Designing a Lottery for a Regret Averse Consumer

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Abstract

This paper investigates how a lottery operator might profit from a consumer's regret aversion. We derive the conditions under which an expected profit maximising monopolist finds it optimal to supply a "realistic" lottery game to a regret averse consumer and we interpret the results with a measure of regret aversion, identifying the central relationship between the consumer's regret aversion and intrinsic risk aversion in the determination of the lottery contract. We parameterise regret aversion and we calculate the optimal design for consumers with different degrees of regret aversion. We show that expected profit is increasing with the consumer's regret aversion.

1 Introduction

There is extensive evidence in the psychology literature to suggest that the anticipation of regret influences decision-making in a variety of settings in which the decision-maker receives information on the results of his foregone choices (e.g. Larrick and Boles, 1995; Zeelenberg *et al.*, 1996; Ritov, 1996; Zeelenberg and Beattie, 1997; and Inmam and Zeelenberg, 1998). One example of this appears to be in the demand for tickets to the postcode lottery (Zeelenberg and Pieters, 2004). The postcode lottery is like a regular lottery except that a player's ticket, should he buy one, is his postcode, typically a six character combination of letters and numbers that identifies the local area of one's home address (the postcode usually

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identifies around 20 addresses). A draw is made and if the player's postcode matches the winning postcode and he has bought a ticket, the player wins a stake of the jackpot prize. The fact that the player knows what his combination would be were he to buy a ticket and the fact that he would almost certainly discover whether or not his ticket were to win if any of his neighbours played means he is very likely to find out whether the decision he makes is the (*ex post*) right one. Given this source of feedback, an aversion to anticipated regret may induce a player to buy a ticket to the postcode lottery to protect himself from the large regret he would experience if he did not buy a ticket and his postcode were drawn as the winning postcode.

In this paper we ask how a postcode lottery operator might design a lottery so as to exploit a consumer's regret aversion and make as much money from the sale of a ticket as possible. This question is relevant because if an aversion to anticipated regret is a significant factor in the demand for tickets to the postcode lottery, then it is something the operator should be thinking about when it designs the lottery game. Moreover, we may learn something about how to model the consumer's preferences by considering the operator's pricing problem, since different preferences will give rise to different optimal lottery designs.

Using the theory of regret averse preferences from our previous work (Gee, 2010) to characterise the consumer's preferences, we model the situation in which an expected profit maximising monopolist designs a lottery and sells a ticket to a regret averse consumer. There are two main aims of this exercise. First we would like to see how far the theory can take us in explaining the existence of a market for lotteries and, second, we would like to determine what structure should be added to the theory to obtain the most realistic predictions about lottery games.

The theory of regret averse preferences in Gee (2010) provides axiomatic foundations for a representation that has previously been used in a number of applications (e.g. Braun and Muermann, 2004; Laciana and Weber, 2008; and Michenaud and Solnik, 2008). The theory accomodates two types of regret aversion, which the author refers to as Type I and Type II regret aversion. Type I regret aversion reflects an aversion to ex post comparisons of one's realised outcome with outcomes that could have been achieved had one chosen differently. It is analogous to the definition of regret aversion in Sarver (2008) and, for a given act selection, it implies a preference for smaller menus. That is, if the decision maker chooses a given act, he prefers to choose it from a menu that has fewer alternatives. The reason is that, for a given act selection, the existence of more alternatives only increases the liklihood that the decision maker will experience regret. Type II regret aversion, on the other hand, reflects a disproportionate distaste for large regrets: the decision maker does not like regrets of any size, but he *really* dislikes large regrets[1]. This is analogous to the definition of regret aversion in classic regret theory (Loomes and Sugden, 1982; Bell, 1982). The desire of the Type II regret averse decision maker to avoid large regrets means his preferred act selection from a given menu may differ from that of the expected utility maximiser, as he gives special weight to the acts that protect him from the greatest regrets. The current application, in which a consumer purchases a ticket to protect himself from the large regret he would feel if he failed to buy a ticket and his postcode won, is an example of this latter type of regret aversion.

Preferences are characterised as follows[2]. Suppose the consumer has to select an act from a menu M, where an act is a mapping from the set of states, S, to $\mathbb{R}$ (we are dealing with money outcomes), then the consumer selects act $a^* \in M$ that maximises

$$V(a, M) = \int_S \left(u\left(a\left(s\right)\right) - R\left(\max_{b \in M} u\left(b(s)\right) - u\left(a\left(s\right)\right) \right) \right) dP\left(s\right), \qquad (1)$$

where $u : \mathbb{R} \to \mathbb{R}$ is a twice differentiable, increasing and concave utility function over money outcomes, $R : \mathbb{R}_+ \to \mathbb{R}_+$ is a twice differentiable, increasing and convex regret externality and P is a probability measure over states $s \in S$. The concavity of u reflects an intrinsic risk aversion in the consumer's preferences - in the absence of concerns about regret the consumer is risk averse - and the convexity of R reflects the consumer's Type II regret aversion - the anticipation of a large regret has a particularly large impact on the consumer's evaluation of an act.

We investigate what further restrictions should be placed on u and R to arrive at the most realistic predictions about lottery designs. In doing so it is helpful to consider two extreme lottery classes. First, the *trivial lottery* is the lottery that guarantees a zero-net payoff to both the monopolist and the consumer. We identify this solution class with the case where there is no market for a lottery. The other extreme lottery class is the *infinite lottery*. This is the class of lotteries in which the monopolist sets an infinitely large jackpot prize, a zero probability of winning and a positive price for a ticket - the monopolist essentially demands that the consumer give him money for nothing. While neither the trivial lottery nor the infinite lottery provide an acceptable explanation for the existence of a market for lotteries, the monopolist may find it optimal to set either of them, depending on the precise nature of the consumer's regret averse preferences. We look for solutions that lie between these two extremes, optimal lottery contracts that entail a strictly positive jackpot prize, a strictly

[1] Consider the "lottery loser" who committed suicide because of the "despair" caused by his decision to not purchase his usual combination of numbers the one week he thought they came up. See *The Independent*, 16th June 1995.

[2] We consider the special case of the Gee (2010) representation that deals with money outcomes. See footnote 13.

positive ticket price and a probability of a win between zero and one. If the solution to the monopolist's optimisation problem is a lottery design of this kind, we say there is a *regret equilibrium.* As it represents what we deem a "realistic" lottery result, the regret equilibrium and, in particular, the conditions under which it exists, provides the focus of this paper.

We show that the nature of the optimal lottery contract depends critically on the trade-off between the consumer's desire to insure himself against risk (as measured by the concavity of u) and the consumer's desire to insure himself against regret (as measured by convexity of R). To formalise this comparison we appeal to the following measure of Type II regret aversion, which has also been used in Michenaud and Solnik (2008),

$$\rho(x) = u'(x) \frac{R''(u(x))}{1 + R'(u(x))}. \tag{2}$$

For a given u, $\rho(x)$ is a measure of the curvature of R at $u(x)$ and we associate higher values of ρ with higher degrees of Type II regret aversion. We show that for there to be a non-trivial solution to the monopolist's optimisation problem, $\rho(x)$ must exceed the Arrow-Pratt measure of absolute risk aversion,

$$\lambda(x) = -\frac{u''(x)}{u'(x)}, \tag{3}$$

for some values of x. That is, for the consumer to take a bet that has a negative expected value, the measure of regret aversion must exceed the measure of risk aversion for *some* wealth values. We impose a condition on the consumer's preferences to ensure that the optimal lottery is not the infinite lottery and we call this the finite solution condition. An implication of the finite solution condition is that the coefficient of regret aversion must not exceed the coefficient of risk aversion as $x \to \infty$. Thus we identify the balance of risk aversion and regret aversion that is necessary for the existence of a regret equilibrium (Theorem 1).

We give special consideration to the class of preferences for which $\rho(x) > \lambda(x)$ at low levels of x and $\rho(x) < \lambda(x)$ at high levels of x and in Theorem 2 we show that there will be a regret equilibrium if the consumer's preferences belong to this class, as long as they also satisfy the finite solution condition. In this way we identify some general conditions on the consumer's preferences that guarantee the existence of a regret equilibrium.

Using the measure of regret aversion in Equation 2 we parameterise regret preferences in a *constant relative regret aversion* (CRReA) specification. It is our understanding that this is the first time regret aversion has been successfully parameterised. In Theorem 3 we show that if the consumer is sufficiently intrinsically risk averse and sufficiently regret averse then the constant relative regret aversion specification, when combined with constant relative risk aversion, will definitely produce a regret equilibrium. We also show that the *constant absolute regret aversion* specification can only generate a regret equilibrium if the consumer's intrinsic absolute risk aversion is increasing in final wealth. As we find this assumption unrealisitic, we do not view the CAReA specification as appealing as the CRReA specification. We use the CRReA parameterisation to compare the optimal lottery designs for consumers with differing degrees of regret aversion. Since the monopolist's expected net revenue can be interpreted as the regret premium net of the risk premium, we would expect it to be increasing in the consumer's regret aversion. In Theorem 4 we present a result that confirms this prediction for CRReA preferences.

The remainder of the paper is structured as follows. In Section 2 we describe the background to the paper in more detail. We first describe the set-up of a typical state lottery and how the slightly different set-up of the postcode lottery encourages the decision-maker to anticipate post-decisional regret. We then discuss how to best model the lottery operator by considering what its objectives are and what constraints it faces. In Section 3 we present the formal model of the lottery design and in Section 4 we set about solving it. In Section 5 we provide two theorems that relate the existence of a regret equilibrium to the balance of the consumer's intrinsic aversion to risk and his aversion to regret. We parameterise regret aversion in Section 6 and in Section 7 we use this parameterisation to compare the optimal lottery designs for consumers with varying degrees of regret aversion. We finish with some conclusions in Section 8. All tables and figures are collected in Appendix A and all proofs are in Appendix B.

2 Background

2.1 State lotteries

In a typical state lottery a player selects a combination of numbers using numbers from a given list. A common example is the 6/49 format, in which a player chooses 6 numbers from the list of 1 to 49, giving him a choice of approximately 14×10^6 combinations (this format is used in the UK, France, Germany and Canada, for example).

At a fixed time and date a combination is randomly drawn as the winning combination. If a player has bought a ticket for the draw and some of the numbers in his combination match the numbers in the winning combination, the player wins a cash prize. The more numbers that match up, the bigger is the prize. When there is a complete match between the player's ticket and the winning combination, the player wins the jackpot prize. The order of the numbers in the combination does not matter, so, using the 6/49 format as an example, the combination 1-2-3-4-5-6 identifies the same ticket as the combination 6-5-4-3-2-1.

A common feature of state lotteries is what is known in the British National Lottery as the *Lucky Dip* ticket. This is a ticket in which the player does not choose the combination himself but accepts a combination that is randomly generated by a computer. Wolfson and Briggs (2002) conducted a survey on the British National Lottery and they found that 56% of respondents use the same numbers each time they play while only 3% of respondents "always" and only 5% of respondents "usually" use the Lucky Dip option. While many lottery players like to use the same numbers for each draw, there remains the option to randomise by purchasing a Lucky Dip ticket.

The jackpot prize is usually much larger than the prizes given out for getting, say, 3 or 4 or 5 matching numbers. Table 1 gives estimates of the prizes for the UK National Lottery game, Lotto. Each prize estimate in Table 1 is the mean payout to winners of that prize over the period June to November 2008. Except for the case when the player matches 5 balls and a bonus ball, which is not much more likely than the player winning the jackpot, there is a noticeable difference in prize value between the jackpot prize and the other prizes. Even when adjusted for the probability of winning the prize, the consolation prizes are markedly smaller than the jackpot prize.

