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Welfare losses under Cournot competition ☆

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Abstract

In a market for a homogeneous good where firms are identical, compete in quantities and produce with constant returns, the percentage of welfare losses (PWL) is small with as few as five competitors for a class of demand functions which includes linear and isoelastic cases. We study markets with positive fixed costs and asymmetric firms. We provide exact formulae of PWL and robust constructions of markets were PWL is close to one in these two cases. We show that the market structure that maximizes PWL is either monopoly or dominant firm, depending on demand. Finally we prove that PWL is minimized when all firms are identical, a clear indication that the assumption of identical firms biases the estimation of PWL downwards.

Keywords: Welfare losses; Asymmetric firms; Fixed costs

1. Introduction

In his classical contribution Cournot (1838, Chapter 8) established that when the number of firms in a market tends to infinity, oligopolistic equilibrium tends to perfect competition. As a corollary, Welfare Losses (WL), measured as the difference between social welfare in the optimal and the equilibrium allocation, tend to zero. But, what happens when the number of firms is finite? Is perfect competition a good approximation or, on the contrary are WL significant? (see Hotelling (1938) and Yarrow (1985) for an early treatment of this problem).

As a first cut to the problem, assume that all firms are identical and costs and demand are linear. It is easily calculated that the percentage of WL under Cournot competition, denoted by PWL, is $1/(1+n)^2$ where n is the number of firms. Thus, despite the fact that monopoly and duopoly entail large PWL this magnitude goes to zero pretty quickly: a market composed by 7 identical firms ("the seven sisters") produces a PWL of 1.56% only.[1] This poses a serious question: were WL systematically small a simple equilibrium concept like perfect competition may be preferable as a description of markets unless an additional argument is made in favor of

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[1] This formula shows that once linearity is assumed, as done implicitly by Harberger (1954), WL seldom goes up to big numbers except if the number of firms is very small. A list of other empirical papers measuring WL in oligopoly can be found in Tullock (2003) p. 2.

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the Cournot model (e.g. that the distribution of the surplus in Cournot and perfect competition is very different). Moreover, the motivation for public policies dealing with efficiency is lost under small WL.

Let us first comment on papers that deal with our problem. McHardy (2000) studies a model with quadratic demand and presents numerical calculations. He finds that WL can be up to 30% larger than those in the linear model, which is encouraging but still does not solve the problem. Anderson and Renault (2003) calculate PWL under the assumptions made above except that they assume an inverse demand function of the form $p=A-bx^{\alpha}$, (x is aggregate output and p market price).[2] They do not study if PWL differs substantially from those in the linear model. Johari and Tsitsiklis (2005) show that if firms are identical, average costs are not increasing and the inverse demand function is concave, PWL is bounded above by $1/(2n+1)$, which is still not very large because a market with seven firms achieves, *at least*, 93.33% of maximum welfare.

Our paper is a quest for markets where oligopoly produces large WL. Specifically, the purpose of our paper is twofold: to provide workable formulae for PWL which depend, as far as possible, on magnitudes that are observable.[3] And to use these formulae to construct markets where the Cournot equilibria yields large PWL.[4]

In Section 2 we consider the Baseline Model, which is that of Anderson and Renault. We might expect that for suitable values of α, WL were much higher than those in the linear case. However, by using numerical methods we find that the maximum PWL obtained in this case is not very different from the one obtained in the linear case. Moreover, for some values of α, PWL is arbitrarily small. Thus, the consideration of a more general class of demand functions does not bring significant WL associated with oligopoly, but on the contrary it adds to the suspicion that WL under oligopoly may be small. We then turn our attention to fixed costs and heterogeneous firms.[5]

In Section 3 we consider free entry with a fixed cost. We provide formulae for the maximal and the minimal PWL where this magnitude depends on the number of firms and α. We show that when α and the fixed cost are not observable, for any exogenously given observation on market price, output, average variable cost and number of firms, PWL can be chosen arbitrarily (Proposition 1). In particular when α tends to infinity, PWL can be chosen to be arbitrarily close to one. This result implies that any given price-marginal costs margin, or elasticity of demand, is compatible with *any* PWL. When the fixed cost can be observed, the observed variables must fulfill a condition which implies that entry is blockaded. We show that any observation fulfilling this condition is compatible with many – but not all – PWL (Proposition 2).

In Section 4 we consider heterogeneous firms. We provide a formula for PWL where this magnitude depends (positively) on the share of the largest firm, (negatively) on the Hirschman–Herfindahl concentration index, denoted by H, and on α. We find that there are markets with a large number of firms where PWL is close to one whereas H is close to zero (Proposition 3). This shows that H is not a reliable measure of WL.[6] More importantly, it implies that the concept of a large economy must be taken with care because seemingly innocuous departs from a model where all firms are small and identical may have serious welfare consequences. Next, we prove that the market structure that maximizes PWL is a dominant firm when $\alpha>0$ and monopoly when $\alpha<0$ (Proposition 4). Thus, monopoly, the target of attacks of our profession from Adam Smith on, is not necessarily the worst outcome in terms of WL. Finally we prove that PWL is minimized when firms are identical (Proposition 5). This shows that proper care of the heterogeneity of firms is essential to obtain estimates of PWL that are not biased towards small PWL.

Finally, in Section 5 we offer some thoughts about our results. Our main conclusion is twofold. On the one hand, the search for WL in actual markets should focus on economies of scale and asymmetric firms, two facts that are seldom considered in the applied literature. On the other hand the Cournot model can easily produce large WL. Other important points are the characterization of the best and the worst possible market structures from the welfare point of view when firms are different

[2] This form of demand generalizes both linear ($\alpha=1$) and isoelastic (with elasticity of demand $1/\alpha$) forms and allows for computation of equilibria.

[3] The parameter α, which can be estimated but not observed, enters in the formula of PWL in Anderson and Renault (2003), so it is unavoidable in the more general set ups considered in this paper.

[4] Johari and Tsitsiklis (2005) offer an example of a market where PWL is arbitrarily close to one but in which the inverse demand function is not differentiable.

[5] Other attempts to find higher WL focus on issues outside market competition like "X-Inefficiency", Leibenstein (1966) and Rent-Seeking, Tullock (1967).

[6] That social welfare is increasing in the marginal cost of small firms was first pointed out by Lahiri and Ono (1988). For a criticism of the idea that concentration is generally bad for social welfare see Daughety (1990), Farrell and Shapiro (1990) and Cable et al. (1994).

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and the construction of a "large" market where PWL is arbitrarily close to one.[7]

It goes without saying that important causes of WL are not considered here, i.e. product differentiation, investment, R&D, location, etc. The analysis of the impact of these variables on WL requires the consideration of games that are more complicated than those considered here and, consequently, they are left for future research.

2. The Baseline Model

There is a representative consumer with a utility function $U = Ax - \frac{bx^{\alpha+1}}{\alpha+1} - px$ where x is aggregate output, p is the market price, $b\alpha>0$ and $\alpha>-1$. The maximization of utility generates an inverse demand function $p=A-bx^{\alpha}$. Notice that if $\alpha<0$, $b<0$, and $A=0$ we have an isoelastic function $p=-bx^{\alpha}$. The linear case occurs if $\alpha=1$.

There are n identical firms each producing a single output denoted by x_i, $i=1, \ldots, n$. Thus $x\equiv \sum_{i=1}^{n} x_i$. Marginal cost is constant and denoted by c. Profits for firm i are $\pi_i\equiv(p-c)x_i$. Defining $\alpha\equiv A-c$ we have that $\pi_i\equiv(a=bx^{\alpha})x_i$. Assume $ab>0$ and $-A\alpha<cn$. These assumptions guarantee that output and market price are positive in equilibrium (see Eq. (2.1)).

