How Important is Intra-household Risk Sharing for Savings and Labor Supply?*

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Abstract

While it is recognized that the family is a risk-sharing institution, little is known about the quantitative effects of this source of insurance on savings and labor supply. In this paper, we present a model where workers (females and males) are subject to idiosyncratic employment risk and where capital markets are incomplete. A household is formed by a female and a male, who decide on consumption, savings and labor supplies. In a calibrated version of our model we find that intra-household risk sharing has its largest impact among wealth-poor households. While the wealth-rich use mainly savings to smooth consumption across unemployment spells, wealth-poor households rely on spousal labor supply. For instance, for low-wealth households, average hours worked by wives of unemployed husbands are 8% higher than those worked by wives of employed husbands. This response in wives' hours makes up 9% of lost family income. We also study the crowding out effects of the public unemployment insurance program on the extent of risk sharing within the household, and on consumption losses upon an unemployment spell.

Keywords: Intra-household risk sharing; Multi-person households; Idiosyncratic unemployment risk; Incomplete markets; Precautionary motive.

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1 Introduction

There is a vast literature studying workers' precautionary responses to employment risk under incomplete asset markets. Understanding these responses is important for assessing both the aggregate implications of this type of risk and public insurance policies. For instance, recent empirical evidence on patterns of insurance against employment risk has shed light on how some sources of insurance are crowded out by public insurance. More specifically, Cullen and Gruber (2000) and Engen and Gruber (2001) use a panel of U.S. households to estimate the extent to which two sources of insurance —the accumulation of financial assets and spousal labor supply— respond to changes in the generosity of public unemployment insurance (UI). They find significant crowding out effects on both, a finding that is of paramount importance for public policy evaluation.[1]

The workhorse model of idiosyncratic risk under incomplete markets, the Aiyagari-Huggett setup, assumes, however, the *bachelor household* formulation, where a single breadwinner partially insures against employment risk by accumulating precautionary wealth when working.[2] This model is thus silent about risk sharing within the household, an insurance arrangement recently emphasized in the empirical literature (see, e.g., Blundell *et al.* 2008 and Shore 2010).

In this paper we introduce within-household risk sharing into the Aiyagari-Huggett model and assess the extent and effects of spousal labor supply as insurance against unemployment shocks. In our model economy, households are made up by two workers who pool risks and make decisions on individual consumptions, labor supplies and joint savings. Risk sharing within the two-person household is assumed to be efficient. There is a firms sector producing a homogeneous good with capital and labor services. Finally, there is a government collecting labor income taxes and paying UI benefits to unemployed workers.

To answer our question on the insurance role of spousal labor supply, we use the model just outlined to conduct a number of simulations. We first focus on wealth accumulation and compute the elasticity of the average household's assets-to-income ratio with respect to the generosity of UI. This elasticity provides us with a measure of both the precautionary saving motive and the crowding-out effect of UI on household savings, a measure that can be compared to recent empirical estimates. Then, we compute the added worker effect, namely, the response of wives' labor supply to husbands' unemployment spells, and the extent to which it is crowded out by UI. We assess both the change in hours of work by females brought about by an unemployment spell of their husbands, as well as the fraction of lost family income made up by this change in hours worked. Next, we look at the role of within-household risk sharing in shaping consumption losses upon an

[1] See, e.g., Attanasio and Rios-Rull (2000), Golosov and Tsyvinski (2007) and Chetty and Saez (2010) for analyses on the optimal level of social insurance when other forms of private insurance are also available.

[2] see Flodén and Lindé (2001), Marcet *et al.* (2007) and Pijoan-Mas (2006), among others.

unemployment spell, a question that has also received attention in the empirical literature. All these effects of within-household risk sharing on spousal labor supply and consumption losses are computed for households with different levels of wealth. Since risk sharing has its largest impact among low-wealth households, special attention is devoted to the group of households with asset holdings below two months of average income. This is the group that Zeldes (1989) termed as liquidity-constrained households, and that represented almost 20% of all U.S. households in 2001. Finally, and in order to put our findings in perspective, we compare the results of our simulations with those obtained in the workhorse model with bachelor households.

Our main findings show that within-household risk sharing has sizable effects. First, our two-person household model generates less precautionary savings than the bachelor model, and comes closer to replicating empirical estimates of the elasticity of the average household's assets-to-income ratio with respect to UI. Engen and Gruber (2001) estimate this elasticity to be -0.28. When calibrated to match key average values for the US economy, our two-person household model yields a value for this elasticity equal to -0.39, while the bachelor household model yields -0.70. This finding suggests that abstracting from risk sharing at the level of the household introduces an important bias in this elasticity. Second, we find that spousal labor supply is a significant source of insurance for wealth-poor households, while it is not for the wealth-rich. For instance, among liquidity-constrained households average hours worked by wives of unemployed husbands are 8.6% higher than those worked by wives of employed husbands. This response in wives' hours makes up on average 9.6% of lost family income. Third, the implications of family insurance for consumption losses upon unemployment are also sizable, especially for low-wealth households. We find that the average liquidity-constrained male lacking insurance from the family (bachelor household) suffers a drop in consumption of 30% upon a job loss, while the same male suffers a drop of only 8% when given access to family insurance (two-person household). When we compute the fraction of household income loss that transmits to household consumption loss for liquidity-constrained households, we find 35% under no family insurance, and 17% with family insurance.

We include two applications and one extension of our model. First, we compute the value of intra-household risk sharing. As suggested by our results above, this value is higher for individuals in low-wealth households. For example, the value of family insurance to an unemployed individual with an employed spouse and no assets represents more than 5% of per period consumption of a similar individual with no family insurance. This value decreases with the level of wealth. Second, we study the consequences of family insurance for optimal UI and find that it is one of its key determinants. We compute optimal UI for households with different wealth levels with and without intra-household risk sharing, and find that family insurance creates a wedge in optimal replacement rates that is decreasing in the level of wealth. For instance, the optimal replacement rate for the average two-person household with no assets is 15%, while this rate is 60% for a similar household lacking family insurance. This wedge closes at wealth levels equal to six months of average income. As an extension, we introduce marital shocks in our benchmark model so that married and single households co-exist. Within this framework we study the robustness of our results on intra-household risk sharing, the implications of marital risk on households' savings, and compute consumption losses upon employment and/or marital shocks.

The literature on uninsurable idiosyncratic income risk with the bachelor household formulation is too large to summarize here. A recent example is the paper by Low *et al.* (2008), where individuals (they focus only on males) are subject to a rich array of idiosyncratic shocks differing in their available insurance opportunities. Kotlikoff and Spivak (1981) is one of the first papers in economics to study the role of the family as provider of insurance. They focus on longevity risk and show that within-household risk sharing closes much of the utility gap between no annuities and complete annuities. A more recent exception to the use of the bachelor household formulation is the work of Attanasio *et al.* (2005), who present a partial equilibrium, two-person household model to assess the response of female labor market participation (extensive margin) to idiosyncratic earnings risk within the family. Heathcote *et al.* (2010) use a life-cycle, two-person household model to study the welfare implications of recent changes in the U.S. wage structure, namely, the rising college premium, the narrowing wage gender gap and the increasing wage volatility. They find that, on average, recent cohorts of households enjoy welfare gains, as the new structure of wages translates into higher educational attainment. Heathcote *et al.* (2009) study the rise in women's participation within a two-person household model with household bargaining.

The remaining of the paper is organized as follows. Section 2 describes the economic environment, defines the steady-state equilibrium, and presents some properties of decision rules for two-person households. The model economy is parameterized and calibrated, and preliminary results on the role played by within-household risk sharing are shown. Section 3 contains results on the extent and effects of insurance from the family. The effects of changes in UI on savings, spousal labor supply and consumption losses upon unemployment are assessed. Section 4 presents some extensions and applications, and Section 5 concludes.

2 The Economic Environment

Consumers The economy is populated by a continuum of measure two of infinitely-lived consumers. Half of this population is referred to as females (f), and the other half as males (m). They supply time to work in the firms sector and face idiosyncratic risk in the form of employment shocks. Employment shocks, s, take on values in $S \equiv \{0, 1\}$ and follow a Markov chain with transition matrix Π^i, for $i = f, m$. Thus, $\pi^i_{s'|s}$ is the probability for an agent of gender i to receive employment shock s' tomorrow conditional on employment shock s today. These probabilities satisfy $\sum_{s'} \pi^i_{s'|s} = 1$, $\pi^i_{s'|s} > 0$, and $\pi^i_{1|1} \geq \pi^i_{1|0}$ for $i = f, m$. The long-run probabilities of the two employment shocks in S are denoted by q^i_0 and q^i_1. As explained further below, we also allow for employment shocks to be correlated across spouses in the two-person household model. There are no others shocks in the economy.

Capital markets are incomplete. Households can save in a non-state-contingent asset, a, that pays the risk-free interest rate r. There is a borrowing constraint represented by $a \geq \underline{a}$.

Lifetime preferences for an agent of gender i over stochastic consumption and leisure streams are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t, l_t), \quad \text{for} \quad i = f, m, \tag{2.1}$$

where c_t denotes consumption and l_t is leisure. We make the following assumptions on U^i: **A1)** Utility $U^i(c, l) : R_+ \times [0, 1] \to R$ is bounded, continuous and twice continuously differentiable in its interior. **A2)** U^i is separable in consumption and leisure. **A3)** U^i is strictly increasing and strictly concave in each of its arguments. Moreover, $\lim_{c \to 0} U^i_c(c, l) = +\infty$, and $\lim_{l \to 0} U^i_l(c, l) = +\infty$.

Firms The aggregate good is produced by competitive firms with neoclassical production function $F(K, L)$, where K is the aggregate stock of capital and L is aggregate labor. Capital depreciates at rate $\delta > 0$. Aggregate labor is defined as $L \equiv \lambda L^m + (1 - \lambda)L^f$, where $0 < \lambda < 1$ defines the productivity of women relative to men. Given r and gross wage rates, w^f and w^m, the firm's first-order conditions are:

$$F_K(K, L) = r + \delta \tag{2.2}$$

$$\lambda F_L(K, L) = w^m \tag{2.3}$$

$$(1 - \lambda)F_L(K, L) = w^f. \tag{2.4}$$

Public Unemployment Insurance (UI) A worker of gender i hit by the unemployment shock, $s^i = 0$, receives a gender-neutral replacement rate, b, of gender-i workers' average earnings, $w^i L^i$. UI is financed on a period-by-period basis by taxing labor income at flat rate τ.

2.1 The Bachelor versus the Two-person Household Model

We consider two different risk-sharing arrangements, each of them defining in turn a different type of household. We start out by presenting the problem of the bachelor household. This is the definition of the household that has dominated not only the literature on precautionary savings, but also most of the macroeconomic literature. A single breadwinner chooses consumption, leisure and asset holdings in order to maximize his/her own lifetime utility. In most studies adopting this framework, the income process is estimated using data on males. The second type of household we study is a dynamic version of the two-person household model pioneered by Chiappori (1988). A household is formed by two individuals who decide on individual consumptions, labor supplies and savings. In order to understand the consequences of intra-household risk sharing we compare the allocations generated by these two household arrangements.

Bachelor Households A household formed by a single agent of gender i solves

$$v^i(s,a;w^i,r) = \max_{c,l,h,a'} \left\{ U^i(c,l) + \beta \sum_{s'} \pi^i_{s'|s} v^i(s',a';w^i,r) \right\} \tag{2.5}$$

$$s.t. \quad c + a' = (1-\tau)w^i hs + be^i(1-s) + (1+r)a \tag{2.6}$$

$$c \geq 0,\ 0 \leq l,h \leq 1,\ l+h \leq 1, \text{ and } a' \in [\underline{a}^i, \bar{a}], \tag{2.7}$$

where $\pi^i_{s'|s}$ are the elements of Π^i and h denotes hours worked. UI income received when unemployed is the replacement rate, b, times average labor earnings of gender-i workers, $e^i \equiv w^i L^i$. A version of this model, where there is a measure one of same-gender workers, is the workhorse model in the literature of uninsurable idiosyncratic risk, precautionary savings and labor supply.[3] By construction, the bachelor household does not engage in insurance activities with other workers beyond public unemployment insurance (UI). Own savings and UI are the only sources of insurance available to this type of household.

Two-person Households We now consider two-person households formed by an egotistical female and an egotistical male who share labor market risk and reach efficient intra-household allocations. Following the literature initiated by Chiappori 1988 (see Chiappori and Donni 2010 for a recent survey), the utility of each individual in the household carries a weight, reflecting her/his relative power in the household. Under full commitment to future intra-household allocations, individual weights are set when the household is formed and remain unchanged thereafter. Thus, transitory shocks, which are small relative to lifetime income, are assumed to have no effect on individual weights. Only variables known or predicted at the time of household formation can affect those weights.[4] In this paper, we assume that Pareto weights are parameters, and write the Pareto weight on females' utility as $\mu \in (0,1)$. Then, in the language of Browning et al. (2006), ours is a unitary model of the household: demand functions are independent of distribution factors and satisfy the Slutsky conditions.

Household-level state variables for the two-person household are the vector of employment shocks, $\boldsymbol{s} = (s^f, s^m)$, and the level of asset holdings, a. The state space of a household is $X = S \times S \times [\underline{a}, \bar{a}]$. The transition matrix for $\boldsymbol{s}$ is denoted by Π. In the case of correlated employment shocks within

[3] See, e.g., Flodén and Lindé (2001), Marcet *et al.* (2007) and Pijoan-Mas (2006).