While the probability of a ticket winning a given prize is fixed, the value of the prize may vary. This is because the lottery operator usually sets aside a pot of money for each prize class and this pot is split between claimants of the prize[3]. So if 2 ticket-holders were to match all 6 numbers, then the total jackpot funds would be split between 2 people. As it would be impossible to know how many other people would buy a given ticket, there is some uncertainty about what the individual's payout would be should he win the jackpot. However, because the probability of a ticket winning the jackpot prize is so low, there are usually few, if any, winners. For example, in Italy, where the winning probability is particularly low, there were 87 consecutive draws between January and August 2009 in which no ticket won the jackpot prize. Jackpot uncertainty is therefore relatively low, especially in games with particularly long odds.

Table 2 illustrates the range of jackpot odds found in lottery games[4]. The Italian Super-Enalotto is the lottery with the smallest chance of winning and, correspondingly, it has the largest jackpot prize. In August 2009 a single ticket owner won a jackpot prize of €146.9 million, Europe's biggest ever payout. At the other end of the spectrum, Ireland's 6/45 format, also used in Australia, gives the best chance of winning the jackpot prize. However,

[3] The prize for matching 3 balls in the UK National Lottery is fixed at £10, but all other prizes may vary from draw to draw.

[4] We use the mean payout to jackpot winners (i.e. winnings per claimant) from June to November 2008 as the estimate of the jackpot prize.

at odds of 1 in 8,145,060, it is still a very long shot bet. As one would expect, the jackpot prize in the Irish lottery is much lower than in Italy, averaging at around €4 million per winner.

So while the prize structures and the odds of winning differ slightly from one lottery to another, the basic formats of lotteries are the same and the motivation to play is clear - the chance to win a very large amount of money from a very small stake.

2.2 The postcode lottery

The postcode lottery was introduced in 1989 in the Netherlands as the National Postcode Loterij (hereafter the Dutch Postcode Lottery) and it is now also available in Sweden (Svenska PostkodLotteriet) and in the UK (The People's Postcode Lottery). These "Charity Lotteries" were all set up by the same company, Novamedia, which has the stated mission "to set up and operate Charity Lotteries all over the world to raise funds for charities and increase awareness for their work."[5]

In the postcode lottery the player's ticket, should he purchase one, is the postcode of his home address, which is a 6 character combination of letters and numbers (e.g. CB3 9DD). The winning postcode is randomly drawn from the entered postcodes and if the ticket-holder's postcode matches the winning postcode then the ticket-holder wins a stake of the jackpot prize, or the *street prize*. The total street prize, which is £25,000 in the UK, is split between the holders of the winning ticket, all of whom live in the same local area. If the ticket-holder's postcode does not match the winning postcode exactly but the first 4 characters of his postcode match the first 4 characters of the winning postcode, then the ticket-holder wins a stake of the smaller *sector prize*.

The feature of the postcode lottery that makes it interesting from the point of view of this paper is the strength of the post-decisional feedback. This feedback arises because the postcode is not a unique identifier of one's address. Rather it identifies in the region of 20 addresses in the same local area. What this means is that if you do not buy a ticket and you find out that one of your neighbours has won the jackpot prize, then you know that had you bought a ticket then you too would have won a stake of the jackpot prize. Furthermore, since the player's choice is between buying a ticket and not buying a ticket, rather than choosing from a large set of combinations, as it is in a typical state lottery, it seems likely that he regret not purchasing a ticket for a draw if his postcode were drawn as the winning postcode.

While it is natural to think that people may regret not purchasing a ticket to the postcode

[5] Novamedia website (www.novamedia.nl).

lottery when their neighbours win, the question we are interested in is whether players anticipate this regret and whether they alter their behaviour in anticipation of it.

Zeelenberg and Pieters (2004) study participation in the Dutch State Lottery with the aim of addressing this question. Using data from questionaires on participants' intention to play the postcode lottery and their attitudes towards regret, the experience of playing the lottery and the social norm, the authors argue that an aversion to anticipated regret is a significant factor in the demand for tickets to the Dutch Postcode Lottery. Furthermore they argue that an aversion to anticipated regret is *not* a significant factor in the demand for tickets to the Dutch State Lottery and they point to the different levels of post-decisional feedback in these lottery games as the reason for this division. In the Dutch State Lottery, as with other state lotteries, there is insufficient post-decisional feedback for the person who never buys a ticket. He may not know what combination he would have chosen had he decided to play (he may even have chosen the Lucky Dip ticket) and he may not pay any attention to the draw. In the postcode lottery it is much harder to avoid feedback on the result of the one's foregone choice.

If for some reason a player of a state lottery knew what combination he would have chosen had he played and if he knew he would discover the result of the draw, then there would be sufficient post-decisional feedback present for anticipated regret to play as important a role in his purchasing decision as in the postcode lottery. Consider, for example, a woman who uses the birthdays of her 6 children to select her lottery ticket and who watches the draw religiously every Saturday night. She knows that if she did not play then she would easily be able to verify whether her ticket would have been a winner upon observing the result of the lottery draw. For this woman, regret aversion may well be a strong motivation in her purchasing decision, but this is only because it is combined with other behavioural and cognitive attributes. For example, she may feel there is some cost to her changing her routine or she may not realise that she could be better off by avoiding the decision problem altogether. We accept that it may be valid to use an aversion to anticipated regret to explain participation in state lotteries, but for the rest of this paper we focus on the postcode lottery example as our motivation. The advantage of this example is that even a relatively sophisticated regret averse consumer, who realises that there is benefit to avoiding lottery games altogether and who is able to nullify the potential for anticipated regret in the purchasing decision in state lotteries by having no preconceived thoughts about what his combination would be if he played, cannot escape the effects of anticipated regret in the postcode lottery decision problem. He knows that if he were to play, his ticket combination would be his postcode and he knows that if that combination were to win, he would almost certainly find out about it. Therefore the postcode lottery has the potential to capture a larger audience of regret averse individuals than regular state lotteries because even the more sophisticated regret averse individual may find it ex ante optimal to buy a ticket.

We would like to model how a lottery operator would design a lottery for a regret averse consumer, but to do this we must first gain an understanding of the objectives of the lottery operator and the environment in which it operates. We now look at what institutional features of lotteries, both state lotteries and postcode lotteries, imply about how we should model the lottery design.

2.3 The lottery operator

In general, lottery games are strictly regulated. In some countries the government licences monopolist rights to provide lottery services (e.g. UK, Finland, Iceland), in some countries the state has a large stake in the lottery company (e.g. FDJ in France), while in other countries the national lottery is a 100% state owned monopoly (e.g. Belgium, Estonia, Hungary). The heavy regulation owes much to the traditional opinion that gambling is a social bad and that as such its consumption should not be decided by the market. But given the relatively accepted view of lottery play as a form of entertainment, many governments have used lotteries to achieve certain aims. For some governments, lotteries represent the safest way for individuals to exercise their natural desire to gamble. For example, in Estonia "the primary purpose of the state lottery monopoly is to satisfy the popular demand of the gambling impulse, but to do this calculatedly and by offering so-called soft or less addictive forms of gambling" (Laansoo and Nit, 2009 p40). By far the biggest reason for the existence of state lotteries, however, is the raising of funds for either government spending or for charity. These funds can be substantial. On average the profits from US state lotteries accounted for 1.1% of US state own-source revenue in 2007, but this figure was as high as 5.1% in Rhode Island and 6.8% in West Virginia[6]. When Italy risked defaulting on its debt in 1992 the Italian government even used lotteries as a fiscal measure and expanded the scale of state lottery products (Croce *et al.*, 2009). The Dutch Postcode Lottery, which was founded as a means to raise money for charity, distributed a total of €244 million euros among 66 charitable organisations in 2008 and it has dispensed more than €2.7 billion euros to its beneficiaries since its inception in 1989[7].

However, while lotteries offer a seemingly easy source of revenue, raising funds through the sale of tickets can be controversial because it represents a particularly regressive form of taxation (see, for example, Hansen, 2005). There is therefore something of a trade-off facing

[6] North American Association of State and Provincial Lotteries; U.S. Census Bureau; Tax Foundation calculations.

[7] Dutch Postcode Lottery Factsheet.

governments with respect to lottery policy. On the one hand there is a large potential revenue to be gained from running a lottery. On the other hand governments are wary of promoting lottery play too much because they do not want people to develop gambling problems and because they do not want to exploit vulnerable members of society (the poor and those with gambling problems). As a result, the advertising of lottery play is heavily regulated. In Finland, the law makes sure that the gambling monopolists (e.g. the monopolist lottery organisation and the monopolist slot machine organisation) do not compete with each other so that there are no aggressive sales promotions (Jaakkola, 2009) and in Belgium, Loterie Nationale tickets must have the odds of winning printed on the back.

Independent checks on lottery operations ensure the lotteries are fair. In the UK the National Lottery Commission established a set of rigourous procedures for the draws to ensure that they are "fair, random and honest"[8]. These procedures are overseen and approved by an independent adjudicator and viewers of the draw are made aware of the independent adjudicator's approval, so not only is the lottery fair, but players are assured that it is fair.

These institutional features of lotteries have the following implications for the modelling of the lottery operator. First, we can assume the lottery operator is a monopolist, so it can set the contract and take all of the rent. Second, the lottery operator is trustworthy in the sense that there are no doubts about it reneging on the agreement - the contract is honoured, the ticket wins with the stated probability and the full sum is paid in the event of a win. Third, the lottery operator has no difficulties in raising money for the jackpot prize - it is a very wealthy organisation and there is no upper bound on the prize it can offer for a winning ticket (this is more true for state lotteries than it is for postcode lotteries). Finally, since the main aim is to raise funds for government or charity, we can assume the lottery operator wants to maximise net revenue.

3 The Set-up

We now turn our attention to the formal model. We consider a scenario in which a monopolist designs a lottery and sells a ticket to a consumer. The lottery has the format of a simple postcode lottery - the consumer chooses to either purchase a ticket or to not purchase a ticket and there are two possible events: either the ticket wins or the ticket does not win. Once the consumer has made his choice, the uncertainty is resolved and it is revealed whether or not the ticket is a winner. This information is revealed to both parties, whether the consumer purchases a ticket or not.

The format of our lottery model differs from the postcode lottery and from regular state

[8] National Lottery Commission website.

lotteries in one notable respect - we neglect the grading of prizes. That is, we consider only the event that the ticket wins the jackpot and the event that the ticket wins nothing. We feel this simplification is valid for the purpose of this model because we believe lottery play is motivated primarily by the prospect of winning the jackpot prize.

3.1 Preliminaries

S is a set of states with a known probability measure, $P(S)$. We denote by W the event that the ticket wins and by L the event that the ticket loses. W and L are such that $W \cup L = S$ and $W \cap L = \emptyset$ and we denote by $q = P(W)$ the probability with which the winning event occurs.

The consumer must choose either the act of buying a ticket, B, or the act of not buying a ticket, D, and the net money payoff he receives from selecting either of these acts depends on what event occurs. We assume that $D(W) = D(L)$ and we normalise this quantity to zero. As such, the occurrence of state $s \in S$ is meaningful only in as far as it determines the ex post value of the lottery ticket.

For the act of purchasing a ticket, we define $x, p \in \mathbb{R}$ such that $B(W) = x$ and $B(L) = -p$. p represents the price of the lottery ticket while x represents the cash value difference between the jackpot prize and the price of the lottery ticket. We call this quantity, x, the winner's surplus. To avoid unnecessary complication, we make the simplifying assumption that there is no uncertainty about the value of the jackpot prize. That is, x is known with certainty[9].

