

Asset Pricing with Heterogeneous Inattention

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Abstract

Does households' limited attention to the stock market affect asset prices? I address this question introducing an observation cost in a multi-agent production economy with incomplete markets and idiosyncratic labor income risk. In this environment, inattention changes endogenously over time and across agents. The model generates a limited equity market participation, a realistic distribution of wealth and stock return predictability. Furthermore, inattention implies countercyclical dynamics for both the stock returns volatility and the Sharpe ratio. Nonetheless, the equity premium is still low, around 1%. Finally, I find that inattention affects asset prices only when borrowing constraints are tight enough.

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Does households' limited attention to the stock market affect asset prices? I address this question introducing an observation cost in a multi-agent production economy with incomplete markets and idiosyncratic labor income risk. In this environment inattention changes endogenously over time and across agents. The presence of the observation cost improves the overall performance of the model, generating limited equity market participation, stock return predictability and countercyclical patterns for both the stock returns volatility and the Sharpe ratio. Yet, inattention cannot account for the bulk of the dynamics of stock prices.

This paper studies the role of households' inattention by relaxing the assumption that agents are always aware of the state of the economy. Despite standard models postulate that households continuously collect information on the stock market and derive optimal consumption/savings plans, in the data we observe a different pattern. For example, Ameriks *et al.* (2003) show that households plan infrequently, with wealthy agents planning more often than poor ones. Alvarez *et al.* (2012) use data from two Italian surveys and find that the median household pays attention to the stock market every 3 months. Furthermore, there is a sizable heterogeneity in inattention across households: 24% of agents observe the financial portfolios less than twice per year, whereas 20% of them do it more often than once per week. Finally, Rossi (2010), Da *et al.* (2011), Sichertmann *et al.* (2012), Vlastakis and Markellos (2012) and Andrei and Hasler (2013) find that the allocation of attention is time-varying, although the sign of the relation between inattention and financial returns is ambiguous.¹ This evidence has motivated a new strand of literature, which concentrates on infrequent planning and limited attention as potential solutions to the equity premium puzzle. A priori,

¹Few other papers show that investors' allocation of attention affects stock prices, e.g. Barber and Odean (2008), Della Vigna and Pollet (2009), Hirshleifer *et al.* (2009) and Mondria *et al.* (2010).

these factors could improve the performance of standard models by increasing the risk of holding stocks and implying a low correlation between consumption and equity returns. Nonetheless, the literature finds inconclusive results. On one hand, Lynch (1996), Gabaix and Laibson (2002), Rossi (2010) and Chien *et al.* (2011, 2012) show that models embodying inattention or infrequent planning can match the level and the dynamics of stock and bond prices. On the other hand, Chen (2006) and Finocchiaro (2011) find that although these features do increase the volatility of stock returns, they have no effects on the equity premium.

In this paper I evaluate whether the observed duration of households' inattention can account for the equity premium and the dynamics of asset prices. I develop a model that plugs the inattention of Reis (2006) in the environment of Krusell and Smith (1997, 1998). Basically, I consider a production economy with incomplete markets and heterogeneous agents, who incur in an observation cost whenever they collect information on the state of the economy and formulate a new plan for consumption and financial investment. This feature creates a trade-off: attentive households take better decisions, but also bear higher costs. As a result, they decide to plan at discrete dates and stay inattentive meanwhile. Inattentive agents do not gather new information and follow by inertia pre-determined paths of consumption and investment. To discipline the role of infrequent planning, I calibrate the observation cost to match the actual duration of inattention for the median household, as estimated by Alvarez *et al.* (2012).

Looking at the results of the model, I find that inattention differs across agents and co-moves with financial returns. The level of inattention depends negatively on households' wealth - in line with the evidence of Ameriks *et al.* (2003) - because poor agents face disproportionately higher observation costs. The cyclicity of inattention depends on the marginal gain and the marginal

cost of being attentive and actively investing in the stock market. Both forces are countercyclical, but they asymmetrically affect different agents. Poor households plan in expansions because they cannot afford the observation cost in bad times. Instead, wealthy agents plan in recession to benefit of the higher expected return to equity. Overall the level of inattention is countercyclical. Second, the participation to the equity market is limited because the observation cost is de facto a barrier to the investment in stocks. In turn, limited participation implies a more realistic wealth distribution since only wealthy stockholders can benefit of the returns to equity. In the benchmark model, inattention impedes 27% of households to participate in the stock market and raises the Gini index of wealth by 56%. Third, the volatility of stock returns is high and countercyclical. The observation cost increases volatility by an order of thirteen because it acts as a capital adjustment cost. Indeed, inattentive agents cannot immediately adjust their financial positions to the realizations of the aggregate productivity shock. Furthermore, the limited participation to the equity market intensifies the inelasticity in the supply of capital. More interestingly, the countercyclical dynamics of inattention implies time-varying adjustment costs which are more stringent in bad times. As a result, the volatility of stock returns peaks in recessions. Inattention has two further effects on stock prices. On one hand, it generates a weak correlation between equity returns and consumption growth, through the slow dissemination of information across agents. On the other hand, it induces large variations in the excess returns and the Sharpe ratio of equity. Such dynamics are usually obtained through consumption habits or heteroskedastic consumption growth. Instead, here they are just the by-product of the observation cost, that concentrates the aggregate risk on a small measure of agents. Indeed, at each point of time there are few attentive investors that trade stocks and bear

the whole aggregate risk of the economy. In addition, inattentive agents create a residual aggregate risk by consuming too much in bad times and too little in good times. Such behavior forces attentive stockholders to switch their consumption away from times in which their marginal utility is high. As a result, they command a higher premium for clearing the goods market, especially in recessions. In this respect, the model endogenizes the limited stock market participation and heterogeneity in trading technologies that Guvenen (2009) and Chien *et al.* (2011, 2012) take as exogenous to replicate the dynamics of asset prices. Fourth, in the benchmark model the equity premium is still around 1%. The price of risk is low because households react to the observation cost, accumulating savings and deleveraging out of stocks. This mechanism explains why increasing the magnitude of the observation cost does not even alter the level of the Sharpe ratio. Finally, I find that the effects of inattention on asset prices crucially depend on the specification of the borrowing constraints. When they are loose enough, all agents participate in the stock market following buy-and-hold positions, as already pointed out in Chen (2006). Households dilute the observation cost by trading more infrequently, without the risk of hitting the borrowing constraint. In this environment inattention plays no role at all and the model fails in matching asset prices.

As a remainder of the paper, Section I discusses the related literature while I introduce the model and characterize its equilibrium in Section II. Section III is devoted to the calibration and the computation of the model, while I discuss its quantitative predictions in Section IV. Finally, Section V concludes.

I Related Literature

This paper adds to the literature on the equity premium puzzle. Since the seminal paper of Mehra and Prescott (1985), many solutions have been proposed: long-run risk (Bansal and Yaron, 2004), consumption habits (Campbell and Cochrane, 1999), and limited stock market participation (Guisan, 2009), among others. The emphasis of this paper is on households' inattention to the stock market. In the literature, households' inattention is usually achieved either by making agents gathering information and planning financial investment at discrete dates (e.g., Duffie and Sun, 1990; Lynch, 1996; Gabaix and Laibson, 2002; Chen, 2006; Reis, 2006 and Finocchiaro, 2011), or through learning with capacity constraints (as in Sims, 2003; Peng, 2005; Huan and Liu, 2007 and Mondria, 2010).² I follow the first strand of the literature because of my emphasis on the effects of inattention on agents' portfolio decisions. Indeed, I study a heterogeneous agent economy, where any household can react to the risk of inattention by modifying its portfolio. This feature avoids having a representative agent which in equilibrium holds anyway the market portfolio. Models featuring learning with capacity constraint can be extended to the case of heterogeneous agents only by neglecting the existence of higher-order beliefs, as discussed in Porapakarm and Young (2008).³ Yet, Angeletos and La'O (2009) show that higher-order beliefs do a crucial role in the dissemination of information across agents. Instead, models in which inattention is modeled as agents gathering information at discrete times do not suffer of this problem.

²The notion of inattention is also closely tied to the concept of information acquisition, e.g. Grossman and Stiglitz (1980), Peress (2004) and Hirshleifer et al. (2011), and the one of learning and uncertainty, see Veronesi (1999) and Andrei and Hasler (2013).

³When agents have imperfect common knowledge and differ in their information set, they need to forecast other agents' forecast, and so on so forth. In such a case, equilibrium prices do not depend only on the infinite dimension distribution of agents across wealth, but also on the infinite dimension distribution of beliefs.

My paper differs from the literature on inattention on two main dimensions. First, I discipline the role of infrequent planning by calibrating the observation cost to match the actual duration of inattention for the median household. In this way, I can evaluate whether the observed level of inattention can quantitatively account for the dynamics of asset prices. Second, I identify the mechanisms tempering or amplifying the effects of the observation cost on stock prices. In this respect, this paper mirrors the analyses that Pijoan-Mas (2007) and Gomes and Michaelides (2008) carried out for habits and agents heterogeneity.

II The Model

In the continuous-time economy there is a representative firm that uses capital and labor to produce a consumption good. On the other side, there is unit measure of ex-ante identical agents. Households are ex-post heterogeneous because they bear an uninsurable idiosyncratic labor income risk. Moreover, they face a monetary observation cost whenever collecting information on the states of the economy and choosing consumption and savings. As a result, agents decide to plan infrequently and stay inattentive meanwhile.

II.A The Firm

The production sector of the economy constitutes of a representative firm, which produces a homogeneous consumption good $Y_t \in \mathbf{Y} \subset \mathbb{R}_+$ using a Cobb-Douglas production function

$$Y_t = z_t N_t^{1-\eta} K_t^\eta$$

where $\eta \in (0, 1)$ denotes the capital income share. The variable $z_t \in \mathbf{Z} \subset \mathbb{R}_+$ follows a stationary continuous Markov process with transition probabilities $\Gamma_z(z', z) = \Pr(z_h = z' | z_t = z)$, for any $h \geq t$. The firm hires $N_t \in \mathbf{N} \subset \mathbb{R}_+$ workers at the wage w_t , and rents from households the stock of physical capital $K_t \in \mathbf{K} \subset \mathbb{R}_+$ at the instantaneous interest rate r_t^a . Physical capital depreciates at a rate $\delta \in (0, 1)$ after production. At every point of time, after the realization of the shock z , the firm chooses capital and labor to maximize profits π_t , where

$$d\pi_t = \left[z_t N_t^{1-\eta} K_t^\eta - (\delta + r_t^a) K_t - w_t N_t \right] dt$$

The first order conditions yield the equilibrium rental rate and wage

$$\begin{aligned} r_t^a &= \eta z_t N_t^{1-\eta} K_t^{\eta-1} - \delta \\ w_t &= (1 - \eta) z_t N_t^{-\eta} K_t^\eta \end{aligned} \tag{1}$$

Both prices depend on the realization of the aggregate productivity shock z_t . I intentionally abstract from any adjustment cost to focus on inattention as the only source of slowly-moving capital, as in Duffie (2010).

II.B Households

The economy is populated by a measure one of ex-ante identical households. They are infinitely lived, discount the future at the positive rate $\rho \in (0, 1)$ and maximize lifetime utility

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} U(c_t) dt$$

where $c_t \in \mathbf{C} \subset \mathbb{R}_+$ denotes consumption at time t . The utility function is a CRRA, $U(c) = \frac{c^{1-\theta}}{1-\theta}$, where θ denotes the risk aversion of households.