If firms compete in the manner of Cournot, the first order condition of profit maximization yields $a-bx^{\alpha}-b\alpha x^{\alpha-1}x_i=0$. It is easy to check that the second order condition holds and that equilibrium is symmetric. Thus Cournot equilibrium output and market price are

$$x^* = \left(\frac{an}{b(n+\alpha)}\right)^{\frac{1}{\alpha}} \quad \text{and} \quad p^* = \frac{A\alpha + cn}{n+\alpha}. \tag{2.1}$$

Social welfare, denoted by W, is the sum of industry profits and the utility of the representative consumer, i.e. $W = ax - b\frac{x^{\alpha+1}}{1+\alpha}$.The optimal aggregate output is found by maximizing W, namely

$$x^o = \left(\frac{a}{b}\right)^{\frac{1}{\alpha}}. \tag{2.2}$$

Social welfare in equilibrium and in the optimal allocation, are, respectively

$$W^* = \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}} \alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{\alpha+1}{\alpha}}(\alpha+1)} \quad \text{and} \quad W^o = \frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)}. \tag{2.3}$$

[7] Other points that have already been noticed in the literature are the importance of the functional form of demand and the failure of the H index and the price-marginal cost ratio to capture WL.

From Eq. (2.3), the percentage of WL denoted by PWL is

$$PWL \equiv \frac{W^o - W^*}{W^o} = 1 - \frac{n^{\frac{1}{\alpha}}(n+\alpha+1)}{(n+\alpha)^{\frac{\alpha+1}{\alpha}}} \equiv L(\alpha, n), \tag{2.4}$$

see Anderson and Renault (2003) p. 262. The following properties of $L(\cdot,\cdot)$ are easily proved:

i) $\lim_{n\to\infty} L(\alpha,n)=0$.
ii) $\lim_{\alpha\to -1} L(\alpha,n)=0$.
iii) $\lim_{n\to\infty} L(\alpha,n)=0$.
iv) $L(\alpha,\cdot)$ decreases with n.
v) $L(\cdot,n)$is quasi-concave in α.

i) is the usual property of large economies, as noticed in the Introduction. The explanation of ii) is that when $\alpha\to -1$, the market produces in the limit an infinity amount of surplus, so the loss caused by oligopoly tends to zero. iii) is caused by the fact that when $\alpha\to\infty$, inverse demand is flat so firms cannot influence price and optimal and equilibrium output are identical. ii) and/or iii) imply that there are markets where, for a given n, PWL is as small as we wish, something that is impossible in the case of quadratic utility functions. iv) shows that, when there are no technological issues at stake, the more competition, the better. Finally v) follows from the fact that Anderson and Renault (2003) proved that W^o/W^* is quasi-concave on α. So W^*/W^o is quasi-convex and $-W^*/W^o$ is quasi-concave, so it is $1-W^*/W^o$.

We now study PWL as a function of α. Table 1 below shows, for selected values of n, the maximum PWL, denoted by $\overline{PWL}$, and PWL when the demand function is linear, denoted by PWLL (see Corchón (2006) for details). Notice that iv) above guarantees that for n larger than 10, $\overline{PWL}$ will be smaller than 2.2%.

Notice that the general form of the utility function does not help much to obtain significant WL. Given this and that PWL can be much smaller than $\overline{PWL}$ (i.e. when α is close to -1 or to ∞) we conclude that the consideration of a more general class of utility functions *alone* is not helpful to finding significant WL.

3. Fixed costs and free entry

In this section we assume that in order to produce, firms must incur a fixed cost, denoted by k, and that there is an infinity number of potential firms. The number of active firms in equilibrium is denoted by n. Given n, output is determined as in the previous section. We assume that the decision of entry is prior to the decision

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Table 1

n	1	2	3	4	5	6	7	8	9	10
$\overline{\text{PWL}}$	.27	.118	.076	.058	.044	.0357	.032	.027	.024	.022
PWLL	.25	.11	.0625	.04	.027	.02	.0156	.012	.01	.008

on output.[8] Thus, equilibrium under free entry implies that if firms are in the market, firm n has non-negative profits but firm $(n+1)$ has non-positive profits, formally

$$\frac{\alpha a^{\frac{1+\alpha}{\alpha}} n^{\frac{1-\alpha}{\alpha}}}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{1+\alpha}{\alpha}}} \geq k \geq \frac{\alpha a^{\frac{1+\alpha}{\alpha}}(n+1)^{\frac{1-\alpha}{\alpha}}}{b^{\frac{1}{\alpha}}(n+\alpha+1)^{\frac{1+\alpha}{\alpha}}}. \tag{3.1}$$

Welfare in a Cournot equilibrium with free entry is

$$W^* = \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}} \alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{\alpha+1}{\alpha}}(\alpha+1)} - nk, \tag{3.2}$$

where n satisfies Eq. (3.1). When social welfare is maximized, aggregate output is given by Eq. (2.2). And the optimal number of firms never exceeds one because the existence of a fixed cost implies that is optimal to produce x^o in one firm. Thus, social welfare in the optimal allocation with one firm is

$$W^o = \frac{\alpha a^{\frac{\alpha+1}{\alpha}}}{b^{\frac{1}{\alpha}}(\alpha+1)} - k. \tag{3.3}$$

Assuming $\alpha a^{\frac{\alpha+1}{\alpha}} > kb^{\frac{1}{\alpha}}(\alpha+1)$, i.e. that the fixed cost is small enough, one active firm is socially optimal because it yields more social welfare than no firms. Thus PWL can be written as

$$\text{PWL} = \frac{\frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)} - \frac{a^{\frac{\alpha+1}{\alpha}} n^{\frac{1}{\alpha}}\alpha(n+\alpha+1)}{b^{\frac{1}{\alpha}}(n+\alpha)^{\frac{1+\alpha}{\alpha}}(\alpha+1)} + (n-1)k}{\frac{a^{\frac{\alpha+1}{\alpha}}\alpha}{b^{\frac{1}{\alpha}}(\alpha+1)} - k}. \tag{3.4}$$

In order to have a formula, in which PWL depends on observable variables, we substitute k for its upper and lower bounds in Eq. (3.1). It is clear that PWL is increasing on k. Thus, the maximal PWL, denoted by MA (α, n), occurs for the maximum value of k, namely

$$\text{MA}(\alpha,n) \equiv \frac{(n+\alpha)^{\frac{1+\alpha}{\alpha}} - n^{\frac{1}{\alpha}}(n+\alpha+1) + (n-1)n^{\frac{1-\alpha}{\alpha}}(\alpha+1)}{(n+\alpha)^{\frac{1+\alpha}{\alpha}} - \alpha^{\frac{1-\alpha}{\alpha}}(\alpha+1)}. \tag{3.5}$$

Minimal PWL, denoted by MI(α, n), occurs for the minimum value of k, namely

$$\text{MI}(\alpha,n) \equiv \frac{(n+\alpha+1)^{\frac{1+\alpha}{\alpha}} - \frac{n^{\frac{1}{\alpha}}(n+\alpha+1)^{\frac{1+2\alpha}{\alpha}}}{(n+\alpha)^{\frac{1+\alpha}{\alpha}}} + (n-1)(n+1)^{\frac{1-\alpha}{\alpha}}(\alpha+1)}{(n+\alpha+1)^{\frac{1+\alpha}{\alpha}} - (n+1)^{\frac{1-\alpha}{\alpha}}(\alpha+1)}. \tag{3.6}$$