We assume that $x \geq 0 \geq -p$. For the consumer's choice problem to be meaningful it must be that $x > 0 > -p$ (if even only one of these inequalities binds then one act weakly statewise dominates the other and so the choice problem is trivial[10]). We call the lottery in which $x = p = 0$ the *trivial lottery*.

The monopolist's problem is to choose the lottery contract, $\mathbf{v} = (x, p, q) \in \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 1]$. The selection of $\mathbf{v}$ is not trivial because the monopolist has to pay for whatever the consumer wins. If the consumer selects B then the monopolist pays the consumer x in event W and $-p$ in event L (that is, the monopolist receives the payment of p from the consumer if L occurs). If the consumer selects D he rejects the lottery and so the monopolist makes a zero payout regardless of what event occurs. The payoff information is summarised in Table 3.

The timing is as follows. At time $t = 0$ the monopolist chooses $\mathbf{v} \in \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 1]$.

[9] There may be uncertainty about the size of the jackpot prize because there may be more than one holder of the winning ticket.

[10] This is guaranteed by the irrelevance of statewise dominated alternatives, which is a feature of this representation of regret aversion (see Gee, 2010).

At time $t = 1$, with full knowledge of $\mathbf{v}$, the consumer decides whether or not to purchase a ticket. At time $t = 2$ it is revealed which event occurs and the monopolist and the consumer receive their payoffs. There is no discounting between periods so we incorporate the sequence of payments (the consumer paying for a ticket and then the monopolist paying out a prize after the lottery has been played) into one final payoff[11].

3.2 The monopolist's preferences

We assume the monopolist is a risk-neutral expected utility maximser and we assume that he behaves optimally. The monopolist's aim is then to set $\mathbf{v}$ so as to maximise expected net revenue, where expected net revenue is equal to

$$E\left[\Pi\left(\mathbf{v}\right)\right] = \begin{cases} p - q(x+p) & \text{if the consumer selects } B \\ 0 & \text{if the consumer selects } D. \end{cases} \tag{4}$$

We assume the monopolist has complete and costless access to capital so there is no cost to raising the prize money and no limit to the amount that can be offered. The only cost that the monopolist incurs is the payout to the consumer if the consumer buys a ticket and event W occurs. The monopolist's reservation payoff is the same as the payoff he receives from the consumer not purchasing a ticket - zero with certainty.

Since the trivial lottery, $\mathbf{v} = (0, 0, q)$, guarantees the monopolist a zero payoff, there is no need to consider the scenario in which the monopolist wishes to induce the consumer to select act D. The payoffs associated with that scenario are the same as when the monopolist sets the trivial lottery, so if it is ever optimal for the monopolist to set a lottery contract that the consumer would reject, then it must also be optimal to set the trivial lottery. Therefore we characterise the monopolist's optimisation problem as the maximisation of the first line of Equation 4 subject to the constraint that the consumer is willing to purchase a ticket and we characterise the trivial solution as the case where there is no market for a lottery. We now look at when the consumer is willing to purchase a ticket.

3.3 The consumer's preferences

3.3.1 The expected utility maximising consumer

Suppose that the consumer is a risk-averse expected utility maximiser with vNM utility, u. If the consumer is willing to select B, the relationship between x, p and q must be such that

[11] Allowing for time discounting in the model changes only the labeling of the prizes if the discount rates are the same for the monopolist and the consumer.

$$E(u) = qu(x) + (1-q)u(-p) \geq u(0). \tag{5}$$

Since u is concave, there is no feasible $\mathbf{v}$ that can produce a positive level of expected net revenue for the monopolist. This is a familiar result[12]. A risk-averse expected utility maximiser will not take a bet that has a negative expected value. Therefore to induce a risk-averse individual to buy a lottery ticket, the expected winnings must be positive and as a corollary the monopolist's expected net revenue must be negative. If the consumer were a risk loving expected utility maximiser, he would be willing to take a bet that has a negative expected value and so there would be an opportunity for the risk neutral monopolist to earn a positive expected net revenue by selling a ticket to the consumer. However, we would like to maintain the assumption of intrinsic risk-aversion. We will show that even when u is concave, the monopolist can profit from selling a lottery ticket if the consumer is regret averse.

3.3.2 The regret averse consumer

Suppose now that the consumer has regret averse preferences as in Gee (2010). When he makes his act selection the consumer considers not only the payoff he receives in each state but also the alternative payoff he would have received had he selected the ex post best act in that state. If the alternative payoff is better than the payoff the consumer receives, he feels regret and this tempers his enjoyment of the payoff he receives. We represent regret averse preferences as follows. Given a menu of acts M, the decision-maker chooses act $a^* \in M$ so as to maximise

$$V(a, M) = \int_S \left(u(a(s)) - R\left(\max_{b \in M} u(b(s)) - u(a(s)) \right) \right) dP(s), \tag{6}$$

where u is a vNM utility function, R is an increasing and convex regret externality and P is a probability measure over states[13]. We assume that u and R are twice differentiable and that $u', R', R'' > 0$ and $u'' < 0$ and we refer to these as the *standard conditions* on u and R.

The ex ante evaluation of act $a \in M$ goes as follows. In state $s \in S$ the consumer assigns a vNM utility evaluation to the payoff he receives when he selects act a and state s occurs. If

[12] We do not provide a formal proof but it can be shown easily using Jensen's inequality on the consumer's participation constraint.

[13] We have slightly abused the notation in Gee (2010). In the original version, $a \in A$ is a mapping from S to the set of lotteries and u is a mapping from the set of lotteries over outcomes to $\mathbb{R}$. Here a is a mapping from S to $\mathbb{R}$ and u is a vNM utility function over money outcomes, which maps from $\mathbb{R}$ to $\mathbb{R}$. While it is not completely true to the original representation, our notational shortcut is valid because the outcomes of the Anscombe-Aumann acts in our application are degenerate lotteries over $\mathbb{R}$.

$a(s) < b(s)$ for some $b \in M$ the consumer experiences regret and the overall utility assigned to the selection of a in s is diminished by amount R. R is a function of the difference between the best possible vNM utility assignment in state s and the vNM utility assignment from selecting act a in s. Furthermore, R is convex, so large regrets cause particularly large reductions to the vNM utility assignment. The convexity of R reflects the consumer's Type II regret aversion and it implies that his act selection may not be the same as the Type II regret neutral expected utility maximiser (note that if R is linear the representation in 6 is a positive affine transformation of the expected utility representation for a consumer with vNM utility u).

For a given lottery contract, $\mathbf{v}$, the consumer will purchase a ticket only if

$$qu\left(x\right)+(1-q)u\left(-p\right)+qR\left(u(x)\right)-(1-q)R\left(-u\left(-p\right)\right)\geq 0. \tag{7}$$

Here we have used a uniqueness property of the regret representation, which ensures that it is invariant to additions of an intercept term to the choiceless utility function, u, to set $u\left(0\right)=0$.

Equation 7 shows that the condition for the regret averse consumer to buy a ticket can be written in the same way as the expected utility condition in Equation 5, but with the addition of a net expected externality term that is made up of the difference between the expected value of regret from not buying the ticket when the ticket wins and the expected value of regret from buying the ticket when the ticket does not win. Call this term $\Gamma\left(x,p,q\right)$.

$$\Gamma\left(x,p,q\right)=qR\left(u(x)\right)-(1-q)R\left(-u\left(-p\right)\right). \tag{8}$$

The value of Γ relative to the value of expected choiceless utility will determine how the regret averse consumer behaves differently from the expected utility maximiser.

How the regret averse consumer behaves differently then depends on the particular format of the bet. For example, there are some bets that the risk averse expected utility maximiser is willing to take, but that the regret averse consumer is not willing to take. These are bets that satisfy $E\left(u\right)\geq 0$ and $E\left(u\right)+\Gamma<0$. On the other hand there are bets that the risk averse expected utility maximiser is *not* willing to take but that the regret averse consumer *is* willing to take. These are bets that satisfy $E\left(u\right)<0$ and $E\left(u\right)+\Gamma\geq 0$. This second category describes the sort of bets that we are interested in. Since the monopolist cannot make a positive expected net revenue from a risk averse expected utility maximiser, we have to look for bets that the expected utility maximiser will not take on to explain the market for a lottery.

Let us look more closely then at when and how the regret averse consumer acts differently from the expected utility maximiser. We first notice that the left-hand side of Equation 7 is increasing in x and decreasing in p, so an increase in the value of the jackpot prize makes buying more attractive while an increase in the price of a ticket makes buying less attractive. This is as we would expect. We assume that u and R are *strictly* increasing functions (recall the standard conditions on u and R), so, for a given x, if there is a price at which the consumer is indifferent between buying and not buying then the consumer will strictly prefer to buy a ticket at any price below this value and strictly prefer to not buy a ticket at any price above it.

Now suppose the monopolist sets the price at some level at which the expected utility maximiser would be indifferent between buying a ticket and not buying a ticket. That is, he sets $\mathbf{v}$ so that Equation 5 binds (we ignore for now what level of expected net revenue this would generate for the monopolist). Then the regret averse consumer will buy a ticket only if $\Gamma \geq 0$. Substituting (binding) Equation 5 into the value of Γ, we can derive the purchasing condition for the regret averse consumer as: $V(B, \{B, D\}) > V(D, \{B, D\})$ only if

$$\Gamma = qR(u(x)) - (1-q)R\left(\frac{q}{1-q}u(x)\right) > 0. \tag{9}$$

Since R is strictly convex and $x > 0$, the following preference rules follow:

$$V(B, \{B, D\}) > V(D, \{B, D\}) \text{ if } 0 < q < \frac{1}{2}; \tag{10}$$

$$V(B, \{B, D\}) = V(D, \{B, D\}) \text{ if } q = \frac{1}{2}; \tag{11}$$

$$V(B, \{B, D\}) < V(D, \{B, D\}) \text{ if } \frac{1}{2} < q < 1. \tag{12}$$

So when $q = \frac{1}{2}$ the regret averse consumer behaves the same way as the expected utility maximiser (this is analogous to the result in Loomes and Sugden (1982, p823) and it is a result that follows from Axiom 8 in Gee (2010)). When $0 < q < \frac{1}{2}$ the regret averse consumer displays less risk aversion than the expected utility maximiser (this is analogous to the result in Bell (1982, p971)) and when $\frac{1}{2} < q < 1$ the regret averse consumer displays more risk aversion than the expected utility maximiser.

The intuition behind these results is as follows. The consumer's Type II regret aversion, as manifested in the convexity of R, makes him apply special attention to states of the world in which there is the potential to be very "wrong", and the "right" decision in these states appears relatively more attractive to the consumer *ex ante*. When q is small - when there is a small probability of a large win and a large probability of a small loss - the consumer pays special attention to W as this is the event with the larger payoff difference between acts.

The "right" decision in W is buying a ticket, so the regret averse consumer is more inclined to buy a ticket than the expected utility maximiser. On the other hand, when q is large - when there is a large probability of a small win and a small probability of a large loss - the consumer pays special attention to L as this is the event with the larger payoff difference between acts. The "right" decision in L is not buying a ticket and so in this case the regret averse consumer is *less* inclined to buy a ticket than the expected utility maximiser.

The above results imply that the monopolist may be able make a positive expected net revenue from selling a ticket to a regret averse consumer, but only if the ticket is more likely to lose than it is to win (i.e. only if $q < \frac{1}{2}$). The results also imply that regret aversion cannot explain the demand for short-odds betting. In fact Equation 12 shows that the regret averse consumer is even less willing to take on a short-odds bet than the expected utility maximiser.

