On-the-Job Search in a Matching Model with Heterogenous Jobs and Workers *

Juan J. Dolado$^a$, Marcel Jansen$^b$ and Juan F. Jimeno$^c$

$^a$Universidad Carlos III de Madrid and CEPR.
$^b$Universidad Carlos III de Madrid.
$^c$Universidad de Alcalá, FEDEA and CEPR.

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Abstract

This paper considers a matching model with heterogenous jobs and workers. Some workers may accept jobs for which they are over-qualified, but the innovation of this study is that we allow these workers to perform on-the-job search. In our economy skill-mismatch therefore has a transitory nature and we show that this has relevant implications for employment, wage inequality and the economy’s response to shocks.

Keywords: on-the-job search, skills, unemployment, wage inequality

JEL Codes: J1, J24, J41

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1 Introduction

This paper considers a simple matching model with heterogeneous jobs and workers. Two-sided heterogeneity arises as firms choose between creating two types of jobs (skilled and unskilled) and the labour force consists of two types of workers (high and low-educated). High-educated workers can perform both jobs, being more productive in skilled jobs, whereas low-educated workers can only be employed in unskilled jobs.¹

In equilibrium some high-educated workers may accept unskilled jobs for which they are over-educated, but the innovation in this paper is that we allow these mismatched workers to perform on-the-job search. In our economy skill mismatch therefore has a transitory nature and we show that this has relevant implications for employment, wages and the reaction of the labour market to shocks.

Labour economists have long recognised the importance of job-to-job transitions, but it is only recently that the literature on equilibrium unemployment has begun to explore its implications systematically.² Here we focus on the implications of over-education and on-the-job search for the equilibrium distribution of wages and employment, stressing their effects on the labour market position of low-educated workers.

Our model builds on the random matching model of Albrecht and Vroman (2002) (henceforth AV) which addresses similar issues. The key difference with AV is that we allow for on-the-job search by over-educated workers while in their model only the unemployed search for jobs. Thus, in AV’s economy, mismatch is considered to be permanent. This allows them to characterize two types of equilibria, i.e. one in which high-educated workers match with both types of jobs (a cross-skill matching equilib-

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¹For simplicity, workers’ skills are assumed to be perfectly correlated with their educational attainments. Moreover, the latter are exogenously determined.
²Broadly speaking, the literature on on-the-job search can be divided in two strands. One strand uses models in the vein of Burdett and Mortensen (1998) to explain how on-the-job search may give rise to wage differentials among identical workers; see Mortensen (2003) for an excellent overview of this literature. The second strand incorporates on-the-job search in the standard matching framework to study the implications for the wage distribution, turnover and the cyclical dynamics of unemployment and vacancies. Examples are Burgess (1993), Pissarides (1994), Albrecht and Vroman (2002), Shimer (2003, 2004), Moscarini (2003) and Nagypál (2003).
rium) and another in which they refuse to take unskilled jobs (an \textit{ex-post segmentation} equilibrium); Shifts in the productivity differential across jobs or in the skill composition of the population lead to abrupt changes in unemployment rates and wages as the economy moves between equilibria (see also Acemoglu, 1999). In our economy, by contrast, over-educated workers can exert \textit{costless} search for a better job without an intervening spell of unemployment.

A first consequence of the introduction of costless on-the-job search is that it rules out equilibria with ex post segmentation. Therefore, unlike AV, we obtain a unique type of equilibrium in which high-educated workers always accept unskilled jobs. This result is a logical consequence of on-the-job search and we show that it adds realism to the response of the labour market to shifts in productivity or in the skill composition of the population.

Second, since mismatched workers are part of the pool of job seekers, firms have a stronger incentive to create skilled jobs than in an economy without on-the-job search in which only the unemployed workers search. Due to the assumption of random matching, low-educated workers will therefore have a lower exit rate out of unemployment, and so the “crowding-out” of low-educated workers is stronger when mismatch is a transitory phenomenon.

Third, the higher separation rate of mismatched workers reduces the overall stability of employment relations in the lower segment of the labour market and we show that this introduces a further negative externality on low-educated workers. In particular, whenever the job-to-job transitions by mismatched workers are frequent, firms with unskilled jobs need to return more frequently to the labour market and since this is costly they will create fewer unskilled jobs.\footnote{In principle, firms may try to reduce worker turnover by paying a higher wage \textit{e.g.} Shimer, 2004). However, we ensure that firms with unskilled jobs cannot match the wage of firms with skilled jobs. For more details see section 3.3.}

The above arguments suggest that the distinction between worker flows and job flows in the unskilled segment of the labour market is particularly harmful for low-educated workers. The only way to avoid these negative effects would be to assume
that high-educated workers are more productive on both jobs, but this is ruled out in our setup.⁴

Finally, we show that on-the-job search has important implications for the distribution of wages and income. It widens the income differences between high- and low-educated workers, and it gives rise to a much larger wage inequality among high-educated workers than in the absence of on-the-job search. In fact, one of our main results shows that mismatched workers earn a lower wage than low-educated workers on the same job. This is the opposite of the result in a model without job-to-job transitions. The higher wage of mismatched workers in models without on-the-job search is due to the better outside option of high-educated workers. This better outside option also explains why these workers may find it optimal to refuse an unskilled job. By contrast, in our model, high-educated workers retain the option of a skilled job while employed in an unskilled one. This feature implies that they will receive a lower wage than in the absence of on-the-job search. Nonetheless, since they quit more often than low-educated workers, firms with unskilled jobs prefer to hire the latter and pay mismatched workers a lower wage.

Furthermore, our simulations suggest that the above effects give rise to a substantial increase in the variance of wages. Thus, on-the-job search seems to add another plausible mechanism to explain the widening of within-education and within-occupation wage dispersions which has been observed notably in the US and in many other OECD countries since the 1980s. Moreover, the “crowding-out” effects of job competition for low-skilled jobs and the prediction about a wage penalty for over-qualified workers may be relevant factors behind the strong pressures in the UK and other EU countries for reforms in the system of tertiary education (see The Economist, January, 2004).

The rest of the paper is organised as follows. Section 2 presents some empirical

⁴AV make the same assumption. Although there is anecdotal evidence about simple tasks that can be better performed by low-educated workers and vice versa, on average it seems sensible to assume equal productivity in unskilled jobs. If high-educated workers were assumed to be more productive in both types of jobs, then mismatched workers would exert a positive externality on low-educated workers. Gautier (2002) considers the effects of on-the-job search in this alternative setup, but in his model wages do not react to the aggregate state of the labour market.
evidence that motivates our theoretical analysis. Section 3 lays out the model. Section 4 discusses the properties of the steady-state equilibrium with on-the-job search and derives the result about the wage penalty for mismatched workers. Section 5 focuses on the comparative statics of the model in response to skilled- biased technical change and skill upgrading. In particular, in contrast to the predictions in AV, we show that an increase in the productivity of skilled jobs always lowers the unemployment rate of high-educated workers and lead to much higher wage dispersion. The effect on the unemployment rate of the low-educated workers is ambiguous, although, when we simulate the model in Section 6 with realistic parameter values, it increases. As for skill upgrading, an increase in the share of high-educated workers in the population typically reduces the unemployment rate of high-educated workers (i.e., a cohort-size effect), despite the increase in their supply, whilst it raises the unemployment rate of the low-educated workers and widens the within-group wage dispersion, due to the more intense job competition for unskilled jobs. Finally, Section 7 concludes. Proofs of the main propositions are gathered in an Appendix.

2 Evidence

2.1 Job-to-job flows and mismatch

Recent evidence suggests that job-to-job moves are an important element of the total inflows into employment. For example, Fallick and Fleischman (2001) report that in the U.S. in 1999 more than four million workers changed employer during an average month, about the same number as the workers who left the labour force from employment and more than twice the number who moved from employment to unemployment. Further, despite the fact that the hazard rate of a job-to-job transition decreases with the education level of the worker (from 3.5% of employment for less-educated workers to 2.0% for high-educated workers), these flows account for a much larger share of total separations for college workers (50%) than for high-school dropouts (30%).