II.B.1 Idiosyncratic Shocks

As in Pijoan-Mas (2007), households bear an idiosyncratic labor income risk which consists of two components. First, agents are hit by a shock $e_t \in \mathbf{E} \subset \{0, 1\}$, which determines their employment status. A household has a job when $e_t = 1$ and is unemployed when $e_t = 0$. I assume that e_t follows a stationary continuous Markov process with transition probabilities

$$\Gamma_e(z, z', e, e') = \Pr(e_h = e' | e_t = e, z_t = z, z_h = z'), \quad h \geq t$$

The shock is idiosyncratic and washes out in the aggregate. Yet, its transition probabilities depend on the aggregate productivity shock. As a consequence, both the idiosyncratic uncertainty and the unemployment rate of the economy rise in recessions.⁴ Second, when a household is given a job, it faces a further shock $\xi_t \in \mathbf{\Xi} \subset \mathbb{R}_+$, which determines the efficiency units of hours worked. This shock is orthogonal to the aggregate productivity shock and follows a stationary continuous Markov process with transitional probabilities

$$\Gamma_\xi(\xi, \xi') = \Pr(\xi_h = \xi' | \xi_t = \xi), \quad h \geq t$$

When a household is unemployed, it receives a constant unemployment benefit $\bar{w} > 0$. Households' labor income l_t is then

$$l_t = w_t \xi_t e_t + \bar{w} (1 - e_t)$$

⁴I define such a structure for the employment shock following Mankiw (1986), who shows that a countercyclical idiosyncratic uncertainty accommodates a higher price of risk. Without such feature, incomplete markets would not affect the equity premium, as discussed in Krueger and Lustig (2010). Anyway, Storesletten et al. (2007) find that in the data labor income risk does peak in recessions.

II.B.2 Market Arrangements

Households own the capital of the economy. Each agent holds $a_t \in \mathbf{A} \equiv [\underline{a}, \infty]$ units of capital, which is either rented to the firm or traded among households. Capital is risky and yields the rate r_t^a , as defined in (1). Agents can also invest in a one-period non-contingent bond $b_t \in \mathbf{B} \equiv [\underline{b}, \infty]$, which is in zero net supply. The bond yields a risk-free rate r_t^b . Households face exogenous borrowing constraints for both assets and cannot go shorter than \underline{b} in the risk-free bond and \underline{a} for the risky equity. When these values equal zero, no short position is allowed at all. Hereafter, I define households' financial portfolio as $f_t = a_t + b_t \in \mathbf{F} \subset [\underline{f}, \infty]$, which is bounded below by a borrowing constraint \underline{f} . The share of the portfolio invested in stocks is then $\alpha_t = a_t/f_t \in \mathbf{A} \subset [0, 1]$. Instead, I denote financial wealth as $\omega_t = e^{r_t^a} a_{t-} + e^{r_t^b} b_{t-} = e^{\alpha_{t-}(r_t^a - r_t^b) + r_t^b} f_{t-} \in \mathbf{\Omega} \subset \mathbb{R}$. In equilibrium, the optimal choices of the agents depend on the levels of both labor income and financial wealth.

In this framework, markets are incomplete because agents cannot trade claims which are contingent on the realizations of the idiosyncratic shock. As long as the labor income risk cannot be fully insured, agents are ex post heterogeneous in wealth, consumption and portfolio choices.

II.B.3 Observation Cost

Agents incur in a monetary observation cost proportional to their labor income χl_t whenever acquiring information on the state of the economy and defining the optimal choices on consumption and savings. This cost is a reduced form for the financial and time opportunity expenditures bore by households to figure out the optimal composition of the financial portfolio. The observation cost induces the

agents to plan infrequently and stay inattentive meanwhile. Planning dates are defined as discrete dates D_i , $i \in \mathbb{N}_0$, such that $D_{i+1} \geq D_i$ for any i . At a planning date D_i , households pay the cost χl_{D_i} , collect the information on the states of the economy and decide the next planning date D_{i+1} . Hereafter, I refer to $d_{i+1} = D_{i+1} - D_i$ as the duration of inattention and consider it as one of the choices of households. At planning dates, households define also the level of consumption throughout the period of inattention⁵ $c \equiv [c_{D_i}, c_{D_{i+1}})$ and both the amount and the composition of the financial portfolio, f_{D_i} and α_{D_i} . Instead, at non planning dates, households are inattentive and follow the pre-determined plan for consumption. I assume that the financial portfolio of inattentive households is re-balanced every period to match the initial share of risky assets α_{D_i} .⁶

In the model, attentive households observe the states of the economy, while inattentive ones do not. These states include the realizations of the aggregate productivity and the idiosyncratic labor income shock. On one hand, it is reasonable to assume that agents are not fully aware of the actual realization of the aggregate shock.⁷ On the other hand, inattentive agents cannot observe even their labor income. This condition is required to preserve the computational tractability of the model. Indeed, if households could also observe their stream of labor income, then they would always gather some new information. Hence, agents would make their decision on whether to be inattentive on a continuous

⁵In Abel et al. (2007, 2013) and Rossi (2010), households finance consumption through checking account funds which pay a lower interest rate than risk-free bonds. Instead, I assume that inattentive agents finance instantaneous consumption with their financial wealth. I abstract from any liquidity and cash-in-advance constraint to isolate the effects of the observation cost on asset prices.

⁶This assumption, which is also made in Gabaix and Laibson (2002), Abel et al. (2007) and Alvarez et al. (2012), is consistent with the empirical evidence on weak portfolio re-balancing across households. For example, Brunnermeier and Nagel (2008) show that the share of the financial portfolio invested in risky assets moves very slowly over time, and it is not very sensitive to changes in households' wealth. Ameriks and Zeldes (2004) study a ten-year panel of households and document that around 60% of them changed the composition of the portfolio at most once.

⁷For example, the statistics on the gross domestic product are released with a lag of a quarter.

basis. Furthermore, agents could infer the dynamics of the aggregate states by exploiting the correlation between aggregate productivity and labor earnings, implying an additional learning dynamics within the model. These features would inflate the states and the mechanisms of the model making it computationally infeasible. Nevertheless, to mitigate the assumption that households do not observe their labor income, I postulate that inattention breaks out exogenously when the employment status changes, from worker to unemployed or vice versa. Changes in employment status are interpreted as major events which capture the attention of agents and require them to change previous plans on consumption and savings. In such a case, households are forced to become attentive and pay the observation cost. This assumption implies that each household is always aware of its employment status. As a result, labor income is only partially unknown to inattentive agents.⁸ Then, I define one further condition on the behavior of inattention. To maintain the existence of credit imperfections, I postulate that inattention breaks out exogenously when agents are about to hit the borrowing constraints. In such a case, an unmodeled financial intermediary calls the attention of the agents which are forced to become attentive and pay the observation cost. These two assumptions affect the outs from inattention. Indeed, a household that at time D_i decides not to observe the states of the economy for d_{i+1} periods will cease to be inattentive at the realized new planning date $\Lambda(D_{i+1})$, which is the minimum between the desired new planning date D_{i+1} and the periods in which either the employment status of the household changes, $\iota \equiv \{j \in [D_i, D_{i+1}) : e_j \neq e_{j-}\}$, or the household is about to hit the borrowing

⁸Unemployed inattentive agents are aware of their earnings while employed inattentive agents have an unbiased expectation about their labor income. Note that the observation cost is calibrated to imply a length of inattention for the median agent which equals a quarter. Therefore, the median agent does not gather full information about her labor income just for three months.

constraint, $\psi \equiv \{j \in [D_i, D_{i+1}) : b_{j+} < \underline{b} \text{ or } a_{j+} < \underline{a} \text{ or } f_{j+} < \underline{f}\}$. I define the realized duration of inattention as $\lambda(D_{i+1}) = \Lambda(D_{i+1}) - D_i$.

II.B.4 Value Function

To define the aggregate states of the households' problem, I first denote with ζ_t the fraction of inattentive agents in the economy in period t . Then, I introduce the distribution of the agents μ_t - defined over households' idiosyncratic states $\{\omega_t, e_t, \xi_t\}$ - which characterizes the probability measure on the σ -algebra generated by the Borel set $J \equiv \Omega \times \mathbf{E} \times \Xi$. Roughly speaking, μ_t keeps track of the heterogeneity among agents by defining the financial wealth and labor income of each household. I stack the fraction of agents ζ_t and the distribution of agents μ_t in the vector $\gamma_t \equiv \{\zeta_t, \mu_t\}$. In this environment, γ_t is an aggregate state because prices depend on it. Krusell and Smith (1997, 1998) discuss how prices depends on the entire distribution of agents across their idiosyncratic states. The further addition of the observation cost makes the prices to depend also on the measure of inattentive agents at each point of time. Indeed, this object signals active investors about the degree of the informational frictions in the economy. The distribution γ_t evolves over time following the law of motion described by the Kolmogorow forward equation

$$d\gamma(\omega_t, e_t, \xi_t; z_t) = H\left(\gamma(\omega_t, e_t, \xi_t; z_t), g^c(\omega_t, e_t, \xi_t; z_t, \gamma_t), g^b(\omega_t, e_t, \xi_t; z_t, \gamma_t), \dots, g^a(\omega_t, e_t, \xi_t; z_t, \gamma_t), g^d(\omega_t, e_t, \xi_t; z_t, \gamma_t), z^t\right)$$

where $g^c(\cdot)$, $g^b(\cdot)$, $g^a(\cdot)$, $g^d(\cdot)$ denote the optimal policy function for consumption, savings in the risk-free bond, investment in the risky stock and inattention,

respectively, z^t is the history of the aggregate shock $\{z_0, \dots, z_t\}$ and $H(\cdot)$ is the Kolmogorov forward operator. The latter describes how the households' optimal policy functions modify over time the distribution of agents across their idiosyncratic states μ and the number of inattention agents ζ . Basically, the operator $H(\cdot)$ pins down $d\gamma_t$ taking as given the initial value of γ_t itself, and the realizations of the aggregate shock z_t . Hereafter, I will not explicitly characterize this operator, but I would rather solve for it numerically in the quantitative analysis.⁹

The structure of the problem should also take into account how the information is revealed to the agents. The state variables of this economy $x_t \equiv \{\omega_t, e_t, \xi_t; z_t, \gamma_t\}$ are random variable defined on a filtered probability space (X, F, P) . X denotes the set including all the possible realizations of x_t , F is the filtration $\{F_t, t \geq 0\}$ consisting of the σ -algebra that controls how the information on the states of the economy is disclosed to the agents, and P is the probability measure defined on F . Hereafter, I define the expectation of a variable v_t conditional on the information set at time k as $\mathbb{E}_k[v_t] = \int v_t dP(F_k) = \int v(x_t) dP(F_k)$. The state vector $P(v_t|F_k) = P(v_t|x_k)$ is a sufficient statistics for the probability of any variable v_t because of the Markov structure of x_t .¹⁰ The presence of observation costs and inattentive agents implies some measurability constraints on the expectations of households. Namely, a planning date D_i defines a new filtration \mathcal{F}_s such that $\mathcal{F}_s = F_{D_i}$ for $s \in [D_i, \Lambda(D_{i+1}))$. The meaning of this condition is the following. Consider a household that at time D_i decides to be inattentive until time D_{i+1} . As a result, its new realized planning date will be $\Lambda(D_{i+1})$, which depends on the probability of hitting the borrowing constraints and changing the

⁹To the best of my knowledge, no paper has explicitly solved the Kolmogorov forward equation in a heterogeneous agents model with aggregate uncertainty yet. For a characterization of the equation in the presence of *only* idiosyncratic uncertainty, see Achdou et al. (2013).