4 Solving for the optimal lottery contract

From this preliminary analysis of the regret averse consumer, we can see that the monopolist will have to set a long shot bet if he wants to make a positive expected net revenue from the sale of a lottery ticket to a regret averse consumer. But what exactly will the lottery contract look like?

The monopolist's optimisation problem is to choose $\mathbf{v}$ so as to maximise the first line of Equation 4 subject to Equation 7 and the solution to this problem, the optimal lottery contract, will depend on the functional forms of u and R. In this section we set about solving for the optimal lottery contract.

The first thing we can say is that the participation constraint in Equation 7 must be binding at a solution. To see why, recall that for a given x, the left hand side of Equation 7 is decreasing in p. Since, for a given x, the monopolist's expected net revenue (Equation 4) is increasing in p, Equation 7 must be binding at a solution. If it is not then the monopolist can increase the price of the ticket by a small amount without changing the consumer's preference ordering over acts and earn a strictly higher level of expected net revenue, in which case the starting point could not have been a maximum. Rearranging the binding participation constraint gives the following expression for q:

$$q = \frac{u(-p) - R\left(-u(-p)\right)}{u(x) + R(u(x)) - u(-p) + R(-u(-p))}. \tag{13}$$

Substituting Equation 13 into the monopolist's objective function, we arrive at the modified monopolist's optimisation problem:

$$\max_{x,p\in\mathbb{R}_+} \Omega\left(x,p\right) = \frac{p\left(u(x)+R\left(u(x)\right)\right)+x\left(u(-p)-R\left(-u(-p)\right)\right)}{u(x)+R(u(x))-u(-p)+R(-u(-p))}. \tag{14}$$

Equation 14 is the function we will try to maximise to determine the optimal lottery contract.

4.1 Extreme solutions

The exact solution to Equation 14 will depend on the functional forms of u and R. One possibility is that the solution will be the trivial lottery, in which the monopolist sets $\mathbf{v} = (0,0,q)$. In this case the monopolist sets a contract such that there is a zero net payment in all states and this is equivalent to there being offered no lottery at all. We know that this will be the solution when the consumer is a risk averse expected utility maximiser, or if the consumer is Type II regret neutral (i.e. if R is linear).

Another possibility is what we call the *infinite lottery*. This is the solution in which the monopolist sets $\mathbf{v} = (\infty, p, 0)$. The monopolist offers an infinitely large jackpot prize and charges a positive ticket price, but there is a zero probability that the ticket will win. This contract is equivalent to the monopolist asking the consumer to give him money for nothing. While it may be crazy to accept this contract, for some specifications of u and R that satisfy the standard conditions (i.e. $u', R', R'' > 0, u'' < 0$), it will be the solution to Equation 14. The monopolist will set this contract and the consumer will buy a ticket.

We would like to rule out optimal lottery designs of this kind because they imply a very fundamental and undesirable form of irrationality in the consumer's preferences. For this reason we impose the following condition on the consumer's preferences.

Condition 1 (Finite solution condition)

$$\Omega\left(0,0\right) \geq \Omega\left(\infty,p\right)$$

for all $p \geq 0$.

This condition ensures that it is never optimal to set the infinite lottery. Moreover, it says that any lottery that promises an infinitely large jackpot prize yields a non-positive expected net revenue.

Proposition 1 *The finite solution condition obtains if and only if*

$$\lim_{x\to\infty} u'\left(x\right)\left(1+R'(u(x))\right) \leq u'\left(0\right)\left(1+R'(0)\right). \tag{15}$$

Proposition 1 tells us that the finite solution condition is equivalent to the condition that the slope of $\tau(x) = u(x) + R(u(x))$ be smaller at x in the limit to infinity than at zero. We can interpret this as a condition that, on average, u must be more concave than R is convex, so that, in a sense, risk aversion dominates regret aversion in the limit to infinity. We will later show more formally that if regret aversion does dominate risk aversion as final wealth tends to infinity then the finite solution condition cannot hold.

We have identified two extreme solution classes. On the one hand we have the trivial lottery - the situation in which there is no market for a lottery; and on the other hand we have the infinite lottery - the situation in which the monopolist is able to manipulate the consumer with a ludicrous lottery contract. Neither are particularly interesting cases and neither do a good job at explaining the market for lotteries. We are interested in the class of solutions that lies between these extreme cases, the class of solutions in which the monopolist offers a lottery with a positive jackpot prize, a positive ticket price and a positive probability of a win, and we define a regret equilibrium as a solution of this kind.

Definition 1 (Regret Equilbrium) *A regret equilibrium is a triple, (x^*, p^*, q^*), where $x^*, p^* > 0$ and $q^* \in (0, 1)$, and (x^*, p^*, q^*) maximises Equation 4 subject to Equation 7.*

5 The balance of aversions

We next demonstrate that the existence of a regret equilibrium depends on the relationship between the consumer's intrinsic risk aversion and regret aversion. We will formalise this relationship with a theorem, but we first define two measures of aversion.

$$\lambda(x) = -\frac{u''(x)}{u'(x)}; \tag{16}$$

$$\rho(x) = u'(x)\frac{R''(u(x))}{1 + R'(u(x))}. \tag{17}$$

λ and ρ denote measures of the consumer's intrinsic risk aversion and regret aversion respectively. λ is the standard Arrow-Pratt measure of risk aversion and it measures the curvature of the consumer's intrinsic utility function, u. ρ has a similar feel to it as λ. For a given u, ρ measures the curvature of R, which is how we measure Type II regret aversion. Furthermore, just as λ is a unique measure of risk attitudes, in the sense that if we apply a positive affine transformation to u then λ is unaffected, so is ρ a unique measure of regret attitudes, in the sense that that if we apply the transformation up to which (u, R) is unique, then ρ is unaffected. Following Michenaud and Solnik (2008) we refer to ρ as the coefficient of absolute regret aversion.

We considered the behavioural meaning of ρ in Gee (2010), showing that, when evaluated at $x = 0$, $\rho(x)$ features in the approximation of the regret averse individual's certainty equivalent for a small risk. In particular, the certainty equivalent for a small risk, z, with mean $\overline{z}$, can be approximated as:

$$CE(z) \approx \overline{z} - \frac{\sigma_z^2}{2}\lambda(0) - \frac{V_{z,CE(z)}}{2}\rho(0), \tag{18}$$

where

$$V_{z,CE(z)} = \int_{S_1} \left(z\left(s\right) - \overline{z}\right)^2 dP(s) - \int_{S_2} \left(z\left(s\right) - \overline{z}\right)^2 dP(s), \tag{19}$$

S_1 is the subset of S in which $z\left(s\right) \leq CE(z)$, S_2 is the subset of S in which $z\left(s\right) > CE(z)$, and, for the calculation of u, the initial wealth is set to $w + \overline{z}$.

The term $V_{z,CE(z)}$ in Equation 18 is a measure of the skew of z relative to the certain outcome. In combination with $\rho(0)$, this term determines by how much the regret averse individual's evaluation of the risk differs from the expected utility maximiser's evaluation of the risk. When the individual is Type II regret neutral ($R'' = 0$), the certainty equivalent is identical to that of the expected utility maximiser.

We now state a theorem giving some conditions on the relationship between ρ and λ that are necessary for existence of a regret equilibrium.

Theorem 1 (Necessity) *A regret equilibrium,* $\mathbf{v}^* = (x^*, p^*, q^*)$, *exists only if*

$$\rho(x) > \lambda(x)$$

for some $x < x^*$ *and there is no* d *such that*

$$\rho(x) > \lambda(x)$$

for all $x > d$.

Any preference specification that fails to satisfy the conditions in Theorem 1 cannot generate a regret equilibrium. Thus we have identified a trade-off between intrinsic risk aversion and regret aversion in the determination of the regret equilibrium. The finite solution condition requires that intrinsic risk aversion dominate regret aversion in the limit $x \to \infty$. If $\rho(x) > \lambda(x)$ in the limit then the solution will be the infinite lottery[14]. However, the first and

[14]It is not necessary that $\lambda(x) > \rho(x)$ as $x \to \infty$ because there are some circumstance under which a

second order conditions for the identification of a local maximum imply that the individual must show a sufficient degree of regret aversion over values of x prior to the jackpot solution. If the balance is right, there will be a regret equilibrium, otherwise the solution will be the uninteresting trivial solution or the unrealistic infinite lottery. This result highlights the point that regret aversion alone is not sufficient to explain the market for lotteries. Rather it is the relationship between regret aversion and intrinsic risk aversion that matters.

For example, consider the preference specification illustrated in Figure 1. In this example the consumer is Type II regret averse, $\rho(x) > 0$ for all $x > 0$, but there is no regret equilibrium. The solution is the trivial lottery and the reason is that $\lambda(x) > \rho(x)$ for all $x > 0$. Another example is illustrated in Figure 2. This is the case of logarithmic u and quadratic R. The necessary conditions are met and in this case there is a regret equilibrium, $\mathbf{v}^* = (1.201w, 0.053w, 0.0391)$. The optimal lottery design for this consumer involves a jackpot of approximately 125% of the consumer's wealth, a 3.91% probability that the ticket will win and a ticket price of 5.3% of the consumer's wealth. By selling this lottery ticket, the monopolist earns an expected net revenue of approximately 0.4% of the consumer's wealth[15].

5.1 A single crossing point

Notice that in the example illustrated in Figure 2 there is a neat relationship between λ and ρ. At low levels of x we observe $\rho(x) > \lambda(x)$, there is a single value of x at which $\rho(x) = \lambda(x)$ and at high levels of x we observe $\lambda(x) > \rho(x)$.

There are no restrictions on how many times the sign of $\rho(x) - \lambda(x)$ may change for there to be a regret equilibrium; there may be many regions in which it is positive and many regions in which it is negative and there may still be a regret equilibrium as long as the necessary conditions are met. We do not require a single crossing point like in Figure 2 for there to be a regret equilibrium. However, when there is such a simple relationship between λ and ρ we can say more about the existence of a regret equilibrium.

Theorem 2 (Single crossing point) *If the finite solution condition obtains and there is some $d > 0$ such that*

$$\lambda(x) < \rho(x)$$

for $x < d$, and

$$\lambda(x) > \rho(x)$$

regret equilibrium may hold if the sign of $\lambda(x) - \rho(x)$ oscillates between postive and negative in the limit to infinity. See page 34.

[15] Notice that the results for x and p in this example are proportions of the consumer's wealth. This is because we assume u is logarithmic. Such a representation may be advantageous for estimations if we wish to treat the wealth parameter as a free variable.

for $x > d$, then there exists a regret equilibrium.

Theorem 2 states some simple sufficiency requirements for the existence of a regret equilibrium, requirements that cover a broad class of preference specifications. In particular, providing the finite solution condition holds, there will be a regret equilibrium if there is a single crossing point between λ and ρ.

6 Parameterising regret aversion

Now that we know some conditions under which a regret equilibrium exists, we are in a position to investigate how the nature of the optimal lottery design varies with varying degrees of regret aversion. However, before proceeding we must first establish how we will make comparative statements about levels of regret aversion in our representation. For this reason we would like to parameterise regret aversion in our preference representation. We next consider two possible parameterisations and we investigate the nature of the optimal lottery designs for both.

6.1 Constant absolute regret aversion

Consider the coefficient of absolute regret aversion, $\rho(x)$, and suppose it is constant for all $x > 0$. That is, we hypothesise a class of preferences in which u and R are such that $\rho(x) = \bar{\rho}$ for all $x > 0$. We call this the *constant absolute regret aversion* (CAReA) specification. Given CAReA we can use $\bar{\rho}$ as a measure of regret aversion and we can make comparative statements about degrees of regret aversion using the values of $\bar{\rho}$.