We now state the properties of and MA(·,·) and MI(·,·) that correspond to i)–iv) in the previous section.

i') $\lim_{n\to\infty}\text{MI}(\alpha, n) = \lim_{n\to\infty}\text{MA}(\alpha, n) = 0$.
ii') $\lim_{\alpha\to-1}\text{MI}(\alpha, n) = \lim_{\alpha\to-1}\text{MA}(\alpha, n) = 0$.
iii') $\lim_{\alpha\to\infty}\text{MI}(\alpha, n) = \frac{n-1}{n}, \lim_{\alpha\to\infty}\text{MA}(\alpha, n) = 1$.
iv') Neither MI(α,·) nor MA(α,·) are monotonic on n.

i') implies that $\lim_{k\to 0}\text{PWL} = 0$, since Eq. (3.1) implies that when $k\to 0$, $n\to\infty$. Variations of this result have been obtained by Dasgupta and Ushio (1981), Fraysse and Moreaux (1981) and Guesnerie and Hart (1985). i') and ii') are identical to i) and ii) in the previous section. However iii') is very different from iii) because it says that markets with very large $\alpha's$ could be very inefficient. For large values of α, the contrast between monopoly and markets with a large number of firms is striking: In the former it is possible to construct examples where PWL is arbitrarily small and in the latter such examples are not possible. This is due to the fact that when n is very large, there are large WL due to the discrepancy between the equilibrium and the optimal number of firms, which is one. Finally iv') is proved by means of an example available under request. The reason for this – apparently paradoxical – result is that k changes in order to maintain the free entry condition (3.1).

We now show that, if k and α are unknown, PWL is arbitrary even if certain variables – like price, output, marginal cost and number of firms – are observed and we require that they correspond to the values in a Cournot Equilibrium with free entry for some parameters defining demand and costs. To formalize this, we say that a *Market* is a list of real numbers (A, c, b, α, k) such that $k>0$, $(A-c)\ \alpha>0$, $\alpha>-1$, $\alpha b>0$, $-A\alpha<cn$ and $\alpha(A-c)^{\frac{\alpha+1}{\alpha}} > kb^{\frac{1}{\alpha}}(\alpha+1)$. An *Observation* is a list $(\mathscr{P}, \mathscr{R}_i, \mathscr{C}, \mathscr{N})$ where $\mathscr{P}$ is market price, $\mathscr{R}_i$ is output of firm i, $\mathscr{C}(<\mathscr{P})$ is the marginal cost and $\mathscr{N}$ is the number of active firms. The last variable is a positive integer and

[8] López-Cuñat (1999) has shown that, under conditions that are met here, the equilibrium considered in this paper is a subset of an equilibrium when both decisions are simultaneous (like in Novshek (1980) and Ushio (1983)).

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the others are positive real numbers. We assume that c is observable because under constant returns, the marginal cost equals the average variable cost which, in principle, can be observed (wages, raw materials, etc.). Now we have the following:

Proposition 1. *Given an observation* $(\mathscr{P},\mathscr{R}_i,\mathscr{C},\mathscr{N})$, *and a number such that* $v=\mathrm{MA}(\hat{\alpha},n),\hat{\alpha}\in(-1,0)\cup(0,\infty)$, *there is a market* $(\hat{A},\mathscr{C},\hat{b},\hat{\alpha},\hat{k})$ *such that* $(\mathscr{P},\mathscr{R}_i,\mathscr{N})$ *is a Cournot equilibrium with free entry for this market (i.e. they fulfill* Eqs. (2.1) *and* (3.1)*), and* PWL$=v$.

Proof. For k equal to the maximum value in Eq. (3.1), PWL is given by Eq. (3.5). Let v and $\hat{\alpha}$ be such that $\mathrm{MA}(\hat{\alpha},\mathscr{N})=v$. Now set

$$\hat{A}=\frac{\mathscr{P}(\mathscr{N}+\hat{\alpha})-\mathscr{C}\mathscr{N}}{\hat{\alpha}},\qquad \hat{k}=\frac{\hat{\alpha}\left(\hat{A}-\mathscr{C}\right)^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}\mathscr{N}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(\mathscr{N}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}},$$

$$\hat{b}=\frac{\left(\hat{A}-\mathscr{C}\right)\mathscr{N}}{\mathscr{N}^{\hat{\alpha}}\mathscr{R}_i^{\hat{\alpha}}(\mathscr{N}+\hat{\alpha})}$$

This system can be solved easily because the first equation determines $\hat{A}$, the last equation determines $\hat{b}$ and with these values of $\hat{A}$ and $\hat{b}$ the remaining equation determines $\hat{k}$. By construction $\hat{A}\hat{\alpha}=\mathscr{P}(\mathscr{N}+\hat{\alpha})-\mathscr{C}\mathscr{N}$, so $\left(\hat{A}-\mathscr{C}\right)\hat{\alpha}=\mathscr{P}(\mathscr{N}+\hat{\alpha})-\mathscr{C}(\mathscr{N}+\hat{\alpha})>0$ Then, from the last equation $\hat{\alpha}\hat{b}>0$ and the remaining equation implies $\hat{k}>0$. Also $\hat{A}\hat{\alpha}+\mathscr{C}\mathscr{N}=\mathscr{P}(\mathscr{N}+\hat{\alpha})>0$. Finally we will show that $\hat{\alpha}\left(\hat{A}-\mathscr{C}\right)^{\frac{\hat{\alpha}+1}{\hat{\alpha}}}>\hat{k}\hat{b}^{\frac{1}{\hat{\alpha}}}(\hat{\alpha}+1)$. Given the definitions of the parameters, this inequality reads $(\mathscr{N}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}-\mathscr{N}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}(\hat{\alpha}+1)>0$. Call $\Psi(\hat{\alpha},\mathscr{N})$ the left hand side of the previous inequality and extend the function to allow n to take real values. Notice that $\Psi(\hat{\alpha},1)=(\hat{\alpha}+1)\left((\hat{\alpha}+1)^{\frac{1}{\hat{\alpha}}}-1\right)>0$. Also $\lim_{n\to\infty}\Psi(\hat{\alpha},\mathscr{N})=\infty$. Then, if $\Psi(\hat{\alpha},\mathscr{N})\leq 0$ there must be a value of $\mathscr{N}$ say $\bar{\mathscr{N}}$ for which $\frac{\partial\Psi(\hat{\alpha},\bar{\mathscr{N}})}{\partial\mathscr{N}}=0$ and $\Psi(\hat{\alpha},\bar{\mathscr{N}})\leq 0$. The former is equivalent to $(\bar{\mathscr{N}}+\hat{\alpha})^{\frac{1}{\hat{\alpha}}}\bar{\mathscr{N}}=\bar{\mathscr{N}}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}(1-\hat{\alpha})$. If $\hat{\alpha}=1$ this is impossible. If $\hat{\alpha}\neq 1$ plugging this equation in the definition of $\Psi(\cdot,\cdot)$ we obtain $\Psi(\hat{\alpha},\bar{\mathscr{N}})=(\bar{\mathscr{N}}+\hat{\alpha})^{\frac{1}{\hat{\alpha}}}\frac{\hat{\alpha}}{1-\hat{\alpha}}(-\hat{\alpha}+1-2\bar{\mathscr{N}})\neq 0$. Thus $\Psi(\hat{\alpha},\bar{\mathscr{N}})<0\Leftrightarrow\hat{\alpha}\in(0,1)$. However for $\hat{\alpha}\in(0,1)$, $(\bar{\mathscr{N}}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}\geq\bar{\mathscr{N}}^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}$, so $\Psi(\hat{\alpha},\bar{\mathscr{N}})\geq\bar{\mathscr{N}}^{\frac{1}{\hat{\alpha}}}\left(\bar{\mathscr{N}}-\frac{1+\hat{\alpha}}{\bar{\mathscr{N}}}\right)\geq\bar{\mathscr{N}}^{\frac{1}{\hat{\alpha}}}\left(\bar{\mathscr{N}}-\frac{2}{\bar{\mathscr{N}}}\right)>0$. Thus, $\Psi(\hat{\alpha},\mathscr{N})>0$.