¹⁰The variables e_t , ξ_t and z_t are Markov process by construction, while γ_t and ω_t inherits the Markov property from e_t , ξ_t and z_t .

employment status. Any decision made throughout the duration of inattention is conditional on the information at time D_i , because the household does not update its information set until it reaches the new planning date $\Lambda(D_{i+1})$. Taking into account this measurability constraint, I write the agents' problem as

$$\max_{c,D,\alpha,f} \mathbb{E}_0 \left[\sum_{i=0}^{\infty} \int_{D_i}^{\Lambda(D_{i+1})} e^{-\rho t} U(c_t) dt \right] \quad (2)$$

$$\text{s.t. } \{c, D, \alpha, f\} \text{ are } \mathcal{F} - \text{adapted} \quad (3)$$

$$\omega_t + l_t = c_t + f_t \quad (4)$$

$$d\omega_t = \left(\left[\alpha_{D_i} (r_t^a(z_t, \gamma_t) - r_t^b(z_t, \gamma_t)) + r_t^b(z_t, \gamma_t) \right] f_t + (l_t - c_t) \right) dt \quad (5)$$

$$\omega_{\Lambda(D_{i+1})_+} = \omega_{\Lambda(D_{i+1})} - \chi l_{\Lambda(D_{i+1})} \quad (6)$$

$$d\gamma(\omega_t, e_t, \xi_t; z_t) = H \left(\gamma(\omega_t, e_t, \xi_t; z_t, \gamma_t), g^c(\omega_t, e_t, \xi_t; z_t, \gamma_t), g^b(\omega_t, e_t, \xi_t; z_t, \gamma_t) \dots \dots g^a(\omega_t, e_t, \xi_t; z_t, \gamma_t), g^d(\omega_t, e_t, \xi_t; z_t, \gamma_t), z^t \right) \quad (7)$$

$$c_t \geq 0, \quad \alpha_t f_t \geq \underline{a}, \quad (1 - \alpha_t) f_t \geq \underline{b}, \quad f_t \geq \underline{f} \quad (8)$$

where Equation (3) defines the measurability constraint and Equation (4) denotes the budget constraint of the agents, who use wealth and labor income to consume and invest in the two assets. Equation (5) derives the evolution of households' wealth. Note that the share of risky capital in the financial portfolio is kept constant at α_{D_i} throughout the period of inattention. Then, Equation (6) shows how households pay the observation cost at planning dates, while Equation (7) defines the evolution over time of the distribution of agents γ_t . Finally, Equation (8) states the positivity constraint on consumption and the borrowing constraints. Following Reis (2006), I combine (5) and (6) to get rid of the discontinuities in

$\omega_{\Lambda(D_{i+1})}$ as follows

$$\begin{aligned} \omega_{\Lambda(D_{i+1})_+} = & \int_0^{\lambda(D_{i+1})} (l_s - c_s) e^{\int_{s+}^{\lambda(D_{i+1})} r_k^P(z_k, \gamma_k; \alpha_{D_i}) dk} ds \\ & + \omega_{D_i} e^{\int_{0+}^{\lambda(D_{i+1})} r_s^P(z_s, \gamma_s; \alpha_{D_i}) ds} - \chi l_{D_{i+1}} \end{aligned}$$

where the return to the portfolio is $r^P(z, \gamma; \alpha) = \alpha [r^a(z, \gamma) - r^b(z, \gamma)] + r^b(z, \gamma)$.

Now I can state the problem (2) - (8) in a recursive way as follows

$$V(\omega, e, \xi; z, \gamma) = \max_{d, [c_0, c_d], \alpha, f} \mathbb{E}_0 \left[\int_0^\lambda e^{-\rho t} U(c_t) dt + e^{-\rho \lambda} V(\omega', e', \xi'; z', \gamma') \right]$$

$$\text{s.t. } \omega + l = c + f$$

$$\omega' = \int_0^\lambda (l_s - c_s) e^{\int_{s+}^\lambda r_k^P(z_k, \gamma_k; \alpha) dk} ds + \omega e^{\int_{0+}^\lambda r_s^P(z_s, \gamma_s; \alpha) ds} - \chi l'$$

$$\begin{aligned} \gamma'(\omega, e, \xi; z) = & H\left(\gamma(\omega, e, \xi; z), g^c(\omega, e, \xi; z, \gamma), g^b(\omega, e, \xi; z, \gamma), g^a(\omega, e, \xi; z, \gamma) \dots \right. \\ & \left. \dots g^d(\omega, e, \xi; z, \gamma), z^\lambda\right) \end{aligned}$$

$$c \geq 0, \quad \alpha f \geq \underline{a}, \quad (1 - \alpha)f \geq \underline{b}, \quad f \geq \underline{f}$$

where I denote with the primes subscript the variables defined at time $\Lambda(D_{i+1})$ or $\Lambda(D_{i+1})_+$. Reis (2006) shows that the measurability constraint holds as long as the optimal choices $\{d, [c_0, c_d], \alpha, f\}$ are made only upon the information given by $\{\omega, e, \xi; z, \gamma\}$.

II.C Equilibrium

II.C.1 Definition of Equilibrium.

A competitive equilibrium for this economy is a value function V and a set of policy functions $\{g^c, g^b, g^a, g^d\}$, a set of prices $\{r^b, r^a, w\}$, and a Kolmogorov

Forward equation $H(\cdot)$ for the measure of agents such that¹¹:

- Given the prices $\{r^b, r^a, w\}$, the law of motion $H(\cdot)$, and the exogenous transition matrices $\{\Gamma^z, \Gamma^e, \Gamma^\xi\}$, the value function V and the set of policy functions $\{g^c, g^b, g^a, g^d\}$ solve the household's problem;
- The bonds market clears, $\int g^b(\omega, e, \xi; z, \gamma) d\mu = 0$;
- The capital market clears, $\int g^a(\omega, e, \xi; z, \gamma) d\mu = K'$;
- The labor market clears, $\int e\xi d\mu = N$;
- The Kolmogorov Forward function $H(\cdot)$ is generated by the optimal decisions $\{g^c, g^b, g^a, g^d\}$, the transition matrices $\{\Gamma^z, \Gamma^e, \Gamma^\xi\}$ and the history of aggregate shocks z .

II.C.2 First-Order Conditions.

Gabaix and Laibson (2002) consider an environment where agents are exogenously inattentive for a given number of periods. In their model, the Euler equation for consumption holds just for the mass of attentive agents because inattentive households are off their equilibrium condition. Instead, here the Euler equations of both attentive and inattentive agents hold in equilibrium. Indeed, the Euler equation of an agent at a planning date t is a standard stochastic intertemporal condition that reads

$$\mathbb{E}_t \left[M_{\lambda,t} e^{\int_t^\lambda \alpha_t [r_s^a(z,\gamma) - r_s^b(z,\gamma)] + r_s^b(z,\gamma) ds} \right] = 0$$

where λ denotes the next date in which the household will gather new information and define a new consumption/savings plan, and $M_{\lambda,t} = e^{-\rho(\lambda-t)} \frac{U'(c_\lambda)}{U'(c_t)}$ is the

¹¹With an abuse of notation, I neglect the dependence of the value function, the policy functions, the set of prices and the Kolmogorov Forward equation on the states of the households' problem.

households' stochastic discount factor. This condition posits that the optimal share of stocks in the portfolio is the one which equalizes the expected discounted flow of returns from stocks and bonds throughout the period of inattention. The Euler equation is not satisfied with equality for borrowing constrained agents. Instead, the Euler equation of an inattentive agent between time s and q , with $t < s < q < \lambda$ is deterministic and equals

$$M_{q,s} e^{\int_s^q \alpha_s [r_k^a(z,\gamma) - r_k^b(z,\gamma)] + r_k^b(z,\gamma) dk} = 0$$

Inattentive agents do not gather any new information on the states of the economy and therefore they behave as if there were no uncertainty. Agents gets back to the stochastic inter-temporal conditions as soon they reach a new planning date, and update their information set. Therefore, as agents alternate between periods of attention and inattention, they also shift from stochastic to deterministic Euler equations. Finally, the optimal condition on the choice of inattention is

$$\frac{\partial U(c_\lambda)}{\partial \lambda} \frac{\partial \lambda}{\partial d} = \rho \frac{\partial \lambda}{\partial d} \mathbb{E}_t [V(\omega', e', \xi'; z', \mu')] - \frac{\partial \mathbb{E}_t [V(\omega', e', \xi'; z', \mu')]}{\partial \lambda} \frac{\partial \lambda}{\partial d}$$

On the left-hand side there is the marginal cost of being inattentive in terms of current consumption. It just states the utility derived from consumption in case a household keeps inattentive and does not update its consumption plan until the realized planning date λ , which depend on the choice of inattention d . On the right-hand side, there is the marginal benefit of being inattentive which consists of two terms. The first is simply the gain from becoming attentive, gathering new information and updating the optimal choices. Instead, the second

one refers to the wedge between the marginal gain a household would get at the optimal time d and in the instant immediately after it. The observation cost χ compares implicitly in the right-hand side through next-period level of wealth. In equilibrium, the optimal choices of d equalizes the marginal cost and the marginal benefit of being inattentive. In the model, the choice of d triggers different realized duration of inattention λ . For example, a household that plans not to observe the stock market for a very long period is aware that it could be exogenously forced to become attentive whenever hitting the borrowing constraints or changing the employment status. In Section IV.F I show that the derivative of λ with respect to d plays an important role in this environment.

III Calibration

The calibration strategy follows Krusell and Smith (1997, 1998) and Pijoan-Mas (2007). Some parameters (e.g., the risk aversion of the household) are set to values estimated in the literature, while others are calibrated to match salient facts of the U.S. economy. The idiosyncratic labor income risk is defined to target the cross-sectional distribution of labor income. It is important to have a realistic variation in labor income because the choice of inattention, and consequently all the effects of the observation cost on asset prices, depends on the budget of households. Then, the aggregate shock is calibrated to match the volatility of aggregate output growth, while the observation cost is defined to replicate the duration of inattention of the median household. Finally, despite I set one period of the model to correspond to one month, I report the asset pricing statistics aggregated at the annual frequency to be consistent with the literature.

The parameters set to values estimated in the literature are the capital share

of the production function η , the capital depreciation rate δ , and the risk aversion of the household θ . I choose a capital share $\eta = 0.40$, as suggested by Cooley and Prescott (1995). The depreciation rate equals $\delta = 0.0066$ to match a 2% quarterly depreciation. The risk aversion of the household is $\theta = 5$, which gives an intertemporal elasticity of substitution of 0.2, at the lower end of the empirical evidence. Then, I set the constraint on wealth \underline{f} to be minus two times the average monthly income of the economy, and households can reach this limit by short selling either bond or capital, that is, $\underline{b} = \underline{f}$ and $\underline{a} = \underline{f}$.¹² Finally, I calibrate the first parameter, the time discount rate of the household, to match the U.S. annual capital to output ratio of 2.5, and find $\rho = 0.0049$.

III.A Aggregate Productivity Shock

I assume that the aggregate productivity shock follows a two-state first-order Markov chain, with values z_g and z_b denoting the realizations in good and bad times, respectively. The two parameters of the transition function are calibrated targeting a duration of 2.5 quarters for both states. The values z_g and z_b are instead defined to match the standard deviation of the Hodrick-Prescott filtered quarterly aggregate output, which is 1.89% in the data. These values are therefore model dependent, and vary with the specification of the environment.