A problem with the CAReA specification is that, when combined with the constant or decreasing absolute risk aversion class of intrinsic utility functions, it is impossible to generate a regret equilibrium. This is because if we have $\lambda(x) > \bar{\rho}$ for large values of x, we must have $\lambda(x) > \bar{\rho}$ for all values of x. So either regret aversion will dominate risk aversion in the limit to infinity, in which case the finite solution condition will be violated, or risk aversion will dominate regret aversion everywhere, in which case the solution will be the trivial lottery. This property of CAReA preferences is problematic because it implies that the only way a regret equilibrium can be supported is if λ increasing over some wealth levels. Since we view this as an unrealistic assumption, we conclude that the CAReA specification is not the right way to parameterise regret aversion.

6.2 Constant relative regret aversion

Another possible parameterisation is what we call the *constant relative regret aversion* (CR-ReA) specification. For this class of preferences, we set

$$\frac{R''(u(x))}{1+R'(u(x))} = c \tag{20}$$

where $c > 0$ is the coefficient of relative regret aversion. The coefficient of absolute regret aversion is then equal to

$$\rho(x) = cu'(x). \tag{21}$$

For a given u, we can represent regret aversion using the constant, c. Since u is concave, ρ is decreasing in final wealth, so the individual's sensitivity to regret decreases with final wealth levels under this specification. We view this as intuitively appealing - less wealthy people have more to lose from making large financial errors, so it seems natural that they should be more sensitive to feelings of regret that arise from financial decisions. Furthermore, considering that we seek to use an aversion to anticipated regret to explain the market for lotteries, it is worth noting that the majority of tickets to state lotteries are bought by people from low wealth backgrounds (see, for example, Clotfelter *et al.*, 1999). The assumption that one's sensitivity to anticipated regret decreases in final wealth appears to be consistent with this observation.

Under CRReA preferences the decision-maker's preferences are represented by:

$$R(u) = A\left(e^{cu} - 1\right) - u, \tag{22}$$

where $A > \frac{1}{c}$.

Now suppose the consumer's intrinsic risk preferences take the constant relative risk aversion (CRRA) form,

$$u(x) = \frac{(w+x)^{1-r} - w^{1-r}}{1-r}, \tag{23}$$

where $r \neq 1$ and where w is the consumer's initial wealth level[16]. We combine CRRA risk attitudes with CRReA regret attitudes and for notational ease we refer to these preferences as *CRRA2* preferences.

Theorem 3 *Suppose the consumer has CRRA2 preferences with parameters r and c. Then there is a regret equilibrium if $r > 1$ and $c > c^{\dagger} = rw^{r-1}$.*

[16] One appealing feature of this specification is that λ is decreasing in final wealth, which we view as more realistic than constant or increasing intrinsic absolute risk aversion.

Theorem 3 states that a regret equilibrium will exist if the CRRA2 consumer is sufficiently intrinsically risk averse and sufficiently regret averse. $c^{\dagger}$ is the threshold value of c that must be met for there to exist a regret equilibrium. Since the coefficient of relative regret aversion is meaningful only in terms of the parameters of a given intrinsic utility specification, it is no surprise that $c^{\dagger}$ should be a function of r and w. If $r < 1$, that is, if the consumer is not sufficiently intrinsically risk averse, we get the infinite lottery because regret aversion dominates risk aversion as the size of the prize approaches infinity; and if $c < c^{\dagger}$, if the consumer is not sufficiently regret averse, we get the trivial lottery.

Theorem 3 shows that, as well as having certain desirable properties, the CRRA2 preference specification guarantees the existence of a regret equilibrium for a wide range of intrinsic risk aversion and regret aversion parameters. We can now use this specification to formally compare optimal lottery designs for consumers with differing degrees of regret aversion, using c as the measure of regret aversion.

7 Example lottery designs

Tables 4-6 describe the optimal lottery designs for CRRA2 consumers with varying degrees of regret aversion. For each set of results we identify the threshold value, $c^{\dagger} = rw^{r-1}$, and we express the different degrees of regret aversion as proportions (> 1) of $c^{\dagger}$. For values of $c < c^{\dagger}$, the optimal lottery design is the trivial lottery. That is, there is no market for a lottery. In each example we set $w = 250,000$.

The results appear to suggest that, for a given u, the monopolist makes a higher level of expected profit the higher is the consumer's regret aversion, as measured by the coefficient of relative regret aversion, c. In Theorem 4 we show that this pattern is true of CRReA regret preferences in general and not just CRRA2 preferences.

Theorem 4 *Suppose the consumer has CRReA preferences with coefficient of relative regret aversion $c > 0$ and intrinsic risk preferences such that there is a regret equilibrium. Then the monopolist's expected profit is strictly increasing in c.*

Theorem 3 identified a subset of CRReA preferences in which we know there will be a regret equilibrium. Theorem 4 now tells us that the monopolist's expected profit at the equilibrium is higher the more regret averse is the consumer. The reason is that the regret averse consumer is willing to pay to insure himself against the possibility of regret and the more regret averse he is, the more he is willing to pay. Realising that this is the case, the monopolist can extract rent from the consumer in the form of expected profit by designing a contract that exposes the consumer to the possibility of regret and that offers the consumer an option to insure against the regret (i.e. the option to purchase a lottery ticket).

Perhaps less expected is the result that the probability with which the ticket wins in the optimal lottery design is not decreasing in the consumer's regret aversion. This may appear counterintuitive because we associate higher regret aversion with an increasing preference for skew and so we may expect the optimal lottery design to have longer odds the more regret averse the consumer. Our calculations appear to contradict that hypothesis. Rather it appears that as the jackpot prize increases, both the price of the ticket and the probability of a win increase too.

8 Conclusions

We considered participation in the postcode lottery as an example of how an aversion to anticipated regret may affect risk attitudes when there is a guarantee of feedback on the results of one's foregone choices. We used our representation of regret preferences from Gee (2010) to model the market for such a lottery and we investigated what constraints on this representation are required to generate the most realistic predictions.

The finite solution condition requires that, in order to avoid an implausible situation in which the lottery designer will find it optimal to set no limit on the jackpot prize and set a winning probability of zero while still selling tickets at a positive price, there must be some limit on the consumer's regret aversion. In particular, the coefficient of absolute regret aversion, ρ, must not exceed the coefficient of absolute (intrinsic) risk aversion, λ, as final wealth tends to infinity.

For the optimal lottery to be non-trivial, the coefficient of absolute regret aversion must exceed the coefficient of absolute risk aversion at some wealth levels. We showed that if the consumer's preferences have a single crossing point property, that is, if $\rho > \lambda$ for low levels of final wealth and $\rho < \lambda$ for high levels of final wealth, with the sign of $(\rho - \lambda)$ changing just once, there will be a regret equilibrium in which the monopolist sets a long shot bet with a strictly positive jackpot prize and ticket price.

We paramaterised regret averse preferences using the constant relative regret aversion specification, which has the feature that the consumer's aversion to anticipated regret is lower the lower is his initial wealth. We showed that when the consumer's intrinsic risk preferences are CRRA this specification generates a regret equilibrium if the coefficients of relative risk aversion and relative regret aversion are large enough. We then made comparisons of the optimal lottery designs for consumers who share the same intrinsic aversion to risk but who have differing levels of regret aversion and we showed that the monopolist's expected profit is increasing with the consumer's regret aversion.

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A Tables and Figures

Table 1: The format of the UK National Lottery game, Lotto (estimated prizes are equal to the mean payout to each winner in the period June-November 2008)

Balls matching	Probability	Prize (£)
6	7.151×10^{-8}	$2,940,269$
5 + bonus ball	4.291×10^{-7}	$143,815$
5	1.802×10^{-5}	$1,609$
4	9.681×10^{-4}	63
3	1.754×10^{-2}	10

Table 2: Lottery formats from around the world (estimated jackpot is based on the mean payout to each winner in the period June-November 2008)

Lottery	Probability of jackpot win	Jackpot prize	Ticket price
UK	7.151×10^{-8}	£$3,000,000$	£1
USA Megamillions	5.691×10^{-9}	$\$40,000,000$	\$1
Ireland	1.228×10^{-7}	€$4,000,000$	€1.50
Italy	1.606×10^{-9}	€$45,000,000$	€1

Table 3: Net payments from the monopolist to the consumer

	Win	Lose
Buy	x	$-p$
Don't Buy	0	0

Table 4: The optimal lottery design for a CRRA2 consumer when r=1.05

	c	Jackpot prize	Price	Probability of winning	Return (%)
$c^{\dagger}\times$	1.01	$21,000$	6.80	0.00032	0.0014
	1.03	$238,515$	515	0.0022	0.22
	1.1	$5,315,000$	$15,000$	0.0026	7.87

Table 5: The optimal lottery design for a CRRA2 consumer when r=10

	c	Jackpot prize	Price	Probability of winning	Return (%)
$c^{\dagger}\times$	1.01	400	0.40	0.001	0.0021
	1.05	$1,000$	8.80	0.0087	0.27
	1.3	$12,160$	360	0.0291	1.71
	1.6	$37,280$	$2,380$	0.0549	14.0

Table 6: The optimal lottery design for a CRRA2 consumer when r=40

	c	Jackpot prize	Price	Probability of winning	Return (%)
$c^{\dagger}\times$	1.04	192	0.5	0.0026	7.14×10^{-3}
	1.15	$1,220$	18.4	0.0151	0.012
	1.5	$4,185$	185	0.0426	3.63
	3	$13,080$	1080	0.058	29.8

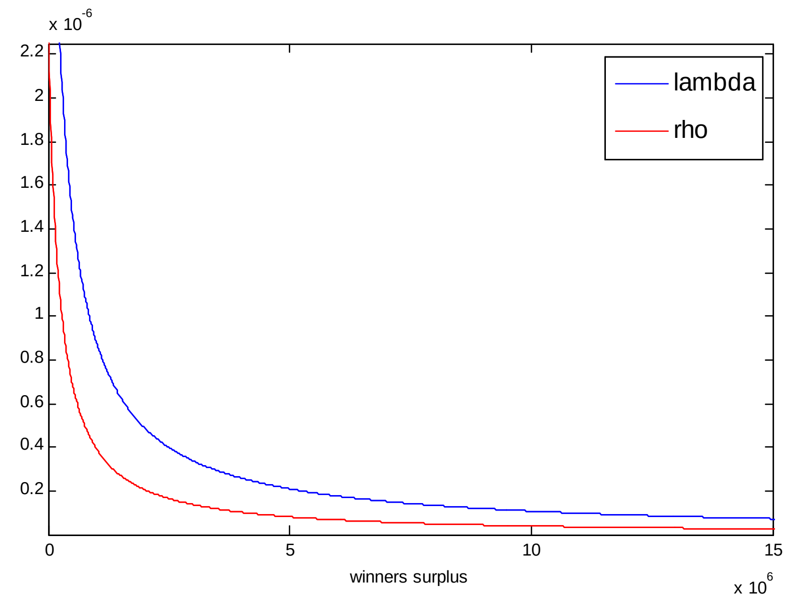

Figure 1: Preferences that generate a trivial lottery

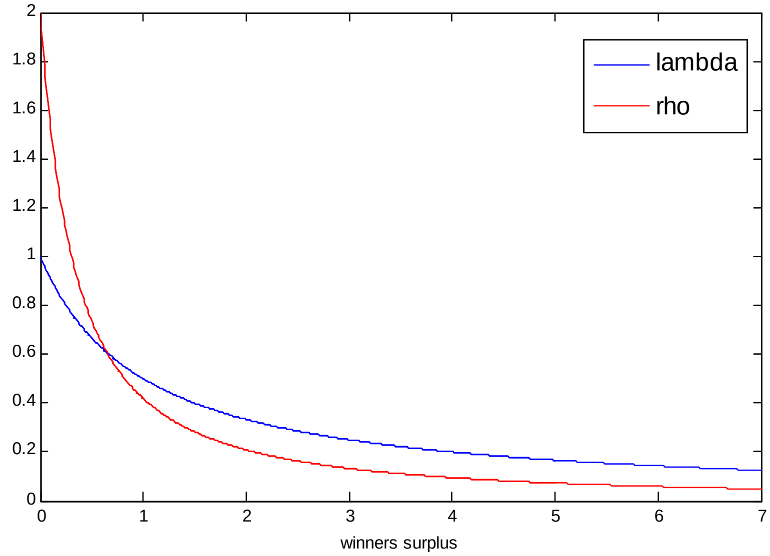

Figure 2: Preferences that generate a regret equilibrium

B Technical Appendix

B.1 Notation

We define $\theta(u) = u + R(u)$ for $u \geq 0$, and we apply the standard conditions on u and R:

(i) $u(t)$ and $R(u)$ are continuous and twice differentiable;

(ii) $u(0) = 0$, $u' > 0$, $u'' < 0$; and

(iii) $R(0) = 0$, $R' > 0$, $R'' > 0$.