Plugging the values of $\hat{A}$ and $\hat{b}$ into Eq. (2.1) we obtain

$$x^*=\left(\frac{(\hat{A}-\mathscr{C})\mathscr{N}}{\hat{b}(\mathscr{N}+\hat{\alpha})}\right)^{\frac{1}{\hat{\alpha}}}=\mathscr{N}\mathscr{R}_i\text{ and }p^*=\frac{\hat{A}\hat{\alpha}+\mathscr{C}\mathscr{N}}{\mathscr{N}+\hat{\alpha}}=\mathscr{P}.$$

From the first inequality in Eq. (3.1) (with equality) and the definition of k it follows that

$$\frac{\hat{\alpha}(\hat{A}-\mathscr{C})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}n^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(n+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}}=\frac{\hat{\alpha}\left(\hat{A}-\mathscr{C}\right)^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}\mathscr{N}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{\hat{b}^{\frac{1}{\hat{\alpha}}}(\mathscr{N}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}}$$

$$\Leftrightarrow\frac{n^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(n+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}}=\frac{\mathscr{N}^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(\mathscr{N}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}},$$

which has $n=\mathscr{N}$ as a solution so the proof is complete. □

There are two main implications of this result. On the one hand it points out the necessity of a good estimate of α in order to judge the efficiency of a market. Notice that first order conditions of profit maximization imply that the elasticity of demand equals $\frac{\mathscr{N}(\mathscr{P}-\mathscr{C})}{\mathscr{P}}$ so neither the elasticity of demand, nor price-marginal costs margins are related to α and/or PWL. On the other hand, together with the second part of iii'), it allows for markets yielding PWL arbitrarily close to one, the main theoretical goal of this paper. The explanation of this, is that we have constructed a market in which, in equilibrium, profits are zero and, when tends to infinity, consumer surplus is also zero since from Eq. (2.1) we have that

$$U=\frac{\alpha}{(\alpha+1)b^{\frac{1}{\alpha}}}\left(\frac{na}{n+a}\right)^{\frac{1+\alpha}{\alpha}},\text{ so }\lim_{\alpha\to\infty}\frac{\alpha}{(\alpha+1)b^{\frac{1}{\alpha}}}\left(\frac{na}{n+\alpha}\right)^{\frac{1+\alpha}{\alpha}}=0.$$

The intuition of the latter equation is that large values of make inverse demand flatter and flatter so consumer surplus goes to zero when α goes to infinity. The difference with iii) in the previous section – where $\lim_{\alpha\to\infty}L(\alpha,n)=0$ – arises from the fact that in the latter industry profits are not zero, but when α tends to infinity they tend to α.

We now consider the case where fixed costs are observable. In this case an observation is a list $(\mathscr{P},\mathscr{R}_i,\mathscr{C},\mathscr{N},\mathscr{T})$ such that $\mathscr{T}\leq\mathscr{R}_i(\mathscr{P}-\mathscr{C})$ (i.e. profits are non-negative). Consider the following condition that guarantees that no firm will like to enter:

Definition 1. Observation $(\mathscr{P},\mathscr{R}_i,\mathscr{C},\mathscr{N},\mathscr{T})$ and α fulfill condition BE (Blockaded Entry) if

$$\left(\frac{\mathscr{N}+\alpha+1}{\mathscr{N}+\alpha}\right)^{\frac{1+\alpha}{\alpha}}\left(\frac{\mathscr{N}}{\mathscr{N}+1}\right)^{\frac{1-\alpha}{\alpha}}>\frac{\mathscr{R}_i(\mathscr{P}-\mathscr{C})}{\mathscr{T}}.$$

The right hand side can be interpreted as the rate of (gross) profits. BE just says that the rate of profits cannot be larger than a certain number which depends on α and $\mathscr{N}$. The condition is more illuminating in several special cases. For instance if $\alpha\to\infty$ condition

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BE reads $\mathcal{T}(\mathcal{N}+1)>\mathcal{N}\mathcal{R}_i(\mathcal{P}-\mathcal{C})$. When $\alpha \to -1$ condition BE reads $\mathcal{T}(\mathcal{N}+1)^2>\mathcal{N}^2\mathcal{R}_i(\mathcal{P}-\mathcal{C})$. Finally when $\alpha=1$, BE reads, $\mathcal{T}(\mathcal{N}+2)^2>(\mathcal{N}+1)^2\mathcal{R}_i(\mathcal{P}-\mathcal{C})$.

Proposition 2. *Given an observation* $(\mathcal{P},\mathcal{R}_i,\mathcal{C},\mathcal{N},\mathcal{T})$ *and a number* v *such that* $v=\mathrm{MI}(\hat{\alpha},\mathcal{N}),\hat{\alpha}\in(-1,0)\cup(0,\infty)$, *if BE holds, there is a market* $(\hat{A},\mathcal{C},\hat{b},\hat{\alpha},\hat{k})$ *such that* $(\mathcal{P},\mathcal{X}_i,\mathcal{N})$ *is a Cournot equilibrium with free entry for this market (i.e. they fulfill Eqs.* (2.1) and (3.1)*), and* $\mathrm{PWL}\geq v$.

Proof. (Virtually identical to the proof of Proposition 1). For k equal to the minimum value in Eq. (3.1), PWL is given by Eq. (3.6). Choose $\hat{\alpha}$ such that $v=\mathrm{MI}(\hat{\alpha},\mathcal{N})$. Set

$$\hat{A}=\frac{\mathcal{P}(\mathcal{N}+\hat{\alpha})-\mathcal{C}\mathcal{N}}{\hat{\alpha}},\ \hat{b}=\frac{(\hat{A}-\mathcal{C})\mathcal{N}}{\mathcal{N}^{\hat{\alpha}}r_i^{\hat{\alpha}}(\mathcal{N}+\hat{\alpha})}$$

This system can be solved, as we showed before. Plugging these values of $\hat{A}$ and $\hat{b}$ into Eq. (2.1) we obtain the required values of x^* and p^*. Finally, the left hand side of the free entry condition Eq. (3.1) holds by the definition of an observation. And when we plug the values of $\hat{A}$ and $\hat{b}$ obtained above, the second inequality of Eq. (3.1) reads

$$\mathcal{T}\geq\frac{\mathcal{R}_i(\mathcal{P}-\mathcal{C})(\mathcal{N}+\hat{\alpha})^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}\left(\frac{\mathcal{N}+1}{\mathcal{N}}\right)^{\frac{1-\hat{\alpha}}{\hat{\alpha}}}}{(\mathcal{N}+\hat{\alpha}+1)^{\frac{1+\hat{\alpha}}{\hat{\alpha}}}},$$

which under BE holds. When the above equation holds with equality, $\mathrm{PWL}=\mathrm{MI}(\hat{\alpha},\mathcal{N})=v$, so $\mathrm{PWL}\geq v$. □

Comparing these with the results obtained in the previous section we see that the consideration of fixed costs allows the possibility of finding large PWL. This is because in this case, we add the misallocation due to the wrong number of firms to the misallocation due to the wrong output. The former comes up to very large numbers because in our model the optimal number of firms is one.[9] But preferences play a role too: In the linear case, values of PWL arbitrarily close to one cannot be obtained for a given n. The reason is that the utility of the representative consumer when $\alpha=1$ is always positive.