[4] Mazzocco (2007) uses CEX data to test, and reject, the hypothesis of intra-household commitment. Since our model abstracts from permanent shocks and assumes only transitory shocks to labor income, we will initially retain, for the sake of analytical tractability, the assumption of commitment. We discuss below the implications of this assumption for our results.

the household, a typical element of Π is written as $\pi^f_{s^{f\prime}|s^f}[(1-\varsigma)\pi^m_{s^{m\prime}|s^m} + \varsigma\mathcal{I}_{\{s^{f\prime}=s^{m\prime}\}}]$, where $\mathcal{I}$ is an indicator function and parameter $\varsigma \in [0,1]$ pins down the extent of positive correlation in employment shocks within the household. Under uncorrelated shocks, matrix Π is simply obtained as $\Pi \equiv \Pi^m \otimes \Pi^f$. The vector of gross wages for the household, (w^f, w^m), is denoted by $\boldsymbol{w}$.

The maximization problem of a two-person household with female's Pareto weight μ is,

$$\begin{aligned} V(\boldsymbol{s}, a; \mu, \boldsymbol{w}, r) &= \max_{c^f, c^m, l^f, h^f, l^m, h^m, a'} \Bigg\{ \mu U^f(c^f, l^f) + (1-\mu)U^m(c^m, l^m) \\ &\qquad + \beta \sum_{\boldsymbol{s'}} \pi_{\boldsymbol{s'}|\boldsymbol{s}} V(\boldsymbol{s'}, a'; \mu, \boldsymbol{w}, r) \Bigg\} \end{aligned} \tag{2.8}$$

s.t.

$$c^f + c^m + a' = \sum_{i=f,m} (1-\tau) w^i h^i s^i + \sum_{i=f,m} b e^i (1-s^i) + (1+r)a \tag{2.9}$$

$$c^f, c^m \geq 0, \quad 0 \leq l^f, l^m, h^f, h^m \leq 1, \; l^f + h^f \leq 1, \; l^m + h^m \leq 1 \text{ and } a' \in [\underline{a}, \bar{a}] \tag{2.10}$$

where $\pi_{\boldsymbol{s'}|\boldsymbol{s}}$ are the elements of Π, and h^f and h^m denote hours worked by the female and the male, respectively; e^i denotes gender-i workers' average labor earnings.

The attitude towards risk of the two-person household depends both on individual preferences for risk and on the Pareto weight. In the Appendix we derive the coefficient of risk aversion for this household under CRRA utility functions. (For a two-period model with uncertainty see Mazzocco 2004.)

It is important to note that our assumption of egoistical preferences is not crucial. Actually, Browning *et al.* (2006) show that under caring preferences of the form $U^i(c^i, l^i) + \nu^i U^j(c^j, l^j)$, where $0 < \nu^i \leq 1$, is agent i's caring parameter, the utility function of the household can be written down as for the case of egotistical preferences, after a re-definition of Pareto weights.

We now present the first-order conditions to the maximization problem (8)-(10). Risk sharing within the household implies that the ratio of marginal utilities of consumption equals relative Pareto weights, and is thus independent of the realized vector of employment shocks. That is,

$$\mu U^f_c = (1-\mu) U^m_c. \tag{2.11}$$

This equation defines individual risk-sharing rules, which, for a given level of household consumption, specify how much is consumed by each of its members. It is straightforward to show that the first-order derivatives of the risk-sharing rules are positive and given by the product of the household's coefficient of absolute risk aversion and the individual's coefficient of absolute risk tolerance.[5] Therefore, the member of the household showing higher risk tolerance will be the one absorbing most of the variation in total household consumption. (In the Appendix we present the

[5] Risk tolerance is defined as the reciprocal of risk aversion.

derivatives of the risk-sharing rules for the case of CRRA utility functions.)

First-order conditions to female and male labor supply are, respectively,

$$\frac{U_l^f}{U_c^f} \geq (1-\tau) w^f s^f \tag{2.12}$$

$$\frac{U_l^m}{U_c^m} \geq (1-\tau) w^m s^m, \tag{2.13}$$

with strict inequalities if labor choices are non interior. Moreover, if the labor supply decision is interior for both household members then

$$\frac{U_l^f}{w^f} = \frac{1-\mu}{\mu} \frac{U_l^m}{w^m}. \tag{2.14}$$

The first-order condition to savings is,

$$U_c^f = \beta(1+r) \sum_{s'} \pi_{\boldsymbol{s'}|\boldsymbol{s}} U_c^{f\prime} \qquad \text{if } a' > \underline{a} \tag{2.15}$$

$$U_c^f \geq \beta(1+r) \sum_{s'} \pi_{\boldsymbol{s'}|\boldsymbol{s}} U_c^{f\prime} \qquad \text{if } a' = \underline{a}. \tag{2.16}$$

Propositions 1 and 2 below present some properties of the solution to the maximization problem of a household with Pareto weight μ for the case where workers' earnings when employed are higher than UI benefits when unemployed, i.e., for $i = f, m$, $(1-\tau)w^i h^i \geq b e^i$ for $s^i = 1, s^j \in S$ and $a \in [\underline{a}, \bar{a}]$.

Proposition 1. Assume **A1–A3**, $\boldsymbol{w} > 0$, $(1+r) > 0$, $\beta(1+r) \leq 1$. Then:

- **(a)** $V(\boldsymbol{s}, a; \mu)$ is strictly increasing and strictly concave in a. Decision rules $c^f(\boldsymbol{s}, a; \mu)$, $c^m(\boldsymbol{s}, a; \mu)$, $l^f(\boldsymbol{s}, a; \mu), l^m(\boldsymbol{s}, a; \mu)$ and $a'(\boldsymbol{s}, a; \mu)$ are continuous in a and strictly positive.
- **(b)** Decision rules for consumption, $c^f(\boldsymbol{s}, a; \mu)$ and $c^m(\boldsymbol{s}, a; \mu)$, are strictly increasing in a. Decision rules for savings, $a'(\boldsymbol{s}, a; \mu)$, and leisure, $l^f(s^f = 1, s^m, a; \mu)$, $l^m(s^m = 1, s^f, a; \mu)$, are increasing in a.
- **(c)** Decision rules for consumption are increasing in the own and the spouse's employment shock: $c^j(s^j = 1, s^i, a; \mu) \geq c^j(s^j = 0, s^i, a; \mu)$ and $c^j(s^j, s^i = 1, a) \geq c^j(s^j, s^i = 0, a; \mu)$.
- **(d)** Decision rules for leisure are increasing in the spouse's employment shock: $l^j(s^j = 1, s^i = 1, a) \geq l^j(s^j = 1, s^i = 0, a; \mu)$.
- **(e)** If $\beta(1+r) \leq 1$, then for all $a \in [\underline{a}, \overline{a}]$, $a'(s^f = 0, s^m = 0, a; \mu) \leq a$ (with strict inequality if $\underline{a} < a < \overline{a}$ and $\beta(1+r) < 1$).

We can prove some results on the asymptotic properties of the consumption program, savings and labor supply of a household with Pareto weight μ, for different values of wages, $\boldsymbol{w}$, and of the interest rate, r. We extend results by Marcet *et al.* (2007) for the bachelor household to our two-person household model. We also extend the results to non-homogeneous utility functions. With this aim, let us denote by $\widetilde{a}(\mu)$ the minimum level of asset holdings for which household income becomes independent of employment shocks. The value of $\tilde{a}(\mu)$ and the proof of Proposition 2 are presented in the Appendix.

Proposition 2: Assume **A1** $-$ **A3**, $\overline{a} > \widetilde{a}(\mu)$, $\boldsymbol{w} > 0$ and $(1+r) > 0$. Then:

- **(a)** If $\beta(1+r) \leq 1$, for any $a \leq \widetilde{a}(\mu)$, $a'(\boldsymbol{s}, a; \mu) \leq \widetilde{a}(\mu)$.
- **(b)** If $\beta(1+r) = 1$, for any $a \geq \widetilde{a}(\mu)$ and any $\boldsymbol{s}$ we have $a'(\boldsymbol{s}, a; \mu) = a$, and $c^f(\boldsymbol{s}, a; \mu) + c^m(\boldsymbol{s}, a; \mu) = a\, r$ such that $\mu U_c^f = (1-\mu)\, U_c^m$. Moreover, $l^i(s^i = 0, s^j, a; \mu) = 1$, and $l^i(s^i = 1, s^j, a; \mu) = l^i(s^i = 1, s^j, \tilde{a}; \mu)$, for $i = f, m$.
- **(c)** If $\beta(1+r) = 1$ and $a \leq \widetilde{a}(\mu)$, then $a_t \xrightarrow{a.s.} \widetilde{a}(\mu), c_t^i \xrightarrow{a.s.} \widetilde{c}^i(\mu)$, and $l_t^i \xrightarrow{a.s.} 1$ for $s^i = 0$, and $l_t^i \xrightarrow{a.s.} l^i(s^i, s^j, \tilde{a}; \mu)$ for $s^i = 1$, $i = f, m$.

In the case $\beta(1+r) < 1$, the household can reach any value of asset holdings from any initial condition in finite time, and a stationary distribution arises in the long run. Moreover, in the case $\beta(1+r) = 1$ and $a \leq \widetilde{a}(\mu)$, capital accumulation in the long run is bounded and it converges asymptotically to $\widetilde{a}(\mu)$. This is in contrast to the case of inelastic labor supply where savings asymptotically grow to infinity if $\beta(1+r) = 1$. As it should be apparent from these results, the endogenous labor-leisure decision changes the asymptotic behavior of consumption and wealth by removing income uncertainty. That is, when household wealth is high enough that the restriction $(1-\tau)w^i h^i \geq be^i$ becomes binding for both workers, employment shocks no longer affect household income. Hence, under non-stochastic income, unbounded asset accumulation is no longer optimal when $\beta(1+r) = 1$.

If we set $\overline{a} > \max_{\mu \in (0,1)} \widetilde{a}(\mu)$ and choose initial capital holdings for all households with relative Pareto weight μ such that $a_0(\mu) \leq \widetilde{a}(\mu)$, then the upper bound on asset holdings, which was imposed to guarantee existence and uniqueness of the value function, is never binding.

2.2 The Steady-State Equilibrium

A stationary equilibrium in the two-person household economy where all households have a common Pareto weight μ is defined as follows. Let $\psi(B; \mu)$ be a probability measure, defined on the Borel sigma algebra $\mathcal{B}$, describing the mass of households at each $B \in \mathcal{B}$. Denote by $P(\boldsymbol{s}, a, B; \mu)$ the probability that a household at state $(\boldsymbol{s}, a)$ will transit to a state that lies in $B \in \mathcal{B}$ in the next period. The transition function P can be constructed as,

$$P(\boldsymbol{s}, a, B; \mu) = \sum_{\boldsymbol{s}' \in B_{\boldsymbol{s}}} \Pi_{\boldsymbol{s}'|\boldsymbol{s}}\, \mathcal{I}_{\{a'(\boldsymbol{s}, a; \mu) \in B_a\}},$$

where $\mathcal{I}$ is an indicator function and $B_{\boldsymbol{s}}$ and B_a are the projections of B on $S \times S$ and $[\underline{a}, \overline{a}]$, respectively. We are now ready to define the stationary equilibrium.

Definition: A stationary recursive competitive equilibrium with incomplete markets in the economy with two-person households is a list of functions $\left\{V, c^f, c^m, l^f, h^f, l^m, h^m, a', K, L^f, L^m\right\}$, a measure of households ψ and a set of prices $\left\{r, w^f, w^m\right\}$, tax rate $\{\tau\}$ and UI replacement rate $\{b\}$ such that:

1) For given prices, tax rate and UI replacement rate, V is the solution to (2.8) – (2.10), and $c^f(\boldsymbol{s}, a; \mu)$, $c^m(\boldsymbol{s}, a; \mu)$, $l^f(\boldsymbol{s}, a; \mu)$, $h^f(\boldsymbol{s}, a; \mu)$, $l^m(\boldsymbol{s}, a; \mu)$, $h^m(\boldsymbol{s}, a; \mu)$ and $a'(\boldsymbol{s}, a; \mu)$ are the associated optimal policy functions.

2) For given prices, K, L^f and L^m satisfy the firm's first-order conditions (2.2) – (2.4).

3) Aggregate factor inputs are generated by the policy functions of the agents:

$$K = \int_X a'(\boldsymbol{s}, a; \mu) d\psi, \tag{2.17}$$

$$L^f = \int_X s^f h^f(\boldsymbol{s}, a; \mu) d\psi, \tag{2.18}$$

$$L^m = \int_X s^m h^m(\boldsymbol{s}, a; \mu) d\psi. \tag{2.19}$$

4) The time-invariant stationary distribution ψ is determined by the transition function P as

$$\psi(B; \mu) = \int_X P(\boldsymbol{s}, a, B; \mu) d\psi \quad \text{for all} \quad B \in \mathcal{B}. \tag{2.20}$$

5) The government budget is balanced: $q_0^f b w^f L^f + q_0^m b w^m L^m = \tau w^f L^f + \tau w^m L^m$.

Under assumptions **A1** $-$ **A3** the interest rate in the stationary equilibrium must be such that $\beta(1 + r) < 1$.