For $u < 0$, the function $\theta(u)$ is undefined. We also make the following definitions:

- $\tau(x) = \theta(u(x))$; and
- $\eta(p) = \theta(-u(-p))$.

We will use the functions $\tau(x)$ and $\eta(p)$ to solve the monopolist's optimisation problem and then work backwards to evaluate what the solution implies about the properties of R. Note that since $-u(-p) \neq u(p)$ we require both functions, $\tau(x)$ and $\eta(p)$, to characterise the consumer's preferences in the intended way.

Based on the standard conditions we put on u and R, we can state the following about $\tau(x)$ and $\eta(p)$:

1. τ and η are continuous and twice differentiable

2. Initial conditions
$$\tau(0) = \eta(0) = 0; \tag{24}$$

3. First order properties
$$\begin{aligned} \tau'(0) &= \eta'(0); & (25) \\ \tau'(x) &> 0 \text{ for } x \geq 0; & (26) \\ \eta'(p) &> 0 \text{ for } p \geq 0; & (27) \end{aligned}$$

4. Second order properties
$$\eta''(p) > 0 \text{ for } p \geq 0. \tag{28}$$

Equations 24 and 28 also imply
$$\eta'(p) > \frac{\eta(p)}{p} \text{ for } p > 0. \tag{29}$$

B.1.1 Derivations of the properties of $\tau(x)$ and $\eta(p)$

We derive the initial conditions and the first, second and third order properties of $\tau(x)$and $\eta(p)$.

$$\begin{aligned} \tau(x) &= u(x) + R(u(x)) \\ \tau(0) &= u(0) + R(0) = 0 \\ \tau'(x) &= u'(x)(1 + R'(u(x))) > 0 \\ \tau''(x) &= u''(x)(1 + R'(u(x))) + (u'(x))^2 R''(u(x)) \\ \tau'''(x) &= u'''(x)(1 + R'(u(x))) + 3u'(x)u''(x)R''(u(x)) + (u'(x))^3 R'''(u(x)) \end{aligned}$$

$$\begin{aligned} \eta(p) &= -u(-p) + R(-u(-p)) \\ \eta(0) &= -u(0) + R(0) = 0 \\ \eta'(p) &= u'(-p)(1 + R'(-u(-p))) > 0 \\ \eta''(p) &= -u''(-p)(1 + R'(-u(-p))) + (u'(-p))^2 R''(-u(-p)) > 0 \\ \eta'''(p) &= u'''(-p)(1 + R'(-u(-p))) - 3u''(-p)u'(-p)R''(-u(-p)) + (u'(-p))^3 R'''(-u(-p)) \end{aligned}$$

B.1.2 The monopolist's optimisation problem

Using this notation we can rewrite the monopolist's optimisation problem as

$$\max_{x,p,q} \Omega(x,p,q) = -qx + (1-q)p \tag{30}$$

subject to:

$$\begin{aligned} q\tau(x) - (1-q)\eta(p) &\geq 0; & (31)\\ x &\geq 0; & (32)\\ p &\geq 0; & (33)\\ 0 &\leq q \leq 1. & (34)\end{aligned}$$

Given that Equation 31 must be binding at a solution, we write

$$q = \frac{\eta(p)}{\tau(x) + \eta(p)}. \tag{35}$$

Straight away we can see that $q < \frac{1}{2}$ if and only if $\tau(x) > \eta(p)$. Now, substituting Equation 35 into Equation 30, we arrive at the modified monopolist's optimisation problem:

$$\max_{x,p \in \mathbb{R}_+} \Omega(x,p) = \frac{p\tau(x) - x\eta(p)}{\tau(x) + \eta(p)}. \tag{36}$$

B.2 Proof of Proposition 1

For $x > 0$ write Equation 36 as

$$\Omega(x,p) = \frac{p\frac{\tau(x)}{x} - \eta(p)}{\frac{\tau(x)}{x} + \frac{\eta(p)}{x}}. \tag{37}$$

Claim 5 *Suppose $\frac{\tau(x)}{x} \to \infty$ as $x \to \infty$. Then $\exists p > 0$ such that $\Omega(\infty, p) = p$.*

Proof. For any $p > 0$ such that $\eta(p)$ is finite, $\Omega(\infty, p) \to \frac{p\frac{\tau(x)}{x}}{\frac{\tau(x)}{x}} = p$ if $\frac{\tau(x)}{x} \to \infty$ as $x \to \infty$. ■

Claim 5 implies the finite solution condition holds only if $\lim_{x\to\infty} \frac{\tau(x)}{x} < \infty$. If $\frac{\tau(x)}{x} \to \infty$ as $x \to \infty$ the solution will be the infinite lottery.

Claim 6 *Suppose* $\lim_{x\to\infty}\frac{\tau(x)}{x}<\infty$. *Then* $\Omega(\infty,p)\leq\Omega(0,0)=0$ *for all* $p\geq 0$ *if and only if*

$$\lim_{x\to\infty}\frac{\tau(x)}{x}\leq\tau'(0). \tag{38}$$

Proof. From Equation 37, if $\lim_{x\to\infty}\frac{\tau(x)}{x}<\infty$ then $\Omega(\infty,p)\leq 0$ for all $p\geq 0$ if and only if

$$p\lim_{x\to\infty}\frac{\tau(x)}{x}-\eta(p)\leq 0 \tag{39}$$

for all $p\geq 0$. It is easy to see that $\Omega(\infty,0)\leq\Omega(0,0)$ so we focus on the case where $p>0$.

For all $p>0$, Equation 39 holds if and only if

$$\lim_{x\to\infty}\frac{\tau(x)}{x}-\frac{\eta(p)}{p}\leq 0. \tag{40}$$

Now since $\eta(p)$ is convex, $\frac{\eta(p)}{p}$ is increasing in p, so for a given x, $\left(\frac{\tau(x)}{x}-\frac{\eta(p)}{p}\right)$ is maximised as $p\to 0$. Therefore

$$\lim_{x\to\infty}\frac{\tau(x)}{x}-\frac{\eta(p)}{p}\leq 0 \tag{41}$$

for all $p>0$ if and only if

$$\lim_{x\to\infty}\frac{\tau(x)}{x}-\lim_{p\to 0}\frac{\eta(p)}{p}\leq 0; \tag{42}$$

$$\Leftrightarrow\ \lim_{x\to\infty}\left(\frac{\tau(x)}{x}-\eta'(0)\right)\leq 0 \tag{43}$$

$$\Leftrightarrow\ \lim_{x\to\infty}\left(\frac{\tau(x)}{x}-\tau'(0)\right)\leq 0 \tag{44}$$

Equation 43 follows because $\eta(0)=0$ and so $\lim_{p\to 0}\frac{\eta(p)}{p}=\eta'(0)$ (L'Hôpital's rule) and Equation 44 follows because $\eta'(0)=\tau'(0)$. ■

Claim 7 *If* $\lim_{x\to\infty}\frac{\tau(x)}{x}<\infty$ *then* $\lim_{x\to\infty}\frac{\tau(x)}{x}=\lim_{x_0\to\infty}\tau'(x_0)$.

Proof. Let $y=\frac{1}{x}$ and define $g(y)=\tau(x)$. Then $y\to 0$ as $x\to\infty$. Take the Laurent series of $g(y)$ centred around $y=0$:

$$g(y)=\sum_{n=-\infty}^{\infty}a_n y^n \tag{45}$$

and use this to write:

$$\frac{\tau(x)}{x}=yg(y)=\sum_{n=-\infty}^{\infty}a_n y^{n+1}=...\frac{a_{-2}}{y}+a_{-1}+a_0 y+a_1 y^2+...$$

$\lim_{x\to\infty}\frac{\tau(x)}{x}$ is finite if and only if $a_n = 0$ for $n < -1$. If $a_n \neq 0$ for some $n < -1$, then as $x \to \infty$, $y \to 0$, and so $\frac{\tau(x)}{x}$ will blow up. Therefore $\lim_{x\to\infty}\frac{\tau(x)}{x}$ is finite if and only if

$$\frac{\tau(x)}{x} = yg(y) = \sum_{n=-1}^{\infty} a_n y^{n+1} = a_{-1} + a_0 y + a_1 y^2 + ...$$

and so $\lim_{x\to\infty}\frac{\tau(x)}{x} = a_{-1}$.

Take $g(y) = \sum_{n=-1}^{\infty} a_n y^n$, make the substitution $y = \frac{1}{x}$ and differentiate with respect to x.

$$\tau'(x) = a_{-1} - a_1\frac{1}{x^2} - 2a_2\frac{1}{x^3} + ...$$

Taking the limit to infinity we get $\lim_{x\to\infty}\tau'(x) = a_{-1}$.

Combining these two results gives

$$\lim_{x\to\infty}\frac{\tau(x)}{x} = \lim_{x_0\to\infty}\tau'(x_0)$$

as wanted. ■

Claim 8 *If* $\lim_{x_0\to\infty}\tau'(x_0) < \infty$ *then* $\lim_{x\to\infty}\frac{\tau(x)}{x} = \lim_{x_0\to\infty}\tau'(x_0)$.

Proof. Using the Laurent series for $\tau(x)$ as above

$$g(y) = \sum_{n=-\infty}^{\infty} a_n y^n, \tag{46}$$

and taking the first derivative with respect to x gives

$$\tau'(x) = -y^2 g'(y) = \sum_{n=-\infty}^{\infty} -na_n y^{n+1}.$$

If $\lim_{x_0\to\infty}\tau'(x_0) < \infty$ then it must be that $a_n = 0$ for $n < -1$, otherwise as $x_0 \to \infty$, $\tau'(x_0)$ will blow up. But we have shown that if $a_n = 0$ for $n < -1$, then $\lim_{x\to\infty}\frac{\tau(x)}{x} = \lim_{x_0\to\infty}\tau'(x_0)$. Therefore if $\lim_{x_0\to\infty}\tau'(x_0) < \infty$ then $\lim_{x\to\infty}\frac{\tau(x)}{x} = \lim_{x_0\to\infty}\tau'(x_0)$, as wanted. ■

Claim 9 $\tau'(x)$ *is finite.*

Proof. First note that

$$\tau'(x) = u'(0)\left(1 + R'(0)\right). \tag{47}$$

Then consider the following:

- $R''(t) > 0$ for $t \geq 0 \Longrightarrow R'(d) > R'(0)$ for $d > 0 \Longrightarrow R'(0) < \infty$.