4. Non-identical firms

Suppose that firms have different costs. Let c_i be the marginal cost of firm i. Without loss of generality let $c_1\leq c_i$ for all i. Let $a_i\equiv A-c_i$. We will assume that for all i, $(n+\alpha-1)\alpha_i>\sum_{j\neq i}a_j$, $b\sum_{j=1}^{n}a_j>0$ and $-A\alpha<\sum_{i=1}^{n}c_i$. These assumptions imply that, in equilibrium, all firms produce a positive output and market price is positive (see Eq. (4.1) below). Cournot equilibrium is easily shown to be unique and given by

$$x_i^*=\frac{1}{\alpha}\left(\frac{\sum_{j=1}^{n}a_j}{b(n+\alpha)}\right)^{\frac{1}{\alpha}}\left(\frac{a_i(n+\alpha)}{\sum_{j=1}^{n}a_j}-1\right), x^*=\left(\frac{\sum_{j=1}^{n}a_j}{b(n+\alpha)}\right)^{\frac{1}{\alpha}} \text{ and } p^*=\frac{A\alpha+\sum_{i=1}^{n}c_i}{n+\alpha}. \tag{4.1}$$

Social welfare is $W=Ax-b\frac{x^{\alpha+1}}{\alpha+1}-\sum_{i=1}^{n}c_ix_i=\sum_{i=1}^{n}a_ix_i-b\frac{x^{\alpha+1}}{\alpha+1}$. In equilibrium,

$$W^*=\frac{1}{\alpha}\sum_{i=1}^{n}a_i\left(\frac{\sum_{j=1}^{n}a_j}{b(n+\alpha)}\right)^{\frac{1}{\alpha}}\left(\frac{a_i(n+\alpha)}{\sum_{j=1}^{n}a_j}-1\right)-\frac{b}{\alpha+1}\left(\frac{\sum_{i=1}^{n}a_i}{b(n+\alpha)}\right)^{\frac{\alpha+1}{\alpha}}, \tag{4.2}$$

which when all a_i's are identical reduces to Eq. (2.3). In the optimal allocation only the technology in the hands of Firm 1 is used and accordingly

$$x^o=\left(\frac{a_1}{b}\right)^{\frac{1}{\alpha}} \text{ and } W^o=\frac{\alpha a_1^{\frac{\alpha+1}{\alpha}}}{(\alpha+1)b^{\frac{1}{\alpha}}}. \tag{4.3}$$

[9] Overentry may also occur even if the marginal cost is increasing, see von Weizsäcker (1980), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

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In order to have a workable expression for PWL that depends on observable variables alone, let us define s_i as the market share of firm i. Clearly, and $\sum_{i=1}^{n} s_i=1$ and $s_1 \geq s_i$, $i=2,\ldots, n$. Then, from Eq. (4.1),

$$s_i \equiv \frac{x_i}{x} = \frac{a_i(n+\alpha) - \sum_{j=1}^{n} a_j}{\alpha \sum_{j=1}^{n} a_j} \Rightarrow a_i = \frac{(\alpha s_i + 1)\sum_{j=1}^{n} a_j}{n+\alpha}. \tag{4.4}$$

We will say that a list of market shares $(s_1, s_2, \ldots, s_n)$ is *a Market Structure*. It is clear from Eq. (4.4) that any vector $(a_1, a_2, \ldots a_n)$ yields a unique market structure compatible with Cournot equilibrium and that given a market structure we can construct a vector $(a_1, a_2, \ldots, a_n)$ (in fact an infinity number of vectors) whose Cournot equilibrium yields this market structure. Given this, we will focus on market structure that has the advantage of being observable.

Plugging the last part of Eq. (4.4) into Eq. (4.2) and after lengthy calculations we obtain PWL as a function of and the market structure, namely

$$\mathrm{PWL} = \frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}} - (\alpha+1)\sum_{i=1}^{n} s_i^2 - 1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}} \equiv P\left(s_1, \sum_{i=1}^{n} s_i^2, \alpha\right). \tag{4.5}$$

When all firms are identical, Eq. (4.5) reduces to Eq. (2.4). It is noteworthy that PWL here depends only on three variables:

– α.
– The market share of the largest firm s_1.
– The Hirschman–Herfindahl index of concentration denoted by $H \equiv \sum_{i=1}^{n} s_i^2$.[10]

Eq. (4.5) allows computation of PWL from s_1 and H assuming that demand is linear or isoelastic (where α is the inverse elasticity of demand). It also allows to plot PWL as a function of α for actual market structures and see what this function looks like, see Corchón (2006) for a simple application to the Spanish gasoline market.

Notice the following properties of $P(\,)$ as defined by Eq. (4.5):[11]

i") $\lim_{\alpha \to -1} \mathrm{P}(s_1, H, \alpha)=0$.
ii") $\lim_{\alpha \to \infty} P(s_1, H, \alpha) = \frac{1}{s_1}\left(s_1 - \sum_{i=1}^{n} s_i^2\right)$.
iii") $P(\cdot, H, \alpha)$ is increasing on s_1.
iv") $P\,(s_1, \cdot, \alpha)$ is decreasing on H.
v") $\lim_{\alpha \to 0} \mathrm{PWL}(s_1, H, \alpha) = \frac{e^{s_1} - 1 - H}{e^{s_1}}$.

i") is identical to i). When firms are identical ii") reduces to ii). Point iii') agrees with the received wisdom: the larger the dominant firm, the closer to monopoly, and hence the larger the PWL is. However, iv") is counterintuitive because it says the larger the concentration, the lower the WL. The reason is that when H increases, production is shifted to the less efficient firms which causes social welfare to fall. Finally v") allows us to extend $P(s_1, H, \cdot)$ to $\alpha=0$ preserving continuity.

We now discuss why the approach followed in the previous section will not work here. An *Observation* is a list $(\mathcal{P}, \mathcal{R}_{\circledR}, \ldots, \mathcal{R}_{\mathcal{N}}, \mathcal{C}_{\circledR}, \ldots, \mathcal{C}_{\mathcal{N}})$ where $\mathcal{P}$ is market price and $\mathcal{R}_i$ and $\mathcal{C}_i(<\mathcal{P})$ are the output and the marginal cost of firm i. A *Market* is a list (A, $c_1, \ldots, c_n$, b, α) such that $(n+\alpha-1)a_i > \sum_{j \neq i} a_j$, $\alpha > -1$, $b\sum_{j=1}{}^{n} a_j > 0$, $b\alpha > 0$, and $-A\alpha < \sum_{i=1}^{n} c_i$. Clearly, not all observations are compatible with the model. In particular, the number of variables in an observation is $2n+1$ and the number of parameters defining a market is $n+3$. With $n>2$, the number of parameters will be, in general, unable to generate the required observations. Also, first order conditions of profit maximization imply that

$$\frac{\mathcal{R}_{\mathcal{J}}}{\mathcal{R}_{\mathcal{J}}} = \frac{\mathcal{P} - \mathcal{C}_{\mathcal{J}}}{\mathcal{P} - \mathcal{C}_{\mathcal{J}}}.$$

This relation may fail even for the case $n=2$. Given this, we will study how PWL depends on α, n and the market structure focussing our attention on limiting cases, i.e. when PWL is maximal or minimal. Our first result is that when

[10] In fact, s_1 and H are not independent but we prefer to write Eq. (4.5) in this way to highlight the role of H in the formula.

