Hiring Through Referrals

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In progress – Please do not cite

April 2010

1 Introduction

It is well-known that social networks play an important role in labor markets. Approximately half of all American workers report learning about their current job through their social network: friends, acquaintances, relatives etc. The exact proportion differs across data sets or studies but it is substantial across the board. For instance, Corcoran et al (1980) report that, according to the PSID, 50% of all workers find their job through their social network; the NLSY puts that proportion to 30% while the 24 studies that are surveyed by Bewley (1999) range from 30% to 60%. Similarly, on the other side of the market, more than half of all employers report using the social networks

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of their current employees when hiring. Again, the exact proportions differ across studies but are everywhere substantial (e.g. 53% according to EOPP as reported by Holzer (1987) and 60% according to Bewley (1999). Finally, it is worth noting that referrals are ubiquitous in that they occur in large numbers in every occupation, industry and educational level.

However, social networks are not typically included in models of the labor market. This begs the question: do these models miss something important by ignoring social networks? To answer this question, one needs to study a model with networks and see if it yields predictions or insights that differ from the usual model. In this paper I introduce social networks in a fairly standard search model of the labor market and derive new empirical predictions as well as policy implications. To do this I first build a model where the use of social networks alleviates various frictions that are present in the labor market. Second, I examine a particular occasion where a model that includes a social network yields different theoretical predictions than one without. To be more precise, the use of social networks creates complementarities in some actions by the workers (e.g. investment in human capital) which might lead to the existence of suboptimal equilibria. Finally, I show that following a policy which targets groups that socialize among themselves (namely, affirmative action) can “leverage” their social networks and achieve a policy goal in a cost-effective way.

The literature has identified two main reasons for why referrals are so widely-used. The first is that a referred worker is on average a better worker,
presumably in some dimension that is unobservable, at least at the time of search. This idea is supported by interviews that have been conducted with employers or firms' human resource departments by Rees (1966), Doeringer and Piore (1971) and, more recently, Bewley (1999). There is also statistical evidence that, conditional on observables, referred workers are more productive (e.g. see the case study of a call-center by Castilla (2005)) and that referred workers have lower quit/lay off rates (Loury (1983, 2006)) which suggests a higher match quality. Overall, this evidence seems to suggest that, at least in some occasions, referrals lead to better workers conditional on a hire. They therefore alleviate an informational problem in the market: firms would like to hire better workers but it’s harder to do so in the open market.

The second reason has to do with increasing the inflow of (perhaps, eligible) workers to the firm. In a variety of studies, firms are asked which methods of search they used and which methods led to a hire. Holzer (1987) and Blau and Robins (1990) report that referred job candidates are more likely to accept a job offer. Furthermore, Castilla (2006) and Fernandez and Weinberg (1997) find that referred job candidates have greater probability of going through an interview even after controlling for observables. On the worker side, Holzer (1988) reports that a person’s social network is the most effective method of search, when compared with employment services, direct applications etc. In sum, this evidence suggest the presence of search frictions which make it time-consuming to find a worker/firm that is eligible to
match and that referrals can mitigate that friction.

2 The Model

2.1 Basics

There are two types of agents, workers and firms, both of which are risk-neutral. Each worker belongs to one of two types: high or low. There measure of each type is equal and normalized to one: $\mu_H = \mu_L = 1$. The measure of firms is determined by free entry. Each firm has a position for one worker, with the exception of the instant of expansion which is described in detail below.

There are two distinct stages: an initial network formation stage and a subsequent infinite horizon search-production stage.

2.2 Network Formation

Network formation proceeds in two steps. First, every worker announces the measure of links that he wants to create with workers of either type. Then, the links are realized, subject to the availability of willing “counterparties.”

Fix worker $i$ of type $j$. His announcement consists of a pair of numbers: $\hat{n}_{ij} = \{\hat{n}_{ij}^H, \hat{n}_{ij}^L\}$ where $\hat{n}_{ij}^k$ is the measure of links that $i$ wants to create with workers of type $k$. Aggregating across workers of type $j$, let $\hat{N}_j^k \equiv \int_{i \in j} \hat{n}_{ij}^k di$ denote the aggregate “demand” for links with type-$k$ from type-$j$ workers.
(or, equivalently, it represents the “supply” of links for type-$k$ with type-$j$ workers).

Turning to the realized network, let $n_{ij}^k$ be the realized number of links for worker $i$. Start with links with the same type: if $\hat{N}_{jj}^j = 0$ then there is no demand for within-$j$ links and $n_{ij}^k = 0$. If on the other hand $\hat{N}_{jj}^j > 0$ then there is sufficient demand for any number of links and $n_{ij}^k = \hat{n}_{ij}^k$. The key point to note is that once there is positive aggregate demand for own-type links, an individual’s choice is unconstrained.