- $u''(t) < 0$ for $t \geq -w$, where w is the individual's wealth. We assume the individual has strictly positive wealth (otherwise he cannot buy a lottery ticket). Therefore $u'(d) > u'(0)$ for some d such that $-w < d < 0$ and so it must be that $u'(0) < \infty$.

Since $u'(0), R'(0) < \infty$, we conclude that $\tau'(0) < \infty$. ■

Proposition 2 *Suppose* $\lim_{x\to\infty} \frac{\tau(x)}{x} < \infty$. *Then* $\Omega(\infty, p) \leq \Omega(0,0) = 0$ *for all* $p \geq 0$ *if and only if*

$$\lim_{x\to\infty} \tau'(x) \leq \tau'(0). \tag{48}$$

Proof. Claim 6 and Claim 7. ■

B.2.1 Proof of necessity for Proposition 1

Proof. From Claim 5, $\Omega(\infty, p) \leq \Omega(0,0)$ implies that $\lim_{x\to\infty} \frac{\tau(x)}{x} < \infty$. From Proposition 2 this implies $\lim_{x\to\infty} \tau'(x) \leq \tau'(0)$ for all $p \geq 0$. ■

B.2.2 Proof of sufficiency for Proposition 1

Proof. From Claim 9, $\lim_{x\to\infty} \tau'(x) \leq \tau'(0)$ implies $\lim_{x\to\infty} \tau'(x)$ is finite and from Claim 8 this implies $\lim_{x\to\infty} \frac{\tau(x)}{x}$ is finite. From Proposition 2 this implies $\Omega(\infty, p) \leq \Omega(0,0) = 0$ for all $p \geq 0$. ■

B.2.3 Conditions for finiteness

This completes the proof of Proposition 1. We next show when $\lim_{x\to\infty} \frac{\tau(x)}{x}$ (and thus $\lim_{x\to\infty} \tau'(x)$) will be finite.

1. $\lim_{x\to\infty} \frac{\tau(x)}{x}$ will be finite if $\tau(x)$ is concave in the limit to infinity (i.e. if there exists some d such that $\tau''(x) < 0$ for all $x > d$)

2. $\lim_{x\to\infty} \frac{\tau(x)}{x}$ will be finite only if $\tau(x)$ is not convex in the limit to infinity (i.e. only if there does not exist some d such that $\tau''(x) > 0$ for all $x > d$)

3. $\lim_{x\to\infty} \frac{\tau(x)}{x}$ may be finite or infinite if $\tau(x)$ is neither convex nor concave in the limit to infinity (i.e. if there is no d such that $\tau''(x) < 0$ for all $x > d$ and no d_0 such that $\tau''(x) > 0$ for all $x > d_0$).

B.3 First and second order conditions for the identification of a regret equilibrium

B.3.1 The derivation of the first order condition

The monopolist's optimisation problem is

$$\max_{x,p\in\mathbb{R}} \Omega\left(x,p\right) = \frac{p\tau(x) - x\eta(p)}{\tau(x)+\eta(p)} \tag{49}$$

Taking the first derivative of Ω with respect to x and setting it equal to zero gives

$$\frac{d\Omega}{dx} = \frac{\eta}{\left(\tau+\eta\right)^2}\left(\left(x+p\right)\tau' - \tau - \eta\right) = 0. \tag{50}$$

similarly for p,

$$\frac{d\Omega}{dp} = \frac{\tau}{\left(\tau+\eta\right)^2}\left(\tau + \eta - \left(x+p\right)\eta'\right) = 0. \tag{51}$$

In a regret equilibrium $x^*, p^* > 0$ and since $\tau\left(x\right) > 0$ and $\eta\left(p\right) > 0$ for $x, p > 0$ we can simplify the first order conditions into the following expression:

$$\tau'\left(x^*\right) = \eta'\left(p^*\right) = \frac{\tau\left(x^*\right) + \eta\left(p^*\right)}{x^* + p^*}. \tag{52}$$

B.3.2 The derivation of the second order condition

The nature of the turning point in the solution characterised by the first order conditions depends on the definiteness of the following Hessian matrix evaluated at (x^*, p^*).

$$H = \begin{bmatrix} \frac{d^2\Omega}{dx^2} & \frac{d^2\Omega}{dxdp} \\ \frac{d^2\Omega}{dxdp} & \frac{d^2\Omega}{dp^2} \end{bmatrix} \tag{53}$$

If H is positive (negative) definite then the solution is a local minimum (maximum). The elements of H for any (x, p) are:

$$\frac{d^2\Omega}{dx^2} = \frac{2\tau'}{\tau+\eta}\left(\frac{d\Omega}{dx}\right) + \frac{x\eta}{(\tau+\eta)^2}\tau'' \tag{54}$$

$$\frac{d^2\Omega}{dp^2} = \frac{2\eta'}{\tau+\eta}\left(\frac{d\Omega}{dp}\right) - \frac{p\tau}{(\tau+\eta)^2}\eta'' \tag{55}$$

$$\frac{d^2\Omega}{dxdp} = -\frac{\eta'}{\eta}\left(\frac{\tau-\eta}{\tau+\eta}\right)\left(\frac{d\Omega}{dx}\right) + \frac{\eta}{(\tau+\eta)^2}(\tau'-\eta') \tag{56}$$

$$\frac{d^2\Omega}{dpdx} = -\frac{\tau'}{\tau}\left(\frac{\tau-\eta}{\tau+\eta}\right)\left(\frac{d\Omega}{dp}\right) + \frac{\tau}{(\tau+\eta)^2}(\tau'-\eta') \tag{57}$$

Evaluated at (x^*, p^*) these values are:

$$\frac{d^2\Omega}{dx^2}(x^*, p^*) = \frac{x^*\eta^*}{(\tau^*+\eta^*)^2}\tau''(x^*) \tag{58}$$

$$\frac{d^2\Omega}{dp^2}(x^*, p^*) = -\frac{p^*\tau^*}{(\tau^*+\eta^*)^2}\eta''(p^*) < 0 \tag{59}$$

$$\frac{d^2\Omega}{dxdp}(x^*, p^*) = 0 \tag{60}$$

$$\frac{d^2\Omega}{dpdx}(x^*, p^*) = 0 \tag{61}$$

where $\tau^* = \tau(x^*)$ and $\eta^* = \eta(p^*)$. The nature of a turning point, (x^*, p^*), can therefore be characterised as follows:

(x^*, p^*) is a local maximum if

$$\tau''(x^*) < 0, \tag{62}$$

and only if

$$\tau''(x^*) \leq 0. \tag{63}$$

B.3.3 Proofs relating to the FOC and SOC

Proposition 3 *Given the standard conditions on u and R, the first order condition in Equation 52 obtains only if*

$$\tau'(0) < \tau'(x^*) < \frac{\tau(x^*)}{x^*}. \tag{64}$$

Proof. First let us rewrite Equation 52 as

$$\tau(x^*) - x^*\tau'(x^*) + \eta(p^*) - p^*\eta'(p^*) = 0. \tag{65}$$

Equation 29 tells us that $\eta(p) - p^*\eta'(p^*) < 0$, so, from Equation 65, at a regret equilibrium it must be that $\tau(x^*) - x^*\tau'(x^*) > 0$. We also know that $\eta'(p^*) > \eta'(0) = \tau'(0)$ and hence we need $\tau'(x^*) > \tau'(0)$ to satisfy $\tau'(x^*) = \eta'(p^*)$. Therefore a necessary condition for Equation 52 to hold is

$$\tau'(0) < \tau'(x^*) < \frac{\tau(x^*)}{x^*} \tag{66}$$

as wanted. ■

Equation 64 implies that τ must be convex for some values of x and concave for other values of x. From the second order condition, τ is concave at x^*, so for 64 to hold we need there to be at least one region in $[0, x^*)$ in which τ is convex before τ becomes concave.

Proposition 4 *Given the standard conditions on u and R, the first order condition in Equation 52 obtains only if*

$$\frac{\eta(p^*)}{p^*} < \eta'(p^*) = \tau'(x^*) < \frac{\tau(x^*)}{x^*}. \tag{67}$$

Proof. Equations 29, 52 and 64. ■

Since τ is continuous in x and η is continuous in p, we know there exists a value, h, where $0 < h < 1$, such that

$$\tau'(x^*) = \eta'(p^*) = h\frac{\tau(x^*)}{x^*} + (1-h)\frac{\eta(p^*)}{p^*}. \tag{68}$$

It then follows that a necessary condition for the satisfaction of the first order condition in Equation 52 is

$$h = \frac{x^*}{x^* + p^*}. \tag{69}$$

For this value of h and this value alone, the pair (x^*, p^*) solves Equation 52.

Notice that, according to the expression for Ω in Equation 36, our condition in Equation 67 implies that any solution to Equation 52 gives $\Omega > 0$ [17]. That is, any solution to Equation 52 produces a strictly positive expected net revenue for the monopolist.

Given the finite solution assumption and the fact that $\Omega(0,0) = 0$, this result tells us that it is sufficient to prove that there exists a pair that solves Equation 52 (the first order

[17] This may be clearer if we rewrite Equation 36 as $\Omega(x,p) = \frac{xp}{\tau(x)+\upsilon(p)}\left(\frac{\tau(x)}{x} - \frac{\upsilon(p)}{p}\right)$.

condition) and satisfies Equation 62 (the second order condition) to prove there exists a regret equilibrium.

B.4 Proof of Theorem 1

Proof. First note that since

$$\tau''(x) = u''(x)\left(1 + R'(u(x))\right) + \left(u'(x)\right)^2 R''(u(x)),$$

we can say the following:

$$\tau''(x) \lesseqqgtr 0 \Longleftrightarrow \lambda(x) \gtreqqless \rho(x).$$

The first part of Theorem 1 then follows from Proposition 3. The second part of Theorem 1 follows from Claim 5 and point 2 on page 34. ■

B.5 First regret equilibrium existence theorem

Theorem 10 (Existence) *Consider the function $\tau(x) = u(x) + R(u(x))$, where u and R satisfy the standard conditions. If the finite solution condition obtains and there is a unique point, $\underline{c} \in \mathbb{R}_{++}$, such that $\tau'(\underline{c}) = \frac{\tau(\underline{c})}{\underline{c}} > \tau'(0)$, and $\tau'' < 0$ for $x \geq \underline{c}$, then there exists a regret equilibrium.*

Proof. Suppose there is a function, $\tau : \mathbb{R} \to \mathbb{R}$, and a single point on $\mathbb{R}_{++}$, $\underline{c}$, such that $\tau'(\underline{c}) = \frac{\tau(\underline{c})}{\underline{c}} > \tau'(0)$ and such that $\tau'' < 0$ for $x \geq \underline{c}$ (we assume there are no more convex regions of τ after x^*. This is not necessary but it simplifies the proof). We define $\overline{c}$ such that $\overline{c} > \underline{c}$ and $\tau'(\overline{c}) = \tau'(0) = \eta'(0)$. Since $\tau'' < 0$ for $x > \underline{c}$, $\overline{c}$ is unique if it exists and it exists if the finite solution assumption holds.**

Assuming $\overline{c}$ exists, we claim there exists a bijection $\pi : [\underline{c}, \overline{c}] \to [0, \widehat{c}]$ such that $\tau'(x) = \eta'(\pi(x))$, where $\widehat{c} = \pi(\underline{c})$. We know there is a unique value of $p = \pi(x)$ for all $x \in [\underline{c}, \overline{c}]$ because $\tau'' < 0$ for $x \geq \underline{c}$ and $\eta'' > 0$ for all $p \geq 0$, so $\pi'(x) < 0$ for $\underline{c} \leq x \leq \overline{c}$.