Similarly, in the U.K. job-to-job moves account for at least 40% of all hires (Pis-
sarides, 1994). Elsewhere in Europe these moves appear to be less frequent than in the U.K. but of a similar magnitude as in the U.S.. For example, Burda and Wyplosz (1994) estimate that in Germany in 1987, job-to-job moves represented around 15 percent of employment inflows while the remaining inflows are shared equally between unemployment and inactivity flows.\(^5\)

Finally, since the focus of our study is on job-to-job transitions by over-educated workers, it is useful to report some recent statistics from Eurostat (2003) about the degree of over-education in the EU countries and its relation to on-the-job search. Figure 1a confirms that mismatched workers tend to search more frequently on-the-job than appropriately matched workers as shown by the positive differentials between the proportions of on-the-job seekers in the respective populations reported in the vertical axis.\(^6\) Furthermore, the difference in the search intensity of these two groups is positively correlated with the overall degree of skill mismatch (horizontal axis). Figure 1b, in turn, links the degree of over-education to shifts in the educational attainment of the labour force. It shows that skill mismatch is particularly prominent in countries that have undergone an intense upgrading of tertiary education over the last two decades. In those countries, predominantly in the Southern Mediterranean region, the growth in supply of university graduates seemingly outpaced the growth in demand, giving rise to 15 to 25 percent of skill mismatch at the highest educational level (categories ISCED 5 and 6).\(^7\)


\(^6\)The data for Figures 1a and b come from an ad hoc module carried out by Eurostat in the EU LFS 2000 designed to collect specific information on the transition from school to working life in EU countries. A job mismatch is defined in Figure 1a as a job outside the field of education of school leavers, held for at least two years, which is considered to have lower skill requirements than those corresponding to the workers’ educational attainments. The education upgrading in Figure 1b is measured by the ratio of the fraction of the population aged 25-34 with tertiary education and the corresponding proportion of the population aged 45-54. Data correspond to 1999 (see OECD 2001).

\(^7\)For example, Oliver and Raymond (2003) show that the proportion of overeducated workers with a college degree in Spain (Diplomados and Licenciados) increased from 14% in 1985 to 21% in 1998. During this period, the share of the Spanish population (25-64) with a tertiary educational attainment...
Figure 1a: Job search intensity and overeducation

Figure 1b: Mismatch and educational upgrading.

raised from 15% to 23%. For evidence about over-education and crowding-out in the Spanish labour market since the mid-1980s, see Alba-Ramírez (1993) and Dolado et al. (2000).
2.2 Mismatch and wages

As mentioned in the Introduction, our model with costless on-the-job search leads to the testable prediction that mismatched workers earn lower wages than properly matched workers in the same unskilled job. The empirical evidence on this issue is mixed. However, it is not clear whether these existing studies include sufficient controls for possible productivity differentials between these two categories of workers in unskilled jobs. The latter is crucial to determine a possible wage penalty for over-educated workers since our results are based on the assumption that high and low-educated workers are equally productive in those jobs.

To provide some further evidence on this issue, we estimate a Mincerian wage equation in order to test whether such a wage penalty for over-education is present when we control for equal productivity on unskilled jobs. The data for ten European countries is taken from the 1998 wave of the European Commission Household Panel (ECHP). We define high-educated workers as those individuals who have completed a college degree, whereas low-educated workers are those with a lower educational degree. Moreover, for each country the sample is limited to full-time workers aged 25 to 34 who work at least 15 hours per week. Finally, since the ECHP includes eight types of occupations in decreasing order of complexity - ranging from Legislators, Senior Officials and Managers (occupation 1) to Elementary Occupations (occupation 9) - for simplicity, we define skilled (unskilled) jobs to those occupations with complexity

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8 Hartog and Oosterbeek (1988) and Gaultier et al. (2002) for the Netherlands, and Sicherman (1991) for the U.S. conclude that over-educated workers earn more than correctly allocated workers. But, De Groot (1993, 1996), Verdugo and Verdugo (1989) and Alba-Ramírez and Blázquez (2003) obtain the opposite result for the U.K, the U.S and Spain, respectively. For example, Verdugo and Verdugo (1989) find the wage return to an extra year of required education is 6.2%, while the penalty for a year of over-education is 8.0% in the U.S.

9 Since educational variables are updated every four years in the survey, starting from 1994, this is the last available ECHP wave with such a characteristic. The ten selected countries (listed in Table 1) are those for which all the variables used in the estimation of equation (1) below were available on a homogenous basis. The sample is restricted to individuals from 25 to 34 years of age. The choice of a restricted age range is justified by the absence of work experience in the ECHP where only potential experience is available.
level below (above) the mid-category of *Service Workers and Shop and Market Sales* (occupation 5). Low-educated workers in the sample are restricted to be only those working in unskilled jobs.

Although the ECHP does not offer direct information about the educational requirements of jobs, it offers a number of questions from which one can infer the type of job match, and improve our knowledge on the productivity of workers of either type performing unskilled jobs. The relevant questions are:

(Q1) *Do you consider you have qualifications to do a more demanding job then the one you have now?*

(Q2) *Has your formal education provided you with the skills needed to perform your current job?*

(Q3) *Is your formal education closely (or very closely) related to your current job?*

The answer to the first question allows us to divide the sample of high-educated individuals into over-educated (affirmative reply) and the rest (negative reply). In order to mimic the assumption in our model whereby over-education only affects high-educated workers, we exclude from our sample those low-educated workers who consider themselves to be over-educated. Hence, the reference individuals are low-educated workers who replied negatively to Q1 and work in unskilled jobs. Furthermore, to control for equal productivity, besides including in the equation the standard co-variates used in this type of studies (age, gender, tenure, etc.), we restrict the sample to individuals who reply affirmatively to both Q2 and Q3. Thus, by doing so, we expect to improve our control on the equal productivity assumption, since both high and low-educated workers declare themselves to be capable of performing the tasks required by their unskilled jobs.\(^{10}\)

The resulting *Mincerian* wage equation takes the form:

\(^{10}\)The idea behind this filtering of the data is to exclude cases where college education (e.g. a degree in Law) may not have provided a high-educated worker with the necessary skills to perform an unskilled job (e.g., as a mechanic) and therefore where lower productivity may be the reason behind their lower wages.
\[ \text{LnW} = a_0 + a_1 \text{DE} + a_2 [\text{DE} \times \text{DO} \times \text{DS}] + a_3 [\text{DE} \times \text{DO} \times \text{DU}] + a'x + \varepsilon \]  

The dependent variable \( \text{LnW} \) is the log of log hourly wages (in Euros), \( x \) is a vector of control variables and \( \varepsilon \) an i.i.d. error term.\(^{11}\) Furthermore, (1) includes 4 dummy variables: \( \text{DE} \) takes the value 1 for high-educated workers and 0 otherwise; \( \text{DO} \) equals 1 for “over-educated” workers and 0 otherwise; finally, the dummy variables \( \text{DS} \) and \( \text{DU} \) indicate the high-educated workers performing skilled and unskilled jobs, respectively.

Accordingly, in the above wage expression, the average college premium is given by \( a_1 \), while \( a_3 \) measures the change in the return to college education if the individual is over-educated and employed in an unskilled job. Thus, if our theoretical prediction about the wage penalty holds, then the sum of \( a_1 \) and \( a_3 \) must be negative. As is customary, we use a one-sided test to test the null hypothesis of \( H_0 : (a_1 + a_3) = 0 \) against the one-sided alternative of \( H_1 : (a_1 + a_3) < 0 \).