2.3 Parameterization and Calibration

We assume identical preferences for females and males and parameterize their common instantaneous utility function as follows,[6]

$$U(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \varphi\, \frac{l^{1-\gamma} - 1}{1 - \gamma}. \tag{2.21}$$

Since our exercise is on intra-household insurance, and since insurance is tightly connected to wealth, it is important that we match the proportion of low-wealth households in the U.S. Thus,

[6] In a previous version of this paper we assumed different preferences for females and males. Our main results are however unaffected by this assumption.

and following ideas in Krusell and Smith (1998), we assume that households are heterogeneous in terms of the discount factor, β. Namely, there is a fraction, say χ, of impatient households with discount factor β_L. Patient households use discount factor $\beta_H > \beta_L$.

The production technology is the standard Cobb-Douglas function, $F(K, L) = K^{\alpha}L^{1-\alpha}$, where labor is $L \equiv \lambda L^m + (1-\lambda)L^f$. Parameter α is the capital's share of income and λ pins down relative gross wages, since $w^f/w^m = (1-\lambda)/\lambda$.

Parameter Values We set values to five preference parameters: $\beta_L, \beta_H, \sigma, \varphi, \gamma$; three technology parameters: α, λ and δ; and the four parameters in the two transition matrices Π^f and Π^m. The public insurance program contains parameter b (the tax rate, τ, balances the program budget). Finally, we have the borrowing limit, $\underline{a}$, the Pareto weight, μ, and the fraction of impatient households, χ .

One period is set to one quarter. We choose $\underline{a} = 0$ so that households face a no-borrowing constraint. We set $\sigma = 2$, which is a standard value in the macro literature. The depreciation rate of capital is set at $\delta = 0.025$, and the capital's share of income, α, equal to 0.36. We pin down the value of λ from *a priori* information on the gender wage gap. We set λ equal to 0.575, corresponding to a wage gap of 0.74, the value reported by Heathcote *et al.* (2010) for the 2004 U.S. economy. The UI replacement rate, b, is set at 0.3, which is the average value for this rate in the U.S. (see OECD 2010).

Transition probabilities for idiosyncratic employment shocks are assumed to be identical for females and males, as the average difference between female and male unemployment rates over the period 1980-2009 is practically zero. We set $\pi^i_{1|1} = 0.09$ and $\pi^i_{1|0} = 0.06$ for $i = f, m$, which match an average employment rate of 93%, after normalizing with the participation rate.[7] In our benchmark economy within-household employment shocks are uncorrelated, an assumption that is supported by SIPP data (Survey of Income and Program Participation). Indeed, within-household unemployment correlation for households where husband and wife report different occupation is 0.05. For households reporting same occupation this correlation is 0.23. However, only 3.2% of the households report same occupation for husband and wife. (For a detailed explanation on the calculation of these correlations, see Shore and Sinai 2010.) We will study the sensitivity of our results to positive correlation in employment shocks within the household.

The remaining six parameters, γ, ρ, μ, β_H, β_L, and χ, are set to match the following six targets: *1)* Married working females' average hours of work represent 28% of their discretionary time.[8] *2)* Married working males' average hours of work represent 40% of their discretionary time (see

[7]These transition probabilities are taken from Imrohoroglu (1989), Krusell and Smith (1998) and Marcet *et al.* (2007).

[8]Mazzocco *et al.* (2008) use PSID data and obtain mean annual hours worked by working married females and males equal to 1660 and 2312, respectively. We assume 16 hours of daily discretionary time.

footnote 8). *3)* Estimates for males' Frisch elasticity in the presence of borrowing constraints range from 0.2 to 0.6 (see Domeij and Flodén 2006). We will target a Frisch elasticity for males of 0.5.[9] It should be noted that this value, along with the first target, imply a Frisch elasticity for females of 0.85, which is in line with empirical estimates (see, e.g., Blundell and MaCurdy 1999). *4)* The capital-to-output ratio is 10. *5)* The fraction of liquidity-constrained households in the U.S. (using Zeldes' 1989 definition of liquidity constrained as holding non-housing wealth below two months of average income) averages 17% for the period 1983-2004, as reported by Gorbachev and Dogra (2010). *6)* Finally, we target the ratio of wealth held by the bottom quintile in the wealth distribution over total wealth to be less than one percent. Table 1 presents our benchmark economy.

[Insert Table 1 here]

Aggregate values at the steady-state equilibrium are: $Y = 1.2722$, $K = 12.6814$, $L = 0.3490$, and $r = 0.0111$. This steady state matches relatively well the bottom tail of the wealth distribution, but it does less well at matching the upper tail. We could improve on the upper tail by introducing three different discount factors, as in Krusell and Smith (1998), instead of only two. However, since our focus is on the effects of intra-household risk sharing at the level of the household, and since these effects are small for households with high levels of wealth, it is not crucial that we match the upper part of the wealth distribution. Hence, our model contains the structure needed to conducting the study on intra-household risk sharing.

2.4 Policy Functions: A First Look at the Effects of Intra-household Risk Sharing

We start by presenting policy functions in our model economy and then assess the effects of intra-household risk sharing on households' savings and hours worked. The left panel of Figure 1 presents savings policy functions (for convenience we plot the net change in asset holdings $a' - a$), for households with low discount factor (top chart on left panel) and with high discount factor (bottom chart on left panel). Among impatient households, positive net savings are observed only when the two spouses are employed and hold low levels of assets. Households with one or two unemployed spouses choose non-positive net savings. Among patient households, those with two employed spouses choose positive net savings at all values in the support of the equilibrium distribution of assets, except for its upper bound where net savings are zero. Patient households

[9] For the assumed utility function, eq. (21), the Frisch elasticity of labor supply for a worker of gender i is $\frac{1}{\gamma}\frac{1-h^i}{h^i}$. We match the targeted elasticity by evaluating the individual one at average hours worked. We have also computed the average elasticity (by averaging out Frisch elasticities across individuals of gender i) and obtained roughly the same value.

with at least one of the spouses unemployed choose negative net savings at a large set of low asset holdings. Negative net savings are larger in households where the male is unemployed.

Policy functions for hours are shown in the right panel of Figure 1 (top chart for impatient households and bottom chart for patient ones). For both groups, hours decrease with household wealth. As asset holdings approach the borrowing limit, policy functions for hours bend upwards, capturing the fact that asset-poor households use labor supply to smooth consumption more intensively. Hours increase if the spouse is unemployed, both for females and males, and the increase is especially marked for females in asset-poor households. For example, a female in a household with no assets will supply almost half of her available time to work if the spouse is unemployed, as opposed to 0.36 when the spouse is employed, which represents a decline of 28%.

[**Insert Figure 1 here**]

The savings and hours effects of intra-household risk sharing are shown in Figures 2 and 3. Figure 2 plots excess savings of two bachelors (each with wealth $a/2$) over a two-person household (with wealth a). The two top charts show excess savings for impatient and patient households across employment shocks. Clearly, although risk sharing affects the savings decisions of all households across the wealth distribution, its effects are strongest among wealth-poor households. The bottom charts of Figure 2 show average excess savings.

[**Insert Figure 2 here**]

The top chart of Figure 3 plots excess hours worked by two bachelors (each with wealth $a/2$) over hours worked by a two-person household (with wealth a). (Since there is not much difference in excess hours between patient and impatient households we plot the average of the two.) For all households where only the male is employed, intra-household risk sharing increases household hours. For households where the female is employed, with the exception of low-wealth households with the male unemployed, intra-household risk sharing decreases household hours. The bottom chart of the Figure shows the average of excess hours across households along the employment distribution. As it is apparent, the effects of intra-household risk sharing on hours are strongest among wealth-poor households.

[**Insert Figure 3 here**]

3 Intra-household Risk Sharing and Public Insurance

The use of savings and labor supply as insurance mechanisms depends on the generosity of public insurance. The extent to which the ability to share risks within the household shapes the crowding out effects of public insurance is explored in this section.

3.1 Household Financial Assets and the Generosity of UI

An implication of our model, as of any model with uninsurable income risk, is that household asset holdings increase with income uncertainty. Since the level of UI is directly correlated with household income risk, Engen and Gruber (2001) exploit the variation in generosity of unemployment insurance schedules across U.S. states to test this implication and to estimate the extent of the precautionary savings motive. These authors use SIPP data —which follows a cross section of households over a period of 2.5 years— in combination with data on UI available to these households under their state/date insurance system. They regress household financial assets (normalized by average household income) on the generosity of UI, controlling for a vector of demographic and economic characteristics of the household. They find an elasticity of the average household's assets-to-income ratio with respect to UI equal to -0.28. That is, reducing the replacement rate of UI by 50% would increase the household's assets-to-income ratio by 14%.

We use our model economy to compute the elasticity of the average assets-to-income ratio with respect to UI. This exercise serves two purposes. First, we test the ability of our two-person household model to account for this estimated measure of the precautionary savings motive. Second, we also compute this elasticity in the bachelor household model and then assess the role of within-household risk sharing in shaping the effects of changes in UI on savings. Variation in UI in the bachelor household model amounts to larger changes in household income risk and, consequently, to larger effects on savings.

In order to mimic the empirical exercise conducted by Engen and Gruber (2001), who rely on the exogenous variation in UI across U.S. states, we proceed as follows. The level of UI in our benchmark economy is interpreted as the average value across all states. Then, in order to embed variation in UI across states, we vary the replacement rate and compute households' assets-to-income ratios holding equilibrium prices unchanged, a strategy in accordance with the existence of a unique financial and labor market across states. Then, we average out these assets-to-income ratios using the long-run distribution of households over asset holdings. However, this long-run distribution is let to change with UI in order to capture that variation in UI across states is permanent, and it has already shaped the distribution of wealth within each state. Thus, our exercise compares the differential asset-to-income ratio of households across states that provide these households with differing levels of UI, which is exactly what Engen and Gruber (2001) do in their empirical work. The results of this exercise are presented in Table 2. Our two-person household model yields an elasticity of the assets-to-income ratio with respect to UI equal to -0.39, accounting thus fairly well for the elasticity estimated by Engen and Gruber (2001). On the contrary, the bachelor household model yields an elasticity of 0.70, which is more than twice the estimated value. This shows that intra-household risk sharing plays a crucial role in the determination of the elasticity of the assets-to-income ratio and, therefore, that this source of insurance is key when assessing the crowding out effects of public insurance.

[Insert Table 2 here]

The relative success of our model at matching this elasticity lends support to the view that the two-person household embeds the most relevant non-market insurance arrangements available to individuals. Indeed, some authors have emphasized that the extended family, friends and other social networks play only a negligible insurance role (see, e.g., Blundell *et al.* 2008). The results in this section are robust to the introduction of correlated employment shocks within the household. The elasticity of the assets-to-income ratio with respect to UI in the two-person household economy increases only to -0.42 when the correlation in employment shocks is set to 0.3.

3.2 Spousal Labor Supply as Insurance

The policy functions shown above suggest that spousal labor supply is a potentially important source of household self-insurance. The change in a household member's labor supply induced by an unemployment spell of another household member —the added worker effect— has been largely studied in the empirical literature. Special attention has been paid to the labor supply response of married women to their husband's unemployment spells.[10]

The early literature on the added worker effect (see Cullen and Gruber 2000 for a short review) has singled out liquidity constraints as one of the main reasons married women increase hours worked during their husband's unemployment spells. Empirical estimates have, however, produced mixed results, failing to find strong support for this effect.[11] Cullen and Gruber (2000), using SIPP data for married couples aged between 25 and 54 years old, report means for wives' monthly hours worked during husbands' spells of employment and unemployment, respectively. Conditional on working women, these authors find that wives of unemployed husbands work a mean of 149 hours per month, as opposed to 132.4 hours by wives of employed husbands. When non-working wives are included, average hours are 98.2 and 97.9, respectively.

We now study the response of female labor supply to males' unemployment spells in our model

[10]The main argument in favor of restricting the attention to labor supply of women is that they are the secondary wage earners in most households (according to Cullen and Gruber 2000, in 87% of married couples in the U.S. the husband earns more and in 73% the husband works more hours).

[11]Stephens (2002) estimates the added worker effect taking into account not only the current period of the husband's job loss but also the periods before and after a job loss. This author finds small pre-displacement effects but large, persistent post-displacement effects.

economy. In order to highlight the role of liquidity constraints for wives' labor supply responses we look at the group of liquidity-constrained households separately (we follow Zeldes (1989) and define a household as liquidity constrained if its non-housing wealth is less than two months of average income). Table 3 below reports the added worker effect in our model economy. For the group of liquidity-constrained households, average hours worked by wives of unemployed husbands are 8.6% higher than those worked by wives of employed husbands, an increase comparable to that found by Cullen and Gruber (2000) in their sample of working women. When all households are taken into account the increase in hours is only 2.7%. This shows that spousal labor supply is an insurance mechanism used mainly by wealth-poor households, but less so by the wealth rich.

[Insert Table 3 here]

How effective is wives' labor supply as insurance against income fluctuations due to husbands' unemployment? To answer this question we compute, for each level of assets, a, the fraction of lost family income that is made up by the wife's response to the husband's unemployment spell,

$$\frac{(1-\tau)w^f[h^f(0,1,a)-h^f(1,1,a)]}{(1-\tau)w^m h^m(1,1,a)-be^m},$$

where $h^f(0,1,a)$ denotes hours worked by a female with an unemployed husband and $h^f(1,1,a)$ is female hours if the husband is employed. The denominator represents lost income due to husband's unemployment. The numerator is the increase in income due to the wife's response in hours. For the group of liquidity-constrained households, the wives' response makes up about 9.6% of lost family income, while this number is only 2.5% when we consider all households. Again, this shows that wealth-rich households smooth consumption over the husband's unemployment spell using savings. Liquidity-constrained households must rely, however, on female labor supply.