[11] As we mentioned before, we take s_1 and H as independent when in fact they are not.

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α, n, and the market structure can be chosen simultaneously, PWL can be arbitrarily close to one and at the same time the concentration index H arbitrarily low.

Proposition 3. *There exists (α, n, s_1, …, s_n) for which PWL is arbitrarily close to one and H is arbitrarily close to zero.*

Proof. From iv") the maximal PWL occurs when $s_2=s_3=,\ldots,=s_n$. Denoting these shares by y, we have that $s_1+(n-1)y=1$. Plugging this in Eq. (4.5) we have that

$$P(s_1,n,\alpha)\equiv\frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}-(\alpha+1)\left(s_1^2+\frac{(1-s_1)^2}{n-1}\right)-1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}. \tag{4.6}$$

PWL is increasing on n so the maximum PWL obtains when n is arbitrarily large, i.e.

$$\lim_{n\to\infty}P(s_1,n,\alpha)=\frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}-(\alpha+1)s_1^2-1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}. \tag{4.7}$$

We easily compute $\lim_{\alpha\to\infty}\lim_{n\to\infty}P(s_1,\ n,\ \alpha)=\lim_{n\to\infty}\lim_{\alpha\to\infty}P(s_1,\ n,\ \alpha)=1-s_1$. Thus when α and n are very large and s_1 very small, PWL is arbitrarily close to one (since limits are interchangeable our procedure is robust). The restriction $s_1\geq s_i$, $i=2$, …, n when firms 2, …, n are identical, is equivalent to $ns_1\geq 1$. This inequality holds when the order of magnitude at which n tends to ∞ is larger than the order of magnitude at which s_1 tends to 0.

Finally, it can be easily shown that when firms 2 to n are identical,

$$H=\frac{ns_1^2+1-2s_1}{n-1}=\frac{s_1^2+\frac{1}{n}-2\frac{s_1}{n}}{1-\frac{1}{n}},$$

which when $n\to\infty$ and $s_1\to 0$ tend to zero. □

From the previous proof it follows that for n and α large, PWL $\simeq 1-\sqrt{H}$ which highlights the point made before about the relationship between concentration and WL.

It can be shown that if one of the variables in our construction is held fixed, can be made large, but not close to one, and is again far from being a reliable measure of Corchón (2006), pp. 19–21. We now perform a more demanding exercise where PWL is studied by varying only one variable, either the market structure or α.

We first concentrate on how market shares affect PWL. A market structure such that $s_1>s_2=,\ldots,=s_n>0$ will be called *a Dominant Firm*. A limit case of a dominant firm is *Monopoly* where only s_1 is positive.

Proposition 4. *For $\alpha>0$, PWL is maximized when the market structure is a dominant firm with $s_1=\frac{n+3}{2n+2}$ if $\alpha=1$ and $s_1=\frac{-n-1+\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n}$ if $\alpha\neq 1$. For $\alpha<0$ the market structure that maximizes PWL is monopoly.*

Proof. The maximum of PWL in Eq. (4.5) over $\sum_{i=1}^{n}s_i=1$ exists (by Weierestrass' theorem). As mentioned before, it occurs when $s_2=s_3=,\ldots,=s_n$. So, let us consider PWL as given by Eq. (4.6). The extrema of this expression with respect to s_1 can be located, either when $\frac{\partial P(s_1,n,\alpha)}{\partial s_1}=0$ or in the bounds of the interval in which s_1 must lie, namely $s_j\leq s_i\leq 1$ for all $j>1$. Since $(n-1)s_j\leq s_1$ the previous inequality can be written as $\frac{1}{n}\leq s_1\leq 1$. Now, rewrite Eq. (4.6) as follows:

$$P(s_1,n,\alpha)=1-\frac{(\alpha+1)\left(ns_1^2-2s_1+1\right)+n-1}{(n-1)(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}}.$$

$$\frac{\partial P}{\partial s_1}=\frac{s_1^2(n-n\alpha^2)-s_1(2+2n+2\alpha+2n\alpha)+2+\alpha(3+n+\alpha)+n}{(n-1)(\alpha s_1+1)^{\frac{1}{\alpha}+2}} \tag{4.8}$$

$$\frac{\partial P}{\partial s_1}=0\Leftrightarrow s_1^2\left(n-n\alpha^2\right)=2s_1(1+n+\alpha+n\alpha)-2-\alpha(3+n+\alpha)-n \tag{4.9}$$

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We have three possible cases: If $\alpha=1$, the solution to Eq. (4.9) is $s_1^*=\frac{n+3}{2n+2}\in\left[\frac{1}{n},n\right]$. Then, the maximum must be located either at $s_1=\frac{1}{n}$, at $s_1=1$ or at $s_1=\frac{n+3}{2n+2}$. We easily compute,

$$P(1,n,1)=\frac{1}{4},\ P\left(\frac{1}{n},n,1\right)=\frac{1}{(n+1)^2},\ P\left(\frac{n+3}{2n+2},n,1\right)=\frac{n+1}{3n+5}.$$

From these expressions we obtain the desired result.

If $\alpha>1$ from the first order condition we obtain two solutions,

$$s_1^*=\frac{-n-1\pm\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n}. \tag{4.10}$$

Clearly only the solution with a plus sign in front of the square root is feasible. We will show that for this solution $s_1^*\in\left[\frac{1}{n},1\right]$. If $\frac{1}{n}>s_1^*$ we would have $\alpha^2(n-1)+n^2(\alpha-1)-\alpha n+1<0$ which is impossible because the left hand side achieves a minimum when $n=2$ and $\alpha=1$. Similarly, if $s_1^*>1$, $\alpha n-\alpha-n+1<0$, which again is impossible.

Finally, notice that since there is only one value of s_1 for which $\frac{\partial P(s_1,n,\alpha)}{\partial s_1}=0$ the shape of $P(\cdot,n,\alpha)$ is determined by the sign of $\frac{\partial P(s_1,n,\alpha)}{\partial s_1}$ at $s_1=\frac{1}{n}$ and $s_1=1$. From Eq. (4.8),

$$\operatorname{sign}\frac{\partial P\left(\frac{1}{n},n,\alpha\right)}{\partial s_1}=\operatorname{sign}\left(n+\alpha+n\alpha-\frac{1}{n}+\alpha^2-\frac{2}{n}\alpha-\frac{1}{n}\alpha^2\right) \tag{4.11}$$

which is positive because the expression on the right hand side is increasing in α and for $\alpha=-1$ equals to zero. Also from Eq. (4.8) we obtain that

$$\operatorname{sign}\frac{\partial P(1,n,\alpha)}{\partial s_1}=\operatorname{sign}\left(\alpha-n\alpha+\alpha^2-n\alpha^2\right)=\operatorname{sign}(\alpha(1+\alpha)(1-n)) \tag{4.12}$$

which is negative so the interior solution is indeed a maximum.