III.B Idiosyncratic Labor Income Shock

Employment Status. The employment shock e follows a two-state first-order Markov chain, which requires the calibration of ten parameters that define four

¹²In Guvenen (2009) the borrowing constraints equal 6 months of labor income. Instead, Gomes and Michaelides (2008) rule out any short sale. In Section IV.F, I evaluate different values for the borrowing constraints.

transition matrices two by two. I consider the ten targets of Krusell and Smith (1997, 1998). I first define four conditions that create a one-to-one mapping between the state of the aggregate shock and the level of unemployment. That is, the good productivity shock z_g comes always with an unemployment rate u_g , and the bad one z_b with an unemployment rate u_b , regardless of the previous realizations of the shock. In this way, the realization of the productivity shock pins down the unemployment rate of the economy. The four conditions are

$$1 - u_g = u_g \Gamma_e(z_g, z_g, 0, 1) + (1 - u_g) \Gamma_e(z_g, z_g, 1, 1)$$

$$1 - u_g = u_b \Gamma_e(z_b, z_g, 0, 1) + (1 - u_b) \Gamma_e(z_b, z_g, 1, 1)$$

$$1 - u_b = u_g \Gamma_e(z_g, z_b, 0, 1) + (1 - u_g) \Gamma_e(z_g, z_b, 1, 1)$$

$$1 - u_b = u_b \Gamma_e(z_b, z_b, 0, 1) + (1 - u_b) \Gamma_e(z_b, z_b, 1, 1)$$

The level of the unemployment rate in good time and bad time are defined to match the actual average and standard deviation of the unemployment rate. I compute the two moments using data from the Bureau of Labor Statistics from 1948 to 2012, and obtain 5.67% and 1.68%, respectively. Under the assumption that the unemployment rate fluctuates symmetrically around its mean, I find $u_g = 0.0406$ and $u_b = 0.0728$. Two further conditions come by matching the expected duration of unemployment, which equals 6 months in good times and 10 months in bad times. Finally, I set the job finding probability when moving from the good state to the bad one as zero. Analogously, the probability of losing the job in the transition from the bad state to the good one is zero.

Unemployment Benefit. I set the unemployment benefit \bar{w} to be 5% of the average monthly labor earning.

Efficiency Units of Hour. The efficiency units of hour ξ follows a three-state first-order Markov chain. The values of the shock and the transition function are calibrated to match three facts on the cross-sectional dispersion of labor earnings across households: the share of labor earnings held by the top 20% and the bottom 40% of households, and the Gini coefficient of labor earnings. The data, taken from Díaz-Giménez et al. (2011), characterize the distribution of earnings, income and wealth in the United States in 2007. Table I reports the calibrated values and the transition function of the shock ξ , while Table II compares the three statistics on the distribution of labor earnings in the data and in the model.

III.C Observation Cost

The observation cost is calibrated to match the duration of inattention of the median household in a year, which Alvarez et al. (2012) estimate to be around 3 months. Accordingly, I set the fixed cost to $\chi = 0.024$. It amounts to 2.4% of households' monthly labor earnings. For example, if the average household earns an income of around \$3,000 per month, then the cost equals \$72.

III.D Computation of the Model

The computation of heterogeneous agent models with aggregate uncertainty are known to be cumbersome because the distribution μ , a state of the problem, is an infinite dimensional object. I follow Krusell and Smith (1997, 1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008) by approximating the distribution μ with a finite set of moments of the distribution of aggregate capital K . The approximation can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of μ . Nevertheless, this class of models

generates almost linear economies, in which it is sufficient to consider just the first moment of the distribution of capital to have almost a perfect fit for the approximation. The presence of inattention implies two further complications. First, the decision of the agents on how long to stay inattentive requires the evaluation of their value function over a wide range of different time horizons. Second, I need to take into account the fraction of inattentive agents in the economy at every period. This condition adds a further law of motion upon which to find convergence. I report the details of the algorithm in the Appendix.

IV Results

I compare the results of the benchmark model with three alternative calibrations. In the first, the observation cost is zero and there is no inattention. In the second one, the observation cost is more severe and amounts to $\chi = 0.048$. Finally, I consider an economy in which agents are more risk averse, with $\theta = 8$. I calibrate each version of the model to match both the level of aggregate wealth and the volatility of aggregate output growth. Results are computed from a simulated path of 3,000 agents over 10,000 periods.

IV.A Inattention

The observation cost is calibrated to give a 3 months per year duration of inattention for the median household. It turns out that such a cost prevents a third of agents from gathering information on the stock market. Table III shows that in the model, in any given month, the average fraction of inattentive agents in the economy equals 39%. Furthermore, Figure 1 shows that there is a negative correlation between wealth and inattention, in line with the empirical evidence of

Ameriks et al. (2003) and Alvarez et al. (2012). There is also a sizable dispersion of inattention across agents, because poor agents cannot afford the observation cost and end up being more inattentive. For example, the wealthiest 20% of households observe the states of the economy every period, while the poorest 20% stay inattentive for 8 months on average. Such behavior implies that in the model inattention behaves as both a time-dependent and a state-dependent rule. Indeed, at each point of time households set a time-dependent rule, deciding how long to stay inattentive. Yet, when a household becomes wealthier, it opts for shorter periods of inattention. Thus, inattention looks as if it were conditional on wealth.¹³

When studying the dynamics of inattention over the cycle, I find that it depends on two forces. On one hand, the countercyclical equity premium induces agents to plan in recessions because the cost of inattention in terms of foregone financial returns is lower in good times. On the other hand, the severity of the observation cost fluctuates as a function of households' wealth. In recessions, agents are poorer and cannot afford the observation cost. The results point out that the former channel dominates in wealthy agents, whose inattention is procyclical. For example, in the model the agents at the 75-th percentile of the wealth distribution are on average inattentive for 1 month in good times and 0.7 months in bad times. Instead, the direct cost of inattention affects relatively more poor agents, which prefer to plan in expansions. The agents at the 25-th percentile of the wealth distribution are on average inattentive for 5.5 months in good times and 6 months in bad times. Overall, inattention is countercyclical: both the duration of inattention for the median agent and the fraction of inattentive

¹³Reis (2006) labels this property of inattention as “recursive time-contingency”. See Alvarez et al. (2012) and Abel et al. (2007, 2013) for further characterizations of the dynamics of inattention over time.

agents in the economy rise in recession. Such a result can also be interpreted as a foundation to the countercyclical dynamics of uncertainty. Indeed, the two concepts are intimately tied: when agents pay less attention to the states of the economy, the dispersion of their forecasts over future returns rises, boosting the level of uncertainty in the economy.¹⁴

Increasing the size of the observation cost to $\chi = 0.048$ extends the duration of inattention for the median agent up to 3.3 months. Also a risk aversion of $\theta = 8$ does increase the duration of inattention, which goes up to 3.7 months. This last result is in line with the evidence provided by Alvarez *et al.* (2012), who show that more risk averse investors observe their portfolio less frequently. This outcome is the net result of two counteracting forces. Agents with a higher risk aversion changes their portfolio towards risk-free bonds, decreasing the need of observing the stock market. At the same time, more risk averse agents have a stronger desire for consumption smoothing, which induce them to keep track of their investments more frequently. In the model, the first channel offsets the second one, implying a longer duration of inattention for more risk averse agents.

IV.B Stock Market Participation

The observation cost induces a large fraction of households not to own any stock. As reported in Table IV, 26.6% of households do not participate to the equity market. Favilukis (2013) shows that in 2007 the actual share of stockholders equals 59.4%. Hence, the observation cost accounts for 44.8% of the observed number of non-stockholders. Unlike in Saito (1996), Basak and Cuoco (1998), and Guvenen (2009), here the limited participation does not arise exogenously.

¹⁴See Veronesi (1999) for evidence on the countercyclical patten of uncertainty.

Indeed, in the economy without inattention virtually all households access the market. Therefore, the observation cost is de facto a barrier to the investment in stocks, as the fixed participation cost does in the environment of Gomes and Michaelides (2008). This result points out to a new rationale to the limited stock market participation: it is not just the presence of trading costs that matters, but also the fact that processing all the information required to invest optimally in the financial markets is not a trivial task at all. In addition, the model successfully predicts that stockholders are on average wealthier than non-stockholders. As Figure 2 shows, stockholders tend to be the wealthiest agents of the economy. For example, the poorest 7.3% of households do not hold any risky capital because they are the most inattentive agents of the economy. However, the model fails in reproducing the higher consumption growth volatility of stockholders with respect of non-stockholders. Mankiw and Zeldes (1991) find that the consumption growth of stockholders is 1.6 times as volatile than the one of non-stockholders. Instead, in the benchmark model the ratio of the consumption growth of stockholders over the one of non-stockholders equals 0.78. Indeed, stockholders turn out to be wealthy agents that are still able to self-insure their consumption stream, experiencing thereby a lower volatility than non-stockholders. I find that even higher observation costs and risk aversion cannot fully account for the observed participation rate and the higher consumption growth volatility of stockholders. Also Guvenen (2009) finds that a low participation rate is not enough to generate a higher volatility of consumption for stockholders, unless it is assumed that stockholders have a higher intertemporal elasticity of substitution than non-stockholders.

IV.C The Distribution of Wealth

The observation cost spreads also the distribution of households' wealth, which is defined as the sum of financial wealth ω_t and labor earnings l_t . Table V reports that the Gini index equals 0.41 in the economy with no inattention. This value is exactly half the value of 0.82 that Díaz-Giménez et al. (2011) find in the data. Indeed, the distribution is too concentrated around the median: there are too few poor and rich agents. This is no surprise. Krusell and Smith (1997, 1998) already discuss how heterogeneous agent models have a hard time to account for the shape of the wealth distribution. Yet, when I consider the observation cost of the benchmark model, the Gini coefficient goes up by 56% to 0.64. Inattention generates a more dispersed distribution through the limited participation to the stock market and the higher returns to stock. Poor agents cannot afford the observation cost and end up being more inattentive. Accordingly, they decide not to own any stock and give up the higher return to risky capital.

The model describes well the wealth distribution at the 20-th, 40-th and 60-th quantiles, but it falls short in replicating the right end. Increasing the size of the observation cost or the risk aversion of households improves just slightly the performance of the model.