The results are reported in Table 1. Besides the shares of high-educated and over-educated workers, the estimates of \( a_1 \) and the sample sizes, the estimates of \( (a_1 + a_3) \) and the p-values of the previous test are displayed in column 5. As can be observed, the estimates of \( (a_1 + a_3) \) are negative in eight out of the ten cases and the null hypothesis is rejected at a 5% significance level for five countries. Interestingly, these

\(^{11}\)The vector \( x \) of co-variates in the estimation of wage equations (Section 2.3) contains the following variables: MALE, AGE (Sample restricted to individuals aged between 25 and 34 years old), CIVIL STATUS, EDUCATION (DE=1 if individual has completed tertiary education), TYPE OF FIRM (Private/Public), SIZE OF FIRM (Small=1 if the individual works in a private firm with <20 employees; Medium =1 if individual works in private firm with 20-500 employees), SENIORITY (<1 year = 1 if individual has worked in current job for < 1 year; 1-5 years= 1 if individual has worked in current job for 1-5 years), TRAINING (=1 if individual receives on-the-job training provided by the employer), PERMANENT CONTRACT (=1 if individual has a permanent contract), SECTOR (Twelve dummies; Reference= Agriculture, hunting and forestry+ fishing), OCCUPATION (Unskilled (DU)=1 from occupation 5 to occupation 9; Skilled (DS)=1 from occupation 1 to occupation 4), OVER-EDUCATION (DO=1 if individual has DE=1 and replies YES to Q1), and UNEMPLOYMENT (=regional unemployment rate in 1998)
countries are among the countries with the strongest intensity of on-the-job search and the highest index of over-education in Figure 1b. Thus, our evidence seems to provide some empirical support for the wage penalty.

Table 1. Over-education and wages. Regression results

<table>
<thead>
<tr>
<th>Country</th>
<th>High-educated (%)</th>
<th>Over-educated (%)</th>
<th>(\hat{a}_1) (St. error)</th>
<th>(\hat{a}_1 + \hat{a}_3) (p-value*)</th>
<th>No. of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>7.3</td>
<td>15.9</td>
<td>0.127 (0.013)</td>
<td>−0.033 (0.043)</td>
<td>831</td>
</tr>
<tr>
<td>Belgium</td>
<td>15.8</td>
<td>15.8</td>
<td>0.133 (0.017)</td>
<td>−0.014 (0.123)</td>
<td>703</td>
</tr>
<tr>
<td>Denmark</td>
<td>21.2</td>
<td>16.2</td>
<td>0.119 (0.019)</td>
<td>−0.029 (0.234)</td>
<td>640</td>
</tr>
<tr>
<td>Finland</td>
<td>15.2</td>
<td>15.4</td>
<td>0.143 (0.016)</td>
<td>−0.012 (0.183)</td>
<td>776</td>
</tr>
<tr>
<td>France</td>
<td>15.6</td>
<td>17.7</td>
<td>0.113 (0.008)</td>
<td>0.019 (0.342)</td>
<td>1,283</td>
</tr>
<tr>
<td>Greece</td>
<td>16.3</td>
<td>22.1</td>
<td>0.089 (0.018)</td>
<td>−0.056 (0.023)</td>
<td>792</td>
</tr>
<tr>
<td>Italy</td>
<td>9.4</td>
<td>23.6</td>
<td>0.083 (0.008)</td>
<td>−0.032 (0.027)</td>
<td>1,643</td>
</tr>
<tr>
<td>Netherlands</td>
<td>25.8</td>
<td>16.5</td>
<td>0.133 (0.011)</td>
<td>0.012 (0.321)</td>
<td>1,339</td>
</tr>
<tr>
<td>Portugal</td>
<td>9.6</td>
<td>20.7</td>
<td>0.126 (0.017)</td>
<td>−0.034 (0.036)</td>
<td>940</td>
</tr>
<tr>
<td>Spain</td>
<td>20.6</td>
<td>22.6</td>
<td>0.101 (0.009)</td>
<td>−0.052 (0.021)</td>
<td>1,482</td>
</tr>
</tbody>
</table>

Note: *p-value for \(H_0: (a_1 + a_3) = 0\) vs. \(H_1: (a_1 + a_3) < 0\).

After this brief discussion of the stylized facts on over-education and mismatch, we now proceed to construct a model that can account for these facts.

3 The Model

3.1 Main assumptions

Consider an economy populated by a continuum of workers with measure normalised to unity. Workers are of two types and the distribution of types is exogenous. A fraction \(\mu \in (0, 1)\) of the population is less-educated (l) while the rest of the population is high-educated (h). All workers are risk-neutral, infinitely-lived and discount the future at the common rate \(r\). Time is continuous and the analysis is restricted to steady states.
Production requires a job and a worker. There are two types of jobs: skilled jobs \( (s) \), and unskilled jobs \( (n) \). Unskilled jobs are less productive but they can be performed by both types of workers, while a skilled job requires an \( h \)-type worker. The details of the production technology are summarised in the following diagram:

<table>
<thead>
<tr>
<th>Workers / Jobs</th>
<th>Unskilled</th>
<th>Skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>l-type</td>
<td>( y(n) )</td>
<td>0</td>
</tr>
<tr>
<td>h-type</td>
<td>( y(n) )</td>
<td>( y(s) )</td>
</tr>
</tbody>
</table>

where \( y(s) > y(n) \), so that \( h \)-type workers have a comparative advantage in skilled jobs. Firms can open at most one job. The type of job is determined \textit{ex ante} before the firm meets workers and there is free entry of firms.

Finally, job destruction is exogenous and follows a Poisson process with arrival rate \( \delta \) that is common to both types of jobs.\(^{12} \) Whenever a job is destroyed, the worker becomes unemployed while the job becomes vacant.

### 3.2 Matching

The labour market is characterised by matching frictions. As in AV’s model, the matching process is assumed to be \textit{undirected} but we allow for on-the-job search by high-educated workers in unskilled jobs. In our economy, mismatch is therefore a transitory rather than a permanent state. The resulting job-to-job flows are captured by the dashed arrow from unskilled jobs to skilled jobs in Figure 2.

The total flow of matches between a worker and a firm is determined by a CRS matching function

\[
m[v(n) + v(s), u(l) + u(h) + e(n, h)],
\]

\(^{12}\)Notice that the effective separation rates will be different due to the voluntary quits of mismatched workers. In line with the empirical evidence, skilled jobs are therefore on average more stable than unskilled jobs.
where $u(j)$ is the mass of unemployed workers of type $j$ ($= l, h$), $v(i)$ is the mass of vacancies of type $i$ ($= n, s$) and $e(n, h)$ is the mass of (mismatched) high-educated workers performing unskilled jobs. As usual we assume that $m[.,.]$ is strictly increasing in both arguments and we denote the “labour market tightness” by $\theta = [v(n) + v(s)]/[u(l) + u(h) + e(n, h)]$. Accordingly, the matching rate of a firm is equal to $q(\theta) = m(1, 1/\theta)$ while the matching rate of a worker is given by $\theta q(\theta)$. The properties of $m[.,.]$ guarantee that $q(\theta)$ is decreasing in $\theta$, while $\theta q(\theta)$ is increasing in $\theta$.

![Figure 2: The flow diagram](image)

The introduction of job-to-job flows implies that we need a few additional assumptions about the quit decisions of mismatched workers. In what follows, we assume that workers need to abandon their current employer before they can start negotiations with an alternative employer. Moreover, we assume that workers only move if they expect a higher wage at the new firm.\(^{13}\) These two assumptions imply that mismatched workers will only leave their current employer if they have located a firm with a skilled job.

Formally, let $\phi (= u(l)/[u(l) + u(h)])$ denote the proportion of low-educated workers in the mass of unemployed, and let $\psi (= [u(l) + u(h)]/[u(l) + u(h) + e(n, h)])$ denote the

\(^{13}\)These assumptions coincide with the assumptions in Pissarides (1994). Furthermore, they guarantee that all job-to-job movements are efficient. For a model of on-the-job search with matching wage offers see Postel-Vinay and Robin (2002).
fraction of unemployed workers in the total mass of job seekers which differs from the mass of unemployed because of the presence of on-the-job seekers. Thus, an unskilled job meets an unemployed job seeker at rate $\psi q(\theta)$, while skilled jobs meet a high-educated worker at rate $(1 - \psi \phi)q(\theta)$. Finally, to define the matching rates of workers, we introduce the share of unskilled vacancies $\vartheta = v(n)/[v(n) + v(s)]$. Thus, $l$-type workers exit unemployment at rate $\vartheta q(\theta)$, while high-educated job seekers meet a skilled job at rate $(1 - \vartheta)q(\theta)$.