Spousal Labor Supply and the Generosity of UI According to some authors, the moderate to nil added worker effect found in the data may be partially explained by the presence of public insurance. That is, unemployment payments during the husband's unemployment spell crowd out wife's labor supply. Cullen and Gruber (2000) estimate this effect using SIPP data on unemployment experiences of husbands and the concurrent labor supply of their wives, coupled with information on cross-household variation in potential UI. They find that a 50% reduction in potential UI of the husband (75 USD per week) would imply an increase in monthly hours worked by the wife (conditional on working) of 13.42 hours, which amounts to an increase of about 9%. They also find a differentially larger response of wives' labor supply among liquidity-constrained households.

We use our model economy to compute the crowding-out effects of UI on wives' labor supply. As there is no cross-household variation in UI in our model, to mimic the empirical exercise of Cullen and Gruber (2000) we compute the change in hours worked by the wife of an unemployment husband if their UI replacement rate drops by 50%. To embed the fact that the drop in UI represents cross-household variation and not an economy-wide, permanent drop, we compute the average change in hours worked by the wife using prices and the wealth distribution of our benchmark economy. The 50% reduction in household UI income increases wife's hours of work by 9.87% in the group of liquidity-constrained households. This increase is only 1.92% when all households are considered. The relatively higher sensitivity of spousal labor supply to UI among liquidity-constrained households found in our model in is line with the finding of Cullen and Gruber (2000).

It should be noted that the Cullen and Gruber (2000) estimated increase in hours of work by the wife after a 50% reduction in UI income received by the unemployed husband is not statistically significant, thus hindering the assessment of our model's predictions.

3.3 Consumption Loss Upon Unemployment

Under imperfect capital markets, the loss of the job implies a reduction in the level of individual consumption. The degree of transmission of unemployment shocks to consumption depends on factors such as the generosity of UI, the level of accumulated wealth and on whether or not risks are shared within the household.

In this section we use our benchmark economy to assess the contribution of intra-household risk sharing to individual consumption insurance, as measured by the degree of transmission of unemployment shocks to consumption. We do so by comparing individual consumption losses upon unemployment in the two-person household model to those in the bachelor household model. We compute the percentage change in consumption upon unemployment, $\triangle c/c$, at all asset levels in the support of the corresponding equilibrium distribution. In the two-person household economy, individual consumption losses for females and males, both with an employed spouse and with an unemployed spouse, are computed as,

$$\frac{c^j(s^j=0,s^i,a)-c^j(s^j=1,s^i,a)}{c^j(s^j=1,s^i,a)},$$

for $j=f,m$, $i=f,m$ and $i\neq j$, both for $s^i=1$ and $s^i=0$. For the bachelor economy, individual consumption losses upon unemployment are computed as, $(c^j(0,a)-c^j(1,a))/c^j(1,a)$ for $j=f,m$.

In panel (a) of Table 4 we report average individual consumption losses, both for the group of liquidity-constrained individuals and for all individuals. We use the respective equilibrium asset and employment distributions to average out individual consumption losses. The results show that intra-household risk sharing provides important consumption smoothing opportunities, especially for liquidity-constrained individuals. Thus, the average consumption loss for a liquidity-constrained female in the bachelor economy is -23.23%, against -4.5% in the two-person household economy. For a liquidity-constrained male, intra-household risk sharing reduces his consumption loss from -30.09% to -8.74%. These numbers imply that the family is an important provider of consumption insurance for a significant fraction of individuals. These results are robust to correlation in employment shocks within the household. For instance, a correlation of 0.3 implies consumption losses in the two-person household economy of -4.85% and -9.05% for liquidity-constrained females and males, respectively.

[**Insert Table 4 here**]

We also study household insurance by computing the fraction of household income loss (due to an unemployment shock) that transmits to household consumption loss. To do this we compute income and consumption losses upon an unemployment shock for each household across the asset and employment distributions. Then, we average out the percentage of income loss that is transmitted to consumption loss across all households. Panel (b) of Table 4 presents our results for the two household arrangements.

The fraction of income loss that transmits to consumption loss in the group of liquidity-constrained households is non-negligible, even in the two-person household economy. For an average household in this group, 17.31% of the household income lost due to an unemployment shock is absorbed by consumption. This result is consistent with the empirical finding of Blundell *et al.* (2008) about the degree of insurability of transitory income shocks. They find that the impact of these shocks on consumption is small when estimated from all households in their sample, but it is found to be larger, about 0.2, in the subsample of wealth-poor households (these authors define a household as wealth poor if its wealth is in the bottom 20 percent of the distribution of initial wealth).

The fraction of income loss that transmits to consumption loss under a 0.3 correlation of employment shocks within the household increases to 18.39% for liquidity-constrained households.

Consumption Loss and the Generosity of UI We now turn to the sensitivity of household consumption losses upon unemployment with respect to the generosity of UI. Browning and Crossley (2001), using a Canadian panel, estimate this sensitivity exploiting legislative changes introduced in 1993 and 1994 that reduced the replacement rate by about five percentage points. In total, $19,000$ individuals who had experienced a job separation either before or after the policy reform were interviewed several times after the job loss. Browning and Crossley (2001) use a sample of 18054 respondents and obtain two main results. First, the level of UI has small average effects on household consumption losses upon unemployment. In particular, a 10 percentage-point reduction in UI leads to an average fall in consumption of 0.8%.[12] Since a reduction of 10 percentage

[12] Gruber (1997) uses U.S. data on food consumption from the Panel Study of Income Dynamics (PSID) and finds a larger mean effect of UI on consumption losses upon unemployment. This author estimates that a 10 percentage-point increase in UI reduces the fall in consumption by 2.65%.

points represents a cut in benefits of 17%, this implies an elasticity of consumption with respect to benefits of $(0.8/17) = 0.047$. Second, the consumption effects of UI are not homogeneous across households, being substantially larger within the group of liquidity-constrained households at the time of job separation. To estimate how consumption losses upon unemployment respond to UI benefits among liquidity-constrained households, these authors add the following two variables to the estimated equation: a) the replacement rate crossed with "no assets at the separation date"; and b) the replacement rate crossed with "spouse not employed" crossed with "not eligible for Social Assistance". The coefficients on these two variables are 15.68 and 26.91, respectively, when estimated for the subsample of job losers entitled to positive benefits. The implied elasticity of consumption losses with respect to benefits among liquidity-constrained households is then $0.8657 \times (1.568/17) + 0.1343 \times (1.568 + 2.691)/17 = 0.1134$, where 0.1343 is the fraction of liquidity-constrained respondents with a non-employed spouse who were not eligible for Social Assistance. The remaining fraction, 0.8657, either had an employed spouse or were eligible for Social Assistance, or both.

Table 5 below presents the elasticities of consumption loss with respect to UI in our model economies with and without intra-household risk sharing. Since the variation exploited by Browning and Crossley (2001) is a legislative change that reduced UI permanently for all households in the economy, in our experiment we compute the stationary equilibrium for a lower replacement rate and then compare consumption losses upon unemployment with those in the benchmark economy. Also, our quantitative exercise explicitly acknowledges the panel dimension of the empirical exercise conducted by Browning and Crossley (2001). We compute the relative change in consumption from the period prior to the unemployment shock to the period in which the job separation is realized. Saving decisions in the pre-unemployment period are used for the computation of consumption upon unemployment. That is, consumption of an individual of gender j in the period before unemployment is $c^j(s^j = 1, s^i, a)$ and consumption at the time of the job loss is $c^j(s^j = 0, s^i, a')$, where $a' = a'(s^j = 1, s^i, a)$. We then weigh consumption levels in both periods using the stationary distribution of employment shocks for the spouse (in the two-person household economy). Our two-person household economy yields an elasticity of consumption loss with respect to UI for liquidity-constrained households which is close to the value estimated by Browning and Crossley (2001). In the bachelor household economy, this elasticity is twice the estimated value.

[Insert Table 5 here]

It is important to note that the empirical exercise in Browning and Crossley (2001) uses Canadian data, while our baseline parameter values have been chosen to match U.S. stylized facts. Then, rather than trying to account for this elasticity, our exercise in this section aims at shedding further light on the implications of intra-household risk sharing.

4 Applications and Extensions

The Value of Intra-household Risk Sharing In order to assess the value of intra-household insurance, we first remove from our benchmark economy all intra-household transfers which are not related to risk sharing. This is accomplished by setting the gender wage gap to zero and the relative Pareto weight to one, so that any transfer within the household is driven by risk pooling. We then compute the increase in bachelor consumption that would leave an individual in a two-person household indifferent between remaining in the household and splitting up with half of the household's assets to remain bachelor thereafter. I.e., we compute the value of ζ that solves,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^{tp}, l_t^{tp}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i((1+\zeta)c_t^{bach}, l_t^{bach}), \tag{4.1}$$

where a superscript *tp* refers to allocations in the two-person household economy from initial conditions (s^i, s^j) and a_0, and a superscript *bach* refers to allocations attained by the deviating individual from initial conditions s^i and $a_0/2$. The results vary widely depending upon own and spouse's employment status, discount factor and asset holdings. For instance, the value of intra-household insurance for an impatient, unemployed individual with an employed spouse and no assets represents 5.16% of per period consumption of a similar bachelor individual. This number goes down to 0.75% if the individual is employed. The value of intra-household insurance for the unemployed decreases if the spouse is also unemployed. Also, the value of family insurance is lower to patient than to impatient individuals. When aggregating over employment shocks and discount factors, the welfare gain within the group of liquidity-constrained households averages 0.2387%. Our model suggests that the value of intra-household insurance is significant to wealth-poor, unemployed individuals and that it decreases with wealth, especially among patient households. It is worth noting that the value of family insurance to unemployed individuals with no assets is higher than the average value of removing aggregate business cycle fluctuations, as estimated in the literature. Finally, we have also computed the value of remaining in the two-person household without removing first intra-household transfers not associated with risk-sharing (i.e., using the relative Pareto weight and gender wage gap of the baseline economy). In this case, the value of ζ for a male, even if unemployed, changes from positive to negative at relatively low levels of household wealth. Since our model abstracts from public consumption within the household, from children and preferences for marriage, the cost for the male of those transfers not related with risk sharing outweighs the gains from family insurance.

Optimal Unemployment Insurance We carry out two exercises to ascertain how access to insurance from the family shapes the welfare effects of UI, and how it affects the optimal provision of public insurance. First, we compute optimal unemployment benefits in the two-person household and the bachelor household economies. Then, we compute the welfare effects of a 10 percentage points reduction in the replacement rate in both economies. These exercises are carried out following a strategy similar to the one used by Krusell *et al.* (2010). That is, ours is a general equilibrium analysis where we compare steady states with different levels of UI, but instead of measuring the welfare effects of UI by comparing ex ante utility across steady states, we compare utilities of moving each household, along with its level of wealth, to a steady state with different UI. We do not condition on the employment status of the household, instead we look at the average household with wealth a when comparing utilities across steady states.

Our results show that insurance provided by the spouse is a key determinant of optimal public insurance. We find that the optimal replacement rate decreases with the level of wealth, both for the two-person and the bachelor household. Also, the differential in optimal replacement rates between these two household arrangements diminishes with wealth. The optimal replacement rate for an average two-person household with no assets is about 15%, while this rate is 60% for the average bachelor household with no assets, i.e., a difference of 45 percentage points. If we look now at households with levels of wealth equal to two months of average income, the differential in optimal replacement rates between the two-person and the bachelor household goes down to 30 percentage points. The differential keeps on decreasing up to a level of wealth equal to six months of average income and then vanishes. The reason for a vanishing differential is that optimal UI becomes zero under both household arrangements when a is sufficiently large. On the one hand, wealth reduces the demand for public insurance because households can use their savings to smooth consumption; on the other hand, positive UI implies lower wages as it reduces the aggregate level of capital. These two effects explain why optimal UI goes to zero as wealth increases in both economies.

In our second exercise, we compute the welfare effects (in consumption equivalent units) of reducing the replacement rate by 10 percentage points from the value in our baseline economy. For the average two-person household with no assets this decrease in UI leads to an increase in welfare of 0.015%, while the average bachelor household experiences a decrease in welfare of -0.164%. Among the group of liquidity-constrained households, the reduction in UI increases welfare of two-person households by 0.034%, and decreases welfare of bachelor households by -0.035%.

Marital Shocks As shown by Mazzocco (2007), the assumption of commitment might not be supported by empirical evidence. Relaxing this assumption imposes further restrictions on intra-household allocations, as husband and wife participation constraints must be met. For the two-person household to be sustained, these constraints impose that both members must be made better off relative to divorce, in all periods and for all individual shocks and wealth levels. This may have non-negligible effects on the extent of insurance provided by the family.