The realization of links with a different type is slightly different. To determine whether a worker of type $j$ can link with a type-$k$, we need to consider the demand for $j$ links by $k$-types ($\hat{N}_{jk}^j$), which can also be thought of as the “supply” of $k$-type links. As before, if $\hat{N}_{jk}^j = 0$ then $n_{ij}^k = 0$. If $\hat{N}_{jk}^j > 0$ then the realized number of links will depend on the relation between demand and supply: if $\hat{N}_{jk}^j \geq \hat{N}_{jj}^k$ then the supply of links with $k$-types exceeds demand and every type-$j$ worker can get his desired number of links: $n_{ij}^k = \hat{n}_{ij}^k$. However, if $\hat{N}_{jk}^j < \hat{N}_{jj}^k$ then the $j$ types will be rationed proportionally: $n_{ij}^k = (\hat{N}_{jk}^j/\hat{N}_{jj}^k)\hat{n}_{ij}^k$.

To summarize, the mapping between desired links and realized links is fully described by the following equations:
\[ n_{ij}^j = \hat{n}_{ij}, \quad \text{if} \quad \hat{N}_j^j > 0 \]  
\[ n_{ij}^j = 0, \quad \text{if} \quad \hat{N}_j^j = 0 \]  
\[ n_{ij}^k = \min[1, \hat{N}_j^k / \hat{N}_j^j] \hat{n}_{ij}^j. \] (3)

The cost of network formation is given by \( c(\hat{n}_{ij}^H + \hat{n}_{ij}^L). \) That is, the cost is paid on the desired links, and it depends on the sum of the links to be created. The cost is assumed to be strictly convex with \( \lim_{n \to 1} c(n) = \infty. \)

The analysis will focus on equilibria where the network-formation strategies are symmetric, segregated and lead to non-trivial networks.

\[ \text{Symmetry:} \quad \hat{n}_{ij}^k = \hat{N}_j^k, \quad \forall i, j, k \]  
\[ \text{Segregation:} \quad \hat{n}_{ij}^k = 0, \quad i \neq k \]  
\[ \text{Non-trivial:} \quad \hat{N}_j^j > 0 \] (6)

### 2.3 Labor Market

Time runs continuously, the horizon is infinite and the future is discounted at rate \( r. \) Only steady states are examined. The labor market is characterized by search frictions: it takes time for workers and firms to meet. Further, it is characterized by informational frictions: all workers regardless of type search in the same market. Vacancies are created by new firms that enter
to the market or by old firms that expand. Creating a vacancy costs $K$ which is used to buy capital. Firms and workers meet through two separate channels: the formal market ($M$) and informal referral networks ($R$). When unemployed, a worker receives $b$. When vacant a firm receives 0.

2.3.1 Formal Market

Let $v$ denote the measure of vacancies and $u_H$ and $u_L$ denote the measure of unemployed workers of high and low type, respectively. Two kinds of events may happen in the market: a vacancy meets a high type worker or it meets a low type worker. The rate at which event occurs is given by a constant returns to scale matching function $M(v, u_j)$. In particular the matching function takes the Cobb-Douglas form: $M(v, u_j) = Av^αu_j^{1-α}$ for some $A > 0$ and $\eta \in (0, 1)$. The rate at which matches of either type occurs is therefore given by

$$M(v, u_H, u_L) = M(v, u_H) + M(v, u_L)$$

which also exhibits constant returns to scale.

The rate at which a firm meets with a type-$j$ worker is given by:

$$\alpha_{F_j} = \frac{M(v, u_j)}{v}$$

while the rate at which a type-$j$ worker meets with a firm through the formal
The key assumption about the matching structure is that, conditional on \( v \), the two types of workers do not interact. This turns out to be very useful for characterizing the equilibrium. Note that from the firm’s point of view there are informational frictions in the market: conditional on meeting with a worker, the firm would strictly prefer an \( H \) rather than an \( L \). Put differently, there would higher firm entry if all of the unemployed were of type \( H \) rather than \( L \).

### 2.3.2 Referrals

Referrals hiring work as follows. At exogenous rate \( \rho \) a firm that employs a worker gets the opportunity to open a new position (“replicates”) in which case it has the option of using the informal network of its existing employee in order to hire. If the firm chooses to use the referral network, a randomly chosen link of the incumbent worker meets with the firm.

