_1}{2})\right)$$

and thus $\frac{\partial B^J_\beta}{\partial \Delta_1} > 0$ since $\psi' > 1$. Given that $\Delta_1 \leq \Delta_1^{\text{max}}$ in $S'_\beta$, we conclude that in $S'_\beta$ it is optimal to set $(\Delta_1, \Delta_2) = (\Delta_1^{\text{max}}, 0)$ and therefore $r_{FF} = 0$. Notice that $B^J_\beta(\Delta_1^{\text{max}}, 0) = 2pY_S - p(2 - p)\Delta_1^{\text{max}} = 2pY_S - 2\rho - (1 - p)^2k$.

**Claim 3** In $S'_\gamma$ all $(\Delta_1, \Delta_2)$ which satisfy (17) with equality are optimal.

Suppose that $(\Delta_1, \Delta_2) \in S'_\gamma$, which implies $c^J(\Delta_1, \Delta_2) = 2\rho + (1 - p)^2k$. Then (13) reduces to (16) and after plugging this into (12) we find

$$B^J_\gamma(\Delta_1, \Delta_2) = 2pY_S - p(2 - p)\left(2\rho + (1 - p)^2k + (1 - p)^2\Delta_1\right) - p^2(1 - p)^2\Delta_2 - 2(1 - p)^2\psi\left(\rho + \frac{(1 - p)^2k - p(2 - p)\Delta_1}{2}\right)$$

Since $\frac{\partial B^J_\gamma}{\partial \Delta_1} = p(2 - p)(1 - p)^2\left(-1 + \psi'(\rho + \frac{(1 - p)^2k - p(2 - p)\Delta_1}{2})\right) > 0$ and $\frac{\partial B^J_\gamma}{\partial \Delta_2} = p^2(1 - p)^2\left(-1 + \psi'(\rho + \frac{(1 - p)^2k - p(2 - p)\Delta_1}{2})\right) > 0$, we infer that (17) binds. By replacing (17) binding into $B^J_\gamma$ we obtain a constant function equal to $2pY_S - 2\rho - (1 - p)^2k$. Hence, all points in which (17) binds are optimal in $S'_\gamma$.

**Claim 4** $(\hat{\Delta}_1, \hat{\Delta}_2) = (0, 0)$ if $k \geq 2(\psi(\rho) - \rho)$ and $(\hat{\Delta}_1, \hat{\Delta}_2) = (\Delta_1^{\text{max}}, 0)$ otherwise. In particular, $(\hat{\Delta}_1, \hat{\Delta}_2)$ never belongs to $S'_\gamma$.

It is easy to verify that $2pY_S - 2\rho - (1 - p)^2k < 2pY_S - 2\rho - (1 - p)^2k$; hence, $(\hat{\Delta}_1, \hat{\Delta}_2)$ will never be in $S'_\gamma$ because the best contract in $S'_\beta$ is better than the best contract in $S'_\gamma$. In comparing $(\Delta_1, \Delta_2) = (0, 0)$ with $(\Delta_1, \Delta_2) = (\Delta_1^{\text{max}}, 0)$, it is straightforward to see that $(0, 0)$ is better if and only if $k \geq 2(\psi(\rho) - \rho)$. Finally, the condition $\max\{r_{SS}, r_{SF}\} \leq Y_S$ we mentioned after introducing the lender’s program at the beginning of section 4 is satisfied if and only if $\max\{2\rho, \Delta_1^{\text{max}}\} \leq Y_S$, which is equivalent to $\frac{2\rho + (1 - p)^2k}{p(2 - p)} \leq Y_S$, or condition (ii) in A1'.
Proof of Lemma 8

First notice that \( \delta \) is defined if and only if \( \Delta \in \left(\frac{1-p}{2p}, \frac{(1-p)(1+3p)}{2p}\right) \). For \( \Delta \in \left(\frac{1-p}{2p}, \frac{(1-p)(1+3p)}{2p}\right) \), we find \( q_0 = 2 - 2\sqrt{\frac{\rho \delta }{p \Delta - (1-p)k}} \), \( C_\delta = 2\rho + 4\sqrt{p(1-p)k(2p\Delta - (1-p)^2k)} + 2(1-p)(1-2p)k - 2p\Delta \) and \( C_\delta < \min\{C_\alpha, C_\gamma\} \). However, \( C_\delta > C_\beta = 2\rho + (1-p)^2k + p^2\Delta \) holds for any \( \Delta \in \left(\frac{1-p}{2p}, \frac{(1-p)(1+3p)}{2p}\right) \) because \( C_\delta > C_\beta \) is equivalent to

\[
p^2(2+p)^2\Delta^2 + (1-p^2)(1-3p)^2k^2 - 2p(2+p)(1-p)(1-3p)k\Delta - 4p(1-p)k(2p\Delta - (1-p)^2k) > 0
\]

or to

\[
a\Delta^2 + b\Delta + c > 0 \tag{26}
\]

with \( a = p^2(2+p)^2, b = 2pk(1-p)(3p-2)(p+1), c = k^2(5p^2 - 2p + 1)(1-p)^2 \). Inequality (26) is satisfied for any \( \Delta \) because \( a > 0 \) and \( b^2 - 4ac = 16p^4k^2(1-p)^2(p^2 - 3p - 6) < 0 \). ■

Proof of Proposition 5

The proof covers both the case of \( (1-p)(3-p)k \leq 2p \) and the case of \( (1-p)(3-p)k > 2p \). The proof of the former consists of four claims. Claim 5 takes care of the case in which \( (1-p)(3-p)k > 2p \).