Several authors have studied risk sharing in environments with no commitment. For example, Ligon et al. (2000) study the interaction of an individual storage technology with risk sharing between households. They find that the introduction of such a storage technology increases the value of autarky and can thus reduce risk sharing and welfare. Attanasio and Rios-Rull (2000) show that the provision of public insurance against aggregate risk may crowd out risk sharing between households, and also reduce overall welfare. Mazzocco et al. (2007) study joint decisions of savings, labor supply and marital status in an economy with no commitment. Their model also includes human capital accumulation, home production, children and changing match quality. Lack of commitment within the household implies that savings and labor supply affect both the decision power of husband and wife and the value of divorce, which allows the authors to jointly account for household allocations and marital transitions.

In this paper, we do not solve for intra-household allocations under limited commitment. Instead, we introduce exogenous marital shocks into our benchmark economy and assess its implications for savings and labor supply. It must be noted that this exercise does not embody all the implications of lack of commitment, but it does capture, arguably, the most relevant one, namely, the end of the marriage and hence of insurance from the family.

We follow the work of Cubeddu and and Ríos-Rull (2003) to model some features of the economy with marital shocks. Two-person households are now assumed to face an exogenous probability of dissolution, say $\phi_d > 0$. When the separation shock hits, household members become single, each with half of the household assets. We assume no divorce costs. Single individuals face a per period probability, $\phi_m > 0$, of being matched with another single of opposite gender. Upon matching, agents share their assets and form a two-person household. While the decision to share assets and form a two-person household is assumed to be exogenous, agents are matched in such a way that relaxing this assumption would not prevent most households from being formed. We restrict matches to be segmented on wealth levels, in the sense that prospective spouses for a single agent with wealth a are assumed to have wealth close to a. When we compute the model, a level of wealth close to a means that it is in the same bin of the grid on asset holdings.[13] We choose ϕ_d so that half of the population lives as married couples (according to the 2001 U.S. census, 54.8% and 44% of men and women over 25 years of age were married, respectively) and that the average duration of marriage is 7 years. Note that in our dynastic model, if 50 percent of the population lives as married households, we must have that $\phi_d = \phi_m$.

We now assess the effects of risk of divorce on savings, on the extent of the precautionary motive

[13] Even though different divorce rules to the one assumed here, and/or an endogenous marriage decision may have important quantitative effects, we believe our model provides a useful benchmark to assessing the effects of marital transitions.

and on consumption losses. The savings effects of marital risk have received some attention in the literature. An important research question in this literature is whether an increase in the risk of marriage dissolution leads to higher household savings.[14] The exercise we conduct in this section consists of computing the change in savings that would follow from the removal of marital risk, both for liquidity-constrained households and for all households in the economy. That is, we start with $\phi_d = 0.024$ and compute savings across two-person households. Then we set the probability of divorce to zero and compute savings for these households. We find that savings go down by 4.2584% within the group of liquidity-constrained households, and 0.0292% when all households are considered. The removal of marital risk eliminates a need for precautionary savings and, therefore, decreases households savings. This result is consistent with the findings in González and Özcan (2008).

We now compute the elasticity of the assets-to-income ratio with respect to UI in this economy with marital risk, which is a measure of the precautionary motive. This elasticity is -0.52, against the -0.39 we found under no marital risk. As expected, the extra risk brought about by marital shocks increases the responsiveness of household asset holdings to changes in UI.

Our final exercise computes consumption losses upon unemployment and/or divorce shocks. That is, we take an agent who is married and employed in period $t-1$, and who transits to period t either as: (i) married and unemployed, (ii) single and employed, or (iii) single and unemployed. We compute the average loss in consumption of this generic agent within the group of liquidity-constrained households. The consumption loss upon the unemployment shock, (i), is -6.42%. The marriage shock, (ii), implies a loss of consumption of -2.13%, and a combined unemployment and marriage shock, (iii), brings about a loss of consumption of -34.32%. In our framework without public goods within the household and without divorce costs, the unemployment shock has larger consumption effects than a marriage shock. The two shocks combined amplify consumption losses.

Three remarks concerning the scope of this version of the model, where married and single households co-exist, are in order. First, as indicated above, under limited commitment one of the participation constraints may bind without implying the breakdown of the marriage, a feature our model is silent about. The agent whose participation constraint binds is likely to be employed. This agent will see his/her power within the household increase, thus weakening the savings and labor supply effects of intra-household insurance, especially among low-wealth households. Second, if we interpret cohabitation as a form of marriage without family insurance, and assume that singles in our model are cohabiting couples (i.e., two singles under the same roof), then our model accounts for the differential in savings between cohabiting and married couples observed in the

[14] See González and Özcan (2008) for a recent empirical study using the legalization of divorce in Ireland in 1996. They find results suggesting that the risk of divorce brought about by the law was followed by an increase in the propensity to save of married couples.

U.S. Recent work by Negrasa and Oreffice (2010) uses the 2000 U.S. Census and shows that cohabiting heterosexual couples save more than their married counterparts. The explanation for this differential provided by our model is higher precautionary savings. Third, if we interpret singles in our model as non-cohabiting adults, then our model cannot account for the higher savings rate of married versus single households. The explanation of this failure is that our model abstracts from important drivers of household savings. Two such omissions are life-cycle considerations and households' housing investment. For example, savings for retirement are an important omission in our model. In the U.S., single, non-cohabiting workers are typically the young, at an early stage of their careers. This group saves less for retirement than married, mid-career workers (a popular rule of thumb is to set the saving rate for retirement at one half of your age, so that a 30 years old worker will save 15% of his/her income and a 40 years old 20%). Also, life expectancy is known to be higher for married than for single individuals. The increased life expectancy should encourage the married household to save more in order to store up funds for the longer life time of consumption after retirement. Another example of an omitted driver for savings in our model is saving for children's education, an omission that clearly biases downwards the saving rate of married households in our model economy. A final example is savings in health savings accounts, which also contain an important life-cycle component.

5 Concluding Remarks

In this paper we assess quantitatively the effects of intra-household risk sharing on savings and labor supply. With this purpose, we present a model economy where households are formed by a female and a male who face idiosyncratic unemployment risk and lack access to a complete capital market. Our model is a dynamic version of the standard two-person household model developed by Chiappori and co-authors since the 1980's, which assumes efficient risk sharing within the household. Equipped with this model, we explore the quantitative effects of within-household risk sharing on households' savings and labor supplies, on the crowding out effects of public insurance and on consumption losses upon unemployment. In light of our results, we conclude that intra-household risk sharing has large quantitative effects on all the margins explored. Importantly, we find that our model economy accounts relatively well for key elasticities of savings and spousal labor supply with respect to UI, as estimated by Engen and Gruber (2001) and Cullen and Gruber (2000), respectively. We also show that the standard bachelor model of the household fail to match those elasticities. A conclusion we draw from the exercise in this paper is that ignoring risk sharing at the level of the household introduces an important bias not only on the extent of the precautionary motive but also on the distortionary effects of public insurance programs.

The model presented in this paper can be used to address a number of related questions. In particular, we plan to use versions of this model to shed further light on a recent debate about gender-based taxation. A number of scholars have argued in favor of taxing females and males differently on the grounds of their different elasticities of labor supply. The interplay of income tax rates with Pareto weights within the household is bound to introduce tradeoffs that have been so far overlooked in this debate. Another extension worth pursuing is the consideration of permanent income shocks under lack of commitment to future household allocations. As mentioned above, under no commitment participation constraints may bind, a feature that is likely to affect the role played by intra-household risk sharing. Finally, in light of recent results on the degree of household insurability against different types of shocks and the evolution of consumption and income inequality (see, e.g., Blundell *et al.* 2008), we need models that can account for the observed ability of households to insure different kinds of risks. Models with two-person households and perfect risk sharing within the household are a first step in this direction.

References

[1] Aiyagari, S.R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics,* 109. pp. 659-684.

[2] Attanasio, O. and Rios-Rull, J.V. (2000), "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare? *European Economic Review,* 44, pp. 1225-1258.

[3] Attanasio, O., Low, H. and Sanchez-Marcos, V. (2005), "Female Labor Supply as Insurance Against Idiosyncratic Risk," *Journal of the European Economic Association,*

[4] Blundell, R. and MaCurdy, T.E. (1999), "Labor Supply: A Review of Alternative Approaches," in: Ashenfelter, Q., Card, D. (Eds.), Handbook of Labor Economics, vol. 3A. Elservier, Ambsterdam.

[5] Blundell, R., Pistaferri, L. and Preston, I. (2008), "Consumption Inequality and Partial Insurance," *American Economic Review,* 98(5), pp. 1887-1921.

[6] Browning, M., Chiappori, P.A. and Lechene, V. (2006), "Collective and Unitary Models: A Clarification," *Review of Economics of the Household,* 4, pp. 5-14.

[7] Browning, M. and Crossley, T.F. (2001), "Unemployment Insurance Benefit Levels and Consumption Changes," *Journal of Public Economics,* 80, pp. 1-23.

[8] Chetty, R. and Saez, E. (2010), "Optimal Taxation and Social Insurance with Endogenous Private Insurance," *American Economic Journal: Economic Policy,* 2, pp. 85-114.

[9] Chiappori, P.-A. (1988), "Rational Household Labor Supply," *Econometrica,* 56, pp. 63-89.

[10] Chiappori, P.-A. and Donni, O. (2010), "Non-unitary Models of Household Behavior: A Survey of the Literature," in: Molina, A. (eds.) Household Economic Behaviors, Berlin: Springer.

[11] Cubeddu, L. and Ríos-Rull, J.-V. (2003), "Families as Shocks," CEPR Discussion Paper No. 3924.

[12] Cullen, J.B. and Gruber, J. (2000), "Does Unemployment Insurance Crowd out Spousal Labor Supply?," *Journal of Labor Economics,* 18, pp. 546-572.

[13] Domeij, D. and Floden, M. (2001), "The labor-supply elasticity and borrowing constraints: why estimates are biased," *Review of Economic Dynamics,* 9, pp. 242-262.

[14] Engen, E.M. and Gruber, J. (2001), "Unemployment Insurance and Precautionary Savings," *Journal of Monetary Economics,* 47, pp. 545-579.

[15] Flodén, M. and Lindé, J. (2001), "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics,* 4, pp. 406-437.

[16] Golosov, M. and Tsyvinski, A. (2007), "Optimal Taxation with Endogenous Insurance Markets," *Quarterly Journal of Economics,* 122, pp. 487-534.

[17] González, L., and Özcan, B. (2008), "The Risk of Divorce and Household Saving Behavior," IZA DP No. 3726.

[18] Gorbachev, O. and Dogra, K. (2010), "Evolution of Consumption Volatility for the Liquidity Constrained Households over 1983-2004," mimeo.

[19] Gruber, J. (1997) "The Consumption Smoothing Benefits of Unemployment Insurance," *American Economic Review,* 87, pp. 192-205.

[20] Heathcote J., Storesletten, K. and Violante, G.L. (2010), "The Macroeconomic Implications of Rising Wage Inequality in the U.S.," *Journal of Political Economy,* 118(4), pp. 681-722.

[21] Heathcote J., Storesletten, K. and Violante, G.L. (2009), "The Macroeconomics of the 'Quiet Revolution'," mimeo.

[22] Huggett, M. (1993), "The Risk-free Rate in Heterogenous-agent Incomplete Insurance Economies," *Journal of Economic Dynamics and Control,* 17, pp. 953-969.

[23] Imrohoroglu, A. (1989), "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy,* 97(6), pp. 1364-1383.

[24] Kotlikoff, L.J. and Spivak, A. (1981), "The Family as an Incomplete Annuities Market," *Journal of Political Economy,* 89, pp. 372-391.

[25] Krusell, P., Mukoyama, T. and Şahin, A. (2010), "Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations," *Review of Economic Studies,* 77(4), pp. 1477-1507.

[26] Krusell, P. and Smith Jr., A. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy,* 106, pp. 867-896.

[27] Ligon, E., Thomas, J. P. and Worrall, T. (2000), "Mutual Insurance, Individual Savings, and Limited Commitment," *Review of Economic Dynamics,* 3, pp. 216-246.

[28] Low, H., Meghir, C. and Pistaferri, L. (2008), "Wage Risk and Employment Risk over the Life Cycle," WP08/06 The Institute For Fiscal Studies.

[29] Marcet, A., Obiols-Homs, F. and Weil, P. (2007), "Incomplete Markets, Labor Supply and Capital Accumulation", *Journal of Monetary Economics*, 54(8), pp. 2621-2635.

[30] Mazzocco, M. (2004), "Savings, Risk Sharing and Preferences for Risk," *American Economic Review,* 94(4), pp. 1169-1182.

[31] Mazzocco, M. (2007), "Household Intertemporal Behavior: A Collective Characterization and a Test of Commitment," *Review of Economic Studies,* 74(3), pp. 857-895.

[32] Mazzocco, M., Ruiz, C. and Yamaguchi, S. (2008), "Labor Supply, Wealth Dynamics and Marriage Decisions," California Center for Population Research (CCPR).

[33] Negrasa, B., Oreffice, S. (2010), "Sexual Orientation and Household Savings: Do Homosexual Couples Save More?," IZA DP No. 4961.

[34] Pijoan-Mas, J. (2006), "Precautionary Savings or Working Longer Hours?," *Review of Economic Dynamics,* 9, pp. 326-352.

[35] Shore, S.H. (2010), "For Better, for Worse: Intra-household Risk-sharing over the Business Cycle," *Review of Economics and Statistics,* 92(3), pp. 536-548.

[36] Shore, S.H. and Sinai, T. (2010), "Commitment, Risk, and Consumption: Do Birds of a Feather Have Bigger Nests?" *Review of Economics and Statistics,* 92(2), pp. 408-424.