IV.D Asset Pricing Moments

IV.D.1 Stock and Bond Returns

The Panel A of Table VI reports the results of the model on the level and the dynamics of stock returns, bond returns and the equity premium. First, I discuss the standard deviations because the observation cost increases the volatility of stock returns by an order of magnitude. In the benchmark model the standard

deviation of returns is 7.11%, which is around a third of the value observed in the data, 19.30%. Nonetheless, without inattention the standard deviation would be just 0.54%. The observation cost boosts the volatility of returns because it acts as a capital adjustment cost. Indeed, inattention makes the supply of capital to be inelastic along two dimensions. On one hand, inattentive agents follow pre-determined path of capital investment and cannot adjust their holdings to the realizations of the aggregate shock. On the other hand, the limited participation to the equity market shrinks the pool of potential investors. As far as the volatility of the risk-free rate is concerned, I find a standard deviation of 6.09%, which is very close to its empirical counterpart, that equals 5.44%. Note that standard models usually deliver risk-free rates which fluctuate too much. For example, Jermann (1998) and Boldrin et al. (2001) report a standard deviation between 10% and 20%. The mechanism that prevents volatility to surge is similar to the one exploited by Guvenen (2009). Basically, poor agents have a strong desire to smooth consumption, and their high demand of precautionary savings offsets any large movements in bond returns. Although in Guvenen (2009) the strong desire for consumption smoothing is achieved through a low elasticity of intertemporal substitution, here it is the observation cost that forces poor and inattentive agents to insure against the risk of infrequent planning. When looking at the level of the equity premium reported in Panel B of Table VI, I find that the model generates a wedge between stock returns and bond yields which is too low. It equals 0.93% while in the data it is 6.17%. Since the model does not suffer of the risk-free rate puzzle of Weil (1989), the weakness is entirely in the level of stock returns. In the model the average stock returns is 2.77%, around a third of the value observed in the data. Again, the observation cost goes a long way forward in explaining the equity premium, because the model with no inattention has a differential be-

tween stock and bond returns of 0.001%. Indeed, the limited participation to the stock market concentrates the entire aggregate risk of the economy on a smaller measure of stock-holders, who accordingly demand a higher compensation for holding equity. Furthermore, inattention exacerbates the curvature of the value function of the agents. Figure 3 - 4 show that the value function of inattentive agents is much more concave than the one of attentive agents. Moreover, the curvature of inattentive agents is much more responsive to aggregate conditions. Indeed, while the risk aversion of attentive agents is rather constant along the cycle, the risk aversion of inattentive agents rises dramatically in recessions. As a result, inattention amplifies the risk associated to holding stocks, especially in bad times. These mechanisms explain why inattention generates an equity premium several orders of magnitude higher than in a model without observation costs. Yet, the improvements are not enough to explain the puzzle. Doubling the size of the observation cost does not yield any better result: the Sharpe ratio does not even change. So, observation costs should be unreasonably high to provide a premium as it is in the data. Only a higher risk aversion of $\theta = 8$ seems to deliver better asset pricing moments, with an average stock returns of 4.07% and a 0.20 Sharpe ratio which implies an equity premium of 1.87%. These results confirm the findings of Gomes and Michaelides (2008) and Guvenen (2009), in which limited participation to the stock market is not sufficient to imply a high equity premium. Both papers introduce heterogeneity in the intertemporal elasticity of substitution to increase the volatility of consumption growth of stockholders, which is then key to generate a high price of risk.

IV.D.2 Cyclical Dynamics

Inattention generates countercyclical variations in stock returns volatility, the equity premium and the Sharpe ratio, as shown in Panel C of Table VI. Since the observation cost bites more strongly in recessions, there are very few active investors in the economy which implies that the quantity of capital is low and very responsive to the investment of the marginal attentive stockholder. Instead, when the observation cost goes to zero the volatility becomes acyclical. Therefore, in this setting the observation cost mimics the role of countercyclical uncertainty in Veronesi (1999), which induces the volatility to be asymmetric over the cycle, peaking in recessions. Also the equity premium is countercyclical and displays a sizable variation over the cycle. It equals 0.67% in good times and 1.19% in bad times. This result is in line with the empirical evidence on a positive risk-return trade-off.¹⁵ Again, this dynamics is driven by inattention since the equity premium does not move over the cycle in the economy with no observation costs. The same applies to the Sharpe ratio. In the benchmark model it fluctuates between 0.12 in expansions and 0.18 in recessions, while it sticks to 0.02 when there is no inattention. Hence inattention generates countercyclical variations in the price of risk which are usually obtained through consumption habits and long-run risk. The model implies two further successful predictions. First, Lettau and Ludvigson (2010) document that in the data the volatility of the Sharpe ratio is around 50%. They also show that neither consumption habits nor long-run risk can generate such high variations in the price of risk. They refer to this evidence as the “Sharpe puzzle variability puzzle”. In this respect, the result of Table VI

¹⁵The evidence on the sign of the risk-return relationship is mixed. Lettau and Ludvigson (2010) shows that while the unconditional correlations are weakly negative, the conditional correlation provides evidence in favor of a strong positive relationship.

shows that inattention accounts for 11% of the observed standard deviations in the Sharpe ratio and therefore provides an alternative source of risk that might help reconciling the empirical evidence on the volatility of the price of risk. Second, in the model both the level and the volatility of the excess returns can be predicted using the consumption-wealth ratio. This is in line with the broad literature that provides evidence in favor of the predictability of stock returns, see for example Campbell (1991) and Cochrane (1991). Furthermore, Lettau and Ludvigson (2001a,b) exploit the insight of Campbell and Mankiw (1989) on the cointegrating relationships between consumption growth and wealth growth to show that a ratio between the two variables does predict future stock returns. Basically, when the log consumption-wealth ratio increases, either the expected consumption grows less quickly, or future returns are expected to be high. Table VII shows that in the model the consumption-wealth ratio predicts the excess return at a four year horizon, despite there is no predictability at shorter frequencies. The picture on the volatility of the equity premium is completely reversed: the consumption-wealth ratio can predict it just at a horizon of one year, but not afterwards. These results confirms the findings of Hirshleifer *et al.* (2011), in which stock return predictability is driven by the limited attention of investors.

IV.D.3 Consumption Growth

The model yields mixed predictions on the dynamics of aggregate consumption, which I report in Table VIII. Panel A shows that inattention does not substantially increases the standard deviation of aggregate consumption growth, which keeps around 0.60 while in the data it equals 0.76. Indeed, despite inattention forces agents to sharp changes in consumption at planning dates, in a general equilibrium agents are aware of it and optimally respond to the observation cost

by choosing even smoother consumption paths. Overall, these two counteracting forces offset each other. Nonetheless, in the model attentive agents experience a slightly lower volatility of consumption growth than inattentive agents. Indeed, as long as inattentive agents and non-stockholders overlap, the counterfactual prediction that stockholders insure relatively better their stream of consumption turns into a lower volatility for attentive agents. More interestingly, the observation cost disconnects the movements in stock returns and aggregate consumption growth. For example, in the model without inattention, the correlation between stock returns and consumption is 0.77. This number falls to 0.38 in the benchmark model and further down to 0.31 with the higher risk aversion coefficient, getting closer to the empirical value of 0.22 (on a quarterly basis). Stock returns and consumption are not so correlated because inattentive agents do not react to changes in the current realizations of the productivity shocks, and in turn to current values of asset prices. The mechanism reminds of Lynch (1996), in which the the lack of synchronization across agents weakens the correlation between equity and consumption. There is also another newsworthy pattern that emerges out of Panel B. Attentive agents display higher than average correlations between consumption growth and stock returns. Indeed, inattentive agents follows pre-determined path of consumption, do not react to realizations of the aggregate shock and tend to consume too much in bad states and too low in good states. Such behavior generates an additional source of risk since attentive agents are forced to give up consumption in recessions, which are times in which their marginal utility of consumption is highest. As a result, they command a higher premium for clearing the goods market. This mechanism is akin to the one studied in Chien *et al.* (2011, 2012), where passive investors which do not re-balance their portfolio raise the risk bore by active investors. When

looking at the dynamics of aggregate consumption growth, Panel C shows that the model delivers a series which is not i.i.d. As in Peng (2005), the frictions in the dissemination of information rationalizes the presence of predictability in consumption growth. On one hand, Hall (1978) posits that consumption growth paths formed by rational agents should be unpredictable. On the other hand, Campbell and Mankiw (1990) find the presence of serial dependence. In the model, the consumption growth paths of agents conditional on their information sets are unpredictable. Still, an econometrician - who can observe all the information of the economy which has not been updated by agents yet - can find evidence of sizable positive autocorrelations. Therefore, it is the different information set between the econometrician and the agents which determines or not the predictability of consumption. Finally, the model fails in generating a dynamics of aggregate consumption growth consistent with the data, because consumption growth is homoskedastic and too persistent. Indeed, a Lagrange Multiplier test rejects the presence of heteroskedasticity in the simulated series of consumption growth at any confidence level. Furthermore, in the model the first autocorrelation of consumption growth measured at the quarterly frequency is 0.31, while in the data it equals 0.20.

IV.E Decomposing the Price of Risk

The observation cost is not enough to generate an equity premium as high as it is in the data, because in a general equilibrium households take it into account when making their optimal portfolio choices. As in Heaton and Lucas (1996), agents respond to the additional fluctuations in consumption due to inattention by reducing their exposure to aggregate risk. In this Section, I disentangle the

three main mechanisms through which households' optimal reaction to the observation cost impedes the price of risk to rise: 1) switches across consumption and precautionary savings; 2) changes in the composition of the financial portfolio and 3) shifts in the set of the agents pricing stocks and bonds. To identify these channels, I follow Pijoan-Mas (2007) by comparing five different equilibria. First, I consider the economy with no observation cost. Second, I formulate an economy where the observation cost equals $\chi = 0.024$ and take as given the optimal choices of agents in the economy with no inattention. So, households suffer the observation cost but cannot react to it. Third, I compute a new equilibrium with the observation cost in which agents can modify the composition of the financial portfolio, but not the allocations in consumption and savings. Fourth, I consider the benchmark model with observation costs where agents can entirely decide their optimal policies. Finally, I take the last equilibrium focusing only on households with interior solutions, which are eventually those pricing the two assets. Table IX shows the coefficient of variation of the marginal value of wealth over different percentiles of its distribution under all these scenarios.¹⁶

The price of risk peaks when moving from the economy without observation cost to one in which $\chi = 0.024$ and the agents are forced to follow the optimal policies of the economy without inattention. For the median households, it surges from 0.002 to 0.49, reaching even 0.74 on the right tail of the distribution. Since agents cannot modify their choices, they command a very high premium to bear the risk of being inattentive and owning stocks. The price of risk decreases as long as we allow agents to modify first their portfolio and then the whole set of choices (i.e., consumption/savings and the composition of the portfolio), and it eventually reaches 0.13 for the median agent that prices risk in the benchmark

¹⁶I compare the scenarios using the law of motion of prices of the economy with no observation cost.

economy. I therefore uses the entries of Table IX at the median to disentangle the different channels through which households' reaction reduces the prices of risk. The most important one is the reaction of consumption to the observation cost, which explains 61.1% of the fall in the price of risk. Indeed, households take into account the risk of being inattentive by increasing precautionary savings. The second most important channel is the changes in the financial portfolio, which accounts for 25% of the difference in the price of risk. Again, inattentive agents shifts their portfolio towards the risk-free bond. Finally, the changes in the set of agents pricing risk matter too, accounting for 13.9% of the difference in the price of risk between the two setups considered here. Indeed, in this environment the equilibrium prices are defined by the stochastic discount factor of the households with interior solutions for both bonds and stocks. As far as stockholders are wealthy, they can self-insure their stream of consumption, implying a low price of risk. If the volatility of consumption growth of stockholders were higher than the one of non-stockholders, the changes in the set of households pricing risk would have boosted the equity premium rather than tempering it.

IV.F The Role of Borrowing Constraints

Chen (2006) considers a Lucas-tree economy where heterogeneous agents face an observation cost, finding that the equity premium is zero and inattention does not prevent agents from owning stocks. Instead, in my model the equity premium is around 1% and the number of agents that do not participate on the equity market is substantial. In this Section I show that such contradictory results can be rationalized by the interaction between the observation cost and the borrowing constraints. In what follows, I compare three economies which differ only for the

level of the borrowing constraints. The first one is the benchmark model, where the borrowing constraints equal minus two times the average monthly income. In the second case, I consider an economy in which agents cannot borrow at all while in the last set up the constraints are loose and equal minus four times the average monthly income of households. In Panel A of Table X, I report the fraction of stockholders, the Gini coefficient of the distribution of wealth and the equity premium implied by these three economies. When agents cannot borrow at all, the stock market participation falls to 60.5%, further spreading the distribution of wealth, whose Gini index is 0.73, and the equity premium is 6.06%. These numbers match almost perfectly their empirical counterparts. Instead, in the economy with loose financial constraints, almost all households own stocks. Furthermore, the wealth distribution is more concentrated and the equity premium is around zero. This exercise highlights that the definition of the borrowing constraints changes starkly the results of the model. Most importantly, the interaction between borrowing constraints and the observation cost plays a non negligible role. Indeed, Panel B of Table X shows the result of the same exercise applied to an economy without observation cost, that is, $\chi = 0$. In this case, tight constraints do increase the equity premium but just up to 4.85%. Therefore, borrowing constraints can generate a high price of risk not only per se, as pointed out in Pijoan-Mas (2007) and Gomes and Michaelides (2008), but also for their interaction with the observation cost.