Suppose that the average number of links of a worker of type \( j \) is \( N_j \). Then the rate at which a type-\( j \) worker with \( n = (n^H, n^L) \) links is referred to a job is:

\[
\alpha_{Mj} = \frac{M(v, u_j)}{u_j}
\]
\[ \alpha_{Rj} = \frac{\rho n^H (1 - u_H)}{N_H} + \frac{\rho n^L (1 - u_L)}{N_L} \]  

(10)

In words, each \( H \)-type link is employed with probability \( u_H \) and in that state he gets the opportunity to refer someone from his network at rate \( \rho \), hence the numerator of the first ratio. The referral is then sent at random to one his links, hence the denominator. And similarly for the links of type \( L \).

As mentioned above, the meeting between a firm and a worker need not lead to a match. Regardless of the outcome of the meeting between the referred worker and the firm, the two positions split after the expansion and the “new” position is sold off. Furthermore, it is assumed that the buyer of the position makes a take-it-or-leave-it offer so that the incumbent firm receives none of the surplus that is created by the position’s creation. This assumption is for simplicity only and can be easily relaxed.

2.3.3 Matching

When a firm and a worker of type \( j \) meet the match-specific productivity \( p \) of the match is drawn from some distribution \( F_j(p) \). The distribution takes the form \( F_j(p) = 1 - e^{-p/\pi_j} \), i.e. it is exponential with parameter \( \pi_j \) which equals the average draw. It is assumed that \( \pi_H > \pi_L \).

Every payoff relevant characteristic is common knowledge during the meeting of the worker and the firm. In particular, they both observe \( p \)
and the firm knows the worker’s type and his network. They decide whether to form the match. If they do, the surplus is split through Nash bargaining where \( \beta \in (0, 1) \) denotes the worker’s bargaining power. The productivity \( p \) remains constant for the duration of the match. Given the above assumptions, it is easy to see that there is a reservation productivity \( p_j \) such that a match between a type-\( j \) worker and a firm is consummated if and only if \( p \geq p_j \).

A match is destroyed at exogenous rate \( \delta \). It is assumed that \( \delta > \rho \) which guarantees that some of the vacancies in the market are created by new firms.

This model examines the steady state of this labor market. The flows out of unemployment equal

\[
u_j (\alpha_{M_j} + \alpha_{R_j})(1 - F(p_j)) = (1 - u_j) \delta
\]

\[
(11)
\]

2.4 Equilibrium definition

The agents’ decisions can be summarized as follows. During the network formation stage, each worker decides his desired measure of links of his own type: \( n_H \) and \( n_L \) for each type. In the labor market, the equilibrium outcomes are the reservation productivity levels \( \{p_H, p_L\} \), the steady state unemployment levels \( \{u_H, u_L\} \) and the number of vacancies \( v \).

**Definition 2.1** An equilibrium is \( \{n_H, n_L, p_H, p_L, u_H, u_L, v\} \).
The next section shows that an equilibrium exists and that it is unique.

3 Equilibrium Characterization

The equilibrium will be characterized by, essentially, backwards induction. Taking the structure of the social network especially segregation and symmetry, as given, the equilibrium will be derived in the labor market. Then, network formation will be examined.

3.1 Labor Market Equilibrium

Consider a worker of type \( j \) and denote his value of being unemployed by \( U_j \). The unemployed worker’s flow utility is \( b \). Job opportunities appear through the formal channel at rate \( \alpha_{Mj} + \alpha_{Rj} \). Then the match-specific productivity is drawn and, if it is above the reservation productivity, the match is formed. Let \( W_j(p) \) denote the value of a type-\( j \) worker who is employed at a match with productivity \( p \). When employed, the worker’s flow utility is equal to the wage \( w_j(p) \) which is the outcome of Nash bargaining. The match is destroyed at rate \( \delta \). The worker’s value functions are given by the following equations:

\[
\begin{align*}
  rU_j &= b + (\alpha_{Mj} + \alpha_{Rj}) \int_{L_j}^{\infty} [W_j(p) - U_j] dF_j(p) \\
  rW_j(p) &= w_j(p) + \delta(U_j - W_j(p))
\end{align*}
\]

(12)  (13)
Now consider a firm. When vacant, it searches for workers and it meets with type-\(j\) workers at rate \(\alpha_{Fj}\). Then the productivity is drawn and, if it is high enough, the match is formed. When producing, the firm’s flow payoffs are \(p - w_j(p)\). The match is destroyed at rate \(\delta\).

\[
\begin{align*}
   rV & = \alpha_{FH} \int_{P_H}^{\infty} [J_H(p) - V] dF_H(p) + \alpha_{FL} \int_{P_L}^{\infty} [J_L(p) - V] dF_L(p) \\
   rJ_j(p) & = p - w_j(p) + \delta(V - J_j(p)) 
\end{align*}
\]