[37] Stephens Jr., M. (2002), "Worker Displacement and the Added Worker Effect," *Journal of Labor Economics,* 20(3), pp. 504-537.

[38] Zeldes, S. P. (1989), "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy,* 97(2), pp. 305-346.

Table 1. Baseline Parameters

Description	Parameter	Value	Description	Parameter	Value
Relative risk aversion	σ	2	Discount factor (patient)	β_H	0.989
Regulates Frisch elasticity	γ	3	Discount factor (impatient)	β_L	0.982
Utility weight	φ	1.27	Share of impatient households	χ	0.20
Female Pareto weight	μ	0.575	UI replacement rate	b	0.3
Capital share	α	0.36	Relative wages	λ	0.575
Capital depreciation rate	δ	0.025			

Table 2. Unemployment Insurance and Financial Assets

	Elasticity of average assets-to-income ratio w.r.t. UI replacement rate
Data (Engen and Gruber 2001)	−0.28
Two-person Household Economy	−0.39
Bachelor Household Economy	−0.70

Notes: This table shows the response in household asset holdings to the generosity of UI.

Table 3. Female Monthly Hours of Work and Male Employment Status

	Liquidity-constrained households	All households
Employed Husband	166.0	145.0
Unemployed Husband	180.2	148.8

Notes: This table shows average monthly hours of work by working females in households with employed and unemployed males in our baseline economy with two-person households.

Table 4. Individual and household consumption loss upon unemployment

(a) Individual consumption loss upon unemployment:

	Two-person Household		Bachelor Household	
	Liquidity-constrained individuals	All individuals	Liquidity-constrained individuals	All individuals
Females, $\triangle c^f/c^f$	-4.5%	-0.92%	-23.23%	-2.46%
Males, $\triangle c^m/c^m$	-8.74%	-1.78%	-30.09%	-2.44%

(b) Fraction of household income loss that transmits to household consumption loss:

	Two-person Household		Bachelor Household	
	Liquidity-constrained households	All households	Liquidity-constrained households	All households
$\triangle c/\triangle y$	17.31%	3.55%	35.66%	3.34%

Notes: Panel (a) of this table presents individual insurance as measured by the percentage of consumption lost upon an unemployment shock. Panel (b) presents household insurance as measured by the degree of transmission of income loss to consumption upon an unemployment shock.

Table 5. Elasticity of Household Consumption Loss to UI

	Liquidity-constrained households	All households
Data (Browning and Crossley 2001)	−0.1134	−0.047
Two-person Household Economy	−0.0954	−0.0175
Bachelor Household Economy	−0.2262	−0.0212

Notes: Sensitivity of household consumption loss upon unemployment with respect to the generosity of UI.

(a) Savings Policy Functions

(b) Policy Functions for Hours Worked

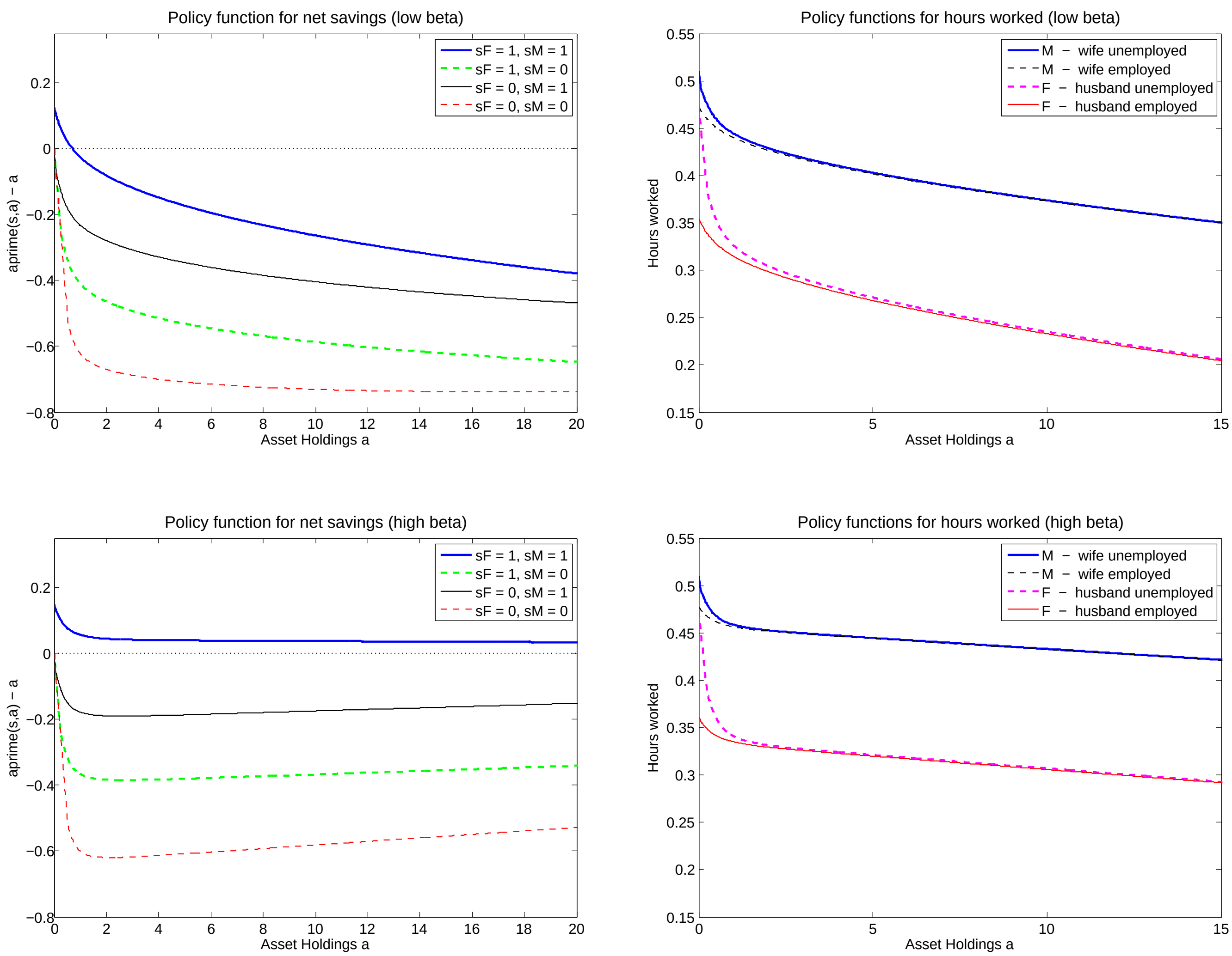
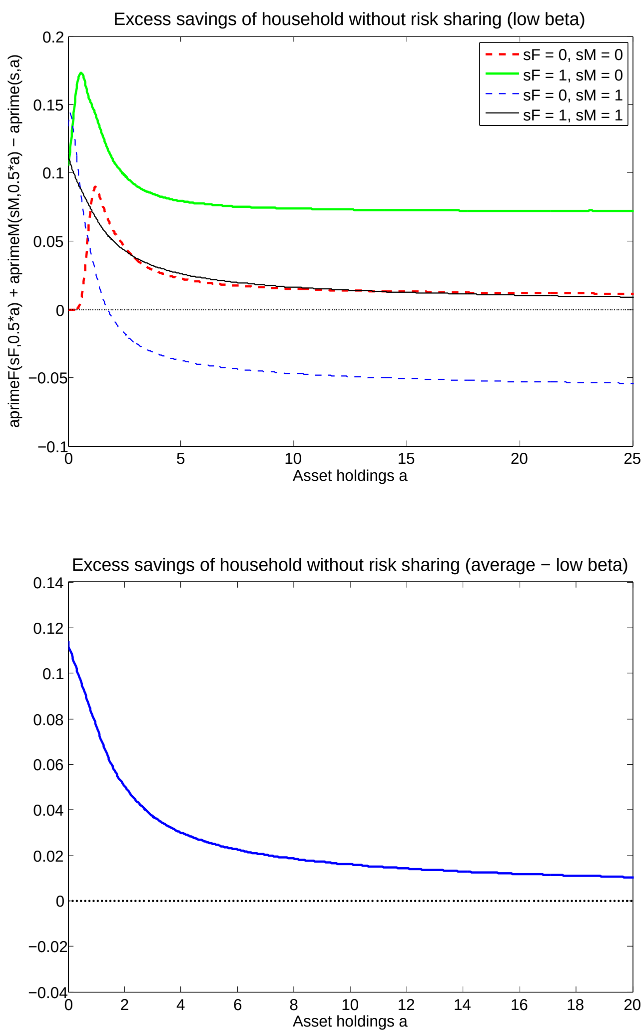


Figure 1: Policy Functions in the Two-person Household Economy

(a) Excess Savings (Low Beta)



(b) Excess Savings (High Beta)

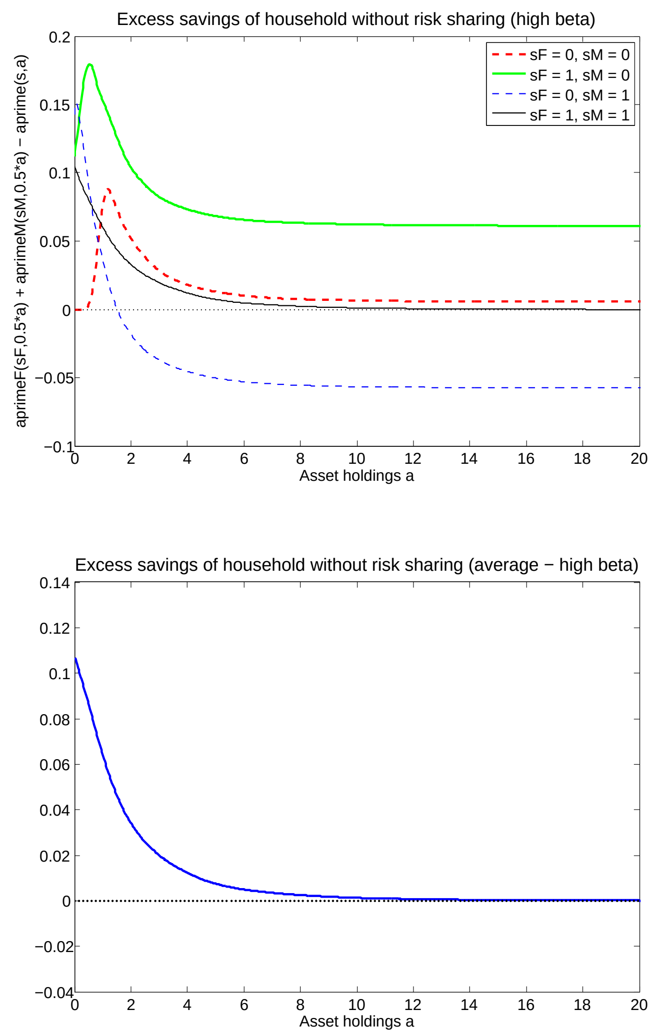
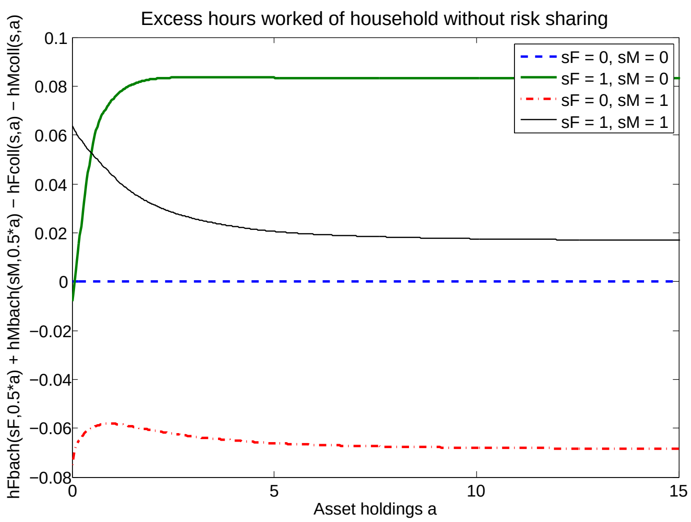


Figure 2: Savings Effect of Intra-household Risk Sharing

Excess Hours Worked



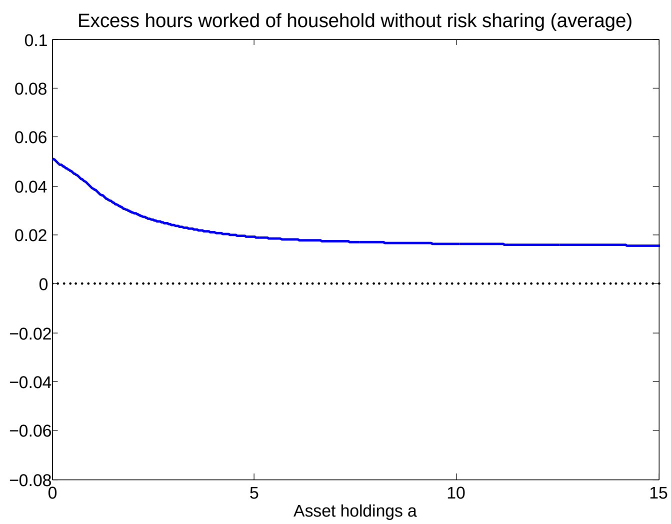


Figure 3: Hours Effect of Intra-household Risk Sharing

APPENDIX (not for publication)

I. Proofs of Propositions 1 and 2

Proof of Proposition 1:

(a) The proof of this part follows from the Contraction Mapping Theorem and Theorem 3 and Corollary 2 in Denardo (1967).