V Conclusion

A recent strand of literature studies the role of agents' infrequent planning and limited attention to the stock market on asset prices, finding inconclusive results.

Although inattention unambiguously increases the wedge between stock and bond returns, it is not clear yet whether it can account for the equity premium puzzle. In this paper, I evaluate the quantitative performance of inattention on asset prices in a production economy with heterogeneous agents and uninsurable labor income risk. I consider a monetary observation cost which generates a level of households' inattention and infrequent planning which is endogenous, heterogeneous across agents and time-varying. To discipline the role of infrequent planning, I calibrate the observation cost to match the actual duration of inattention of the median household. I find that the observation cost improves the performance of the model over several dimensions. Inattention spreads the wealth distribution toward realistic values and induces households not to hold stocks, pointing a new rationale for the limited stock market participation. Households do not own stocks because investing in equity is not a trivial task at all. Then, I show that inattention induces the volatility of stock returns to be high and countercyclical. It also generates sizable countercyclical variations in the equity premium and in the price of risk. Indeed, on one hand the aggregate risk is concentrated on a small measure of agents. On the other hand, inattentive agents create a residual risk by consuming too much in recessions and too little in expansions. Thus, attentive agents that actively invest in stocks command a countercyclical compensation to bear such additional source of risk. Nevertheless, any effect of inattention on the dynamics of stock prices vanishes as long as borrowing constraints are loose enough. This result suggest that models featuring inattention should carefully take into account the imperfections credit markets to deliver predictions consistent with the data. Furthermore, the model fails in delivering a consumption growth for stockholders less volatile than the one of non-stockholders, and an aggregate consumption growth which is homoskedastic

and too persistent. Also the equity premium is still too low, around 1%. Interestingly, increasing the magnitude of the observation cost does not alter the Sharpe ratio. Indeed, in such a case households reduce the equilibrium price of risk by extending the duration of their inattention, accumulating more precautionary savings and disinvesting out of stocks. Overall, although inattention improves the performance of the model, it cannot quantitatively account for the observed dynamics of stock prices and excess returns yet.

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A Computational Appendix

This Section describes the steps and the details of the computational algorithm I used to numerically solve the model. The algorithm is an extension to the case of inattention of the standard heterogeneous agent model with aggregate uncertainty and two assets, which has been already implemented by Krusell and Smith (1998), Pijoan-Mas (2007) and Gomes and Michaelides (2008).

It is well known that the numerical computation of heterogeneous agent model with aggregate uncertainty and two assets is very cumbersome. The reason is twofold. First, one of the endogenous aggregate state of the problem is given by the distribution of the agents over their idiosyncratic states μ , which is an infinite dimensional object. Indeed, as noted by Krusell and Smith (1997), agents need to know the entire distribution μ in order to generate rational expectations on prices.¹⁷ To circumvent this insurmountable curse of dimensionality, the state space has to be somehow reduced. I follow Krusell and Smith (1997) by approximating the entire distribution μ by a set of moments $m < \infty$ of the stock of aggregate capital K . The approximation can be interpreted as if the agents of the economy were bounded rational, ignoring higher-order moments of μ . As in previous studies, I find that $m = 1$ is enough to have an almost perfect approximation of μ . That is, the mean of aggregate capital \bar{K} is a sufficient statistics that capture virtually all the information that agents need to forecast future prices. Second, when extending the basic Krusell and Smith (1997) algorithm to the case of an economy with two assets, the market for bonds does not clear at all dates and states. Indeed, the total bondholdings implied by the model is almost

¹⁷Current prices depend just on the actual realization of the aggregate shock z and the mean of the distribution μ . Instead, future prices depend on future realization of the aggregate shock z and the entire distribution μ , including all its higher-order moments.

a random walk. As far as total bondholdings experience large movements over time, it is not always possible to achieve the clearing of the market. I therefore follow the modified algorithm of Krusell and Smith (1998), where agents perceive the bond return as a state of the economy. The equilibrium bond return is then the one in which the bond return perceived by the agents and the one implied by the optimal decisions of the agents coincide.

The presence of the observation cost adds two further complications. First, agents have to decide their optimal duration of inattention. This step requires the derivation of the household's maximization procedure not just in one case (i.e., today vs. the future), but in a much wider set of alternatives. Indeed, the household can decide whether to be attentive today and tomorrow, whether to be attentive today and inattentive for the following period, or to be attentive today and inattentive for the following two periods, and so on and so forth. Accordingly, I define a grid over all the potential durations of inattention that agents can pick up, solve the model over each grid points and eventually take the maximum among the different value functions to derive the optimal choice of inattention. Second, prices depend not just on the measure of agents μ , but also on the severity of informational frictions in the economy. Indeed, when every agent is attentive, the model shrinks down to the standard Krusell and Smith (1998). Instead, where there is a (non-negligible) measure of inattentive agent, which is the case at the core of my analysis, the model departs from the standard setting. As far as the presence of observation costs pin down different equilibria, and therefore different path of futures prices, agents are required to be aware of the extent of the frictions in the economy whenever taking their optimal choices on consumption and savings. I conjecture that at each point of time the number of inattentive agents in the economy is a sufficient state that

describes the magnitude of the informational frictions. Hence, the algorithm posits that, before taking their optimal decisions, attentive agents look at the number of inattentive agents to understand how “inattentive” is the economy at that precise point of time, and what kind of rational expectation on the future paths of prices they have to derive.

The computation of the model requires the convergence upon six forecasting rules which predict the future mean of the stock of aggregate capital, the future price of the bond and the future number of inattentive agents for both the aggregate shocks z_b and z_g . The procedure yields a set of twenty two different parameters upon which to converge. This algorithm is very time-consuming and makes at the moment computationally infeasible any extension of the model that inflates either the mechanisms or the number of states. For example, the assumption that inattentive agents do not gather information about their idiosyncratic shocks is required by this computational constraint.

In what follows, I first describe the computational algorithm in Section A.A. Then, I discuss the problem of the household given the forecasting rule on future prices in Section A.B. Finally, Section A.C concentrates on the derivation of the equilibrium forecasting rules. I also show that the substitution of the entire distribution μ with the first moment of aggregate capital \bar{K} yields an almost perfect approximation.

A.A Algorithm

The algorithm works around nine main steps, as follows:

1. Guess the set of moments m_t of aggregate capital K_t upon which to approximate the distribution of agents μ_t ;

2. Guess the functional forms for the forecasting rule of the set of moments m_t , the number of inattentive agents in the economy ζ_t and the risk-free return to bond r_t^b ;
3. Guess the parameters of the forecasting rules;
4. Solve the household's problem;
5. Simulate the economy:
 - (a) Set an initial distribution of agents over their idiosyncratic states ω , e and ξ ;
 - (b) Find the interest rate r^{b*} that clears the market for bonds. Accordingly, guess an initial condition $r^{b,0}$, solve the household's problem in which agents perceive the bond return $r^{b,0}$ as a state, and obtain the policy functions $g^c(\omega, e, \xi; z, m, \zeta, r^{b,0})$, $g^b(\omega, e, \xi; z, m, \zeta, r^{b,0})$, $g^a(\omega, e, \xi; z, m, \zeta, r^{b,0})$ and $g^d(\omega, e, \xi; z, m, \zeta, r^{b,0})$. Use the policy functions on bondholdings g^b to check whether the market clears, that is, whether the total holdings of bond equals zero. If there is an excess of bond supply, then change the initial condition to $r^{b,1} < r^{b,0}$. If there is an excess of bond demand, then change the initial condition to $r^{b,1} > r^{b,0}$. Iterate until the convergence on the interest rate r^{b*} that clears the market.
 - (c) Derive next period distribution of agents over their idiosyncratic states ω , e and ξ using the policy functions $g^c(\omega, e, \xi; z, m, \zeta, r^{b*})$, $g^b(\omega, e, \xi; z, m, \zeta, r^{b*})$, $g^a(\omega, e, \xi; z, m, \zeta, r^{b*})$ and $g^d(\omega, e, \xi; z, m, \zeta, r^{b*})$ and the law of motions for the shocks z , e and ξ .
 - (d) Simulate the economy for a large number of periods T over a large measure of agents N . Drop out the first observations which are likely to be influenced by the initial conditions.

6. Use the simulated series to estimate the forecasting rules on m_t , ζ_t and r_t^b implied by the optimal decisions of the agents;
7. Check whether the coefficients of the forecasting rules implied by the optimal decisions of the agents coincide with the one guessed in step (3). If they coincide, go to step (8). Otherwise, go back to step (3);
8. Check whether the functional forms of the forecasting rule as chosen in step (2) give a good fit of the approximation of the state space of the problem. If this is the case, go to step (9). Otherwise, go back to step (2);
9. Check whether the set of moments m_k of aggregate capital K yields a good approximation of the distribution of agents μ . If this is the case, the model is solved. Otherwise, go back to step (1).

A.B Household's Problem

I solve the household's problem using value function iteration techniques. I discretize the state space of the problem as follows. First, I guess that the first moment of aggregate capital is a sufficient statistics describing the evolution of the distribution of agents μ . Later on, I evaluate the accuracy of my conjecture. Then, I follow Pijoan-Mas (2007) by stacking all the shocks, both the idiosyncratic and the aggregate ones, in a single vector ϵ , which has 8 points: four points - one for unemployed agents and three different level for employed agents - for each aggregate shock z . For the wealth ω I use a grid of 60 points on a logarithmic scale. Instead, for the possible durations of inattention d , I use a grid of 30 points: the first 25 points are equidistant and goes from no inattention at all, 1 month of inattention until 2 years of inattention. The following four grid points are equidistant on a quarterly basis. In this respect, the assumption made in the

model on when inattention breaks out exogenously are very helpful in the definition of the grid. Indeed, agents will not choose too long durations of inattention because they take into account the probability of being called attentive due to a change in their employment status or because they hit the borrowing constraints. For example, in the benchmark model the largest point of the grid yields a duration of inattention of 3 years. Yet, this choice is hardly picked up by households in the simulations done to solve the model. Without the two assumptions on the exogenous ending of inattention, then some households could theoretically be inattentive forever, which would require a wider grid for the choice variable d . Then, for the grids of the first moment of aggregate capital \bar{K} and the number of inattentive agents ζ I use 6 points since the value function does not display a lot of curvature along these dimensions. To sum up, any value functions is computed over a total of 518,400 different grid points. Furthermore, the state space is inflated in the case of the problem in which households perceive the bond return as a state of the economy. I use a grid for r^b formed by 10 points, which yields a total of 5,184,000 grid points. Decisions rules off the grid are evaluated using a cubic spline interpolation around along the values of wealth ω and a bilinear interpolation around the remaining endogenous state variables. Finally, the solution of the model is simulated from a set of 3,000 agents over $T=10,000$ time periods. In any evaluation of the simulated series, the first 1,000 observations are dropped out.