Consider the following equation.

$$A(x, p, a) = a\left(\frac{\tau}{x} - \tau'\right) + (1 - a)\left(\frac{\eta}{p} - \eta'\right) \tag{70}$$

for $x > \underline{c}$ and $0 \leq a \leq 1$, where $p = \pi(x)$.

Since $\tau'' < 0$ for $x \geq \underline{c}$, we know that for $x > \underline{c}$,

$$
\begin{aligned}
(x - \underline{c})\,\tau'(x) \quad &< \quad \tau(x) - \tau(\underline{c}) && (71)\\
&\Longrightarrow \quad x\tau'(x) < \tau(x) - \underline{c}\left(\tau'(\underline{c}) - \tau'(x)\right) && (72)\\
&\Longrightarrow \quad x\tau'(x) < \tau(x) && (73)
\end{aligned}
$$

where Equation 72 makes use of the definition of $\underline{c}$ (i.e. that $\tau'(\underline{c}) = \frac{\tau(\underline{c})}{\underline{c}}$) and Equation 73 makes use of the fact that $\tau'' < 0$.

Since $\frac{\tau}{x} - \tau' > 0$ and $\frac{\eta}{p} - \eta' < 0$ (see Equation 29), then for an arbitrary $x > \underline{c}$ and $p = \pi(x) < \widehat{c}$, there is always a value of a, $0 \leq a \leq 1$, that solves $A(x,p,a) = 0$. Let $a^*(x)$ be the root of $A(x,p,a)$, keeping x, p fixed. Then

$$
\begin{aligned}
a^*(x) \quad &= \quad \frac{\eta' - \frac{\eta}{p}}{\frac{\tau}{x} - \frac{\eta}{p} - \tau' + \eta'} && (74)\\
&= \quad \frac{\eta' - \frac{\eta}{p}}{\frac{\tau}{x} - \frac{\eta}{p}}. && (75)
\end{aligned}
$$

The value of the denominator reduces to $\frac{\tau}{x} - \frac{\eta}{p}$ because, by the definition of $p = \pi(x)$, $(\tau' - \eta') = 0$.

The function $a^*(x)$, has the following properties:

1)$\lim\limits_{x \to \underline{c}} a^*(x) = 1$

2)$\lim\limits_{x \to \overline{c}} a^*(x) = 0$

where the second property follows from the use of L'Hôpital's Rule on $\eta' - \frac{\eta}{p}$ as $p \to 0$.

Now consider the function:

$$
S(x, \pi(x)) = a^*(x) - \frac{x}{x + \pi(x)}. \tag{76}
$$

This function has the following properties:

1)$\lim\limits_{x \to \underline{c}} S(x,p) = 1 - \frac{\underline{c}}{\underline{c} + \widehat{c}} = \frac{\widehat{c}}{\underline{c} + \widehat{c}} > 0$

2)$\lim\limits_{x \to \overline{c}} S(x,p) = -1$

Therefore, given that $S(x,p)$ is continuous, there exists a pair, $\left(x^C, p^C\right)$ such that $\underline{c} < x^C < \overline{c}$, $0 < p^C < \widehat{c}$, and $S\left(x^C, p^C\right) = 0$ (Intermediate value theorem).

According to Equation 69 this implies there is a turning point $\left(x^C, p^C\right)$, with $q^C = \frac{\eta\left(p^C\right)}{\tau(x^C) + \eta(p^C)}$. Since $\tau''(x^C) < 0$, this is a local maximum, and it yields a positive expected net revenue. Therefore there exists a regret equilibrium.

**Strictly speaking, the finite solution condition does *not* guarantee there is a value $\bar{c}$ such that $\bar{c} > \underline{c}$ and $\tau'(\bar{c}) = \tau'(0)$ because the finite solution condition includes the case where $\lim_{x\to\infty} \tau'(x) = \tau'(0)$. However, in this case we can say there is a value $\bar{c}'$ such that $\bar{c}' > \underline{c}$ and $\tau'(\bar{c}') = \tau'(0) + \delta$ for an arbitrarily small and positive δ. We then continue with the rest of the proof using $\bar{c}'$ instead of $\bar{c}$ and making the appropriate adjustments to the notation. Since a turning point must be such that $\tau'\left(x^C\right) > \tau'(0)$ and since δ is arbitrarily small, we know that $x^C < \bar{c}'$, which is all that is required for the rest of the proof. We also use this caveat in the proof of Theorem 11 when we invoke the finite solution condition. ■

B.6 Second regret equilibrium existence theorem

Definition 2 *A function, $f : \mathbb{R} \to \mathbb{R}$, is strictly convex-concave if f is twice differentiable and there is some point $d > 0$ such that $f''(t) > 0$ for all $0 < t < d$ and $f''(t) < 0$ for all $t > d$.*

Theorem 11 (Sufficiency) *If the function $\tau(x) = u(x) + R(u(x))$ is strictly convex-concave and satisfies the finite solution assumption, $\lim_{x\to\infty} \tau'(x) \leq \lim_{x\to 0} \tau'(x)$, then there exists a regret equilibrium.*

Proof. Suppose τ is strictly convex-concave. Then $\frac{\tau(x)}{x} < \tau'(x)$ for $0 < x < d$.

Since $\frac{d}{dx}\left(\frac{\tau(x)}{x}\right) = \frac{1}{x}\left(\tau'(x) - \frac{\tau(x)}{x}\right) > 0$ for $0 < x < d$, it must be that $\lim_{x\to 0} \tau'(x) < \frac{\tau(d)}{d} < \tau'(d)$.

As $\tau''(x) < 0$ for all $x > d$ and $\frac{d}{dx}\left(\frac{\tau(x)}{x}\right) > 0$ for x immediately after d, then $\frac{d}{dx}\left(\tau'(x) - \frac{\tau(x)}{x}\right) < 0$ immediately after d. Eventually there will be a point $\underline{c} > d$ where $\tau'(\underline{c}) = \frac{\tau(\underline{c})}{\underline{c}}$. This point, $\underline{c}$, must be finite because:

1. $\lim_{x\to\infty} \frac{\tau(x)}{x} \leq \lim_{x\to 0} \tau'(x)$ (finite solution condition); and

2. $\lim_{x\to 0} \tau'(x) < \frac{\tau(d)}{d} < \frac{\tau(x)}{x}$ for $d < x < \underline{c}$.

At $\underline{c}$ we have $\tau'(\underline{c}) = \frac{\tau(\underline{c})}{\underline{c}} > \lim_{x\to 0} \tau'(x)$ and $\tau''(x) < 0$ for $x \geq \underline{c}$.

With the same caveat as in Theorem 10 we use the finite solution condition to identify a point $\bar{c}$ such that $\tau'(\bar{c}) = \lim_{x\to 0} \tau'(x)$.

By Theorem 10 there exists a regret equilibrium with $x^* \in (\underline{c}, \bar{c})$. ■

B.7 Proof of Theorem 2

Proof. First note that $\tau(x)$ is convex-concave if there is some $d > 0$ such that

$$\lambda(x) < \rho(x) \tag{77}$$

for $x < d$, and

$$\lambda(x) > \rho(x). \tag{78}$$

Given that the finite solution condition holds, Theorem 11 states there exists a regret equilibrium. ■

B.8 Proof of Theorem 3

Proof. If the consumer has CRRA2 preferences, then Theorem 11 states there will be a regret equilibrium if the finite solution condition holds and there is some $d > 0$ such that

$$\begin{aligned} r(w+x)^{r-1} &< c \qquad \text{for } x < d\text{; and} & (79)\\ r(w+x)^{r-1} &> c \qquad \text{for } x > d. & (80) \end{aligned}$$

These requirements are met if and only if $r > 1$ and $c > c^{\dagger} = rw^{r-1}$. ■

B.9 Proof of Theorem 4

Proof. If there is a regret equilibrium then $\mathbf{v}^* = (x^*, p^*, q^*)$ is such that $x^*, p^* > 0, q^* \in (0,1)$ and $\mathbf{v}^*$ solves the monopolist's constrained optimisation problem. Given CRReA preferences, the monopolist's expected profit at the regret equilibrium as a function of c is

$$M(c) = \Omega\left(x^*(c), p^*(c); c\right) = \frac{p\tau\left(x^*\right) - x\eta\left(p^*\right)}{\tau\left(x^*\right) + \eta\left(p^*\right)}. \tag{81}$$

By the envelope theorem we have

$$\begin{aligned} \frac{dM(c)}{dc} &= \frac{\partial M(c)}{\partial c}, & (82)\\ &= \frac{\frac{\partial\tau(x^*(c))}{\partial c}\left(p^* + x^*\right)\eta\left(p^*\right) - \frac{\partial\eta(p^*(c))}{\partial c}\left(p^* + x^*\right)\tau\left(x^*\right)}{\left(\tau\left(x^*\right) + \eta\left(p^*\right)\right)^2}. & (83) \end{aligned}$$

Since $x^*, p^* > 0$ it follows that

$$\frac{dM(c)}{dc} \gtreqless 0 \Longleftrightarrow \frac{\partial\tau\left(x^*(c)\right)}{\partial c}\eta\left(p^*\right) \gtreqless \frac{\partial\eta\left(p^*(c)\right)}{\partial c}\tau\left(x^*\right). \tag{84}$$

Regret preferences are CRReA so:

- $\frac{\partial\tau(x^*(c))}{\partial c} = u(x^*)Ae^{cu(x^*)}$,
- $\eta\left(p^*\right) = A\left(e^{-cu(-p^*)} - 1\right)$,

- $\frac{\partial \eta(p^*(c))}{\partial c} = -u(-p^*)Ae^{-cu(-p^*)}$,
- $\tau(x^*) = A\left(e^{cu(x^*)} - 1\right)$.

We can therefore rewrite Equation 84 as

$$\frac{dM(c)}{dc} \gtreqless 0 \Longleftrightarrow \frac{u(x^*)e^{cu(x^*)}}{e^{cu(x^*)} - 1} \gtreqless \frac{-u(-p^*)e^{-cu(-p^*)}}{e^{-cu(-p^*)} - 1}, \tag{85}$$

or

$$\frac{dM(c)}{dc} \gtreqless 0 \Longleftrightarrow f(u(x^*)) \gtreqless f(-u(-p^*)), \tag{86}$$

where

$$f(y) = \frac{ye^{cy}}{e^{cy} - 1}. \tag{87}$$

It can easily be shown that $f'(y) > 0$. Therefore Equation 86 implies

$$\frac{dM(c)}{dc} \gtreqless 0 \Longleftrightarrow u(x^*) \gtreqless -u(-p^*), \tag{88}$$

which implies

$$\frac{dM(c)}{dc} \gtreqless 0 \Longleftrightarrow \tau(x^*) \gtreqless \eta(p^*), \tag{89}$$

for $c > 0$.

Now we know from EQUATION 10 that $q^* < \frac{1}{2}$ and from EQUATION 37 that $q^* = \frac{\eta(p^*)}{\tau(x^*)+\eta(p^*)}$, so it follows that $\tau(x^*) > \eta(p^*)$ and so $\frac{dM(c)}{dc} > 0$ as wanted. ■