### 3.3 Bargaining

Match formation is voluntary and wages are determined by a standard linear sharing rule. The fixed surplus share of the worker is denoted by $\beta \in (0, 1)$. Formally, let $U(j)$ denote the value of unemployment for a worker of type $j$, $V(i)$ the value of a vacant job of type $i$, $W(i, j)$ the value of employment for a worker of type $j$ on a job of type $i$ and $J(i, j)$ the value to the firm of filling a job of type $i$ with a worker of type $j$. Accordingly, the surplus of a match between a job of type $i$ and a worker of type $j$ can be expressed as $S(i, j) = W(i, j) + J(i, j) - V(i) - U(j)$ and, when a match is consummated, the wage $w(i, j)$ satisfies the following standard sharing rule\(^{14}\)

\[
(1 - \beta)[W(i, j) - U(j)] = \beta[J(i, j) - V(i)]. \tag{2}
\]

The above expression assumes that a worker’s threat point in the negotiations coincides with her outside option as an unemployed worker. Thus, an $h$-type worker cannot use employment in an unskilled job to bid up her wage in a skilled job.\(^{15}\)

\(^{14}\)We are grateful to Robert Shimer for pointing out that the equivalence between the Nash bargaining solution and (2) may break down in models with on-the-job search (see Shimer, 2003). The reason is that firms may try to reduce worker turnover by paying a higher wage. Here we ignore this possibility. Nonetheless, in our simulations we will restrict attention to parameter configurations for which $w(s, h) > y(n)$. A firm with an unskilled job therefore cannot match a wage offer from a firm with a skilled job without making a loss.

\(^{15}\)This is consistent with our assumption that there is no recall of wage offers.
3.4 Asset values

The model is conveniently summarised by the asset value equations of workers and firms. Let \( b \in [0, y(n)) \) denote the flow value of home production by unemployed workers. In steady state, the asset value of an \( h \)-type unemployed, \( U(h) \), then satisfies

\[
    rU(h) = b + \theta q(\theta) \left[ \theta \max(W(n,h) - U(h), 0) + (1 - \theta)(W(s,h) - U(h)) \right].
\]  

(3)

Similarly, the asset value of a mismatched worker, \( W(n,h) \), satisfies

\[
    rW(n,h) = w(n,h) + \delta[U(h) - W(n,h)] + \theta q(\theta)(1 - \theta)(W(s,h) - W(n,h)),
\]  

(4)

where the last term on the right-hand side corresponds to the expected gains from on-the-job search.

Combining the above expressions, it is easy to show that \( W(n,h) - U(h) \) satisfies:

\[
    W(n,h) - U(h) = \max \left\{ \frac{w(n,h) - b}{r + \delta + \theta q(\theta)}, 0 \right\} \tag{5}
\]

which is positive when \( w(n,h) > b \). Thus, in equilibrium, unemployed \( h \)-type workers will match with unskilled job whenever these jobs pay more than \( b \). In the next section we will show that this is always the case under our assumption that \( y(n) > b \).

The rest of the asset value equations of workers are standard:

\[
    rW(s,h) = w(s,h) + \delta[U(h) - W(s,h)] \tag{6}
\]

\[
    rW(n,l) = w(n,l) + \delta[U(l) - W(n,l)] \tag{7}
\]

\[
    rU(l) = b + \theta q(\theta)(W(n,l) - U(l)) \tag{8}
\]
Now consider firms. We assume that the cost of maintaining a vacant job during a period of unit length is equal to \( c \). The expected profit from opening an unskilled job, \( V(n) \), can then be written as follows:

\[
rV(n) = -c + \psi q(\theta) [\phi (J(n, l) - V(n)) + (1 - \phi) \max \{J(n, h) - V(n), 0\}] ,
\]

while the asset value of opening a skilled job, \( V(s) \), satisfies

\[
rV(s) = -c + q(\theta)(1 - \psi \phi)(J(s, h) - V(s), 0) .
\]

Similarly, the asset values of filled jobs satisfy

\[
rJ(n, l) = y(n) - w(n, l) + \delta[V(n) - J(n, l)]
\]

\[
rJ(s, h) = y(s) - w(s, h) + \delta[V(s) - J(s, h)]
\]

\[
rJ(n, h) = y(n) - w(n, h) + \delta[V(n) - J(n, h)] + \theta q(\theta)(1 - \vartheta)[(V(n) - J(n, h)].
\]

The last term on the right-hand side of (13) reflects the loss of profits when a \( h \)-type worker quits to a skilled job. In the next section we analyse how these quits affect the equilibrium allocation.

4 Equilibrium

We are now in a position to derive the equilibrium. For the moment we assume that \( h \)-type workers match with all jobs. At the end of Section 4.4 we will show that this is the only possible type of equilibrium when firms offer both types of jobs.\(^{16}\)

\(^{16}\)In this section we will also derive the necessary condition to rule out the corner solution in which \( \vartheta = 1 \).
Formally, a steady state equilibrium corresponds to a vector \( \{\theta, \vartheta, \phi, \psi, u\} \) that solves the pair of free entry conditions \( V(j) = 0, \ j = s, n \) and that satisfies the appropriate steady state conditions for \( u(l), u(h) \) and \( e(n, h) \):

\[
\partial q(\theta) \phi u = \delta (\mu - \phi u) \tag{14}
\]

\[
q(\theta) (1 - \phi) u = \delta [1 - \mu - (1 - \phi) u] \tag{15}
\]

\[
\partial q(\theta) (1 - \phi) u = [\delta + \theta q(\theta) (1 - \vartheta)] e(n, h), \tag{16}
\]

### 4.1 Analysis

A first step in the derivation of the equilibrium free entry conditions, is to derive the expressions for the surplus in the three types of employment relations.

Let us start with the case of \( l \)-type workers. Combining (8), (7), (9) and (11) and imposing the free-entry condition \( V(n) = 0 \), we can show that \( S(n, l) \equiv J(n, l) + W(n, l) - U(l) - V(n) \), satisfies

\[
(r + \delta) S(n, l) = y(n) - r U(l).
\]

Together with (2) this implies that \( w(n, l) \) is given by

\[
w(n, l) = r U(l) + \beta [y(n) - r U(l)]. \tag{17}
\]

Similarly, for \( h \)-type workers on skilled jobs we obtain

\[
(r + \delta) S(s, h) = y(s) - r U(h)
\]

\[
w(s, h) = r U(h) + \beta [y(s) - r U(h)], \tag{18}
\]
where we have used (2), (3), (6), (10) and (12) together with the free-entry condition \( V(s) = 0 \). The above wage expressions are standard (e.g. Pissarides, 2000). On the contrary, for mismatched workers we obtain:

\[
[r + \delta + \theta q(\theta)(1 - \vartheta)]S(n, h) = y(n) - rU(h) + \theta q(\theta)(1 - \vartheta)\beta S(s, h),
\]

while the associated wage \( w(n, h) \) satisfies

\[
w(n, h) = rU(h) + \beta[y(n) - rU(h)] - (1 - \beta)\theta q(\theta)(1 - \vartheta)\beta S(s, h). \tag{19}
\]

Equation (19) shows that mismatched workers earn less than a share \( \beta \) of the flow surplus \( y(n) - rU(h) \).\(^{17}\) The reason is that these workers retain the option of employment in a skilled job through on-the-job search. A firm with an unskilled job therefore does not have to compensate a mismatched worker for the expected gains from employment in the skilled segment of the labour market.

Next, substituting the three surplus expressions into (3) and (8) yields the expressions for workers’ outside options:

\[
rU(l) = \frac{(r + \delta)b + \theta q(\theta)\vartheta \beta y(n)}{r + \delta + \theta q(\theta)\vartheta \beta} \tag{20}
\]

\[
rU(h) = \frac{(r + \delta)\lambda_3 b + \theta q(\theta)\beta \left[\vartheta (r + \delta) y(n) + (1 - \vartheta) \lambda_2 y(s)\right]}{\lambda_1 \lambda_2} \tag{21}
\]

where \( \lambda_1 = r + \delta + \theta q(\theta)(1 - \vartheta)\beta \), \( \lambda_2 = r + \delta + \theta q(\theta)(1 - \vartheta + \vartheta \beta) \), and \( \lambda_3 = r + \delta + \theta q(\theta)(1 - \vartheta) \). Inspection of these equations shows that high-educated workers have a better higher outside option than less-educated workers when \( \vartheta < 1 \).