The household's problem used in step (4) of the algorithm modifies the standard structure presented in the text to allow for the approximation of the measure of agents μ with the first moment of aggregate capital \bar{K} . Then, I postulate three forecasting rules (R_1, R_2, R_3) for the mean of aggregate capital \bar{K} , the number of inattentive agents ζ and the return of the bond r^b , respectively. The household's

problem reads

$$\begin{aligned}
V(\omega, \epsilon, \bar{K}, \zeta) &= \max_{d, [c_0, c_d], \alpha, f} \mathbb{E}_0 \left[\int_0^\lambda e^{-\rho t} U(c_t) dt + e^{-\rho \lambda} V(\omega', \epsilon', \bar{K}', \zeta') \right] \\
\text{s.t. } \quad \omega + l &= c + f \\
\omega' &= \int_0^\lambda (l_s - c_s) e^{\int_{s+}^\lambda r_k^P(z_k, \bar{K}_k, \zeta_k; \alpha) dk} ds + \omega e^{\int_{0+}^\lambda r_s^P(z_s, \bar{K}_s, \zeta_s; \alpha) ds} - \chi l' \\
\bar{K}' &= R_1(\bar{K}, \zeta, [z_0, z_\lambda]) \\
\zeta' &= R_2(\bar{K}, \zeta, [z_0, z_\lambda]) \\
r^b &= R_3(\bar{K}, \zeta, [z_0, z_\lambda]) \\
c &\geq 0, \quad \alpha f \geq \underline{a}, \quad (1 - \alpha)f \geq \underline{b}, \quad f \geq \underline{f}
\end{aligned}$$

Instead, in step (5b) of the problem the households perceive the return of the bond r^b as a state of the economy, as follows

$$\begin{aligned}
V(\omega, \epsilon, \bar{K}, \zeta, r^b) &= \max_{d, [c_0, c_d], \alpha, f} \mathbb{E}_0 \left[\int_0^\lambda e^{-\rho t} U(c_t) dt + e^{-\rho \lambda} V(\omega', \epsilon', \bar{K}', \zeta') \right] \\
\text{s.t. } \quad \omega + l &= c + f \\
\omega' &= \int_0^\lambda (l_s - c_s) e^{\int_{s+}^\lambda r_k^P(z_k, \bar{K}_k, \zeta_k, r_k^b; \alpha) dk} ds + \omega e^{\int_{0+}^\lambda r_s^P(z_s, \bar{K}_s, \zeta_s, r_s^b; \alpha) ds} - \chi l' \\
\bar{K}' &= R_1(\bar{K}, \zeta, [z_0, z_\lambda]) \\
\zeta' &= R_2(\bar{K}, \zeta, [z_0, z_\lambda]) \\
r^b &= R_3(\bar{K}, \zeta, [z_0, z_\lambda]) \\
c &\geq 0, \quad \alpha f \geq \underline{a}, \quad (1 - \alpha)f \geq \underline{b}, \quad f \geq \underline{f}
\end{aligned}$$

I use this problem to simulate the economy given the return to the bond r^b as a perceived state for the households. I follow Gomes and Michaelides (2008) by

aggregating agents' bond demands and determining the bond return that clears the market through linear interpolation. This value is then used to recover the implied optimal decisions of the agents, which are then aggregated to form the aggregate variables that becomes state variables in the following time period.

A.C Equilibrium Forecasting Rules

I follow Krusell and Smith (1997, 1998) by defining log-linear functional forms for the forecasting rules of the mean of aggregate stock capital \bar{K} , the number of inattentive agents ζ and the bond return r^b . Namely, I use the following law of motions:

$$\log \bar{K} = \alpha_0(z) + \alpha_1(z) \log \bar{K} + \alpha_2(z) \log \zeta$$

$$\log \zeta = \beta_0(z) + \beta_1(z) \log \bar{K} + \beta_2(z) \log \zeta$$

$$r^b = \gamma_0(z) + \gamma_1(z) \log \bar{K} + \gamma_2(z) \log \zeta + \gamma_3(z) (\log \bar{K})^2 + \gamma_4(z) (\log \zeta)^2$$

The parameters of the functional forms depend on the aggregate shock z . Indeed, there is a set of three forecasting rule for each of the two realizations of the aggregate shock z , resulting in a total of six forecasting rules and twenty two parameters, upon which to find convergence.

I find the equilibrium forecasting rules as follows. First, I guess a set of initial conditions $\{\alpha_0^0(z), \alpha_1^0(z), \alpha_2^0(z), \beta_0^0(z), \beta_1^0(z), \beta_2^0(z), \gamma_0^0(z), \gamma_1^0(z), \gamma_2^0(z), \gamma_3^0(z), \gamma_4^0(z)\}$. Then, given such rules I solve the household's problem. I take the simulated series to then re-estimate the forecasting rules, which yields a new set of implied parameters $\{\alpha_0^1(z), \alpha_1^1(z), \alpha_2^1(z), \beta_0^1(z), \beta_1^1(z), \beta_2^1(z), \gamma_0^1(z), \gamma_1^1(z), \gamma_2^1(z), \gamma_3^1(z), \gamma_4^1(z)\}$. If the two sets coincide (up to a numerical wedge), then these values correspond to

the equilibrium forecasting rules. Otherwise, I use the latter set of coefficients as a new initial guess.

For the benchmark specification of the model, I find the following equilibrium forecasting rules for $z = z_g$

$$\log \bar{K} = 0.101 + 0.976 \log \bar{K} - 0.249 \log \zeta \quad \text{with } R^2 = 0.993761$$

$$\log \zeta = -0.208 + 0.037 \log \bar{K} + 0.861 \log \zeta \quad \text{with } R^2 = 0.995890$$

$$r^b = 1.042 - 0.077 \log \bar{K} + 0.016 \log \zeta + \\ + 0.011 (\log \bar{K})^2 + 0.006 (\log \zeta)^2 \quad \text{with } R^2 = 0.998717$$

and the following equilibrium forecasting rules for $z = z_b$

$$\log \bar{K} = 0.084 + 0.986 \log \bar{K} - 0.240 \log \zeta \quad \text{with } R^2 = 0.994014$$

$$\log \zeta = -0.229 + 0.040 \log \bar{K} + 0.851 \log \zeta \quad \text{with } R^2 = 0.997081$$

$$r^b = 1.036 - 0.073 \log \bar{K} + 0.021 \log \zeta + \\ + 0.009 (\log \bar{K})^2 + 0.009 (\log \zeta)^2 \quad \text{with } R^2 = 0.998965$$

Note that the R^2 are all above 0.99. This result points out that approximating the distribution of agents μ with the first moment of aggregate capital \bar{K} implies basically no discharge of relevant information that agents can use to forecast future prices.

B Tables and Figures

Table I: Parameters shock efficiency units of hour

	$\xi_1 = 15$	$\xi_2 = 4$	$\xi_3 = 1$
	$\Gamma_\xi(\xi_1, \cdot)$	$\Gamma_\xi(\xi_2, \cdot)$	$\Gamma_\xi(\xi_3, \cdot)$
$\Gamma_\xi(\cdot, \xi_1)$	0.9850	0.0025	0.0050
$\Gamma_\xi(\cdot, \xi_2)$	0.0100	0.9850	0.0100
$\Gamma_\xi(\cdot, \xi_3)$	0.0050	0.0125	0.9850

Note: The efficiency unit of hours ξ follows a first-order Markov chain with transition function Γ_ξ .

Table II: The distribution of labor earnings

Target	Model	Data
Share earnings top 20%	62.1%	63.5%
Share earnings bottom 40%	4.4%	4.2%
Gini index	0.57	0.64

Note: the data are from Díaz-Giménez et al. (2011).

Table III: Inattention

Inattention	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
A. Duration of inattention (months per year)					
Median	3.0	0	3.3	3.7	3.0
Median - good times	2.8	0	3.0	3.3	-
Median - bad times	3.2	0	3.5	4.0	-
75th percentile - good times	1.0	0	1.1	1.2	-
75th percentile - bad times	0.7	0	0.8	0.7	-
25th percentile - good times	5.5	0	5.8	6.2	-
25th percentile - bad times	6.0	0	6.2	6.6	-
B. Fraction of inattentive agents					
Median	0.39	0	0.41	0.45	-
Median - good times	0.36	0	0.38	0.41	-
Median - bad times	0.42	0	0.44	0.49	-

Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. Good times denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z = z_b$. The fraction of inattentive agents are reported in percentage values. Data are from Alvarez et al. (2012).

Table IV: Participation to the stock market

Variable	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
% Stockholders	73.4	98.5	70.5	64.9	40.6
$\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$	0.78	0.37	0.80	0.88	1.6

Note: the variable χ defines the observation cost and θ is the risk aversion of agents, which equals 5 in the benchmark model. The ratio $\frac{\sigma(\Delta \log c_S)}{\sigma(\Delta \log c_{NS})}$ compares the standard deviation of the consumption growth of stockholders $\sigma(\Delta \log c_S)$ with the standard deviation of consumption growth of non-stockholders $\sigma(\Delta \log c_{NS})$. Data are from Mankiw and Zeldes (1991) and Favilukis (2013).

Table V: The distribution of wealth

% wealth held by	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
20th percentile	2.3	5.5	1.9	1.5	1.1
40th percentile	6.6	14.2	6.3	5.7	4.5
60th percentile	16.7	31.6	16.1	14.3	11.2
90th percentile	51.8	29.4	53.0	59.3	71.4
Gini index	0.64	0.41	0.66	0.69	0.82

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. Data are from Díaz-Giménez et al. (2011).

Table VI: Asset pricing moments

Variable	Moment	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
A. Stock and bond returns						
Stock return	Mean	3.16	1.23	3.37	4.07	8.11
	Std. dev.	7.11	0.35	7.36	10.08	19.30
Risk-free return	Mean	1.84	1.22	1.86	1.90	1.94
	Std. dev.	6.09	9.01	6.02	5.73	5.44
B. Equity premium						
Equity premium	Mean	0.93	0.01	1.01	1.87	6.17
	Std. dev.	6.64	0.48	7.21	9.35	19.49
Sharpe ratio	Mean	0.14	0.02	0.14	0.20	0.32
	Std. dev.	0.05	0.01	0.07	0.10	0.47
C. Cyclical dynamics						
Stock returns	Std. dev. - good times	7.02	0.55	7.22	7.81	-
	Std. dev. - bad times	7.20	0.55	7.50	8.16	-
Equity premium	Mean - good times	0.67	0.01	0.88	1.98	-
	Mean - bad times	1.19	0.01	1.24	2.36	-
Sharpe ratio	Mean - good times	0.12	0.02	0.10	0.15	-
	Mean - bad times	0.18	0.02	0.20	0.25	-

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in expectation and reported in annualized percentage values. Annual returns are defined as the sum of log monthly returns. The equity premium is the $r^e = \mathbb{E}[r^a - r^b]$. The Sharpe ratio is defined as the ratio between the equity premium and its standard deviation. Good times denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times denote the periods in which the aggregate productivity shock is $z = z_b$. Data are from Campbell (1999), Guvenen (2009) and Lettau and Ludvigson (2010).