Claim 1 The optimal \( \Delta \) in \([0, \frac{(1-p)^2}{2p}]\) is 0.
If \( \Delta \in [0, \frac{(1-p)^2}{2p}] \), then \( r_F = \rho \) and \( B_\rho(\Delta) = 2pY_\rho - 2p(\rho + \Delta) - 2(1-p)\psi(\rho) \) is decreasing with respect to \( \Delta \); \( B_\rho(0) = 2pY_\rho - 2p - 2(1-p)\psi(\rho) - \rho \).

Claim 2 For \( \Delta \in \left[\frac{(1-p)^2}{2p}, \frac{2(1-p)}{p}\right] \), the borrowers’ payoff is monotone (increasing or decreasing).
If \( \Delta \in \left[\frac{(1-p)^2}{2p}, \frac{2(1-p)}{p}\right] \), then \( c'(\Delta) = C_\beta \) and this implies \( r_F = \rho + \frac{(1-p)^2}{2}k - \frac{p(2-p)}{2}\Delta \); notice that \( r_F > 0 \) for any \( \Delta \in \left[\frac{(1-p)^2}{2p}, \frac{2(1-p)}{p}\right] \). The borrowers’ payoff is then

\[
B_{\beta}\!(\Delta) = 2p\rho - (2p^{(1-p)^2}k + (2p^2 - 2p-\Delta)) - 2(1-p)\psi(\rho + \frac{(1-p)^2}{2}k - \frac{p(2-p)}{2}\Delta)
\]

and it is monotone with respect to \( \Delta \) since \( \psi \) is linear.

Claim 3 The optimal \( \Delta \) in \( \left[\frac{(1-p)^2}{2p}, \frac{2p}{p}\right] \) is \( \frac{2p(1-p)^2}{2p} \).
If \( \Delta \in \left[\frac{2(1-p)}{p}, \frac{2p}{p}\right] \), then \( c'(\Delta) = C_\gamma \) and \( r_F = \rho + \frac{1-p^2}{2}k - p\Delta \). Hence, the borrowers’ payoff is

\[
B_{\gamma}(\Delta) = 2p\rho - (2p^{(1-p)^2}k + 2(1-p)\Delta) - 2(1-p)\psi(\rho + \frac{1-p^2}{2}k - p\Delta)
\]
and
\[ \frac{dB^I_\gamma(\Delta)}{d\Delta} = 2p(1-p) \left( -1 + \psi'(\rho + \frac{1-p^2}{2p}k - p\Delta) \right) > 0 \]

\( B^I_\gamma \) is maximized in \( \left[ \frac{2(1-p)k}{p}, \frac{2p+(1-p^2)k}{2p} \right] \) at \( \Delta = \frac{2p+(1-p^2)k}{2p} \) with \( B^I_\gamma(\frac{2p+(1-p^2)k}{2p}) = 2pY_S - 2p - (1-p^2)k \).

**Claim 4** \( \hat{\Delta} = 0 \) if \( (1+p)k \geq 2(\psi(\rho) - \rho) \) while \( \hat{\Delta} = \frac{2p+(1-p^2)k}{2p} \) otherwise.

Since the objective function is monotone when \( \Delta \in \left[ \frac{(1-p)^2k}{p(2-p)k}, \frac{2(1-p)^2k}{p} \right] \), the optimum \( \hat{\Delta} \) is either in \( [0, \frac{(1-p)^2k}{p(2-p)k}] \) or in \( \left[ \frac{2(1-p)k}{p}, \frac{2p+(1-p^2)k}{2p} \right] \). By claims 1 and 3, then \( \hat{\Delta} = 0 \) if \( B^I_\alpha(0) \geq B^I_\gamma(\frac{2p+(1-p^2)k}{2p}) \), which is equivalent to \( (1+p)k \geq 2(\psi(\rho) - \rho) \); \( \hat{\Delta} = \frac{2p+(1-p^2)k}{2p} \) if \( (1+p)k < 2(\psi(\rho) - \rho) \).

**Claim 5** \( \hat{\Delta} = 0 \) if \( pp+(1-p)^2k \geq (1-p)(2-p)(\psi(\rho) - \rho) \) while \( \hat{\Delta} = \frac{2p+(1-p)^2k}{p(2-p)} \) otherwise.

We can argue like in claims 1 and 2 above to find that the borrowers’ payoff is decreasing with respect to \( \Delta \) for \( \Delta \in [0, \frac{(1-p)^2k}{p(2-p)k}] \) and is monotone for \( \Delta \in \left[ \frac{(1-p)^2k}{p(2-p)k}, \frac{2p+(1-p^2)k}{2p} \right] \). Hence, \( \hat{\Delta} \in \{0, \frac{2p+(1-p^2)k}{p(2-p)}\} \) and a simple comparison of payoffs shows that \( \hat{\Delta} = 0 \) if \( pp+(1-p)^2k \geq (1-p)(2-p)(\psi(\rho) - \rho) \).

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