Finally, to complete the derivation of the equilibrium, we need to substitute the surplus expressions into the two free entry conditions:

\[
rV(n) = -c + \psi q(\theta)(1 - \beta) \left[\phi S(n, l) + (1 - \phi)S(n, h)\right] = 0,
\]

---
\(^{17}\)Pissarides (1994) obtained a similar result in a model with two types of jobs and homogenous workers. In his model on-the-job search takes place at short tenures. However, he does not provide a comparison of the wages of workers who are engaged in on-the-job search and those who are not.
\[ rV(s) = -c + q(\theta)(1 - \psi \phi)(1 - \beta) S(s, h) = 0. \]

Together with (20) and (21), this yields:

\begin{align*}
\frac{c}{q(\theta)} &= \psi(1 - \beta) \left[ \phi \frac{y(n) - b}{r + \delta + \theta q(\theta) \beta} + (1 - \phi) \frac{y(n) - b}{\lambda_2} \right], \quad (22) \\
\frac{c}{q(\theta)} &= (1 - \beta)(1 - \psi \phi) \left[ \frac{y(s) - b}{\lambda_1} - \theta q(\theta) \beta y(n) - b \right]. \quad (23)
\end{align*}

Conditions (14)-(16), (22) and (23) fully characterise the equilibrium allocations. The conditions for existence and uniqueness will be discussed in Section 4.5. But first we want to highlight two important implications stemming from the assumption of on-the-job search.

4.2 Implications of on-the-job search

Our first objective is to show that \( h \)-type workers never find it optimal to refuse an unskilled job. In particular, after substitution of (21) into the expression for \( S(n, h) \) we arrive at the following solution:

\[ S(n, h) = \frac{y(n) - b}{r + \delta + \theta q(\theta)(1 - \psi \phi)}, \quad (24) \]

which is completely independent of the productivity in skilled jobs. Thus, given our assumption that \( y(n) > b \), the match surplus \( S(n, h) \) is always strictly positive.

**Proposition 1** In equilibrium, all matches between \( h \)-type workers and unskilled jobs are consummated.

**Proof.** The positive value of \( S(n, h) \) implies that \( W(n, h) - U(h) = \beta S(n, h) > 0 \) and \( J(n, h) - V(n) = (1 - \beta) S(n, h) > 0 \). Thus, match formation is mutually beneficial. □

The above result validates our claim that high-educated workers will never refuse unskilled jobs. The introduction of costless on-the-job search therefore rules out equilibria with ex-post segmentation.
A second implication of on-the-job search is that firms with unskilled jobs prefer less-educated applicants. These less-educated workers have the same productivity as high-educated applicants, but on average they remain with the firm for a longer period. This preference for less-educated workers follows immediately from a comparison between $S(n, h)$ and $S(n, l)$, where the latter is given by:

$$S(n, l) = \frac{y(n) - b}{r + \delta + \theta q(\theta) \beta \vartheta}.$$  

This shows that $S(n, l) > S(n, h)$ because firms discount the flow profit from a match with a mismatched worker at a higher rate than the flow profit from a match with a less-educated worker.

The third implication is less evident. Namely, in equilibrium mismatched workers earn less than less-educated workers.

**Proposition 2** In any equilibrium with $\vartheta < 1$, it must be true that $w(n, h) < w(n, l) < w(s, h)$.

**Proof.** The second inequality follows immediately from the assumption that $y(s) > y(n)$, so that $U(h) > U(l)$ and $w(s, h) > w(n, l)$ if $\vartheta < 1$. For the first inequality we rewrite $U(h)$ as:

$$rU(h) = b + \theta q(\theta) \beta [\vartheta S(n, h) + (1 - \vartheta)S(s, h)].$$  

Substituting this expression into (19) shows that $w(n, h)$ reduces to:

$$w(n, h) = b + \beta [y(n) - b] + \theta q(\theta) \beta \vartheta S(n, h).$$  

Similarly, $U(l)$ satisfies:

$$rU(l) = b + \theta q(\theta) \beta \vartheta S(n, l),$$

which together with (17) implies that:
\[ w(n,l) = b + \beta[y(n) - b] + \theta q(\theta) \beta \theta S(n,l). \] (26)

Comparing (25) and (26) immediately shows that \( w(n,h) < w(n,l) \) because \( S(n,h) < S(n,l) \).

Proposition 2 contrasts with the predictions of models without job-to-job flows. In those models high-educated workers have to sacrifice the option of employment in a skilled job if they decide to accept an unskilled job. Firms therefore have to compensate the high-educated workers for their entire opportunity cost \( U(h) \) and since \( U(h) \) is bigger than \( U(l) \), this would mean that mismatched workers would earn more than less-educated workers (for details see AV).

The above argument shows that on-the-job search raises the degree of within-group wage inequality among high-educated workers for given aggregate labour market conditions. Nonetheless, it should be recognised that this result is partly due to our assumption that on-the-job search is costless. Imagine for example that the arrival rate of job offers to employed workers is equal to \( \lambda \theta q(\theta) \) where \( 0 \leq \lambda \leq 1 \). The extreme case in which \( \lambda = 1 \) would correspond to costless on-the-job search, while \( \lambda = 0 \) would correspond to the model of AV with infinite costs of on-the-job search. In between of these extremes we may therefore expect that there exists a value \( \lambda^* \) such that for any \( \lambda \leq \lambda^* \) firms would need to pay mismatched workers at least the same as less-educated workers.\(^{18}\)

4.3 Existence and uniqueness

Our results showed that there exists only one type of equilibrium when firms offer both types of jobs. In this section we will derive the conditions that guarantee the existence and uniqueness of this equilibrium.

\(^{18}\)For an example in which employed workers have a lower arrival rate of job offers than unemployed job seekers, see Moscarini (2003).
4.3.1 Existence

To ensure existence of an equilibrium with \( \vartheta < 1 \), we need to rule out the possibility of a corner solution in which no firm is willing to open a skilled job. Starting from an outcome in which all firms create unskilled jobs and \( V(n) = 0 \), we therefore need to ensure that \( V(s) \) is positive.

Formally, let \( \theta^* \) denote the labour market tightness when \( \vartheta = 1 \) and \( V(n) = 0 \). Given the reasonable assumption that employed workers only search on-the-job when \( \vartheta < 1 \), we find that \( V(s) \) is positive when:

\[
y(s) - b > \left[ 1 + \frac{\mu(r + \delta)}{(1 - \mu)(r + \delta + \beta\theta^*q(\theta^*))} \right] (y(n) - b).
\]

(27)

According to equation (27), existence requires that skilled jobs are more productive since they have a lower matching rate. Furthermore, this required productivity differential increases with \( \mu \).

4.3.2 Uniqueness

To establish uniqueness, we solve the steady state conditions for \( \vartheta, u \) and \( \psi \) in terms of \( \theta \) and \( \phi \). Substituting these solutions into free entry conditions (22) and (23) yields a system of two equations in two unknowns that can easily be solved.

First, solving (14) and (15) for \( \vartheta \) and \( u \) yields:

\[
u(\theta, \phi) = \frac{\delta}{\delta + \theta q(\theta)} \frac{1 - \mu}{1 - \phi},
\]

(28)

\[
\vartheta(\theta, \phi) = \frac{(1 - \phi)\theta q(\theta)\mu + \delta(\mu - \phi)}{\theta q(\theta)\phi(1 - \mu)},
\]

(29)

with \( \partial \vartheta(\cdot)/\partial \phi < 0 \) and \( \partial \vartheta(\cdot)/\partial \theta > 0 \) (because \( \phi > \mu \)).\(^{20}\) Next, the solution for the

\(^{19}\)This is identical to one of the existence conditions in AV.

\(^{20}\)Less-educated workers are over-represented in the pool of unemployment since their exit rate out of unemployment is lower than the exit rate of high-educated workers.
share of unemployed job seekers $\psi$ follows from (16). Since the ratio $e(n,h)/u$ is equal to $(1 - \psi)/\psi$, this yields:

$$\psi(\theta, \phi) = \left[ 1 + \frac{\vartheta(\theta, \phi)(1 - \phi)q(\theta)}{\delta + (1 - \vartheta(\theta, \phi))q(\theta)} \right]^{-1}$$

(30)

with $\partial \psi(\cdot)/\partial \theta < 0$ and $\partial \psi(\cdot)/\partial \phi > 0$.