Manipulating equations (13) and (15) yields:

\[
\begin{align*}
   W_j(p) & = \frac{w_j(p) + \delta U}{r + \delta} \\
   J_j(p) & = \frac{p - w_j(p) + \delta V}{r + \delta} 
\end{align*}
\]

Denote the surplus of a match between a firm and a type-\(j\) worker with productivity \(p\) by \(S_j(p)\). The surplus can be calculated by:

\[
\begin{align*}
   S_j(p) & = W_j(p) - U_j + J_j(p) - V \\
   & = \frac{w_j(p) + \delta U_j}{r + \delta} - U_j + \frac{p - w_j(p)}{r + \delta} - V \\
   & = \frac{p - rU_j - rV}{r + \delta}
\end{align*}
\]
The cutoff productivity levels are such that \( S_j(p_j) = 0 \). This leads to \( p_j = rU_j + rV \) and hence:

\[
S_j(p) = \frac{p - p_j}{r + \delta} \quad (21)
\]

The wage is determined by Nash bargaining:

\[
w_j(p) = \arg\max [W_j(p) - U_j]^\beta [J_j(p) - V]^{1-\beta} \quad (22)
\]

Setting the first order conditions to zero yields:

\[
W_j(p) - U_j = \beta S_j(p) = \frac{\beta(p - p_j)}{r + \delta} \quad (23)
\]

\[
J_j(p) - V = (1 - \beta) S_j(p) = \frac{(1 - \beta)(p - p_j)}{r + \delta} \quad (24)
\]

Introducing the above equation inside the worker’s value of unemployment and integrating by parts leads to:
\[ rU_j = b + (\alpha_{Mj} + \alpha_{Rj}) \int_{p_j}^{\infty} \frac{\beta(p - p_j)}{r + \delta} dF_j(p) \]  
\[ = b + \frac{(\alpha_{Mj} + \alpha_{Rj})\beta}{r + \delta} \int_{p_j}^{\infty} [1 - F_j(p)] dp \]  
(25)  

Similarly, we can rewrite the firm’s value of a vacancy as:

\[ rV = \frac{\alpha_{FH}(1 - \beta)}{r + \delta} \int_{p_H}^{\infty} [1 - F_H(p)] dp + \frac{\alpha_{FL}(1 - \beta)}{r + \delta} \int_{p_L}^{\infty} [1 - F_L(p)] dp \]  
(26)  

To calculate the flows note that network segregation and symmetry imply that \( \alpha_{Rj} = (1 - u_j) \rho \). Therefore, the unemployment rate of each type of worker is implicitly defined by the following equation as a function of the number of vacancies and reservation productivity of that type:

\[ u_j[M(v, u_j)/u_j + \rho(1 - u_j)]e^{-p_j/\pi_j} - \delta(1 - u_j) = 0 \]  
(28)  

Free entry by firms implies that \( V = K \) and therefore \( rU_j = p_j - rK \). We now have the two equilibrium conditions:
\[ p_j = b + rK + \frac{(\alpha_{M_j} + \alpha_{R_j})\beta}{r + \delta} \int_{L_j}^{\infty} [1 - F_j(p)] dp \]  
(29)

\[ K = \frac{\alpha_{F_H}(1 - \beta)}{r + \delta} \int_{L_H}^{\infty} [1 - F_H(p)] dp + \frac{\alpha_{F_L}(1 - \beta)}{r + \delta} \int_{L_L}^{\infty} [1 - F_L(p)] dp \]  
(30)

The following result can be proven.

**Proposition 3.1** The labor market equilibrium exists and it is unique.

### 3.2 Equilibrium in network formation

In the network formation stage, the worker takes as given the aggregate demand for links and the labor market equilibrium that follows. He chooses how many link to create, anticipating that more links increase the hiring rate when unemployed.

The focus on strategies that are symmetric and segregated in the network formation stage means that the referral rate of a type-\( j \) worker is given by

\[ \alpha_{R_j}(n; N_j) = \frac{n(1 - u_j)\rho}{N_j} \]  
(31)

where \( n \) is the number of same-type links and \( N_j \) is the number of links that every other type-\( j \) worker has. Symmetry implies that \( n = N_j \) in equilibrium.

The worker’s objective function is:
\[ rU_j(n) = b + [\alpha_{Mj} + \alpha_{Rj}(n; N_j)] \int_{p_j(n)}^{\infty} [W_j(p) - U_j] dp \quad (32) \]

where the labor market aggregates \((u_j, v, p_j)\) are taken as given.

Proposition 3.2 An equilibrium exists in the network formation stage and it is unique.

4 Conclusions

TBA
References