(b) Case 1: We consider first values of a such that $a'(\boldsymbol{s},a) > \underline{a}$ (interior solution).

(i) $c^f(\boldsymbol{s},a)$, $c^m(\boldsymbol{s},a)$ are strictly increasing in a. Take the envelope condition (using **A2**):

$$V_a(\boldsymbol{s},a;\mu) = \mu U_c^f(c^f(\boldsymbol{s},a),\cdot)(1+r) = (1-\mu)U_c^m(c^m(\boldsymbol{s},a),\cdot)(1+r). \tag{I.1}$$

Since $V(\boldsymbol{s},a,\mu)$ is strictly concave, $V_a(\boldsymbol{s},a;\mu)$ is strictly decreasing in a. It follows that $U_c^i(c^i(\boldsymbol{s},a;\mu),\cdot)$, $i=f,m$, must be strictly decreasing in a as well. Since U^i is strictly concave in c^i, the result follows.

(ii) $a'(\boldsymbol{s},a)$ increasing in a. By contradiction: suppose there were values a_1,a_2 such that $a_2 > a_1$ and $a'(\boldsymbol{s},a_2) < a'(\boldsymbol{s},a_1)$. Then since $c^f(\boldsymbol{s},a)$ is strictly increasing in a (as shown before), it has to be that $c^f(\boldsymbol{s},a'(\boldsymbol{s},a_2)) < c^f(\boldsymbol{s},a'(\boldsymbol{s},a_1))$. As utility is separable and the marginal utility of consumption does not depend on the level of leisure, the following holds:

$$\beta(1+r)E\left[U_c^f(c^f(\boldsymbol{s}',a'(\boldsymbol{s},a_2)),\cdot)\right] > \beta(1+r)E\left[U_c^f(c^f(\boldsymbol{s}',a'(\boldsymbol{s},a_1)),\cdot)\right].$$

However, the Euler equation then implies $U_c^f(c^f(\boldsymbol{s},a_2),\cdot) > U_c^f(c^f(\boldsymbol{s},a_1),\cdot)$, which is a contradiction because $c^f(\boldsymbol{s},a_2) > c^f(\boldsymbol{s},a_1)$.

(iii) $l^f(s^f=1,s^m,a)$ and $l^m(s^m=1,s^f,a)$ increasing in a. Intratemporal optimality requires:

$$\frac{U_l^i}{U_c^i} \geq (1-\tau)w^i s^i, \quad \text{for } i=f,m, \tag{I.2}$$

with strict inequality if the labor choice is non-interior. Since $c^i(\boldsymbol{s},a)$ is strictly increasing in a, $U_c^i(c^i(\boldsymbol{s},a),\cdot)$ is strictly decreasing in a. Hence, $U_l^i(\cdot,l^i(s^i=1,s^j,a))$ has to be decreasing in a, too. This implies that $l^i(s^i=1,s^j,a)$ is increasing in a.

Case 2: Consider now values of a such that $a'(\boldsymbol{s},a) = \underline{a}$ (non-interior solution). In this case the budget constraint reads

$$\begin{aligned} c^f(\boldsymbol{s},a) + c^m(\boldsymbol{s},a) = {} & (1-\tau)w^f(1-l^f(\boldsymbol{s},a))s^f + (1-\tau)w^m(1-l^m(\boldsymbol{s},a))s^m \\ & + be^f(1-s^f) + be^m(1-s^m) + (1+r)a - \underline{a}, \end{aligned} \tag{I.3}$$

The proof is by contradiction:

(i) Suppose that $l^f(\boldsymbol{s}, a)$ is decreasing in a and $l^m(\boldsymbol{s}, a)$ is increasing in a. From intratemporal optimality (I.2) it follows that $c^f(\boldsymbol{s}, a)$ must be decreasing in a and that $c^m(\boldsymbol{s}, a)$ must be increasing in a. This is a contradiction with the first-order condition defining risk-sharing rules, i.e., equation (11) in the paper.

(ii) Suppose that $l^f(\boldsymbol{s}, a)$ is increasing in a and $l^m(\boldsymbol{s}, a)$ is decreasing in a. From intratemporal optimality (I.2) it follows that $c^f(\boldsymbol{s}, a)$ must be increasing in a and that $c^m(\boldsymbol{s}, a)$ must be decreasing in a. This is a contradiction with (11).

(iii) Suppose that $l^f(\boldsymbol{s}, a)$ and $l^m(\boldsymbol{s}, a)$ are decreasing in a. From intratemporal optimality (I.2) it follows that $c^f(\boldsymbol{s}, a)$ and $c^m(\boldsymbol{s}, a)$ must be decreasing in a. This is a contradiction with (I.3).

Hence, $l^f(\boldsymbol{s}, a)$ and $l^m(\boldsymbol{s}, a)$ are increasing in a, and (I.3) implies that c^f and c^m are strictly increasing in a.

(c) As a first step, note from the risk-sharing equation, (11), that individual consumptions within the household move always in the same direction, so that if the husband consumption increases so does consumption of the wife, and *viceversa*. We organize the rest of the proof in two cases.

Case 1: Consider values of a such that $a'(\boldsymbol{s}, a) > \underline{a}$ (interior solution).

As in the proof of Lemma 1 in Huggett (1993), it can be shown by induction that $V_a(s^j = 1, s^i, a) \leq V_a(s^j = 0, s^i, a)$, $\forall s^i$, using $(1-\tau)w^i h^i \geq be^i$, for $s^i = 1, s^j \in S$, and the assumption that $\pi^i_{1|1} \geq \pi^i_{1|0}$. The result then follows immediately from the envelope condition (I.1). Case 2: We consider now values of a such that $a'(\boldsymbol{s}, a) = \underline{a}$ (non-interior solution).

First we show that $c^j(s^j = 1, s^i = 0, a) \geq c^j(s^j = 0, s^i = 0, a)$. Evaluating the budget constraint at these two employment shocks we obtain,

$$
\begin{aligned}
&c^j(s^j = 1, s^i = 0, a) + c^i(s^j = 1, s^i = 0, a) + \underline{a} - (1+r)a - (1-\tau^j)w^j h^j(s^j = 1, s^i = 0, a) - be^i = 0, \\
&c^j(s^j = 0, s^i = 0, a) + c^i(s^j = 0, s^i = 0, a) + \underline{a} - (1+r)a - be^j - be^i = 0. \qquad \text{(I.4)}
\end{aligned}
$$

Since $(1-\tau)w^j h^j(s^j = 1, s^i = 0, a) \geq be^j$, this implies that $c^j(s^j = 1, s^i = 0, a) + c^i(s^j = 1, s^i = 0, a) \geq c^j(s^j = 0, s^i = 0, a) + c^i(s^j = 0, s^i = 0, a)$. The result follows from the first-order condition for consumption, (11).

We now show that $c^j(s^j = 1, s^i = 1, a) \geq c^j(s^j = 0, s^i = 1, a)$. Using the budget constraints at $(s^j = 1, s^i = 1)$ and $(s^j = 0, s^i = 1)$, and eliminating terms we get,

$$
\begin{aligned}
&c^j(s^j = s^i = 1, a) + c^i(s^j = s^i = 1, a) - (1-\tau)w^i h^i(s^j = s^i = 1, a) - (1-\tau)w^j h^j(s^j = s^i = 1, a) \\
&= c^j(s^j = 0, s^i = 1, a) + c^i(s^j = 0, s^i = 1, a) - (1-\tau)w^i h^i(s^j = 0, s^i = 1, a) - be^j. \qquad \text{(I.5)}
\end{aligned}
$$

Suppose, towards a contradiction, that $c^i(s^j = 1, s^i = 1, a) < c^i(s^j = 0, s^i = 1, a)$. Intratemporal optimality (I.2) then requires $h^i(s^j = 0, s^i = 1, a) < h^i(s^j = 1, s^i = 1, a)$, and (2.11) implies $c^j(s^j = 1, s^i = 1, a) < c^j(s^j = 0, s^i = 1, a)$. Since $(1-\tau)w^j h^j(s^i = s^j = 1, a) \geq be^j$, the right-hand side of equation (I.5) is strictly larger than the first three terms on the left-hand side, which immediately leads to a contradiction.

(d) Start from $c^j(s^j = 1, s^i, a) \geq c^j(s^j = 0, s^i, a), \forall a$. Then (11) implies that $c^i(s^j = 1, s^i, a) \geq c^i(s^j = 0, s^i, a)$. The result follows immediately from equations (12) and (13).

(e) By contradiction: suppose there is an $a \in [\underline{a}, \overline{a}]$ such that $a'(s^f = 0, s^m = 0, a) > a$, and

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) = \beta(1+r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right], \quad i = f, m.$$

(The equality follows from $a'(s^f = 0, s^m = 0, a) > a \geq \underline{a}$.) Since (i) $\beta(1+r) \leq 1$, (ii) $c^i(\boldsymbol{s}, a)$ strictly increasing in a and (iii) $c^i(\boldsymbol{s}, a)$ is time-invariant if factor prices are constant, it follows that:

$$\beta(1+r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right] \leq E\left[U_c^i(c^i(\boldsymbol{s}', a), \cdot)\right].$$

Combining these two expressions implies that

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) \leq E\left[U_c^i(c^i(\boldsymbol{s}', a), \cdot)\right].$$

Using part (c) this can only hold if $c^i(\boldsymbol{s}, a)$ is the same for all $\boldsymbol{s} \in S \times S$ and, consequently, $a'(\boldsymbol{s}, a) > a$ for all $\boldsymbol{s}$. Since consumption is strictly increasing in a, this implies that future consumption will be strictly higher in any state $\boldsymbol{s}'$ and, hence,

$$U_c^i(c^i(\boldsymbol{s}, a), \cdot) > E\left[U_c^i(c^i(\boldsymbol{s}', a'(\boldsymbol{s}, a)), \cdot)\right].$$

The Euler equation, however, requires

$$U_c^i(c^i(\boldsymbol{s}, a), \cdot) = \beta(1+r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s, a)), \cdot)\right],$$

which is impossible for $\beta(1+r) \leq 1$.
Strict inequality: suppose there is an $a \in (\underline{a}, \overline{a})$ such that $a'(s^f = 0, s^m = 0, a) = a$. Using part (c) it follows that $a'(\boldsymbol{s}, a) \geq a$ for all $\boldsymbol{s}$. Since consumption is strictly increasing in a, this implies that future consumption will be at least as high as current consumption in any state s' and, hence,

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) \geq E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, , a)), \cdot)\right].$$

The Euler equation, however, requires

$$U_c^i(c^i(s^f = 0, s^m =, a), \cdot) = \beta(1+r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right],$$

(the equality follows from $a'(s^f = 0, s^m = 0, a) = a > \underline{a}$). This is impossible for $\beta(1+r) < 1$.

Proof of Proposition 2:

First we pin down the value of $\tilde{a}(\mu)$. To simplify notation, define $\bar{l}^i \equiv 1 - be^i/[(1-\tau)w^i]$. Since utility is separable in consumption and leisure, we can plug (11) into the first-order conditions to female and male labor supply to obtain

$$U_c^f s^f \leq \frac{U_l^f}{(1-\tau)w^f} \tag{I.6}$$

$$U_c^f s^m \leq \frac{U_l^m}{(1-\tau)w^m}\frac{1-\mu}{\mu}, \tag{I.7}$$

with strict inequalities if $s^i = 1 \wedge l^i = \bar{l}^i$, or $s^i = 0 \wedge l^i = 1$, $i = f, m$. Define $\widetilde{U}_l^i$ as the marginal utility of leisure for individual $i = f, m$, at $l^i = \bar{l}^i$. Also, define

$$\widetilde{U}_c^f(\mu) \equiv \min\left\{\frac{\widetilde{U}_l^f}{(1-\tau)w^f}\ ,\ \frac{\widetilde{U}_l^m}{(1-\tau)w^m}\frac{1-\mu}{\mu}\right\} \tag{I.8}$$

and $\widetilde{U}_c^m(\mu) \equiv \frac{\mu}{1-\mu}\widetilde{U}_c^f(\mu)$. Let $\widetilde{c}^i(\mu)$ be the level of consumption for which the corresponding marginal utility of consumption equals $\widetilde{U}_c^i(\mu)$. Then the level of asset holding $\tilde{a}(\mu)$ mentioned above is defined as

$$\widetilde{a}(\mu) \equiv \frac{1}{r}\Big[\widetilde{c}^f(\mu) + \widetilde{c}^m(\mu)\Big]. \tag{I.9}$$

It can easily be checked that at $\widetilde{a}(\mu)$, equations (9)–(11), (I.6) and (I.7) are satisfied for all possible realizations of s^f and s^m, if consumption levels equal $\widetilde{c}^f(\mu)$ and $\widetilde{c}^m(\mu)$, hours worked equal $l^i(s^i = 1, s^j, \widetilde{a}(\mu); \mu) = \bar{l}^i$ and $l^i(s^i = 0, s^j, \widetilde{a}(\mu); \mu) = 1$, $\forall s^j$, $i = f, m$, and asset holdings remain constant. In the case that $\beta(1+r) = 1$, equation (15) is satisfied, because consumption is constant. Hence, if $\beta(1+r) = 1$, optimal decision rules are

$$c^i(\boldsymbol{s}, \widetilde{a}(\mu); \mu) \ = \widetilde{c}^i(\mu) \tag{I.10}$$

$$l^i(s^i = 1, s^j, \widetilde{a}(\mu); \mu) \ = \bar{l}^i, \qquad \forall s^j \tag{I.11}$$

$$l^i(s^i = 0, s^j, \widetilde{a}(\mu); \mu) \ = 1, \qquad \forall s^j \tag{I.12}$$

$$a'(\boldsymbol{s}, \widetilde{a}(\mu); \mu) \ = \widetilde{a}(\mu), \tag{I.13}$$

for $i = f, m$ and for all $\boldsymbol{s} \in S \times S$. Thus, if the household ever reaches $\widetilde{a}(\mu)$, it will maintain a constant consumption stream. For lower interest rates, constant consumption does not satisfy the first-order condition for asset holdings, and the household never reaches $\widetilde{a}(\mu)$. We can now prove the Proposition.