Finally let us consider the case $\alpha<1$. Suppose that the negative root in Eq. (4.10) is less than one. Then

$$\frac{-n-1-\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}}{\alpha n-n}<1 \quad\Leftrightarrow -\sqrt{1+\alpha n+\alpha^2 n+\alpha n^2}>\alpha n+1,$$

which is impossible. So there is, at most, one interior solution. Suppose first that $\alpha>0$. From Eqs. (4.11)–(4.12) we get that $\operatorname{sign}\frac{\partial P\left(\frac{1}{n},n,\alpha\right)}{\partial s_1}$ is positive and $\operatorname{sign}\frac{\partial P(1,n,\alpha)}{\partial s_1}$ is negative which implies that maximum PWL is achieved at the interior solution. If $\alpha=0$ the positive root in Eq. (4.10) equals one. Finally, if $\alpha<0$, from Eqs. (4.11)–(4.12), we have that $\operatorname{sign}\frac{\partial P\left(\frac{1}{n},n,\alpha\right)}{\partial s_1}$ and $\operatorname{sign}\frac{\partial P(1,n,\alpha)}{\partial s_1}$ are both positive which given that there is, at most one value of s_1 for which $\operatorname{sign}\frac{\partial P(\cdot,n,\alpha)}{\partial s_1}$ switches from positive to negative means that $P(\cdot,n,\alpha)$ is increasing, so it achieves the maximum when $s_1=1$. □

Proposition 4 says that the most deleterious market structure is not always monopoly, the target of the wrath of economists since Adam Smith. In many cases a dominant firm structure is worse because firms other than do not add much competition to the market and they are technologically inefficient. We notice that under maximal PWL,

$$H=\frac{ns_1^2+1-2s_1}{n-1}\ \text{and}\ \mathrm{PWL}=\frac{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}-(\alpha+1)\left(ns_1^2+\frac{(1-s_1)^2}{n-1}\right)-1}{(1+\alpha s_1)^{\frac{\alpha+1}{\alpha}}},$$

so H decreases with n but PWL increases with n. And H increases with s_1 but PWL not necessarily so. Thus, again, the concentration index H is a poor measure of WL.

The maximum PWL for given n and α is obtained by plugging the value of s_1 that maximizes PWL as found in Proposition 4 and denoted by $s(\alpha,n)$, into $P(s_1,n,\alpha)$. Let $P(s(\alpha,n),n,\alpha)\equiv F(\alpha,n)$, say.

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It can be shown that $F(\alpha,\cdot)$ is increasing in n which implies that, for any number of firms, it is possible to find the PWL of, at least, $F(\alpha, 2)$ which for values of $\alpha \in (0, 50]$ never goes below. Finally, we state two limiting properties of $F(\cdot,\cdot)$:

$$\lim_{\alpha\to\infty} F(\alpha,n) = \frac{(\sqrt{n})^3+\sqrt{n}-2n}{(\sqrt{n})^3-\sqrt{n}}.$$

$$\lim_{n\to\infty} F(\alpha,n) = 1-\frac{(\sqrt{\alpha}-1)^2+(\alpha+1)+(\alpha-1)^2}{(\alpha-1)^{\frac{\alpha-1}{\alpha}}(\alpha\sqrt{\alpha}-1)^{\frac{\alpha+1}{\alpha}}}$$

Notice that in both cases PWL is high even for small values of α and n. It is clear that $\lim_{n\to\infty,\ \alpha\to\infty} F(\alpha, n) = \lim_{\alpha\to\infty,\ n\to\infty} F(\alpha, n) = 1$.

We now turn to the study of the market structure that minimizes PWL.

Lemma 1. *Suppose that* $(\hat{s}_1,\hat{s}_2,\ldots,\hat{s}_n)$ *minimizes* $P(s_1, \sum_{i=1}^n s_i^2, \alpha)$. *Then* $\nexists \hat{s}_i, \hat{s}_j, j>1$ *such that* $\hat{s}_1 > \hat{s}_i \geq \hat{s}_j > 0$.

Proof. Increasing $\hat{s}_i$ by a small amount, say dx, and decreasing $\hat{s}_j$ by dx too is feasible — i.e. $\hat{s}_i + dx$ and $\hat{s}_j - dx \in [0, s_1]$ increases H and so decreases PWL which contradicts that is minimized. □

Lemma 1 implies that only three market structures might minimize PWL: 1) All firms produce the same output 2) All firms minus one, say n, produce the same output. 3) A number of firms, say $1, \ldots, m$ with $m<n$ produce the same output, and the remaining firms produce zero output. But the last option cannot minimize PWL since it was established that when all firms are identical, PWL decreases with the number of (active) firms (Property iv) in Section 2). So we are left with options 1 and 2.

Proposition 5. *The market structure that minimizes PWL is when all firms produce the same output.*

Proof. Notice that market structures 1 and 2 can be written as $(x, x, \ldots, 1-(n-1)x)$ with $x \in \left[\frac{1}{n-1}, \frac{1}{n}\right]$, where the lower bound of this interval comes from $1 \geq (n-1)x$. In this case $H=(n-1)x^2+(1-(n-1)x)^2$. Plugging H into Eq. (4.5) we obtain

$$\mathrm{PWL} = 1-\frac{(\alpha+1)\left((n-1)x^2+(1-(n-1)x)^2\right)+1}{(1+\alpha x)^{\frac{\alpha+1}{\alpha}}} \equiv \mathrm{PW}(\alpha,x,n).$$

Now, computing $\frac{\partial \mathrm{PW}(\alpha,x,n)}{\partial x}$ this expression is found to be equal to

$$\frac{-(1+\alpha)}{(1+x\alpha)^{\frac{1+\alpha}{\alpha}}}\left[2n^2x-2nx-2n+2-\frac{(1+\alpha)\left((n-1)x^2+(1-(n-1)x)^2\right)+1}{1+x\alpha}\right]$$

Solving for $\frac{\partial PW(\alpha,x,n)}{\partial x}=0$ we obtain the following. If $\alpha=1$,

$$\frac{\partial \mathrm{PW}(\alpha,x,n)}{\partial x}=0 \Leftrightarrow 4n+4x+2-4n^2x=0 \Leftrightarrow x=\frac{2n+1}{2n^2-2}<\frac{1}{n-1}.$$

So only boundary solutions are feasible and PWL is minimized when $x=\frac{1}{n}$. If $\alpha \neq 1$,

$$\frac{\partial \mathrm{PW}(\alpha,x,n)}{\partial x}=0 \Leftrightarrow x=\frac{-n^2+1\pm\sqrt{n^4+1+2\alpha n^3+\alpha^2n^2-3\alpha n^2-\alpha^2 n-2n^3+\alpha n}}{(\alpha-1)(n^2-n)}.$$

Suppose that $\alpha>1$. Clearly, the negative root is not feasible, so consider the positive root, say x^*. If $x^* \leq \frac{1}{n}$, it must be that $(n-1)\,(\alpha^2+\alpha n-1-n) \leq 0$ which for $n>2$ and $\alpha>1$ is impossible.