The above results imply that the matching rate of unskilled jobs, $\psi(\cdot)q(\theta)$, is decreasing in $\theta$ for given values of $\phi$. In contrast, for the matching rate of skilled jobs $(1 - \psi\phi)q(\theta)$ we obtain two opposite effects. On the one hand, we have the congestion effect captured by the negative sign of $\partial q(\theta)/\partial \theta$. But, on the other hand, there is a composition effect of opposite sign captured by $\partial \psi(\cdot)/\partial \theta < 0$ as the proportion of high-educated workers looking for a skilled job increases. Nonetheless, it can be shown that the congestion effect dominates the composition effect for sufficiently high values of $\mu$. In what follows, we shall therefore assume that $(1 - \psi\phi)q(\theta)$ is decreasing in $\theta$ for given values of $\phi$.$^{21}$

\[\text{Figure 3: Uniqueness}\]

$^{21}$Compared to a standard matching model, this is the only additional restriction that we impose on the matching technology. A sufficient condition for $\partial(1 - \psi\phi)q(\theta)/\partial \theta < 0$ is cumbersome to obtain, but in our numerical simulations we find that this derivative is always negative for values of $\mu \geq 0.5$. 

23
Under this mild assumption we can show that the equilibrium is unique if: (a) there is a majority of unskilled workers \( \mu \geq 0.5 \), (b) workers obtain at least half the surplus of a match \( \beta \geq 0.5 \), and (c) the productivity of skilled jobs, \( y(s) \), is sufficiently large (for details, see Appendix).

This case is illustrated in Figure 3. The free entry curve for unskilled jobs has a positive slope in the plane \( (\theta, \phi) \). As the labour market tightness increases, firms find it more difficult to locate an applicant. To restore zero profits the share of less-educated workers among the unemployed therefore needs to increase. Similarly, for skilled jobs the lower matching rate needs to be compensated by a higher share of high-educated job seekers and so by a lower value of \( \phi \). As a result, the free entry curve for skilled jobs has a negative slope and the two free entry curves intersect at most once.

In the next section we investigate how this unique equilibrium responds to changes in \( y(s) \) and \( \mu \).

5 Comparative statics

In our economy job-to-job flows make the profits of skilled jobs more responsive to changes in the skill distribution. The reason is that high-educated workers stay in the pool of job seekers when they accept an unskilled job. At the same time, however, these flows also insulate the profits of unskilled jobs from changes in \( y(s) \) (see equation 22).

In this section we want to explore how these two features affect the response of the equilibrium to an increase in the productivity of skilled jobs and/or an increase in the supply of high-educated workers. Below we refer to these two changes as “skill-biased technological change” (SBTC) and “skill upgrading” (SU), respectively.

5.1 Skill-biased technological change

The effects of SBTC are illustrated in Figure 4. Due to the increase in \( y(s) \) firms are willing to open more skilled jobs for any given value of \( \phi \). As a result, the \( V(s) = 0 \) locus shifts to the right along the \( V(n) = 0 \) locus, giving rise to an increase in \( \theta \) and \( \phi \).
The increase in the labour market tightness $\theta$ reduces the unemployment rate of high-educated workers, $\bar{u}(h) = \delta/(\delta + \theta q(\theta))$, while we cannot draw any conclusion about the unemployment rate of low-educated workers, $\bar{u}(l) = \phi u/\mu = \delta/(\delta + \vartheta \theta q(\theta))$ since the rise in $\theta$ can be offset by a reduction in $\vartheta$ (recall that $\partial \theta(.,)/\partial \phi < 0$). Nonetheless, in most cases we expect an increase in $\bar{u}(l)$ since the increase in the mass of skilled jobs makes it more difficult for these workers to find an unskilled job and, furthermore, these additional jobs congest the market for firms with unskilled jobs. In sum,

**Proposition 3** *In a unique equilibrium, SBTC reduces $\bar{u}(h)$ and $1-\psi$ while the effect on $\bar{u}(l)$ is ambiguous.*

Proposition 3 contrasts with the predictions of AV. In their model SBTC reduces the employment opportunities of less-educated workers while it has no effect on the unemployment rate of high-educated workers. The job-to-jobs flows of high-educated workers therefore increase the sensitivity of $\bar{u}(h)$ to changes in $y(s).^{22}$

---

22This result is unrelated to our assumption of a common value for $b$ and $c$. In particular, we would obtain the same result if the unemployment income of high-educated workers and the flow cost of skilled vacancies are indexed to $y(s)$. In our model, technological change is only neutral when
5.2 Skill-upgrading

In a frictional labour market we may expect that SU produces a similar effect on job creation as SBTC. The higher probability of meeting a high-educated worker improves the profitability of skilled jobs while the expected profits of unskilled jobs go down. The main difference is therefore on the supply side. In the case of SBTC the supply of labour was unchanged, so that the demand shift unambiguously reduced $\tilde{u}(h)$. By contrast, in the case of SU, the overall effect will depend on the relative size of the demand and supply shift. If the demand shift exceeds the supply shift, $\tilde{u}(h)$ will go down. Otherwise, the unemployment rate of high-educated workers $\tilde{u}(h)$ will go up. Furthermore, even if $\tilde{u}(h)$ goes down, there may still be a higher mass of high-educated job seekers. The latter effect tends to reduce job stability in unskilled jobs and this worsen the labour market position of the low-educated workers even more.

The ambiguous response of the labour market is illustrated in Figure 5. A reduction in $\mu$ shifts both free entry schedules downwards. Hence we obtain a reduction in $\phi$, while $\theta$ may go up or down. Despite these ambiguous effects on $\tilde{u}(l)$ and $\tilde{u}(h)$, however, $y(s)$, $y(n)$, $b$ and $c$ all grow at the same rate. A shock to the relative productivity of workers can therefore move the equilibrium to a different balanced growth rate in which the unemployment rate of high-educated workers is permanently lower than before.

![Figure 5: The effects of skill-upgrading](image-url)
our simulations below show that if $\mu \geq 0.5$, then the demand shift will dominate the supply shift. Hence, as depicted in Figure 5, the shift to the right of the $V(n) = 0$ locus will exceed the shift to the left of the $V(s) = 0$ locus, leading to an increase in $\theta$ in addition to the fall in $\phi$. The increase in $\theta$ implies a reduction of $\tilde{u}(h)$. In other words, an increase in the cohort-size of high-educated workers tends to improve their job finding rate, an effect which echoes the result by Shimer (2001) about the lower unemployment rates of the baby boom generation. Likewise, when $\mu \geq 0.5$, the share of mismatched workers in the mass of job seekers raises, deteriorating therefore the prospects of firms opening unskilled vacancies and leading to a rise in $\tilde{u}(l)$. The novel positive cohort-size effect for high-educated workers is again absent in AV, who also obtain an increase in $\tilde{u}(l)$ but no effect on $\tilde{u}(h)$.

6 Numerical solutions

To complete the analysis we consider a simple numerical example. The aim is to provide an estimate for the effects of SBTC and SU for reasonable values of the parameters. Furthermore, for the same parameter values we also compute the outcome without on-the-job search by setting $e(n,h) = 0$ and $\psi = 1$. This will allow us to compare our results to AV and isolate the impact of job-to-job flows.

The model is calibrated using a standard Cobb-Douglas matching function, $m = [v(n) + v(s)]^{1-\alpha}[u(l) + u(h) + e(n,h)]^\alpha$, together with the following parameter configuration: $\alpha = \beta = 0.5$, $r = 0.03$, $c = 0.5$, $y(n) = 1$, $y(s) = 1.5$, $\delta = 0.1$, $b = 0.1$, and $\mu = 0.75$. Table 2 reports the labour market outcomes in our benchmark economy.