In order to compact notation, we will write $\widetilde{a}(\mu)$ simply as $\widetilde{a}$.

(a) Let us first assume $r > 0$. We prove that $a'(\boldsymbol{s}, \widetilde{a}) \leq \widetilde{a}$. The result then follows from the fact that $a'(\boldsymbol{s}, a)$ is increasing in a, as shown before. From part (c) of Proposition 1,

$$a'(s^f = 0, s^m = 0, \widetilde{a}) \leq \widetilde{a} \tag{I.14}$$

Then, using the budget constraint we obtain,

$$(1+r)\widetilde{a} - c^f(s^f = 0, s^m = 0, \widetilde{a}) - c^m(s^f = 0, s^m = 0, \widetilde{a}) + be^f + be^m \leq \widetilde{a} \tag{I.15}$$

$$c^f(s^f = 0, s^m = 0, \widetilde{a}) + c^m(s^f = 0, s^m = 0, \widetilde{a}) - be^f - be^m \geq r\widetilde{a}. \tag{I.16}$$

From before we know that decision rules for consumption are increasing both in the own and the spousal employment shock. Hence,

$$c^f(\boldsymbol{s}, \widetilde{a}) + c^m(\boldsymbol{s}, \widetilde{a}) - be^f - be^m \geq r\widetilde{a} \;, \qquad \forall \boldsymbol{s}. \tag{I.17}$$

Using the definition of $\widetilde{a}$ from above and the first-order conditions with respect to leisure, we know that $l^i(s^i = 1, s^j, \widetilde{a}) = \bar{l}^i, \forall s^j$, and $l^i(s^i = 0, s^j, \widetilde{a}) = 1, \forall s^j, i = f, m$. As $(1-\tau)w^i(1-\bar{l}^i) = be^i, i = f, m$, it is straightforward to show that:

$$\begin{aligned} c^f(\boldsymbol{s}, \widetilde{a}) + c^m(\boldsymbol{s}, \widetilde{a}) - (1-\tau)w^f(1 - l^f(\boldsymbol{s}, \widetilde{a}))s^f - (1-\tau)w^m(1 - l^m(\boldsymbol{s}, \widetilde{a}))s^m - be^f(1-s^f) \\ -\; be^m(1-s^m) \;\geq c^f(\boldsymbol{s}, \widetilde{a}) + c^m(\boldsymbol{s}, \widetilde{a}) - be^f - be^m, \qquad \forall \boldsymbol{s}. \end{aligned}$$

Plugging this relation into (I.17) and rearranging terms yields $a'(\boldsymbol{s}, \widetilde{a}) \leq \widetilde{a}$.
Case $r \leq 0$: Take $a_1 < a_2$ and thus $c^f(\boldsymbol{s}, a_1) + c^m(\boldsymbol{s}, a_1) < c^f(\boldsymbol{s}, a_2) + c^m(\boldsymbol{s}, a_2)$. Plug in the budget constraints and eliminate terms:

$$\begin{aligned} &(1-\tau^f)w^f(1 - l^f(\boldsymbol{s}, a_1))s^f + (1-\tau)w^m(1 - l^m(\boldsymbol{s}, a_1))s^m + (1+r)a_1 - a'(\boldsymbol{s}, a_1) < \\ &(1-\tau^f)w^f(1 - l^f(\boldsymbol{s}, a_2))s^f + (1-\tau)w^m(1 - l^m(\boldsymbol{s}, a_2))s^m + (1+r)a_2 - a'(\boldsymbol{s}, a_2) \end{aligned}$$

and thus

$$\begin{aligned} a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1) <& (1+r)(a_2 - a_1) + (1-\tau)w^f(l^f(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^f \\ &+ (1-\tau)w^m(l^m(\boldsymbol{s}, a_1) - l^m(\boldsymbol{s}, a_2))s^m. \end{aligned}$$

Divide by $a_2 - a_1$:

$$\begin{aligned} \frac{a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1)}{a_2 - a_1} <& (1+r) + \frac{1}{a_2 - a_1}\Big[(1-\tau)w^f(l^f(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^f \\ &+ (1-\tau)w^m(l^m(\boldsymbol{s}, a_1) - l^m(\boldsymbol{s}, a_2))s^m\Big]. \end{aligned}$$

Since leisure is increasing in a, the last two terms are non-positive. Also, r is non-positive by assumption. Therefore,

$$\frac{a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1)}{a_2 - a_1} < 1.$$

That is, the decision rule for capital accumulation has a slope that is strictly lower than 1 and strictly positive.

(b) Take an arbitrary level of asset holdings $a_0 \geq \widetilde{a}$ and check whether the proposed allocation $\left\{\widehat{c}^f, \widehat{c}^m, \widehat{l}^f, \widehat{l}^m, \widehat{a}'\right\}$ satisfies first-order optimality:

- equation (2.11) is satisfied by definition
- $\widehat{c}^f + \widehat{c}^m = a\, r \geq \widetilde{a}\, r = \widetilde{c}^f + \widetilde{c}^m$; moreover, $\widehat{c}^i \geq \widetilde{c}^i \Longrightarrow \widehat{U}_c^i \leq \widetilde{U}_c^i$, $i = f, m$, which implies by (I.8) that equations (I.6) and (I.7) are satisfied
- the budget constraint (9) holds and
- the Euler equation (15) holds because consumption is constant.

Since the problem is concave, first-order optimality is sufficient for an optimum. Since the policy functions characterize the optimum, the proposed allocation is optimal.

(c) The proof exploits results in Chamberlain and Wilson (2000), which are also used in Marcet, Obiols-Homs and Weil (2007). Part (a) implies that $a_t \leq \widetilde{a}$, $\forall t$, and part (b) of Proposition 1 together with part (b) of Proposition 2 imply that $c_t^i \leq \widetilde{c}^i$, $i = f, m$, so that individual consumption levels are bounded almost surely. The first-order condition to savings (15) and (16) imply that $U_{c,t}^i \geq E_t(U_{c,t+1}^i)$ almost surely, so that $U_{c,t}^i$ is a super-martingale. As $U_{c,t}^i$ is bounded from below by $U_c^i\left(\widetilde{c}^i\right)$, we can apply the martingale convergence theorem, which implies that $U_{c,t}^i$ converges almost surely to a random variable. Suppose, by contradiction, that $U_{c,t}^i$ converged to a value strictly larger than $U_c^i\left(\widetilde{c}^i\right)$, which would imply that consumption levels would converge to values $\widehat{c}^i < \widetilde{c}^i$, so that the consumption-leisure choice would be interior for at least one of the two spouses when employed. In that case labor income would converge to $\iota \equiv (1-\tau)w^f(1-\widehat{l}^f)s^f + (1-\tau)w^m(1-\widehat{l}^m)s^m$, where $\widehat{l}^f$ and/or $\widehat{l}^m$ are strictly smaller than $\bar{l}^f$ and/or $\bar{l}^m$, respectively, and solve (I.6) and (I.7). ι is a non-degenerate random variable with positive variance, which implies that the lower or upper bounds on asset holdings would be violated with positive probability, a contradiction. This follows from the result of Chamberlain and Wilson (2000) that under $\beta(1+r) = 1$ consumption and assets grow with no bound if income is suitably stochastic. Thus, $U_{c,t}^i$ cannot converge to a value strictly larger than $U_c^i\left(\widetilde{c}^i\right)$ and it must converge to $U_c^i\left(\widetilde{c}^i\right)$. Since U_c^i is invertible, consumption will converge to $\widetilde{c}^i$. The budget constraint implies that a_t must converge to $\widetilde{a}$.

II. Household Risk Aversion with Intra-household Risk Sharing

In this appendix we derive the coefficient of risk aversion of the two-person household as a function of individual preferences for risk and the relative Pareto weight. We also show that the derivative of the risk-sharing rule for a household member of gender $i = f, m$, is given by the product of the household's coefficient of risk aversion and the individual's coefficient of risk tolerance.

The coefficient of absolute risk aversion of a bachelor household with instantaneous utility function $U^i(c, l)$ is defined as

$$\rho^i \equiv -\frac{U^i_{cc}}{U^i_c}, \text{ for } i = f, m.$$

When two individuals with different attitudes towards risk form a household and share risks, the household's coefficient of risk aversion is obviously different from the individual ones. Collective household's risk preferences will depend on individual preferences and Pareto weights.

Two-person Household's Risk Aversion

Let us denote the utility function of the two-person household over total household consumption, y, and individual leisures, l^f and l^m, by $u(y, l^f, l^m; \mu)$. This utility function is defined as,

$$\begin{aligned} u(y, l^f, l^m; \mu) &= \max_{c^f, c^m}\{\mu U^f(c^f, l^f) + (1-\mu)U^m(c^m, l^m)\} \\ s.t. &= c^f + c^m = y. \end{aligned}$$

With this utility function we can write the maximization problem solved by the two-person household as,

$$\begin{aligned} \tilde{V}(\boldsymbol{s}, a; \mu) &= \max_{l^f, l^m, a', \tilde{c}} \{u(\tilde{c}, l^f, l^m; \mu) + \beta \sum_{\boldsymbol{s'}} \pi_{\boldsymbol{s'}|\boldsymbol{s}} \tilde{V}(\boldsymbol{s'}, a'; \mu)\} \\ s.t. & \quad \tilde{c} + a' = \sum_{i=f,m} (1-\tau)w^i(1-l^i)s^i + \sum_{i=f,m} be^i(1-s^i) + (1+r)a. \end{aligned}$$

The coefficient of absolute risk aversion of a two-person household with Pareto weight μ can then be defined as,

$$\rho_\mu \equiv -\frac{u_{yy}}{u_y}.$$

Let us assume individual utility functions of the form:

$$U^i(c, l) = \frac{c^{1-\sigma^i} - 1}{1-\sigma^i} + \varphi^i \frac{l^{1-\gamma^i} - 1}{1-\gamma^i},$$

for $i = f, m$. To derive this coefficient of risk aversion let us consider the first-order condition to the static maximization problem embedded into the household problem, i.e.,

$$\mu(c^f)^{-\sigma^f} = (1-\mu)(c^m)^{-\sigma^m}.$$

Taking logarithms on both sides of this equation and differentiating with respect to y yields,

$$\sigma^f \frac{dc^f}{dy}\frac{1}{c^f} = \sigma^m \frac{dc^m}{dy}\frac{1}{c^m}.$$

Using that $\frac{dc^f}{dy} + \frac{dc^m}{dy} = 1$, we can solve for for dc^f/dy as,

$$\frac{dc^f}{dy} = \left(1 + \frac{\sigma^f}{\sigma^m}\frac{c^m}{c^f}\right)^{-1}.$$

Now, if we take the derivative of u^μ with respect to y, and use the first-order condition gives,

$$u_y = \mu(c^f)^{-\sigma^f}.$$

Differentiating this equation with respect to y again yields,

$$u_{yy} = -\sigma^f \mu(c^f)^{-\sigma^f-1}\frac{dc^f}{dy}.$$

Then, the coefficient of absolute risk aversion of a household with Pareto weight μ is,

$$\rho_\mu = \frac{\sigma^f\sigma^m}{\sigma^m c^f + \sigma^f c^m},$$

and the coefficient of relative risk aversion is $\frac{\sigma^f\sigma^m(c^f+c^m)}{\sigma^m c^f+\sigma^f c^m}$.

Now, it is straightforward to show that the derivatives of the sharing rules, $\frac{dc^f}{dy}$ and $\frac{dc^m}{dy}$, are given by the household's coefficient of absolute risk aversion, ρ_μ, times the coefficient of absolute risk tolerance of each individual in the household. From the first-order condition it is easily obtained that

$$\frac{dc^f}{dy} = \left(\frac{\sigma^f\sigma^m}{\sigma^m c^f + \sigma^f c^m}\right)\frac{c^f}{\sigma^f},$$

where the expression within brackets on the right-hand side is the household's coefficient of absolute risk aversion and the second term, c^f/σ^f, is the individual's coefficient of absolute risk tolerance. The same result can be shown for $\frac{dc^m}{dy}$.

References

[1] Chamberlain, G., and Wilson, C. A. (2000), "Optimal Intertemporal Consumption under Uncertainty," *Review of Economic Dynamics*, 3, pp. 365-395.

[2] Denardo, E.V. (1967), "Contraction Mappings in the Theory Underlying Dynamic Programming," *SIAM Review,* 9(2), pp. 165-177.