Suppose that $\alpha<1$. If the negative root is less than or equal to $\frac{1}{n}$, we have that $-\sqrt{n^4+1+2\alpha n^3+\alpha^2n^2-3\alpha n^2-\alpha^2 n-2n^3+\alpha n} \geq (n+\alpha)(n-1)$ which is impossible. Take the positive root. If this root is larger than or equal to $\frac{1}{n-1}$, then $n(1-\alpha) \leq \alpha^2 -$

$3\alpha+2$ or $n \leq \frac{\alpha^2-3\alpha+2}{1-\alpha}$. The right hand side of this inequality has a maximum at 3 when $\alpha \to -1$ Since this value of is never actually achieved, this inequality only may hold when $n=2$. But $\frac{\partial \mathrm{PW}(\alpha,0.5,2)}{\partial x} = \frac{0.5\alpha+1.5}{0.5\alpha+1} > 0$ which means that the minimum is achieved at the boundaries of x. Since in this case these bounds imply monopoly and duopoly, by iv) in Section 2 we achieve the desired result. □

An implication of Proposition 5 is that disregarding firms heterogeneity stacks the deck in favour of small WL. Also, minimal PWL is given by the function $L(\cdot,\cdot)$ in Eq. (2.4). Recall that maximal PWL is given by the function $F(\alpha,\cdot)$ (defined in the second paragraph after the end of Proposition 4). Notice that since $L(\alpha,\cdot)$ is decreasing in α and $F(\alpha,\cdot)$ is increasing in n, the difference between maximal and minimal PWL increases with n for a given α. Also, since $P(\cdot, n, \alpha)$ is continuous in s_1, any PWL between $L(\alpha, n)$ and $F(\alpha, n)$ is reachable by the choice of s_1.

Finally we consider the effect of α alone on PWL. We have little to say about the value of α that maximizes PWL because first order condition of maximization with respect to α is not very informative. However, the continuity of $P(s_1, n, \cdot)$ has an interesting implication. Let $V \equiv \max\left\{\frac{s_1-H}{s_1}, \frac{(1+s_1)^2-2H-1}{(1+s_1)^2}, \frac{e^{s_1}-1-H}{e^{s_1}}\right\}$. The values in the bracket are respectively, $P(s_1, n, 0)$, $P(s_1, n, 1)$ and $\lim_{\alpha\to\infty} P(s_1, n, \alpha)$. Then, we have:

Corollary 1. *Any PWL* $\in (0, V)$ *is obtainable by the choice of* α.

5. Final comments

When one observes public policies on oligopolies one sees some concern about the number and the relative size of firms. But the question of the output set by oligopolists is cause of little or no concern at all. This paper provides some justification to this attitude: We found that WL due to the divergence between equilibrium and optimal output are small, even with as few as four firms in the market as shown in Section 2. On the contrary WL due to the number and relative size of firms can be quite substantive as found in Sections 3 and 4. This conclusion, though, is likely to be exaggerated by our assumption that the optimal number of firms is one. Other important factors are the consideration of product differentiation and other solution concepts, e.g. Bertrand or Stackelberg equilibria, see Cable et al. (1994) for the case of duopoly and quadratic utility. In fact, two of the main conclusions of Cable et al. (1994, p. 98) are that "the particular form of oligopolistic interaction exerts a major influence on the level of welfare" and "the power of inter-firm rivalry to further social welfare is highly sensitive to the degree of product differentiation in he market" (pp. 98–9). Moreover, in a dynamic framework WL can be larger than here because firms may collude. Thus, our results are just a first cut to the problem.

Our results have a number of implications for the applied literature.

1. To measure WL due to oligopolistic output setting may be misguided because these losses are likely to be small. However WL due to overentry or to asymmetric firms can be quite substantial. Lack of consideration of these points biases downwards our estimates of WL.
2. Bresnahan and Reiss (1991) found markets where, as the number of firms increased beyond three, the competitive effect of additional firms on average markups was exhausted, a fact that suggests that the outcome is very close to perfect competition. A possible explanation for their findings is that they considered markets where asymmetries and economies of scale were possibly small (i.e. doctors, dentists, druggists, plumbers and tire dealers). In contrast, Campbell and Hopenhayn (2002) find that this competitive effect persists with a large number of firms in markets were firms are asymmetric (and the product is differentiated). Our findings in this paper may help to understand the difference in results in these two papers.
3. The impact of mergers and collusive agreements on social welfare depends on the characteristics of the market. For instance, with identical firms and no fixed costs our results in Section 2 suggest that anti-trust authorities should not be very concerned with mergers that do not bring the number of competing firms below, say four. However merging from duopoly to monopoly approximately doubles PWL. If firms are not identical or there are fixed costs, traditional measures of concentration fail to capture the full size of WL.
4. WL depend crucially on the parameter that cannot be observed, but can be estimated. Our results point out the importance of the estimation of for the proper account of WL. This may be problematic because to say something empirical about the local (around the actual price) characteristics of the demand curve sounds reasonable, but our approach requires global information about those characteristics.

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References

Anderson, S.P., Renault, R., 2003. Efficiency and surplus bounds in Cournot competition. Journal of Economic Theory 113, 253–264.

Bresnahan, T., Reiss, P., 1991. Entry and competition in concentrated markets. Journal of Political Economy 99 (5), 977–1009.

Cable, J., Carruth, A., Dixit, A., 1994. Oligopoly and welfare. In: Cable, J. (Ed.), Current Issues in Industrial Economics. Macmillan Press, London.

Campbell, J., Hopenhayn, H., 2002. Market size matters. NBER Working Paper 9113.

Corchón, L., 2006. Welfare losses under Cournot competition. Working paper 06–39, Universidad Carlos III, June.

Cournot, A.A., 1838. Recherches sur les principles mathematiques de la thèory des Richesses. Hachette, Paris.

Dasgupta, P., Ushio, Y., 1981. On the rate of convergence of oligopoly equilibria in large markets: an example. Economics Letters 8, 13–17.

Daughety, A., 1990. Beneficial concentration. American Economic Review 80 (5), 1231–1237.

Farrell, J., Shapiro, C., 1990. Horizontal merger: an equilibrium analysis. American Economic Review 80 (1), 107–126.

Fraysse, J., Moreaux, M., 1981. Cournot equilibrium in large markets under increasing returns. Economics Letters 8, 217–220.

Guesnerie, R., Hart, O., 1985. Welfare losses due to imperfect competition. International Economic Review 26 (2), 525–545.

Harberger, A.C., 1954. Monopoly and resource allocation. American Economic Review: Papers and Proceedings 44, 77–87.

Hotelling, H., 1938. The general welfare in relation to problems of taxation and of railways and utility rates. Econometrica 6, 242–269.

Johari, R., Tsitsiklis, J. (2005). Efficiency loss in Cournot games. Mimeo, January 28, 2005.

Lahiri, S., Ono, Y., 1988. Helping minor firms reduces welfare. Economic Journal 1199–1203 December.

Leibenstein, H., 1966. Allocative efficiency versus x-efficiency. American Economic Review 56, 392–425.

López-Cuñat, J.M., 1999. One-stage and two-stage entry Cournot equilibria. Investigaciones Económicas 23 (1), 115–128.

Mankiw, G., Whinston, M., 1986. Free entry and social inefficiency. Rand Journal of Economics 17 (1), 48–58.

McHardy, J.P., 2000. Miscalculations of monopoly and oligopoly welfare losses with linear demand. Hull Economic Research Papers, November.

Novshek, W., 1980. Cournot equilibrium with free entry. Review of Economic Studies 42, 473–486.

Suzumura, K., Kiyono, K., 1987. Entry barriers and economic welfare. Review of Economic Studies 54, 157–167.

Tullock, G., 1967. The welfare costs of tariffs, monopolies, and theft. Western Economic Journal 5, 224–232.

Tullock, G., 2003. The origin rent-seeking concept. International Journal of Business and Economics 2 (1), 1–8.

Ushio, Y., 1983. Cournot equilibria with free entry: the case of decreasing average cost function. Review of Economic Studies 50, 347–354.

von Weizsäcker, C.C., 1980. A welfare analysis of barriers to entry. Bell Journal of Economics 11, 399–420.

Yarrow, G.K., 1985. Welfare losses in oligopoly and monopolistic competition. Journal of Industrial Economics 33 (4), 515–529.

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