The parameter configuration corresponds closely to the parameter choice in AV. The assumption of a linear homogeneous matching function and a matching elasticity of 0.5 are consistent with the empirical evidence presented in Petrongolo and Pissarides (2001). Furthermore, using U.S. data, Blanchard and Diamond (1989) and Shimer (2004) cannot reject the assumption of a Cobb-Douglas matching technology. On the contrary, the assumption that $\alpha = \beta$ is made for convenience. This so-called Hosios’ condition is widely used in the matching literature. The remaining parameters are similar to those in Mortensen and Pissarides (1999). Finally, the choice of $\mu = .75$ is close to the average share of individuals with at most a high-school degree in the US and in some countries of the
Table 2. Equilibrium values in Baseline Model

<table>
<thead>
<tr>
<th>θ</th>
<th>ϑ</th>
<th>φ</th>
<th>ψ</th>
<th>u</th>
<th>̅u(l)</th>
<th>̅u(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.435</td>
<td>0.674</td>
<td>0.810</td>
<td>0.763</td>
<td>0.102</td>
<td>0.111</td>
<td>0.078</td>
</tr>
</tbody>
</table>

The labour market tightness of 1.435 corresponds to an average unemployment duration of 10 months, in line with the average duration of unemployment in some European countries like Spain (see Bentolila and Jimeno, 2003). The latter corresponds to an average unemployment rate of 10.2% while the unemployment rates for high and low-educated workers are equal to 7.8% and 11.1%. These values seem reasonably in line with the EU average during the last fifteen years. Furthermore, in the baseline economy the share of high-educated job seekers, \(1 - \psi\phi\), is equal to 38.3%. This exceeds the baseline value of \(1 - \mu\) by more than 50%. High-educated workers are therefore strongly over-represented in the pool of job seekers. Finally, in our benchmark economy skilled jobs pay a wage \(w(s, h) = 1.36\) while \(w(n, l)\) and \(w(n, h)\) are 0.86 and 0.78, respectively, in line with our predictions in Proposition 2 and the evidence presented in subsection 2.2.

The above wages give rise to a considerable amount of wage inequality. As a proxy for the degree of between-group wage inequality we report the ratio between the average wage of high-educated workers and the wage of less-educated workers. Analogously, the within-education wage inequality is measured by the ratio between the average wage of high-educated workers and their wage on unskilled jobs. Finally, since these proxies do not control for the relative size of the two groups, we also decompose the overall variance of the wage distribution in a permanent component due to between-group wage differences and a transitory component due to within-group wage differences.

In our benchmark economy, the proxies for the between and within-group wage inequality are equal to 1.37 and 1.63, respectively, while the within-group wage differences among the high-educated accounts for roughly 30% of the overall variance in wages.

EU.
Figure 6 illustrates the effects of SBTC. The latter is captured by a gradual increase in $y(s)$ from 1.5 to 2.0. The solid lines describe the outcomes with on-the-job search, while the dashed lines represent the sequence of steady state equilibria without on-the-job search, as in AV. Inspection of the solid lines shows that $\bar{u}(h)$ falls by 1.2 percentage points, while $\bar{u}(l)$ goes up by 1.0 percentage points. Thus, the changes in unemployment are small compared to the changes in relative productivity. On the contrary, the share of high-educated job seekers drops by as much as 6.2 percentage points while the degree of between and within-group wage inequality rise by, respectively, 35% and 39.6%. Hence, the bulk of the adjustment takes place via a change in wages and an increase in the mass of skilled jobs.²⁴

More striking results are obtained when we compare these results to the outcomes without on-the-job search. In this case, the equilibrium switches to ex-post segmentation when $y(s)$ reaches a value 1.78. As a result, the unemployment rate of high-

²⁴In our experiment $w(n,l)$ remains virtually unchanged while $w(s,h)$ increases by 34%.
educated workers jumps to 15.8%, well above the unemployment rate of l-type workers. Obviously, this is not too realistic. Furthermore, the same is true for the dramatic increase in the between-group wage inequality. A strong argument in favour of our model is therefore that it generates a more realistic adjustment to shocks.

Another important difference is the evolution of \( \tilde{u}(l) \). In our baseline model we obtain an unemployment rate of 11.2% for low-educated workers, compared to an unemployment rate of 7.8% without on-the-job search. The explanation was given above. Since mismatched workers are part of the mass of job seekers, firms have a much stronger incentive to open skilled jobs. Equilibrium matching models that abstract from on-the-job search are therefore likely to underestimate the negative effects of skill mismatch for low-educated workers.

Finally, as should be expected, our model produces much more wage inequality than a model without job-to-job flows. Moreover, a large share of this wage inequality is due to the increased earnings-instability of high-educated workers.\textsuperscript{25} The latter is illustrated in Table 3 where we report the total variance of wages and the (unweighted) variance for high-educated workers.

<table>
<thead>
<tr>
<th>( y(s) )</th>
<th>Total ( h )-workers</th>
<th>Total ( h )-workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>0.0303 0.0363</td>
<td>0.00048 0.0018</td>
</tr>
<tr>
<td>1.65</td>
<td>0.0528 0.0514</td>
<td>0.00602 0.0146</td>
</tr>
<tr>
<td>1.80</td>
<td>0.0821 0.0904</td>
<td>0.00247 0.0000</td>
</tr>
</tbody>
</table>

In Figure 7 we report similar findings for the case of SU. In this experiment we gradually increase the share of high-educated workers from 25% to 50% of the population. The numerical results reveal a growing gap between the unemployment rates of high- and low-educated workers along with a widening of the between- and within group variance

\textsuperscript{25}For the contribution of earnings instability to the growth in wage inequality in the U.S., see Gottschalk and Moffit (1994).
wage inequality. The only qualitative difference with the case of SBTC is the evolution of the share of high-educated job seekers. While SBTC led to a reduction in this share, we now observe a strong increase from 34% to 49%. Nonetheless, the increase in the cohort-size of h-type workers gives rise to a small reduction in the unemployment rate of these workers. As in Shimer (2001), our model therefore exhibits cohort-size effects. In particular, the introduction of on-the-job search make labour demand so responsive to supply shifts that high-educated workers face a lower risk of unemployment if their share in the population increases.

Figure 7: The effects of skill upgrading

Following AV all the above results are generated under the assumption of common values for \( b, c \) and \( \delta \). However, in an earlier draft of this paper (see Dolado et al., 2003) we showed that our predictions about the effects of SBTC on \( \hat{u}(l) \) may be sensitive to the assumptions about common separation rates. In particular, with a lower separation rate on skilled jobs, SBTC is less harmful for the less-educated workers since it will draw high-educated workers into more stable jobs, thereby inducing a strong reduction
in the mass of high-educated job seekers. Furthermore, if this is combined with a relatively high cost of skilled vacancies, so that they are initially scarce and SBTC increases their supply sharply, this may even result in a small reduction in $\tilde{u}(l)$. For example, if we consider the case where the cost of opening a skilled vacancy is raised from 0.5 to 1.0, and its separation rate is reduced from 0.10 to 0.075, SBTC (i.e., an increase in $y(s)$ from 1.5 to 2.0) gives rise to a fall in $\tilde{u}(l)$ of about half-a-percentage point. This last observation seems to point at the need for targeted labour market policies that foster the stability of employment in skilled jobs when on-the-job search is a relevant phenomenon.\footnote{A careful analysis of this type of policies is beyond the scope of this paper, but in a companion paper (see Dolado et al., 2004) we deal with the effects of dual employment protection in a model with ex-ante heterogeneous workers.}

\section{Conclusions}

In this paper we consider a simple random matching model with heterogeneous workers and jobs. The novel element is that we allow for on-the-job search by mismatched workers. In contrast to most of the existing literature, skill mismatch therefore has a transitory rather than a permanent nature. We show that our model can account for some of the stylised facts of job-to-job flows, in particular in countries that have recently undergone a large upgrading of their system of tertiary education. Furthermore, the introduction of job-to-job transitions has important implications for the distribution of wages, employment and the economy’s response to shocks.

Two results stand out. First of all, we show that the demand for high-educated workers is much more sensitive to shifts in relative productivity or in the distribution of skills. This aggravates the crowding-out of low-educated workers whose unemployment rate raises, while the unemployment rate of high-educated workers falls in response to skill-biased technological change (skill upgrading). In addition, the higher quit rate of mismatched workers reduces the profits of unskilled jobs which imposes a further negative externality on the low-educated workers. Thus, models that abstract from re-
alistic job-to-job transitions by mismatched workers tend to underestimate the negative effects of over-education for low-educated workers.