Table VII: Predictability of excess returns

Horizon (Years)	Model		Data	
	Coefficient	R^2	Coefficient	R^2
A. Excess returns				
1	3.79	0.01	6.23	0.08
2	3.98	0.03	9.82	0.28
3	5.04	0.09	12.28	0.31
4	5.62	0.19	12.91	0.33
B. Volatility of excess returns				
1	-0.82	0.11	-1.60	0.19
2	-0.77	0.09	-1.97	0.19
3	-0.23	0.02	-1.41	0.08
4	-0.18	0.01	-0.59	0.01

Note: In both panels the independent variable is the logarithm of the consumption growth - wealth growth ratio $\log(c_t) - \log(\omega_t)$. In Panel A, the dependent variable is the logarithm of the expected excess return $r_{t+h}^e = \mathbb{E}_t[r_{t+h}^a - r_{t+h}^b]$. In Panel B, the dependent variable is the standard deviation of the log expected excess return $\left[\sum_{j=1}^h (r_{t+j}^e - \bar{r}^e)^2\right]^{1/2}$, where \bar{r}^e denotes the sample average of the log equity premium. h denotes the yearly horizon of the forecasting equation. The regression is run with standard OLS methods. Data are from Lettau and Ludvigson (2010).

Table VIII: Moments of aggregate consumption growth

	$\chi = 0.024$	$\chi = 0$	$\chi = 0.048$	$\theta = 8$	Data
A. Standard deviations					
Average	0.63	0.61	0.64	0.67	0.76
Attentive agents	0.60	0.61	0.60	0.63	-
Inattentive agents	0.68	-	0.69	0.72	-
B. Correlation with stock returns					
Average	0.38	0.77	0.36	0.31	0.22
Attentive agents	0.41	0.77	0.39	0.34	-
Inattentive agents	0.33	-	0.34	0.28	-
C. Time-series dynamics					
Autocorrelation	0.31	0.09	0.33	0.37	0.20
ARCH Effects	No	No	No	No	Yes

Note: the variable χ defines the observation cost and θ is the risk-aversion of agents, which equals 5 in the benchmark model. All statistics are computed in quarterly values. In the model, the series of consumption and output growth are derived taking the Hodrick-Prescott filter of the logarithm of the simulated series aggregated at the quarterly frequency. The correlation of attentive agents' consumption growth with aggregate output growth is computed pooling the average of the individual correlations of attentive agents over time. The same applies to the correlation of inattentive agents. The autocorrelation reports the persistence of an AR(1) model fitted to the series of consumption growth. ARCH effects are evaluated using a Lagrange Multiplier test upon the fit of a ARMA(1,1)-ARCH(1) model. If the test statistics is greater than the Chi-square table value, the null hypothesis of no ARCH effects is rejected. Data are from Campbell (1999) and Guvenen (2009).

Table IX: Price of risk under different scenarios

Economy / Percentiles	25th	50th	75th	95th
$\chi = 0$	0.002	0.002	0.002	0.002
$\chi = 0.024$ - Choices as in $\chi = 0$	0.46	0.49	0.55	0.74
$\chi = 0.024$ - Consumption as in $\chi = 0$	0.38	0.40	0.45	0.48
$\chi = 0.024$ - Optimal policies	0.16	0.18	0.21	0.24
$\chi = 0.024$ - Pricing agents	0.09	0.13	0.16	0.18

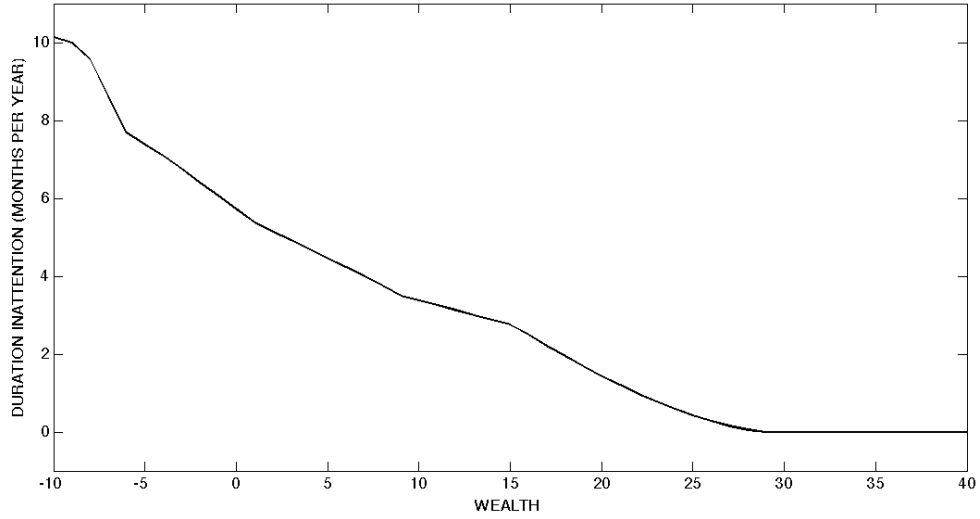
Note: the variable χ defines the observation cost. The risk-aversion of agents is $\theta = 5$ in all the cases. The pricing agents are the ones with interior policy functions for both bonds and capital.

Table X: The role of borrowing constraints

Variable	Benchmark	Tight Constraints	Loose Constraints
A. Inattentive economy - $\chi = 0.024$			
% Stockholders	73.4	60.5	95.1
Gini index wealth	0.64	0.73	0.49
Equity premium	0.93	6.06	0.08
B. Attentive economy - $\chi = 0$			
% Stockholders	98.5	91.2	98.7
Gini index wealth	0.41	0.54	0.36
Equity premium	0.01	4.85	0.004

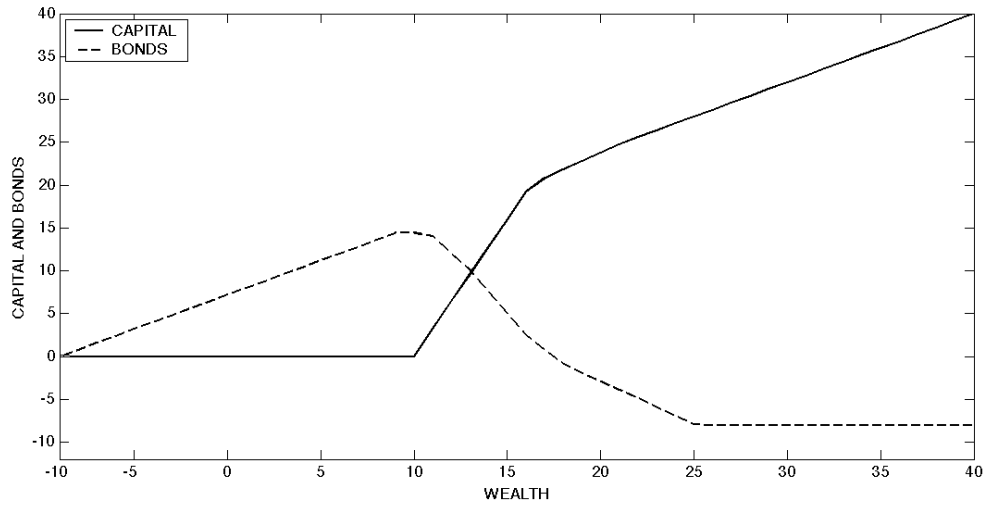
Note: The variable χ defines the observation cost. In the “Benchmark” model, borrowing constraints equal minus two times the average monthly income of households, that is, $\underline{f} = -2\mathbb{E}[l_t]$. The “Tight Constraints” model does not allow short sales, that is, $\underline{f} = 0$. In the “Loose Constraints” model borrowing constraints equal minus four times the average monthly income of households, that is, $\underline{f} = -4\mathbb{E}[l_t]$. The equity premium is reported in annualized percentage values.

Figure 1: Optimal Choice of Inattention



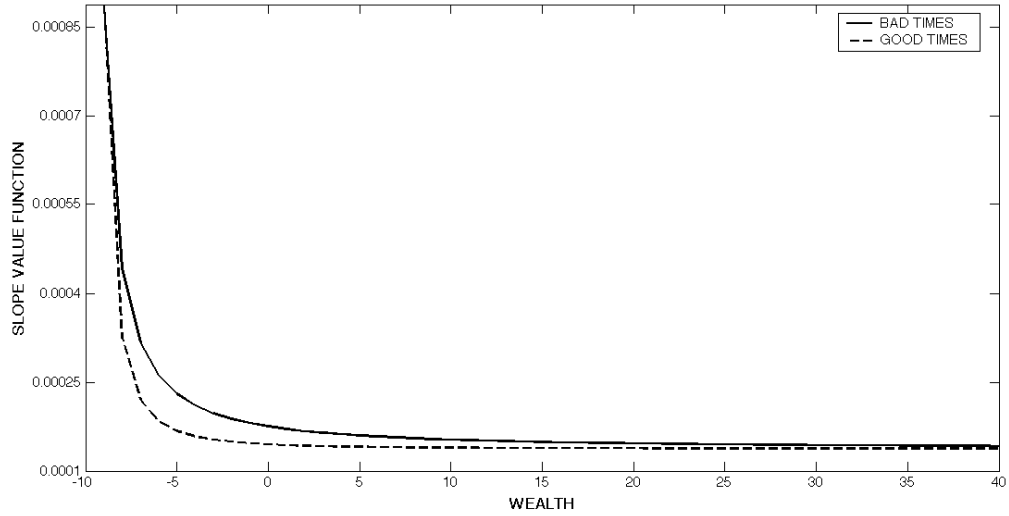
Note: the figure plots the policy function of inattention g^d as a function of wealth ω . The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.

Figure 2: Optimal Portfolio Choices



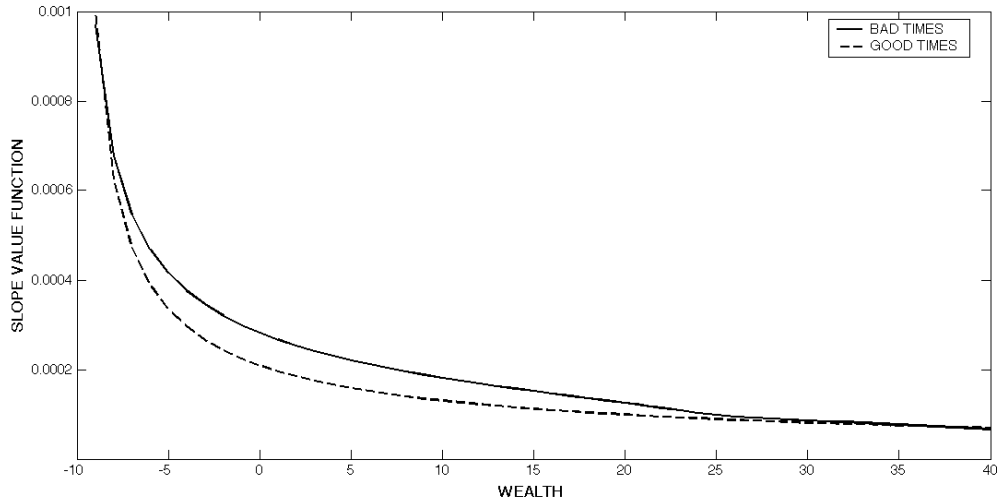
Note: the figure plots the policy functions of investment in risky assets g^a (continuous line) and risk free bonds g^b (dashed line) as a function of wealth ω . The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate shock is $z = z_g$ and the aggregate capital equals its mean.

Figure 3: Slope of the Value Function - Attentive Agents



Note: the figure plots the slope of the value function of attentive agents as a function of wealth ω . Good times (dashed line) denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times (continuous line) denote the periods in which the aggregate productivity shock is $z = z_b$. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate capital equals its mean.

Figure 4: Slope of the Value Function - Inattentive Agents



Note: the figure plots the slope of the value function of attentive agents as a function of wealth ω . Good times (dashed line) denote the periods in which the aggregate productivity shock is $z = z_g$ and bad times (continuous line) denote the periods in which the aggregate productivity shock is $z = z_b$. The idiosyncratic shocks are set to $e = 1$ and $\xi = 4$. The aggregate capital equals its mean.