Second, our model generates substantially more wage inequality than a model without job-to-job transitions. Furthermore, we show our prediction that mismatched workers should receive a lower wage than appropriately matched workers with the same productivity receives some support in the data. On-the-job search therefore seems to offer a plausible channel to explain the widening of within-group wage inequality and the increase in the skill premium which has been observed in many OECD countries.

The model could be extended in a number of ways. One extension would be to endogenise the skill distribution by allowing workers to invest in education. A second extension would be to consider a model of directed search. In that environment workers can target their search to different types of jobs but nonetheless in some cases high-educated may consciously decide to apply for unskilled jobs. Finally, one could consider the possibility of allowing for multiple meetings so that the model can address issues of ranking of applicants by firms.
8 Appendix : Proofs

**Proof of Uniqueness**

Substituting \( \vartheta = \vartheta(\theta, \phi; \mu) \) and \( \psi = \psi(\theta, \phi; \mu) \) from equations (29) and (30) into the free-entry conditions yields two equations in two unknowns, \( \theta \) and \( \phi \), denoted in implicit form by \( V_S(\theta, \phi) = 0 \) and \( V_N(\theta, \phi) = 0 \).

**Skilled jobs:** To show that the \( V_S(\theta, \phi) = 0 \) locus has a negative slope, it is sufficient to show that

\[
\frac{d\phi}{d\theta} = -\frac{\partial V_S/\partial \theta}{\partial V_S/\partial \phi} < 0.
\]

Taking the derivative of \( V_S(\theta, \phi) \) with respect to \( \theta \), dividing all terms by \( \Delta = (1 - \beta)(y(n) - b) \) and denoting the ratio \( \frac{y(s) - b}{y(n) - b} \) by \( R \), yields

\[
\frac{1}{\Delta} \frac{\partial V_S}{\partial \theta} = \frac{q'(\theta)}{\lambda_1} (1 - \psi \phi) \left[ R - \frac{\beta \partial q(\theta)}{\lambda_2} \right] - \frac{q(\theta)}{[\lambda_1] \Delta} (1 - \psi \phi) \left[ R - \frac{\beta \partial q(\theta)}{\lambda_2} \right] \frac{\partial \lambda_1}{\partial \theta} - \frac{q(\theta)}{\lambda_1} (1 - \phi) \beta \frac{\partial q(\theta)}{\lambda_2} \frac{\partial \lambda_2}{\partial \theta} + \frac{q(\theta)}{\lambda_1} R - \frac{\beta \partial q(\theta)}{\lambda_2} \frac{\partial (1 - \psi \phi)}{\partial \theta}.
\]

Since \( q'(\theta) < 0 \), \( \frac{\partial \lambda_1}{\partial \theta} > 0 \), and \( \frac{\partial q(\theta)}{\partial \theta} > 0 \) the first three terms of the above expression are negative whereas the last two terms, given that \( \frac{\partial \lambda_2}{\partial \theta} > 0 \), \( \frac{\partial (1 - \psi \phi)}{\partial \theta} > 0 \), are positive. Nonetheless, combining the the third and fourth terms yield a negative term whilst our assumption that \( \frac{\partial (1 - \psi \phi) q(\theta)}{\partial \theta} < 0 \), ensures that a combination of the first and fifth terms is also negative. Thus, \( \partial V_S/\partial \theta < 0 \).

The corresponding expression for \( \partial V_S(\theta, \phi)/\partial \phi \) is:
\[
\frac{1}{\Delta} \frac{\partial V_S}{\partial \phi} = -\frac{q(\theta)}{[\lambda_1]^2} (1 - \psi \phi) \left[ R - \frac{\beta \partial q(\theta)}{\lambda_2} \right] \frac{\partial \lambda_1}{\partial \phi} \\
+ \frac{q(\theta)}{[\lambda_1]^2} \left[ R - \frac{\beta \partial q(\theta)}{\lambda_2} \right] \cdot \frac{\partial (1 - \psi \phi)}{\partial \phi} \\
+ \frac{q(\theta) (1 - \psi \phi)}{[\lambda_2]^2} \cdot \beta \partial q(\theta) \frac{\partial \lambda_2}{\partial \phi} \\
- \frac{\beta q(\theta) \partial q(\theta)}{\lambda_1 \lambda_2} \frac{\partial \lambda_2}{\partial \phi}
\]

The first two terms are negative since \( \frac{\partial \lambda_1}{\partial \phi} > 0 \) and \( \frac{\partial (1 - \psi \phi)}{\partial \phi} < 0 \) whereas the last two terms are positive since \( \frac{\partial \lambda_2}{\partial \phi} > 0 \) and \( \frac{\partial q(\theta) \partial \lambda_2}{\partial \phi} < 0 \). However, if \( R \) is sufficiently large, then the negative sign of the first two terms will dominate. Thus, \( \partial V_S / \partial \phi < 0 \) when the productivity differential \( \{ y(s) - y(n) \geq y^* \} \), namely, a threshold value such that, in absolute value, the sum of the first two terms exceeds the remaining two terms (which do not depend on \( R \)) in the above derivative.

**Unskilled jobs:** To show that the \( V_N(\theta, \phi) = 0 \) locus has a positive slope, it is sufficient to show that

\[
\frac{d\phi}{d\theta} = -\frac{\partial V_N / \partial \theta}{\partial V_N / \partial \phi} > 0.
\]

As before, taking the derivative of \( V_N(\theta, \phi) \) with respect to \( \theta \) and dividing all terms by \( \Delta = (1 - \beta)(y(n) - b) \) yields

\[
\frac{1}{\Delta} \frac{\partial V_N}{\partial \theta} = q'(\theta) \psi \left[ \frac{\phi}{r + \delta + \theta q(\theta) \psi} + (1 - \phi) \frac{1 - \phi}{\lambda_2} \right] \\
+ \left[ \frac{q(\theta) \phi}{r + \delta + \theta q(\theta) \psi} + \frac{q(\theta) (1 - \phi)}{\lambda_2} \right] \frac{\partial \psi}{\partial \theta} \\
- \left[ \frac{\beta q(\theta) \psi \phi}{r + \delta + \theta q(\theta) \psi} \right] \cdot \frac{\partial \theta q(\theta)}{\partial \theta} + \frac{\psi \theta q(\theta) (1 - \phi)}{[\lambda_2]^2} \cdot \frac{\partial \lambda_2}{\partial \theta},
\]

which is unambiguously negative since \( q'(\theta) < 0 \), \( \frac{\partial \psi}{\partial \theta} < 0 \) and \( \frac{\partial \lambda_2}{\partial \theta} < 0 \). Similarly, the derivative \( \partial V_N(\theta, \phi) / \partial \phi \) is given by
\[
\frac{1}{\Delta} \frac{\partial V_N}{\partial \theta} = \left[ \frac{q(\theta)}{r + \delta + \theta q(\theta) \partial \beta} - \frac{q(\theta)}{\lambda_2} \right] \cdot \frac{\partial \psi \phi}{\partial \phi} \\
\quad - \frac{\beta \psi \phi q(\theta)}{[r + \delta + \theta q(\theta) \partial \beta]^2} \cdot \frac{\partial \theta q(\theta)}{\partial \phi} \\
\quad + \frac{q(\theta)}{\lambda_2} \cdot \frac{\partial \psi}{\partial \phi} - \frac{q(\theta) \psi (1 - \phi)}{[\lambda_2]^2} \cdot \frac{\partial \lambda_2}{\partial \phi},
\]

where the first three terms are positive since \(\frac{\partial \psi \phi}{\partial \phi} > 0\), \(\frac{\partial \theta q(\theta)}{\partial \phi} < 0\), \(\frac{\partial \psi}{\partial \phi} > 0\) and \(\lambda_2 > r + \delta + \theta q(\theta) \partial \beta\) whilst the last term is negative since \(\frac{\partial \lambda_2}{\partial \phi} > 0\). However, if \(\beta \geq 0.5\) and \(\mu \geq 0.5\), so that \(\phi \geq 0.5\), the second term dominates the fourth term in absolute value. Therefore \(\partial V_N / \partial \phi > 0\).
References


